Quantitative Reasoning for Management

Sample Problem Solutions

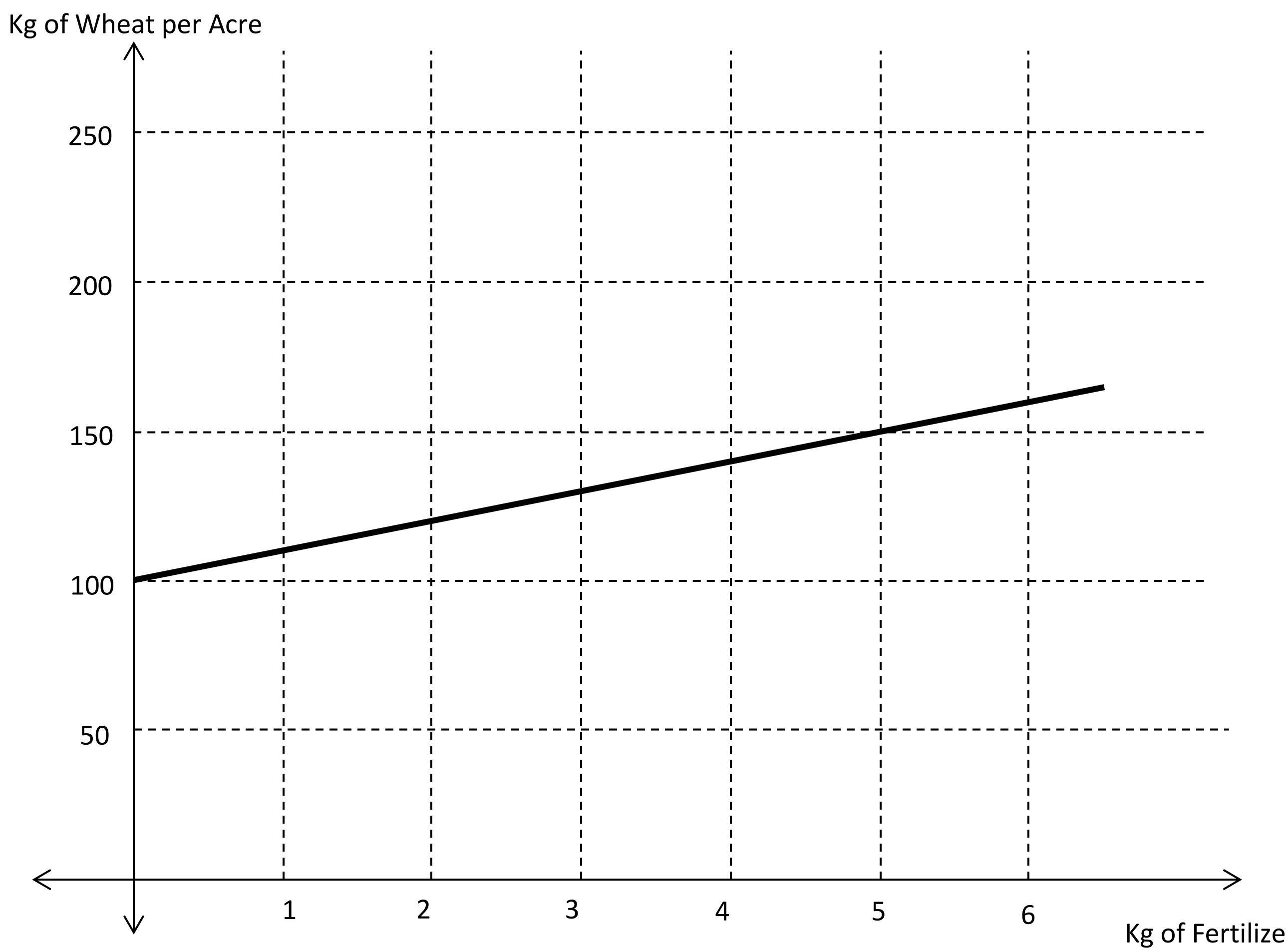
1(a) A graphical representation of regression can be determined by focusing on two things. First, we know that a regression has a general format:

$$Y = b_0 + b_1 X$$

In this case, b_0 represents the vertical intercept of the regression line, and the coefficient b_1 is the slope of the regression line. The regression line in this question is:

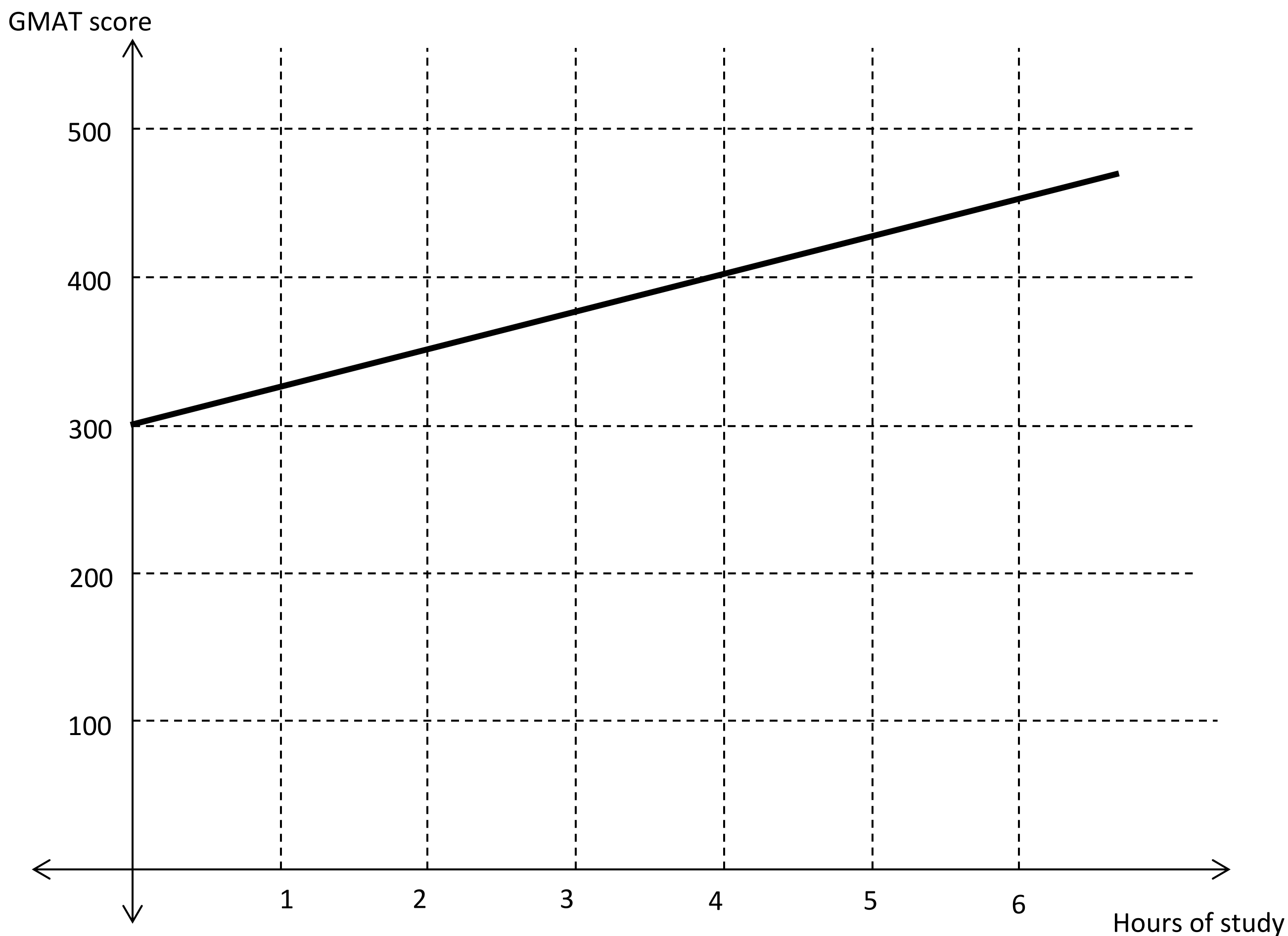
(Kilograms of Wheat Grown per Acre) = 100 + 10(Kilograms of Fertilizer per Acre)

So the vertical intercept is 100, and the slope is 10. In order to draw the appropriate line, it needs to intersect the vertical intercept at 100, and then increase by 10 units for every one-unit increase in our X variable. Or, in the case of our regression, there is a 10-Kg increase in wheat grown for every extra kilogram of fertilizer applied per acre.



To interpret the coefficients, we recognize that the intercept is 100. This means that when there is no fertilizer applied to the field, the average amount of wheat per acre grown is 100. If the slope term is 10, then we recognize that the slope represents the change in Kg of wheat grown (the "Y" variable) because of a one-unit increase in fertilizer (the "X" variable). Therefore, a one-Kg increase in fertilizer per acre will result in a 10-Kg increase in wheat per acre. This is represented on the graph by the fact that when fertilizer increases from 0 to 5, we can see that wheat production increases from 100 to 150 – that is, a 5-Kg increase in fertilizer is related to a 50-Kg increase in wheat production.

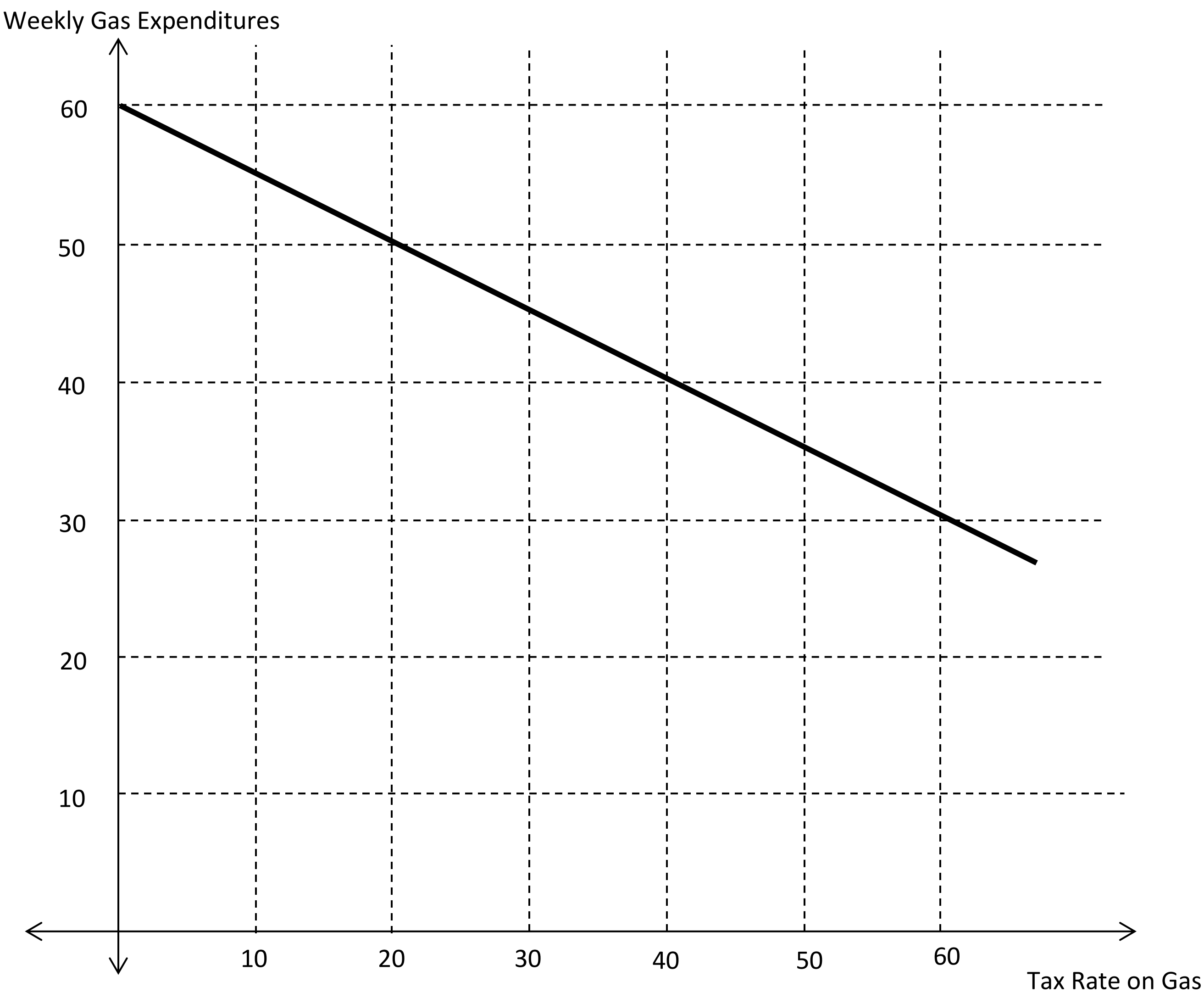
(b) In this regression, we see that the vertical intercept is 300, and the slope is 25. As such, we need to draw a regression line with a vertical intercept at 300, and then increase by 25 units for every one-unit increase in our X variable (or, equivalently, increase by 100 units for every four-unit increase in our X variable). This line is drawn below:



To interpret the coefficients, we recognize that the intercept is 300. This means that when a GMAT test-taker doesn't study at all, they get a score of 300 on average. If the slope term is 25, then we recognize that the slope represents the change in GMAT score (the "Y" variable) for every additional hour of study (the "X" variable). Therefore, a one-hour increase in study time will result in a 25-point increase in GMAT scores. This is represented on the graph by the fact that when hours of study increase from 0 to

4, we can see that the GMAT score increases from 300 to 400 – that is, a four-hour increase in study time is related to a 100-unit increase in GMAT scores.

(c) In this regression, we see that the vertical intercept is 60, and the slope is –0.5. As such, we need to draw a regression line with a vertical intercept at 60, and then decrease by 0.5 units for every one-unit increase in our X variable (or, equivalently, decrease by 5 units for every ten-unit increase in our X variable). This line is drawn below:



To interpret the coefficients, we recognize that the intercept is 60. This means that when taxes on gas are set at zero, then drivers spend on average 60 dollars on gas. If the slope term is –0.5, then we recognize that the slope represents the change in gas expenditures (the "Y" variable) for every one-percentage point increase in the tax rate on gas (the "X" variable). Therefore, a one-percentage-point increase in the tax rate on gas will result in 50-cent decrease in weekly gas expenditures. This is represented on the graph by the fact that when hours of study increase from 0 to 20, we can see that gas expenditures decrease from 60 to 50 – that is, a ten-percentage-point increase in the tax rate is related to a ten-dollar decrease in gas expenditures.