[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1386 **I**

Unique Paper Code : 2372011102

Name of the Paper : Introduction to Probability-DSC-2

Name of the Course : **B.Sc. (H) Statistics (NEP-UGCF)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **5** questions in all.

3. Question No. **1** is compulsory.

4. Attempt **4** more questions selecting any **two** questions each from **Section A** and **Section B**.

1. Attempt any **six** parts :

(i) If A and B are two events and $P(B) \neq 1$, prove that

$$P(A|\bar{B})\frac{[P(A)-P(A\cap B)]}{[1-P(B)]},$$

where $\bar{B}$ denotes the event complementary to B and hence deduce that $P(A \cap B) \geq P(A) + P(B) - 1$

(ii) In a random arrangement of the letters of the word 'Mathematics', find the probability that all the vowels occur together.

(iii) An integer is chosen at random from first two hundred numbers. What is the probability that the integer is either divisible by 6 or 8?

(iv) There is 80% chance that Mohan takes bus to the school and there is a 20% chance that his father drops him to school. The probability that he is late to school is 0.5 if he takes the bus and 0.2 if his father drops him. On a given day, Mohan is late to school. Find the probability that his father dropped him to school on that day.

(iv) A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

(v) A discrete random variable X has the following probability distribution find the value of C and the mean of the distribution.

x	1	2	3	4	5	6	7
P(X=x)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

(vii) Prove that $E[(X - c)^2] = Var(X) + [E(X) - c]^2$, where c is constant.

(viii) The probability mass function of a random variable X is zero except at the points X = 0, 1,2. At these points it has the values $p(0) = 3c^3$, $p(1) = 4c - 10c^2$ and $p(2) = 5c - 1$ for some $c > 0$.

(a) Determine value of c.

(b) Compute the probabilities, $P(X < 2)$ and $P(1 < X \leq 2)$. (5×6)

Section A

2. (a) If A and B are two events such that P(A)=3/4 and P(B)=5/8, then show that

(i) $P(A \cup B) \geq \frac{1}{4}$

(ii) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$

(iii) $P(\bar{A} \cap B) \leq \frac{1}{4}$

(b) One shot is fired from each of the three guns. E1, E2, E3 denote the events that the target is hit by the first, second and third guns respectively. If P(E1) = 0.5, P(E2) = 0.6 and P(3) = 0.8 and E1, E2, E3, are independent events, find the probability that (i) exactly one hit is registered and (ii) at least two hits are registered. (8,7)

3. (a) Why does it pay to bet consistently on seeing 6 at least once in 4 throws of a die, but not on seeing a double six at least once in 24 throws with two dice?

(b) A jar has n chips numbered 1, 2,..., n. A person draws a chip, returns it, draws another, returns it, and so on, until a chip is drawn that has been drawn before. Let X be the number of drawings. Find E(X). (8,7)

4. (a) Prove that for any positive integer m :

$$P(\cap_{i=1}^{m} A_i) \leq P(A_i) \leq P(\cup_{i=1}^{m} A_i) \leq \sum_{i=1}^{m} P(A_i)$$

(b) In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it. (7,8)

Section B

5. (a) A person draw cards one by one from a pack of cards until draw all the aces. Find the expected value of the number of cards drawn.

(b) The length (in hours) X of a certain type of light bulb is a continuous random variable with p.d.f

$$f(x) = \begin{cases} \frac{a}{x^3}, & 1500 < x < 2500 \\ 0, & otherwise \end{cases}$$

(i) Determine the constant a,

(ii) the distribution function of X and

(iii) probability of the event 1600 < x < 1800.

(7,8)

6. (a) The c.d.f. of random variable X is defined by :

$$F(x) = \begin{cases} 0, & x \leq 1 \\ k(x-1)^2, & 1 < x \leq 3 \\ 1, & x > 3. \end{cases}$$

Find (i) value of k (ii) the pdf of X (iii) The mean and median of X.

(b) In a certain society, an individual pays income tax only if his income exceeds 'a', the amount of tax being c(x-a), where $0 < c < 1$. The distribution of the incomes of individuals liable to tax has density function

$$f(x) = \begin{cases} \frac{k}{x^{\theta+1}}; & x > a, \theta > 1 \\ 0; & otherwise \end{cases}$$

Show that the average tax paid is $\frac{ca}{(\theta-1)}$.

(8,7)

7. (a) If two unbiased dice are thrown, then find the expected values of the (i) product of the numbers (ii) square of the sum of numbers.

(b) Let = $f(x) = \begin{cases} \frac{2}{9}(x+1); & -1 \leq x < 2 \\ 0; & elsewhere \end{cases}$

be the pdf of random variable X. Find the distribution function and pdf of $Y = X^2$.

(7,8)