

Sr. No. of Question Paper : 1231  
Unique Paper Code : 2353012002  
Name of the Paper : Mathematical Python  
Type of the Paper : DSE  
Semester : III/V  
Programme : B.Sc. (Hons.) Mathematics  
Duration : 3 Hours

Maximum Marks : 90

**Instructions for the Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting any six parts from question number 1, and any three parts from question numbers 2 to 6 respectively.
3. All commands to be written in Python code.

1. Attempt any 6 parts.

Total marks:  $2.5 \times 6 = 15$

- (a). How do you find the minimum and maximum values in a NumPy array?
- (b). Consider the list, `list1=[1,2,4,5,7,8,3]`. Add an integer '8' at the third index of the list1.
- (c). How do you find the indices of maximum and minimum values in a NumPy array?
- (d). Write a SymPy code to find the sum of  $1^3 + 2^3 + 3^3 + \dots + n^3$  till  $n = 5$ .
- (e). Consider an array,  $\text{array1} = \begin{pmatrix} 2 & 3 & 1 \\ 4 & -6 & 3 \\ -3 & 0 & 7 \end{pmatrix}$ , extract an array  $\begin{pmatrix} -6 & 3 \\ 0 & 7 \end{pmatrix}$  from array1.
- (f). What will be the output of the following code:

```
import numpy as np
arr = np.arange(10)
arr
```

- (g). Use pretty printing to print the following expression

$$\int_0^1 \frac{1}{(x+1)} dx$$

- (h). Write a SymPy code to solve the following equation for  $x$   
 $x^2 + 5x + 4 = 0$

2. Attempt any 3 parts.

Total marks:  $5 \times 3 = 15$

- (a). Using turtle graphics, write a python code that draw 60 hexagons turning right 10 degrees after each hexagon complete.
- (b). Write a python code to plot the functions  $y_1 = \tan x$ ,  $y_2 = \tan 2x$  in a single plot. The code must include the following requirements:
- (i). Legends, grid and a title of the plot.
  - (ii). Different colour for each function.
- (c). Create a  $5 \times 5$  matrix using NumPy, and
- (i). Calculate the sum of all elements.
  - (ii). Find the mean of each row.
  - (iii). Identify the maximum value in each column.
- (d). Write a python code for simple die rolling game, where we keep rolling the die until we have rolled a total of 50.

3. Attempt any 3 parts.

Total marks:  $5 \times 3 = 15$

- (a). Write a NumPy code to find the characteristic polynomial, eigenvalues and eigenvectors for a given matrix  $A$ ,

$$A = \begin{pmatrix} -2 & 0 & 4 \\ 0 & -2 & 4 \\ 4 & -2 & 1 \end{pmatrix}$$

- (b). Use lambda function to evaluate the following function at  $x = 0$ .

$$g(x) = \frac{(x-1)}{(x^2-1)}$$

- (c). Use SymPy to write and evaluate the following expression:

$$\int_0^{\infty} x^y e^{-x} dx$$

- (d). Write a SymPy code to find the derivatives of all orders using loop for the following function

$$f(x) = (ax + b)^4$$

4. Attempt any 3 parts.

Total marks:  $5 \times 3 = 15$

(a). A store sells five types of fruits: apples, bananas, oranges, grapes, and mangoes. Sales figure for a week are as follows: 50, 30, 45, 20 and 60, respectively. Write a python code to plot a basic bar plot showing the sales of each fruit. Label each bar with the fruit name and provide axis labels.

(b). What output will the following code produce:

```
import numpy as np
arr1 = np.array([[ -2, 1, 3],[ -3, 6, 6]])
arr2 = np.array([[ 4, -8, 9],[0, 1, -2]])
print(np.concatenate([arr1, arr2], axis=0))
print(np.concatenate([arr1, arr2], axis=1))
```

(c). Write a SymPy code to find the left hand limit and right hand limit of the following function at the point  $x = 0$ .

$$f(x) = \frac{|x|}{x}$$

(d). Write a SymPy code to solve the following second order differential equation with the

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 0$$

with the given initial conditions,  $y(0) = 1, y'(0) = 0$ .

5. Attempt any 3 parts.

Total marks:  $5 \times 3 = 15$

(a). Write a SymPy code to factorize the expression  $x^2 - y^2$  and to expand  $(x + 2)(x^2 - 3x + 4)$ .

(b). Write a python code for the following:

(i). Using SymPy, find the series expansion of  $\ln(1 + x)$  up to the order  $x^4$ .

(ii). Define a recursive sequence where:

$$a_n = \begin{cases} n + a_{n-1} & , \text{if } n \text{ is even} \\ 2 \times a_{n-1} & , \text{if } n \text{ is odd} \end{cases}$$

with initial term  $a_0 = 1$ .

(c). Write a NumPy code to solve the following system of linear equations using the inverse of coefficient matrix:

$$\begin{aligned} 5x - 3y + 2z &= -5 \\ 2x - y - z &= 4 \\ 3x - 2y + 2z &= -7 \end{aligned}$$

- (d). Write a SymPy code to find the critical points of the following function and print the points of local maxima and minima.

$$f(x) = x^2 - 4x + 3$$

6. Attempt any 3 parts.

Total marks:  $5 \times 3 = 15$

- (a). Write a python code using SymPy for the following:

- (i). Create an identity matrix of order 3.
- (ii). Create a matrix of all zeros of order  $2 \times 3$ .
- (iii). Create a diagonal matrix of order 4 with 1,2,3,4 as diagonal elements.

- (b). Write a SymPy code to obtain and verify the solution of the following system of linear equations:

$$\begin{aligned} 3x + 4y &= 18 \\ 2x - 5y &= -3 \end{aligned}$$

- (c). For the given matrix:

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{pmatrix}$$

Write a python code using SymPy to perform the following operations;

- (i). Create the matrix  $A$  and find its transpose.
  - (ii). Find the nullspace and column space of the matrix  $A$ .
  - (iii). Find the row reduced echelon form of the matrix  $A$ .
- (d). Write a python code using pandas to create a data frame with the following data by reading as CSV file with the following columns: Food, Calories and Protein.

Food	Calories	Protein
Milk	73	3.9
Yogurt	130	17
Egg	75	6
Beans	113	7.5

- (i). Change name of the column Protein to Protein(grams).
- (ii). Add a new column called Carbs to the data frame with the following values: Milk: 6.4, Yogurt: 7, Egg: 0.4 and Beans: 20.
- (iii). Sort the data frame by Calories in the ascending order.