

## ⇒ Regular Expression:

- \* "Regular expression is a simple expression used to describe the language accepted by the finite Automata."
- \* It is an effective way to represent any language
- \* They are used to represent certain set of strings in an algebraic fashion
- \* Symbols we use 0, 1, a, b, ε, φ etc are Regular expression
- \* Union of 2 regular expression is also regular expression.
- \* Concatination of 2 RE's is also RE
- \* Iteration (closure) of a regular expression is also a R.E i.e if 'b' is regular expression then  $b^*$  is also regular expression.

## Application of Regular Expression

### ① Lexical Analysis:

Task of lexical analyser is to scan the input code and separate out the tokens.

Ex: Identifier is a category of token in the source code and it can be identified by Regular Expression

(letter) (letter+digit)\*

② finding patterns in the text

Regular expression is used for finding patterns from the given text . (pdf, doc, file or any document)

ex: strings ending with digits by a pattern for the given text which can be recognised by the following R.E

as  $[a-z][a-z]^* [0-9]^+$

③ In unix

egrep is the tool (command) used for searching text pattern in the file and list of words containing that pattern.

ex: Assume , file xyz is having the following data

Ram , Dexter , Roman , Then

% egrep 'n' xyz → This will search all words in file xyz having letter 'n'

op: roman  
Then

## Problems on Regular Expression

(2)

- ① write regular expression denoting language with strings having any number of a's over  $\Sigma = \{a\}$

$$\underline{RE = a^*}$$

- ② write RE to accept all strings having either a or b over  $\Sigma = \{a, b\}$  of length '1'

$$\underline{RE = a + b}$$

- ③ write RE to accept all strings that contain any combination of a and b of length 2  $\Sigma = \{a, b\}$

$$RE = (a+b)(a+b) \text{ or } (a+b)^2$$

note:  $L = \{aa, ab, ba, bb\}$

- ④ write regular E to accept all strings that contain any combination of a's and b's of length 3  $\Sigma = \{a, b\}$

$$L = \{aaa, aba, bbb, bab, bba, abb, \dots\}$$

$$RE = (a+b)(a+b)(a+b)$$

- ⑤ string that contains any combination of 'a' and 'b' of any length

$$RE = (a+b)^*$$

- ⑥ string start with 'a' and ends with 'b'

$$RE = a(a+b)^*b$$

⑦ strings begin with ab over  $\Sigma = \{a, b\}$   
 $RE = \underline{\underline{ab}}(a+b)^*$

⑧ string end with ba  
 $RE = (\underline{\underline{a+b}})^*ba$

⑨ string containing substring 'a'  $\Sigma = \{a, b\}$   
i.e. a must be present in the string and before a & after a anything can be present.  
 $RE = (a+b)^* a (a+b)^*$

⑩ string containing one more a's over  $\Sigma = \{a, b\}$   
 $RE = \underline{\underline{aa}}^*$

⑪ string length of length atleast 3 over  $\Sigma = \{a, b\}$  i.e.  $|w| \geq 3$   
 $RE = (a+b)(a+b)(a+b)(a+b)^*$

⑫ represent string of a's & b's having odd length  
 $RE ((a+b)(a+b))^* (a+b)$   
*This will give even + one odd length*

⑬ string of odd length over  $\Sigma = \{1\}$   
 $L = \{1, 111, 11111, 11111111 \dots\}$   
 $RE = 1 (11)^* \text{ or } (11)^* 1$

⑭ string length atleast 3 over  $\Sigma = \{a\}$  (3)

$$RE = \underline{\underline{aaaa}}^*$$

⑮ string of even lengths over  $\Sigma = \{a, b\}$

$$RE = \underline{\underline{(a+b)(a+b))}}^*$$

$$L = \{ \underline{\epsilon}, \underline{aa}, \underline{ab}, \underline{aab}, \underline{abba}, \underline{abba}, \underline{abbb} \dots \}$$

⑯ string of lengths almost 2 over  $\Sigma = \{a, b\}$  i.e maximum length of string = 2. not more than 2

$$RE = \{ \begin{matrix} \epsilon & + a + b & + (a+b)(a+b) \end{matrix} \}$$

length      length      length

⑰ string start over  $\Sigma = \{0, 1\}$  that contain 1

$$RE = \underline{\underline{(0+1)^* 1 (0+1)^*}}$$

⑱ string having exactly 2 a's over  $\Sigma = \{a, b\}$

$$RE = \underline{\underline{b^* ab^* ab^*}}$$

⑲ string of odd length

$$RE = \underline{\underline{(0+1)(0+1)(0+1))}}^*$$

20)  $w$  corresponds to the binary encoding without leading 0's of natural number that are evenly divisible by 4.

i.e. binary number evenly divisible by 4 & there should not be leading 0's.

$$L = \left\{ \underbrace{100}_4, \underbrace{1000}_8, \underbrace{1100}_{(12)}, \underbrace{10000}_{16}, \dots \right\}$$

$$\underline{RE = 1 (0+1)^* 00}$$

21)  $w$  correspond to the binary encoding without leading 0's of natural number that are power of 4.

$$L = \left\{ \underbrace{1}_{4^0=1}, \underbrace{100}_{4^1=4}, \underbrace{10000}_{4^2=16}, \underbrace{1000000}_{4^3=64}, \dots \right\}$$

$$\underline{RE = 1 (00)^*}$$

22)  $w \in \{0-9\}^*$   $w$  corresponding to the decimal encoding without leading 0's of an odd natural number. i.e every odd decimal no. end with 1 or 3 or 5 or 7 or 9 & begin with (1-9) & in between we can have anything

$$\underline{RE = (1-9)(0-9)^* (1+3+5+7+9)}$$

23) string does not contain substring 01

$$\underline{RE = 1^* 0^*}$$

24) starts with a but not having consecutive b's

$$\underline{RE = (\underline{a+a}b)^*}$$

(4)

$$\textcircled{25} \quad L = \{a^n b^m : (m+n) \text{ is even}\}$$

either m and n should be even/m+n odd

$$RE = \underline{(aa)^* (bb)^*} + a\underline{(aa)^* b (bb)^*}$$

$\underbrace{\text{seven a's}}$  } or {  $\begin{cases} \text{odd a's \&} \\ \text{odd b's} \end{cases}$   
 $\begin{cases} \text{even a's} \end{cases}$

m	n	m+n
0	0	0
1	1	2
2	3	4
3	1	4
2	2	4
1	5	6

$$\textcircled{26} \quad L = \{a^n b^m \mid n \geq 4 \text{ \& } m \leq 3\}$$

$$RE = (\underline{\text{at least 4 a's}}) (\underbrace{e+b+bb+bbb}_{\text{almost 3 b's}})$$

$$\textcircled{27} \quad L = \{a^n b^n \mid 0 \leq n \leq 4\}$$

$$RE = \{e + ab + aabb + aaabbb + aaaaabbbbb\}$$

$$\textcircled{28} \quad L = \{a^{2n} b^{2m} \mid n \geq 0 \text{ \& } m \geq 0\}$$

$$RE = \underline{(aa)^*} \underline{(bb)^*}$$

\textcircled{29} a's appear tripler there is no restriction  
on the no of b's

$$RE = \underline{(b^* (aaa)^* b^*)^*}$$

$$\textcircled{30} \quad L = \{w \in \{a, b\}^* \mid |w| \bmod 3 = 0\}$$

$$\underline{(a+b)(a+b)(a+b)^*}$$

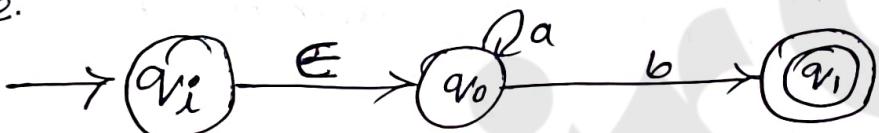
## Converting fsm to Regular Expression

[State elimination method.]

Step1: initial state should not have incoming edge. If exist create one new state and make it as the initial state.

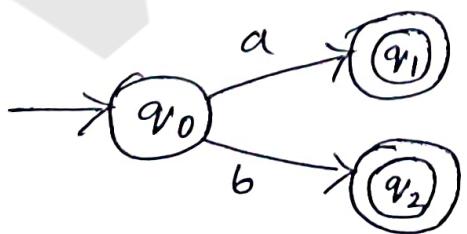


$q_0$  is having incoming edge. so create new state and make it as a initial state, so that we can remove incoming edge.

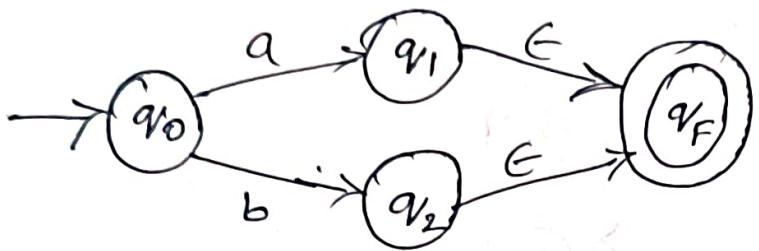


' $q_i$ ' is the new state make  $q_i$  as the initial state and show  $\epsilon$ . transition from  $q_i$  to  $q_0$

Step2 : finite state machine should not have multiple final states. If exists than create one or more new state make this state as the final state and show  $\epsilon$  transition from old final states to new final state



Here fsm is having 2 final states  $q_1$  &  $q_2$  we need to remove multiple final state



$q_F$  is new final state.

Step 3: final state in FSM should not have outgoing edge. If exist then create new state and make it as a final state.



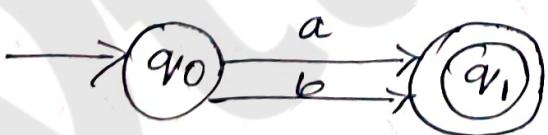
Here  $q_2$  is the final state having outgoing edge.



now  $q_F$  does not contain outgoing edge.

Step 4: Remove every state one after another in any order and at the same time write equivalent R.E  
Finally FA should have one initial & one final state.

CX:

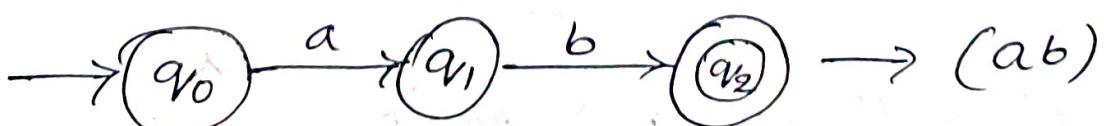


R.E

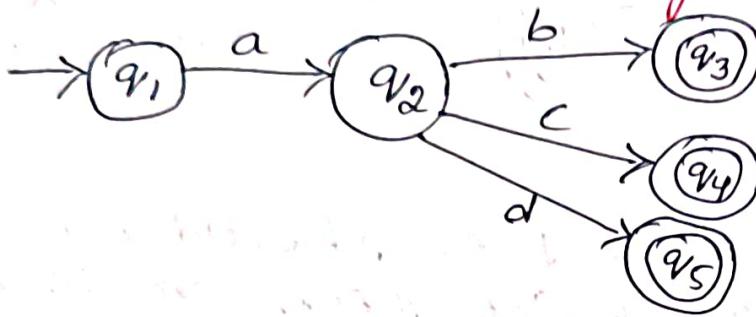
$(a+b)$



$(a^*)$

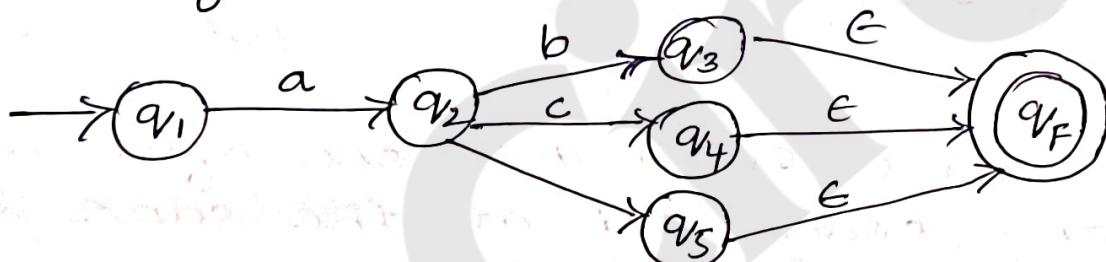


① convert Fsm to regular expression.



Step1: Initial state  $q_1$  does not have any incoming edge. so no modification required.

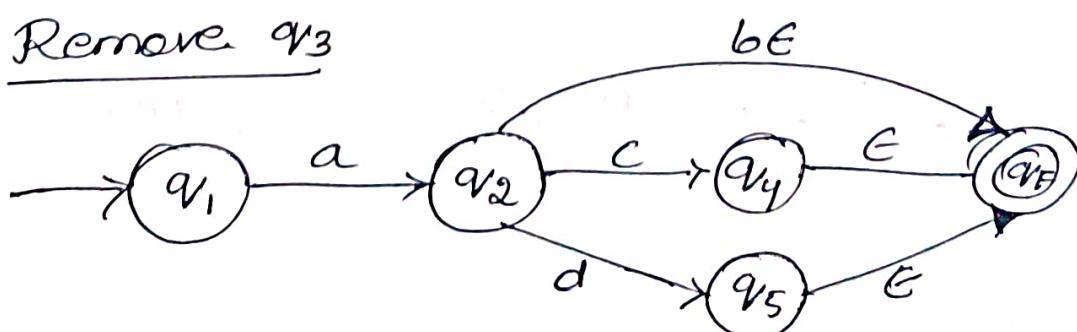
Step2: Here fsm is having 3 final state. This has to be removed. Introduce new final state  $\underline{q_F}$



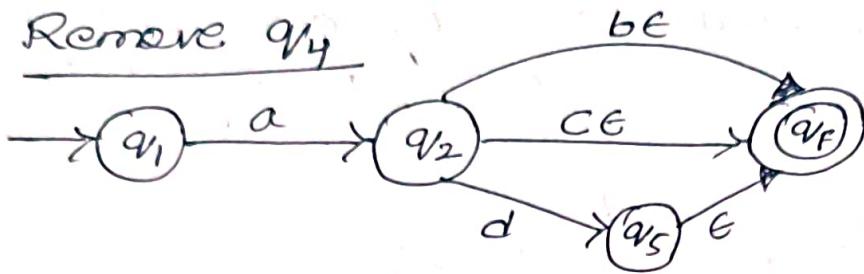
Step3: fsm should not have outgoing edge for final state. In this fsm  $q_F$  does not have any outgoing edge. so no modification is required.

Step4: Eliminate state one after another and write equivalent regular expression till we get one initial and final state.

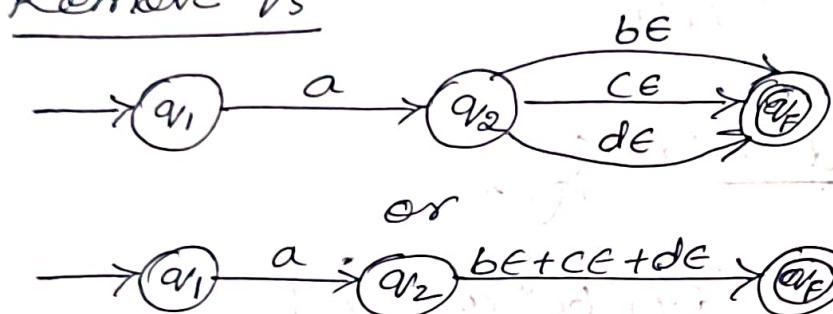
Remove  $q_3$



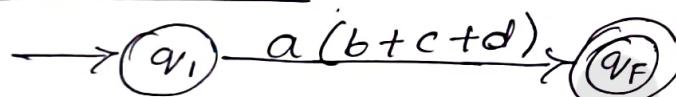
Remove  $q_4$



Remove  $q_5$

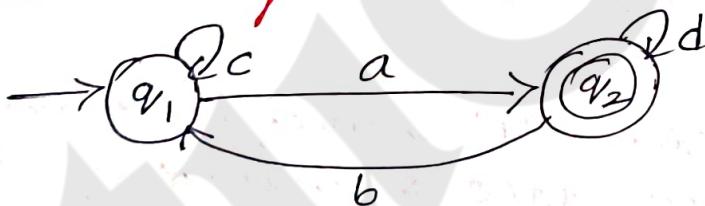


Remove  $q_2$



$$\therefore \boxed{RE = a(b+c+d)}$$

(2) obtain regular expression for the following Fsm

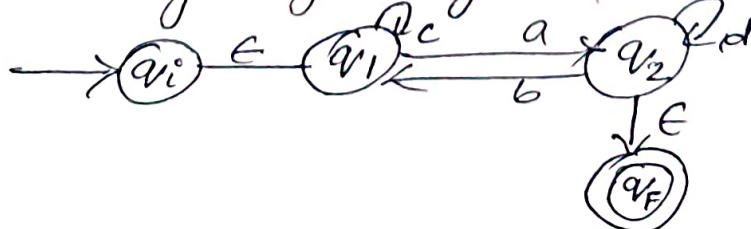


Step 1 :



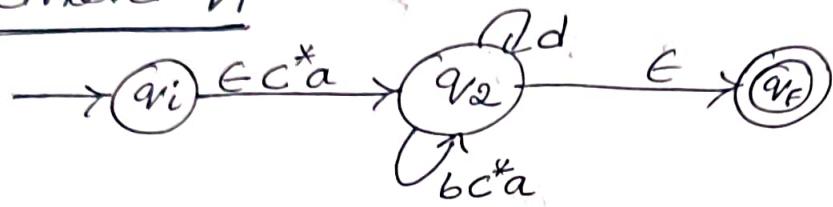
Step 2 : no multiple final state so no modification

Step 3 : outgoing edge exist for  $q_2$

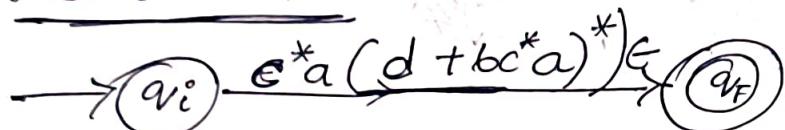


Step 4: Remove state one by one until we get one initial & one final state

Remove  $q_1$

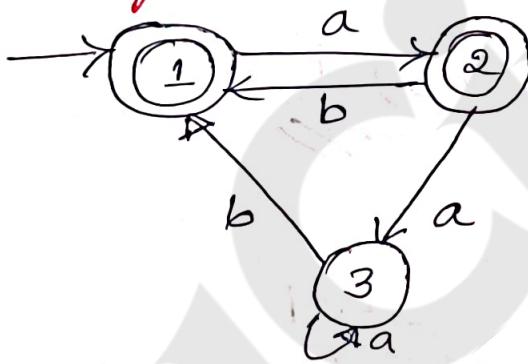


Remove  $q_2$

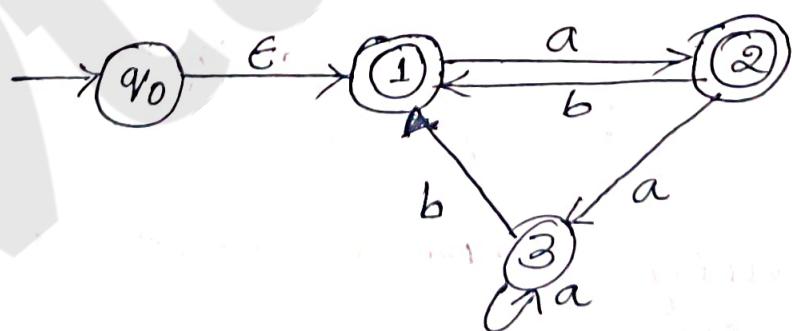


$$\therefore RE = c^*a(d + (bc^*a)^*)$$

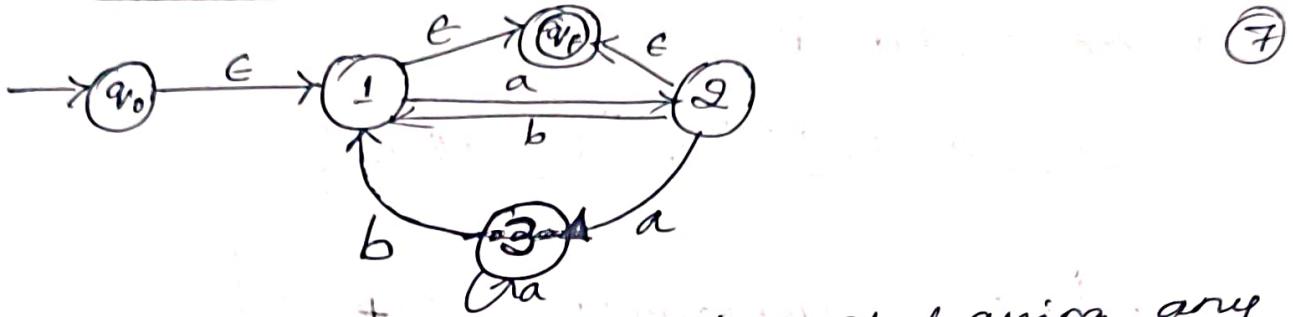
③ obtain regular expression for given psm.



Step 1: Initial state ① having incoming edge so create a new state & make it as initial state



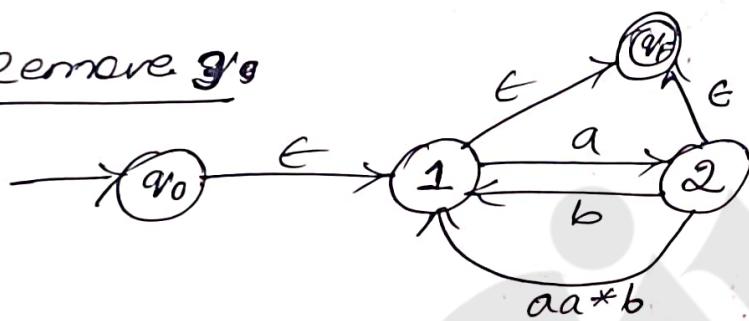
Step 2: PSM having multiple final state so create a new final state



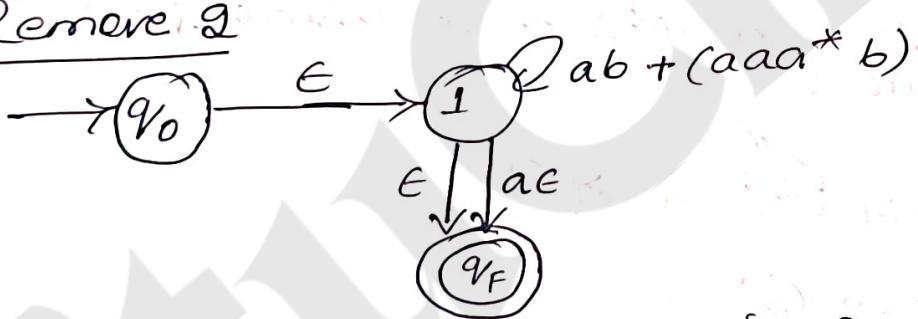
Step 3: Final state  $q_F$  is not having any outgoing edge. so no modification is required.

Step 4: Remove state one by one till we get one initial & one final state.

Remove 3



Remove 2

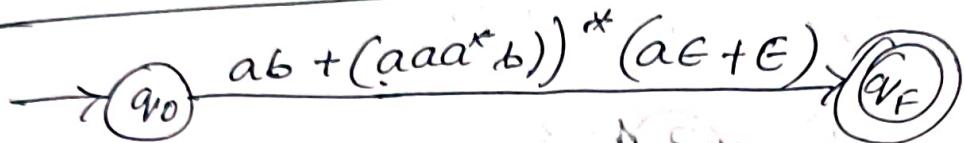


→ state ① to ① there is a loop 'ab' through state ②. since we are removing state ② mention the loop 'ab' to state ①.

→ state ① to ① there is one more loop  $a^{aa^*b}$  through ②. so mention loop  $a^{aa^*b}$  to state ①

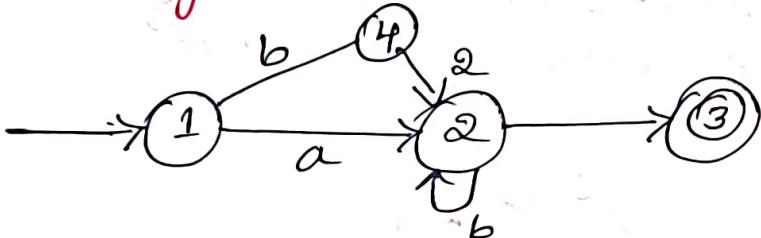
→ from ① to  $q_F$  there is a transition 'ae' through ②. since we are removing ② write transition 'ae' from ① to  $q_F$ .

Remove state ④



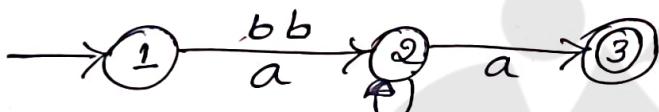
$$\therefore RE = ab + (aaa^*b)^* (a \# \epsilon)$$

(4) Build regular expression from FSM



Step 1:

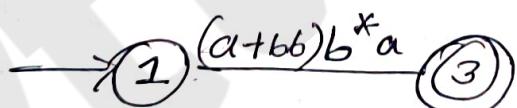
Let replace state 4



Step 2: collapse multiple transitions from state 1 to state 2



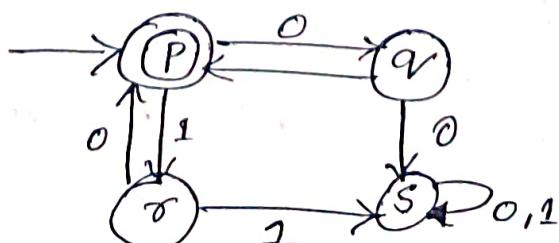
Step 3: remove ②



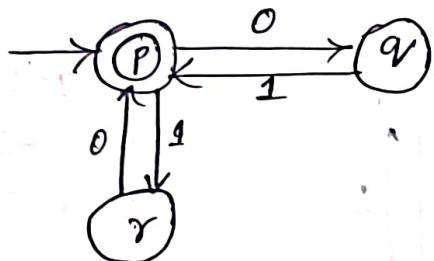
$$\therefore RE = (a+bb)b^*a$$

(5)

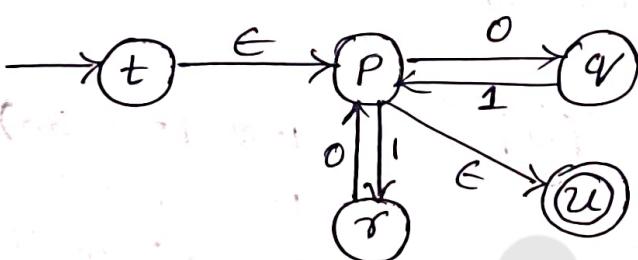
Build RE from FSM



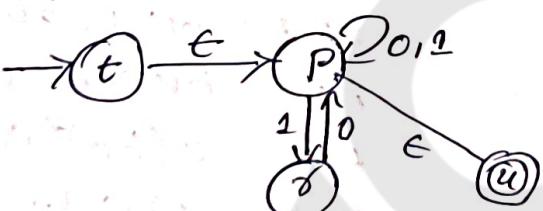
Step 1: Remove state  $s$  as its dead state. (8)



Step 2: Add new state start state  $t$  & new accept state  $u$ . After adding  $t$  and  $u$ .



Step 3: Let remove state  $q$

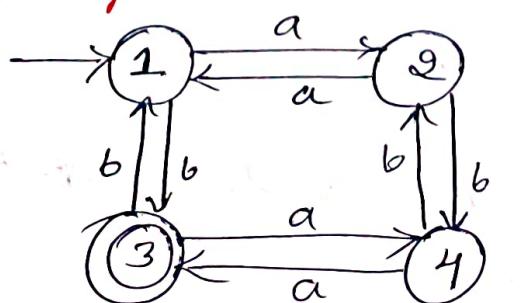


Step 4: Let remove state  $r$

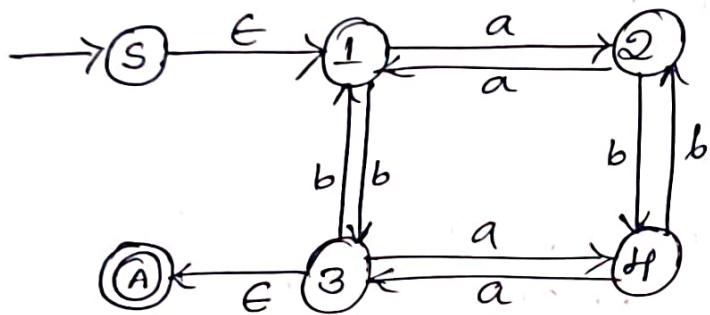


$$\therefore RE = (01 + 10)^*$$

6). Build Regular expression from Fsm

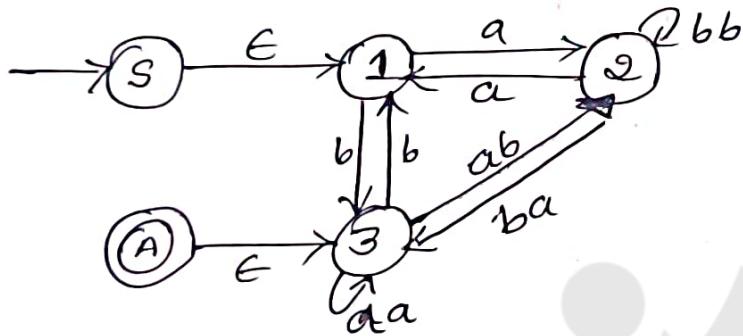


Step 1: Add new start state  $s$  and new accepting state  $A$ .



$$\begin{aligned} 2-4-2 &= bb \\ 3-4-3 &= aa \\ 2-4-3 &= ba \\ 3-4-2 &= ab \end{aligned}$$

Step 2: replace state 4



$$\begin{aligned} 1-2-1 &= a(bb)^*a \\ 1-2-3 &= a(bb)^*ba \\ 3-2-1 &= ab(bb)^*a \\ 3-2-3 &= ab(bb)^*ba \end{aligned}$$

$$\begin{aligned} RE_1 &= bua(bb)^*a \\ RE_2 &= buab(bb)^*a \\ RE_3 &= a(bb)^*a \\ RE_4 &= aa\cup ab(bb)^*ba \end{aligned}$$

Step 3: Let replace state 2

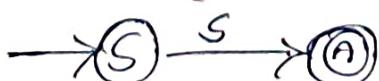


Step 4: Let replace state 3



$$\begin{aligned} RE_5 &= (RE_1)(RE_4)^* \\ RE_6 &= (RE_3) \cup (RE_1)(RE_4)^* \\ &\quad (RE_2) \end{aligned}$$

Step 5: ~~RE~~:  $(RE_6)^*$  let replace state 1



$$RE = (RE_6)^*(RE_5)$$

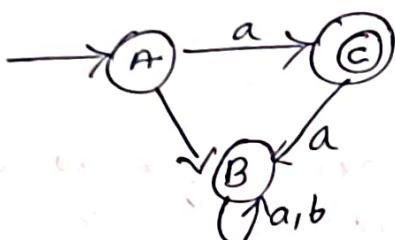
$$RE = a(bb)^*aa$$

$$RE = (RE_3) \cup (RE_1)(RE_4)^* (RE_2)^* ((RE_1)(RE_4)^*)^*$$

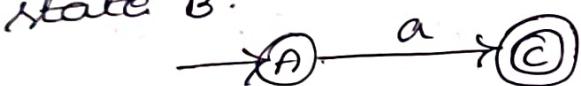
↑ regular expression value.

(9)

⑦ Find RE for following DFSA

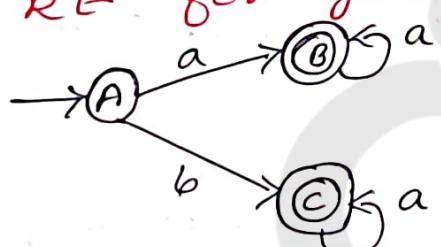


Step 1: since state B is dead state, we eliminate state B.

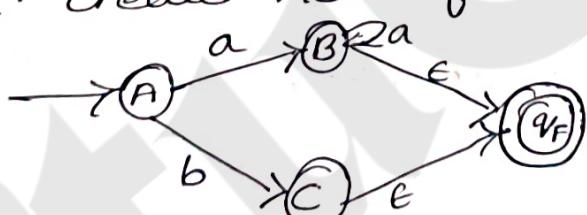


$$\boxed{RE = a}$$

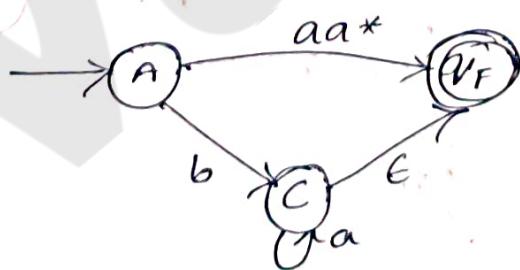
⑧ find RE for given DFSA



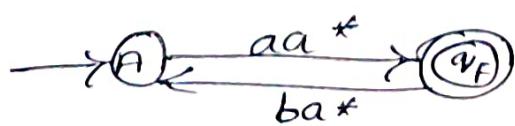
Step 1: create new final state



Step 2: Eliminate B



Step 3: Eliminate C



$$\boxed{RE = aa^* + ba^*}$$

~~update PSM~~  
conversion from Regular expression to FSM  
[using subset method]

(2)

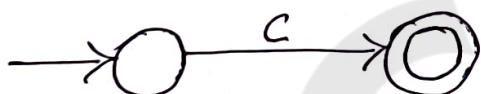
OR

Building FSM from Regular Expression

- Any language that can be defined by regular expression can be accepted by some PSM.
- i.e. According to Kleen's theorem, for every ~~some~~ regular expression, there is an equivalent PSM.

①  $\alpha = RE$  i.e.  $L(\alpha) = L(M)$

$\alpha = c \in \Sigma$ , construct simple PSM



② if  $\alpha$  is any  $\phi$ , we construct simple PSM

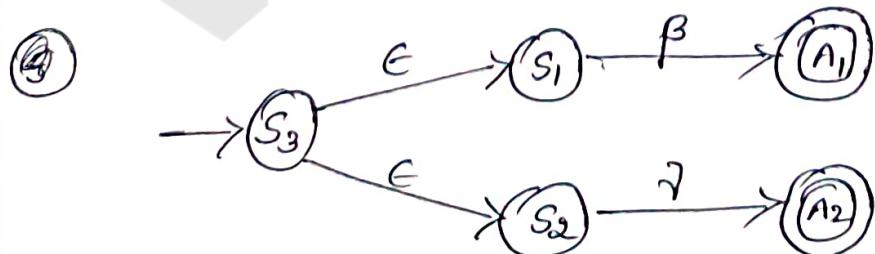


③ if  $\alpha$  is  $\epsilon$ , we construct simple PSM



④ Let  $\beta$  and  $\gamma$  two regular expression

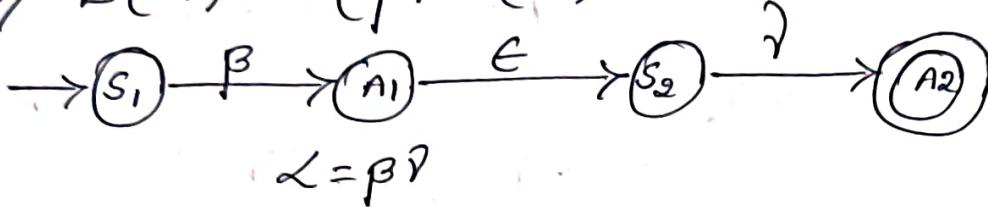
$$L(M) = L(\alpha) = L(\beta) \cup L(\gamma)$$



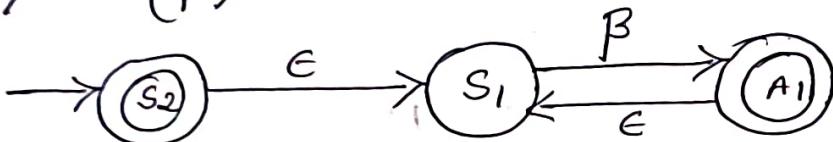
$$\boxed{\alpha = \beta \cup \gamma}$$

(10)

$$5) L(\alpha) = L(\beta) L(\gamma)$$



$$6) L(\beta)^*$$



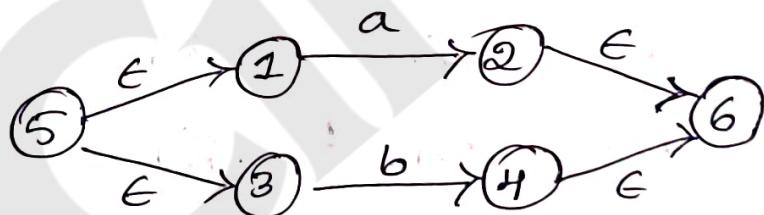
① obtain Fsm for the given regular expression

$$\underline{b(a \cup b)} \text{ or } \underline{b(a+b)}$$

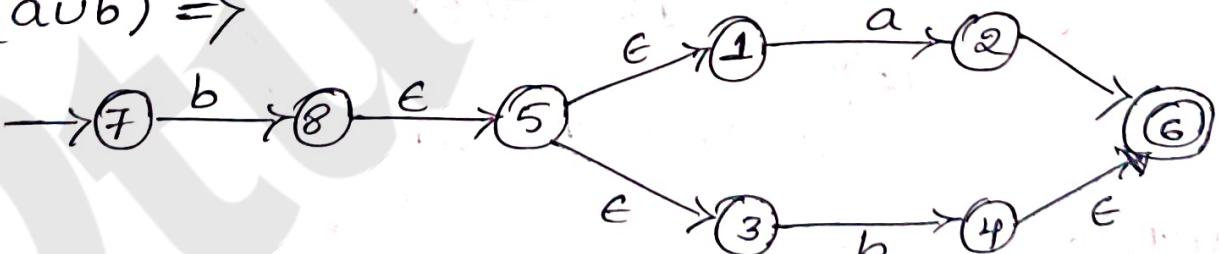
$$a = \begin{array}{c} 1 \\ \xrightarrow{a} \\ 2 \end{array}$$

$$b = \begin{array}{c} 3 \\ \xrightarrow{b} \\ 4 \end{array}$$

$$a+b/a \cup b =$$



$$b(a \cup b) \Rightarrow$$



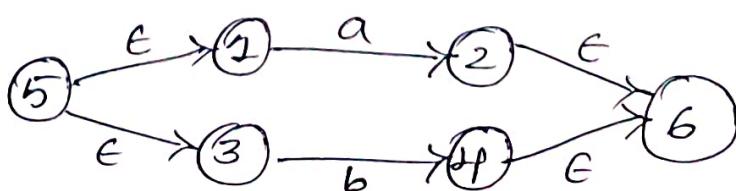
② obtain Fsm for the given regular expression

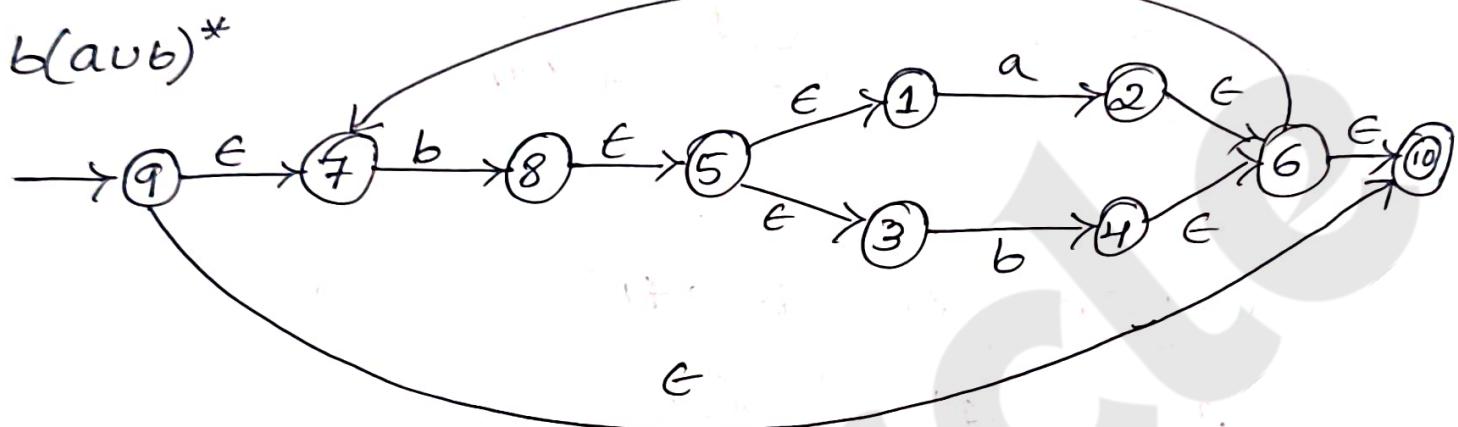
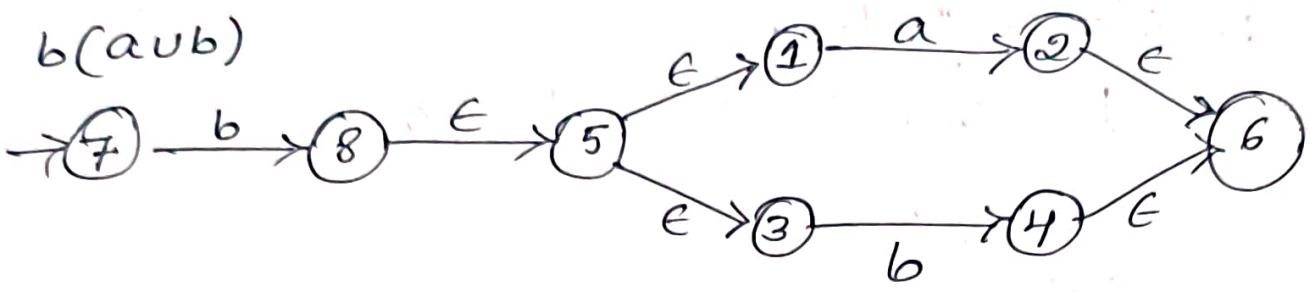
$$a = \begin{array}{c} 1 \\ \xrightarrow{a} \\ 2 \end{array}$$

$$\underline{b(a \cup b)^*}$$

$$b = \begin{array}{c} 3 \\ \xrightarrow{b} \\ 4 \end{array}$$

$$(a \cup b)$$



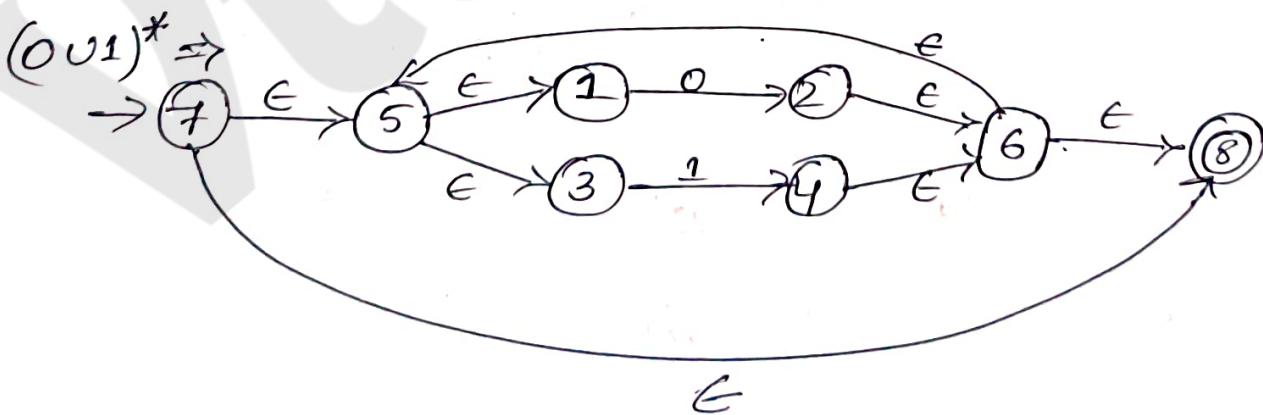
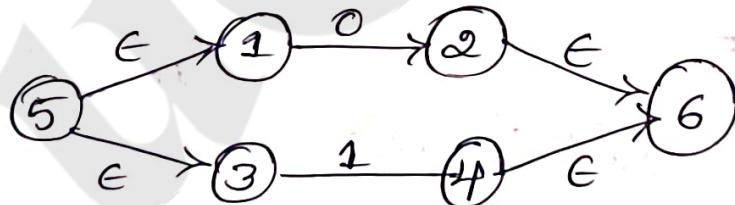


③ obtain fsm from RE  $(0 \cup 1)^*$

$$0 = \quad \textcircled{1} \xrightarrow{0} \textcircled{2}$$

$$1 = \quad \textcircled{3} \xrightarrow{1} \textcircled{4}$$

$$0 \cup 1 =$$



(4) obtain fsm from R.E  $(babua^*)$  (11)

$$a = \textcircled{1} \xrightarrow{a} \textcircled{2}$$

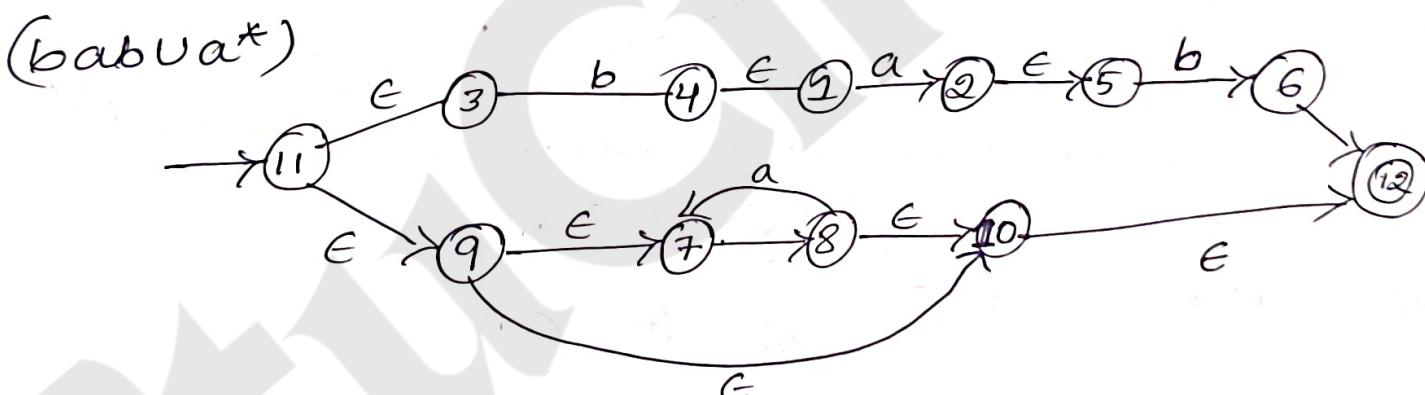
$$b = \textcircled{3} \xrightarrow{b} \textcircled{4}$$

$$b = \textcircled{5} \xrightarrow{b} \textcircled{6}$$

$$bab \quad \textcircled{3} \xrightarrow{b} \textcircled{4} \xrightarrow{\epsilon} \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{\epsilon} \textcircled{5} \xrightarrow{b} \textcircled{6}$$

$$a = \textcircled{7} \xrightarrow{a} \textcircled{8}$$

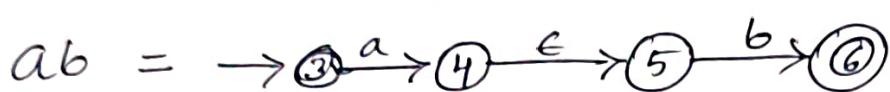
$$a^* = \textcircled{9} \xrightarrow{\epsilon} \textcircled{7} \xrightarrow{a} \textcircled{8} \xrightarrow{\epsilon} \textcircled{10}$$



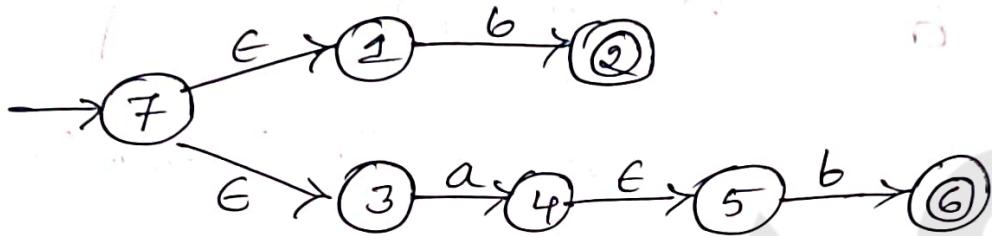
QP Question

- 1)  $10 + (0+11)0^*1$
- 2)  $(a+b)^* abb$
- 3)  $(a^*+ab)(a+b)^*$

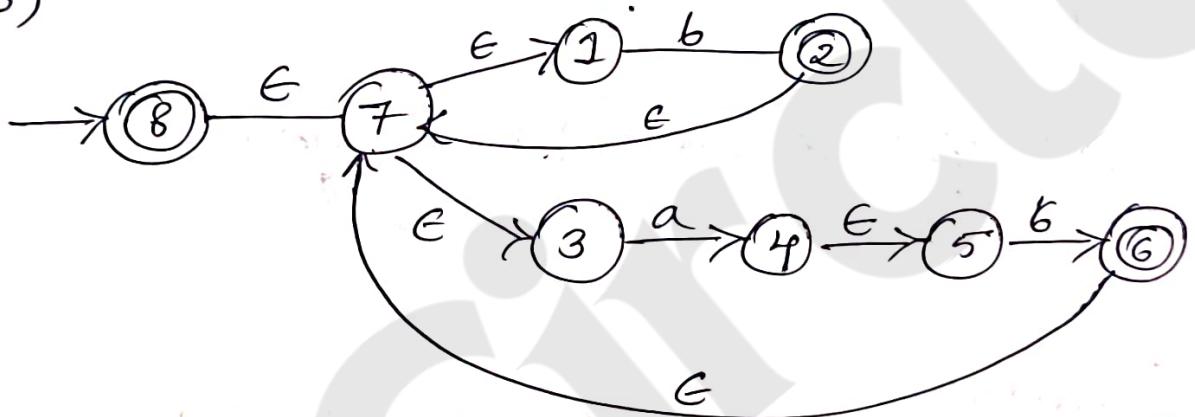
⑤ construct FSM for given RE  $(b \cup ab)^*$



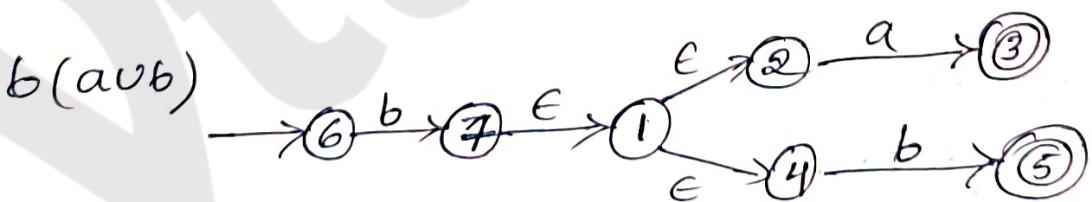
$b \cup ab =$



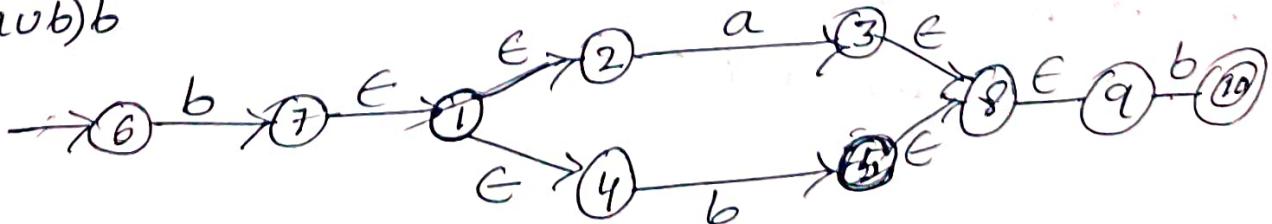
$(b \cup ab)^*$



⑥ construct FSM for given RE  $(b(a \cup b)b)^*$

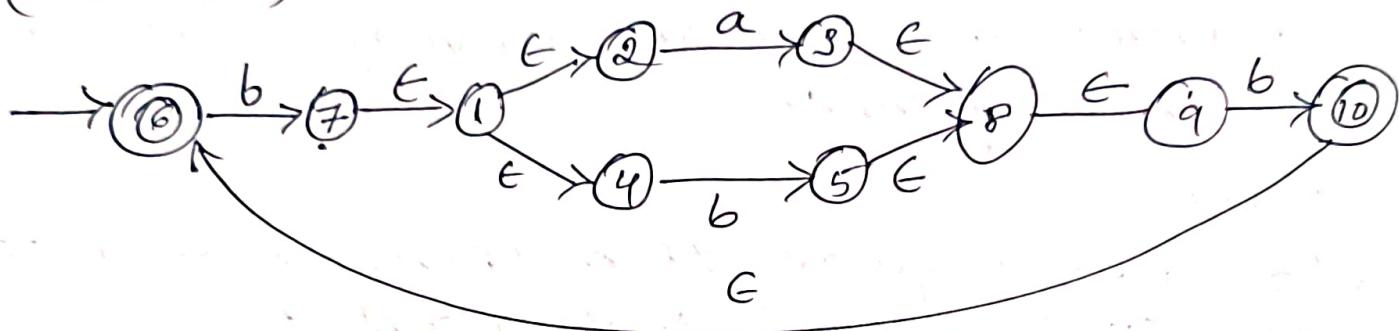


$b(a \cup b)b$



$$(b(a \cup b)b)^*$$

(12)



## Closure property of Regular Expression

Theorem: The regular languages are closed under union, concatenation and Kleen star.

Proof:

It is given that  $L_1$  and  $L_2$  are regular languages so there exists regular expression  $R_1$  and  $R_2$  such that.

$$L_1 = L(R_1); L_2 = L(R_2)$$

By the definition of regular expressions we have.

$\rightarrow R_1 + R_2$  is a regular expression denoting language  $L_1 \cup L_2$

$\rightarrow R_1 R_2$  is a regular expression denoting the language  $L_1 \cdot L_2$

$\rightarrow R_1^*$  is a regular expression denoting the language  $R_1^* = L_1^*$

So the regular languages are closed under union, concatenation and Kleen Star operation.

## Closure under complement

Theorem: The regular language are closed under complement.

### Proof:

- \* If  $L_1$  is regular, then there exist a DFSM  $M_1 = (K, \Sigma, \delta, S, A)$  that accept it.
- \* The DFSM  $M_2 = (K, \Sigma, \delta, S, A)$  namely  $M_1$  with accepting and non accepting states swapped accepts  $\sim(L(M_1))$  because it rejects all the strings that  $M_1$  accepts & accepts all strings that  $M_1$  rejects.

## Closure under Intersection

Theorem: The regular language are closed under intersection.

### Proof:

- \*  $L(M_1) \cap L(M_2) = \sim(\sim L(M_1) \cup \sim L(M_2))$
- we have already shown that all the regular language are closed under both complement and union.
- \* Thus they are closed under intersection.

## Closure under the difference

Theorem: The regular language are closed under set difference.

### Proof:

$$L(M_1) - L(M_2) = L(M_1) \cap \sim L(M_2)$$

- regular language are closed under both complement and intersection.
- thus regular language are closed under set difference.

## Closure under reverse

Theorem: The regular languages are closed under reverse.

Proof:  $L^R = \{ w \in \Sigma^* \mid w = x^R \text{ for some } x \in L \}$

Ex Let  $L = \{001, 10, 111\}$  then  $L^R = \{100, 01, 111\}$

## Closure under substitution

The regular languages are closed under letter substitution

- consider any two alphabets  $\Sigma_1$  &  $\Sigma_2$
- let  $\text{sub}$  be a function from  $\Sigma_1$  to  $\Sigma_2^*$
- then  $\text{let-sub}$  is a letter substitution from  $L_1$  to  $L_2$  iff  $\text{let-sub}(L_1) = \{w \in \Sigma_2^* : \exists y \in L_1 \text{ such that } w = y \text{ except that every character } c \text{ of } y \text{ has been replaced by } \text{sub}(c)\}$

## \* Pumping Lemma Theorem

→ Anything (Any language) that can be represented using FSM or R.E is called Regular Language

→ we use this theorem called pumping lemma to prove the language is not regular.

## Pigeonhole principle:

According to this principle long strings force repeated states

\* State and prove pumping lemma theorem for regular language.

Theorem: If  $L$  is regular language then:

$\exists K \geq 1 (\forall \text{ string } w \in L, \text{ where } |w| \geq K (\exists x, y, z (w = xyz; |xy| \leq K, y \neq \epsilon \text{ and } \forall q \geq 0 (xy^q z \in L))))$

Proof:

If  $L$  is a regular then it is accepted by some DFSM  $M = (K, \Sigma, S, S, A)$

Let  $K$  be  $|K|$

- Let  $w$  be any string in  $L$  of length  $K$  or greater
- By theorem to accept  $w$ ,  $M$  must traverse some loop atleast once.
- We can carve  $w$  up and assign the name  $y$  to the first substring to drive  $M$  through a loop.
- Then  $x$  is the part of  $w$  that precedes  $y$  &  $z$  is the part of  $w$  that follows  $y$ .
- We show that each of the last three condition must then hold:

1.  $|xy| \leq K$

$M$  must not traverse through a loop if it can read  $K-1$  character without revisiting any states. But  $K^{\text{th}}$  character will take  $M$  to a state visited before.

2.  $y \neq \epsilon$

Since  $M$  is deterministic, there are no loops traversed by  $\epsilon$

3.  $\forall q \geq 0 (xy^q z \in L)$

$y$  can be pumped out once and the resulting string must be in  $L$ .

steps to prove language is not regular (14)  
by contradiction method.

1. Assume  $L$  is regular
2. Apply pumping theorem for the given language.
3. choose a string  $w$ , where  $w \in L$  and  $|w| > k$
4. split  $w$  into  $xyz$  such that  $|xy| \leq k$  &  $y \neq \epsilon$
5. choose a value for  $q$  such that  $xy^qz$  is not in  $L$ .
6. our assumption is wrong and hence the given language is not regular.

problems on pumping lemma theorem

[showing that the language is not regular]

1)  $A^n B^n = \{a^n b^n : n \geq 0\}$  is not regular.

Assume  $L$  is regular

$$w = a^k b^k \quad |w| = k+k = 2k \quad |w| > k$$

$w$  is in language

$w$  should satisfy condition of pumping lemma there must exist some  $xyz$

$w = x, y, z$  such that

$$|xy| \leq k \quad y \neq \epsilon \quad \forall q \geq 0 \quad xy^q z \in L$$

$$w = xyz$$

since  $|xy| \leq k$   $y$  must occur within first  $k$  character

$$w = a \underbrace{\dots}_{xy} a \quad b \underbrace{\dots}_{z} b$$

$$= a \underbrace{\dots a}_{x^{k-1}} / a \underbrace{b \dots b}_{y z}$$

$$= a^{k-1} a b^k$$

According to pumping lemma  $\forall q > 0$   
 $xy^q z \in L$

$$= a^{k-1} (a)^q b^k$$

Assume  $q = 2$

$$= a^{k-1} a^2 b^k$$

$$= a^{k-1+2} b^k$$

$$= a^{k+1} b^k \notin L$$

since  $w$  has more  $a$ 's than  $b$ 's

$\therefore$  Hence the language is not regular.

② show that language  $L = \{wwr ; w \in \{a, b\}^*\}$  is not regular

if  $w = abab$

$w^r = baba$

Let  $v = ababbaba$  is a string  $\in L$

$$|v| = 8 \quad \text{let } k = 8$$

$$\therefore |v| > k$$

divide  $v$  into 3 parts  $xyz$  such that  
 $|xy| \leq k$  &  $y \neq \epsilon$  and for  $q > 0$   $xy^q z \in L$

$$v = \underbrace{ab}_{xy} \underbrace{abbaba}_z a$$

$$|xy| \leq 8 \quad \text{i.e. } |ab| \leq 8 \quad \text{i.e. } 2 \leq 8 - 1^{\text{st}} \text{ condition holds}$$

(15)

~~if~~ ~~if~~  $y \neq \epsilon$  because  $y = b$  [second condition holds]

for  $q > 0$   $xy^q z \in L$

for  $q = 0$   $xy^q z = xy^0 z$

$$= ab^0 abbaba$$

$$= aabbaba \notin L$$

$\therefore$  Hence given language is not regular

(3)

The prime number of a is not regular

Let  $L = \text{prime } a = \{a^n : n \text{ is prime}\}$

Let  $w = a^j$  assume  $j$  is smallest prime number of length  $> k+1$   $|w| \geq k+1$

$|xy| \leq k$ ,  $y \neq \epsilon$ ,  $xy^q z \in L \forall q \geq 0$

$w = a^{|x|+|y|+q|y|}$  must be in  $L$

$|x|+|y|+q|y|$  must be prime

$$q = |x| + |y| \quad q \geq 0$$

$$= |x| + |y| + |x| + |y| + q|y|$$

$$= \underbrace{|x| + |y|}_{\text{both factors are } \geq 1} (1 + q|y|) - \text{composite}$$

which is a composite (non prime) when both factors are  $\geq 1$

$$|x| + |y| \geq 1 \quad \therefore |w| = k+1 \quad |y| \leq k$$

$$1 + q|y| \geq 1$$

Hence the result string is not in  $L$

$\therefore$  Hence given language is not regular.

3) The language with more a's & b's is not regular

$$L = \{a^n b^m : n > m\}$$

$$w = a^{k+1} b^k \quad |w| = k+1+k = 2k+1 \\ |w| \geq k$$

$|xy| \leq k$ ,  $y \neq \epsilon$  such that  $xy^q z \in L$   $q \geq 0$

$$w = \underset{y}{a^{k-1}} a \underset{z}{ab^k}$$

$$= a^{k-1} a^q ab^k$$

Here if you pump up (in) number of a's will still be greater number of b's

Here we will pump out  $\neq q \geq 0$   $xy^q z \notin L$

$$q=0 \quad = a^{k-1} a^0 ab^k \\ = a^{k-1} ab^k \\ = a^k b^k \notin L$$

$\therefore$  Hence L is not regular.

4) Even palindrome is not regular

$$\text{Let } L = \text{palindrome} = \{wwr : w \in \{0,1\}^*\}$$

$$w = 0^K 1^K 1^K 0^K \quad |w| = k+k+k+k = 4k \\ \therefore |w| \geq k$$

$|xy| \leq k$ ,  $y \neq \epsilon$ , such that  $xy^q z \in L$ ,  $q \geq 0$

Since  $|xy| \leq k$  y must appear within first k character.

Let  $q=2$

$$\begin{aligned} w &= \frac{o^{k-1}}{x} \frac{a}{y} 1^k 1^k 0^k \\ &= o^{k-1} o^2 1^k 1^k 0^k \\ &= o^{k+1} 1^k 1^k 0^k \end{aligned}$$

first half of the has more length than second half.

- ∴ Hence it is not palindrome of L
- ∴ Hence it is not regular.

⑥ prove that  $L = \{a^{2n} \mid n \geq 1\}$  is not regular

$$L = \{aa, aaaa, aaaceaa, \dots\}$$

Assume  $w = aaaa$

$$|w|=4 \quad \text{let } k=4$$

$$|w| > k$$

$$|aaaa| > 4$$

$$4 > 4$$

we can divide w into 3 parts

$$w = \frac{aaa}{x} \frac{a}{y} \frac{a}{z}$$

I  $|xy| \leq k$  i.e.  $2 \leq 4$  which is true

II  $y \in$  i.e.  $y=a$  which is true

III  $a \geq 0 \quad xy^nz \in L$

$$\begin{aligned} \text{let } q=0 \quad xy^nz &= aaaa \\ &= aaa \notin L \end{aligned}$$

∴ Hence given language is not regular

⑦ prove that  $L = \{a^i b^j | i \leq j\}$  is not regular

$$L = \{aabb, abbb, aabbbb, abbbbb\}$$

assume  $L$  is regular

$$w = aabbbb$$

$$|w|=5 \text{ let } k=5$$

$$|w| \leq k$$

divide  $w$  into 3 parts  $xyz$ .

$$w = \overbrace{aa}^{x} \overbrace{a}^{y} \overbrace{bbb}^{z}$$

I  $|xy| \leq k$  i.e.  $|aa| \leq 5$  i.e.  $2 \leq 5$  true

II  $y \neq \epsilon$  here  $y=a$  which is true

III for  $q \geq 0$   $xy^q z \in L$

$$\text{take } q=0 \quad xy^0 z = aa^0 bbb \in L$$

$$q=1 \quad xy^1 z = aa^1 bbb \in L$$

$$q=2 \quad xy^2 z = aa^2 bbb \notin L$$

$$q=3 \quad xy^3 z = aa^3 bbb \\ = aaaabbb \notin L$$

$\therefore$  Hence given language is not regular