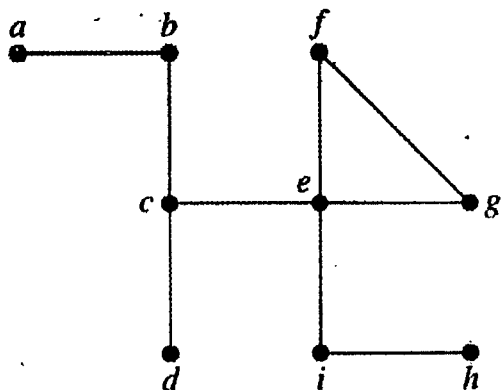(i) Check whether this graph has an Euler Circuit. Give such a circuit, if it exists.

(ii) Check whether this graph has a Hamiltonian Circuit. Give such a circuit, if it exists.

(c) Identify all the cut vertices and cut edges in the graph given below : (8)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4114　　　　H

Unique Paper Code : 2342011202

Name of the Paper : Discrete Mathematical Structures

Name of the Course : **B.Sc. (Hons.) Computer Science (NEP-UGCF-2022)**

Semester : II

Duration : 3 Hours　　　　Maximum Marks : 90

(For admissions of 2022)

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question No. **1 (Section-A)** is compulsory.

3. Attempt any **four** questions from **Section-B**.

4. Parts of a question should be attempted together.

5. Use of simple calculator is allowed.

Section A

1. (a) (i) In how many ways can 6 boys and 5 girls stand in a line, so that no two girls are next to each other?

         (ii) In a class of 40 students, 25 are boys. In how many ways can a 10-student club be formed, if there must be equal number of boys and girls?   (5)

     (b) Given that $f(x) = 3x + 2$ and $g(x) = 2x + 3$, find $f \circ g$ and $g \circ f$ for the functions f and g. Is $f \circ g = g \circ f$? Justify your answer.   (5)

     (c) Given the statement :   (5)

         "If it rains today, I will go to college tomorrow."

         State its converse, contra-positive and inverse.

     (b) Let $A = \{1,3,6,8,21,25\}$ be a non-empty set and R be the partial order relation of divisibility defined on A, i.e., if $(a, b) \in A$, then a divides b
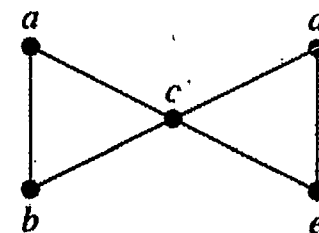
         (i) Draw the Hasse diagram of R.

         (ii) Find the maximal and minimal elements in A.   (8)

7. (a) What is a planar graph?

         Draw a planar graph with at least 3 regions. Verify its planarity using Euler Formula.   (3)

     (b) Consider the graph given below :   (4)

(b) Let a be a numeric function such that :

$$a_r = \begin{cases} 2 & 0 \le r \le 2 \\ r & r \ge 3 \end{cases}$$

Find Δa, ∇a and $S^2 a$.   (8)

5. (a) Using truth table, show that $(p \to (q \to r))$ and $((p \wedge q) \to r)$ are logically equivalent.   (7)

(b) Find the sum and product of the numbers $(102)_3$ and $(201)_3$ in base 3, without converting them into decimal system.   (8)

6. (a) Show that the following argument is valid:

If Mehek is an architect, then she is ambitious. If Mehek wakes up early, then she does not like paranthas. If Mehek is ambitious, then she wakes up early. Then, if Mehek is an architect, she does not like parathas.   (7)

(d) Define graph isomorphism. Is the following pair of graphs isomorphic? Justify your answer.   (5)
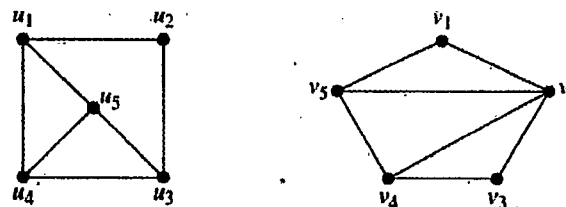
(e) Given the numeric functions a_r and b_r, determine the closed form of $a * b$.   (5)

$$a_r = \begin{cases} 3 & 0 \le r \le 2 \\ 0 & r \ge 3 \end{cases} \qquad b_r = \begin{cases} 2 & 0 \le r \le 3 \\ 0 & r \ge 4 \end{cases}$$

(f) Use the Euclidean algorithm to find $\gcd(111, 201)$.   (5)

Section B

2. (a) Using mathematical induction, prove that for non-negative integer n :   (7)

$$1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$$

(b) (i) Give the bit strings for the sets A = {2, 3,4,5} and B = {1,6,7}, with universal set as U = {1,2,3,4,5,6,7,8}. (8)

(ii) Use bit strings to find the union, intersection and difference of the sets A and B.

3. (a) Prove that a tree with n vertices has n − 1 edges. (7)

(b) In how many ways can 22 different books be distributed among eight students, so that

(i) Six students would have 2 books each and the other two students would have 5 books each.

(ii) Two particular students would get exactly 1 book each.

(iii) Two particular students do not get any book. (8)

4. (a) In a batch of class 11, there are 250 students. A total of 52 students have taken a course in Computer Science, 140 have taken a course in Mathematics, and 56 have taken a course in Economics. Further, 25 have taken courses in both Computer Science and Mathematics, 20 have taken courses in both Computer Science and Economics, and 15 have taken courses in both Mathematics and Economics. There are 120 students that have taken at least one of Computer Science, Mathematics, and Economics.

(i) How many students have taken a course in all three subjects?

(ii) How many students have not taken a course in any of these three subjects?

(iii) How many students opted for Computer Science but not for Mathematics? (7)