opg II-4.

opg 1

$D(g_1)D(g_2) = D(g_1 g_2)$ nødvendig

$(D(g_1 g_2))^T = (D(g_1)D(g_2))^T = D(g_2)^T D(g_1)^T$

Derfor er $D(g_1)^T$ ikke en repr. af G hvis der eksisterer $g_1, g_2 \in G$:

$D(g_1)D(g_2) \neq D(g_2)D(g_1) \quad (\Rightarrow D(g_2)^T D(g_1)^T \neq D(g_1)^T D(g_2)^T)$

opg 2

$g_1 \sim g_2 \Rightarrow g_1^2 \sim g_2^2$:

$g_1 = f g_2 f^{-1} \Rightarrow g_1^2 = (f g_2 f^{-1})(f g_2 f^{-1}) = f g_2^2 f^{-1}$

opg 3

Den definerende repr. ((2x2) matricer) er givet p. 125 (18). $D(g)$ er reel, så derfor er $D^*(g) = D(g)$ og derfor reel ($S = \hat{I}$ kan bruges, så den er ikke pseudo-reel).

opg II.4 (opg 3 fortsat).

Men efterb $\frac{1}{N_G}\sum_{g\in G}\chi^{(r)}(g^2) = 1$, $r=2$

$\chi^{(2)}(g)$ er givet (19) p. 127.

$\chi^{(2)}(I) = 2$, $\chi^{(2)}(-I) = -2$, resten 0.

$$D_4 = \{I, R, \underbrace{R^2}_{=-I}, R^3, \underbrace{r}_{=r_y}, \underbrace{rR}_{=d_2}, \underbrace{rR^2}_{=r_x}, \underbrace{rR^3}_{d_1}\}$$

$rR = Rr^{-1}$, $r^2 = I$, $R^4 = I$.

$I^2 = I$, $R^2 = -I$, $(R^2)^2 = (-I)^2 = I$, $(R^3)^2 = R^6 = R^2 = -I$

$r_x^2 = I$, $r_y^2 = I$, $d_1^2 = d_2^2 = I$.

$$\sum_{g\in G}\chi^{(2)}(g^2) = 6\cdot\chi^{(2)}(I) + 2\,\chi^{(2)}(-I) = 8 = N(D_4)$$

Opg II.4 — opg 4

A_4: karakter-tabellen for A_4:
(15) p. 121.

$\chi^{(3)}(I) = 3$, $\chi^{(3)}((ab)(cd)) = -1$, $\chi^{(3)}((abc)) = 0$

$I^2 = I$, $[(ab)(cd)][(ab)(cd)] = (ab)^2(cd)^2 = I$

$(abc)^2 = (acb)$,

$A_4 = \{I, (ab)(cd), (abc)\}$
$\qquad 1, \quad 3, \quad 8$

$\sum_g \chi^{(3)}(g^2) = 4 \cdot 3 = 12 = N(A_4)$.

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opg 5

karakter tabel for S_4: (17) p. 125

$\chi^{(3)}(I) = 3$, $\chi^{(3)}((ab)(cd)) = -1$, $\chi^{(3)}((abc)) = 0$
$\chi^{(3)}((ab)) = 1$, $\chi^{(3)}((abcd)) = -1$

$I^2 = I$, $((ab)(cd))^2 = I$, $(abc)^2 = (acb)$
$(ab)^2 = I$ $\quad (abcd)^2 = (ac)(bd)$

$\sum_g$ ~~$\chi^{(3)}$~~ $=$

Opg II.4   opg 5 fortsat

S_4

$$\begin{array}{ccccc} I, & (ab)(cd), & (abc) & (ab) & (abcd) \\ 1 & 3 & 8 & 6 & 6 \end{array}$$

$$\sum_{g \in S_4} \chi^{(3)}(g^2) = (1+3+6)\,\chi^{(3)}(I) + 6\,\chi^{(3)}((abc)) + 6\,\chi^{(3)}(\ldots$$

$$= 30 + 0 - 6 = 24 = N(S_4)$$

Opg 6   Karaktertabel p. 128 (2)

$\chi^{(2)}(I) = 2$, $\chi^{(2)}(-1) = -2$, resten 0.

$1^2 = 1$, $(-1)^2 = 1$, resten: $a^2 = -1$: ($i^2 = -1$, $j^2 = -1$, $k^2 = -$

$$\sum_g \chi^{(2)}(g^2) = 2 + 2 + 6 \cdot (-2) = -8 = -N(Q)$$

Opgave I.3

opg 2

$$R = I + A \,,\quad R^T = I - A \qquad (A^T = -A)$$

$$R^T R = I + O(A^2)$$

$$\mathrm{Det}\, R^T R = 1 + O(\varepsilon^2) \,,\quad A = O(\varepsilon).$$

$$\mathrm{Det}\, R^T R = \mathrm{Det}\, R^T\, \mathrm{Det}\, R = \mathrm{Det}\, R\, \mathrm{Det}\, R = (\mathrm{Det}\, R)^2$$

$$\mathrm{Det}\, R = \sqrt{1 + O(\varepsilon^4)} = 1 + O(\varepsilon^4) \,,$$

opg 3

$$\mathcal{J}_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix},\quad \mathcal{J}_x^2 = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{J}_x^{2n+1} = \mathcal{J}_x \cdot (-1)^n \,,\quad \mathcal{J}_x^{2n} = \mathcal{J}_x^2 (-1)^n \,,\ n > 0.$$

$$R_x(\theta_x) = e^{+\theta \mathcal{J}_x} = I + \theta \mathcal{J}_x + \frac{1}{2!}\theta^2 \mathcal{J}_x^2 + \ldots$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \left(1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 \ldots\right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$+ \left(\theta - \frac{1}{3!}\theta^3 + \ldots\right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{pmatrix}$$

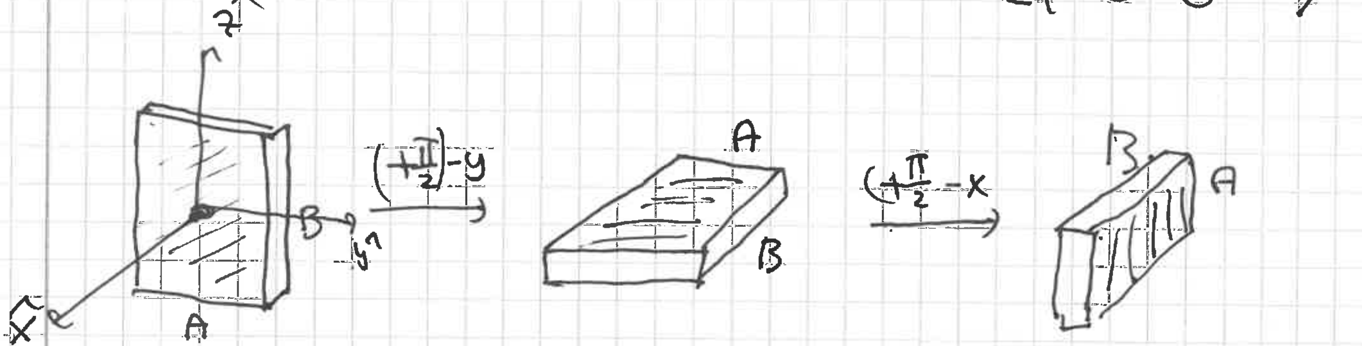
opgave I.3 — opg 3 fortsat.

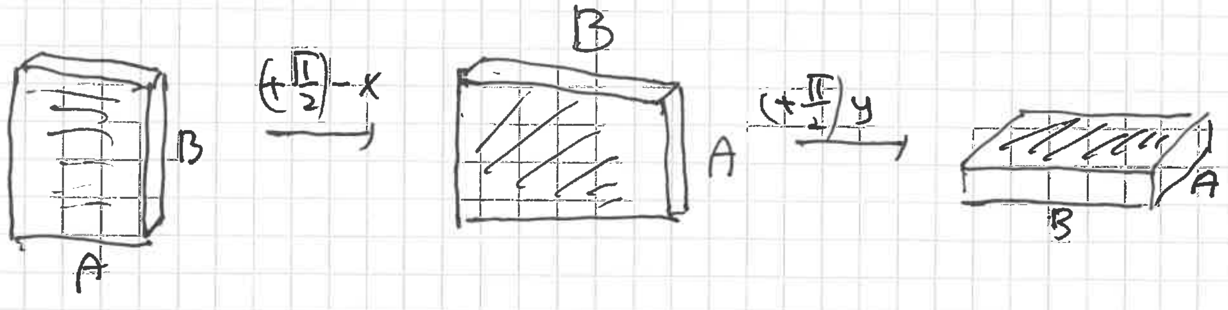
$\theta_x = -\frac{\pi}{2}$, $\theta_y = -\frac{\pi}{2}$

$$R_x\left(-\frac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & +1 & 0 \end{pmatrix}, \quad R_y\left(-\frac{\pi}{2}\right) = \begin{pmatrix} 0 & 0 & +1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$R_x\left(\frac{\pi}{2}\right) R_y\left(\frac{\pi}{2}\right) = \begin{pmatrix} 0 & 0 & -1 \\ +1 & 0 & 0 \\ 0 & +1 & 0 \end{pmatrix}$$

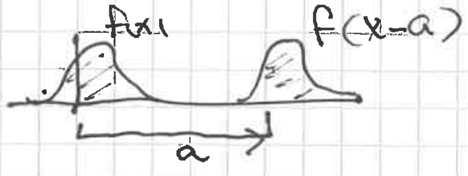
$$R_y\left(\frac{\pi}{2}\right) R_x\left(\frac{\pi}{2}\right) = \begin{pmatrix} 0 & +1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$$





Note $R(\theta) = e^{-\theta_x J_x}$: rotation of body $+\theta_x$

Recall : $T_a(x) = x + a$ $\quad (D(T_a)f)(x) = f(T_a^{-1}(x)) = f(x-a)$



Opgave I.3

Opg 4

$$[J_i, J_j] = i\, C_{ijk} J_k \Rightarrow$$

$$[J_j^\dagger, J_i^\dagger] = -i\, C^*_{ijk} J_k^\dagger \Rightarrow \quad (J^\dagger = J)$$

$$[J_j, J_i] = -i\, C^*_{ijk} J_k \quad \Rightarrow$$

$$[J_i, J_j] = i\, C^*_{ijk} J_k \quad ; \quad C_{ijk} = C^*_{ijk}$$

Opg 5

$$(J_{mn})^{ij} = -i\left[\delta^{mi}\delta^{nj} - \delta^{mj}\delta^{ni}\right]$$

$$[J_{mn}, J_{pq}]^{il} = - \left(\delta^{mi}\delta^{nj} - \delta^{mj}\delta^{ni}\right)\left(\delta^{pj}\delta^{ql} - \delta^{pl}\delta^{qj}\right) + \left(\delta^{pi}\delta^{qj} - \delta^{pj}\delta^{qi}\right)\left(\delta^{mj}\delta^{nl} - \delta^{ml}\delta^{nj}\right)$$

$$= -\delta^{mi}\delta^{ql}\delta_{np} + \delta^{mi}\delta^{pl}\delta_{nq} + \delta_{mp}\delta^{ni}\delta^{ql} - \delta_{mq}\delta^{ni}\delta^{pl} + \delta^{pi}\delta^{nl}\delta_{qm} - \delta^{pi}\delta^{ml}\delta_{qn} - \delta^{qi}\delta^{nl}\delta_{pm} + \delta^{qi}\delta^{ml}\delta_{pn}$$

$$= \delta_{mp}\left[\delta^{ni}\delta^{ql} - \delta^{nl}\delta^{qi}\right] + \delta_{nq}\left[\delta^{mi}\delta^{pl} - \delta^{ml}\delta^{pi}\right] - \delta_{np}\left[\delta^{mi}\delta^{ql} - \delta^{ml}\delta^{qi}\right] - \delta_{mq}\left[\delta^{ni}\delta^{pl} - \delta^{nl}\delta^{qi}\right]$$

$$= i\left(\delta_{mp} J_{(nq)} + \delta_{nq} J_{mp} - \delta_{np} J_{mq} - \delta_{mq} J_{np}\right)$$