Dynamical Systems Homework: Describing a simple Lorenz attractor

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The Lorenz system is a simplified mathematical model which is used to illustrate theories of chaos and non-linear systems. Lorenz system consists of three coupled differential equations, and characterizes a system with three state variables (x,y,z) and three constants σ, ρ, β. Below, you can see the equations:

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

Setting specific values to these three constants will result in chaotic behavior of the system. For example, the figure below is a depiction of the system's phase portrait for the following values: $\sigma = 10$, $\rho = 28$, $\beta = 8/3$.

Fig. 1 shows the phase portrait of the system for the parameter values $\sigma = 10$, $\rho = 28$, $\beta = 8/3$.

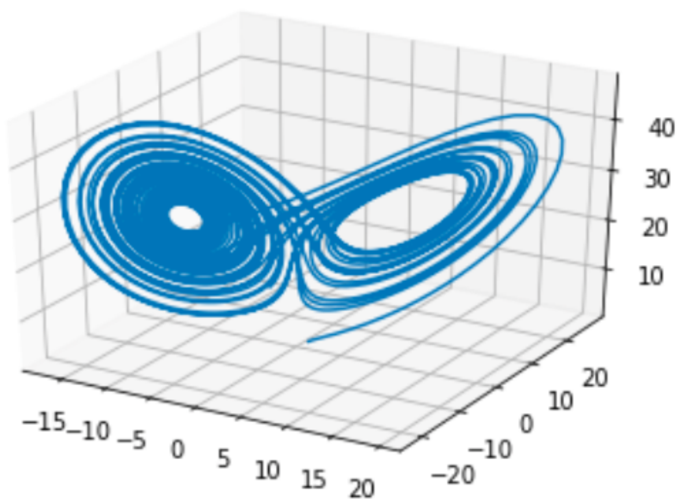

Figure 1: Python simulation of a Lorenz system for $\sigma = 10$, $\rho = 28$, $\beta = 8/3$

The three axes on the plot represent the three states (x,y,z) plotted over the 40 seconds of simulated time. The two circular paths in this figure are attractors. In fact, this system circles around in one area in state-space, then moves to a neighbor area circles around the new area. This system is very sensitive to initial conditions and therefore the number of times it circles in a specific area and the time when it switches to a new area illustrate a chaotic behavior. One of the characteristics of the Lorenz system is that when the value of ρ augments, we see more repulsors in the plot. Figure 2 shows the Lorenz system after changing ρ from 28 to 4000.

Fig. 2 shows the phase portrait of the system for the parameter values $\sigma = 10$, $\rho = 4000$, $\beta = 8/3$.

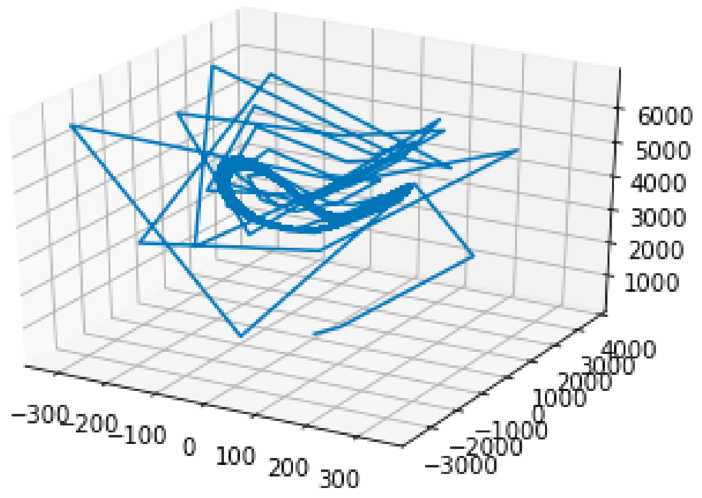

Figure 2: Python simulation of a Lorenz system for $\sigma = 10$, $\rho = 4000$, $\beta = 8/3$

As stated before, this plot reveals the influence of high values of ρ and the appearance of phase portrait.