Linear Algebra
Lecture Note 2

Addition, Subtraction and transpose of a matrix

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We previously learned about vectors

They look like this $x = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

We can tell their dimension with this notation $x \in \mathbb{R}^3$

Today we are learning "what is a matrix?"

They look like this

$$X = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 2 & 0 \end{bmatrix} \quad or \quad Y = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 0 & 3 \end{bmatrix}$$

We can tell their dimension with this notation

$$X \in \mathbb{R}^{2\times 3} \quad or \quad Y \in \mathbb{R}^{3\times 2}$$

Matrices can take on any dimension

Notice that instead of a lowercase letters matrices are denoted with capital letters

Practice

- If A is a 3×4 matrix and B is a 4×2 matrix, what is the dimension of AB?
- Let C be a 5×3 matrix. What is the dimension of C^T (transpose of C)?

If D is a 6×2 matrix and E is a 2×6 matrix, is the product DE defined? If yes, what is its dimension?

Matrix Shapes and Names

A matrix that is wide is called a **Fat matrix**(with is greater than height). For example

$$X = \begin{bmatrix} 1 & 2 & 3 \\ o & 0 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 2 & 3 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}, Z = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

A matrix that is tall is called **Tall matrix**(width is less than height). For example

$$X = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix}, Y = \begin{bmatrix} 0 & 3 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}, Z = \begin{bmatrix} 1 & 6 & 10 \\ 2 & 7 & 11 \\ 3 & 8 & 12 \\ 4 & 9 & 13 \end{bmatrix}$$

A matrix whose width is equal to height is called **Square matrix**. For example

$$X = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, Y = \begin{bmatrix} 1 & 4 & 8 \\ 2 & 5 & 9 \\ 3 & 6 & 10 \end{bmatrix}$$

Just like vectors we can also transpose matrices

The transpose operation is when you flop the vector down

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, then\ x^T = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, then\ y^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, then\ z^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

How to transpose a matrix

$$X = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}, then\ X^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}, then\ Y^T = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

Transpose of a transpose

$$X= \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}, thenX^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$(X^T)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Practice

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Find A^T

Given matrices A and B:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Find $(A + B)^T$.

Given matrices A and B:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Find $(AB)^T$.

Practice

1.If a matrix A has 4 rows and 6 columns, what is its shape?

2. Given the matrix:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

What is its shape?

3. Given the matrix:

$$E = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$$

What is its shape?

4. Given the matrix

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

What is its shape?

Multiplying a matrix by a constant

$$\mathrm{X} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, then \quad 2\mathrm{X} = \begin{bmatrix} 2 \times 1 & 2 \times 0 \\ 2 \times 2 & 2 \times 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 2 \end{bmatrix}$$

1. If matrix A and multiply it by a constant 3.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Find 3A

2. If matrix B and multiply it by a constant -**2**

$$B = \begin{bmatrix} 5 & 7 \\ 1 & 3 \\ 8 & 2 \end{bmatrix}$$

Then find -2B

Addition by a constant

$$\mathrm{X} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, then\ \mathrm{X} + 2 = \begin{bmatrix} 1+2 & 0+2 \\ 2+2 & 1+2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$