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[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1104 **I**

Unique Paper Code : 2222013502

Name of the Paper : Quantum Mechanics – I

Name of the Course : **B.Sc. Hons. – (Physics)_ NEP: UGCF-2022**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt only **five (5)** questions.

3. Question No. **1** is compulsory.

4. **All** questions carry equal marks.

5. Use of non-programmable scientific calculator is allowed.

1. Attempt any six : (6×3=18)

(i) Write the conditions required for physical acceptability of wave function.

(ii) What are stationary states? Why are they called so?

(iii) Let $\psi_0(x)$ and $\psi_2(x)$ are the ground state and second excited state energy eigen-functions of a particle moving in a harmonic oscillator potential with frequency ω. At $t = 0$, the wavefunction of the particle is

$\psi(x,0)=\frac{1}{\sqrt{3}}\psi_0(x)+\psi_2(x)$. Find $\psi(x, t)$ for $t \neq 0$.

(iv) List the four quantum numbers needed to describe an atomic electron? What is their physical significance?

(v) For 6g state of hydrogen atom, what are the values of quantum number n, l, m_l and energy of the state?

(vi) Compute the commutator $[x\ p^2]$.

(vii) Show that (a) $[\hat{L}_x\ \hat{x}]=0$ (b) $[\hat{L}_x\ \hat{y}]=i\hbar\hat{z}$

2. (i) Solve the Schrodinger equation for a particle having energy $E < V_0$ for a square well potential of finite depth V_0. Discuss the graphical representation of the transcendental equations.

(ii) Obtain the mathematical form of position operator in momentum space. (15,3)

3. The potential energy of a simple harmonic oscillator of mass m, oscillating with angular frequency ω is $V(x) = \frac{1}{2} m\omega^2 x^2$.

(i) Write the time independent Schrodinger equation. Using the time independent schrodinger equation, evaluate the energy for the eigenstate

$$\psi_0(\alpha x) = \sqrt{\frac{\alpha}{\pi^{1/2}}} e^{-\alpha^2 x^2}; \alpha = \sqrt{\frac{m\omega}{\hbar}},$$ where ω is the angular frequency of the oscillator.

(ii) Find $\langle p \rangle$ for ψ_0.

(iii) If the harmonic oscillator is in the ground state at t=0, what will be the wavefunction at $t = \frac{\pi}{2\omega}$. (18)

4. Write the Schrodinger equation for a 3D hydrogen atom in spherical polar coordinates. Derive three separate equations for r, θ, ϕ using the method of separation of variables. Solve the equation for θ to obtain the normalized eigenfunctions. (18)

5. (i) What is spin angular momentum? Discuss the experimental observations which could not be accounted for without introducing the spin angular momentum.

(ii) What are Pauli Spin matrices. For $s = \frac{1}{2}$, obtain the matrix form of S_z.

(iii) What is total angular momentum? After defining ladder operator J_+ and J_- obtain (a) $[\hat{J}_+ \, \hat{J}_-]$ (b) $[\hat{J}_+ \, \hat{J}_z]$. (6+6+6)

6. (i) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of the hydrogen atom? Express the answer in terms of Bohr radius.

(ii) At a given instant of time, a system is in the state $Y(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \sin\theta \sin\phi$. Determine the expectation values of L_z and L^2. (9,9)