Quantitative Reasoning for Management
Sample Problem Solutions

1(a) We begin by discussing the general interpretation of each coefficient, and by recognizing that this is a multivariate regression, we interpret each coefficient as the effect of a given variable on stock price, while holding constant all other variables.

(i) To begin, the intercept is the average stock price that would prevail if every other variable – firm profits, firm debt, the industry growth rate and GDP growth rate -- were equal to zero. Under these circumstances, we'd predict that the stock price was \$3.

(ii) The coefficient on "Firm Profits" represents the change in the stock price due to a one-dollar increase in firm profits, while holding constant firm debt, industry growth rate and GDP growth rate. In this case, the effect is to increase the stock price by 0.05 dollars (or five cents).

(iii) The coefficient on "Firm Debt" represents the change in the stock price due to a one-dollar increase in firm debt, while holding constant firm profits, industry growth rate and GDP growth rate. In this case, the effect is to decrease the stock price by 0.12 dollars (or twelve cents).

(iv) The coefficient on "Industry Growth Rate" represents the change in the stock price due to a one-percentage point increase in the industry growth rate, while holding constant firm profits, firm debt, and GDP growth rate. In this case, the effect is to increase the stock price by 0.4 dollars (or forty cents).

(v) The coefficient on "GDP Growth Rate" represents the change in the stock price due to a one-percentage point increase in the GDP growth rate, while holding constant firm profits, firm debt, and industry growth rate. In this case, the effect is to increase the stock price by 10 dollars.

Now, we will discuss the hypothesis testing for each coefficient. Calculating a t-test of the null hypothesis that a coefficient is equal to zero requires the computation of a t-statistic: (Estimated Coefficient – 0)/(Standard Error of the Coefficient). Let's calculate this statistic for each of the regression coefficients.

(i) t-statistic for intercept = (3 – 0)/1 = 3
(ii) t-statistic for coefficient on firm profits = (0.05 – 0)/0.01 = 5
(iii) t-statistic for coefficient on firm debt = (–0.12 – 0)/0.02 = –6
(iv) t-statistic for coefficient on industry growth rate = (0.4 – 0)/0.04 = 10
(v) t-statistic for coefficient on GDP growth rate = (10 – 0)/2 = 5

In order to test the null hypothesis that each coefficient is equal to zero, we compare the absolute value of these statistics to 2 (since, as we recall, we reject the null hypothesis if it is outside of the 95% confidence interval, which is two standard deviations away from the mean). In all of the cases (i) to (v), the absolute value of the t-statistic is larger than two. This indicates that for each case, we can reject the null hypothesis that the coefficient is equal to zero at the 5% level of significance.

(b) In this case, we've received information about all of the variables in our regression. As such, we can use the information to form a prediction about the stock price by determining the conditional mean with the values we've been provided. This requires substituting the values of each variable into our regression:

$$(Stock\ Price) = \beta_0 + \beta_1(Firm\ Profits) + \beta_2(Firm\ Debt) + \beta_3(Industry\ Growth\ Rate) + \beta_4(GDP\ Growth\ Rate)$$

$$\rightarrow (Stock\ Price) = 3 + 0.05(Firm\ Profits) - 0.12(Firm\ Debt) + 0.4(Industry\ Growth\ Rate) + 10(GDP\ Growth\ Rate)$$

$$\rightarrow (Stock\ Price) = 3 + 0.05(10000) - 0.12(4000) + 0.4(5) + 10(2)$$

$$\rightarrow (Stock\ Price) = 3 + 500 - 480 + 2 + 20$$

$$\rightarrow (Stock\ Price) = 45$$

Therefore, we would predict that the stock price is $45.

In addition, of course, we need to recognize that this is guess , and there is an inherent uncertainty about this guess. To describe this uncertainty, we use a 95% confidence interval around our mean, which is calculated as two standard errors of the regression below our conditional mean estimate, and two standard errors of the regression above our conditional mean:

(i) The lower part of the confidence interval is: 45 – 2*(1.5) = 42
(ii) The upper part of the confidence interval is: 45 + 2*(1.5) = 48

Overall, then we would say that our best estimate of the stock price is $45, but to be more careful, we would also say that we are 95% certain that the "true" population stock price under these circumstances would be between $42 and $48 with 95% certainty.

(c) Let's begin answering this question by determining the new prediction of our stock price, given the new values of the variables in our regression:

$$(Stock\ Price) = 3 + 0.05(Firm\ Profits) - 0.12(Firm\ Debt) + 0.4(Industry\ Growth\ Rate) + 10(GDP\ Growth\ Rate)$$

$$\rightarrow (Stock\ Price) = 3 + 0.05(10500) - 0.12(4000) + 0.4(5) + 10(2)$$

$$\rightarrow (Stock\ Price) = 3 + 525 - 480 + 2 + 20$$

$$\rightarrow (Stock\ Price) = 70$$

Therefore, we would predict that the stock price would be $70, given this new information.

Now comes the somewhat more challenging part: can we test whether or not this new information would give us a predicted price that is significantly different than $45? Let's recall what we said about hypothesis testing: we want to think about whether or not our hypothesis is inside or outside of our confidence interval. So let's figure out what the confidence interval would be in this case. Remember that when we calculate conditional means using our regression formula, we can create a confidence interval around this conditional mean using the standard error of the regression.

How does that work? Remember that we can use the standard error of the regression as the equivalent to a standard deviation around a mean. That is, we can calculate a confidence interval by determining the values two standard errors of the regression above and below our conditional mean:

(iii) The lower part of the confidence interval is: 70 – 2*(1.5) = 67
(iv) The upper part of the confidence interval is: 70 + 2*(1.5) = 73

Now, we can ask the question: is our predicted stock price of $70 different from our predicted price of $45 in part (a)? Well, there are many ways to test this idea, but here's one: let's set up a null hypothesis that the stock price in part (b) is $45 (the price we calculated in part (a)):

H_0: The predicted stock price in part (b) is equal to $45

H_a: The predicted stock price in part (b) is not equal to $45

To test this hypothesis, we would ask ourselves: is the conditional mean of the null hypothesis inside of our confidence interval? Clearly, the answer is no. The confidence interval in this case is between $67 and $73, and the price of the null hypothesis ($45) is not within this confidence interval. Therefore we can reject the null hypothesis at the 5% level of significance, and this means that we are willing to say that the stock price we have predicted here in part (b) is different than the price in part (a), at the 5% level of significance.

2(a) We interpret the coefficient on tax rates in the standard way for a bivariate regression: it represents the change in profits due to a one-percentage-point increase in tax rates. In this case, we see that the coefficient suggests that a one-percentage-point increase in tax rates increases firm profits by \$20,000.

In order to answer the question of whether or not these results suggest that tax rates have no effect on profits in the population of the firm's factories, we need to re-cast this question as a formal hypothesis test. And we can do that as follows:

$$H_0: \beta_1 = 0$$

$$H_a: \beta_1 \neq 0$$

To test this hypothesis, we can either use a confidence interval around the estimated regression coefficient, or we can calculate a test statistic. I'll do the latter, but remind you that it's perfectly acceptable to do the former, if you prefer. Here's the test statistic:

$$test\ statistic = \frac{\hat{b}_1 - \hat{\beta}_1}{se(\hat{b}_1)} = \frac{20 - 0}{8} = 2.5$$

Since the test statistic is larger, in magnitude, than 2, then we reject the null hypothesis at the 5% level of significance. Therefore, we reject the hypothesis that there is no effect of tax rates on profits.

(b) The predicted profitability of a factory located in a country where the tax rate is 20% is determined by calculating the conditional mean of profitability when the tax rate is 20%:

$$Profits = 80 + 20(Tax\ rates) = 80 + 20(20) = 480$$

Therefore, our best estimate of this factory is \$480,000.

Our confidence interval can be calculated by determine the values that are two standard errors of the regression (SER, for brevity) above and below this conditional mean. Since our regression output indicates that the standard error of the regression is 100, then our calculations are straightforward – the confidence interval is:

$$Conditional\ mean + 2 * SER = 480 + 2(100) = 680$$

and:

$$Conditional\ mean - 2 * SER = 480 - 2(100) = 280$$

Therefore, this confidence interval is: \$280,000 to \$680,000.

(c)(i) The interpretation of the coefficients on tax rates is: it represents the effect of a one-percent increase in tax rates on the factory's profits, in thousands of dollars, while holding constant the workforce's education level.

(c)(ii) The predicted profitability of a factory located in a country where the tax rate is 30% and the workforce has an average level of education equal to 16 is determined by calculating the conditional mean of profitability for these two values. We use the coefficients estimated for our new regression model to determine this value:

$$Profits = 80 - 2(Tax\ rates) + 10(Education) = 80 - 2(30) + 10(16) = 180$$

Therefore, our best estimate of this factory is $180,000.

Our confidence interval can be calculated by determine the values that are two standard errors of the regression above and below this conditional mean. Since our regression output indicates that the standard error of the regression is 60, then our calculations are straightforward – the confidence interval is:

$$Conditional\ mean + 2 * SER = 180 + 2(60) = 300$$

and:

$$Conditional\ mean - 2 * SER = 180 - 2(60) = 60$$

Therefore, this confidence interval is: $60,000 to $300,000.

(d) Let's begin by determining the conditional mean estimate of the profitability of a factory in Alphatown:

$$Profits = 80 - 2(Tax\ rates) + 10(Education) = 80 - 2(20) + 10(12) = 160$$

Therefore, our best estimate of this factory is $160,000.

Our confidence interval can be calculated by determine the values that are two standard errors of the regression above and below this conditional mean. Since our regression output indicates that the standard error of the regression is 60, then our calculations are straightforward – the confidence interval is:

$$Conditional\ mean + 2 * SER = 160 + 2(60) = 280$$

and:

$$Conditional\ mean - 2 * SER = 160 - 2(60) = 40$$

Therefore, this confidence interval is: $40,000 to $280,000.

In contrast, the firm could also invest in a "riskless" government treasury bill to net a guaranteed profit of $100,000. By asking our advice on the best use of the new funds, the firm is (essentially) asking whether or not the profits on the government treasury bills are different than the profits on the new factory. We see, however, that the treasury bill profits are inside of the confidence interval around our estimated conditional mean of $160,000; formally, we would say that we can't reject the hypothesis that the two options would generate the same profits. Beyond this, we might also use plain language to tell the firm that although we could say with 95% certainty that the new plant may generate a profit of as high as $280,000, it may also run as low as $40,000. If the risk inherent in this variability in our estimate appeals to the firm, then they may prefer the new factory. But if the firm was more interested divesting itself of risk, then the government bond – although it represents a lower average profit – may be the superior option due to its riskless return.