

# 09 March Assignment

May 10, 2023

[ ]: Q1: What are the Probability Mass Function (PMF) and Probability Density Function (PDF)? Explain with an example.

[ ]: ANS -

[ ]: Probability Mass Function [ ] Discrete

The Probability Mass Function (PMF) provides the probability distribution for discrete variables. For example, rolling dice. There are 6 distinct possible outcomes that define the entire sample space {1, 2, 3, 4, 5, 6}. Note that we only have whole numbers, i.e. no 1.2 or 3.75. In the PMF, each discrete variable is mapped to its probability. In an ideal situation rolling a die, each of the six variables has a  $1/6$  probability of being rolled. While the ideal distribution is  $1/6$  for each possible outcome 1 to 6, this rarely occurs in real-world scenarios. If we simulate an increasing number of trials we see that the distribution doesn't match the ideal even with 100 dice rolls. As we increase orders of magnitude the simulated distribution approaches the theoretical PMF.

[ ]: Probability Density Function [ ] Continuous

The probability density function (PDF) shows where observations are more likely to occur in the probability distribution. Perhaps the most important thing to remember to understand PDFs is that the probability of any specific outcome is 0. We have to think in terms of bins or ranges of values to calculate the probability of seeing those values. We can use PDFs to calculate probability by looking at the area under the curve for our interval. Mathematically, this is why a single point has 0 probability, the area of a point is 0. Because we have a continuous variable, we can use integrals to calculate the area under the curve between our interval with the following equation.

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[ ]: Q2: What **is** Cumulative Density Function (CDF)? Explain **with** an example. Why CDF **is** used?

[ ]: ANS -

[ ]: The cumulative distribution function, CDF, **or** cumulant **is** a function derived **from the** probability density function **for** a continuous random variable. It gives the probability of finding the random **variable** at a value less than **or** equal to a given cutoff. Many questions **and** computations about probability distribution functions are **convenient** to rephrase **or** perform **in** terms of CDFs, e.g. computing the PDF of a function of a random variable.

The cumulative distribution function (CDF) calculates the cumulative **probability for** a given x-value. Use the CDF to determine the probability that a random observation that **is** taken **from the** population **will be less than or** equal to a certain value.

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[ ]: Q3: What are some examples of situations where the normal distribution might be **used as** a model? Explain how the parameters of the normal distribution relate to the shape of **the distribution**.

[ ]: ANS -

[ ]: Normal/Gaussian Distribution **is** a bell-shaped graph that encompasses two **basic terms-** mean **and** standard deviation. It **is** a symmetrical arrangement of a data **set in** which most values cluster **in the mean and** the rest taper off symmetrically towards either extreme. Numerous genetic **and** environmental factors influence **the trait**.

[ ]: 1. Height :

The height of people **is** an example of normal distribution. Most of **the people in** a specific population are of average height. The number of people taller **and** shorter than the average height people **is** almost equal, **and** a very small number of people

are either extremely tall or extremely short. Several genetic and environmental factors influence height. Therefore, it follows the normal distribution.

[ ]: 2. Rolling A Dice :

A fair rolling of dice is also a good example of normal distribution. In an experiment, it has been found that when a dice is rolled 100 times, chances to get '1' are 15-18% and if we roll the dice 1000 times, the chances to get '1' is, again, the same, which averages to 16.7% (1/6). If we roll two dice simultaneously, there are 36 possible combinations. The probability of rolling '1' (with six possible combinations) again averages to around 16.7%, i.e., (6/36). More the number of dice more elaborate will be the normal distribution graph.

[ ]: 3. Blood Pressure :

Blood pressure generally follows a Gaussian distribution (normal) in the general population, and it makes Gaussian mixture models a suitable candidate for modelling blood pressure behaviour.

[ ]: 4. IQ :

In this scenario of increasing competition, most parents, as well as children, want to analyze the Intelligent Quotient level. Well, the IQ of a particular population is a normal distribution curve; where the IQ of a majority of the people in the population lies in the normal range whereas the IQ of the rest of the population lives in the deviated range.

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[ ]: Q4: Explain the importance of Normal Distribution. Give a few real-life examples of Normal Distribution.

[ ]: ANS -

[ ]: The normal distribution is the most important probability distribution in statistics because it fits many natural phenomena. For example, heights, blood pressure, measurement error, and IQ scores follow the normal distribution. It is also known as the Gaussian distribution and the bell curve.

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↳the people **in** a specific population are of average height. The number of people taller **and** shorter than the average height people  
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↳factors influence height. Therefore, it follows the normal distribution.

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↳the deviated **range**.

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[ ]: Q5: What **is** Bernaulli Distribution Give an Example. What **is** the difference  
↳between Bernoulli Distribution **and** Binomial Distribution?

[ ]: ANS -

[ ]: The Bernoulli distribution **is** a discrete probability distribution that  
↳describes the probability of a random variable **with** only two outcomes.

In the random process called a Bernoulli trial, the random variable can take  
 ↳ one outcome, called a success, with a probability  $p$ ,  
 or take another outcome, called failure, with a probability  $q = 1-p$ .  
 The success outcome is denoted as 1 and the failure outcome is denoted as 0.  
 The Bernoulli distribution is a special case of the binomial distribution where  
 ↳ a single trial is conducted and the binomial  
 distribution is the sum of repeated Bernoulli trials.

[ ]: Example :

Tossing a coin can result in only two possible outcomes (head or  
 ↳ tail). We call one of these outcomes (head) a success and  
 the other (tail), a failure.  
 The probability of success ( $p$ ) or head is 0.5 for a fair coin. The probability  
 ↳ of failure ( $q$ ) or tail =  $1-p = 1-0.5 = 0.5$ .  
 We have two outcomes:  
 Tail or 0 with a probability of 0.5.  
 Head or 1 with a probability of 0.5 also.  
 This is an example of a probability mass function where we have the probability  
 ↳ for each outcome.

[ ]: The difference between Bernoulli distribution and binomial distribution is  
 ↳ given below: Bernoulli distribution is used when we  
 want to model the outcome of a single trial of an event. If we want to model  
 ↳ the outcome of multiple trials of an event, Binomial  
 distribution is used. It is represented as  $X \sim \text{Bernoulli}(p)$ .

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[ ]: Q6. Consider a dataset with a mean of 50 and a standard deviation of 10. If we  
 ↳ assume that the dataset  
 is normally distributed, what is the probability that a randomly selected  
 ↳ observation will be greater  
 than 60? Use the appropriate formula and show your calculations.

[ ]: ANS -

[ ]:  $z = (x - ) /$

[4]:  $z = (60 - 50) / 10$

[5]:  $z$

[5]: 1.0

[ ]: 15.87

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[ ]: Q7: Explain uniform Distribution with an example.

[ ]: ANS -

[ ]: A continuous probability distribution is a Uniform distribution and is related to the events which are equally likely to occur.  
It is defined by two parameters, x and y, where x = minimum value and y = maximum value. It is generally denoted by  $u(x, y)$ .  
OR  
If the probability density function or probability distribution of a uniform distribution with a continuous random variable X is  $f(x) = 1/(y-x)$ , then It is denoted by  $U(x, y)$ , where x and y are constants such that  $x < a < y$ . It is written as  
X  
 $U(a, b)$

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[ ]: Q8: What is the z score? State the importance of the z score.

[ ]: ANS -

[ ]: A Z-score is an indicator of how closely a value relates to the mean of a set of values. Z-score is quantified by the standard deviations from the mean. The mean score and the data points score are equal when the Z-score is zero. A Z-score of 1.0 indicates a result that is one standard deviation from the mean. Z-scores are binary, with a positive value denoting a score above the mean and a negative value denoting a score below the mean.  
Z-scores within the financial sector can be described as measures of the variability of an observation that traders can use to gauge market volatility.  
The Z-score can also be known as the Altman Z-score model.

[ ]: importance of the z score :

The Z-score is important because it lets statisticians and traders know whether a score falls within the norm for a given data

set or deviates from it. Analysts can also modify scores from multiple data sets using Z-scores to create scores that are more accurately comparable to one another.

Because they allow for a comparison between two scores that do not belong to the same normal distribution, z-scores are significant.

They are also employed to determine the likelihood that a z-score will occur inside a normal distribution. A negative z-score indicates that the raw data is smaller than the mean. The raw score is higher than the average if the z-score is positive.

Among Z-scores benefits are:

A z-score transformation takes into account the mean value as well as the variability of a group or raw scores. Even though the raw data came from various tests, it is still possible to compare them.

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[ ]: Q9: What is Central Limit Theorem? State the significance of the Central Limit Theorem.

[ ]: ANS -

[ ]: Central Limit Theorem Definition The Central Limit Theorem (CLT) states that the distribution of a sample mean that approximates the normal distribution, as the sample size becomes larger, assuming that all the samples are similar, and no matter what the shape of the population distribution.

The central limit theorem is useful when analyzing large data sets because it allows one to assume that the sampling distribution of the mean will be normally-distributed in most cases. This allows for easier statistical analysis and inference.

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[ ]: Q10: State the assumptions of the Central Limit Theorem.

[ ]: ANS -

[ ]: Assumptions of Central Limit Theorem :

Assumptions of central limit theorem are stated below:

-The sample should be drawn randomly following the condition of randomization.

-The samples drawn should be independent of each other. They should **not** influence the other samples.  
-When the sampling **is** done without replacement, the sample size shouldn't exceed 10% of the total population.  
-The sample size should be sufficiently large.

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