This question paper contains **7** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5738**

Unique Paper Code : **2353012006**

Name of the Paper : **Mechanics**

Type of the Paper : **DSE-2**

Name of the Course : **B.Sc. (H)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

Part of the questions to be attempted together.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) (*i*) If two forces of magnitude P and Q act at an inclination θ to each other, prove that the magnitude of the resultant R is given by $R^2 = P^2 + Q^2 + 2PQ\cos\theta$.

(*ii*) Two forces act at a point and are such that if the direction of one is reversed the direction of the resultant is turned through

P.T.O.

a right angle. Show that the two forces must be equal in magnitude. 3½+4

(*b*) (*i*) Forces P, Q, R act along the lines $x = 0$, $y = 0$, $x\cos\theta + y\sin\theta = p$. Find the magnitude of the resultant and the equation of its line of action.

(*ii*) A force, of fixed magnitude R and variable inclination θ to the x-axis, acts in the plane Oxy at the fixed point (a, b). Find its moment about the origin as a function of θ, and obtain the values of θ for which this moment :

(*i*) is a maximum

(*ii*) is a minimum,

(*iii*) vanishes. 4+3½

(*c*) State the principle of Virtual Work. Two heavy particles of weights w, w' are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius a, the axis of which is horizontal. If θ and θ' are the inclinations to the vertical of the radii drawn to the particles, show that :

$$\frac{\sin\theta}{\sin\theta'} = \frac{w'}{w}.$$ 7½

2. (*a*) (*i*) Show that a field of force with components (X, Y) is conservative if and only if

$$\frac{\partial x}{\partial y} = \frac{\partial Y}{\partial x}.$$

(*ii*) Find the C.G. of the area bounded by the parabola $y^2 = 4ax$, the x-axis and latus rectum. 4+3½

(*b*) A portion of a circular disc of radius r is cut of by a straight cut of length $2c$. Find the position of the mass centre of the larger portion. 7½

(*c*) A light ladder is supported on a rough floor and leans against a rough wall. How far up the ladder can a man ascend without the ladder slipping ? 7½

3. (*a*) Derive expression for radial and transverse components of velocity and acceleration of a particle moving along a plane curve. Also, prove that if a particle moves in plane with constant speed, then its acceleration is perpendicular to its velocity. 7½

(*b*) Show that the sum of kinetic energy T and potential energy V remains constant in the absence of non-conservative forces. 7½

(*c*) State D'Alembert's principle. Use it to solve the following problem : three equal particles are joined by light rods to form an equilateral triangle. If the triangle rotates in its plane about its centroid with constant angular velocity, find the tensions in the rods. 7½

4. (*a*) Define the Centrifugal force and the Coriolis force. A automobile travels round a curve of radius r. If h is the height of the center of gravity above the ground and $2b$ the width between the wheels, show that it will overturn if the speed exceeds $\sqrt{\frac{grb}{h}}$, assuming no side-slipping takes place. 7½

(*b*) Mud is thrown off from the tire of a wheel (radius a) of a car travelling at a speed V, where $V^2 > ga$. Neglecting the resistance of air, show that no mud can rise higher than a height

$$\left(a + \frac{V^2}{g} + \frac{ga^2}{2V^2}\right)$$

above the ground. 7½

(*c*) Define a Harmonic oscillator and discuss its motion in case of light damping oscillations by depicting appropriate position-time graph. 7½

5. (*a*) (*i*) Show that the specific gravity of a mixture of n fluids is greater when equal volumes are taken than when equal weights are taken, assuming no change in volume as the result of mixing.

(*ii*) If pure water is added, drop by drop, to a vessel of volume V filled with a salt solution of specific gravity s, which is allowed to overflow. Find the specific gravity of the solution when a volume v of water has been poured in. 3½+4

(*b*) A piston is slowly pushed along a cylinder against water under pressure and to prevent leakage the piston is fitted with a leather collar of breadth l which the water pushes against the walls of the cylinder. If μ is the coefficient of friction between the leather and the cylinder and r is the radius of a section of the cylinder, prove that the fraction of the work done which is spent in overcoming friction is

$$\frac{2\mu l}{r + 2\mu l}.$$ 7½

(*c*) State Kepler's laws. Prove that the square of periodic time of elliptical orbit is proportional to the cube of the semi-major axis of the orbit. 7½

6. (*a*) A vertical circular cylinder of height $2h$ and radius r, closed at the top, is just filled by equal volumes of two liquids of densities ρ and σ. Show that, if the axis be gradually inclined to the vertical, the pressure at the lowest point of the base will never exceed

$$g(\rho + \sigma)\left(r^2 - h^2\right)^{1/2}.$$ 7½

(*b*) A fine glass tube in the shape of an equilateral triangle is filled with equal volumes of three liquids which do not mix, whose densities are in arithmetical progression. The tube is held in a vertical plane and the side that contains portions of the heaviest and lightest liquids makes an angle θ with the vertical. Show that the surfaces of separation divide the sides in the ratio

$$\cos\left(\frac{\pi}{6} - \theta\right) : \cos\left(\frac{\pi}{6} + \theta\right).$$ 7½

(*c*) Equal volumes of three fluids of different densities, which do not mix, together completely fill a circular tube which is kept in a vertical plane. Prove that, if the densities of the fluids are in arithmetical progression, the common surface of the lightest and heaviest fluids is at an extremity of a horizontal diameter of the circle. 7½