BITS Pilani K.K Birla Goa Campus

Effects of Earth, Moon And Jupiter on the eLISA Space-craft Configuration

by

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Study Project Report

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Abstract

This report describes the results obtained from a multi-body simulation of the three eLISA space-crafts under the gravitational effects of the Sun, Earth, Moon and finally Jupiter. These bodies are added one after the other, studying the effects of each on stable triangle configuration of the space-crafts. The variation of the orbits from the ideal one is quantified and optimized using the Nelder-Mead Optimization method.

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Chapter 1

Introduction

The evolved laser Interferometer space antenna is a mission proposed by the European Space Agency. It will be used to detect and measure low frequency gravitational waves in the range of 10^{-5} Hz to 1 Hz. The mission is set to be launched in 2034, but a forerunner mission, the LISA Pathfinder was launched on December 3 2015 to test the technologies that are planned for eLISA [2].

The system involves three space-crafts that follow a earth like Helio-centric orbit around the sun. They form an equilateral triangle with an arm length in the order of one million km. Long arms amplify the effects of low frequency gravitational waves. The interferometer requires the space-crafts to maintain constant distance between [1]. According to Bender *et al.* a stable configuration can be achieved between the space-crafts by launching them in a specific orbit around the sun[3]. This report focuses on how to achieve these orbits.

Chapter 2

The Orbits

The system of the space-crafts and the sun were simulated under the effects of gravity. The effects of the space-crafts on each other were ignored. The initial configuration of the space-crafts were given by Dhurandhar *et al.* [1] as follows:

- The space-crafts follow elliptical orbits around the sun with eccentricity e and semi major axis R. The value of R and e depends on the initial velocity of the space-crafts
- L is the arm length, which is one million km for eLISA
- The first space-craft is moved above the plane of rotation of the earth by $\frac{L}{2}$
- The plane of orbit of the space-craft now makes an angle

$$\epsilon = sin^{-1}\left(\frac{L}{2r}\right)$$

with the plane of orbit of the earth.

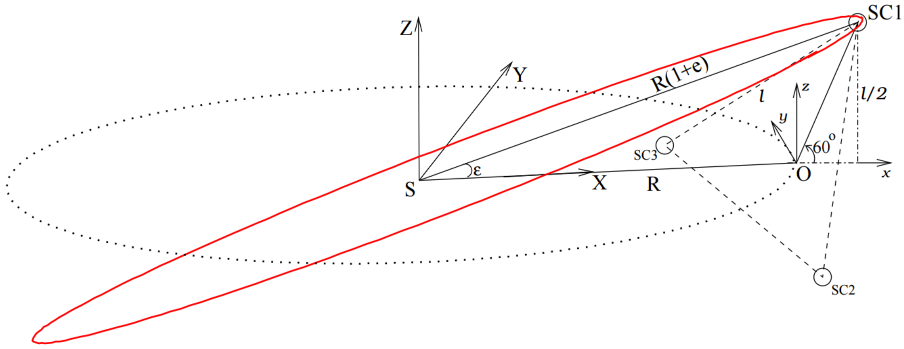

FIGURE 2.1: Configuration of the first Space-craft (Courtesy:[1])

- The orbit of the second space-craft is the same as the first, but it is rotated by 120^0 with respect to the axis perpendicular to the orbital plane of the earth, at the origin
- The orbit of the third space-craft can be determined by using the same procedure as the second space-craft, only it is rotated by 240^0
- In the simulation, the initial position of the earth is at its aphelion.
- The first space-craft is located 50 million kms from earth, L/2 above the orbital plane of earth.
- The the initial position and velocity of the second space-crafts were optimized for minimal variation of distance between the space-crafts. Details are given in the later sections.

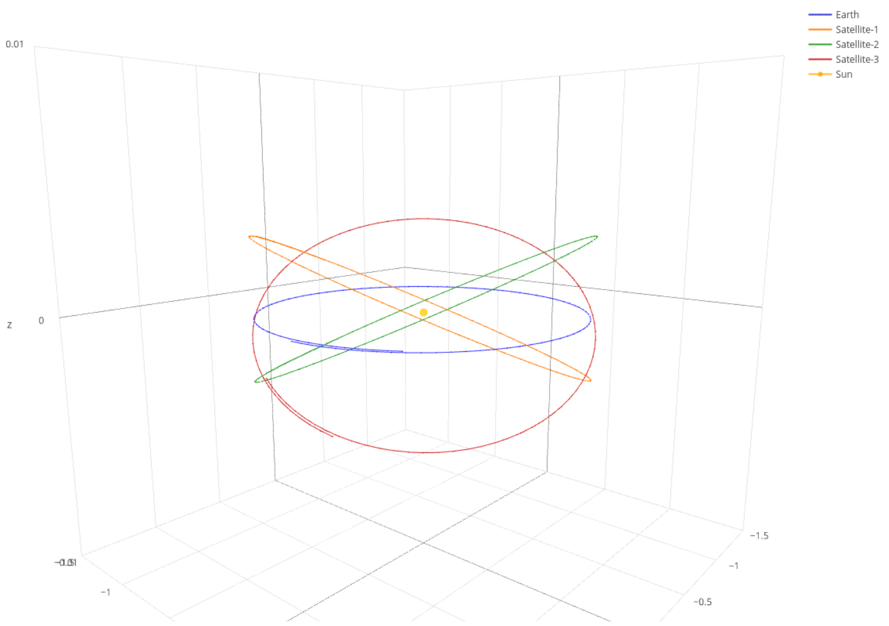

FIGURE 2.2: Orbit of the three Space-crafts (Figure generated in Python)

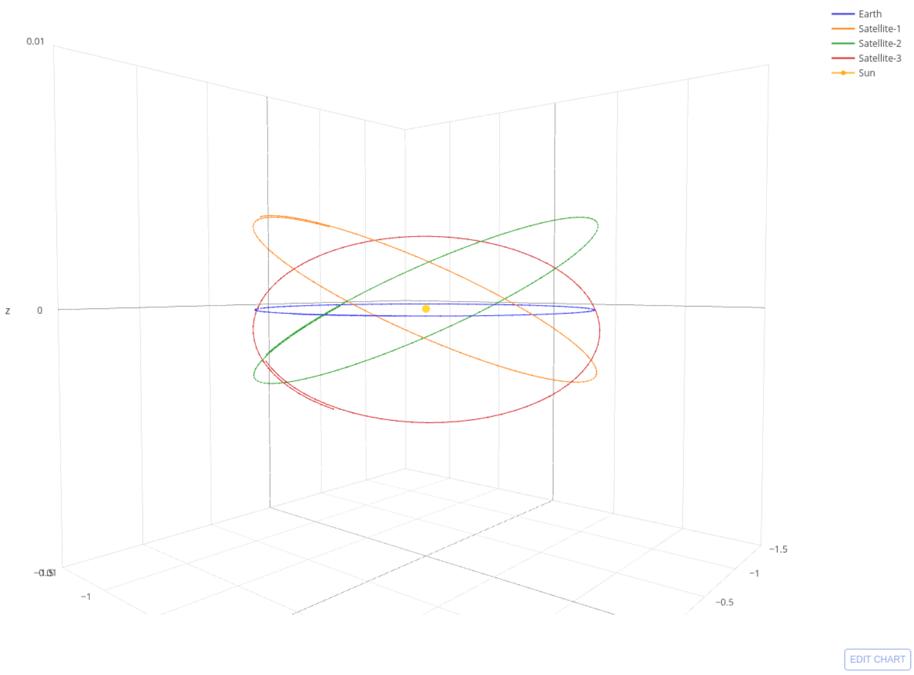

FIGURE 2.3: Orbit of the three Space-crafts (Figure generated in Python)

Chapter 3

Simulation of Orbits

To simulate the orbits of the three spacecrafts under gravity, Newton's equation of gravity was used

$$\vec{F} = G\frac{Mm}{r^2}\hat{r}$$

In **x,y** coordinates, the equations become

$$\vec{F_x} = G\frac{Mm}{r^2}cos(\theta)\hat{i} \qquad \vec{F_y} = G\frac{Mm}{r^2}sin(\theta)\hat{j}$$

From the figure given below, its clear that

$$\vec{F_x} = G\frac{Mmx}{r^3}\hat{i} \qquad \vec{F_y} = G\frac{Mmy}{r^3}\hat{j}$$

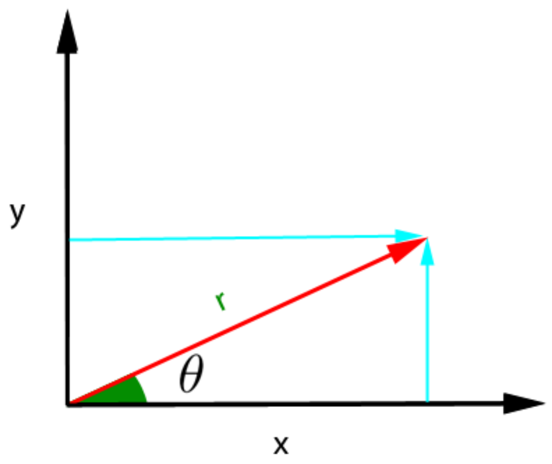

FIGURE 3.1: Splitting the Gravitational force into components

Extending it to three dimensions,

$$m\vec{a}_x = G\frac{Mmx}{r^3}\hat{i} \qquad m\vec{a}_y = G\frac{Mmy}{r^3}\hat{j} \qquad m\vec{a}_z = G\frac{Mmz}{r^3}\hat{k}$$

We get

$$a_x = G\frac{Mx}{r^3} \qquad a_y = G\frac{My}{r^3} \qquad a_z = G\frac{Mz}{r^3}$$

Thus the equations to solve are

$$\frac{dv_x}{dt} = G\frac{Mx}{r^3} \qquad \frac{dv_y}{dt} = G\frac{My}{r^3} \qquad \frac{dv_z}{dt} = G\frac{Mz}{r^3}$$

$$\frac{dx}{dt} = v_x \qquad \frac{dy}{dt} = v_y \qquad \frac{dz}{dt} = v_z$$

These equations were solved using the third order Runge-Kutta method. A time step of one minute was used.

3.1 Effects of Multiple Objects

When the effect of the Sun(M_s), Earth(M_e), Moon(M_m) and Jupiter(M_j) are considered, the equation of motion becomes,

$$\frac{dv_i}{dt} = G\left(\frac{M_s i}{r_s^3} + \frac{M_e i}{r_e^3} + \frac{M_m i}{r_m^3} + \frac{M_j i}{r_j^3}\right) \qquad i = x, y, z$$

$$\frac{dx}{dt} = v_x \qquad \frac{dy}{dt} = v_y \qquad \frac{dz}{dt} = v_z$$

Chapter 4

Optimization of Orbits

The distance between the spacecrafts must be maintained at a constant value of **L**. But the distances actually oscillate around this value. The degree of oscillation depends on the initial velocity and starting phase of the spacecrafts in their respective orbits. To measure the degree of variation of the distances from **L** over time, the following parameter was used:

$$\Phi = \sum_{t=0}^{T}((r_{12}^{t} - L)^2 + (r_{23}^{t} - L)^2 + (r_{31}^{t} - L)^2)$$

Where r_{ij}^{t} is the distance between the spacecraft **i** and **j** at time t.

For minimal variation of the distance between the orbits, the variable Φ had to be minimized. To probe the function for existence of local minimums, the plot of the above equation was plotted for all initial phases of both space-crafts.

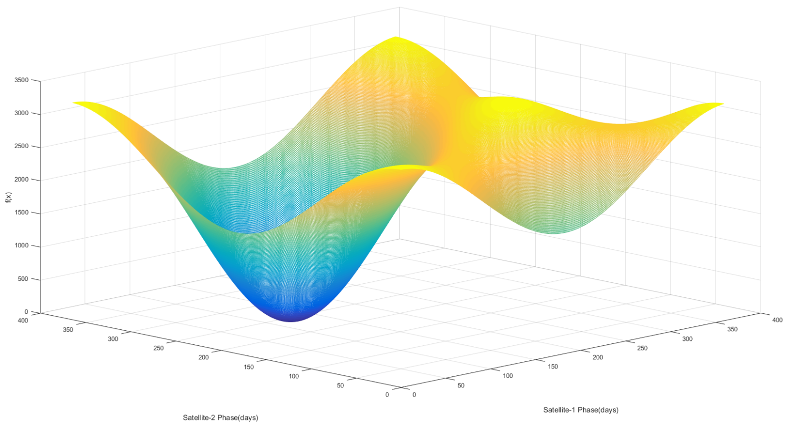

FIGURE 4.1: Value of Φ for different values of Phases (Figure Generated in MATLAB)

It was observed that the global minimum is near the point where Space-craft-two is 243 days ahead (two-thirds of a year) and and Space-craft-three is 121 days ahead of its starting locations. This fact was used in the optimization algorithm to find the global minimum.

4.1 Effects of the Sun

We will first examine the orbits of the spacecraft under the effects of the Sun. This is a two body problem as the forces between the space-crafts are almost negligible. For the simulation, the units of length, mass and time were scaled to AU, M_0 and year. This was necessary as in SI units, the length, mass and time were too large and may lead to an arithmetic overflow error.

$$1 \quad AU = 149597870700 \quad m$$

$$1 \quad M_0 = 1.98855 \times 10^{30} \quad kg$$

$$1 \quad year = 31536000 \quad sec$$

The data for aphelion position and velocity of the planets and the moon were obtained from the NASA factsheet [4]. The bodies were simulated for a year. The following is a snapshot of the orbits.

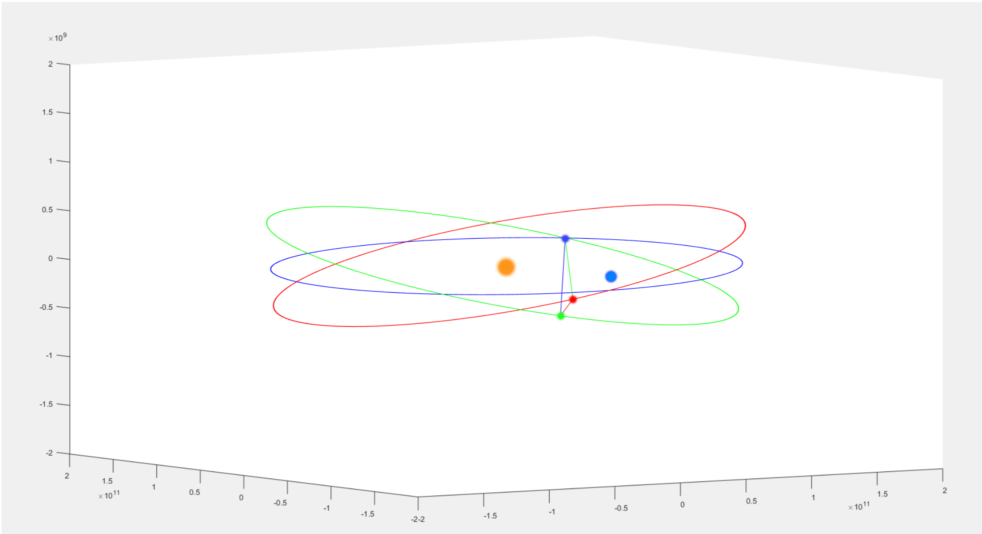

FIGURE 4.2: A snapshot of the Simulation (Figure Generated in MATLAB)

The NelderMead downhill simplex method was used to find the phases with minimal oscillations. The following is a plot of the distance between the three spacecrafts over a year. The orbits were optimized to get the minimum value of Φ. For this system $\Phi = 7.360846$.

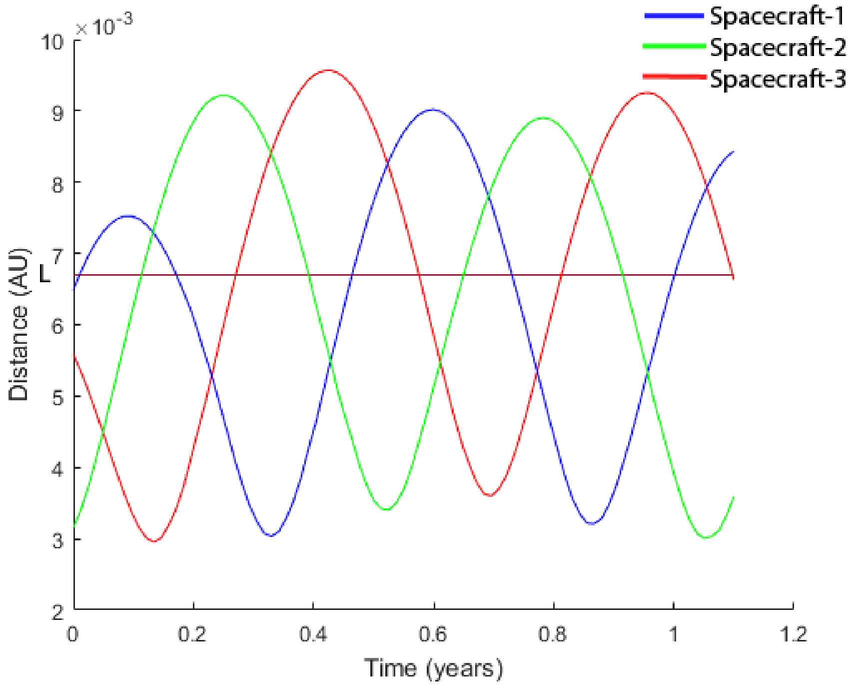

FIGURE 4.3: Distance between the Space-crafts with only the effects of the Sun

4.2 Effects of Earth

In addition to the sun, the code was extended to include the effects of earth. Now it becomes a three body problem under gravity. When the optimal phases that were obtained for the sun only case were used in this system, the following plot was obtained with $\Phi = 8.708102$.

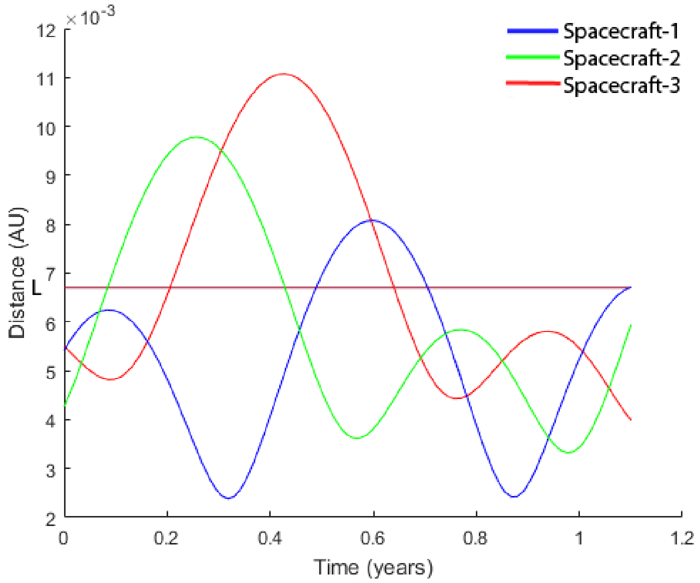

FIGURE 4.4: Distance between the Space-crafts with only the effects of the Sun & Earth

To reduce the value of Φ even further, this system was optimized again using the Nelder-Mead algorithm. With the new values of the phases, the following plot was obtained with $\Phi = 8.510521$

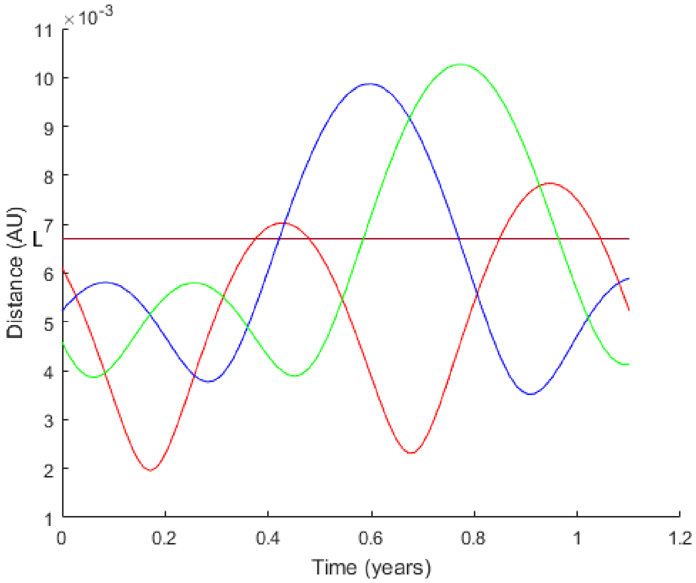

FIGURE 4.5: Distance between the Space-crafts with only the effects of the Sun & Earth with Optimized Phases

4.3 Effects of the Moon

After the earth, the effects of the moon was considered. The phases obtained form the previous result was used in this four body system. The following plot was obtained with $\Phi = 8.510474$

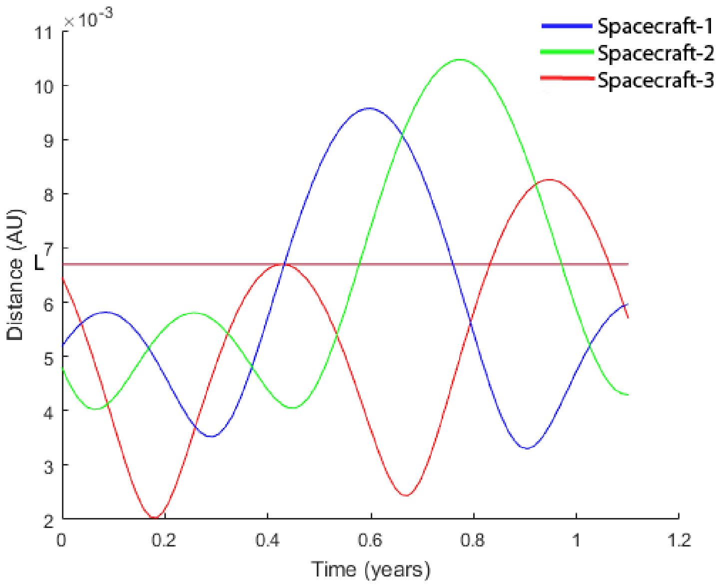

FIGURE 4.6: Distance between the Space-crafts with only the effects of the Sun, Earth & Moon

To reduce the value of Φ even further, this system was optimized again using the Nelder-Mead algorithm. With the new values of the phases, the following plot was obtained with $\Phi = 8.510244$

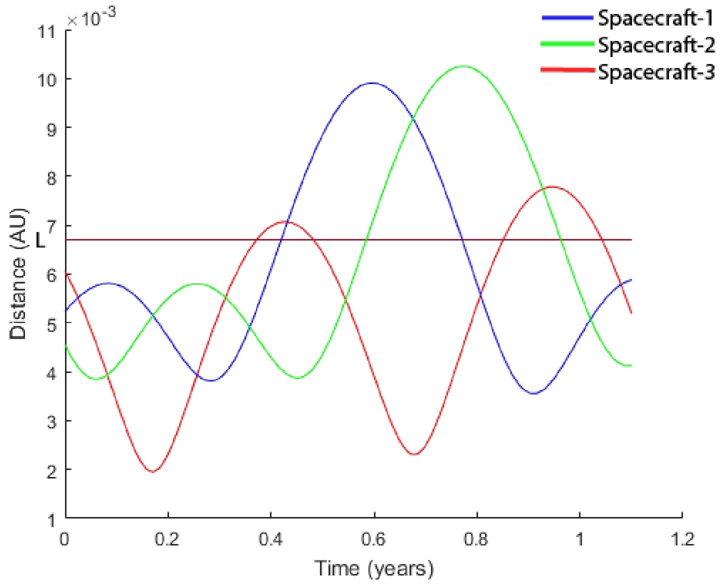

FIGURE 4.7: Distance between the Space-crafts with only the effects of the Sun, Earth & Moon with Optimized Phases

4.4 Effects of Jupiter

To add the effects of Jupiter, its position in space was needed to be located. It is known that Jupiter will reach its Aphelion on 17 Feb 2017 [5]. It is also known that the time taken by Jupiter for one revolution is 11.862 years. The eLISA mission is planned to be launched in 2034. We know that on 2028 December 28th Jupiter will again be at its Aphelion. Using this data, the Jupiter-Sun system was simulated for 5.0064932 years to find Jupiter's position and velocity in 2034.

The following plot shows the position of Jupiter in 2034.

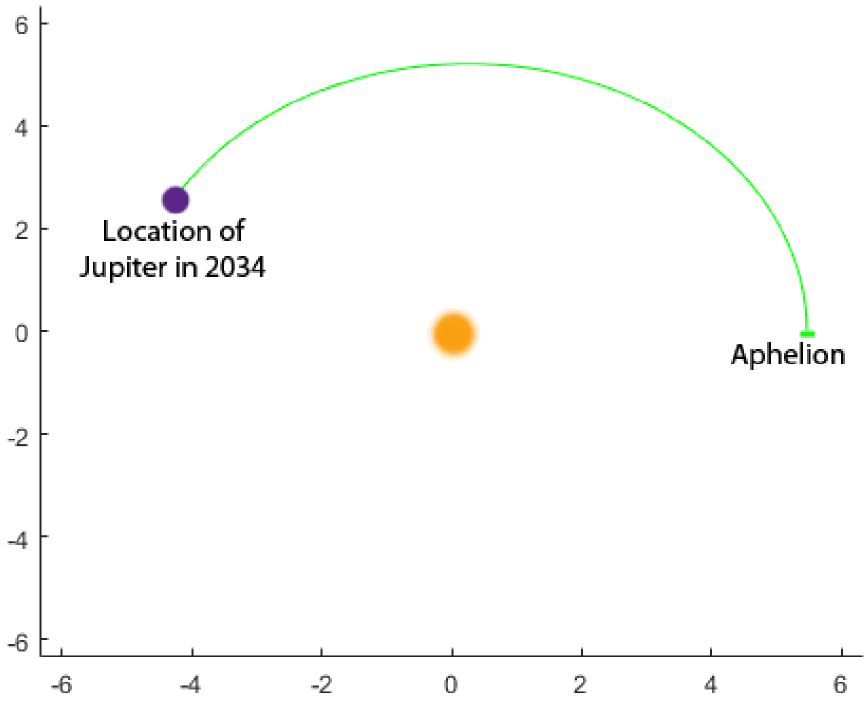

FIGURE 4.8: Position of Jupiter in 2034

Using this data, the effect of Jupiter was also included in the simulation. The following plot was obtained using the previous values of the Phases with $\Phi = 8.697866$

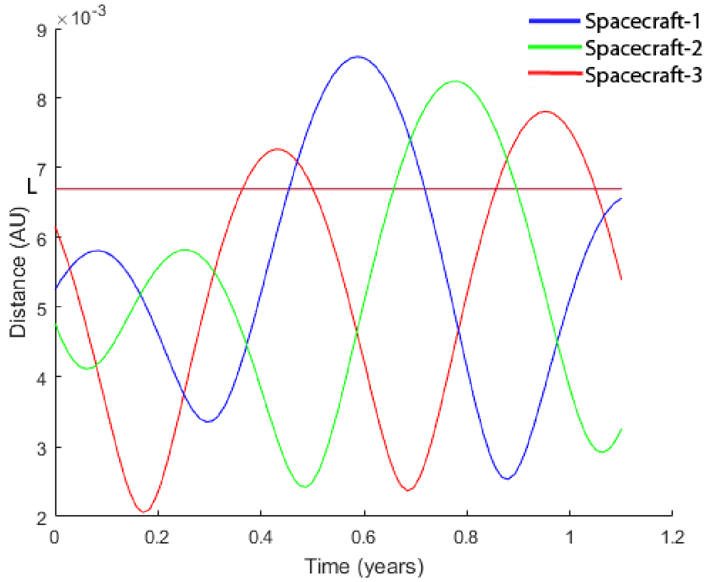

FIGURE 4.9: Distance between the Space-crafts with only the effects of the Sun, Earth, Moon & Jupiter

Finally the phases were optimized again to get the following plot with $\Phi = 7.749617$

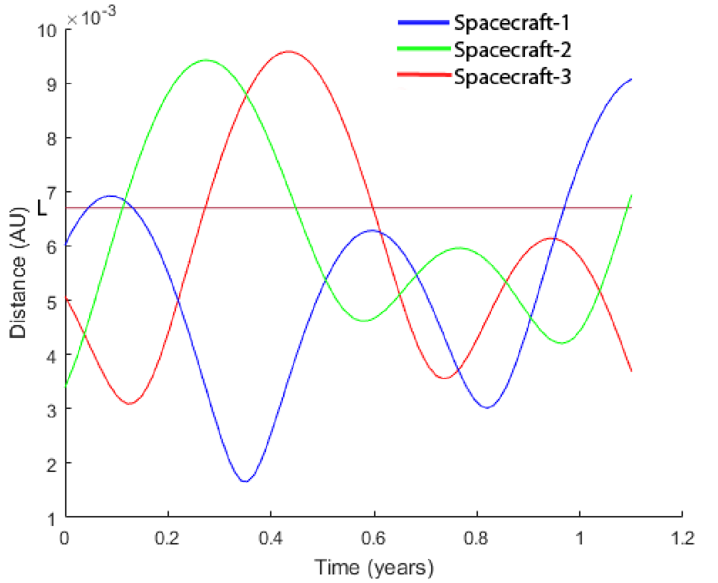

FIGURE 4.10: Distance between the Space-crafts with only the effects of the Sun, Earth, Moon & Jupiter with Optimized Phases

Chapter 5

Conclusion

It is clear from the graphs that deviation from the ideal orbits increases as more bodies are added to to simulation. At least this was the case for the Earth and Moon. However, it is indeed surprising that the deviations reduces when Jupiter is considered too. Although the deviation is higher than what was observed with the Sun only system, it is like Jupiter negates the effects of the Earth and the Moon.

This is just a hypothesis on the stability of the configuration and further research has to be conducted to investigate this.

Chapter 6

Appendix

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