Ryerson University

Faculty of Engineering & Architectural Science

Course Number	ELE532
Course Title	Signals and Systems I
Semester/Year	Fall 2022
Instructor	Dr. D. Androutsos
Section No.	12
Group No.	N/A
Submission Date	2022-11-13
Due Date	2022-11-14

Lab/Tut Assignment NO.	3

Assignment Title	**Fourier Series Analysis**

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(Note: remove the first 4 digits from your student ID)

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Procedures

a) Complete Problems A.1 to A.6

Problem A.1 [1 Mark] Given the periodic signal $x_1(t)$:

$$x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t,$$

derive an expression for the Exponential Fourier Series coefficients D_n.

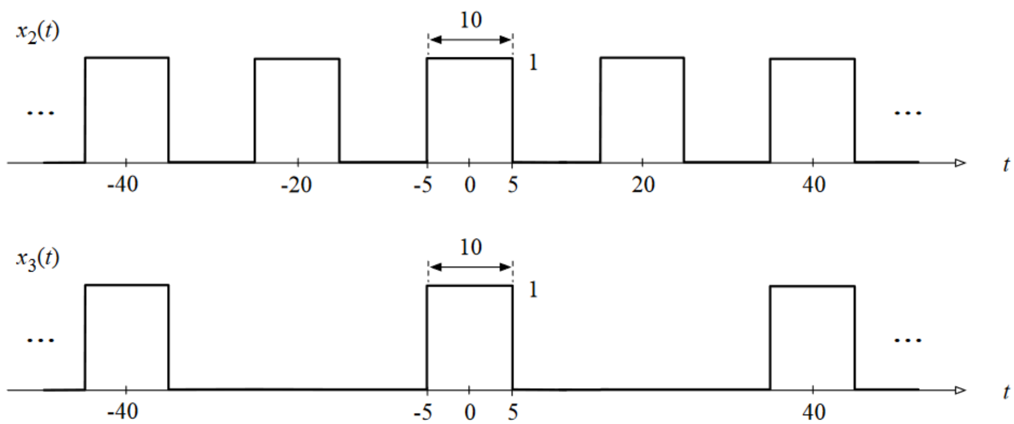


Figure 1: Periodic functions $x_2(t)$ and $x_3(t)$.

Problem A.2 [1 Mark] Repeat Problem A.1 for the periodic signals $x_2(t)$ and $x_3(t)$ shown in Figure 1.

Problem A.3 [3 Marks] Now that you have an expression for D_n, write a MATLAB function that generates D_n for a user specified range of values of n.

Problem A.4 [3 Marks] Generate and plot the **magnitude** and **phase** spectra of $x_1(t)$, $x_2(t)$ and $x_3(t)$ (using the `stem` command) from their respective D_n sets for the following index ranges:

(a) $-5 \leq n \leq 5$;

(b) $-20 \leq n \leq 20$;

(c) $-50 \leq n \leq 50$;

(d) $-500 \leq n \leq 500$.

Note: You can use the MATLAB commands `abs` and `angle` to determine the magnitude and phase of a complex number.

Problem A.5 [3 Marks] Write a MATLAB function that takes a MATLAB generated D_n set and reconstructs the original time-domain signal from which the Fourier coefficients had been derived. For example, given the set of truncated Fourier coefficients $\{D_n, n = 0, \pm 1, \ldots, \pm 20\}$, your code should reconstruct the time-domain signal from this set using Equation (1). **Note:** Use the time variable t defined with the MATLAB command `t=[-300:1:300]`.

Problem A.6 [3 Marks] Reconstruct the time-domain signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ with the Fourier coefficient sets you generated in Problem A.4. Plot each reconstructed signal.

b) Complete Problems B.1 to B.7

Problem B.1 [1 Mark] Determine the fundamental frequencies of $x_1(t)$, $x_2(t)$ and $x_3(t)$.

Problem B.2 [1 Mark] What is the main difference between the Fourier coefficients of $x_1(t)$ and $x_2(t)$?

Problem B.3 [1 Mark] Signals $x_2(t)$ and $x_3(t)$ have the same rectangular pulse shape but different periods. How are these characteristics reflected in their respective Fourier coefficients?

Problem B.4 [2 Marks] The Fourier coefficient D_0 represents the DC value of the signal. Let $x_4(t)$ be the periodic waveform shown in Figure 2. Derive D_0 of $x_4(t)$ from D_0 of $x_2(t)$.

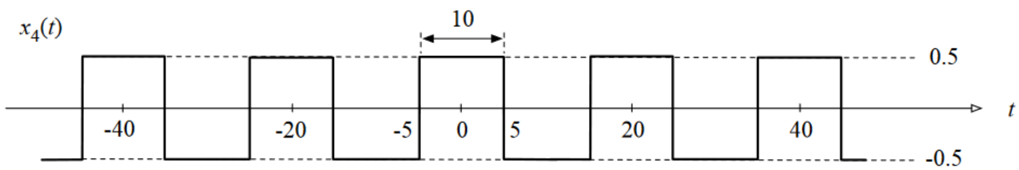


Figure 2: Periodic function $x_4(t)$.

Problem B.5 [2 Marks] Using the results of Problem A.6, explain how the reconstructed signal changes as you increase the number of Fourier coefficients used in the reconstruction. Discuss for both $x_1(t)$ and $x_2(t)$.

Problem B.6 [2 Marks] How many Fourier coefficients do you need to **perfectly** reconstruct the periodic waveforms discussed in this lab experiment?

Problem B.7 [2 Marks] Let $x(t)$ be an arbitrary periodic signal. Instead of storing $x(t)$ on a computer, we consider storing the corresponding Fourier coefficients. When we need to access $x(t)$, we read the Fourier coefficients stored on the computer hard drive and reconstruct the signal. Is this a viable scenario? Explain your answer.

4. Results

Assignment A:

A.1)

ELE532 Lab 3

A1: $x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$

$$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{2}\left(\frac{1}{2}e^{j\frac{\pi}{10}t} + \frac{1}{2}e^{-j\frac{\pi}{10}t}\right)$$

$$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}$$

Fundamental Frequency:

$\frac{3\pi}{10}\cdot\frac{10}{\pi} = 3$ $\qquad T_0 = \frac{2\pi}{\left(\frac{\pi}{10}\right)} = 2\pi\cdot\frac{10}{\pi} = 20$

$\omega_{0_1} = \frac{3\pi}{10} \quad \omega_{0_2} = \frac{\pi}{10}$

$\frac{GCF}{LCM} = \frac{\pi}{10}$

$jn\frac{\pi}{10}t = j\frac{3\pi}{10}t \qquad jn\frac{\pi}{10}t = j\frac{\pi}{10}t$

$n=3 \qquad n=-3 \qquad n=1 \qquad n=-1$

$D_3 = \frac{1}{2} \qquad D_{-3} = \frac{1}{2} \qquad D_1 = \frac{1}{4} \qquad D_{-1} = \frac{1}{4}$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right]e^{-j\frac{\pi}{10}nt}\,dt$$

$$\frac{1}{20}\left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\frac{\pi}{10}} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\frac{\pi}{10}} + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\frac{\pi}{10}} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\frac{\pi}{10}}\right]$$

$$D_n = \frac{1}{2}\left[\text{sinc}[(3-n)\pi] + \text{sinc}[(3+n)\pi] + \frac{1}{2}\text{sinc}[(1+n)\pi] + \frac{1}{2}\text{sinc}[(1-n)\pi]\right]$$

A.2)

A.2: $x_2(t)$: $T_0 = 20$ $\quad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\left[\int_{-5}^{5} (1)\, e^{-jn\frac{\pi}{10}t}\, dt\right] = \frac{1}{20}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\frac{\pi}{2}} + \frac{10}{jn\pi} e^{jn\frac{\pi}{2}}\right] = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)$$

$$D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$x_3(t)$: $T_0 = 40$ $\quad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$ $\quad | \quad D_n = \frac{1}{40}\int_{-5}^{5} (1)\, e^{-jn\frac{\pi}{20}t}\, dt$

$$D_n = \frac{1}{40}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5} \qquad D_n = \frac{1}{40}\left[\frac{-20}{jn\pi} e^{-jn\frac{\pi}{4}} + \frac{20}{jn\pi} e^{jn\frac{\pi}{4}}\right]$$

$$D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

A.3)

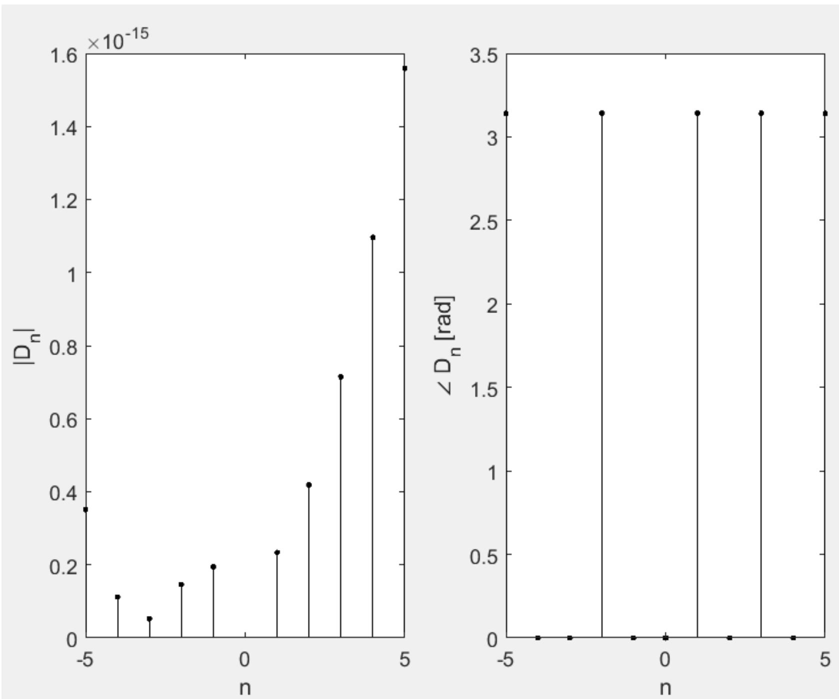
```
function [D] = Dn (d,n)
D1 = [0.5,0.-0.5*1i.0.0/5*1i.0.0.5];
D2 = (1/(n.*pi)*sin((n*pi)/2));
D3 = (1/(n.*pi)*sin((n*pi)/4));

if(d==1)

      D=D1;end

if(d==2)

      D=D2;end

if(d==3)

      D=D3;end
end
```

A.4)

a)

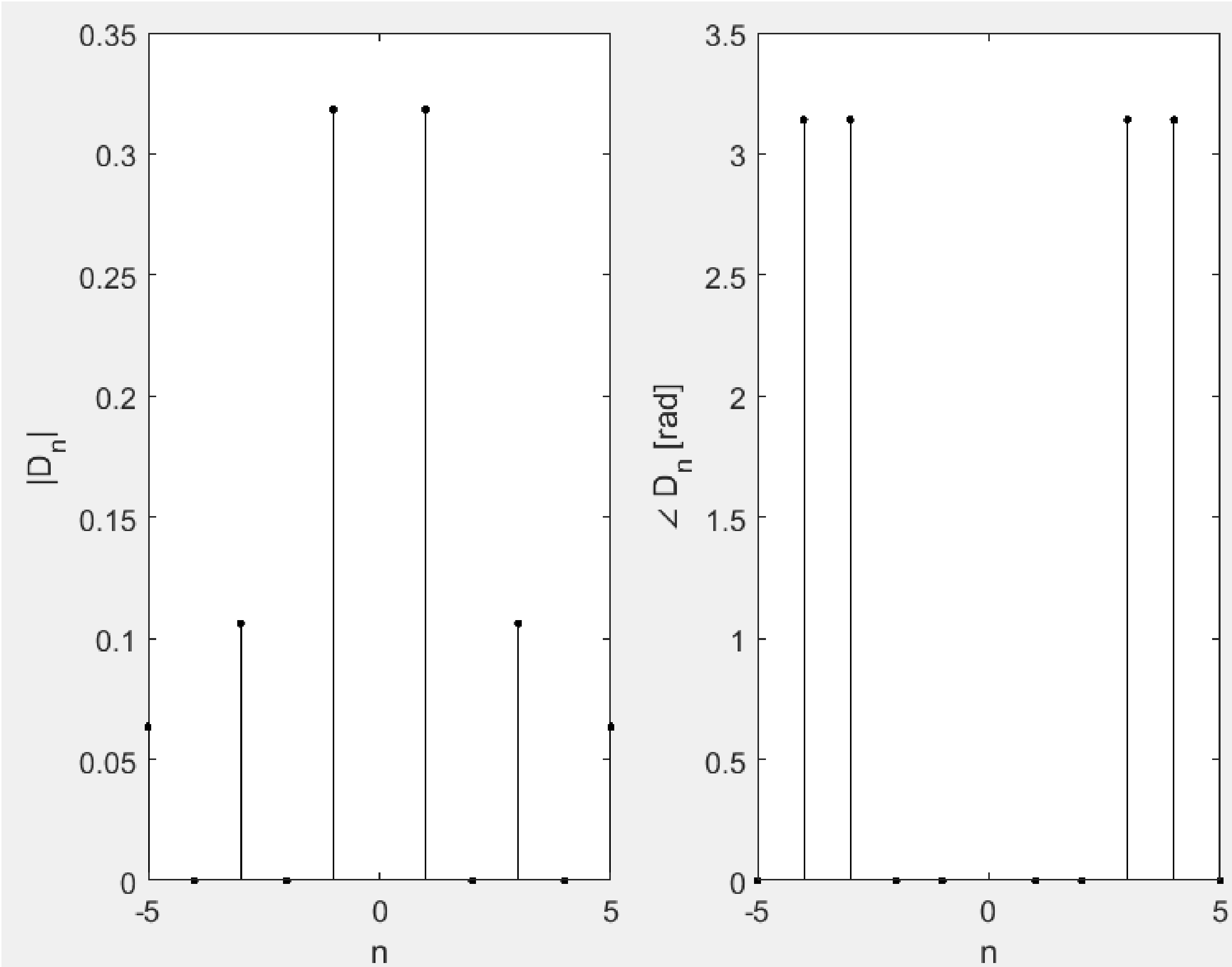
i)

```
clf;
n = (-5:5);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi)) ;

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



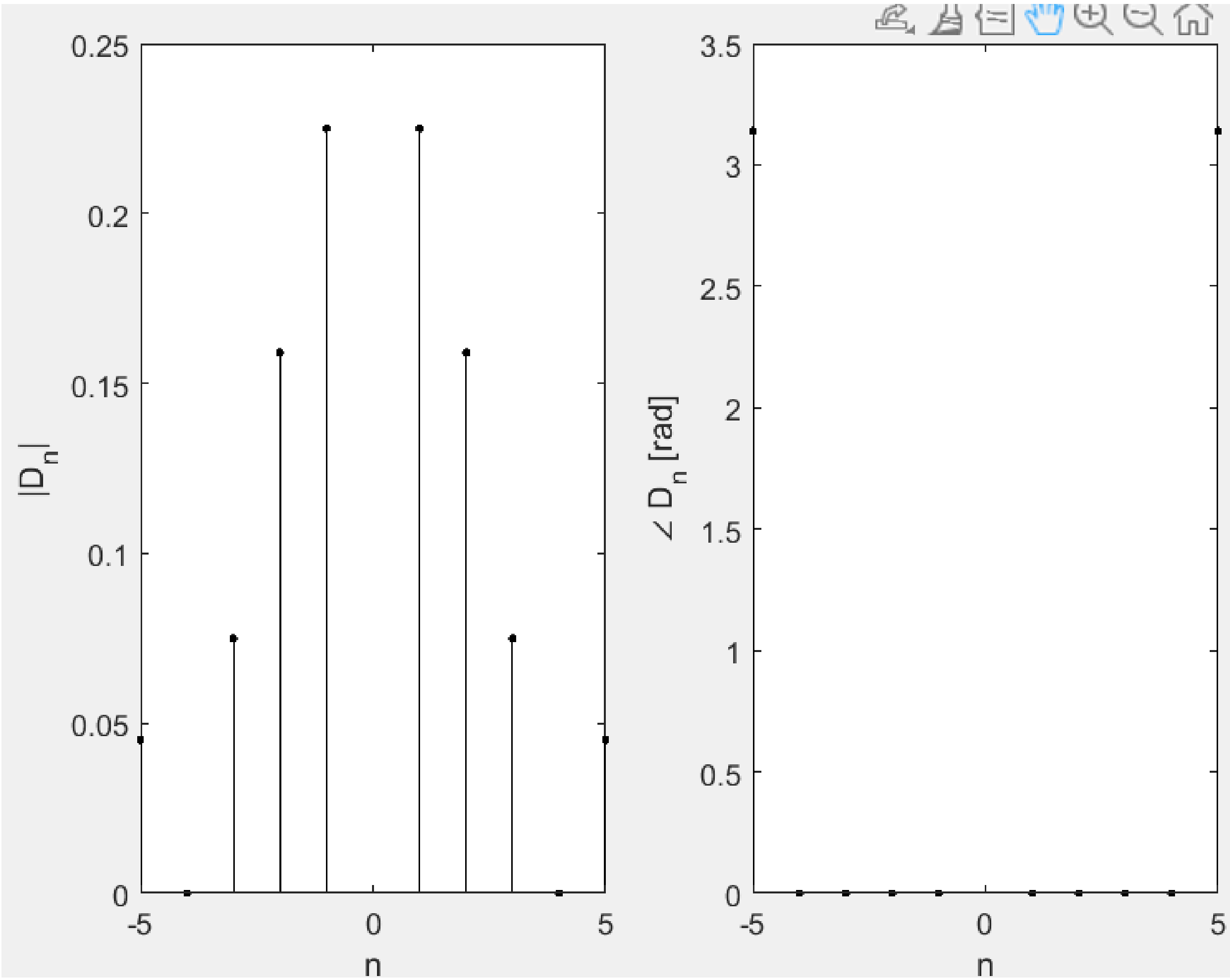
ii)

```
clf;
n = (-5:5);

D_n = (1./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



iii)

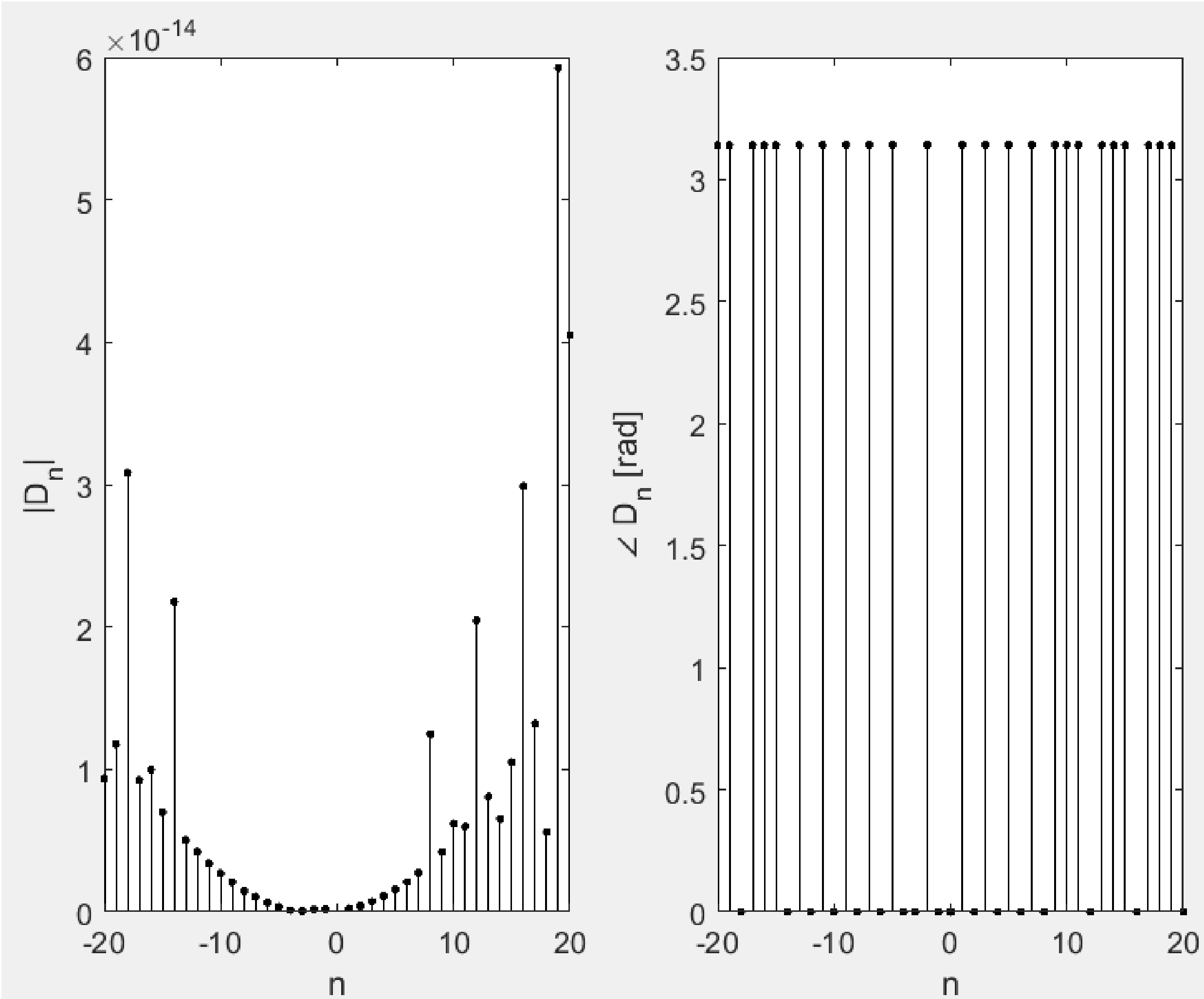
```
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



b)

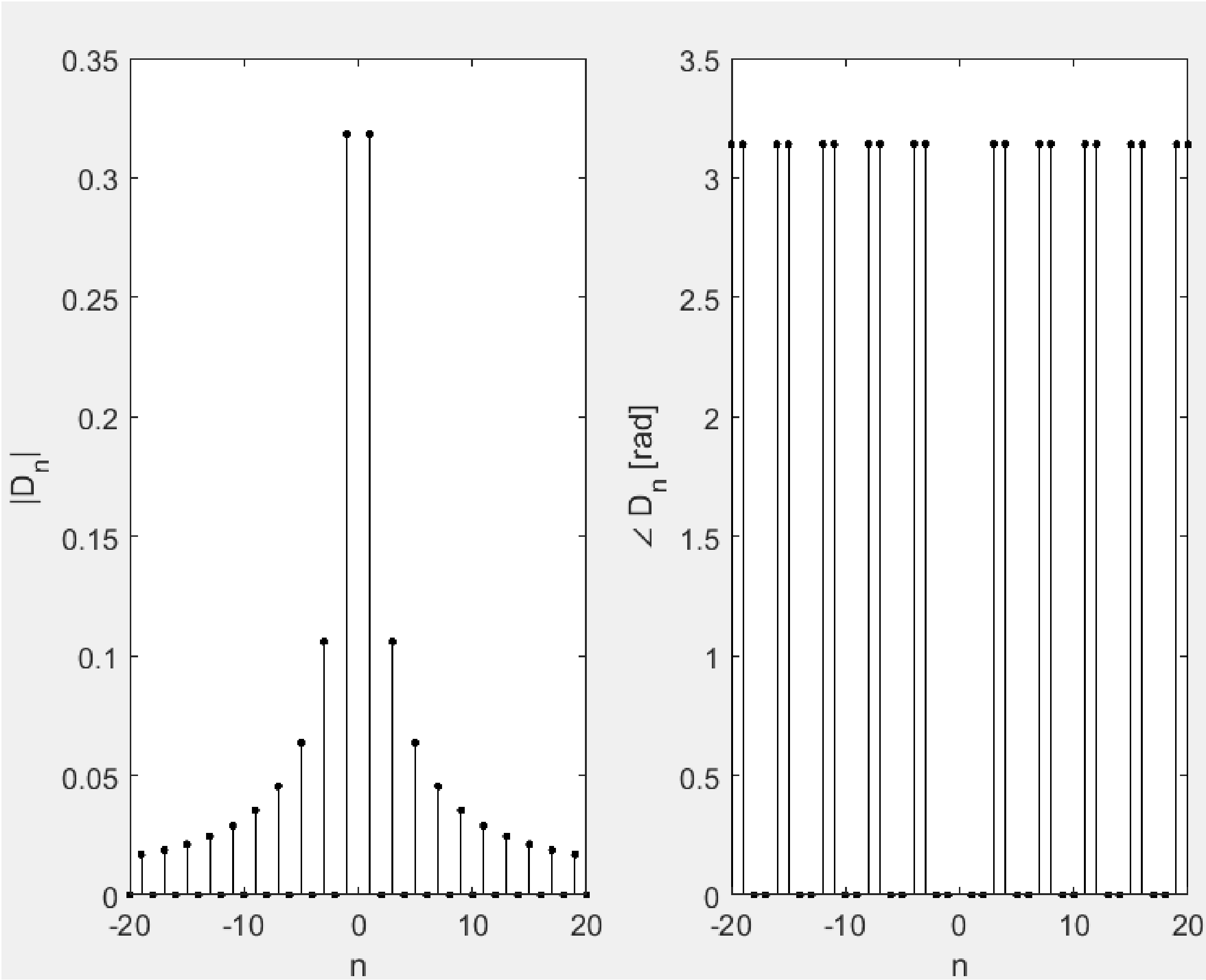
i)

```
clf;
n = (-20:20);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



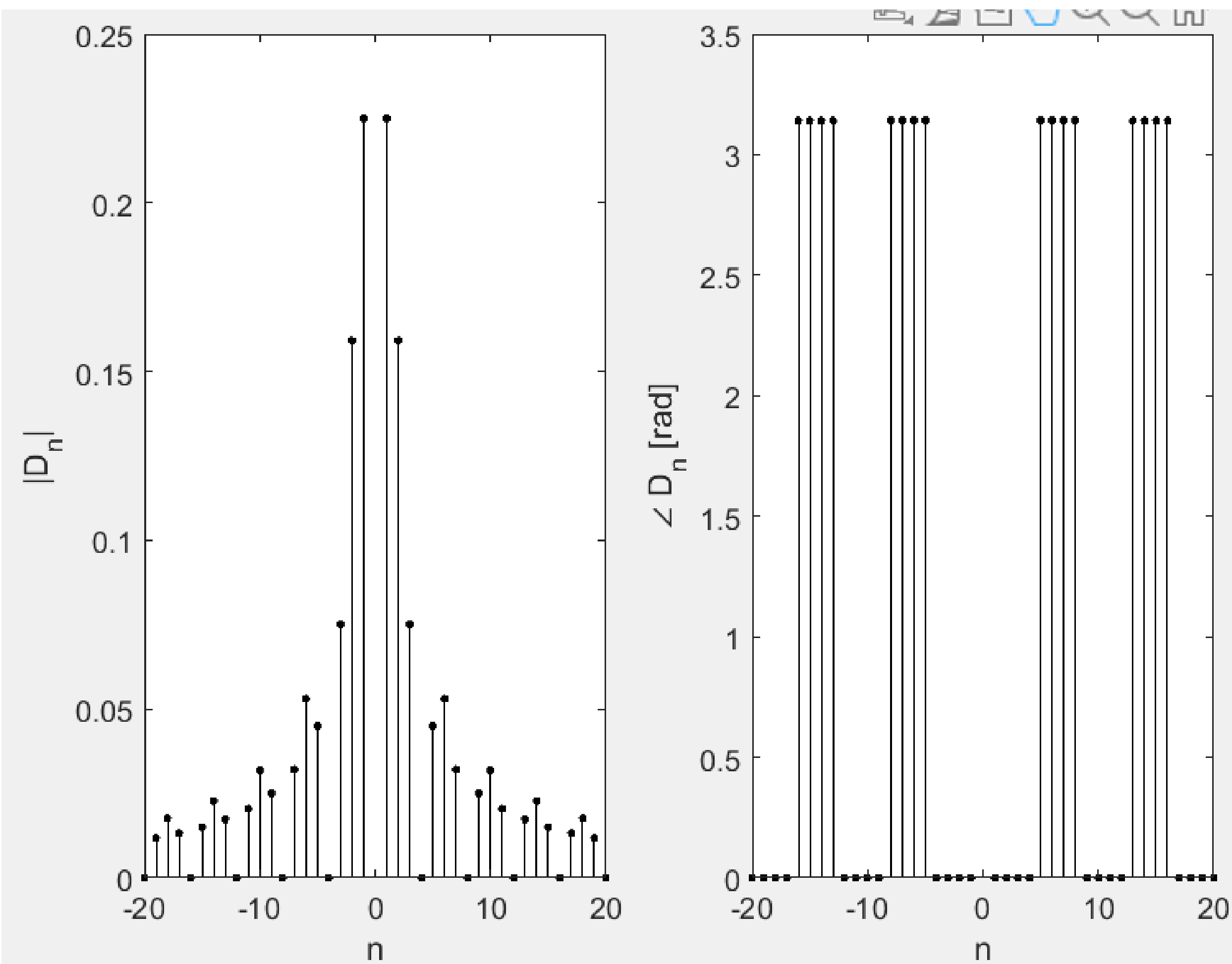
ii)

```
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



iii)

```
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

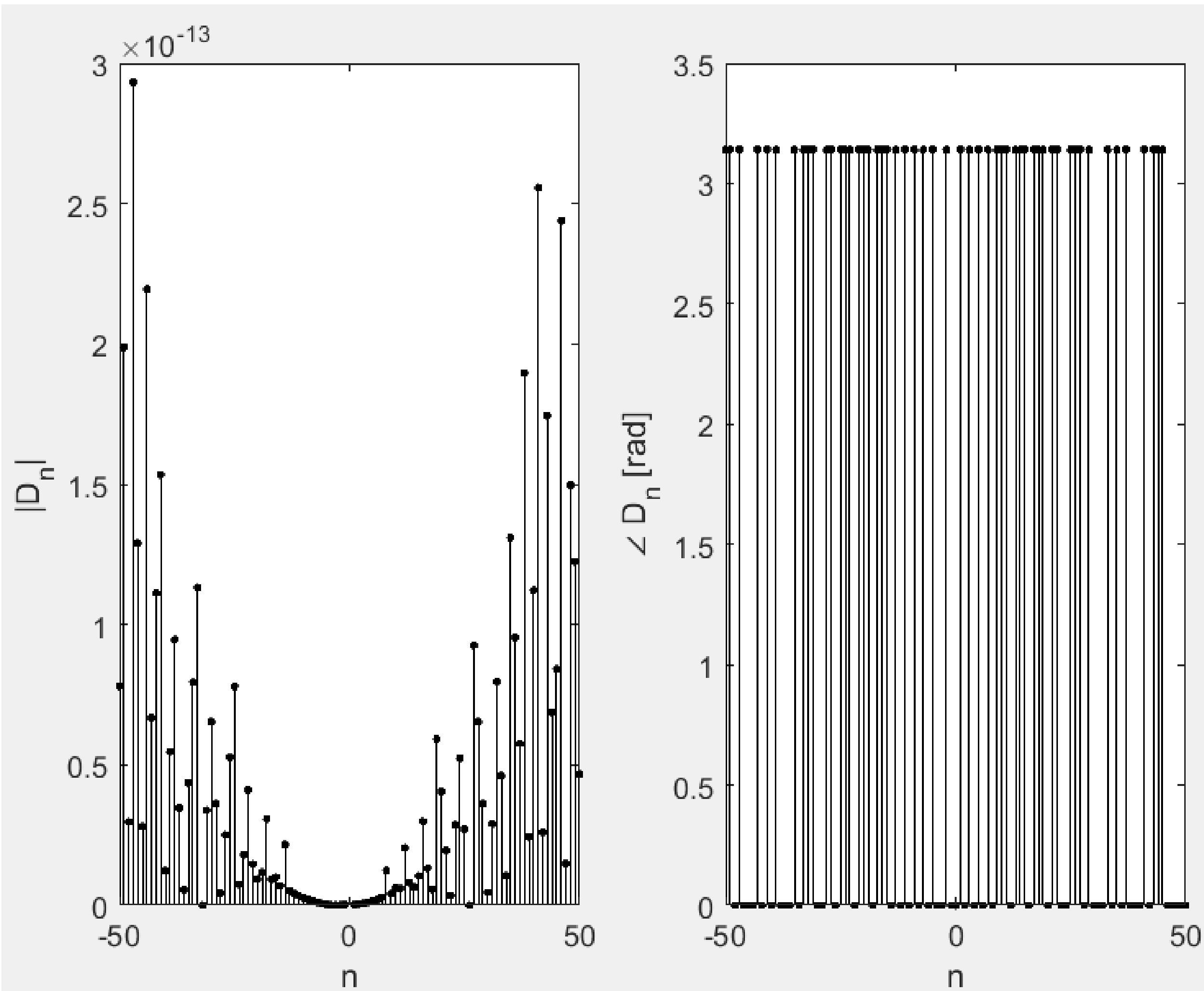


0.25
0.2
0.15
0.1
0.05
0
|D_n|
-20
-10
0
10
20
n
3.5
3
2.5
2
1.5
1
0.5
0
∠ D_n [rad]
-20
-10
0
10
20
n

c)

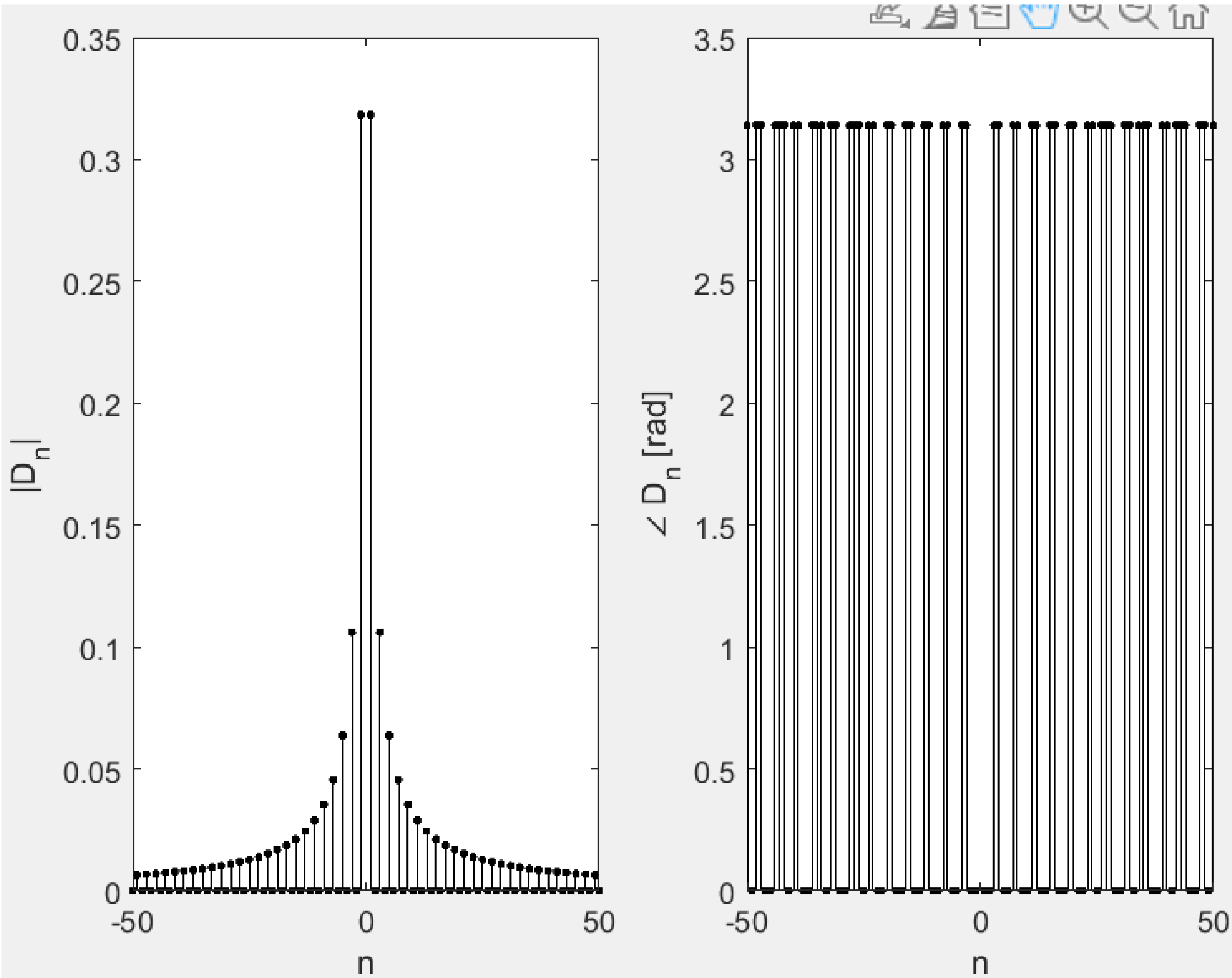
i)

```
clf;
n = (-50:50);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi)) ;

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



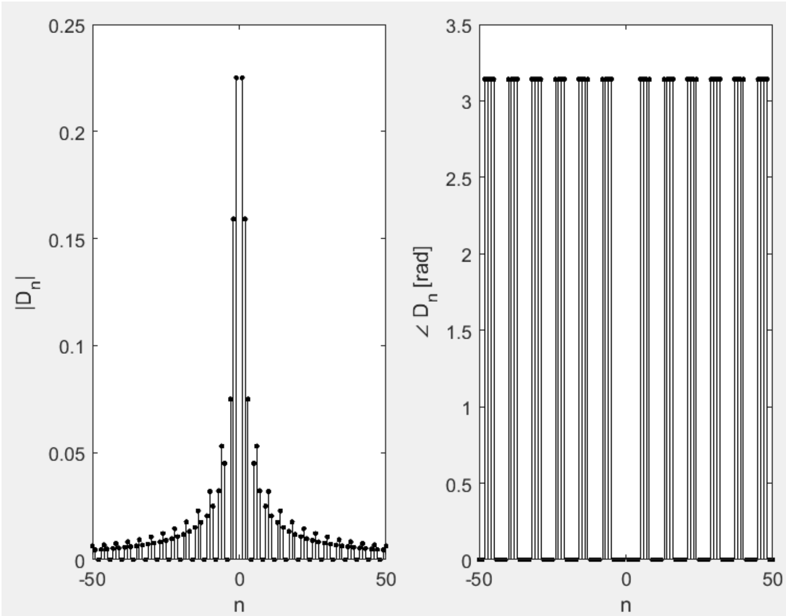
ii)

```
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



iii)

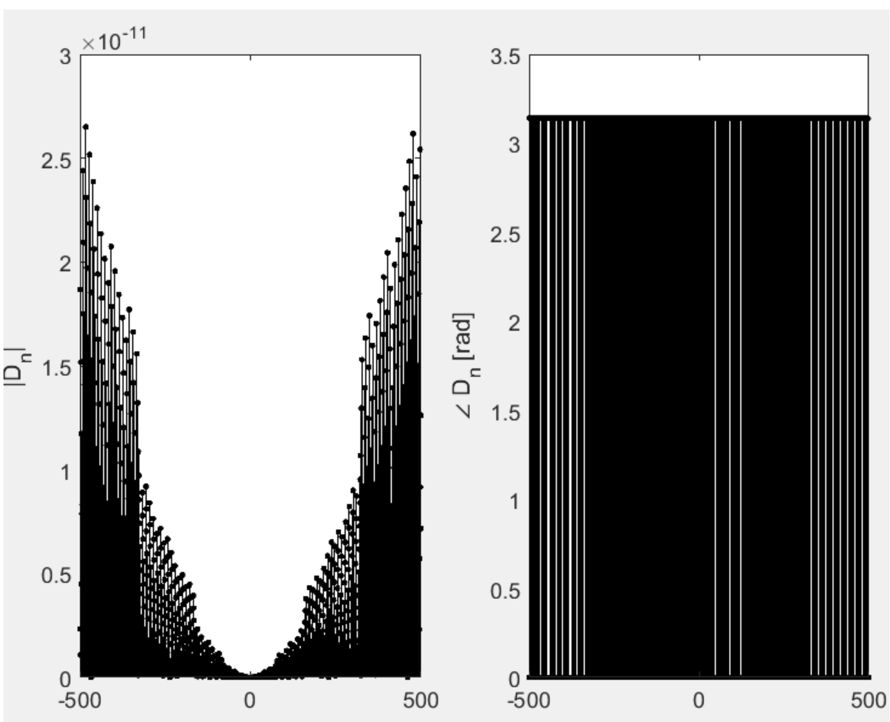
```
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



d)

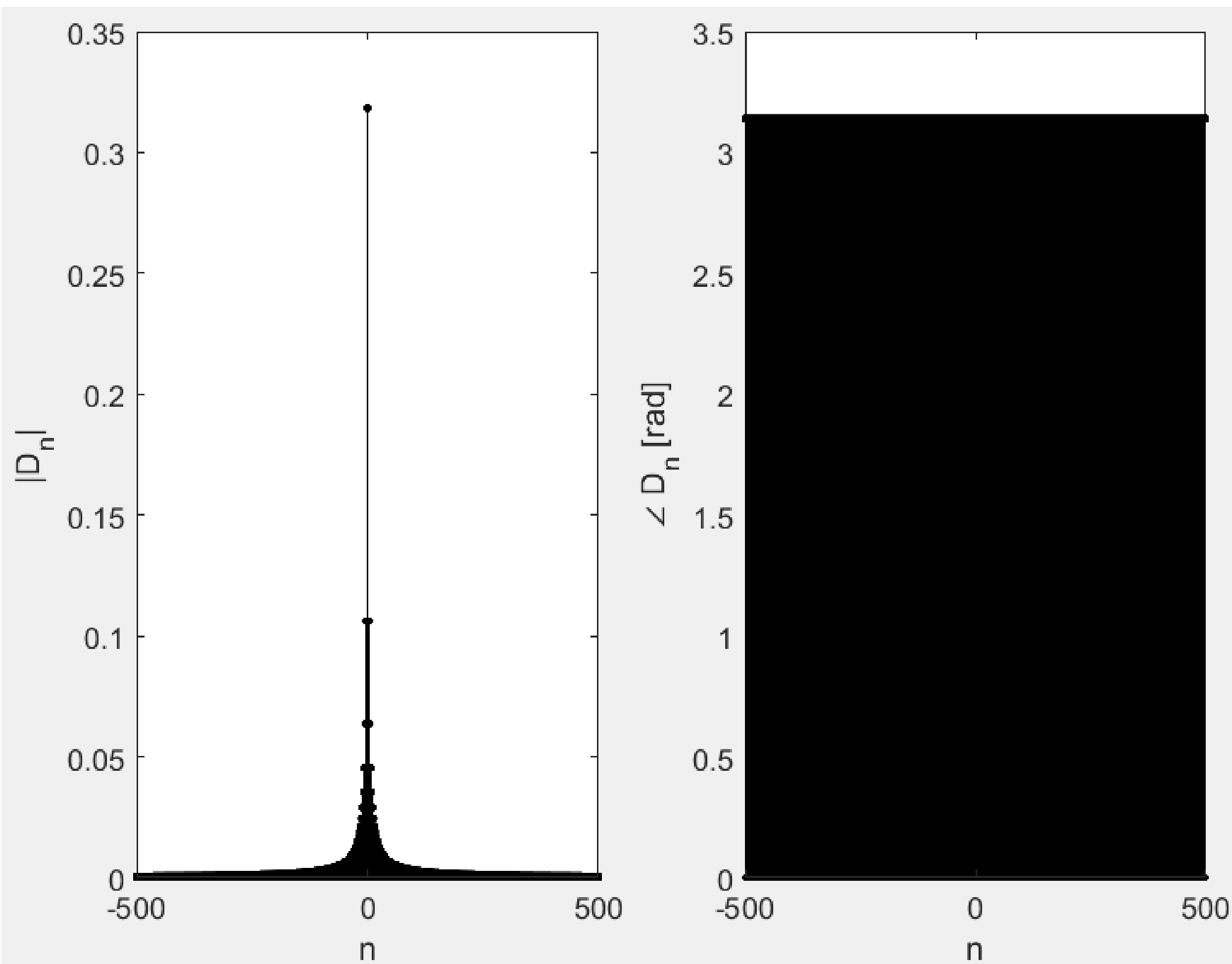
i)

```
clf;
n = (-500:500);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi)) ;

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



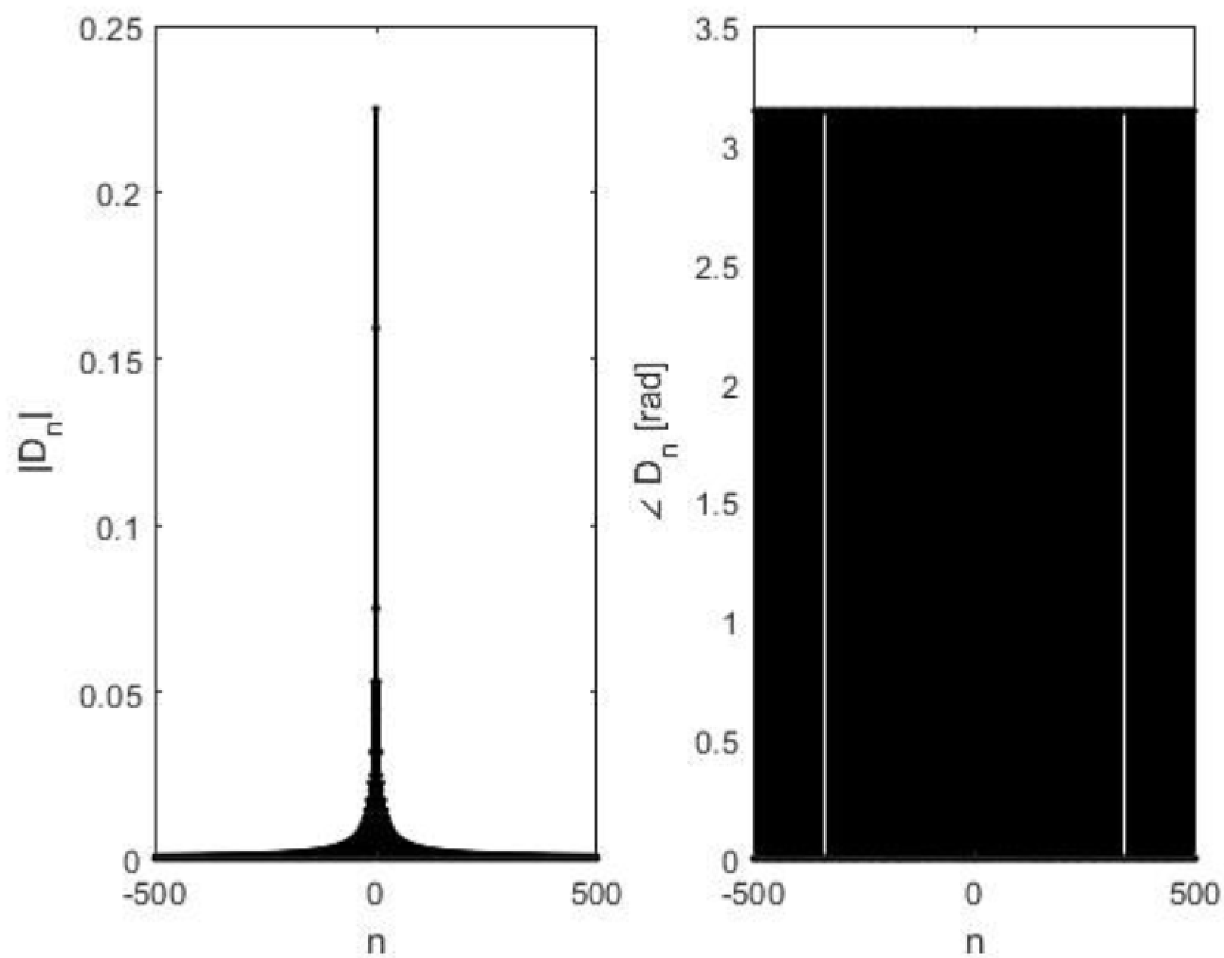
ii)

```
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



iii)

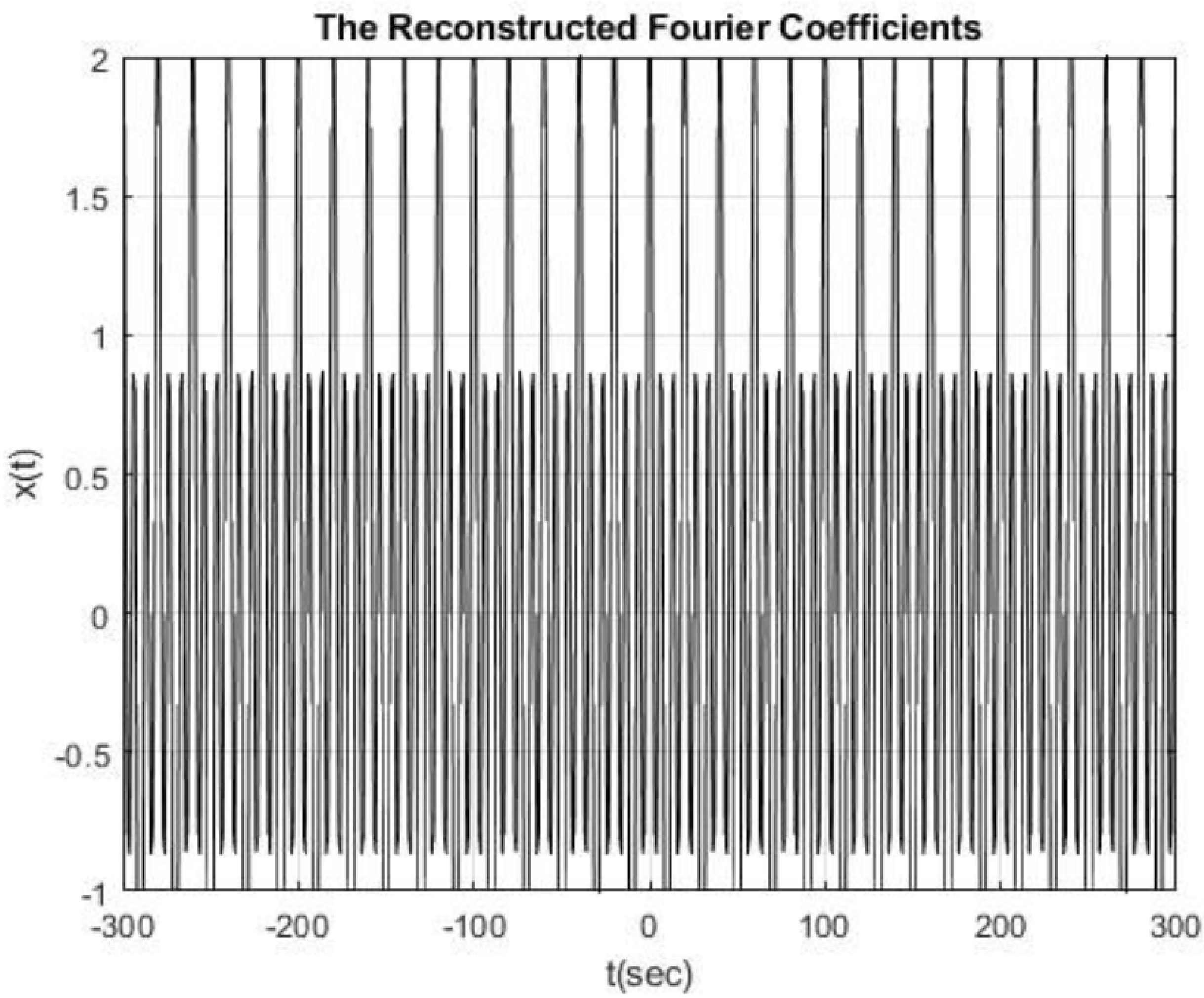
```
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

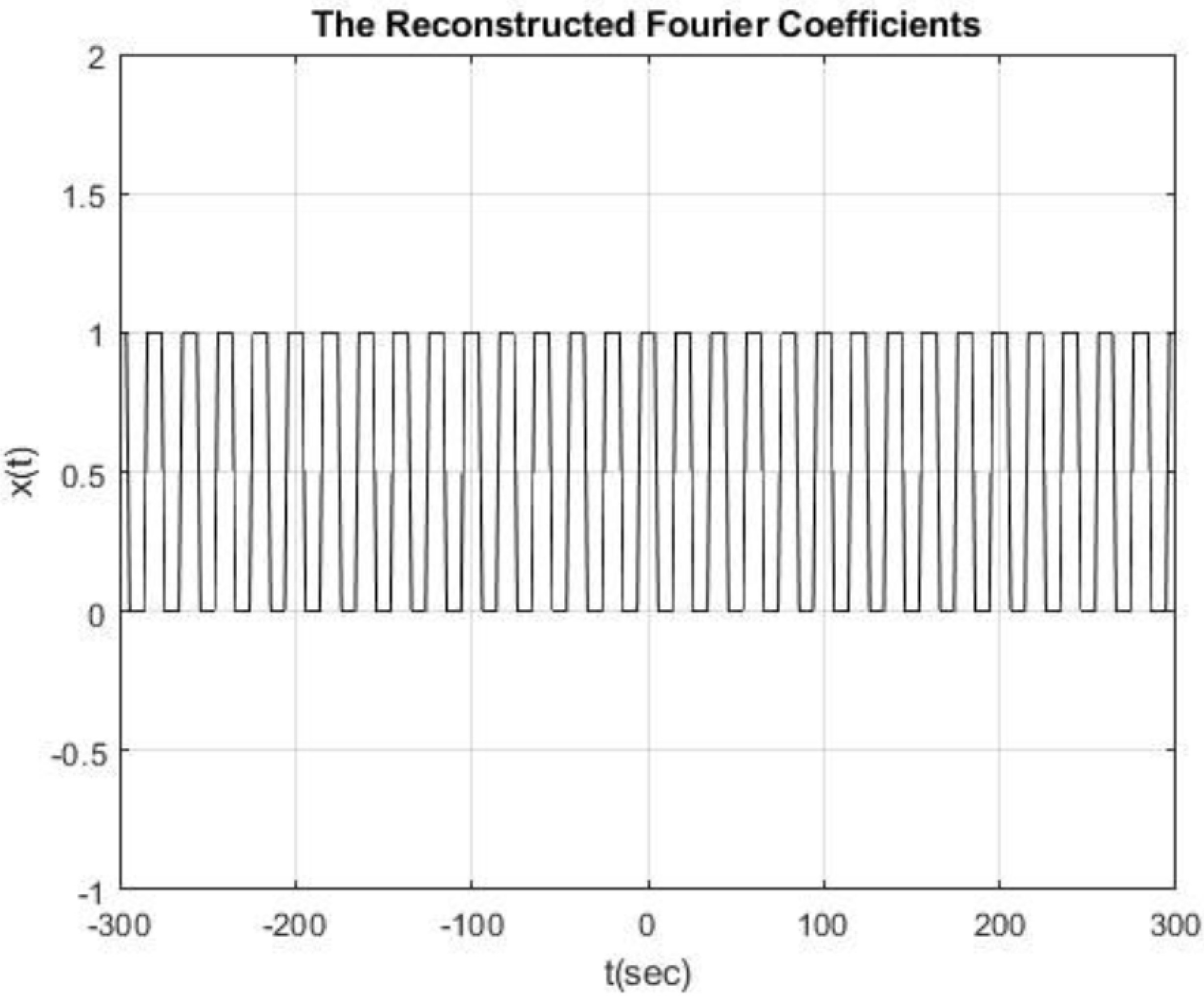


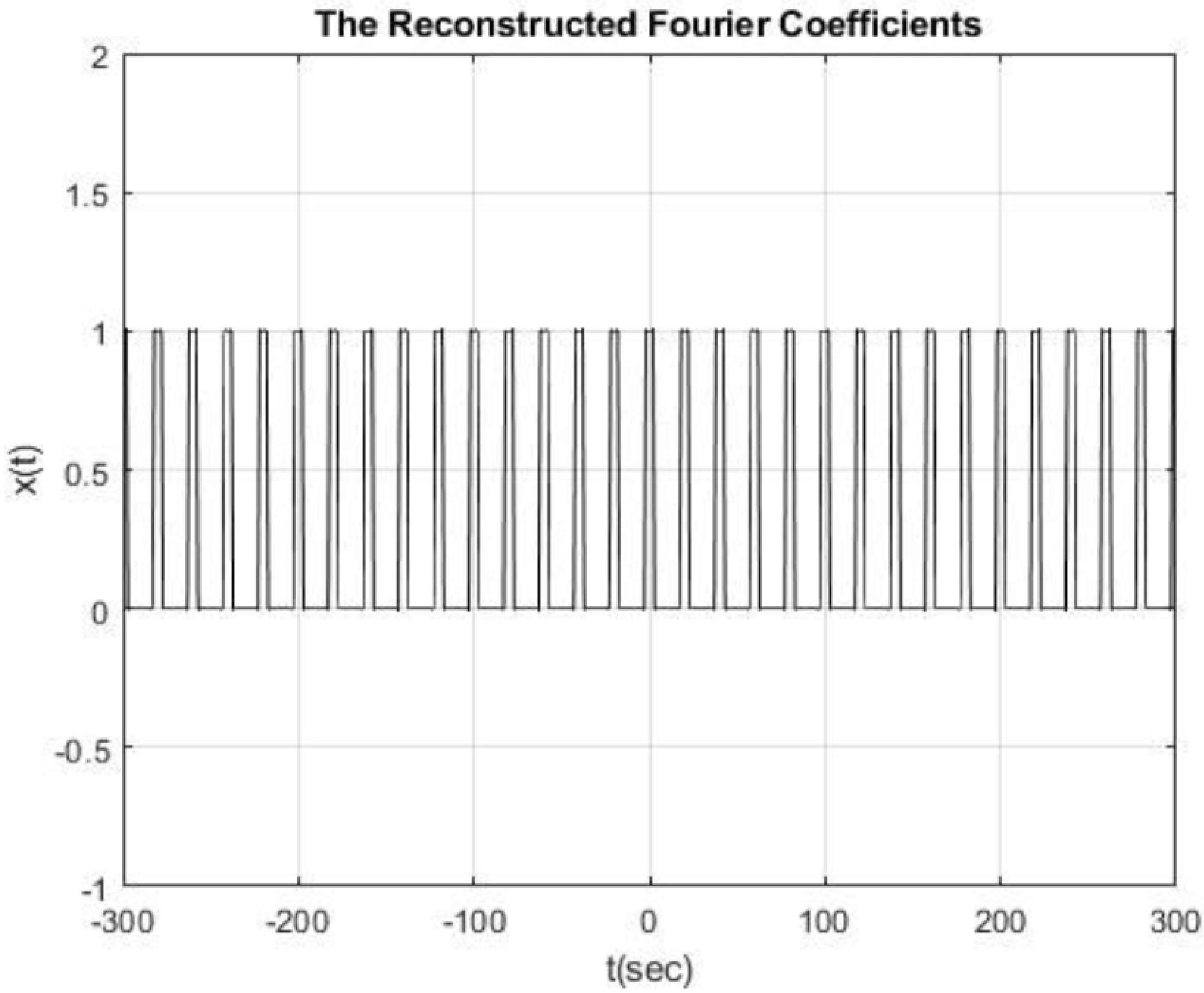
A.5)

```
function [D] = Lab3a5(Dn)
n=-500:500;
D=0.25*sinc(n/4);
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
 x=x+D(i)*exp(j*n(i)*w*t);
 't'
end

figure(5);
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('The Reconstructed Fourier Coefficients');
grid;
```

A.6)


The Reconstructed Fourier Coefficients
2
1.5
1
0.5
0
-0.5
-1
x(t)
-300
-200
-100
0
100
200
300
t(sec)


The Reconstructed Fourier Coefficients
x(t)
t(sec)
2
1.5
1
0.5
0
-0.5
-1
-300
-200
-100
0
100
200
300


The Reconstructed Fourier Coefficients
2
1.5
1
0.5
0
-0.5
-1
x(t)
-300
-200
-100
0
100
200
300
t(sec)

Assignment B:

B.1)

$$x_1(t) = \cos\left(\frac{3\pi}{10}\right)t + \frac{1}{2}\cos\left(\frac{\pi}{10}\right)t,$$

$$\omega_{o1} = \frac{3\pi}{10},\ \omega_{o2} = \frac{\pi}{10}$$

$$\omega_o = \frac{G.C.F\ of\ numerator}{L.C.M\ of\ denominator} = \frac{\pi}{10} = 0.314\ rad/s$$

For $x_2(t) \rightarrow T_o = 20\ s$

$$\omega_o = \frac{\pi}{10} = 0.314\ rad/s$$

For $x_3(t) \rightarrow T_o = 40\ s$

$$\omega_o = \frac{\pi}{20} = 0.157\ rad/s$$

B.2)

The difference between the fourier coefficients of x1(t) and x2(t) is that one has sinc and the other consists of sin functions respectively. Furthermore, x1(t) has four distinct fourier series coefficients, while x2(t) has an infinite number of fourier coefficients for Dn.

B.3)

Signal x3(t) has a smaller fundamental frequency value compared to signal x2(t) for it's Fourier coefficients.

B.4)

Do = 0.5 for signal x4(t), derived from x2(t) .

B.5)

Since x1(t) has a finite number of Dn values, nothing changes if the Fourier coefficients are increased. However, for x2(t) and x3(t), increasing values of Dn results in a higher accuracy.

B.6)

Again, since x1(t) has a finite number of Dn values, we would only need four Fourier series coefficients, in this case, to perfectly reconstruct. However, for x2(t) and x3(t), we would need an infinite number of Dn for perfect reconstruction.

B.7)

Since a periodic signal has an infinite number of Dn values, it is not viable. However, if it is finite like x1(t), then the values of Dn can be stored. However, this is not recommended for signals which have a large amount of finite Dn values as they would tend to utilize an unnecessary amount of space.