DEPARTMENT OF PHYSICS
B. SC. (HONOURS) PHYSICS

DISCIPLINE SPECIFIC CORE COURSE – DSC - 7: MATHEMATICAL PHYSICS III

Course Title & Code	Credits	Credit distribution of the course			Eligibility Criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical		
Mathematical Physics III DSC – 7	4	3	0	1	Class 12^{th} Pass	Should have studied DSC - 1 and DSC - 4 of this program or its equivalent

LEARNING OBJECTIVES

The emphasis of course is on applications in solving problems of interest to physicists. The course will also expose students to fundamental computational physics skills enabling them to solve a wide range of physics problems. The skills developed during course will prepare them not only for doing fundamental and applied research but also for a wide variety of careers.

LEARNING OUTCOMES

After completing this course, student will be able to,

- Determine continuity, differentiability and analyticity of a complex function, find the derivative of a function and understand the properties of elementary complex functions.
- Work with multi-valued functions (logarithmic, complex power, inverse trigonometric function) and determine branches of these functions.
- Evaluate a contour integral using parameterization, fundamental theorem of calculus and Cauchy's integral formula.
- Find the Taylor series of a function and determine its radius of convergence.
- Determine the Laurent series expansion of a function in different regions, find the residues and use the residue theory to evaluate a contour integral and real integral.
- Understand the properties of Fourier transforms and use these to solve boundary value problems.
- Solve linear partial differential equations of second order with separation of variable method.
- In the laboratory course, the students will learn to,
    - create, visualize and use complex numbers
    - use Gauss quadrature methods to numerically integrate proper and improper definite integrals
    - Solve the boundary value problems numerically
    - Compute the fast Fourier transform of a given function

SYLLABUS OF DSC – 7

THEORY COMPONENT

Unit - I **(28 Hours)**

Complex Analysis: The field of complex numbers. Graphical, Cartesian and polar representation. Algebra in the complex plane. Triangle inequality. Roots of complex numbers. Regions in the complex plane – idea of open sets, closed sets, connected sets, bounded sets and domain.

(3 Hours)

The complex functions and mappings. Limits of complex functions. Extended complex plane and limits involving the point at infinity. Continuity and differentiability of a complex function, Cauchy-Riemann equations in Cartesian and polar coordinates, sufficient conditions for differentiability, harmonic functions. Analytic functions, singular points. Elementary functions. Multi-functions, branch cuts and branch points.

(10 Hours)

Integration in complex plane: contours and contour integrals, Cauchy-Goursat Theorem (No proof) for simply and multiply connected domains. Cauchy's inequality. Cauchy's integral formula. Taylor's and Laurent's theorems (statements only), types of singularities (removable poles and essential), meromorphic functions, residues and Cauchy's residue theorem, Jordan Lemma (statement only), evaluation of real integrals by contour integration (excluding integrands with branch points)

(15 Hours)

Unit – II **(9 Hours)**

Fourier Transform: Fourier Integral theorem (Statement only), Fourier Transform (FT) and Inverse FT, existence of FT, FT of single pulse, finite sine train, trigonometric, exponential, Gaussian functions, properties of FT, FT of Dirac delta function, sine and cosine function, convolution theorem. Fourier Sine Transform (FST) and Fourier Cosine Transform (FCT)

Unit – III **(8 Hours)**

Partial Differential Equations: Solutions to partial differential equations (2 or 3 independent variables) using separation of variables: Laplace's equation in problems of rectangular geometry. Solution of wave equation for vibrational modes of a stretched string. Solution of 1D heat flow equation (Wave/Heat equation not to be derived)

References:

Essential Readings:

1) Mathematical methods for Scientists and Engineers, D.A. McQuarrie, Viva Book, 2003
2) Essential Mathematical Methods, K. F. Riley and M. P. Hobson, Cambridge Univ. Press, 2011
3) Mathematical Methods for Physicists, G. B. Arfken, H.J. Weber, F. E. Harris, 7th Edition, Elsevier, 2013
4) Complex Variables and Applications, J. W. Brown and R. V. Churchill, 9th Edition, Tata McGraw-Hill, 2021
5) Complex Variables: Schaum's Outline, McGraw Hill Education, 2009
6) Fourier analysis: With Applications to Boundary Value Problems, Murray Spiegel, McGraw Hill Education, 2017
7) Fourier series and boundary value problems, J. W. Brown and R. V. Churchill, 5th

Edition, Tata McGraw-Hill, 1993.
8) Applied Mathematics for Engineers and Physicists, 3rd edition, L. A. Pipes and L. R. Harvill, Dover Publications.

Additional Readings:

1) Mathematical Physics with Applications, Problems and Solutions, V. Balakrishnan, Ane Books, 2017
2) Complex Variables, A. S. Fokas and M. J. Ablowitz, 8^{th} Edition, Cambridge Univ. Press, 2011
3) Fourier Transform and its Applications, third edition, Ronald New Bold Bracewell, McGraw Hill, 2000
4) A Students Guide to Fourier Transforms: With Applications in Physics and Engineering, 3rd edition, Cambridge University Press, 2015
5) Partial Differential Equations for Scientists and Engineers, S. J. Farlow, Dover Publications, 1993
6) Differential Equations – Theory, technique and practice, George F. Simmons and Steven G. Krantz, Indian Edition McGraw Hill Education Pvt. Ltd, 2014

PRACTICAL COMPONENT

(15 Weeks with 2 hours of laboratory session per week)
The aim of this lab is not just to teach computer programming and numerical analysis but to emphasize its role in solving problems in Physics.

- The course will consist of practical sessions and lectures on the related theoretical aspects of the laboratory.
- Assessment is to be done not only on the programming but also on the basis of formulating the problem.
- The list of recommended programs is suggestive only. More programs may be done in the class with physics applications. Emphasis should be given to formulate a physics problem as mathematical one and solve it by computational methods.
- At least 6 programs must be attempted (taking at least one from each unit). The implementation can be either in Python/ C++/ Scilab. Inbuilt libraries can be used wherever applicable.

Unit 1
Handling of Complex Numbers: Syntax for creating complex numbers in Python/C++/Scilab, accessing real and imaginary parts, calculating the modulus and conjugate of a complex number, complex number arithmetic, plotting of complex numbers as ordered pairs of real numbers in a plane, conversion from Cartesian to polar representation.
Recommended List of Programs:

a) Determine the nth roots of a complex number and represent it in Cartesian and polar form.
b) Transformation of complex numbers as 2-D vectors e.g. translation, scaling, rotation, reflection.
c) Visualisation of mappings of some elementary complex functions w = f(z) from z-plane to w-plane.

Unit 2
Gauss Quadrature Integration Methods: Gauss quadrature methods for integration: Gauss Legendre, Gauss Lagaurre and Gauss Hermite methods.

Recommended List of Programs:

a) Solving a definite integral by Gauss Legendre quadrature method. Application – representation of a function as a linear combination of Legendre polynomials.
b) Solving improper integrals over entire real axis or the positive real axis using Gauss Lagaurre and Gauss Hermite quadrature method. Comparison of results with the ones obtained by contour integration analytically.
c) Comparison of convergence of improper integral computed by Newton Cotes and Gauss Quadrature Methods.

Unit 3

Fast Fourier Transform: Discrete Fourier transform, Any algorithm for fast Fourier transform.

a) Computation of Discrete Fourier Transform (DFT) using complex numbers.
b) Fast Fourier Transform of given function in tabulated or mathematical form e.g function $\exp(-x^2)$.

Unit 4

Numerical Solutions of Boundary Value Problems: Two-point boundary value problems, types of boundary conditions – (Dirichlet, Neumann and Robin), importance of converting a physics problem to dimensionless form before solving numerically. Finite difference method, Shooting method with bisection/Secant/Newton method for solving non-linear equation and using RK methods for solving IVP (The programs developed in the last semester may be used here).

Algorithm for any one numerical method to solve Partial Differential Equations e.g. Finite Difference method, relaxation methods, Crank-Nicolson method

Recommended List of Programs:

(a) The equilibrium temperature of a bar of length L with insulated horizontal sides and the ends maintained at fixed temperatures.
(b) Solve for the steady state concentration profile y(x) in the reaction-diffusion problem given by $y''(x) - y(x) = 0$ with $y(0) = 1, y'(1) = 0$.
(c) Use any numerical method to solve Laplace equation/ Wave equation/ Heat equation.

References (for Laboratory Work):

1) Documentation at the Python home page (https://docs.python.org/3/) and the tutorials there (https://docs.python.org/3/tutorial/).
2) Documentation of NumPy and Matplotlib: https://numpy.org/doc/stable/user/ and https://matplotlib.org/stable/tutorials/
3) Schaum's Outline of Programming with C++, J. Hubbard, 2000, McGraw-Hill Education.
4) An Introduction to Computational Physics, T. Pang, Cambridge University Press, 2010
5) Introduction to Numerical Analysis, S. S. Sastry, 5th Edition, 2012, PHI Learning Pvt. Ltd.
6) Numerical Recipes: The art of scientific computing, William H. Press, Saul A. Teukolsky and William Vetterling, Cambridge University Press; 3rd Edition, 2007
7) Computational Problems for Physics, R. H. Landau and M. J. Páez, CRC Press, 2018