DEPARTMENT OF STATISTICS
B. Sc. (H) Statistics
SEM-IV

Category I

DISCIPLINE SPECIFIC CORE COURSE-10: SAMPLING DISTRIBUTIONS

CREDIT DISTRIBUTION, ELIGIBILITY, AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the Course			Eligibility Criteria	Pre-requisite of the Course (if any)
		Lecture	Tutorial	Practical/ Practice		
Sampling Distributions	4	3	0	1	Class XII with Mathematics	knowledge of probability and probability distributions

Learning Objectives:

The learning objectives of this course are as follows:
• To introduce the modes of convergence and their relation to limit laws, with a focus on the central limit theorem.
• To introduce the concept of sampling distributions and their applications in statistical inference.
• To describe the statistical ideas behind the procedure of hypothesis testing.
• To explain the assumptions and conditions under which to apply different tests of hypothesis about population parameters and draw appropriate conclusions from the analysis.

Learning Outcomes:

After successful completion of this course, students should be able to:
• Understand the basics of convergence theory and its importance in limit laws.
• Apply the concept of the central limit theorem and the relevance of the theorem in inferential statistics.
• Analyze data by using suitable hypothesis testing procedures in real-life applications related to large and small samples.
• Apply the knowledge of the idea of sampling distributions and appreciate their importance in the field of statistics.
• Integrate the knowledge of various sampling distributions like chi-square, t, and F distributions in hypothesis testing problems.

Theory

UNIT I   (10 Hours)
Modes of Convergence and Central Limit Theorem
Convergence in probability, convergence with probability one, convergence in the mean square, convergence in distribution – definitions and relations between the various modes.
Chebyshev's inequality, Weak Law of Large Numbers (WLLN), and Strong Law of Large Numbers (SLLN) along with examples and applications.
Basic idea and relevance of Central Limit Theorem (CLT), De-Moivre Laplace theorem, Lindeberg Levy theorem, Liapunov Theorem (only statement), and applications of CLT.

UNIT II   (4 Hours)
Order Statistics
Basic concept and discussion on the area of applications, probability distribution and cumulative distribution function of a single order statistic, joint probability distribution of two and the general case of all order statistics, distribution of range, and distribution of sample median.

UNIT III   (9 Hours)
Sampling Distributions and Test of Hypotheses
Concepts of parameter, statistic, sampling distribution of a statistic, standard error. Sampling distribution of sample mean, standard errors of the sample mean, sample variance, and sample proportion.
Null and alternative hypotheses, level of significance, Type I and Type II errors, their probabilities and critical region, determination of sample size, confidence intervals, and p-value.
Tests of significance and confidence intervals for - single proportion, difference of two proportions, single mean, difference of two means, and difference of two standard deviations.

UNIT IV   (10 Hours)
Exact Sampling Distribution
Chi-Square distribution: Definition and derivation of the probability distribution of Chi-square distribution with n degrees of freedom, nature of the curve for different degrees of freedom, mean, mode, variance, moment generating function, cumulant generating function, additive property, and limiting form of the Chi-square distribution, Applications of Chi-Square distribution.

UNIT V   (12 Hours)
Exact Sampling Distributions (continued)
Student's t-statistic and Fishers t-statistic: definition and derivation of their sampling distributions, nature and characteristics of graph of t distribution, moments, limiting form and applications of the t distribution.
F-statistic: Definition and derivation of the sampling distribution, the graph of F distribution, moments, and applications of the F distribution. Relationship between t, F, and Chi-square distributions.

PRACTICAL / LAB WORK – 30 Hours
List of Practicals:
1. Large Sample Tests:
   a) Testing of significance and confidence intervals for single proportion and difference of two proportions.
   b) Testing of significance and confidence intervals for single mean and difference of two means.
   c) Testing of significance and confidence intervals for the difference of two standard deviations.
2. Tests based on Chi-Square Distribution:
   a) Testing of significance and confidence intervals for the population variance has a specific value.
   b) Testing for the goodness of fit.
   c) Testing of significance for the independence of attributes.
   d) Testing based on a 2 x 2 contingency table without and with Yates' corrections.
3. Tests based on t- Distribution and F- Distribution:
   a) Testing of significance and confidence intervals for single mean and difference of two means and paired t-test.
   b) Testing of significance and confidence intervals of an observed sample correlation coefficient.
   c) Testing and confidence intervals of equality of two population variances.

Practical work to be conducted using electronic spreadsheet / EXCEL/ Statistical Software Package/ SPSS/ calculators.

ESSENTIAL READINGS
• Gupta, S. C. and Kapoor, V. K. (2020). Fundamentals of Mathematical Statistics, Twelfth Edition, S. Chand and Sons. Delhi.
• Goon, A.M., Gupta, M.K. and Dasgupta, B. (2016). An Outline of Statistical Theory, Volume I, The World Press, Kolkata.
• Mukhopadhyay, P. (2016). Mathematical Statistics, Books and Allied, India.
• Hogg, R.V., Tanis, E.A. and Rao, J.M. (2009). Probability and Statistical Inference, Seventh Edition, Pearson Education, New Delhi.
• Miller, I. and Miller, M. (2006). John E. Freund's Mathematical Statistics with Applications, Eight Edition, Pearson Education, Asia.
• Johnson, R.A. and Bhattacharya, G.K. (2001). Statistics-Principles and Methods, Fourth Edition, John Wiley and Sons.

SUGGESTED READINGS
• Bhat, B.R. (2016). Modern Probability Theory- An Introductory Textbook, Fourth Edition, New Age International Publishers.
• Rohatgi, V. K and Saleh M. E. (2015). An Introduction to Probability and Statistics, Third Edition, John Wiley and Sons, Inc., New Jersey.
• Mood, A.M. Graybill, F.A. and Boes, D.C. (2007). Introduction to the Theory of Statistics, Third Edition, (Reprint), Tata McGraw-Hill Pub. Co. Ltd.