(7)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1160 **I**

Unique Paper Code : 2352012303

Name of the Paper : Discrete Mathematics (DSC-9)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. All questions are compulsory.

3. Questions 1 to 6 have three parts each. Attempt any **TWO** parts from each Question. Each part carries 7.5 marks.

4. Use of the Calculator is not allowed.

1. (a) Suppose $\mathbb{N}_0 = \mathbb{N} \cup \{0\} = \{0,1,2, \ldots\}$. Define $m \leq n$ in $\mathbb{N}_0$, if and only if there exists $k \in \mathbb{N}_0$, such that $n = km$. Prove that $(\mathbb{N}_0, \leq)$ is a partially ordered set. Is $(\mathbb{N}_0, \leq)$ a chain, an antichain or none? Justify your answer. (7.5)

(b) Define when we say that the two sets have the same cardinality. Prove that the sets $(0,1)$ and (a, ∞) have the same cardinality. (7.5)

(c) Draw the Hasse diagrams for the following ordered sets:

(i) $(\wp(X), \subseteq)$, with $X = \{1,2,3\}$. Here $\wp(X)$ is the power set of X.

(ii) Dual of $\oplus\, M_3$, where $M_n = 1 \oplus \bar{n} \oplus 1$.

(iii) **2 x 3**

Here n denotes the chain obtained by giving the set $P = \{0,1,\ldots., n\text{-}1\}$, the order in which $0 < 1 < \ldots < n\text{-}1$ and $\bar{\mathbf{n}}$ for P regarded as an antichain. (3, 2.5, 2)

2. (a) Define maximal element of an ordered set. Give an example of an ordered set which has exactly one maximal element but does not have a greatest (or maximum) element. Give one example of an ordered set with exactly 3 maximal elements. (7.5)

(b) (i) State duality principle in ordered sets.

(ii) Define an order preserving map between two ordered sets and prove that the composite map of two order preserving maps is order preserving. (3, 4.5)

(c) Define bottom and top element in an ordered set. Give one example of an ordered set in which bottom and top both exist and one example in which none of them exist. (7.5)

3. (a) (i) Prove that in a lattice L, for any a, b, c, $d \in L$, $a \leq c$, $b \leq d$ implies that $a \vee b \leq c \vee d$.

(ii) Prove that a lattice L is a chain if and only if every nonempty subset of L is a sublattice of L. (3, 4.5)

(b) Prove that in any lattice L, the following holds

$$((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y, \text{ for all } x, y, z \in L.$$ (7.5)

(c) Let L and K be lattices and $f: L \rightarrow K$ be a map. Show that the following are equivalent:

(i) f is order preserving.

(ii) $(\forall\, a, b \in L), f(a \wedge b) \leq f(a) \wedge f(b)$.

(7.5)

4. (a) Let L and M be two ordered sets and f be an order isomorphism from L onto M. Prove that if L is a lattice, then M is also a lattice, and f is a lattice isomorphism. (7.5)

(b) Prove that the direct product L x K of two distributive lattices L and K is also a distributive lattice. (7.5)

(c) (i) Prove that every sublattice of a modular lattice L is modular.