INTERNSHIP REPORT ON

Modeling and Forecasting Volatility of DS30 Index Using Time Series Models



Shadman Saquib Rahman

Department of Economics, University of Dhaka

Intern, Financial Stability Department, Bangladesh Bank

Internship Report on

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Dhaka-1000, Bangladesh

Submitted by

Shadman Saquib Rahman

Department of Economics, University of Dhaka

Intern, Financial Stability Department, Bangladesh Bank

Supervised by

Uttam Chandra Paul

Joint Director

Financial Stability Department, Bangladesh Bank

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Executive Summary

In this report, the monthly volatility of DS30 index is modelled using time series analysis over from the period January 2013 to June 2024. After estimating and comparing ARMA-ARCH, ARMA-GARCH, ARMA-EGARCH and TGARCH models, results show that over the given period, the EGARCH(3,1) model most accurately estimates the volatility of Bangladesh's primary blue-chip index. Volatility or the fluctuation in stock prices is a key characteristic of the capital market as it not only reflects the changes in the intrinsic value of the security in response to different macroeconomic and firm-level transformations but also because it creates opportunities for investors to earn greater profits. The findings of the report are valuable for analysts, portfolio managers and retail investors trying to estimate or model the volatility of the Dhaka Stock Exchange 30 index or gauge market sentiment. The report also forecasts future volatility of the DS30 based on the model selected for the next six months.

The views, opinions and observations expressed in this report are entirely the author's and do not necessarily reflect that of Bangladesh Bank.

1. Introduction

'Blue chip' refers to, in the business world, trustworthy businesses that are leaders in the industries they operate in and consistently generate profits. An index of affluent, widely recognized, and top businesses is called a blue-chip index. A well-known indicator for the Bangladeshi capital market, the Dhaka Stock Exchange 30 (DS30) index monitors the performance of the most well-established and stable companies in the country. In other words, 30 of the most active and financially secure businesses across several industries are included in the DS30. The free-float market capitalization technique is used to create the index, which was introduced in early 2013. To make sure it still represents the most recent circumstances, it is updated around twice a year. It is crucial for investors, both international and domestic, to understand the performance of the Bangladeshi stock market through the DS30. The current volatility of the index represents the fluctuating intrinsic value of these companies in adapting to the complex interplay of macroeconomic changes and firm-level adjustments.

In Bangladesh, the DS30 Index gives investors access to the biggest and most well-known stocks, ensuring stability and a dominant position in the market. It also reduces exposure to any one industry by providing diversity across several industries and sectors. Because of their larger client bases, steady revenue streams, and stronger balance sheets, the index is less volatile than smaller, less established stocks. By comparing investments to the overall market, it acts as a benchmark for market performance. It does have some drawbacks, though, including a lack of exposure to smaller, rapidly expanding businesses and an underrepresentation of several industries, like technology and healthcare. In general, the DS30 Index might not give a precise view of how the market is doing. Therefore, it is crucial for investors of all stripes—from analysts and portfolio managers to individual retail investors navigating the volatile world of finance—to comprehend the inherent oscillations, or volatility, inside this index.

Through comprehensive analysis of volatility trends, investors can make informed decisions that may open up opportunities for substantial profit. This paper conducts a thorough investigation of the monthly volatility available in the DS30 index using time series models, spanning a substantial timeframe from January 2013 to June 2024.

Some of the models estimated and compared include the ARMA-GARCH, ARMA-ARCH, ARMA-EGARCH, and the adaptable TGARCH model. This report attempts to identify which model best captures the volatility of the DS30 index during the selected period through a rigorous process of estimation and evaluation.

In addition to providing insight on the DS30 index's historical volatility, this report aims to provide investors with a powerful lens through which to see Bangladesh's blue-chip market's future direction.

The report is structured into 5 sections other than the references and appendix section. Section 2, includes a brief literature review to identify the research gap, upon which the methodology section (Section 3) expands upon. Section 4 presents the results of the report in detail and Section 5 discusses the findings of the report and draws an overall conclusion.

2. Literature Review

estimated the volatility of the Dhaka Stock Exchange (DSE) comprising over the period 2002 to 2013 using a GARCH (1,1) model and noted the high period of volatility during 2010 reflecting the stock market crash of 2010-11 and how the variance in the index tapered off afterwards. In a smaller scope, Miah et al. (2016) sampled the closing prices of four companies from the Dhaka Stock Exchange from the period 2000 to 2014 on a daily and monthly frequency and concluded that the price volatility of all four companies followed a random walk process that could not be modelled without further transformation. Later, Miah & Rahman (2016) concluded that for the four companies, GARCH(1,1) model was the most efficient for estimating daily stock returns volatility. Finally, Pervez et al. (2018) evaluated the Weak Form Efficiency of the DSE General Index, DSE Broad Index and DS30 Index from 2004 to 2018 and found that only the latter followed a random walk process while the former were not as efficient. This brief review highlights scope for further research in modelling the volatility of the Dhaka Stock Index and its subindices over not only a longer time frame but also using more diverse models like TGARCH, EGARCH and ARMA-ARCH.

3. Methodology

Data Collection

The daily closing prices for the DS30 index for 10 years from January 1st, 2014 to June 30, 2024 were collected from Investing.com. The data was averaged on a monthly frequency as ' P_i ' to account for missing data. The collected data was arranged into a time series for further analysis using Stata 17 software.

Unit Root Test

To determine whether or not the DS30 index is non-stationary time-series process, Augmented Dickey-Fuller (ADF) test was carried out. Further linear transformations like differencing/detrending were carried out to achieve stationarity (constant mean. autocovariance, homoskedasticity) if necessary.

Box-Jenkins Method for ARIMA Model Selection

Given we achieve stationarity for the DS30 index (transformed or not), we can estimate Autoregressive Integrated Moving Average (ARIMA) models for the index and carry out model selection based on:

- 1. Autocorrelation function (ACF) and Partial autocorrelation (PACF) functions
- 2. Portmanteau Q Test (to ensure estimated residuals follow a white noise process)
- 3. Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)

Lagrange Multiplier Test for ARCH effects

After modelling the conditional mean of the time series using the most efficient ARIMA(p,q) model, we test the homoskedasticity assumption of the estimated residuals using both a histogram and the Autoregressive Conditional Heteroskedasticity (ARCH) Lagrange Multiplier test. If heteroskedasticity is detected from the test, we try model the variance in the estimated residuals, $\hat{\epsilon}$ or the volatility in index returns/value using ARCH, ARIMA-ARCH, ARIMA-GARCH, GARCH, ARIMA-EGARCH, TGARCH models. To estimate the most efficient model the conditional means for the time series that is a component for estimating conditional variance will be with/without ARIMA models before fitting the different conditional heteroskedasticity models.

Model Estimation and Selection

The volatility models we will estimate in the presence of heteroskedasticity will be generally specified as follows:

Autoregressive Integrated Moving Average, ARIMA (p,d,q) Model:

$$R_t = (1 - L)^d P_t$$

Where, 'd' is the number of differencing need to achieve stationarity for the monthly DS30 index values. ' R_t ' is the 't' monthly return from the index if 'd' = 1 (See Results). 'L' is a lag operator where $L^i(X_t) = X_{t-i}$

$$\left(1 - \sum_{i=1}^{p} \varphi_i L^i\right) R_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \varepsilon_t$$

Where, ε_t refers to the white-noise residual term ($\varepsilon_t \sim i.i.d\ N(0,\sigma^2)$). This model is used to estimate the conditional mean, \bar{R} of the time series given that stationarity is achieved.

Autoregressive Conditional Heteroskedasticity, ARCH (q) Model:

Given the variance of the time series is heteroskedastic, we can estimate the conditional variance, σ_t^2 for the residual term ε_t of the time series using Engle's (1982) ARCH model. In broad strokes, the model assumes that σ_t^2 follows an autoregressive process.

$$\varepsilon_t = \sigma_t w_t$$
, where 'w_t' is a white noise process

$$\sigma_t^2 = \omega + \beta_1 \varepsilon_{t-1}^2 + \dots + \beta_q \varepsilon_{t-q}^2 = \beta_0 + \sum_{i=1}^q \beta_i \varepsilon_{t-i}^2$$
, here $\omega > 0$, $\beta_i \ge 0$, $i > 0$

Generalized Autoregressive Conditional Heteroskedasticity, GARCH (p,q) Model:

Bollerslev & Ghysels (1994) specify the GARCH model assuming that σ_t^2 has both an autoregressive and a moving average component as specified below:

$$\sigma_t^2 = \omega + \beta_1 \varepsilon_{t-1}^2 + \dots + \beta_q \varepsilon_{t-q}^2 + \alpha_1 \sigma_{t-1}^2 + \dots + \alpha_p \sigma_{t-p}^2 = \sum_{j=1}^q \beta_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \alpha_i \sigma_{t-j}^2,$$

$$\text{here } \omega > 0, \ \beta_i \ge 0, \ i > 0 \text{ and } \alpha_i \ge 0$$

Exponential Generalized Autoregressive Conditional Heteroskedasticity, EGARCH (p,q) Model:

Nelson (1991) suggested the EGARCH model to estimate the conditional variance of ε_t in a logarithmic form to relax the non-negativity constraint on the estimated parameters that GARCH models impose. As a result, this flexible model is more suitable for modelling market indices, as the author did for the CRSP Value-Weighted Market Index.

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^p \beta_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^q \alpha_i \ln(\sigma_{t-j}^2) + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}}, \text{ assuming } \gamma \neq 0$$

Threshold Generalized Autoregressive Conditional Heteroskedasticity, TGARCH (p,q) Model:

The TGARCH model accounts for the asymmetry in the volatility associated with financial markets (for example, where negative news may result in greater volatility in the DS30 index than positive news) by introducing a synthetic variable, ' d_{t-1} '(Zakoian, 1994).

$$d_{t-1} = \begin{cases} 1, \ \varepsilon_{t-1} < 0 \\ 0, \ \varepsilon_{t-1} \ge 0 \end{cases},$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \beta_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \alpha_i \sigma_{t-j}^2 + \sum_{k=1}^r \gamma_k \varepsilon_{t-k}^2 d_{t-k}$$

Once these models have been estimated, their respective AIC and BIC scores were used and compared to select the most efficient model.

4. Results

The following section shares the results of the tests and models estimated from the methodology outlined in the previous section.

Unit Root Test

The ADF (See <u>Table 2</u> in Appendix) test of the time series of monthly DS30 prices, P_t revealed that the process was a random walk process. So to achieve stationarity, first differencing of P_t was carried out such that returns, $R_t = \Delta P_t = P_t - P_{t-1}$. Further ADF test (<u>Table 2</u>) of returns revealed that stationarity was achieved, so ARIMA (p,1,q) model for prices or ARIMA(p,q) models for returns. <u>Figure 2</u> and <u>Table 1</u> present the descriptive statistics of R_t .

Box-Jenkins ARIMA Model Selection

The ACF and PACF (See Figure 3) show there is significant autocorrelation among the lag terms of R_t and that ARMA(p,q) models may be suitable. Using both the Portmanteau Q-test and the AIC/BIC Model Selection Criterion, Autoregressive (AR), Moving Average (MA) and ARMA(1,1) to ARMA(5,5)

models were estimated and compared. Only the AIC/BIC scores of models that passed the Q-test were considered as the estimated residuals for those specific ARIMA models are uncorrelated. Results (See <u>Table</u> 3) show that ARMA(5,1) is the most efficient model to estimate the conditional mean of the time series of monthly returns of DS30.

Lagrange Multiplier Test for ARCH effects

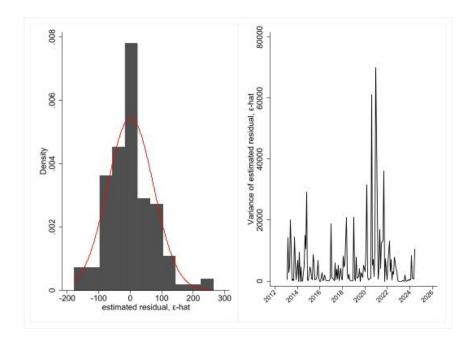
The ARMA(5,1) model can only be used to estimate the average monthly return of DS30 if the homoskedasticity assumption of the estimated residual term, $\bar{\varepsilon}_t$. Otherwise, the conditional heteroskedasticity of ARMA(5,1) model will need to be estimated separately using the models specified earlier (ARCH, GARCH, etc.).

Graphically, plotting the histogram of the estimated residuals of the ARMA(5,1) model hints at a non-normal distribution while the line graph of the variance of the estimated residuals reveals a noisy pattern signaling a violate of the homoskedasticity assumption and prompting more empirical tests like the Lagrange Multiplier test.

The Lagrange Multiplier test for ARCH effects reveal that there is significant clustering of volatility or ARCH effects at lags p = 4 and onwards for our estimated residuals (See <u>Table 4</u> in Appendix). Based on the findings, it can be inferred that not only can ARCH models be used to model volatility of the estimated residuals for the ARMA model but also that GARCH or more flexible EGARCH or TGARCH models may be suitable.

Figure 4

Evidence of heteroskedasticity of the ARMA(5,1) model residuals



Note. Left: Histogram of the residuals, red line shows normal distribution. Right: Noisy variance of estimated residuals $(\bar{\varepsilon}_t^2)$

Model Comparison and Selection

Using AIC/BIC Model selection criteria, a wide range of models were considered to model the volatility of the average monthly returns of DS30 index. Specifically, ARCH, GARCH, EGARCH, TGARCH, ARMA-ARCH, ARMA-GARCH, ARMA-EGARCH and ARMA-TGARCH models were estimated and their AIC/BIC scores tabulated post estimation. From <u>Table 5</u> (See Appendix), it can be concluded that EGARCH(3,1) is the most efficient model to estimate the volatility of DS30 Index over the sample time period. It was observed ARMA integrated volatility models were more efficient with lower AIC/BIC scores.

Model Estimation and Visualization

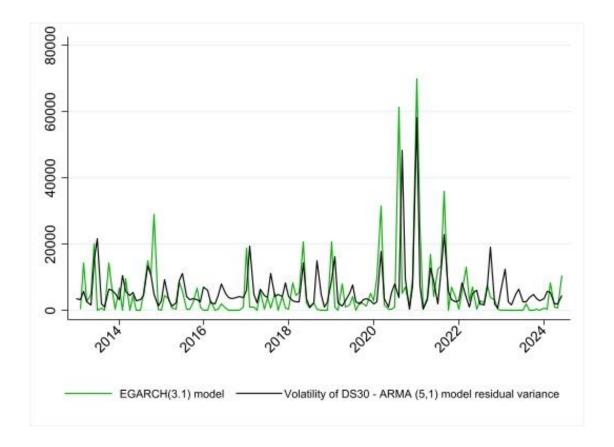
For our estimated EGARCH(3,1) Model, the estimated coefficients are:

$$\ln(\overline{\sigma}_{t}^{2}) = 11.63 + 0.13 * \left| \frac{\overline{\varepsilon}_{t-1}}{\overline{\sigma}_{t-1}} - E\left(\frac{\overline{\varepsilon}_{t-1}}{\overline{\sigma}_{t-1}}\right) \right| - 0.71 * \left| \frac{\overline{\varepsilon}_{t-2}}{\overline{\sigma}_{t-2}} - E\left(\frac{\overline{\varepsilon}_{t-2}}{\overline{\sigma}_{t-2}}\right) \right| + 0.19 * \left| \frac{\overline{\varepsilon}_{t-3}}{\overline{\sigma}_{t-3}} - E\left(\frac{\overline{\varepsilon}_{t-3}}{\overline{\sigma}_{t-3}}\right) \right| + 0.56 * \ln(\overline{\sigma}_{t-1}^{2}) + 0.27 * \frac{\overline{\varepsilon}_{t-1}}{\overline{\sigma}_{t-1}}$$

The equation was structured from the regression output from the model estimation calculations. We can also graphically visualize the how closely the estimated variance from the EGARCH(3,1) match the variance of the estimated residuals of the stationary ARMA(5,1) model as shown below:

Figure 5

Estimated Volatility Juxtaposed To Actual Volatility of Monthly DS30 Returns



Note. Graph is computed from author's calculation.

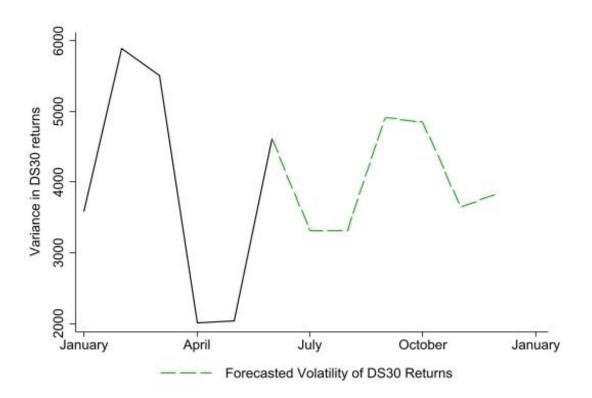
It is apparent that the EGARCH(3,1) model can be used to model the volatility of the DS30 Index monthly returns based on the sampled data. Moreover, it can be used to forecast the future volatility and help investors make informed decisions.

Forecast

One of the major applications of estimating volatility models is forecasting the future levels for risk management purposes from an investor's perspective.

Figure 6

Future Volatility of Monthly DS30 Returns based on EGARCH(3,1) Model



Note. Forecasted volatility for DS30 monthly returns for 2024

The diagram illustrates that the next six months are likely to be less volatile than the previous few months, making DS30 stocks a worthwhile and stable assets which asset managers and investors can consider holding in their portfolio.

5. Conclusion

The results of this report suggest that the volatility of the returns of the DS30 index from the period, January 2014 to June 2024 is best estimated by the EGARCH(3,1) model where compared to alternative time-series models like ARCH, GARCH and the baseline, GARCH(1,1) model. Volatility of the index of Bangladesh's blue chip companies is an important metric for portfolio managers, investors and traders alike as it signals the risk and opportunity associated with their holdings. The empirically test model highlighted can be leveraged by these stakeholders to forecast volatility effectively for an index fewer research has been done. This report is limited by the types of conditional heteroskedasticity models estimated and compared, future research could include the usage of newer models that incorporate machine learning, investigate the time series using daily closing prices over a longer period of time and compare how the volatility differs among different Bangladeshi indices like the Chittagong Stock Exchange or the Dhaka Stock Exchange Broad Index.

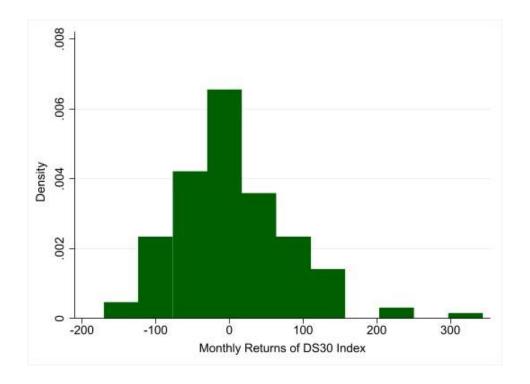
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Appendix

Figure 2

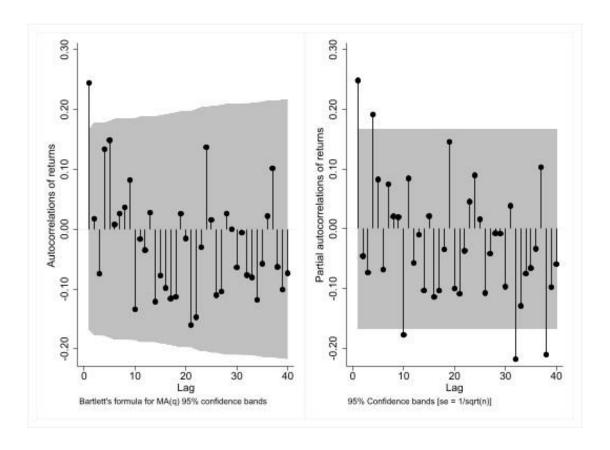
Distribution of Monthly Average Returns of DS30



Note. The figure shows the histogram of the monthly returns of the DS30 Index. Source: Author's calculations

Figure 3

ACF and PACF of Monthly Average Returns of DS30



Note. The ACF and PACF highlight the suitability of ARMA(p,q) model

Table 1

Descriptive Statistics of Monthly Returns of DS30

Observations	137
Mean	2.69
Median	-3.54
Maximum	344
Minimum	170
Std. Dev.	76.6
Skewness	0.97
Kurtosis	5.37

Note. The summary statistics are based on the author's calculations

Table 2

ADF Test of Monthly Average Price and Monthly Returns of DS30

Test Statistics Dicky-Fuller critical value

		1%	5%	10%
Price, P _t	-2.038	-4.028	-3.445	-3.145
Returns, R _t	-6.718	-4.029	-3.445	-3.145

Note. The table shows that the time series for price is a non-stationary process while that for returns is stationary

Table 3

AR, MA, ARMA Model Selection that passed Portmanteau Q test

Conditional Mean Equation	AIC	BIC
AR(1)	1571.969	1577.809
AR(2)	1580.527	1586.366
AR(4)	1577.932	1583.772
AR(5)	1577.266	1583.106
MA(1)	1572.191	1578.031
MA(2)	1580.538	1586.378
MA(4)	1578.095	1583.935
MA(5)	1575.992	1581.832
ARMA(1,1)	1573.814	1582.574
ARMA(1,2)	1573.905	1582.665
ARMA(1,3)	1571.615	1580.375
ARMA(1,4)	1571.371	1580.131
ARMA(1,5)	1570.013	1578.773
ARMA(2,1)	1573.935	1582.694
ARMA(2,4)	1580.063	1588.823
ARMA(2,5)	1577.853	1586.613
ARMA(3,1)	1572.389	1581.149
ARMA(3,3)	1579.425	1588.185
ARMA(3,4)	1578.375	1587.135
ARMA(3,5)	1576.833	1585.593
ARMA(4,1)	1571.541	1580.301
ARMA(4,2)	1579.892	1588.652
ARMA(4,3)	1578.143	1586.903
ARMA(4,4)	1579.902	1588.662
ARMA(4,5)	1577.135	1585.895
ARMA(5,1)	1571.793	1580.553
ARMA(5,2)	1579.184	1587.944
ARMA(5,3)	1578.148	1586.908
ARMA(5,4)	1578.015	1586.775
ARMA(5,5)	1576.966	1585.726

Note. Collected from the author's calculations

Table 4Lagrange Multiplier Test for ARCH effects

Lags(p)	Chi-squared value	df	Prob>Chi2
1	2.472	1	0.116
2	2.708	2	0.258
3	2.832	3	0.418
4	3.571	4	0.467
5	22.206	5	0.001**
6	22.492	6	0.001**
7	22.930	7	0.002**
8	26.538	8	0.001**
9	27.311	9	0.001**
10	27.687	10	0.002**
11	27.344	11	0.004**
12	28.298	12	0.005**
13	28.488	13	0.008**
14	28.371	14	0.013**
15	31.394	15	0.008**

Note. ** indicate Prob.>Chi-squared = 0.05 or less for significant ARCH(p) disturbance

Table 5Volatility Model Comparison and Selection.

Conditional Variance Equations	AIC	BIC
ARCH(1)	1573.169	1581.929
ARCH(2)	-	-
ARCH(3)	1581.997	1590.757
ARCH(4)	1577.139	1585.899
ARCH(5)	1571.963	1580.723
ARCH(6)	1582.399	1591.159
GARCH(1,1)	-	-
GARCH(1,2)	1573.914	1585.594
GARCH(1,3)	1572.271	1583.951
GARCH(1,4)	1569.051	1580.731
GARCH(1,5)	1574.538	1586.217
GARCH(1,6)	1574.622	1586.302
GARCH(2,1)	1573.116	1587.716
GARCH(2,2)	1575.136	1589.736
GARCH(2,3)	-	-

GARCH(3,1)	-	-
EGARCH(1,1)	1562.153	1576.753
EGARCH(1,2)	1564.388	1584.828
EGARCH(1,3)	1562.897	1589.177
EGARCH(1,4)	-	-
EGARCH(2,1)	1547.939	1565.459
EGARCH(2,2)	1562.240	1585.600
EGARCH(2,3)	1564.870	1594.070
EGARCH(2,4)	-	-
EGARCH(3,1)	1548.855	1569.295
EGARCH(3,2)	-	-
TGARCH(1,1)	-	-
TGARCH(2,1)	-	-
TGARCH(3,1)	-	-
ARMA(1,4)-ARCH(1)	1568.938	1583.537
ARMA(1,5)-ARCH(1)	1567.302	1581.902
ARMA(1,5)-ARCH(2)	1573.975	1588.575
ARMA(1,5)-ARCH(3)	1573.750	1588.350
ARMA(1,5)-ARCH(4)	1572.960	1587.560
ARMA(1,5)-ARCH(5)	1561.682	1576.282
ARMA(1,5)-ARCH(6)	1570.999	1585.599
ARMA(1,5)- $GARCH(1,1)$	1569.109	1586.629
ARMA(1,5)-GARCH(1,2)	1562.201	1579.721
ARMA(1,5)-GARCH(1,3)	1567.395	1584.915
ARMA(1,5)-GARCH(1,4)	1557.613	1575.133
ARMA(1,5)-GARCH(1,5)	1568.086	1585.606
ARMA(1,5)-GARCH(1,6)	1568.982	1586.502
ARMA(1,5)-GARCH(1,7)	1567.797	1585.316
ARMA(1,5)-GARCH(1,8)	1567.865	1585.385
ARMA(1,5)-EGARCH(1,1)	-	-
ARMA(1,5)-EGARCH(1,2)	1549.107	1575.387

ARMA(1,5)-EGARCH(1,3)	-	-
ARMA(1,5)-EGARCH(1,4)	1553.87	1591.83
ARMA(1,5)-EGARCH(1,5)	-	-
ARMA(1,5)-EGARCH(2,1)	1569.603	1592.963
ARMA(1,5)-EGARCH(2,2)	-	-
ARMA(1,5)-TGARCH(1,1)	-	-
ARMA(1,5)-TGARCH(1,2)	-	-
ARMA(1,5)-TGARCH(2,1)	-	-

Model with the lowest AIC and BIC score: EGARCH(3,1)

Note. Since the maximum likelihood estimation method is utilized for this process, models for which flat log-

likelihood curves had been encountered cannot be estimated and thus have no AIC/BIC scores.