HDS Exercise set 2_Post

Shabbeer Hassan

Problem 2 – Solution

```r
library(qvalue)

# Generate 5000 p-values
m = 1000
p = 5000
pval <- c(rbeta(m, 1, 100), runif(p-m, 0, 1))

# BH
BH <- p.adjust(pval, method = "BH", n = length(pval))

# Qvalue
Qval <- qvalue(pval)
pi0 <- Qval$pi0  # overall proportion of true null hypotheses

# Linear Reg
lm.fit <- lm(Qval$qvalues ~ BH)
lm.fit$coefficients
```

```
##   (Intercept)            BH
## -8.641789e-15  8.624009e-01
```

On comparison of slope to the pi0 estimate from qval we see that they are similar to each other. To answer this why this might be so, we need to look at the definitions of the terms itself.

p-value = most extreme probability for a test statistic under the null hypothesis, not accounting for multiple comparisons.. BH p-value or pBH = most extreme probability for the above, after accounting for multiple comparisons such that there is an upper-bound to the overall false positive rate at <= p. And finally, q-value is a direct estimate of the FDR associated with pBH.

Now, as in the examples before and other course notes we see that q-values depend also on the estimated fraction of test p-values in the chance or uniform component of the distribution at some pFDR p. Here,

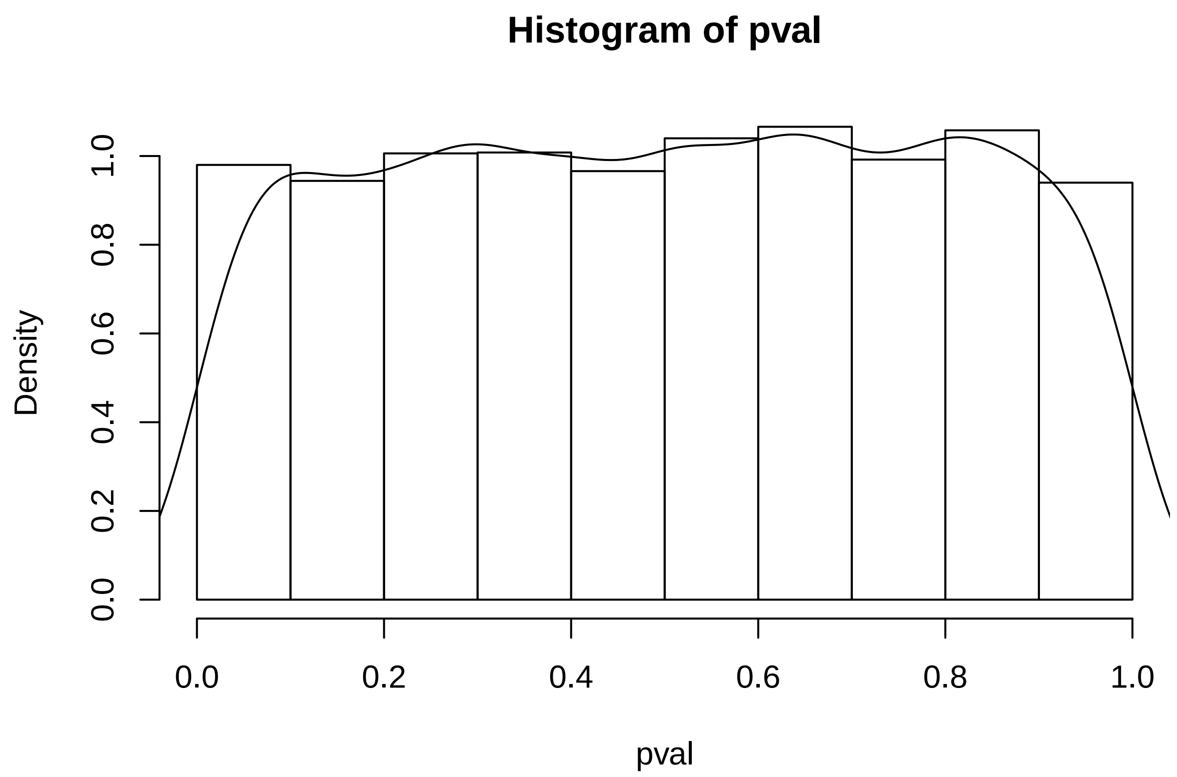
pi0 = estimated probability (overall) of a given result being truly null (i.e., false positive) at p|FDR. And then, q-value = BH p-value * pi0 (probability that test t incorrectly rejects the null at pBH).

So q = pBH * pi0 as seen above in the example, and directly estimates the pFDR for any given test t. In fact, going by the above regression equation and the explanation above, if pi0 is estimated as at or very near 1.0, then pBH and q will be the same for any given test t.

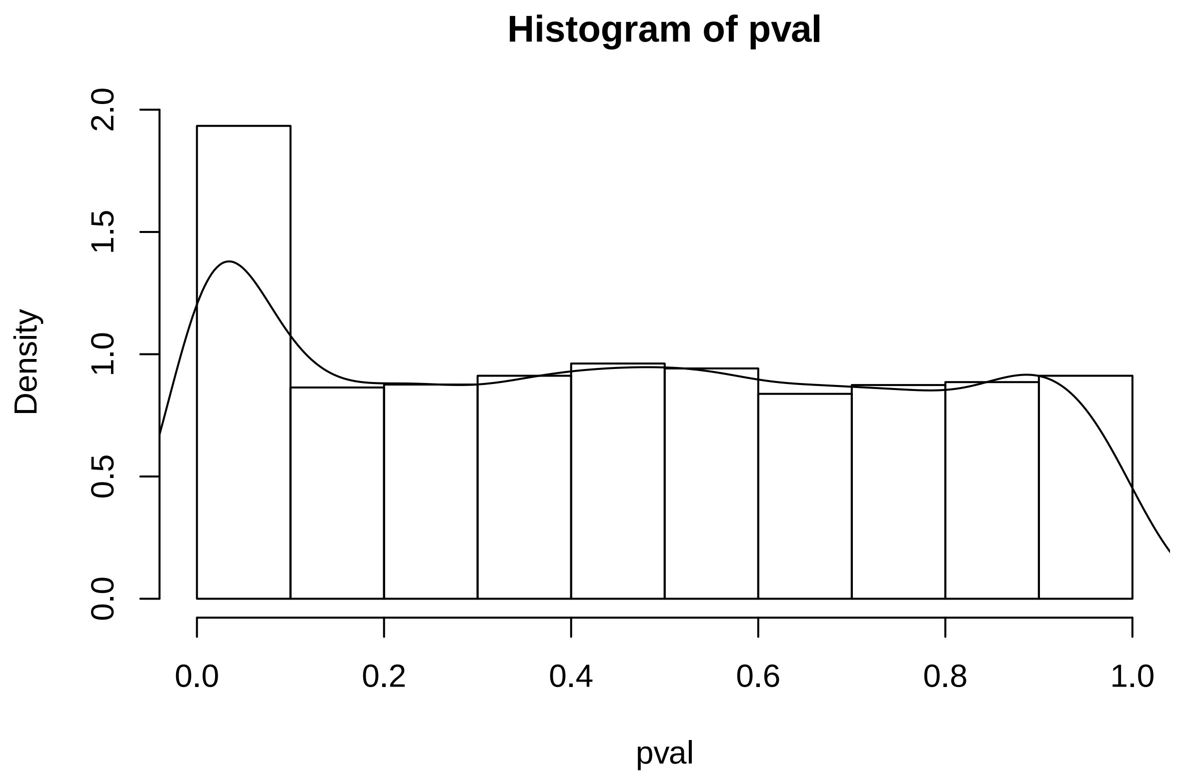
Problem 3 – Solutions

(a)

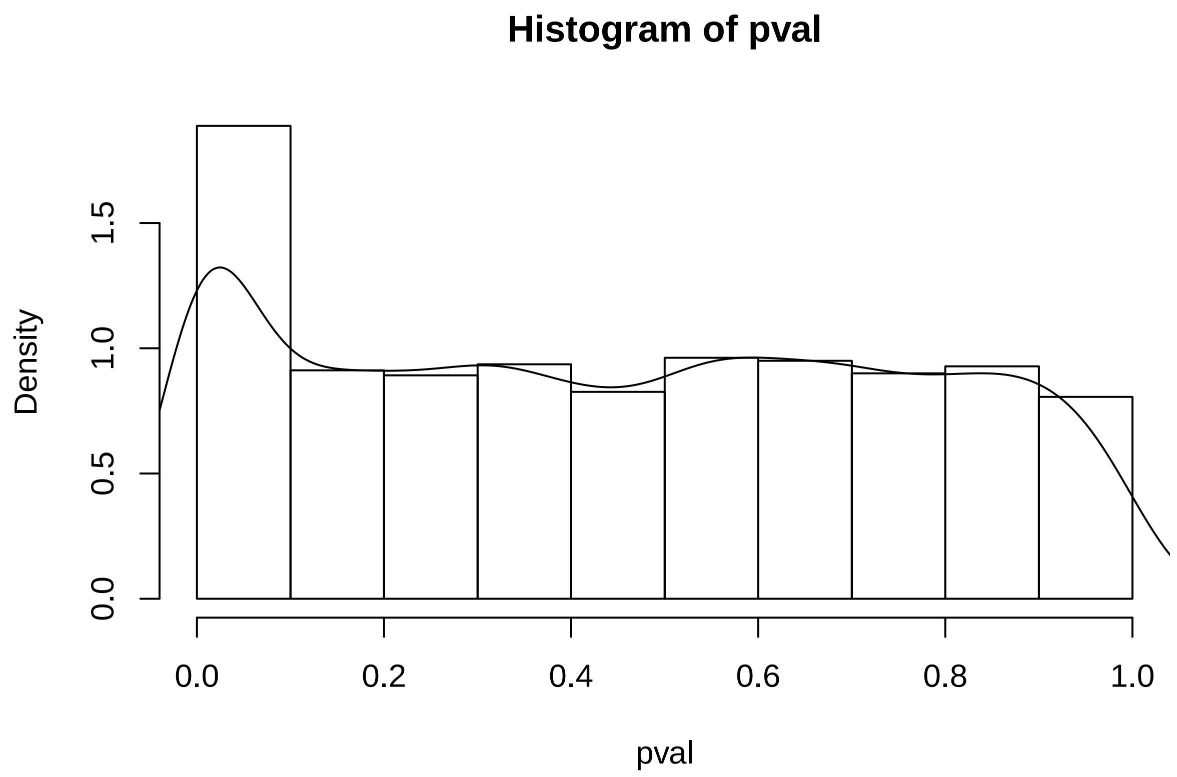
```r
# Generate 5000 p-values
m = 500
p = 5000
#pval <- c(rbeta(m, b.1, b.0), runif(p-m, 0, 1))

b.1 = 1
b.0 = 1
pval <- c(rbeta(m, b.1, b.0), runif(p-m, 0, 1))
hist(pval, xlab = "pval", prob = TRUE)
lines(density(pval))
```



```r
b.1 = 1
b.0 = 100
pval <- c(rbeta(m, b.1, b.0), runif(p-m, 0, 1))
hist(pval, xlab = "pval", prob = TRUE)
lines(density(pval))
```



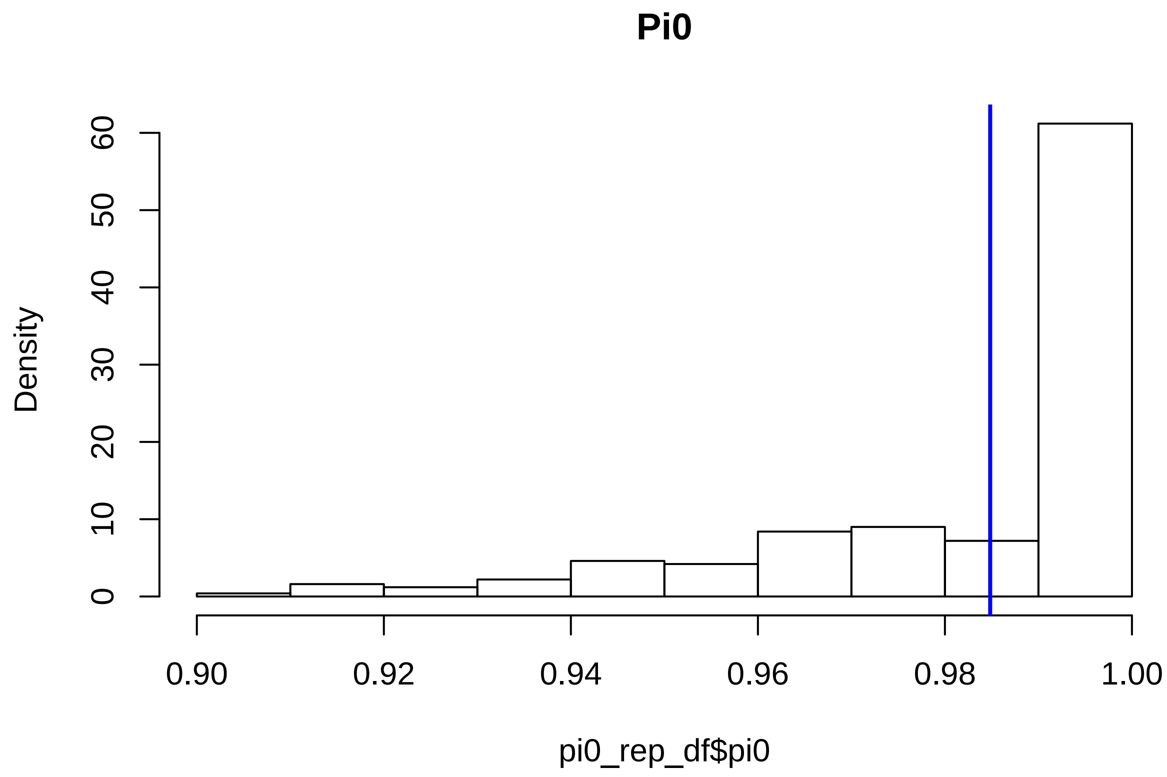
```
b.1 = 1
b.0 = 500
pval <- c(rbeta(m, b.1, b.0), runif(p-m, 0, 1))
hist(pval, xlab = "pval", prob = TRUE)
lines(density(pval))
```



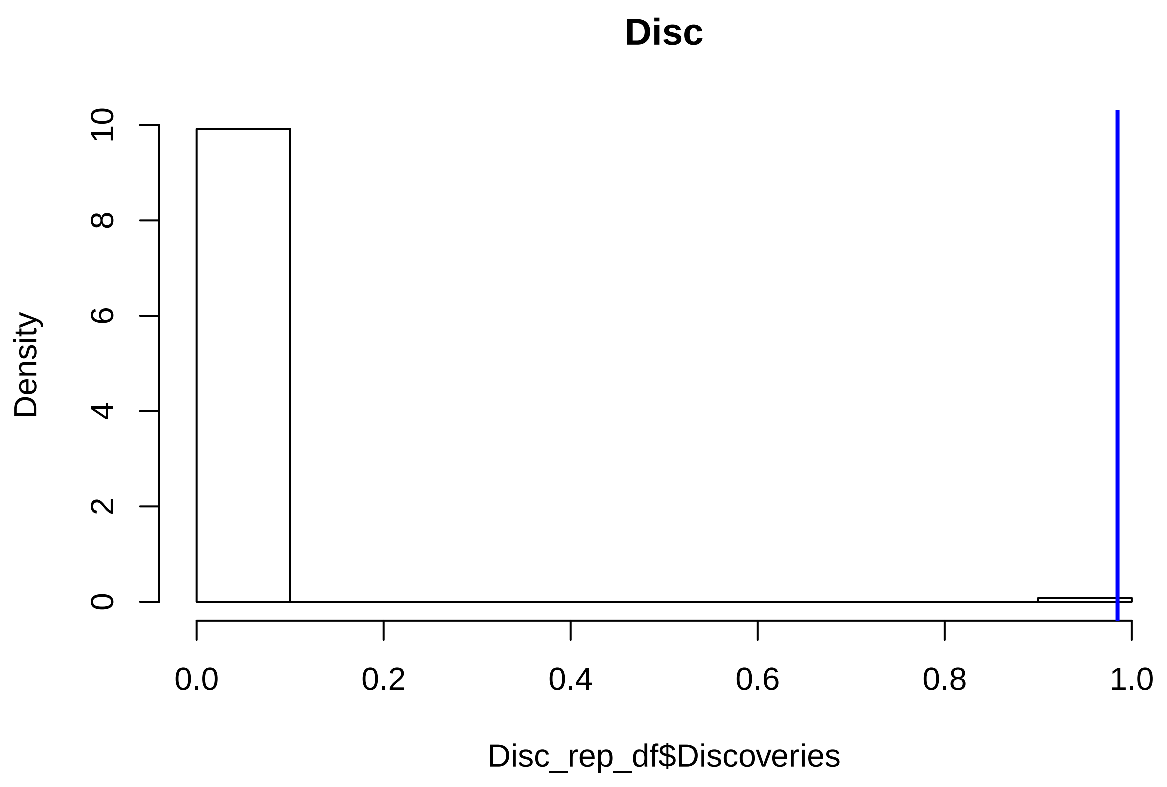
(b)

```
# Generate R = 500 replicates
R = 500
qval_fun <- function(x) {qvalue(p = x)}

#############

b.1 = 1
b.0 = 1

# Generate replicated pvalues and apply qval function over the columns
pval_new <- replicate(R, c(rbeta(m, b.1, b.0), runif(p-m, 0, 1)))
eff <- c(rep(T, m), rep(F, p - m))
qval <- lapply(1:ncol(pval_new), function(x) qvalue(pval_new[,x]))

# Extract variables/elements from the above list
qval_rep <- lapply(qval, `[`, c('qvalues'))
qval_rep_df <- data.frame(matrix(unlist(qval_rep), nrow=length(qval_rep), byrow=T))

pi0_rep <- lapply(qval, `[`, c('pi0'))
pi0_rep_df <- data.frame(matrix(unlist(pi0_rep), nrow=length(pi0_rep), byrow=T))
names(pi0_rep_df) <- names(pi0_rep[[which(lengths(pi0_rep)>0)[1]]])


alpha=0.1
```

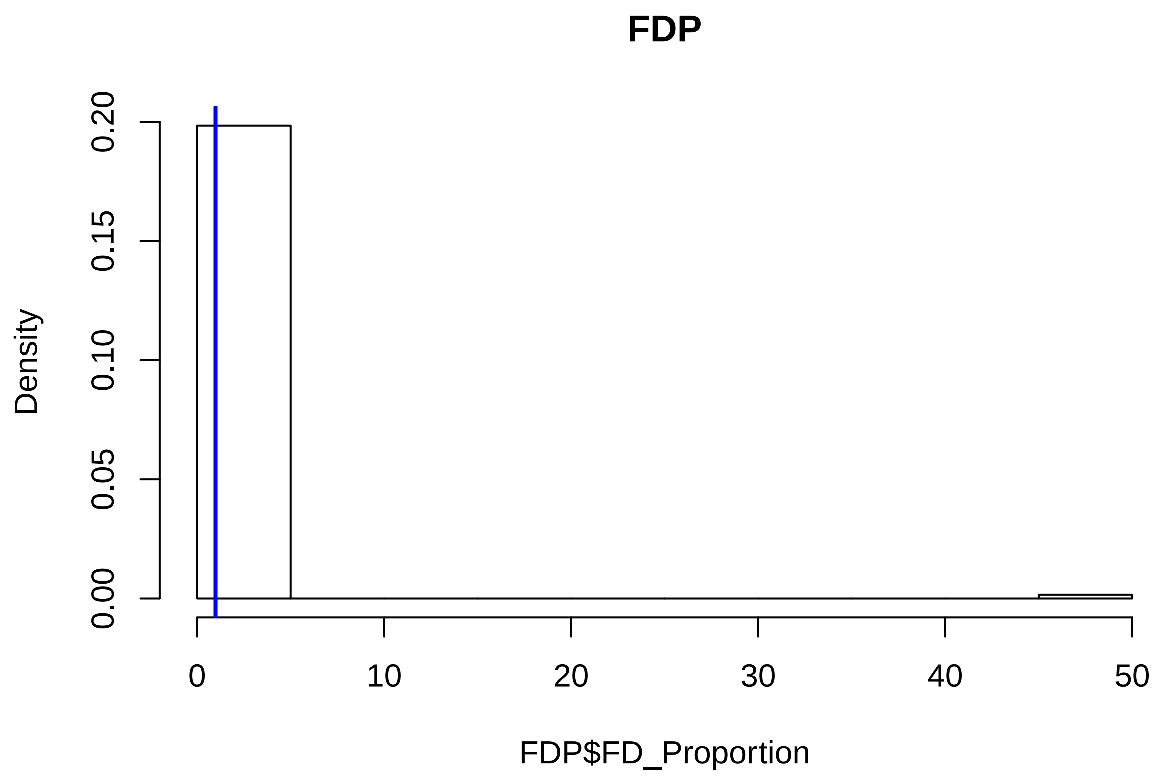
```r
# Discoveries
D_func <- function(x) { (sum(qval_rep_df[x] < alpha)) }
Disc <- lapply(1:ncol(pval_new), D_func)
Disc_rep_df <- data.frame(matrix(unlist(Disc), nrow=length(Disc), byrow=T)) # Extract and generate df f
colnames(Disc_rep_df) <- "Discoveries"


# FDP
FD_func <- function(x) {sum(qval_rep_df[(m + 1):p] < alpha)}
FD <- lapply(1:ncol(pval_new), FD_func)
FD_rep_df <- data.frame(matrix(unlist(FD), nrow=length(FD), byrow=T))
colnames(FD_rep_df) <- "False Discoveries"
FDP <- as.data.frame(ifelse(Disc_rep_df$Discoveries == 0, 0, FD_rep_df$`False Discoveries`/Disc_rep_df$
colnames(FDP) <- "FD_Proportion"

# Power
val <- which(Disc_rep_df$Discoveries < alpha)
Disc_corr <- sum(eff[val] == "TRUE")
pow <- Disc_corr/m

# Histograms
pi_mean <- mean(pi0_rep_df$pi0)
hist(pi0_rep_df$pi0, main = "Pi0", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```



```r
Disc_mean <- mean(Disc_rep_df$Discoveries)
hist(Disc_rep_df$Discoveries, main = "Disc", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```

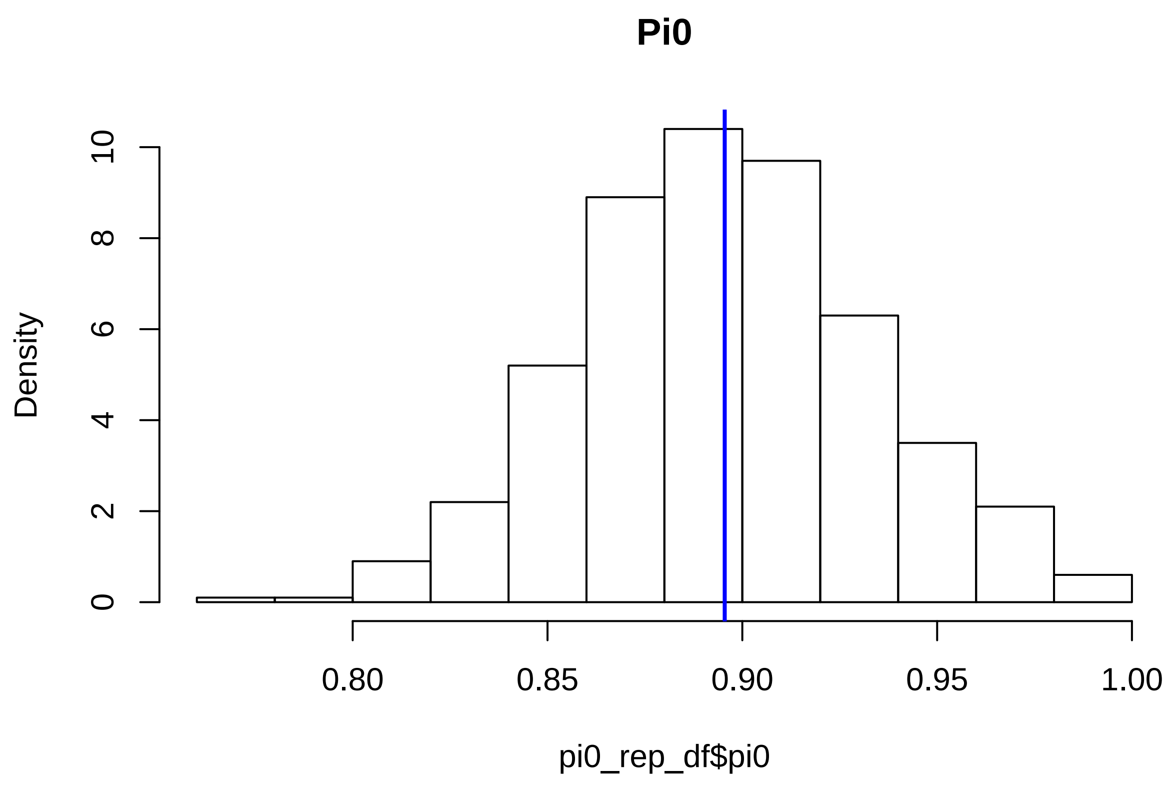


```r
FDP_mean <- mean(FDP$FD_Proportion)
hist(FDP$FD_Proportion, main = "FDP", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```

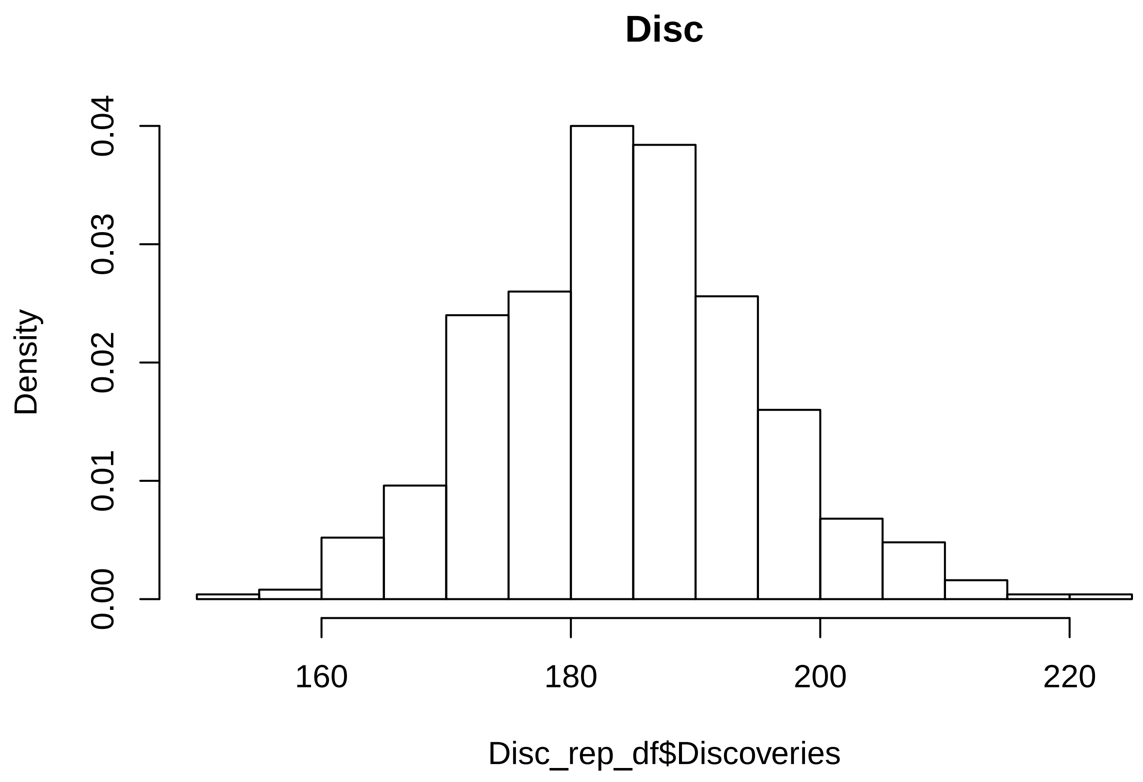


```
##############
b.1 = 1
b.0 = 100

# Generate replicated pvalues and apply qval function over the columns
pval_new <- replicate(R, c(rbeta(m, b.1, b.0), runif(p-m, 0, 1)))
eff <- c(rep(T, m), rep(F, p - m))
qval <- lapply(1:ncol(pval_new), function(x) qvalue(pval_new[,x]))

# Extract variables/elements from the above list
qval_rep <- lapply(qval, `[`, c('qvalues'))
qval_rep_df <- data.frame(matrix(unlist(qval_rep), nrow=length(qval_rep), byrow=T))

pi0_rep <- lapply(qval, `[`, c('pi0'))
pi0_rep_df <- data.frame(matrix(unlist(pi0_rep), nrow=length(pi0_rep), byrow=T))
names(pi0_rep_df) <- names(pi0_rep[[which(lengths(pi0_rep)>0)[1]]])


alpha=0.1

# Discoveries
D_func <- function(x) { (sum(qval_rep_df[x] < alpha)) }
Disc <- lapply(1:ncol(pval_new), D_func)
Disc_rep_df <- data.frame(matrix(unlist(Disc), nrow=length(Disc), byrow=T))
colnames(Disc_rep_df) <- "Discoveries"
```

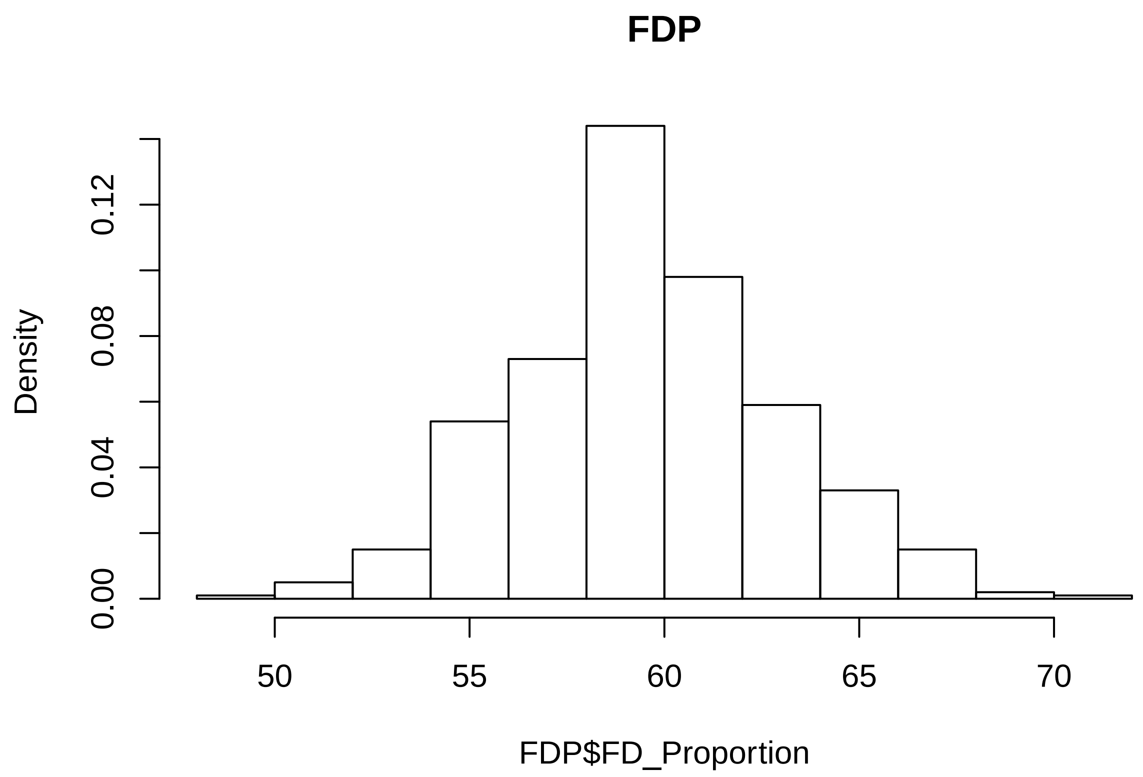
```r
# FDP
FD_func <- function(x) {sum(qval_rep_df[(m + 1):p] < alpha)}
FD <- lapply(1:ncol(pval_new), FD_func)
FD_rep_df <- data.frame(matrix(unlist(FD), nrow=length(FD), byrow=T))
colnames(FD_rep_df) <- "False Discoveries"
FDP <- as.data.frame(ifelse(Disc_rep_df$Discoveries == 0, 0, FD_rep_df$`False Discoveries`/Disc_rep_df$I
colnames(FDP) <- "FD_Proportion"

# Power
val <- which(Disc_rep_df$Discoveries < alpha)
Disc_corr <- sum(eff[val] == "TRUE")
pow <- Disc_corr/m

# Histograms
pi_mean <- mean(pi0_rep_df$pi0)
hist(pi0_rep_df$pi0, main = "Pi0", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```



```r
Disc_mean <- mean(Disc_rep_df$Discoveries)
hist(Disc_rep_df$Discoveries, main = "Disc", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```

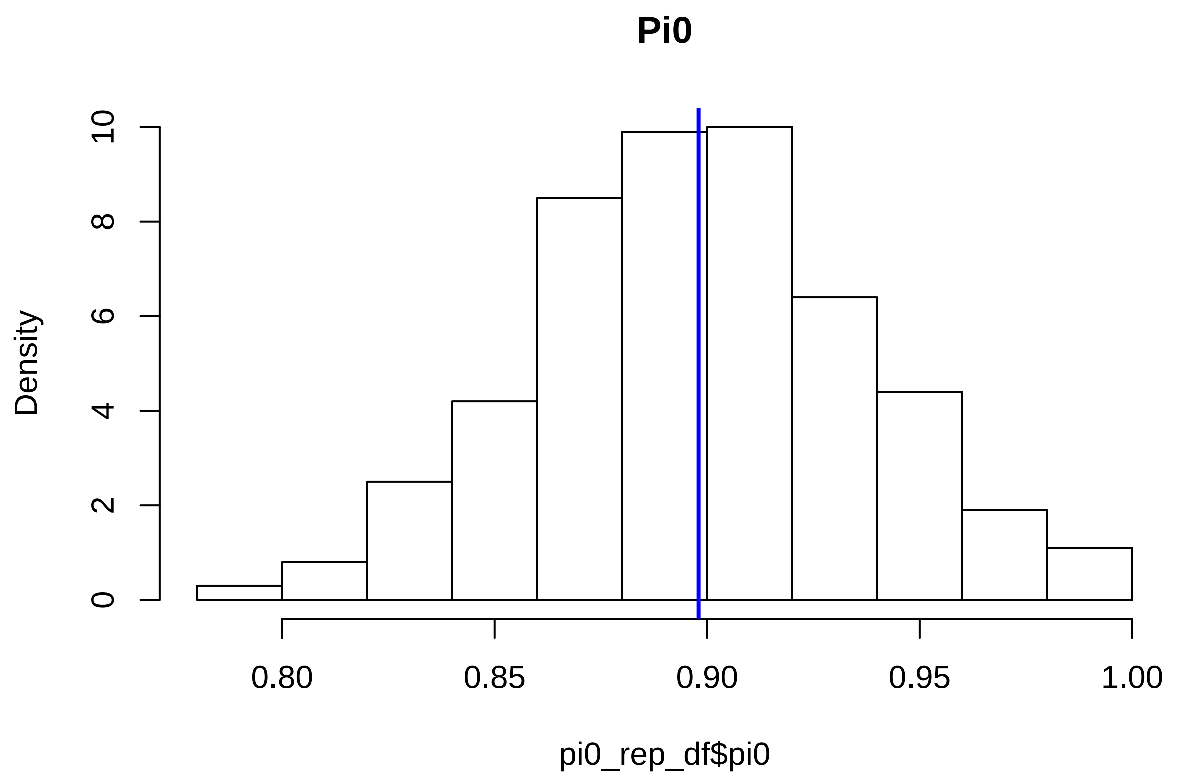


```
FDP_mean <- mean(FDP$FD_Proportion)
hist(FDP$FD_Proportion, main = "FDP", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```

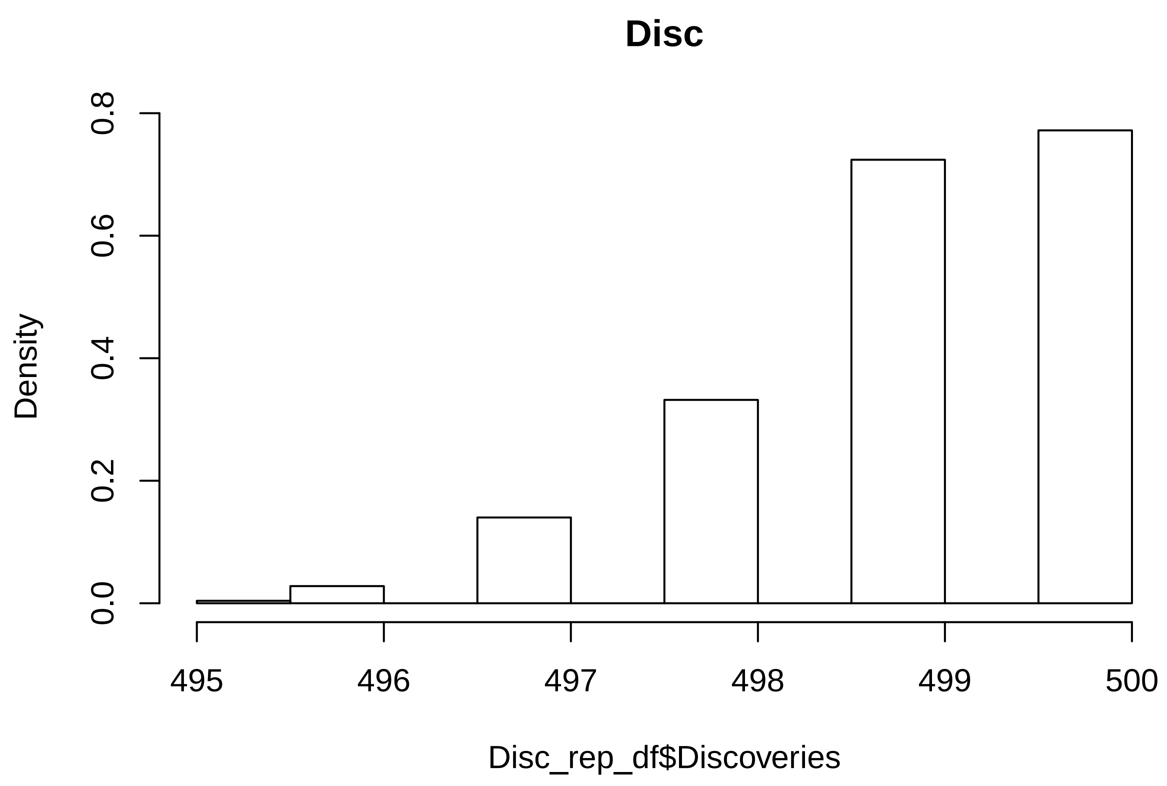


```r
#############

b.1 = 1
b.0 = 500

# Generate replicated pvalues and apply qval function over the columns
pval_new <- replicate(R, c(rbeta(m, b.1, b.0), runif(p-m, 0, 1)))
eff <- c(rep(T, m), rep(F, p - m))
qval <- lapply(1:ncol(pval_new), function(x) qvalue(pval_new[,x]))

# Extract variables/elements from the above list
qval_rep <- lapply(qval, `[`, c('qvalues'))
qval_rep_df <- data.frame(matrix(unlist(qval_rep), nrow=length(qval_rep), byrow=T))

pi0_rep <- lapply(qval, `[`, c('pi0'))
pi0_rep_df <- data.frame(matrix(unlist(pi0_rep), nrow=length(pi0_rep), byrow=T))
names(pi0_rep_df) <- names(pi0_rep[[which(lengths(pi0_rep)>0)[1]]])


alpha=0.1

# Discoveries
D_func <- function(x) { (sum(qval_rep_df[x] < alpha)) }
Disc <- lapply(1:ncol(pval_new), D_func)
Disc_rep_df <- data.frame(matrix(unlist(Disc), nrow=length(Disc), byrow=T))
colnames(Disc_rep_df) <- "Discoveries"
```

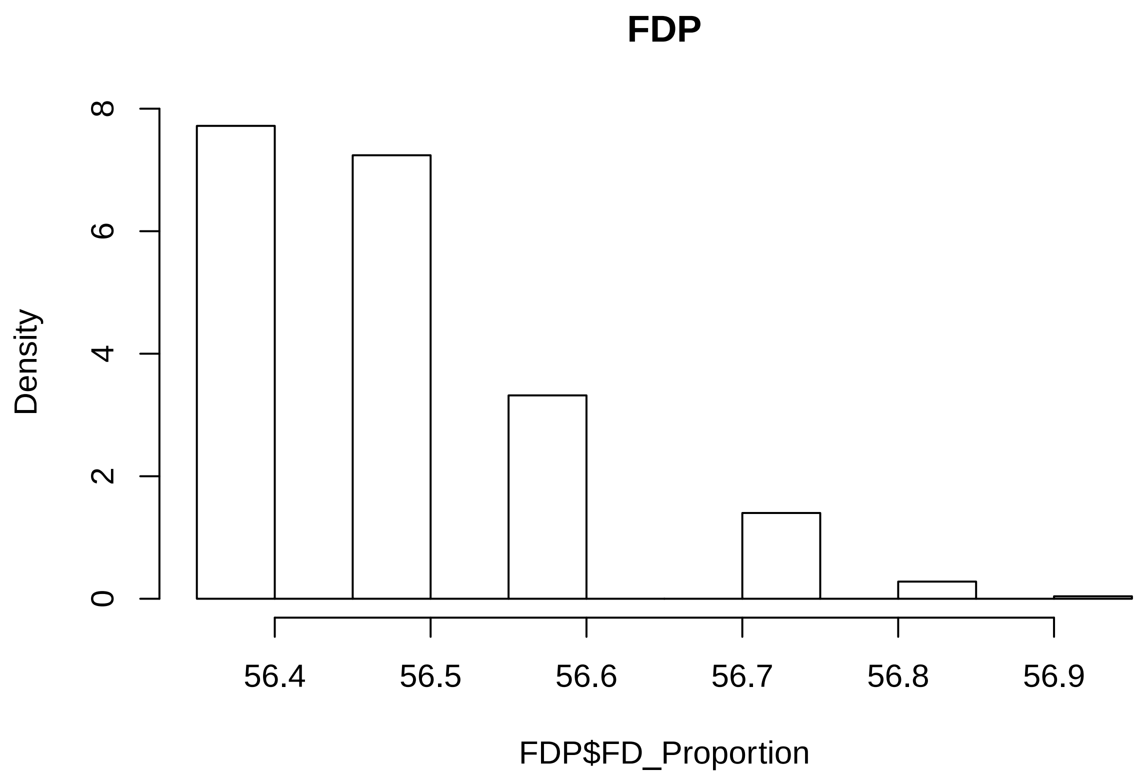
```
# FDP
FD_func <- function(x) {sum(qval_rep_df[(m + 1):p] < alpha)}
FD <- lapply(1:ncol(pval_new), FD_func)
FD_rep_df <- data.frame(matrix(unlist(FD), nrow=length(FD), byrow=T))
colnames(FD_rep_df) <- "False Discoveries"
FDP <- as.data.frame(ifelse(Disc_rep_df$Discoveries == 0, 0, FD_rep_df$`False Discoveries`/Disc_rep_df$
colnames(FDP) <- "FD_Proportion"

# Power
val <- which(Disc_rep_df$Discoveries < alpha)
Disc_corr <- sum(eff[val] == "TRUE")
pow <- Disc_corr/m

# Histograms
pi_mean <- mean(pi0_rep_df$pi0)
hist(pi0_rep_df$pi0, main = "Pi0", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```



```
Disc_mean <- mean(Disc_rep_df$Discoveries)
hist(Disc_rep_df$Discoveries, main = "Disc", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```



```
FDP_mean <- mean(FDP$FD_Proportion)
hist(FDP$FD_Proportion, main = "FDP", prob = T)
abline(v = pi_mean, col = "blue", lwd = 2)
```



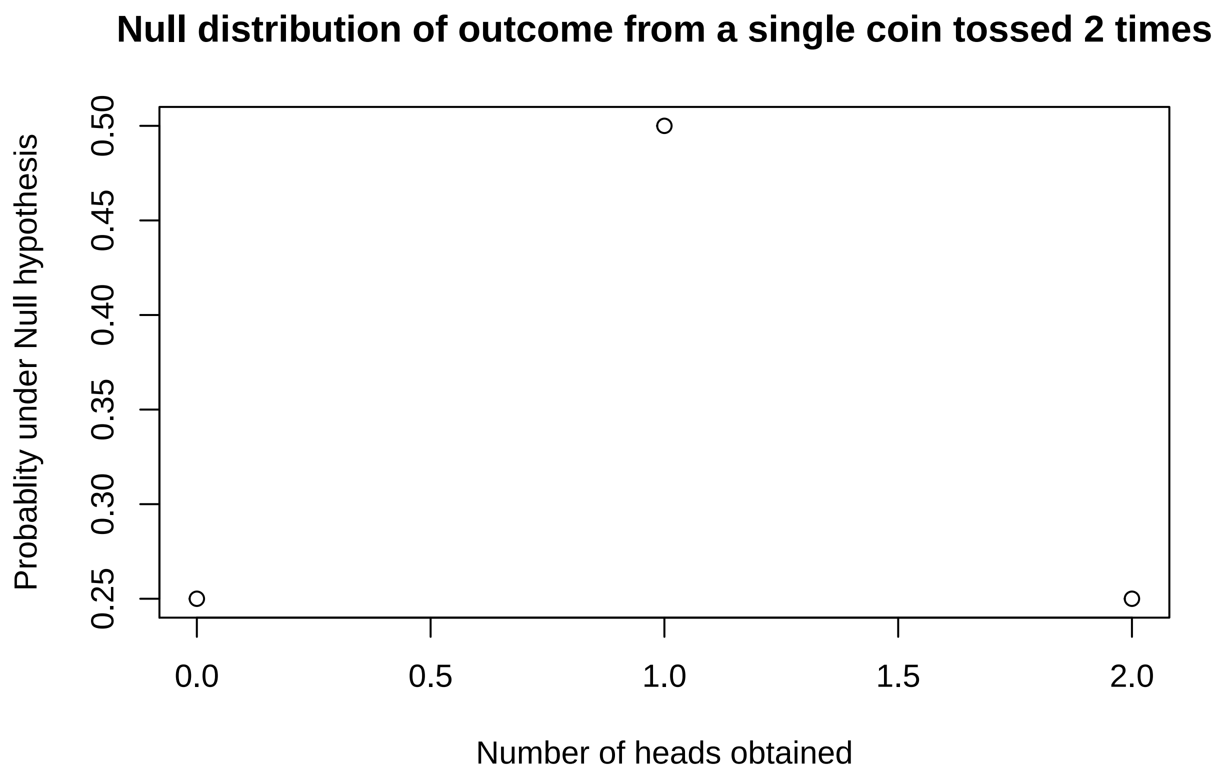
Problem 4 – Solutions

(a)

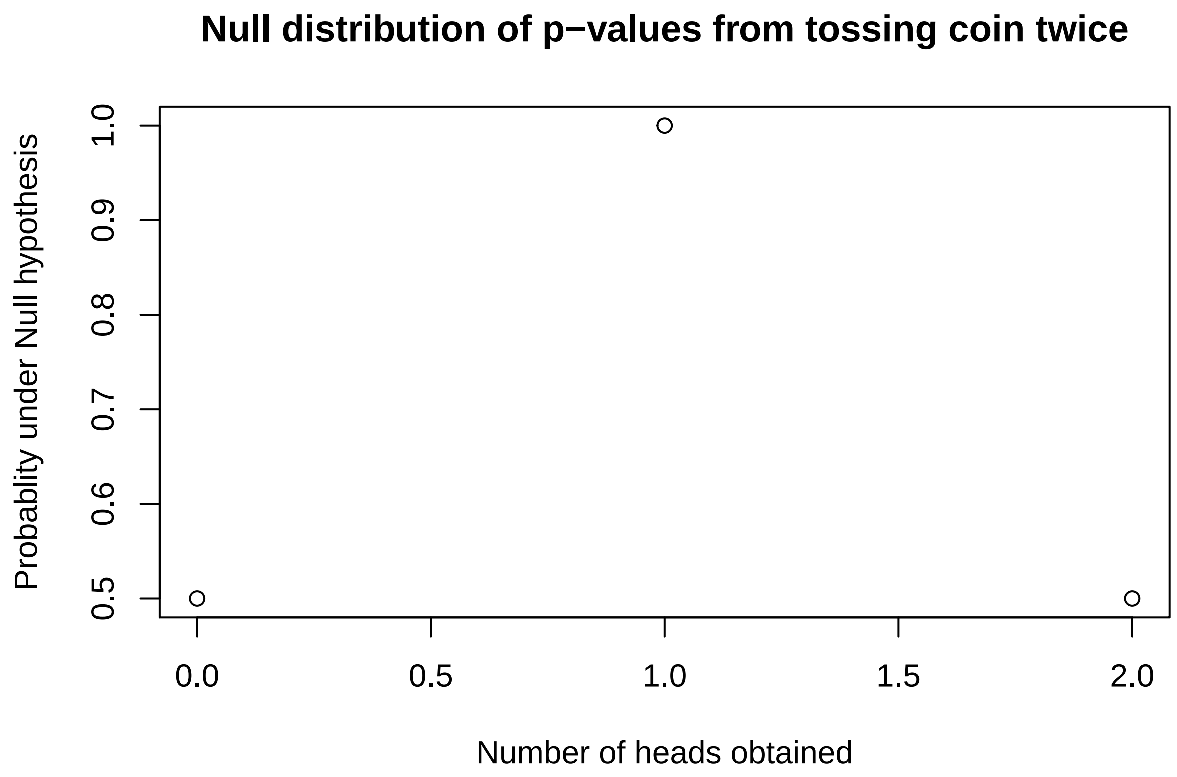
```
# For 1 coin toss
p <- 1

# Fair coin
null_prob <- 0.5

# possible number of heads after 2 tosses
y <- c(0, 1, 2)

# Null hypothesis for 3 possible outcomes aka heads in the tosses
null_prob <- dbinom(y, 2, null_prob)
null_prob
```

```
## [1] 0.25 0.50 0.25
```

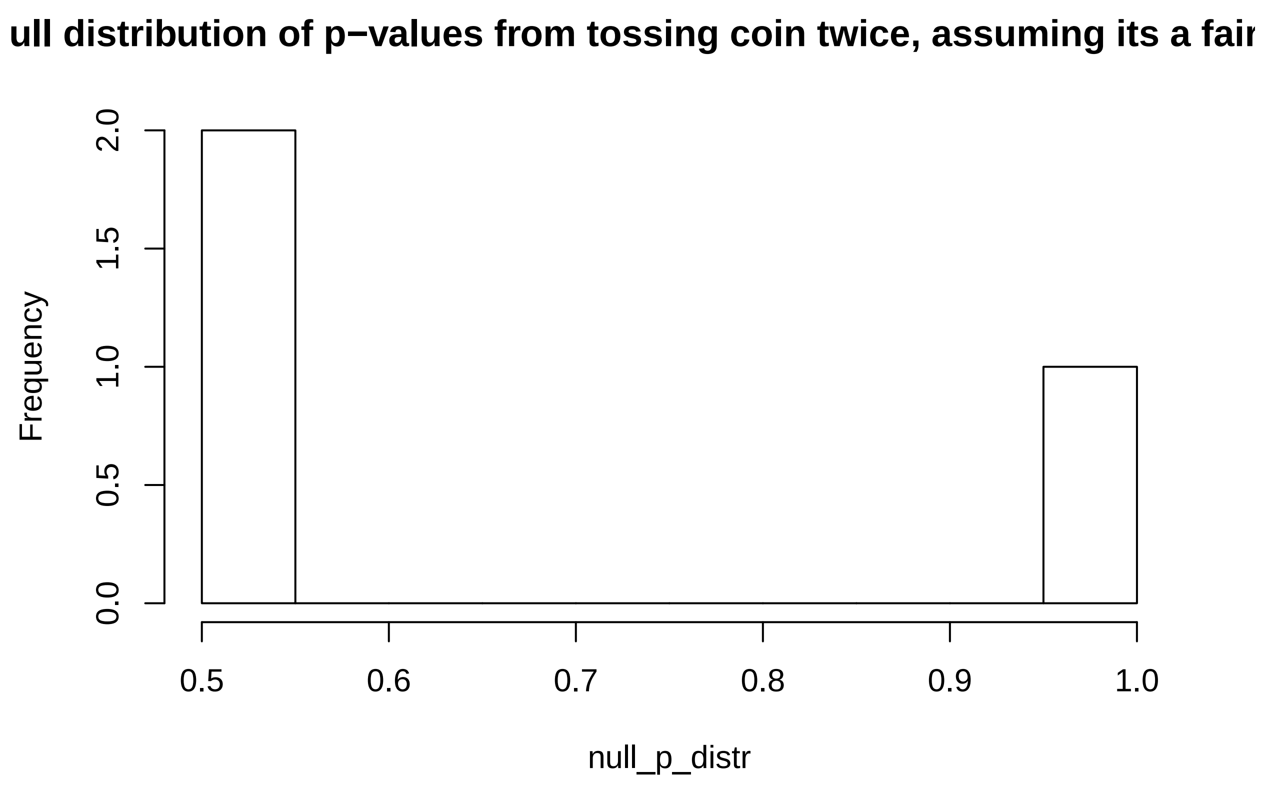
```
# Null distribution of outcome v/s the number of heads obtained
plot(y, null_prob, type='p', main = "Null distribution of outcome from a single coin tossed 2 times", xl
```



```r
# Null distribution of p-values from tossing coin twice, assuming its a fair coin
null_p_distr <- c(0.5, 1, 0.5)

# Null distribution v/s the number of heads obtained
plot(y, null_p_distr, type='p', main = "Null distribution of p-values from tossing coin twice", xlab =
```

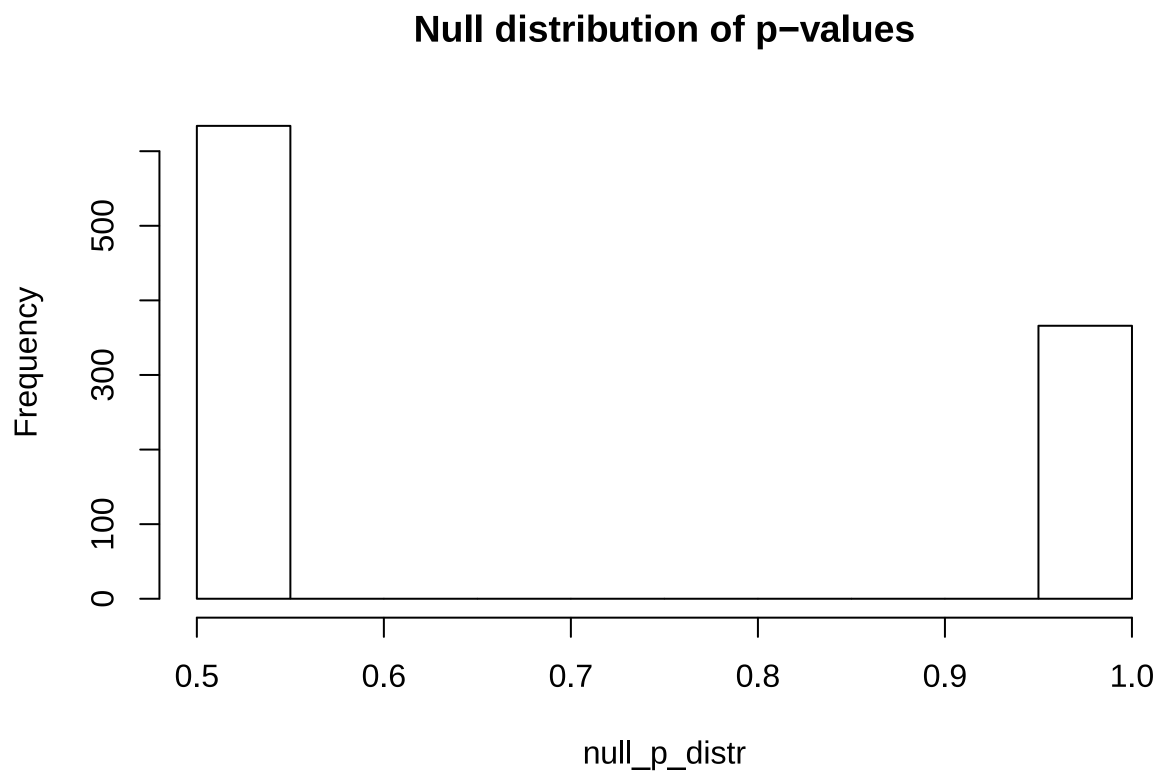


```
# Histogram of the p-values
hist(null_p_distr, 10, main="Null distribution of p-values from tossing coin twice, assuming its a fair
```



(b)

```r
# For 1000 coins
p <- 1000

# Observed counts of outcomes
y <- c(180, 366, 454)

# Distribution of tp-values for the above 1000 coins
null_p_distr <- rep(c(0.5, 1, 0.5), y)
hist(null_p_distr, 10, main="Null distribution of p-values")
```

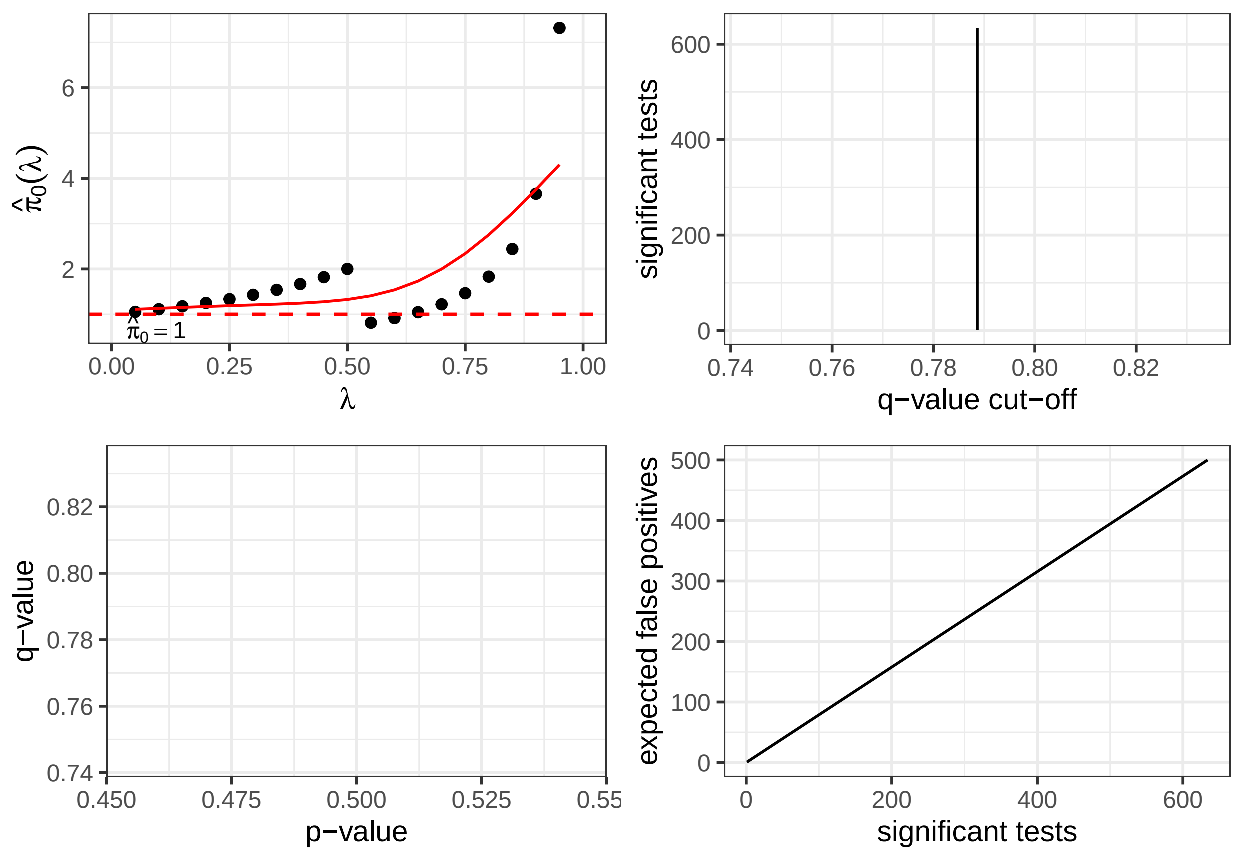


```r
# From the null distribution from part(a) of the question we see that the prob of 1 head is 1 as this i

# Pi0 = (2 * Fair coins)/Total coins

# In the above example, estimate of Pi0 would be
Pi0_est <- (2*366)/p
Pi0_est
```

```
## [1] 0.732
```

(c)

```r
# Given values of lambda
lambda <- c(0.4, 0.5, 0.9)

# Estimate of Pi0
Pi0_est <- as.data.frame(sapply(lambda,function(x){sum(null_p_distr > x)})/p/(1 - lambda))
colnames(Pi0_est) <- "lambda"

View(Pi0_est)

# Only for lambda = 0.5 estimate agrees with part(a).

#Q-values
qVal <- qvalue(null_p_distr)
plot(qVal)
```



Q-values dont seem to work with discrete data at all, as Q-value for obsevations whose pvalue=0.5 is around 0.79. This value is higher than that we got in (b)