CSE 105:
Data Structures and Algorithms-I
(Part 2)

Instructor

Dr Md Monirul Islam

Graph Searching

CT 2 Syllabus:

List, Stack and Queue

Breadth-First Search

```
BFS(G, s)
for each vertex u ∈ G.V − {s}
    u.color = WHITE
    u.d = ∞
    u.π = NIL
s.color = GRAY
s.d = 0
s.π = NIL
Q = ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    u = DEQUEUE(Q)
    for each v ∈ G.Adj[u]
        if v.color == WHITE
            v.color = GRAY
            v.d = u.d + 1
            v.π = u
            ENQUEUE(Q, v)
    u.color = BLACK
```

Review

Whitening

Enqueue the root

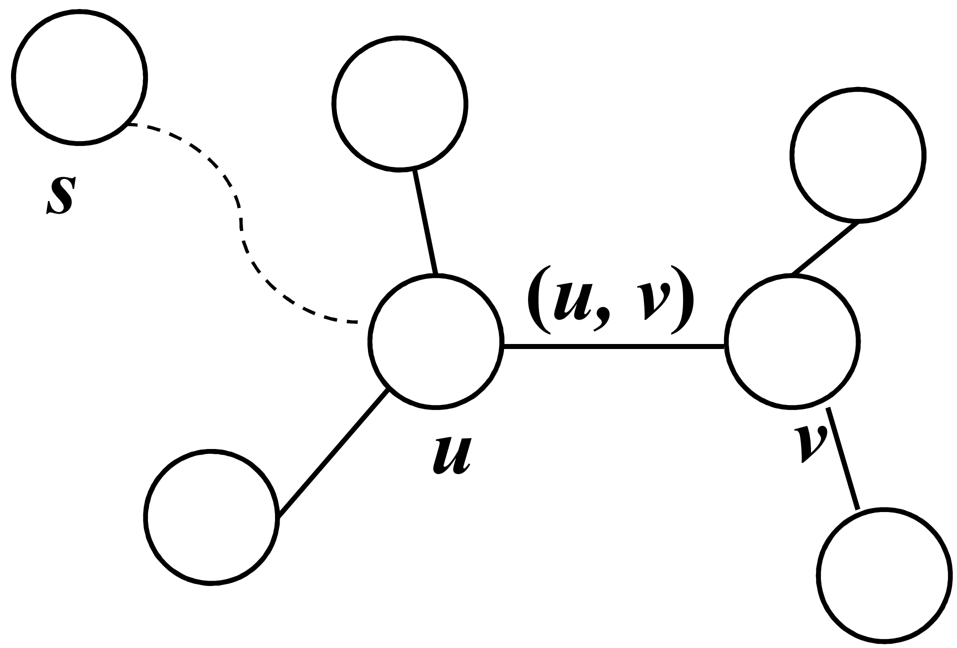
runs until queue is empty

Lemma 22.1

Let G = (V, E) be a directed or undirected graph, and let s Ɛ V be an arbitrary vertex. Then, for any edge (u, v) Ɛ E,

$\delta(s, v) \leq \delta(s, u) + 1$

Review



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 Next       v.π = u
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Lemma 22.2

Let $G = (V, E)$ be a directed or undirected graph, and suppose that BFS is run on G from a given source vertex s Ɛ V. Then upon termination, for each vertex v Ɛ V, the value $v.d$ computed by BFS satisfies $v.d \geq \delta(s, v)$

Review

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Lemma 22.3

Suppose that during the execution of BFS on a graph $G = (V, E)$, the queue Q contains the vertices $\langle v_1, v_2, \cdots, v_r \rangle$ where v_1 is the head of Q and v_r is the tail. Then (1) $v_r.d \le v_1.d + 1$ and (2) $v_i.d \le v_{i+1}.d$ for $i = 1, 2, \cdots, r-1$

Review

v_1	v_2	v_3	.	..	.	.	.		v_r

Vertices in Queue

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Lemma 22.3

Review

Prove: (1) $v_r.d \le v_1.d + 1$

(2) $v_1.d <= v_2.d <= v_3.d <= \cdots <= v_r.d$

Corollary 22.4

Suppose that vertices v_i and v_j are enqueued during the execution of BFS, and that v_i is enqueued before v_j. Then $v_i.d \le v_j.d$ at the time that v_j is enqueued.

v_1	v_2	v_3	.	v_i	..	v_j	.		v_r

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Theorem 22.5 (Correctness of breadth-first search)

Let $G = (V, E)$ be a directed or undirected graph, and suppose that BFS is run on G from a given source vertex $s \in V$. Then, during its execution, BFS discovers every vertex $v \in V$ that is reachable from the source s, and upon termination, $v.d = \delta(s, v)$ for all $v \in V$. Moreover, for any vertex $v \neq s$ that is reachable from s, one of the shortest paths from s to v is a shortest path from s to $v.\pi$ followed by the edge $(v.\pi, v)$.

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Let some vertex v receives other distance

$$v.d \neq \delta(s,v)$$

v is not s.

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By Lemma 22.2, $v.d \geq \delta(s,v)$

Therefore, $v.d > \delta(s,v)$

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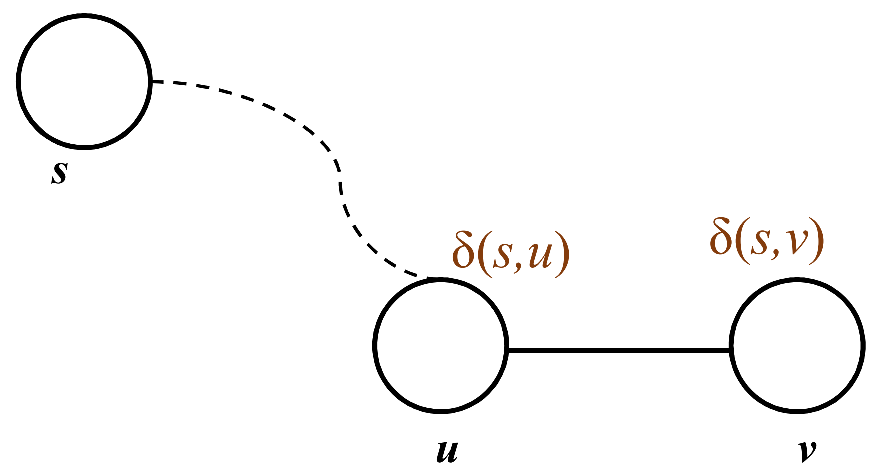
Let u be just the previous vertex on the shortest path to v



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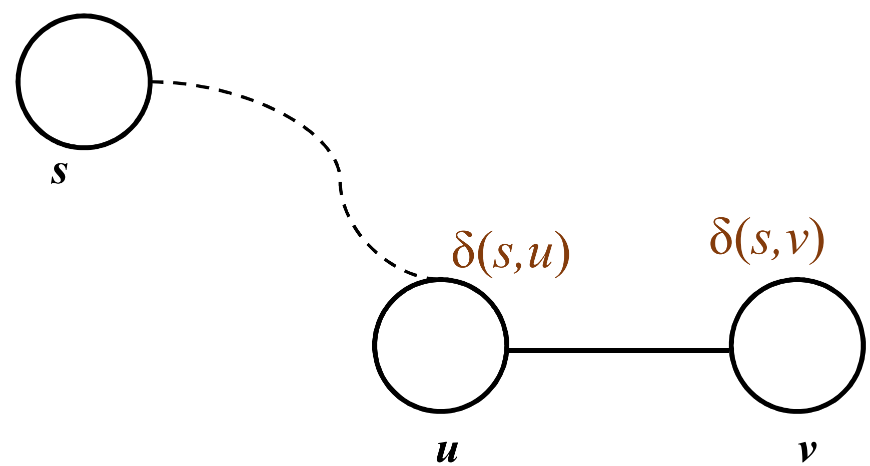

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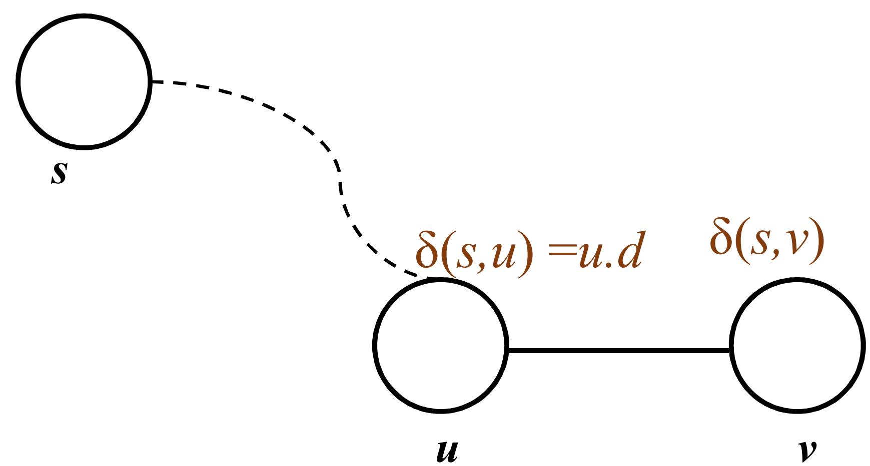
Assume v is the ONLY unlucky vertex: $v.d \neq \delta(s,v)$

For others $u.d = \delta(s,u)$



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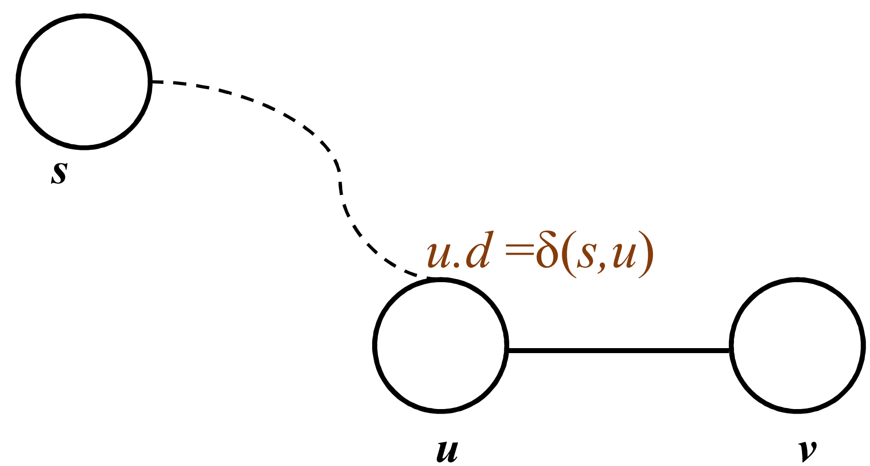
$$\delta(s,v) = \delta(s,u) + 1 \text{ and so, } \delta(s,u) < \delta(s,v)$$

Now, $v.d > \delta(s,v) = \delta(s,u) + 1 = u.d + 1$



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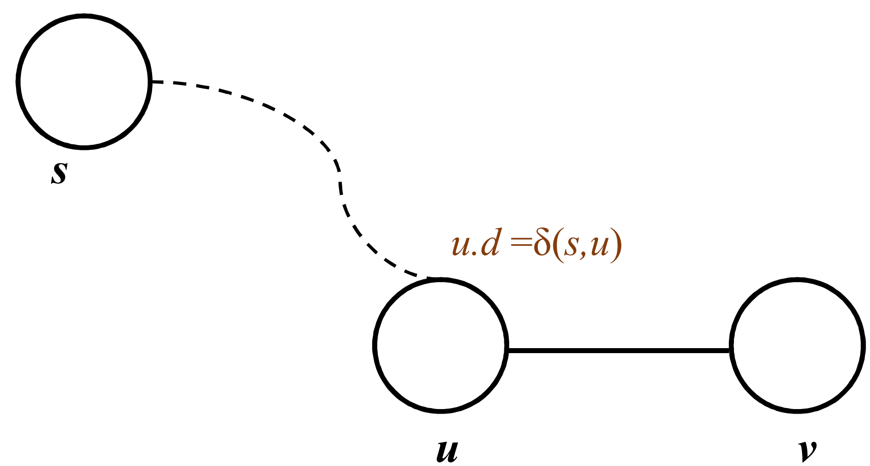

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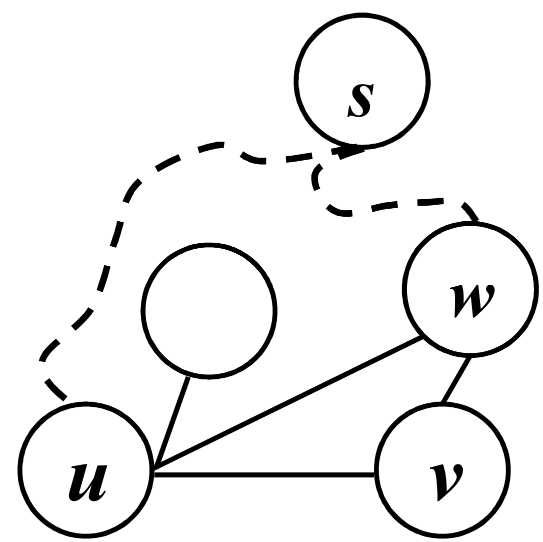
$$v.d \neq \delta(s,v)$$

We got, $v.d > u.d + 1$ (22.1)

When u is dequeued from Q:

Case 1: v is white

- $v.d = u.d + 1$ [Contradicts 22.1]



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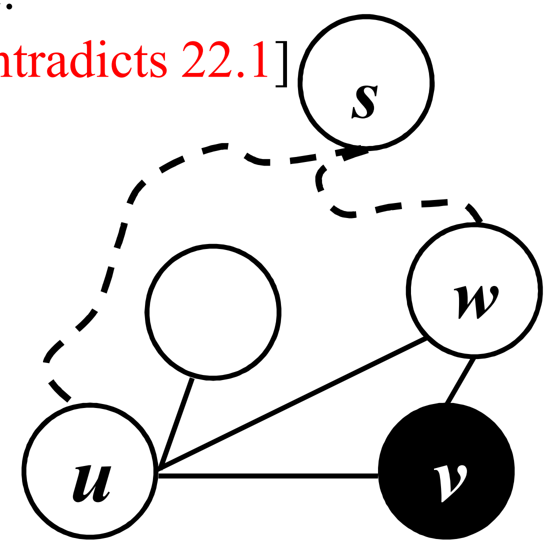
When u is dequeued from Q:

Case 1: v is white

- $v.d = u.d + 1$ [Contradicts 22.1]

Case 2: v is black

- v has been handled before u.
- Cor. 22.4=> $v.d <= u.d$ [Contradicts 22.1]



Corollary 22.4

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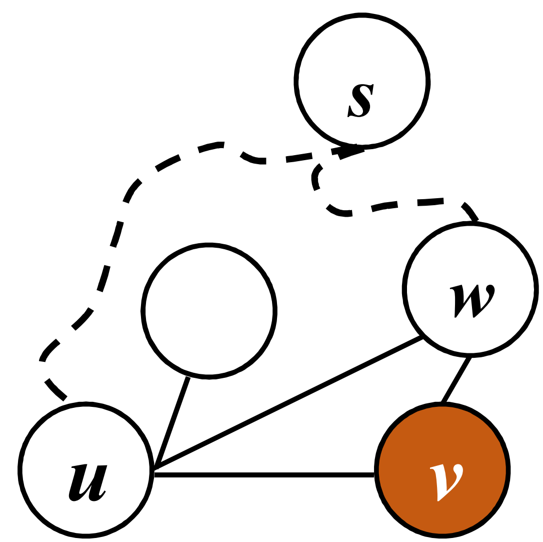
Let some vertex v receives other distance

$$v.d \neq \delta(s,v)$$

We got, $v.d > u.d + 1$ (22.1)

When u is dequeued from Q:

Case 3: v is GRAY



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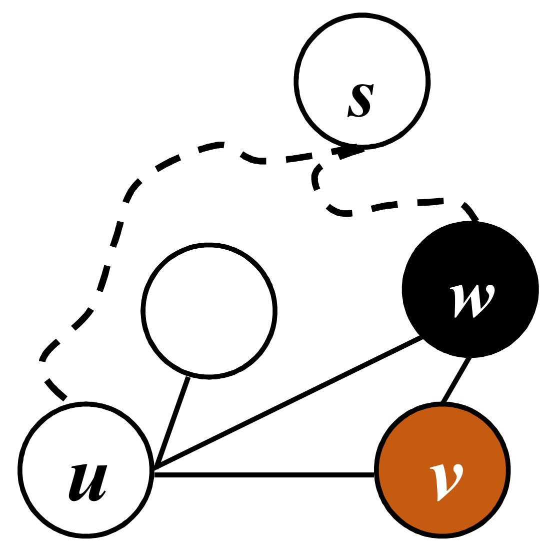
Case 3: v is GRAY

- someone else (not u) ‘painted’ it gray (say that vertex is w)



```
BFS(G, s)
for each vertex u ∈ G.V − {s}
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    u.π = NIL
s.color = GRAY
s.d = 0
s.π = NIL
Q = ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    u = DEQUEUE(Q)
    for each v ∈ G.Adj[u]
        if v.color == WHITE
            v.color = GRAY
            v.d = u.d + 1
            v.π = u
            ENQUEUE(Q, v)
    u.color = BLACK
```

Theorem 22.5 (Correctness of breadth-first search)

Let $G = (V, E)$ be a directed or undirected graph, and suppose that BFS is run on G from a given source vertex $s \in V$. Then, during its execution, BFS discovers every vertex $v \in V$ that is reachable from the source s, and upon termination, $v.d = \delta(s, v)$ for all $v \in V$. Moreover, for any vertex $v \neq s$ that is reachable from s, one of the shortest paths from s to v is a shortest path from s to $v.\pi$ followed by the edge $(v.\pi, v)$.

Let some vertex v receives other distance

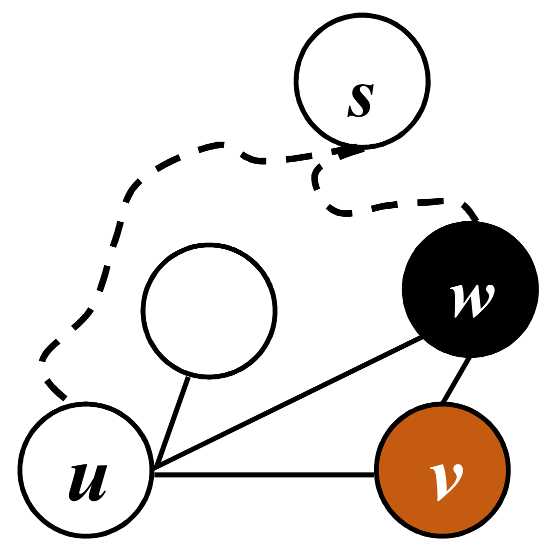
$$v.d \neq \delta(s,v)$$

We got, $v.d > u.d + 1$ (22.1)

When u is dequeued from Q:

Case 3: v is GRAY

- someone else (not u) ‘painted’ it gray (say w)
- $v.d = w.d + 1$ (A)



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Q = Ø
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while Q ≠ Ø
    u = DEQUEUE(Q)
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        if v.color == WHITE
            v.color = GRAY
            v.d = u.d + 1
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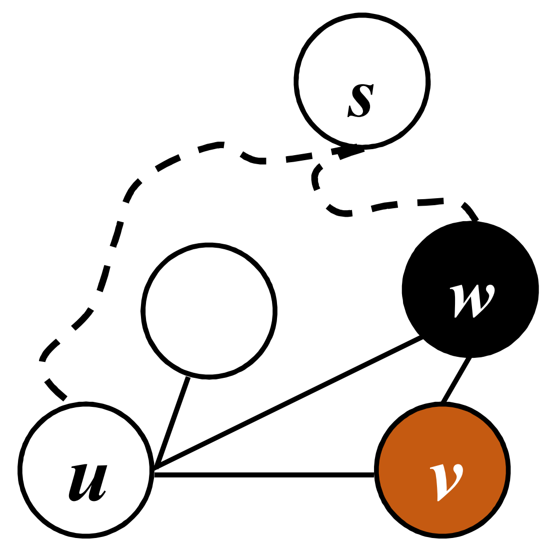
- Cor. 22.4=> $w.d <= u.d$ (B)



Corollary 22.4

Suppose that vertices v_i and v_j are enqueued during the execution of BFS, and that v_i is enqueued before v_j. Then $v_i.d \leq v_j.d$ at the time that v_j is enqueued.

```
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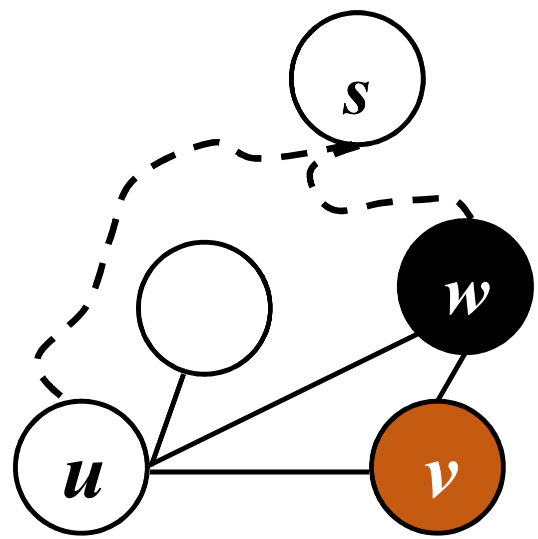
- (A) and (B) => $v.d <= u.d+1$

[Contradicts 22.1]



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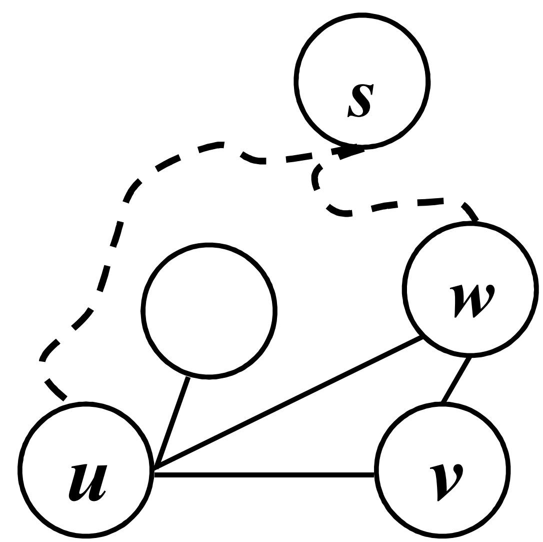
That means, $v.d = \delta(s,v)$



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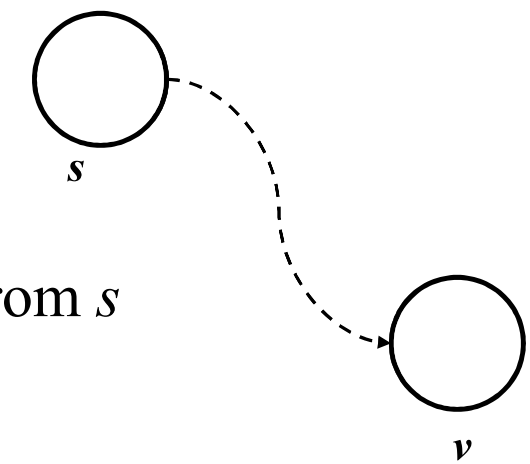

v is reachable from *s*

```
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Q = Ø
ENQUEUE(Q, s)
while Q ≠ Ø
    u = DEQUEUE(Q)
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        if v.color == WHITE
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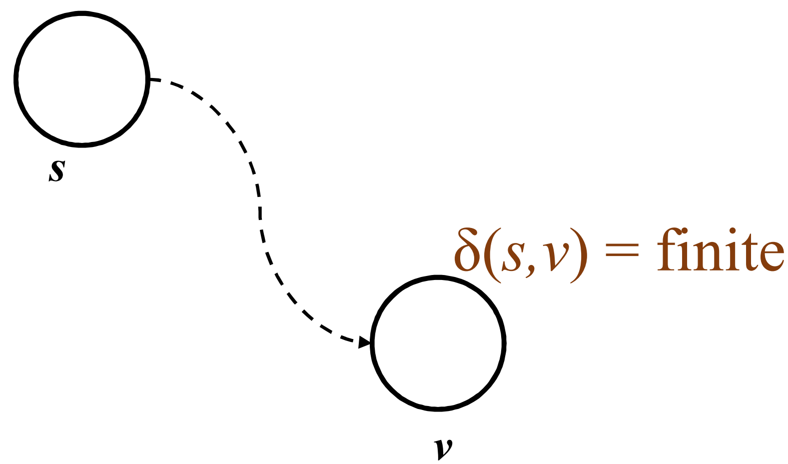

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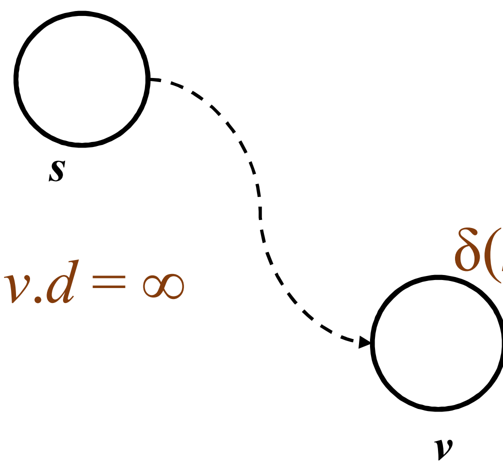

$\delta(s, v)$ = finite

If not discovered, $v.d = \infty$

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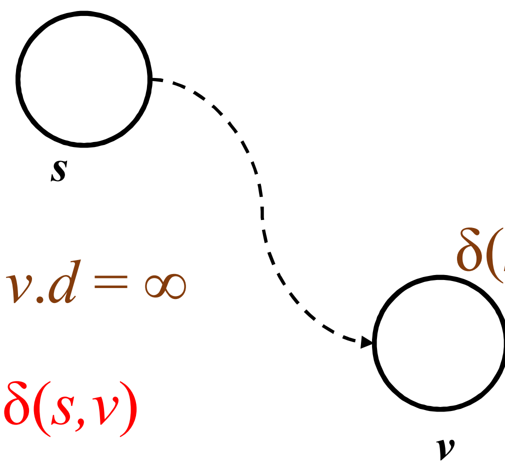

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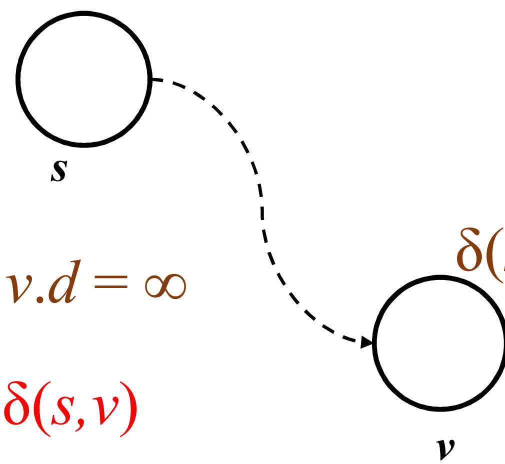

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If not discovered, $v.d = \infty$

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From the 2nd statement, $v.d = \delta(s,v)$

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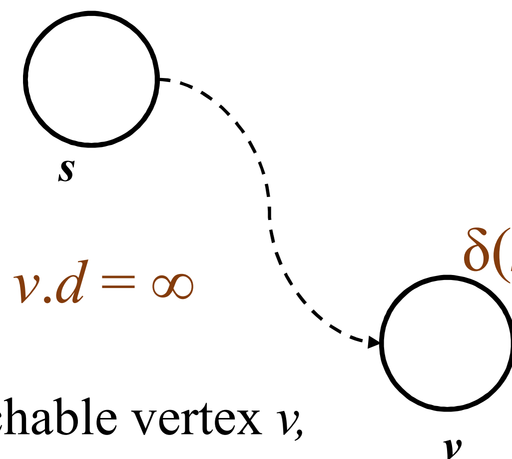

$\delta(s, v)$ = finite

If not discovered, $v.d = \infty$

This means, for a reachable vertex v,
$v.d$ is finite and discovered

```
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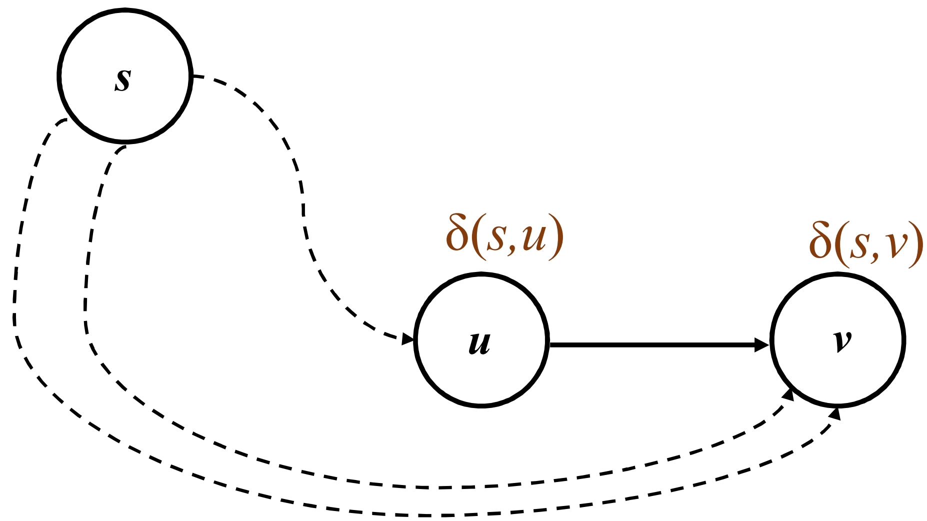

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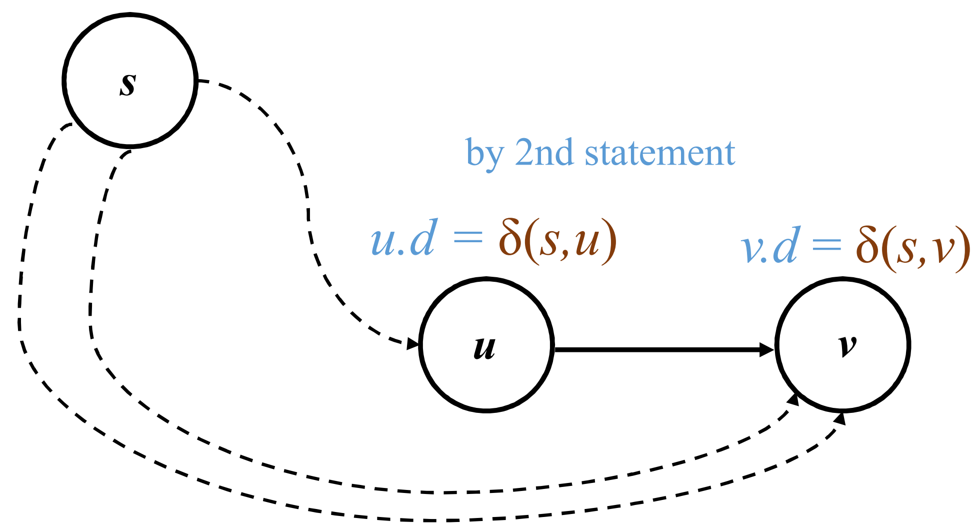

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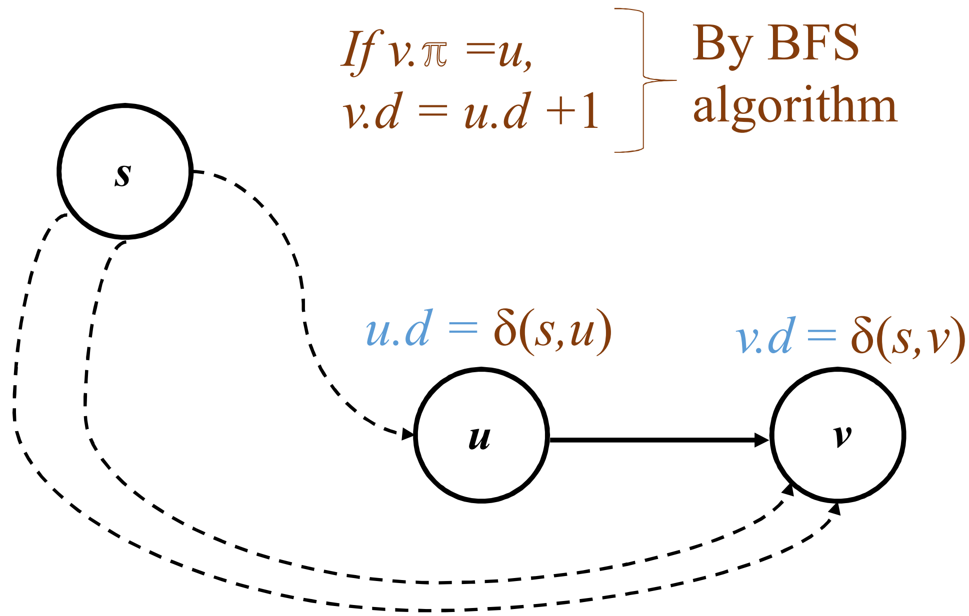

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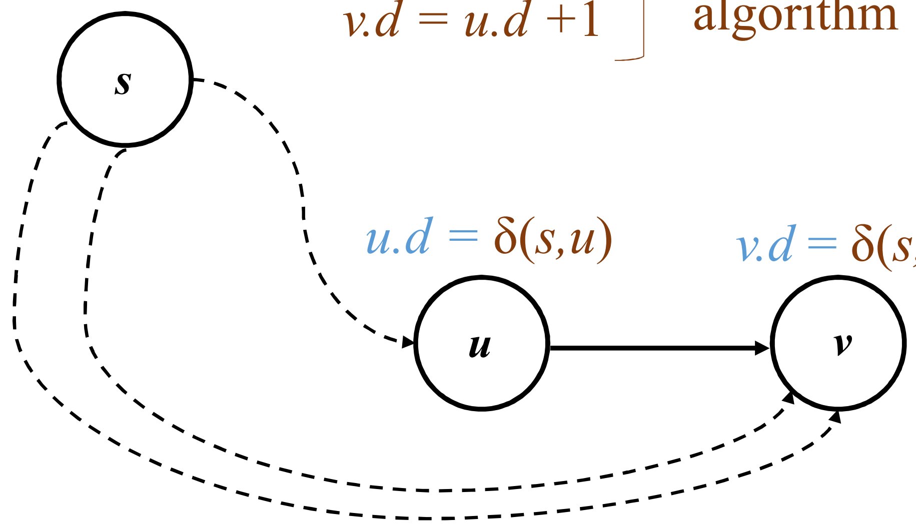
$\left.\begin{array}{l} \textit{If } v.\pi = u, \\ v.d = u.d + 1 \end{array}\right\}$ By BFS algorithm

combining all,

$\delta(s,v) = \delta(s,u) + 1$

$u.d = \delta(s,u)$ $\quad$ $v.d = \delta(s,v)$



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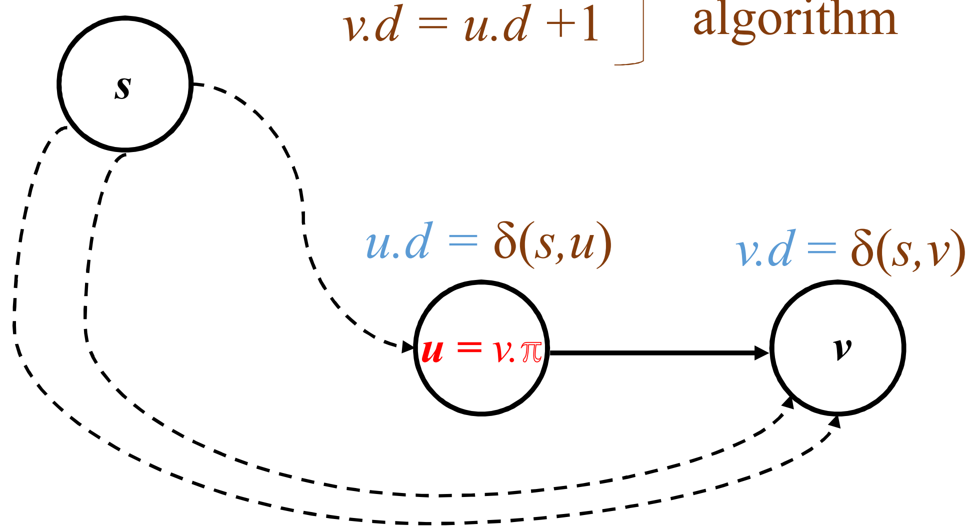
$$\delta(s,v) = \delta(s, v.\pi) + (u, v)$$



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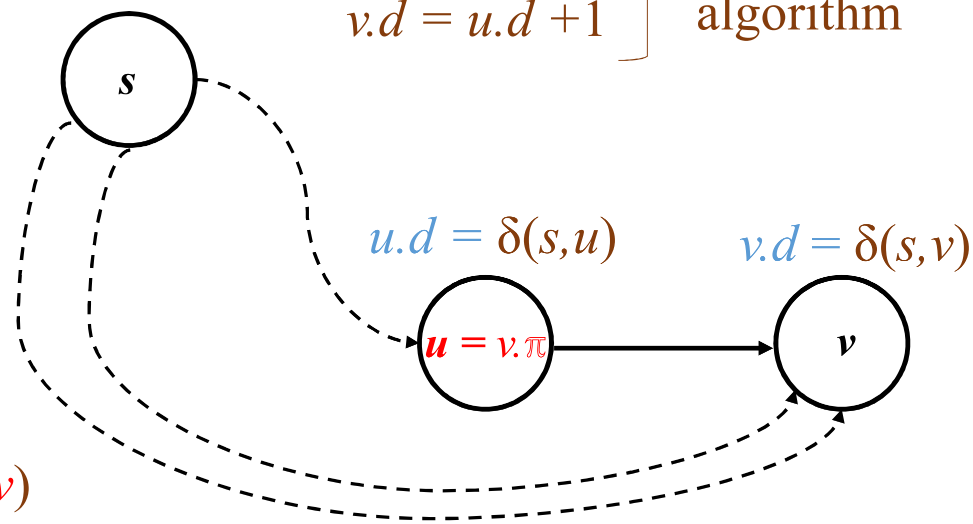
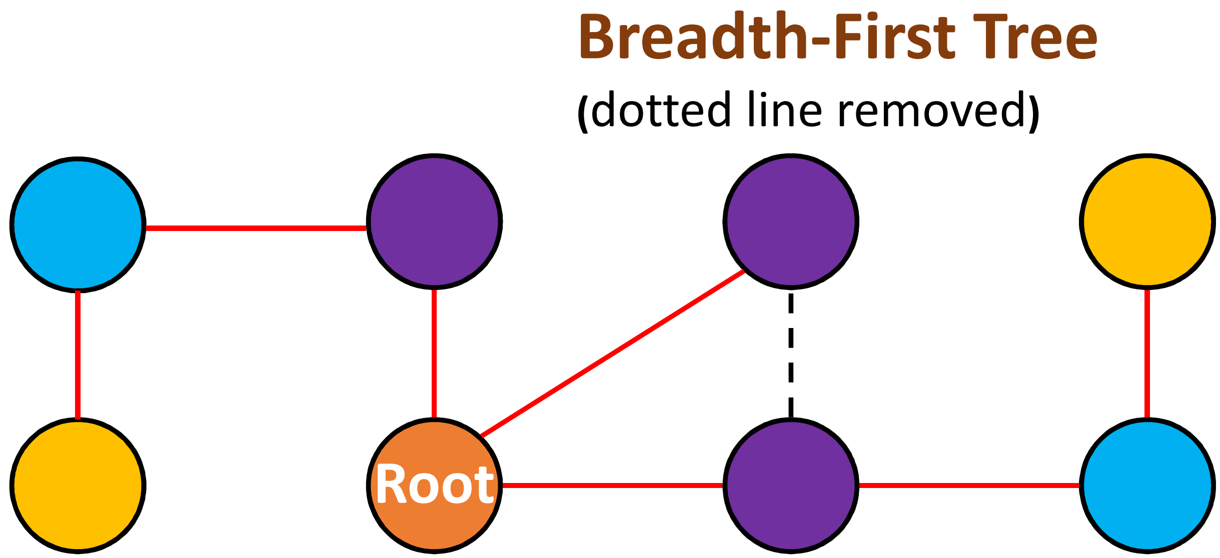
$\delta(s,v) = \delta(s,u) + 1$

$\delta(s,v) = \delta(s,\ v.\pi) + (u,\ v)$

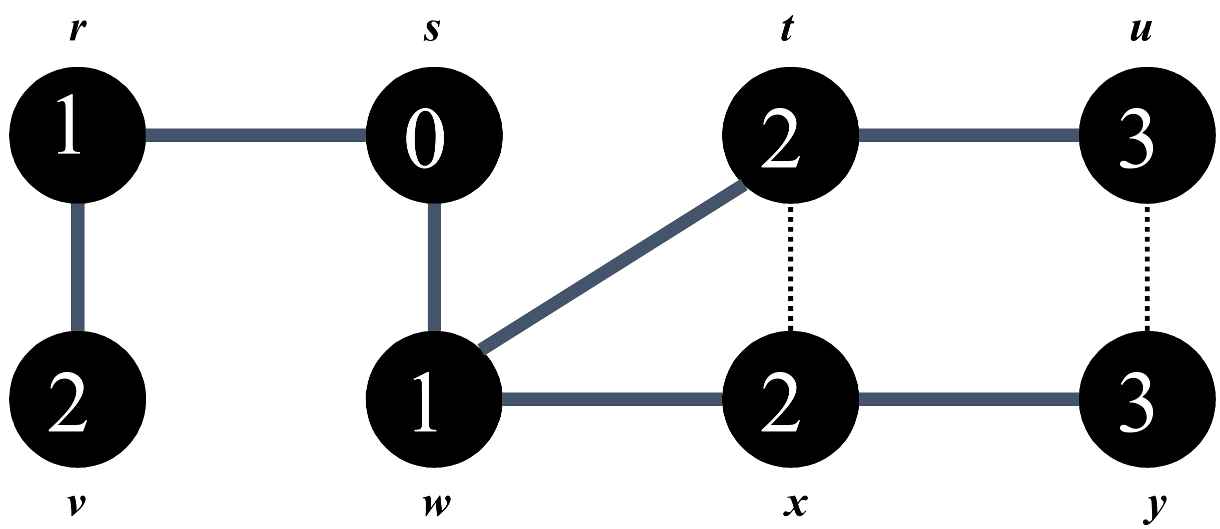
$\delta(s,v) = \delta(s,\ v.\pi) + (v.\pi,\ v)$



Breadth-First Tree



Breadth-First Tree

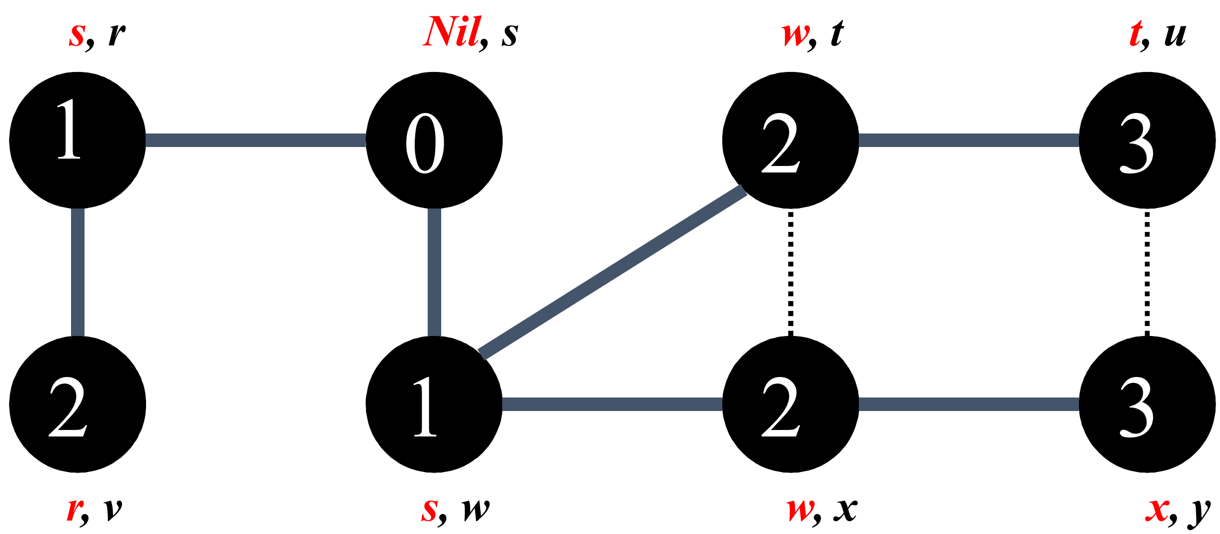


Breadth-First Tree

(dotted lines removed)

Breadth-First Tree

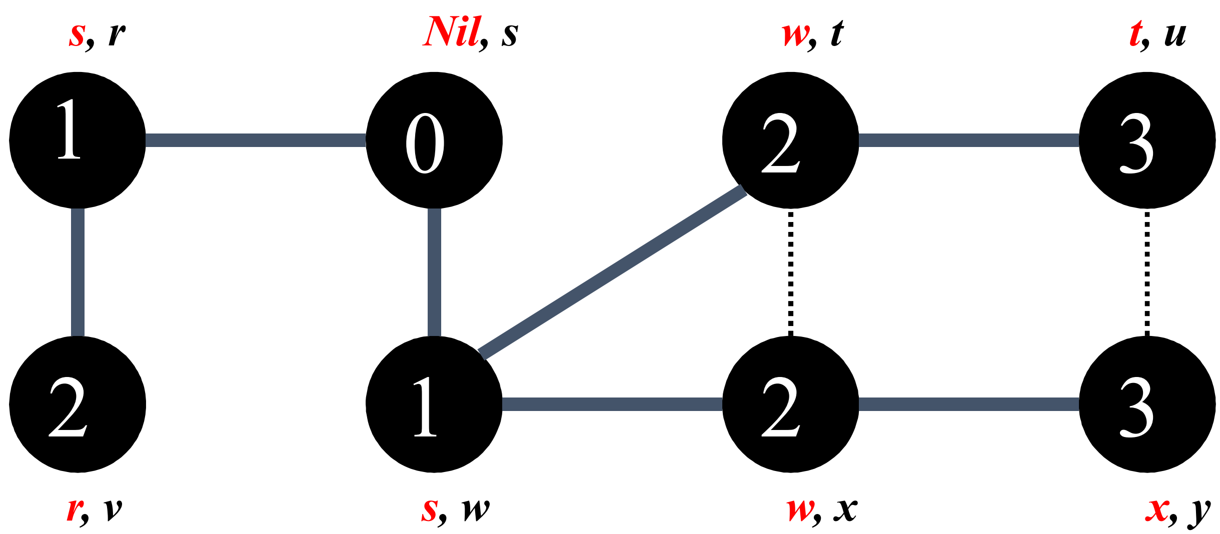
Parent is shown as
u*.π, *u



Breadth-First Tree
(dotted lines removed)

Breadth-First Tree

Parent is shown as
$u.\pi$, u

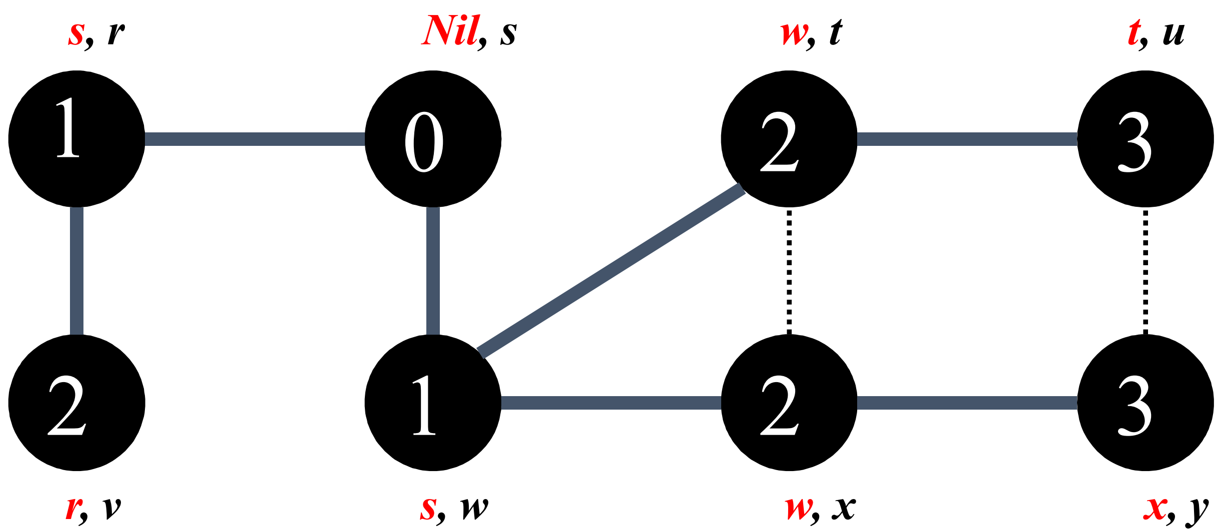


Edges in the Tree: $E_\pi = \{(s, w), (s, r), (w, t), (w, x), (r, v), (t, u), (x, y)\}$

Not included in E_π : $(t, x), (u, y)$

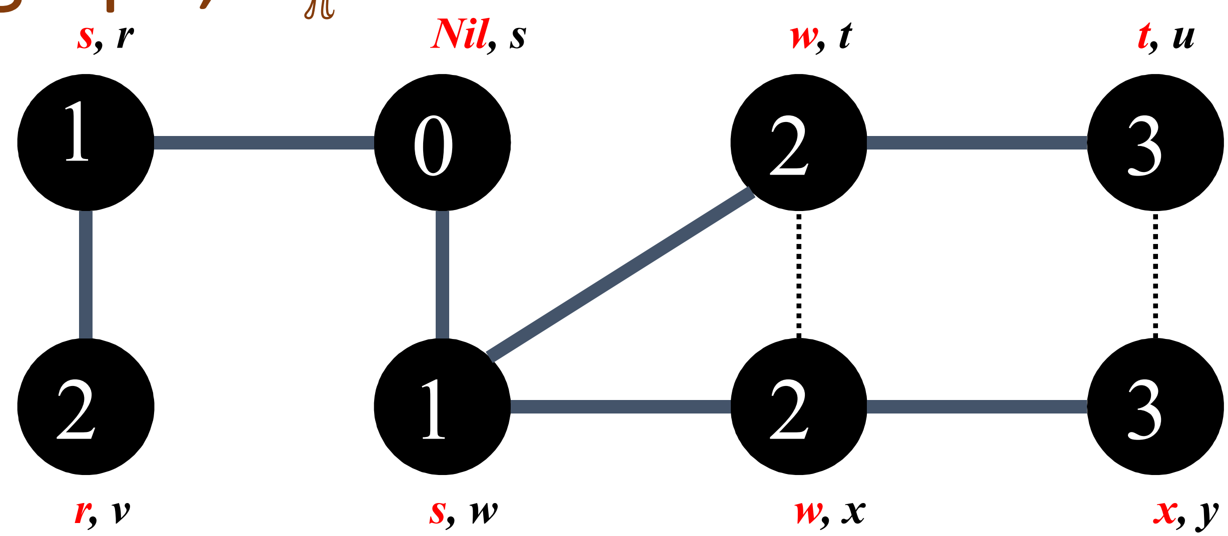
Predecessor Subgraph, G_π

- A graph where all predecessors are defined



Predecessor Subgraph, G_π

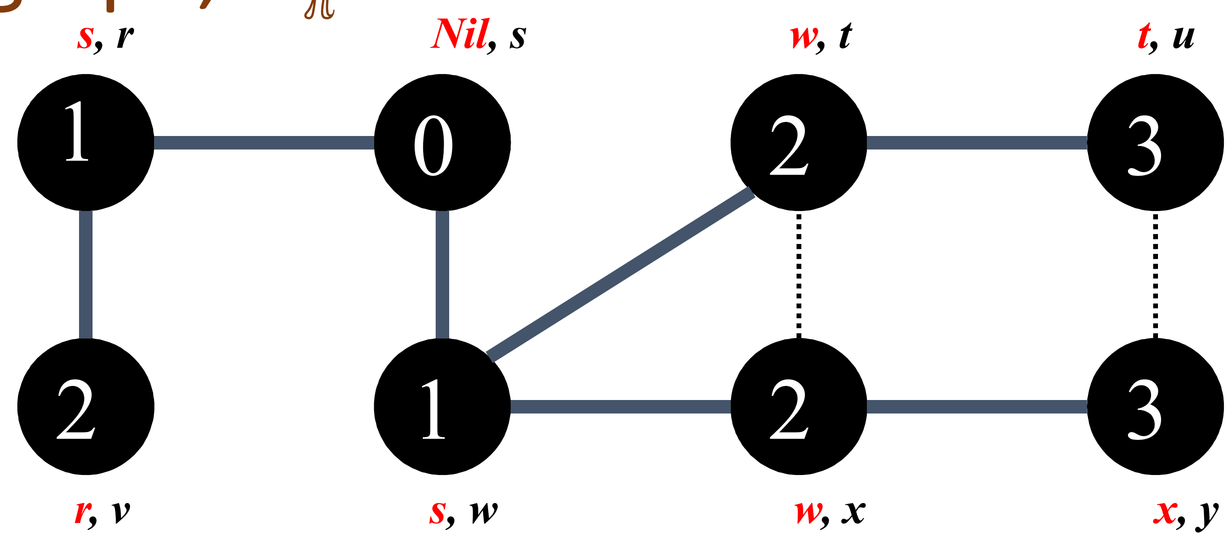
- More formally, for a graph $G = (V, E)$ with source s, we define the ***predecessor subgraph*** of G as $G_\pi = (V_\pi, E_\pi)$, where



$V_\pi = \{v \in V : v.\pi \neq \text{NIL}\} \cup \{s\}$ and

$E_\pi = \{(v.\pi, v) : v \in V_\pi - \{s\}\}$

Predecessor Subgraph, G_π



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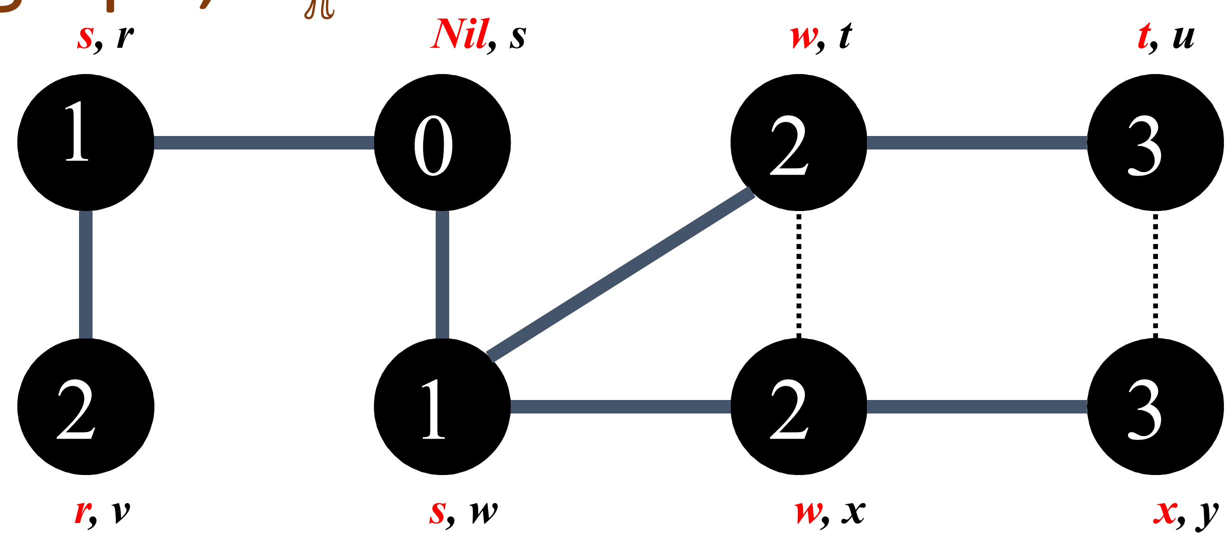
G_π is a tree? How?

Predecessor Subgraph, G_π



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G_π is a tree? How?

V_π consists of (1) vertex s plus

(2) those unique vertices that have a parent

Predecessor Subgraph, G_π

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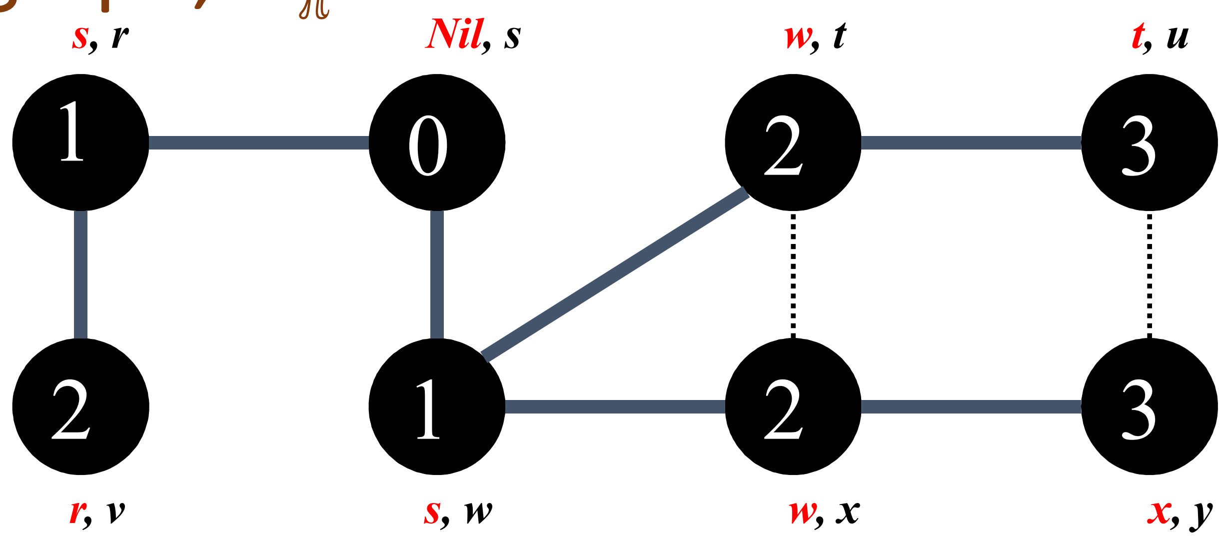

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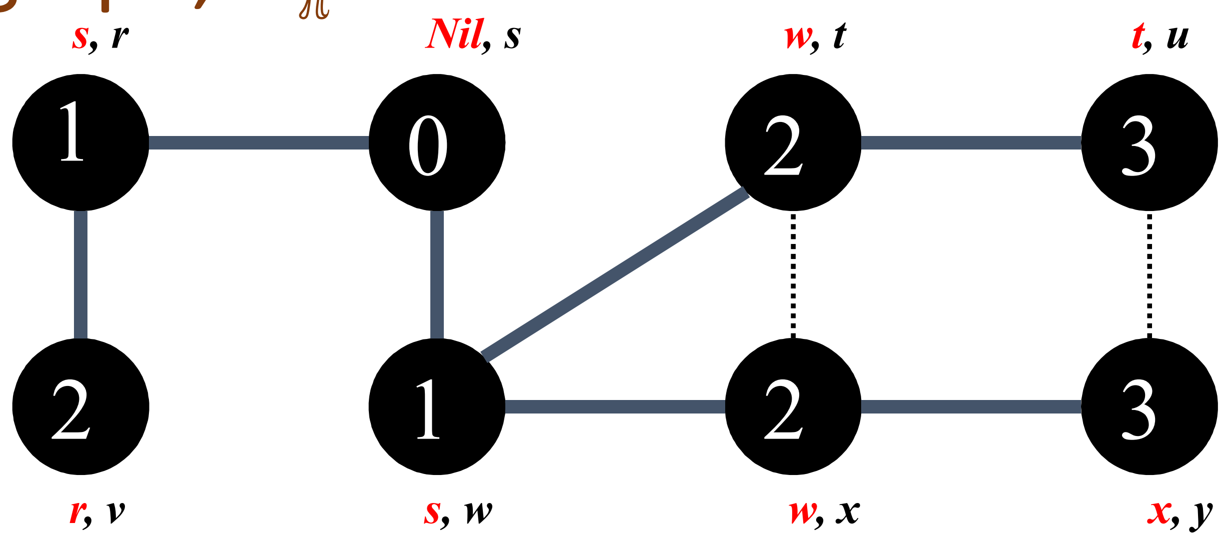
G_π is a tree? How?

E_π consists of edges from vertices of $\{V_\pi - \{s\}\}$ to their parents

ONLY s has NO connection to its parents

Predecessor Subgraph, G_π



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G_π is a tree? How?

$|E_\pi| = |V_\pi| - 1$

That means, G_π is a tree.

Predecessor Subgraph **and** Breadth-First Tree

- More formally, for a graph $G = (V, E)$ with source s, we define the ***predecessor subgraph*** of G as $G_\pi = (V_\pi, E_\pi)$, where

$V_\pi = \{v \in V : v.\pi \neq \text{NIL}\} \cup \{s\}$ and

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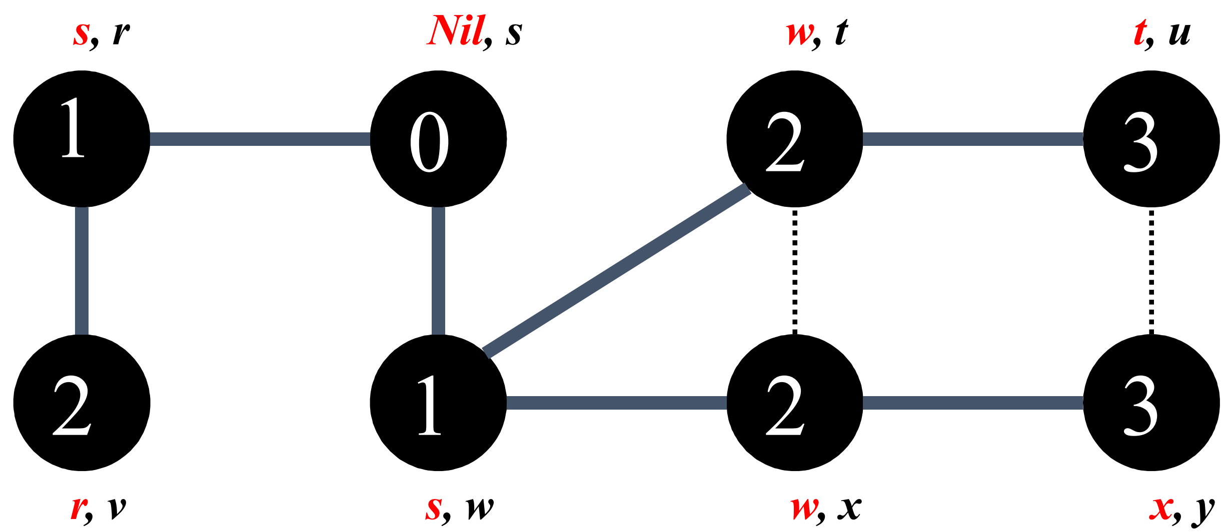

G_π is a ***breadth-first tree*** if V_π consists of the vertices reachable from s and, for all $v \in V_\pi$, the subgraph G_π contains a unique simple path from s to v that is also a shortest path from s to v in G.

```
BFS(G, s)
1  for each vertex u ∈ G.V − {s}
2      u.color = WHITE
3      u.d = ∞
4      u.π = NIL
5  s.color = GRAY
6  s.d = 0
7  s.π = NIL
8  Q = ∅
9  ENQUEUE(Q, s)
10 while Q ≠ ∅
11     u = DEQUEUE(Q)
12     for each v ∈ G.Adj[u]
13         if v.color == WHITE
14             v.color = GRAY
15             v.d = u.d + 1
16             v.π = u
17             ENQUEUE(Q, v)
18     u.color = BLACK
```

Lemma 22.6

When applied to a directed or undirected graph $G = (V, E)$, procedure BFS constructs π so that the predecessor subgraph $G_\pi = (V_\pi, E_\pi)$ is a breadth-first tree.

Line 16 sets $v.\pi = u$ iff (u, v) in E, and $\delta(s, v) < \infty$, that is v is reachable from s.

```
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This proves that V_π consists of all vertices reachable from s.

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As G_π forms a tree, it contains unique simple path from s to every vertex in V_π.

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This proves that V_π consists of all vertices reachable from s.

As G_π forms a tree, it contains unique simple path from s to every vertex in V_π.

Theorem 22.5 proves that each such path is a shortest path.

Depth-First Search

- Explore "deeper" in the graph whenever possible
- Edges are explored out of the most recently discovered vertex v that still has unexplored edges
- When all of v's edges have been explored, backtrack to the vertex from which v was discovered (i.e., its parent)

Depth-First Search

- Explore "deeper" in the graph whenever possible
- Edges are explored out of the most recently discovered vertex *v* that still has unexplored edges
- When all of *v*'s edges have been explored, backtrack to the vertex from which *v* was discovered (i.e., its parent)

- Vertices initially colored white
- Then colored grey when discovered
- Then black when finished

```
DFS(G)
for each vertex u ∈ G.V
    u.color = WHITE
    u.π = NIL
time = 0
for each vertex u ∈ G.V
    if u.color == WHITE
        DFS-VISIT(G, u)
```

```
DFS-VISIT(G, u)
time = time + 1
u.d = time
u.color = GRAY
for each v ∈ G.Adj[u]
    if v.color == WHITE
        v.π = u
        DFS-VISIT(G, v)
u.color = BLACK
time = time + 1
u.f = time
```

```
DFS(G)
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        v.π = u
        DFS-VISIT(G, v)
u.color = BLACK
time = time + 1
u.f = time
```

- records predecessors in π attributes
- Produces multiple trees

- we define the ***predecessor subgraph*** of G as $G_\pi = (V, E_\pi)$, where

  $E_\pi = \{(v.\pi, v) : v \in V \text{ and } v.\pi \neq NIL\}$

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        v.π = u
        DFS-VISIT(G, v)
u.color = BLACK
time = time + 1
u.f = time
```

- Records timestamps for each vertex, *v*
  - Discovery time, *d*: when *v* is discovered
  - Finishing time, *f* : when *v's adjacency list* is finished

```
DFS(G)
for each vertex u ∈ G.V
    u.color = WHITE
    u.π = NIL
time = 0
for each vertex u ∈ G.V
    if u.color == WHITE
        DFS-VISIT(G, u)
```

Θ(V)

Θ(V) EXCLUDING the time required for DFS-VISIT().

```
DFS-VISIT(G, u)
time = time + 1
u.d = time
u.color = GRAY
for each v ∈ G.Adj[u]
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        DFS-VISIT(G, v)
u.color = BLACK
time = time + 1
u.f = time
```

```
DFS(G)
for each vertex u ∈ G.V
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time = 0
for each vertex u ∈ G.V
    if u.color == WHITE
        DFS-VISIT(G, u)
```

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for each v ∈ G.Adj[u]
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        DFS-VISIT(G, v)
u.color = BLACK
time = time + 1
u.f = time
```

$\sum_{v \in V} |Adj[v]| = \Theta(E)$

Θ(V + E)

How many times DFS-Visit() is called?

- The procedure DFS-VISIT is called exactly once for each vertex since:
  - the vertex u on which DFS-VISIT() is invoked must be white
  - the first thing DFS-VISIT does is paint vertex u gray