

$P_N$  refers to the  $N^{\text{th}}$  prime number, e.g.,  $P_1=2$ ,  $P_2=3$ , etc.

$P_N\#$  refers to the  $N^{\text{th}}$  “primorial” – the product of primes 1 through  $N$ :  $P_N\# = \prod_{i=1}^N P_i$

$N$ -rough numbers are numbers not divisible by any prime less than or equal to  $P_N$

I consider a list of gaps between  $N$ -rough numbers, and call that list a *sieve*. The sieve  $S^N$  consists of gaps between numbers with all prime factors above  $P_N$ . Because the gaps will repeat, the sieve need only contain the first repetition of the pattern, so  $\sum S^N = P_N\#$ . The first entry in a sieve is the difference from one, so  $S^N_1 = P_{N+1} - 1$ .

Here are the first few sieves:

$$S^1 = \{2\}$$

$$S^2 = \{4, 2\}$$

$$S^3 = \{6, 4, 2, 4, 2, 4, 6, 2\}$$

$$S^4 = \{$$

And the first few corresponding sets of rough numbers:

$$R^1 = \{3, 5, 7, 9, \dots\}$$

$$R^2 = \{5, 7, 11, 13, 17, 19, 23, 25, \dots\}$$

$$R^3 = \{7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, \dots\}$$

$$R^4 = \{$$

Some properties of these sieves:

$\sum S^N = P_N\#$  — the sum of entries is the primorial.

$1 + S^N_I = P_{N+1}$  — the first candidate is the next prime.

The first composite from  $S^N$  is  $P_{N+1}^2$  — the first several candidates are prime, and the corresponding sieve entries are prime gaps.

For  $N > 2$ ,  $S^N_2 = S^{N-1}_3$  — candidates which are not removed are shifted left.

Going from  $S^{N-1}$  to  $S^N$ , each entry in  $S^{N-1}$  is (effectively) copied  $P_N$  times, and then fewer than  $P_N$  pairs are replaced with their sum — somewhere in  $S^N$  there will be an intact sequence of every entry from  $S^{N-1}$ ; any number which appears as an entry in any sieve will also appear as an entry in every subsequent sieve.

From one sieve to the next, sieve entries multiply right, shift left, and some are merged with neighbors. Once a number appears as an entry it never disappears; fewer copies are lost to merge than are created. As entries move left and merge, only those that make it past the prime gap threshold ( $P_{N+1}^2$ ) are certain to be prime gaps. The proportion of the sieve which consists of consecutive certain prime gaps gets significantly smaller with every sieve.