$$\begin{bmatrix} 3 & -1 & 5 \\ 6 & 4 & 0 \\ 10 & 8 & 6 \end{bmatrix}$$

(b) Convert the following game problem into a pair of linear programming problems for players A and B and provide the optimal strategies of the two players and value of the game.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 2 & 3 & 0 & -2 \\ 1 & 2 & 3 & 2 \end{bmatrix} \end{array}$$

(c) Find the inverse of the matrix using Simplex Method:

$$\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1228 **I**

Unique Paper Code : 2353010008

Name of the Paper : Linear Programming and Applications (DSE)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

1. (a) Solve the following linear programming problem by Graphical Method:

$$\begin{aligned} \text{Minimize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 2 \\ & 5x_1 + 9x_2 \leq 45 \\ & x_2 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

(b) Examine the convexity of the set $S = \{(x_1, x_2): x_2^2 \geq 4x_1\}$. Justify your conclusion. Further prove or disprove that intersection of two convex sets is a convex set.

(c) Solve the following problem by simplex method:

$$\begin{array}{ll} \text{Minimize} & x_1 + 2x_2 - 4x_3 \\ \text{subject to} & x_1 + x_2 + 2x_3 \leq 9 \\ & x_1 + x_2 - x_3 \leq 2 \\ & -x_1 + x_2 + x_3 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{array}$$

2. (a) Consider the following problem:

$$\begin{array}{ll} \text{Minimize} & z = cx \\ \text{subject to} & Ax \geq b, \\ & x \geq 0, \end{array}$$

Let $(x_B, 0)$ be a basic feasible solution corresponding to basis B such that $z_j - c_j \leq 0$ for all non- basic variables x_j. Prove that $(x_B, 0)$ is an optimal basic feasible solution.

	I	II	III	IV	Availability
A	19	30	50	10	7
B	70	30	40	60	9
C	40	8	70	20	18
Requirements	5	8	7	14	

(c) Find the optimal solution of the Assignment Problem with the following cost matrix :

	U	V	X	Y	Z
I	2	9	2	7	1
II	6	8	7	6	1
III	4	6	5	3	1
IV	4	2	7	3	1
V	5	3	9	5	1

6. (a) (i) Define Saddle Point and Mixed Strategy for a "Two-Person Zero Sum" game.

(ii) Use maxmin and minmax Principle to find the saddle point, if it exists, for the following pay-off matrix

$$\text{Minimize } z = 2x_1 + 9x_2 + x_3$$
$$\text{subject to } \quad x_1 + 4x_2 + 2x_3 \geq 5$$
$$3x_1 + x_2 + 2x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0.$$

5. (a) For the following cost minimization Transportation Problem, find the initial basic feasible solution by using North West Corner Rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions in terms of cost:

Machine	I	II	III	IV	V	Supply
A	2	11	10	3	7	4
B	1	4	7	2	1	8
C	3	9	4	8	12	9
Demand	3	3	4	5	6	

(b) Solve the following cost minimization Transportation Problem:

(b) Find all the basic feasible solutions for the following system of equations:

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$

Classify the obtained solutions as degenerate or non-degenerate.

(c) Reduce the feasible solution, $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ of the following system to two different basic feasible solutions:

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14.$$

3. (a) Solve the following problem by Big-M method

$$\text{Maximize } \quad z = 6x_1 + 4x_2$$
$$\text{subject to } \quad 2x_1 + 3x_2 \leq 30$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Is the solution unique? If not, give two different solutions.

(b) Solve the following problem by Two-phase method

$$\begin{aligned} \text{Minimize } z &= -5x_1 + 4x_2 - 3x_3 \\ \text{subject to } \quad & 2x_1 + x_2 - 6x_3 = 20 \\ & 6x_1 + 5x_2 + 10x_3 \leq 76 \\ & 8x_1 - 3x_2 + 6x_3 \leq 50 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(c) Solve the following system of simultaneous equations using simplex method:

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2\,x_1 + x_2 &= 3 \end{aligned}$$

4 (a) (i) Prove that if either the primal or the dual problem has an unbounded objective function value, then the other problem has no feasible solution.

(ii) Show that the dual of the dual is primal for the following primal problem

$$\begin{aligned} \text{Minimize } z &= x_1 + 2x_2 - 3x_3 \\ \text{subject to } \quad & 4x_1 + 5x_2 - 6x_3 = 7 \\ & 8x_1 - 9x_2 + 10x_3 \leq 11 \\ & x_1, x_2 \geq 0\ , x_3 \text{ unrestricted.} \end{aligned}$$

(b) Obtain the dual of the following primal problem:

$$\begin{aligned} \text{Minimize } z &= 3x_1 - 2x_2 + 4x_3 \\ \text{subject to } \quad & 3x_1 + 5x_2 + 4x_3 \geq 7 \\ & 6x_1 + x_2 + 3x_3 \geq 4 \\ & 7x_1 - 2x_2 - x_3 \leq 10 \\ & x_1 - 2x_2 + 5x_3 \geq 3 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(c) Solve the following problem using Complementary Slackness theorem