DEPARTMENT OF PHYSICS & ASTROPHYSICS

Category-I

BSc. (H) Physics

DISCIPLINE SPECIFIC CORE COURSE – 4: MATHEMATICAL PHYSICS II

Course title & Code	Credits	Credit distribution of the course			Eligibility Criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical		
Mathematical Physics II DSC – 4	4	2	0	2	Class XII Pass	-----

LEARNING OBJECTIVES

The emphasis of course is on applications in solving problems of interest to physicists. The course will also expose students to fundamental computational physics skills enabling them to solve a wide range of physics problems. The skills developed during course will prepare them not only for doing fundamental and applied research but also for a wide variety of careers.

LEARNING OUTCOMES

After completing this course, student will be able to,

- Use curvilinear coordinates to solve problems with spherical and cylindrical symmetries
- Represent a periodic function by a sum of harmonics using Fourier series
- Obtain power series solution of differential equation of second order with variable coefficient using Frobenius method
- Understand the properties and applications of Legendre polynomials
- Learn about gamma and beta functions and their applications
- In the laboratory course, the students will learn to
    - Apply appropriate numerical method to solve selected physics problems both using user defined and in-built functions from Scilab/ Python
    - Solve non-linear equations
    - Perform least square fitting of the data taken in physics lab by user defined functions.
    - Interpolate a data by polynomial approximations
    - Generate and plot a function by its series representation
    - Generate and plot Legendre polynomials and verify their properties.
    - Numerically integrate a function and solve first order initial value problems numerically.

SYLLABUS OF DSC – 4

UNIT – I **(13 Hours)**

Orthogonal Curvilinear Coordinates: Orthogonal Curvilinear Coordinates. Scale factors, element of area and volume in spherical and cylindrical coordinate Systems. Derivation of Gradient, Divergence, Curl and Laplacian in Spherical and Cylindrical Coordinate Systems

Fourier Series: Periodic functions, Orthogonality of sine and cosine functions, Convergence of Fourier series and Dirichlet Conditions (Statement only), Expansion of periodic functions in a series of sine and cosine functions and determination of Fourier coefficients, Even and odd functions and their Fourier expansions (Fourier Cosine Series and Fourier Sine Series), Parseval's Identity.

UNIT – II **(17 Hours)**

Frobenius Method and series solution of Differential Equations: Singular Points of Second Order Linear Differential Equations and their importance, Frobenius method for finding series solution and its applications, Legendre Differential Equations and its solution. Properties of Legendre Polynomials: Rodrigues Formula, Generating Function, Orthogonality of Legendre Polynomials, Simple recurrence relations, Expansion of function in a series of Legendre Polynomials.

Some Special Integrals: Beta and Gamma Functions and relation between them, Expression of Integrals in terms of Gamma and Beta Functions.

References:

Essential Readings:

1) Mathematical Methods for Scientists and Engineers, D. A. McQuarrie, 2003, Viva Book.
2) Advanced Engineering Mathematics, Erwin Kreyszig, 2008, Wiley India.
3) Essential Mathematical Methods, K. F. Riley and M. P. Hobson, 2011, Cambridge Univ. Press.
4) Vector Analysis and Cartesian Tensors, D. E. Bourne and P. C. Kendall, 3 Ed., 2017, CRC Press.
5) Vector Analysis, Murray Spiegel, 2nd Ed., 2017, Schaum's Outlines Series.
6) Fourier analysis: With Applications to Boundary Value Problems, Murray Spiegel, 2017, McGraw Hill Education.
7) Differential Equations, George F. Simmons, 2006, Tata McGraw-Hill.
8) Mathematical Methods for Physicists, G. B. Arfken, H. J. Weber, F. E. Harris, 7 Ed., 2013, Elsevier.

Additional Readings:

1) Introduction to Electrodynamics, Chapter 1, David J. Griffiths, 4 Ed., 2017, Cambridge University Press.
2) The Feynman Lectures on Physics, Volume II, Feynman, Leighton and Sands, 2008, Narosa Publishing House.
3) Advanced Engineering Mathematics, D. G. Zill and W. S. Wright, 5 Ed., 2012, Jones and

Bartlett Learning.
4) Introduction to Vector Analysis, Davis and Snider, 6 Ed., 1990, McGraw Hill.
5) Mathematical Tools for Physics, James Nearing, 2010, Dover Publications.
6) Mathematical Physics, A. K. Ghatak, I. C. Goyal and S. J. Chua, 2017, Laxmi Publications Private Limited.

PRACTICAL COMPONENT – 60 Hours

The aim of this laboratory is not just to teach computer programming and numerical analysis but to emphasize its role in solving problems in Physics. The course will consist of practical sessions and lectures on the related theoretical aspects of the laboratory. Assessment is to be done not only on the programming but also on the basis of formulating the problem.

- Every student must perform at least 12 programs covering each unit.
- The list of recommended programs is suggestive only. Students should be encouraged to do more practice. Emphasis should be given to formulate a physics problem as mathematical one and solve by computational methods.
- The implementation can be either in Python/ C++/ Scilab.

Unit 1: Root Finding: Bisection, Newton Raphson and secant methods for solving roots of equations, Convergence analysis.

Recommended List of Programs (At least two):

(a) Determine the depth up to which a spherical homogeneous object of given radius and density will sink into a fluid of given density.
(b) Solve transcendental equations like $\alpha = \tan(\alpha)$.
(c) To approximate nth root of a number up to a given number of significant digits.

Unit 2: Least Square fitting (At least one): Algorithm for least square fitting and its relation to maximum likelihood for normally distributed data.

a) Make a function for least square fitting, use it for fitting given data (*x, y*) and estimate the parameters a, b as well as uncertainties in the parameters for the following cases.
   i. Linear $(y = ax + b)$
   ii. Power law $(y = ax^b)$
   iii. Exponential $(y = ae^{bx})$
b) Weighted least square fitting of given data (*x, y*) with known error/uncertainty-values using user defined function.

Unit 3: Generating and plotting of a function using series representation (At least one):

a) To approximate the elementary functions (e.g. exp(*x*), sin(*x*), cos(*x*), ln(1+*x*), etc.) by a finite number of terms of Taylor's series and discuss the truncation error. To plot the function as well the nth partial sum of its series for various values of *n* on the same graph and visualise the convergence of series.
b) Generating and plotting Legendre Polynomials using series expansion and verifying recurrence relation

Unit 4: **Interpolation:** Concept of Interpolation, Lagrange form of interpolating polynomial, Error estimation, optimal points for interpolation.

Recommended List of Programs (At least one)

(a) Write program to determine the unique polynomial of a degree n that agrees with a given set of $(n+1)$ data points (x_i, y_i) and use this polynomial to find the value of y at a value of x not included in the data.

(b) Generate a tabulated data containing a given number of values $(x_i, f(x_i))$ of a function f(x) and use it to interpolate at a value of x not used in table.

Unit 5: **Numerical Integration:** Newton Cotes Integration methods (Trapezoidal and Simpson rules) for definite integrals, derivation of composite formulae for these methods and discussion of error estimation.

Recommended List of Programs (At least three)

(a) Given acceleration at equidistant time values, calculate position and velocity and plot them.

(b) Use integral definition of ln(x) to compute and plot ln(x) in a given range. Use trapezoidal, Simpson and Gauss quadrature methods and compare the results.

(c) Verify the rate of convergence of the composite Trapezoidal and Simpson methods by approximating the value of a given definite integral.

(d) Verify the Orthogonality of Legendre Polynomials.

(e) To evaluate the Fourier coefficients of a given periodic function (e.g. square wave, triangle wave, half wave and full wave rectifier etc.). To plot the function as well the nth partial sum of its series for various values of n on the same graph and visualise the convergence of series. Study of Gibbs phenomenon.

(f) Verify the properties of Dirac Delta function using its representation as a sequence of functions.

Unit 6: Numerical Solutions of Ordinary Differential Equations: Euler, modified Euler, and Runge-Kutta (RK) second and fourth order methods for solving first order initial value problems (IVP) and system of first order differential equations,

Recommended List of Programs (At least two)

(a) Solve given first order differential equation (Initial value problems) numerically using Euler RK2 and RK4 methods and apply to the following physics problems:
   i. Radioactive decay
   ii. Current in RC and LR circuits with DC source
   iii. Newton’s law of cooling

(b) Write a code to compare the errors in various numerical methods learnt by solving a first order IVP with known solution.

(c) Solve a system of first order IVP numerically using Euler and Runge-Kutta methods. Application to physical problems.

References (for Laboratory work):

1) Documentation at the Python home page (https://docs.python.org/3/) and the tutorials there (https://docs.python.org/3/tutorial/).
2) Documentation of NumPy and Matplotlib: https://numpy.org/doc/stable/user/ and https://matplotlib.org/stable/tutorials/
3) Computational Physics, Darren Walker, 1st Edn., Scientific International Pvt. Ltd (2015).
4) Elementary Numerical Analysis, K. E. Atkinson, 3rd Edn., 2007, Wiley India Edition.
5) An Introduction to Computational Physics, T. Pang, Cambridge University Press (2010).
6) Introduction to Numerical Analysis, S. S. Sastry, 5th Edn., 2012, PHI Learning Pvt. Ltd.
7) Applied numerical analysis, Cutis F. Gerald and P. O. Wheatley, Pearson Education, India (2007).
8) Numerical Recipes: The art of scientific computing, William H. Press, Saul A. Teukolsky and William Vetterling, Cambridge University Press; 3rd edition (2007)
9) Computational Problems for Physics, R. H. Landau and M. J. Páez, 2018, CRC Press.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.