Solutions for Sample Questions for Hypothesis Testing

(a) To think about whether or not age is an important factor, we can think about testing a hypothesis in this case. Specifically, we would think about the following null and alternative hypotheses:

$$H_0: \beta_1 = 0$$

$$H_a: \beta_1 \neq 0$$

Why is this particular null hypothesis useful in this case? Remember what a value of zero for β_1 would imply: an additional year of age would not change average productivity at all. If this were true, then it would suggest that your company should not pay attention to an applicant's age, because it has no bearing on their potential productivity. So let's test that idea:

Method #1: t-test approach

$$t = \frac{\hat{b}_1 - \beta_1}{se(\hat{b}_1)} = \frac{50 - 0}{100} = 0.5$$

In this case, because the magnitude of our t-statistic is less than 2, we would fail to reject the null hypothesis at the 5% level of significance. In common terms, we can't reject the notion that age has no effect on productivity.

Method #2: Confidence interval approach

The 95% confidence interval is:

$$\hat{b}_1 - 2 * se(\hat{b}_1) = 50 - 2 * 100 = -150$$

$$\hat{b}_1 + 2 * se(\hat{b}_1) = 50 + 2 * 100 = 250$$

Therefore, the 95% confidence interval ranges from –150 to 250. Since zero lies within this range, we fail to reject the null hypothesis that age has no effect on productivity.

(b) We will consider the idea that significant industry experience is valuable by approaching this in a somewhat indirect manner. Specifically, we will test the following null and alternative hypotheses:

$$H_0: \beta_2 = 0$$

$$H_a: \beta_2 \neq 0$$

What does this particular null hypothesis imply? It suggests that industry experience has no effect on productivity. But why are we doing this – that is, why are we specifying the hypothesis in this way? If the manager is hypothesizing that industry experience is important, then why test that it's *un*important? This all comes down to the philosophy of the statistical testing -- the best we can do is *reject* an idea. So what idea could we reject if we wanted to show that a variable *had* a meaningful effect? The answer is: we would reject a zero to suggest that a meaningful effect existed. So let's get to it!

Method #1: t-test approach

$$t = \frac{\hat{b}_2 - \beta_2}{se(\hat{b}_2)} = \frac{200 - 0}{40} = 5$$

In this case, because the magnitude of our t-statistic is greater than 2, we would reject the null hypothesis at the 5% level of significance. In common terms, we reject the notion that experience in the industry has no effect on productivity (or, less technically, we reject the zero effect because we think that a positive effect is evident).

Method #2: Confidence interval approach

The 95% confidence interval is:

$$\hat{b}_2 - 2 * se(\hat{b}_2) = 200 - 2 * 40 = 120$$

$$\hat{b}_2 + 2 * se(\hat{b}_2) = 200 + 2 * 40 = 280$$

Therefore, the 95% confidence interval ranges from 120 to 280. Since zero lies outside of this range, we reject the null hypothesis that experience in the industry has no effect on productivity.

(c) Following in the footsteps of part (b), we will again consider the idea that education is valuable by using an indirect approach. Specifically, we will test the following null and alternative hypotheses:

$$H_0: \beta_3 = 0$$

$$H_a: \beta_3 \neq 0$$

Again, let's think about what does this particular null hypothesis implies. It suggests that education has no effect on productivity. And, again, let's remind ourselves why are we specifying the hypothesis in this way. If we want to know whether or not increased educational requirements are important, then our best approach is to see if we can reject that a variable a zero effect of education; if we could do this, then it would suggest that a meaningful effect existed.

<u>Method #1: t-test approach</u>

$$t = \frac{\hat{b}_3 - \beta_3}{se(\hat{b}_3)} = \frac{400 - 0}{100} = 4$$

In this case, because the magnitude of our t-statistic is greater than 2, we would reject the null hypothesis at the 5% level of significance. In common terms, we reject the notion that education has no effect on productivity (or, less technically, we reject the zero effect because we think that a positive effect is evident).

<u>Method #2: Confidence interval approach</u>

The 95% confidence interval is:

$$\hat{b}_3 - 2 * se\left(\hat{b}_3\right) = 400 - 2 * 100 = 200$$

$$\hat{b}_3 + 2 * se\left(\hat{b}_3\right) = 400 + 2 * 100 = 600$$

Therefore, the 95% confidence interval ranges from 200 to 600. Since zero lies outside of this range, we reject the null hypothesis that education has no effect on productivity.

(d) In this question, we're confronted with a different type of hypothesis: that a factor has a zero effect. This is different than the cases presented in the past three sections, where it was hypothesized that a particular factor had a positive effect; in those cases, our hypothesis tests were indirect. We sought to determine if we could reject a zero null hypothesis in order to make some kind of statement about a positive effect of the variable we were studying.

But when analyzing the potential zero effect of a letter of recommendation, we can still use the same approach as we did in parts (a) through (c), except that we are now able to directly address the zero effect with the following null and alternative hypotheses:

$$H_0: \beta_4 = 0$$

$$H_a: \beta_4 \neq 0$$

The null hypothesis implies that a positive letter has no effect on productivity; our ability to reject it or fail to reject it will speak to directly to the HR manager's suggestion. Let's see what happens:

<u>Method #1: t-test approach</u>

$$t = \frac{\hat{b}_4 - \beta_4}{se(\hat{b}_4)} = \frac{150 - 0}{200} = 0.75$$

In this case, the magnitude of our t-statistic is less than 2, so we can't reject the null hypothesis at the 5% level of significance. In common terms, we can't dismiss the idea that a positive letter of recommendation has no effect on productivity.

<u>Method #2: Confidence interval approach</u>

The 95% confidence interval is:

$$\hat{b}_4 - 2 * se\left(\hat{b}_4\right) = 150 - 2 * 200 = -250$$

$$\hat{b}_4 + 2 * se\left(\hat{b}_4\right) = 150 + 2 * 200 = 550$$

Therefore, the 95% confidence interval ranges from –250 to 550. Since zero lies inside of this range, we can't reject the null hypothesis that a positive letter of recommendation has no effect on productivity.

Now, let's proceed to the second hypothesis: that a record of a past dismissal has no effect on productivity. Here, we will test the following null and alternative hypotheses:

$$H_0: \beta_5 = 0$$

$$H_a: \beta_5 \neq 0$$

The null hypothesis implies that a dismissal has no effect on productivity; if we can reject it, then suggest that a meaningful effect existed. Let's see what happens:

<u>Method #1: t-test approach</u>

$$t = \frac{\hat{b}_5 - \beta_5}{se(\hat{b}_5)} = \frac{-250 - 0}{100} = -2.5$$

In this case, the magnitude of our t-statistic is greater than 2 (remember – we don't care about the sign of the t-statistic, but only the magnitude of it), so we can reject the null hypothesis at the 5% level of significance. In common terms, we reject the idea that a prior dismissal has no effect on productivity.

<u>Method #2: Confidence interval approach</u>

The 95% confidence interval is:

$$\hat{b}_5 - 2 * se(\hat{b}_5) = -250 - 2 * 100 = -450$$

$$\hat{b}_5 + 2 * se(\hat{b}_5) = -250 + 2 * 200 = -50$$

Therefore, the 95% confidence interval ranges from –450 to –50. Since zero lies outside of this range, we reject the null hypothesis that a dismissal has no effect on productivity.