1.(a) Find V_x in the following circuit: (A circuit diagram is provided with a 15V voltage source, resistors of 1Ω, 2Ω, 5Ω, and a dependent voltage source $2V_x$.)

To find V_x in the given circuit:

- **Identify the circuit elements:** The circuit consists of a 15V independent voltage source, a 1Ω resistor, a 2Ω resistor, a 5Ω resistor, and a dependent voltage source $2V_x$.
- **Apply Kirchhoff's Voltage Law (KVL):** We can apply KVL to the loop containing the 15V source, the 1Ω resistor, and the 2Ω resistor. Let I be the current flowing clockwise in the main loop. The voltage across the 2Ω resistor is V_x. So, $V_x = I \times 2\Omega$. The voltage across the 1Ω resistor is $I \times 1\Omega$. The voltage across the 5Ω resistor is $I \times 5\Omega$. The dependent voltage source is $2V_x = 2(2I) = 4I$.
- **Write the KVL equation for the entire loop:** $-15V + (I \times 1\Omega) + (I \times 2\Omega) + (I \times 5\Omega) + 2V_x = 0$ Substitute $V_x = 2I$: $-15 + I + 2I + 5I + 2(2I) = 0$ $-15 + I + 2I + 5I + 4I = 0$ $-15 + 12I = 0$ $12I = 15$ $I = \frac{15}{12} = \frac{5}{4} = 1.25A$
- **Calculate V_x:** $V_x = I \times 2\Omega = 1.25A \times 2\Omega = 2.5V$

(b) What is a supermode? Explain with an example.

- A supermode is a concept used in the analysis of coupled systems, particularly in optics and quantum mechanics.

- It refers to a collective mode of oscillation or propagation that involves multiple coupled subsystems vibrating or propagating in a synchronized manner.
- In simpler terms, when two or more individual systems (like waveguides or resonators) are brought close enough to interact, their individual modes can combine to form new, collective modes, known as supermodes. These supermodes can have different characteristics (like propagation constants or frequencies) compared to the original uncoupled modes.
- **Example:** Consider two identical optical waveguides placed parallel to each other.
  - If they are far apart, light propagates independently in each waveguide, each having its own fundamental mode.
  - When they are brought close enough, the evanescent fields of the modes in each waveguide overlap, leading to coupling.
  - Due to this coupling, the individual modes no longer propagate independently. Instead, two new supermodes are formed:
    - **Symmetric Supermode:** Where the electric fields in both waveguides are in phase.

        - **Anti-symmetric Supermode:** Where the electric fields in both waveguides are 180 degrees out of phase.
    - These two supermodes will have slightly different propagation constants, leading to power transfer between the waveguides as light propagates along their length. This phenomenon is fundamental to directional couplers and other integrated optical devices.

(c) Define reactive power. What is its value for a pure resistance.

- **Reactive Power (Q):**
    - Reactive power is the portion of apparent power that is exchanged between the source and the reactive components (inductors and capacitors) in an AC circuit.
    - It represents the power that oscillates back and forth between the source and the reactive elements, and it does not contribute to the net transfer of energy or do any useful work.
    - It is responsible for establishing and maintaining the electric and magnetic fields in capacitors and inductors, respectively.

- Reactive power is measured in Volt-Ampere Reactive (VAR).
- It can be calculated as $Q = V_{rms} I_{rms} \sin(\phi)$, where ϕ is the phase angle between voltage and current.

- **Value for a pure resistance:**
  - For a pure resistance, the voltage and current are always in phase, meaning the phase angle $\phi = 0°$.
  - Therefore, $\sin(\phi) = \sin(0°) = 0$.
  - Substituting this into the reactive power formula: $Q = V_{rms} I_{rms} \sin(0°) = V_{rms} I_{rms} \times 0 = 0$.
  - Hence, the reactive power for a pure resistance is zero. A pure resistor only consumes real power (P).

(d) State Millman's theorem.

- **Millman's Theorem:**
  - Millman's theorem states that for any number of parallel voltage sources with series resistances, the voltage across the parallel combination is equal to the sum of the short-circuit currents of each branch divided by the sum of the conductances of each branch.
  - Alternatively, it can be stated that the voltage at a common node of several parallel branches, where each

branch contains a voltage source and a series impedance, is given by the sum of the ratios of each source voltage to its series impedance, divided by the sum of the reciprocals of all the series impedances.

- **Formula:** If there are 'n' branches connected to a common node, and each branch 'k' has a voltage source V_k and a series impedance Z_k, then the voltage at the common node (V_{node}) with respect to a reference node is: $V_{node} = \frac{\frac{V_1}{Z_1}+\frac{V_2}{Z_2}+\cdots+\frac{V_n}{Z_n}}{\frac{1}{Z_1}+\frac{1}{Z_2}+\cdots+\frac{1}{Z_n}}$
- In terms of conductances ($Y_k = \frac{1}{Z_k}$): $V_{node} = \frac{V_1Y_1+V_2Y_2+\cdots+V_nY_n}{Y_1+Y_2+\cdots+Y_n}$
- This theorem is particularly useful for simplifying circuits with multiple parallel voltage sources.

(e) Calculate the Thevenin's equivalent resistance across terminals AB. (A circuit diagram is provided with a 5V voltage source, current source 5A, and resistors of 10Ω, $10j\Omega$, 2Ω.)

To calculate Thevenin's equivalent resistance (R_{Th}) across terminals AB:

- **Deactivate independent sources:**
  - Voltage sources are short-circuited (replaced by a wire).

  - Current sources are open-circuited (removed from the circuit).
- **Redraw the circuit with deactivated sources:**
  - The 5V voltage source becomes a short circuit.
  - The 5A current source becomes an open circuit.
- **Identify the components remaining and their connections:**
  - The 10Ω resistor is in series with the short-circuited 5V source. Effectively, the 10Ω resistor is connected.
  - The $10j\Omega$ inductor and 2Ω resistor are connected between the node and terminal B.
- **Calculate the equivalent resistance/impedance seen from terminals AB:**
  - Looking into terminals AB, the 10Ω resistor is connected to one side.
  - The $10j\Omega$ inductor is in series with the 2Ω resistor. This combination is connected in parallel with the 10Ω resistor. No, this isn't correct based on typical circuit configurations. Let's assume the 10Ω resistor is in series with the 5V source, the $10j\Omega$ in series with 2Ω, and the current source is connected between some nodes. Without a visual diagram, the exact connection cannot be definitively determined.

- **Assuming a common configuration:** If the 10Ω resistor is in series with the 5V source, and the $10j\Omega$ and 2Ω resistors are connected in a way that they form a separate path, and then the current source is bridging two nodes.
- Let's assume the 10Ω resistor is in series with the 5V source. When the 5V source is shorted, the 10Ω resistor is effectively present.
- Let's assume the $10j\Omega$ inductor and 2Ω resistor are connected to terminals AB.
- Let's assume the 5A current source is connected in parallel with some other components. When it is open-circuited, it is removed.
- **Common interpretation for R_{Th} calculation with no dependent sources:**
  - Short-circuit the 5V voltage source.
  - Open-circuit the 5A current source.
  - Look into terminals AB.
  - If the 10Ω resistor is in series with the 5V source, it remains in the circuit.
  - If the $10j\Omega$ and 2Ω resistors are in parallel with each other, and then this combination is in series

with the 10Ω resistor (or vice-versa), we calculate accordingly.

- **Without a clear diagram, I will make an assumption that 10Ω is parallel to ($10j\Omega$ in series with 2Ω).** This is a common arrangement.

- $Z_{Th} = 10\Omega \mid\mid (2\Omega + j10\Omega)$
- $Z_{Th} = \frac{10\times(2+j10)}{10+(2+j10)}$
- $Z_{Th} = \frac{20+j100}{12+j10}$
- To simplify, multiply by the conjugate of the denominator: $Z_{Th} = \frac{20+j100}{12+j10} \times \frac{12-j10}{12-j10}$ $Z_{Th} = \frac{(20)(12)+(20)(-j10)+(j100)(12)+(j100)(-j10)}{12^2+10^2}$ $Z_{Th} = \frac{240-j200+j1200-j^{21000}}{144+100}$ $Z_{Th} = \frac{240+j1000+1000}{244}$ (since $j^2 = -1$) $Z_{Th} = \frac{1240+j1000}{244}$ $Z_{Th} = \frac{1240}{244} + j\frac{1000}{244}$ $Z_{Th} \approx 5.082 + j4.098\Omega$
- **Note:** If the circuit configuration is different, the calculation will change. The absence of a visual diagram makes the interpretation difficult. Assuming all are connected to A and B.

(f) Define the terms quality factor and bandwidth.

- **Quality Factor (Q):**

- The quality factor, often denoted as Q, is a dimensionless parameter that describes how underdamped an oscillator or resonator is. It characterizes the "quality" or "sharpness" of a resonant circuit.
- In an RLC circuit, it is a measure of the ratio of the energy stored in the circuit to the energy dissipated per cycle. A higher Q factor indicates lower energy loss and a more selective circuit (sharper resonance).
- **Formula for series RLC circuit:** $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$
- **Formula for parallel RLC circuit:** $Q = \frac{R}{\omega_0 L} = \omega_0 CR$
- Where ω_0 is the resonant frequency, L is inductance, C is capacitance, and R is resistance.
- A high Q factor implies a narrow bandwidth and efficient energy storage.

- **Bandwidth (B or BW):**
  - Bandwidth, in the context of resonant circuits, refers to the range of frequencies over which the circuit's response (e.g., current or voltage) is significant, typically defined by the half-power points (or -3dB points).

- The half-power points are the frequencies where the power delivered to the load is half of the maximum power delivered at resonance. Equivalently, these are the frequencies where the current or voltage magnitude is $1/\sqrt{2}$ (approximately 0.707) of its maximum value at resonance.
- It quantifies the frequency range over which a filter or resonant circuit operates effectively.
- **Formula:** $B = \omega_2 - \omega_1$, where ω_1 and ω_2 are the lower and upper half-power frequencies, respectively.
- **Relationship with Q factor and resonant frequency:** For a resonant circuit, bandwidth is inversely proportional to the quality factor: $B = \frac{\omega_0}{Q}$ (for angular frequency) or $B = \frac{f_0}{Q}$ (for linear frequency)
- A smaller bandwidth indicates higher selectivity (only a narrow range of frequencies passes through or resonates), while a larger bandwidth indicates less selectivity.

2.(a) Use mesh analysis to determine i_1, i_2 and i_3 in the given circuit (A circuit diagram is provided with a 24V voltage source, current source 4A, and resistors of 5Ω, 5Ω, 3Ω, 10Ω, 20Ω.)

To determine i_1, i_2, and i_3 using mesh analysis:

- **Identify the meshes and assign mesh currents:** Let's assume three meshes with clockwise currents i_1, i_2, and i_3.
  - Mesh 1: Involves the 24V source, the first 5Ω resistor, and the 3Ω resistor.
  - Mesh 2: Involves the 3Ω resistor, the second 5Ω resistor, the 10Ω resistor, and the 4A current source.
  - Mesh 3: Involves the 10Ω resistor, the 20Ω resistor, and the 4A current source.
- **Handle the current source:** The 4A current source is shared between Mesh 2 and Mesh 3, forming a supermesh.
  - **Supermesh equation:** The current source imposes a constraint on the mesh currents. $i_2 - i_3 = 4A$ (assuming i_2 is in the direction of the current source and i_3 is opposite).
- **Apply KVL to Mesh 1:** $-24 + 5i_1 + 3(i_1 - i_2) = 0$ $-24 + 5i_1 + 3i_1 - 3i_2 = 0$ $8i_1 - 3i_2 = 24$ (Equation 1)
- **Apply KVL to the Supermesh (Mesh 2 and Mesh 3 combined):** Start from a point and go around the supermesh, avoiding the current source. $3(i_2 - i_1) + 5i_2 + 10(i_3 - 0) + 20i_3 = 0$ (assuming the 10Ω and 20Ω are in the rightmost branch, not shared with other meshes besides the supermesh). If the 10Ω is between i_2 and i_3, then $10(i_2 - i_3)$ or $10(i_3 - i_2)$. Let's assume the 10Ω is shared

between i_2 and i_3, and 20Ω is only in i_3. This requires a precise diagram.

Re-interpreting a common mesh configuration based on description:

- Mesh 1: 24V source, 5 Ohm, 3 Ohm. Current i_1.
- Mesh 2: 3 Ohm, 5 Ohm, 10 Ohm. Current i_2.
- Mesh 3: 10 Ohm, 20 Ohm, 4A current source. Current i_3.
- If 4A source is vertical between meshes 2 and 3, and common to both 10 Ohm and 20 Ohm.

Let's assume the 4A current source is in the branch between Mesh 2 and Mesh 3, such that i_3 flows down through it, and i_2 flows up through it.

- $i_3 - i_2 = 4A$ (Supermesh constraint)
- **KVL for Mesh 1:** $-24 + 5i_1 + 3(i_1 - i_2) = 0$ $8i_1 - 3i_2 = 24$ (Equation 1)
- **KVL for the Supermesh (around Mesh 2 and Mesh 3, excluding the current source branch):** $3(i_2 - i_1) + 5i_2 + 10i_3 + 20i_3 = 0$ $-3i_1 + (3+5)i_2 + (10+20)i_3 = 0$ $-3i_1 + 8i_2 + 30i_3 = 0$ (Equation 2)

- **Solve the system of equations:** From the supermesh constraint: $i_3 = i_2 + 4$

Substitute i_3 into Equation 2: $-3i_1 + 8i_2 + 30(i_2 + 4) = 0$ $-3i_1 + 8i_2 + 30i_2 + 120 = 0$ $-3i_1 + 38i_2 = -120$ (Equation 3)

Now we have a system of two equations with two unknowns (i_1, i_2):

a. $8i_1 - 3i_2 = 24$

b. $-3i_1 + 38i_2 = -120$

Multiply Equation 1 by 3 and Equation 3 by 8 to eliminate i_1: $3 \times (8i_1 - 3i_2 = 24) \Longrightarrow 24i_1 - 9i_2 = 72$ $8 \times (-3i_1 + 38i_2 = -120) \Longrightarrow -24i_1 + 304i_2 = -960$

Add the two new equations: $(24i_1 - 9i_2) + (-24i_1 + 304i_2) = 72 - 960$ $295i_2 = -888$ $i_2 = -\frac{888}{295} \approx -3.01A$

Substitute i_2 back into Equation 1: $8i_1 - 3(-3.01) = 24$ $8i_1 + 9.03 = 24$ $8i_1 = 24 - 9.03$ $8i_1 = 14.97$ $i_1 = \frac{14.97}{8} \approx 1.87A$

Calculate i_3: $i_3 = i_2 + 4 = -3.01 + 4 = 0.99A$

Therefore:

- $i_1 \approx 1.87A$
- $i_2 \approx -3.01A$
- $i_3 \approx 0.99A$

(b) For the given network, obtain the equivalent resistance at the terminals a-b (A circuit diagram is provided with resistors of 25Ω, 30Ω, 10Ω, 20Ω, 5Ω, 15Ω.)

To find the equivalent resistance R_{ab} at terminals a-b:

- **Identify series and parallel combinations:** Without a visual diagram, I will assume a common ladder or bridge configuration for these resistors.
  - Let's assume the 25Ω and 30Ω are in series. (This is a common start for complex networks)
  - Let's assume the 10Ω and 20Ω are in series.
  - Let's assume the 5Ω and 15Ω are in series.

  This is an assumption. A typical scenario is to reduce series/parallel combinations systematically. Let's assume a configuration as often seen in textbooks:
  - 20Ω and 5Ω are in series. Let this be $R_A = 20 + 5 = 25\Omega$.
  - This R_A is parallel with the 10Ω resistor. Let this be $R_B = \frac{25\times10}{25+10} = \frac{250}{35} = \frac{50}{7} \approx 7.14\Omega$.
  - This R_B is in series with the 15Ω resistor. Let this be $R_C = 7.14 + 15 = 22.14\Omega$.
  - This R_C is in parallel with the 30Ω resistor. Let this be $R_D = \frac{22.14\times30}{22.14+30} = \frac{664.2}{52.14} \approx 12.74\Omega$.

- Finally, this R_D is in series with the 25Ω resistor, connected to terminals a-b.
- $R_{ab} = 25\Omega + R_D = 25 + 12.74 = 37.74\Omega$.

- **Note:** The exact value will heavily depend on the arrangement of the resistors in the circuit diagram, which is not provided. The above is a plausible interpretation for calculating equivalent resistance by reducing series/parallel branches. If it is a bridge circuit, a different approach (like delta-wye transformation) might be needed. Without a diagram, this is the best general approach assuming simple series/parallel reduction.

(c) In the following circuit, find the values of R, V_1 and V_2, given $i_0 = 15mA$ (A circuit diagram is provided on Source 4, showing a 60mA current source, and resistors $10k\Omega$, $6k\Omega$, R, with voltages V_1, V_2.)

To find R, V_1, and V_2 given $i_0 = 15mA$:

- **Analyze the circuit based on a common configuration:** Assuming the 60mA current source is at the input, and $10k\Omega$, $6k\Omega$, and R are arranged in a way that i_0 flows through one of them, and V_1 and V_2 are node voltages.
- Let's assume a typical current divider or node voltage setup.
  - Let the 60mA current source be connected to a node, say Node A.

- From Node A, there might be branches: one with $10k\Omega$, one with $6k\Omega$, and one with R.
- Let V_1 be the voltage at Node A (with respect to ground).
- Let i_0 be the current flowing through a specific resistor.
- Let V_2 be the voltage at another node.

Assumption based on typical current source/resistor configurations:

- The 60mA current source feeds a node.
- The $10k\Omega$ resistor is connected between the source node (where V_1 is) and ground. So V_1 is the voltage across $10k\Omega$.
- The $6k\Omega$ resistor and resistor R are in series, and this series combination is in parallel with the $10k\Omega$ resistor, connected to the same source node.
- $i_0 = 15mA$ is the current flowing through the series combination of $6k\Omega$ and R.
- V_2 is the voltage across resistor R.

- **Calculate V_1:**
  - The total current from the source is 60mA.

- Current through the $10k\Omega$ resistor is $I_{10k\Omega} = \frac{V_1}{10k\Omega}$.
- Current through the $(6k\Omega + R)$ branch is $i_0 = 15mA$.
- By Kirchhoff's Current Law (KCL) at Node V_1: $60mA = I_{10k\Omega} + i_0$ $60mA = \frac{V_1}{10k\Omega} + 15mA$ $\frac{V_1}{10k\Omega} = 60mA - 15mA = 45mA$ $V_1 = 45mA \times 10k\Omega = 45 \times 10^{-3}A \times 10 \times 10^3\Omega = 450V$
- So, $V_1 = 450V$.

- **Calculate R:**
  - The voltage across the series combination of $6k\Omega$ and R is $V_1 = 450V$.
  - The current flowing through this branch is $i_0 = 15mA$.
  - The total resistance of this branch is $6k\Omega + R$.
  - Using Ohm's Law: $V_1 = i_0 \times (6k\Omega + R)$ $450V = 15mA \times (6k\Omega + R)$ $\frac{450V}{15mA} = 6k\Omega + R$ $\frac{450}{15\times10^{-3}} = 6 \times 10^3 + R$ $30 \times 10^3\Omega = 6 \times 10^3\Omega + R$ $30k\Omega = 6k\Omega + R$ $R = 30k\Omega - 6k\Omega = 24k\Omega$
  - So, $R = 24k\Omega$.
- **Calculate V_2:**
  - V_2 is the voltage across resistor R.

- Using Ohm's Law: $V_2 = i_0 \times R$ $V_2 = 15mA \times 24k\Omega = 15 \times 10^{-3}A \times 24 \times 10^3\Omega = 360V$
- So, $V_2 = 360V$.

Therefore:

- $R = 24k\Omega$
- $V_1 = 450V$
- $V_2 = 360V$

3.(a) Obtain the equivalent impedance of the following network: (A circuit diagram is provided with capacitors, inductors and resistors: $j4\Omega$, $-j\Omega$, 2Ω, 1Ω, $j2\Omega$, $-j2\Omega$.)

To obtain the equivalent impedance:

- **Identify the components and their connections:** Without a diagram, I'll assume a series-parallel combination. Let's list the components:
  - Inductive reactance: $j4\Omega$, $j2\Omega$
  - Capacitive reactance: $-j\Omega$, $-j2\Omega$
  - Resistances: 2Ω, 1Ω
- **Assume a reasonable configuration for calculating equivalent impedance:** Let's consider a configuration where:
  - 2Ω resistor and $-j2\Omega$ capacitor are in series.

- 1Ω resistor and $j2\Omega$ inductor are in series.
- The combination of (2Ω in series with $-j2\Omega$) is in parallel with the combination of (1Ω in series with $j2\Omega$).
- And finally, $j4\Omega$ inductor and $-j\Omega$ capacitor are in series with this parallel combination.
- **Step 1: Calculate the impedance of the first series branch (Z_1).** $Z_1 = 2\Omega - j2\Omega$
- **Step 2: Calculate the impedance of the second series branch (Z_2).** $Z_2 = 1\Omega + j2\Omega$
- **Step 3: Calculate the parallel combination of Z_1 and Z_2 ($Z_{parallel}$).** $Z_{parallel} = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$ $Z_{parallel} = \frac{(2-j2)(1+j2)}{(2-j2)+(1+j2)}$ $Z_{parallel} = \frac{2+j4-j2-j^{2}4}{3}$ $Z_{parallel} = \frac{2+j2+4}{3} = \frac{6+j2}{3} = 2 + j\frac{2}{3}\Omega$
- **Step 4: Add the remaining series components.** Let's assume $j4\Omega$ and $-j\Omega$ are in series with $Z_{parallel}$. $Z_{eq} = Z_{parallel} + j4\Omega - j\Omega$ $Z_{eq} = (2 + j\frac{2}{3}) + j4 - j1$ $Z_{eq} = 2 + j(\frac{2}{3} + 4 - 1)$ $Z_{eq} = 2 + j(\frac{2}{3} + 3)$ $Z_{eq} = 2 + j(\frac{2+9}{3})$ $Z_{eq} = 2 + j\frac{11}{3}\Omega$ $Z_{eq} \approx 2 + j3.67\Omega$

- **Note:** The solution is highly dependent on the circuit configuration which is not provided. This is a common way to approach such problems by assuming series and parallel combinations.

(b) Find I_0 using mesh analysis. (A circuit diagram is provided with a $10\angle 0^\circ A$ current source, a $50\angle 30^\circ V$ voltage source, and components $-j2\Omega$, 6Ω, $j4\Omega$, 8Ω.)

To find I_0 using mesh analysis:

- **Identify meshes and assign mesh currents:** Let's assume three meshes.
    - Mesh 1: Contains the $10\angle 0^\circ A$ current source. Let this be I_1.
    - Mesh 2: Contains the $-j2\Omega$ and 6Ω components. Let this be I_2.
    - Mesh 3: Contains the $j4\Omega$, 8Ω components, and the $50\angle 30^\circ V$ voltage source. Let this be I_3.
    - Let I_0 be a current in one of the branches, for instance, the current through the 8Ω resistor.
- **Handle the current source:** If the $10\angle 0^\circ A$ current source is directly in Mesh 1, then $I_1 = 10\angle 0^\circ A$. If it's a shared branch between two meshes, it forms a supermesh. Let's assume it's in the leftmost branch, so $I_1 = 10\angle 0^\circ A$.
- **Write KVL equations for each mesh:**

- **Mesh 1:** Since there's a current source, we don't apply KVL directly to this mesh if it's the only element setting the current. If it's between two meshes, it's a supermesh. Let's assume the $10\angle 0^\circ A$ source is in the far left, defining I_1. So, $I_1 = 10\angle 0^\circ A$.
- **Mesh 2:** Let's assume the components are connected such that I_2 flows through $-j2\Omega$ and 6Ω. And that $-j2\Omega$ is shared with I_1. $(-j2)(I_2 - I_1) + 6I_2 + j4(I_2 - I_3) = 0$ (This assumes a specific layout) This depends on how I_0 is defined and where the components are.

Let's assume a common structure for Mesh Analysis with a current source:

- Mesh 1: I_1
- Mesh 2: I_2
- Mesh 3: I_3 (Let I_0 be I_3 or a combination)

Let's assume:

- The $10\angle 0^\circ A$ current source is between Mesh 1 and Mesh 2. So, $I_1 - I_2 = 10\angle 0^\circ A$ (or $I_2 - I_1 = 10\angle 0^\circ A$ depending on direction). Let's assume it forces I_1 to be $10\angle 0^\circ A$ through a branch.
- The $-j2\Omega$ is common between Mesh 1 and Mesh 2.
- The 6Ω is in Mesh 2.

- The $j4\Omega$ is common between Mesh 2 and Mesh 3.
- The 8Ω is in Mesh 3.
- The $50\angle 30^{\circ} V$ voltage source is in Mesh 3.
- Let I_0 be I_3.

Supermesh approach: Let Mesh 1, Mesh 2, and Mesh 3 be defined. If the $10\angle 0^{\circ} A$ current source is the only element in the branch between Mesh 1 and Mesh 2: $I_1 - I_2 = 10\angle 0^{\circ}$ (if I_1 is current through the branch and I_2 is opposite)

Let's try a different assumption for typical mesh analysis with current sources: Assume I_1, I_2, I_3 are mesh currents.

- $10\angle 0^{\circ} A$ source is between Mesh 1 and Mesh 2. Let I_0 be I_3.
- Supermesh equation: $I_1 - I_2 = 10\angle 0^{\circ}$ (Assuming I_1 is the current through the source in its direction).
- KVL for Supermesh (around the outer loop of Mesh 1 and Mesh 2): Assume $-j2\Omega$ is in Mesh 1 and 6Ω is in Mesh 2. $j4\Omega$ is shared between Mesh 2 and Mesh 3. 8Ω in Mesh 3. $-j2\Omega \cdot I_1 + 6\Omega \cdot I_2 + j4\Omega(I_2 - I_3) = 0$ $-j2I_1 + (6 + j4)I_2 - j4I_3 = 0$ (Equation A)

- KVL for Mesh 3: $j4(I_3 - I_2) + 8I_3 + 50\angle 30^\circ = 0$ (assuming source aids I_3) $-j4I_2 + (j4 + 8)I_3 = -50\angle 30^\circ$ (Equation B)

Now we have 3 unknowns (I_1, I_2, I_3) and 3 equations:

c. $I_1 = I_2 + 10\angle 0^\circ$ (or $I_2 = I_1 - 10\angle 0^\circ$)

d. $-j2I_1 + (6 + j4)I_2 - j4I_3 = 0$

e. $-j4I_2 + (8 + j4)I_3 = -50\angle 30^\circ$

Substitute (1) into (2): $-j2(I_2 + 10) + (6 + j4)I_2 - j4I_3 = 0$ $-j2I_2 - j20 + (6 + j4)I_2 - j4I_3 = 0$ $(6 + j4 - j2)I_2 - j4I_3 = j20$ $(6 + j2)I_2 - j4I_3 = j20$ (Equation C)

Now solve system of (B) and (C): C) $(6 + j2)I_2 - j4I_3 = j20$ B) $-j4I_2 + (8 + j4)I_3 = -50\angle 30^\circ$

From (C): $I_2 = \frac{j20 + j4I_3}{6 + j2}$ Substitute into (B):

$-j4\left(\frac{j20 + j4I_3}{6 + j2}\right) + (8 + j4)I_3 = -50\angle 30^\circ$ $\frac{-j^{280} - j^{216}I_3}{6 + j2} + (8 + j4)I_3 = -50\angle 30^\circ$ $\frac{80 + 16I_3}{6 + j2} + (8 + j4)I_3 = -50\angle 30^\circ$

Multiply by $(6 + j2)$: $80 + 16I_3 + (8 + j4)(6 + j2)I_3 = -50\angle 30^\circ(6 + j2)$ $80 + 16I_3 + (48 + j16 + j24 + j^{28})I_3 = -50(\cos 30^\circ + j\sin 30^\circ)(6 + j2)$ $80 + 16I_3 + (48 + j40 - 8)I_3 = -50(0.866 + j0.5)(6 + j2)$ $80 + 16I_3 + (40 + j40)I_3 = -50(5.196 + j1.732 + j3 + j^{21})$ $80 + (16 + 40 + j40)I_3 = -50(5.196 + j4.732 - 1)$ $80 + (56 + j40)I_3 = -50(4.196 + j4.732)$ $80 + (56 +$

$j40)I_3 = -209.8 - j236.6$ $(56 + j40)I_3 = -209.8 - j236.6 - 80$ $(56 + j40)I_3 = -289.8 - j236.6$ $I_3 = \frac{-289.8 - j236.6}{56 + j40}$ Convert to polar form: Numerator: $M_N = \sqrt{(-289.8)^2 + (-236.6)^2} = \sqrt{84084.04 + 55979.56} = \sqrt{140063.6} \approx 374.25$ $\theta_N = \text{atan2}(-236.6, -289.8) = -140.7^\circ$ or 219.3° $I_N \approx 374.25\angle -140.7^\circ$

Denominator: $M_D = \sqrt{56^2 + 40^2} = \sqrt{3136 + 1600} = \sqrt{4736} \approx 68.82$ $\theta_D = \text{atan2}(40{,}56) \approx 35.5^\circ$ $D \approx 68.82\angle 35.5^\circ$

$I_3 = \frac{374.25\angle -140.7^\circ}{68.82\angle 35.5^\circ}$ $I_3 \approx \frac{374.25}{68.82}\angle(-140.7^\circ - 35.5^\circ)$ $I_3 \approx 5.438\angle -176.2^\circ A$

If $I_0 = I_3$, then $I_0 \approx 5.438\angle -176.2^\circ A$. **Note:** The definition of I_0 in the diagram is crucial. My assumption for I_0 is as I_3.

(c) Using node analysis, calculate V_1 and V_2 in the following circuit (A circuit diagram is provided with a 600V voltage source, 15A current source, and resistors of 50Ω, 10Ω, 30Ω.)

To calculate V_1 and V_2 using node analysis:

- **Identify nodes and assign node voltages:** Let V_1 and V_2 be the unknown node voltages. Choose a reference node (ground).
- **Apply KCL at each non-reference node.**

Let's assume a common configuration:

- Node V_1: Connected to the 600V source (possibly through a resistor), 50Ω resistor, and 10Ω resistor.
- Node V_2: Connected to the 10Ω resistor, 30Ω resistor, and 15A current source.
- Let the 600V source be connected to Node 1 via some resistor (e.g., R_s). If it's a direct connection to V_1, then $V_1 = 600V$. Let's assume it's connected in series with a resistor, or it defines a supernode.
- **Assumption:** The 600V source is connected such that V_1 is a node, and the 600V source is between V_1 and ground, making $V_1 = 600V$. This is simplest.
    - If V_1 is directly connected to the 600V source (e.g., it's the positive terminal with the negative terminal at ground), then $V_1 = 600V$.
- **Alternative (Supernode):** If the 600V source is *between* two non-reference nodes, or between a non-reference node and ground with a resistor, then we apply KCL to the supernode.
- **Let's assume a common structure where V_1 is a node that the 600V source *connects to* the circuit, and V_2 is another node.** Assume the 600V source is in series with the 50Ω resistor, connecting to Node V_1. Assume the 10Ω resistor is between V_1 and V_2.

Assume the 30Ω resistor is between V_2 and ground. Assume the 15A current source is connected to Node V_2 (flowing out of it to ground).

- **Node V_1 equation (KCL):** Current leaving V_1 through 50Ω (towards the 600V source): $\frac{V_1-600}{50}$ Current leaving V_1 through 10Ω (towards V_2): $\frac{V_1-V_2}{10}$ Sum of currents leaving $V_1 = 0$: $\frac{V_1-600}{50} + \frac{V_1-V_2}{10} = 0$ Multiply by 50 to clear denominators: $(V_1 - 600) + 5(V_1 - V_2) = 0$ $V_1 - 600 + 5V_1 - 5V_2 = 0$ $6V_1 - 5V_2 = 600$ (Equation 1)
- **Node V_2 equation (KCL):** Current leaving V_2 through 10Ω (towards V_1): $\frac{V_2-V_1}{10}$ Current leaving V_2 through 30Ω (towards ground): $\frac{V_2}{30}$ Current leaving V_2 due to 15A source: $15A$ (if it's leaving, or $-15A$ if entering) Assume 15A source is entering V_2. So, current leaving is $-15A$. $\frac{V_2-V_1}{10} + \frac{V_2}{30} - 15 = 0$ Multiply by 30 to clear denominators: $3(V_2 - V_1) + V_2 - 450 = 0$ $3V_2 - 3V_1 + V_2 - 450 = 0$ $-3V_1 + 4V_2 = 450$ (Equation 2)
- **Solve the system of equations:**

  f. $6V_1 - 5V_2 = 600$

  g. $-3V_1 + 4V_2 = 450$

Multiply Equation 2 by 2: $2 \times (-3V_1 + 4V_2 = 450) \Longrightarrow -6V_1 + 8V_2 = 900$

Add this new equation to Equation 1: $(6V_1 - 5V_2) + (-6V_1 + 8V_2) = 600 + 900$ $3V_2 = 1500$ $V_2 = \frac{1500}{3} = 500V$

Substitute $V_2 = 500V$ back into Equation 1: $6V_1 - 5(500) = 600$ $6V_1 - 2500 = 600$ $6V_1 = 600 + 2500$ $6V_1 = 3100$ $V_1 = \frac{3100}{6} = \frac{1550}{3} \approx 516.67V$

Therefore:

- $V_1 \approx 516.67V$
- $V_2 = 500V$

4.(a) Find the rms value, average value and form factor for the given waveform (A graph of v(t) versus t is provided, showing a triangular waveform peaking at 10, repeating from 0-2, 5-7, 10-12.)

To find the RMS value, average value, and form factor for the given triangular waveform:

- **Analyze the waveform:**
  - The waveform is a triangular wave.
  - It peaks at $V_{max} = 10$.

- It repeats every 5 units of time (e.g., from 0-5, 5-10, 10-15).
- It's a half-wave rectified triangle if it goes to 0, or a full triangle if it goes negative. The description "peaking at 10, repeating from 0-2, 5-7, 10-12" suggests segments of a triangle. Let's assume it's a periodic waveform defined over one period.
- A period T from the description "0-2, 5-7, 10-12" is confusing. It implies a pattern.
- If it means from 0 to 2 is one cycle, 5 to 7 is another, etc., then the period is $T = 2$ (length of each active pulse).
- However, if it's "0-2" (ramp up/down) then "2-5" (zero), then "5-7" (ramp up/down), the period would be 5.
- Let's assume the simplest interpretation: A triangular pulse from $t = 0$ to $t = 2$ peaking at 10, and then zero for $t = 2$ to $t = 5$. Then it repeats. So the period $T = 5$.
- The waveform can be defined as:
  - $v(t) = 5t$ for $0 \leq t \leq 1$ (ramp up to 10)

  - $v(t) = -5(t-2)$ for $1 \le t \le 2$ (ramp down from 10 to 0) or $v(t) = 10 - 5(t-1) = 15 - 5t$
  - $v(t) = 0$ for $2 < t \le 5$

- **1. Average Value (V_{avg}):** $V_{avg} = \frac{1}{T}\int_0^T v\,(t)dt$ The integral is the area under one cycle divided by the period. Area of the triangle from $t = 0$ to $t = 2$ is $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 2 \times 10 = 10$. The period $T = 5$. $V_{avg} = \frac{10}{5} = 2V$

- **2. RMS Value (V_{rms}):** $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v\,(t)^2 dt}$ We need to integrate $v(t)^2$ over the active part (0 to 2). For $0 \le t \le 1$: $v(t) = 10t$ (assuming peak at t=1, slope 10, not 5t) Let's re-evaluate the waveform definition based on "peaking at 10, repeating from 0-2". This means the base is 2 and the peak is 10. The function is: * $v(t) = 10t$ for $0 \le t \le 1$ (slope is 10) * $v(t) = 10 - 10(t-1) = 20 - 10t$ for $1 \le t \le 2$ (slope is -10) * $v(t) = 0$ for $2 < t \le 5$ (period $T = 5$)

  $\int_0^T v\,(t)^2 dt = \int_0^1 (10t)^2 dt + \int_1^2 (20 - 10t)^2 dt + \int_2^5 0^2\, dt$ $= \int_0^1 100\, t^2 dt + \int_1^2 (400 - 400t + 100t^2) dt =$ $[100\frac{t^3}{3}]_0^1 + [400t - 400\frac{t^2}{2} + 100\frac{t^3}{3}]_1^2 = (\frac{100}{3} - 0) +$ $[(400(2) - 200(2^2) + \frac{100}{3}(2^3)) - (400(1) - 200(1^2) +$

$\frac{100}{3}(1^3))] = \frac{100}{3} + [(800 - 800 + \frac{800}{3}) - (400 - 200 + \frac{100}{3})] = \frac{100}{3} + [\frac{800}{3} - (200 + \frac{100}{3})] = \frac{100}{3} + \frac{800}{3} - 200 - \frac{100}{3} = \frac{800}{3} - 200 = \frac{800-600}{3} = \frac{200}{3}$

Now, $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v\,(t)^2 dt} = \sqrt{\frac{1}{5} \times \frac{200}{3}} = \sqrt{\frac{40}{3}} = \sqrt{13.333} \approx 3.65V$

Alternative approach for RMS of triangular pulse: For a triangular pulse of peak V_p and duration τ, the RMS value is $V_p/\sqrt{3}$ if it's a full triangle from 0 to τ. If it's zero for some part of the period T: For a single triangular pulse of duration t_1 in a period T, $V_{rms} = V_p/\sqrt{3} \times \sqrt{t_1/T}$. Here, $V_p = 10$, pulse duration $t_1 = 2$, period $T = 5$. $V_{rms} = \frac{10}{\sqrt{3}} \times \sqrt{\frac{2}{5}} = \frac{10}{\sqrt{3}} \times \sqrt{0.4} \approx \frac{10}{1.732} \times 0.632 \approx 5.773 \times 0.632 \approx 3.65V$. This matches the integration.

- **3. Form Factor (FF):** $FF = \frac{V_{rms}}{V_{avg}}\,FF = \frac{3.65V}{2V} = 1.825$

Therefore:

  - RMS Value (V_{rms}) $\approx 3.65V$
  - Average Value (V_{avg}) $= 2V$
  - Form Factor (FF) ≈ 1.825

(b) Given $v(t) = 112\cos(\omega t + 10°)V$ and $i(t) = 4\cos(\omega t - 50°)A$, find the average power and the reactive power.

To find the average power and reactive power:

- **Convert to phasor form:**
  - Voltage phasor: $\mathbf{V} = 112\angle 10° V$
  - Current phasor: $\mathbf{I} = 4\angle -50° A$
- **1. Calculate Average Power (P):** Average power is given by $P = V_{rms}I_{rms}\cos(\theta_v - \theta_i)$. The peak values are given, so $V_{rms} = \frac{112}{\sqrt{2}}$ and $I_{rms} = \frac{4}{\sqrt{2}}$. Phase angle difference $\phi = \theta_v - \theta_i = 10° - (-50°) = 10° + 50° = 60°$.

  $P = \frac{112}{\sqrt{2}} \times \frac{4}{\sqrt{2}} \times \cos(60°)$ $P = \frac{112\times4}{2} \times 0.5$ $P = 224 \times 0.5 = 112W$
- **2. Calculate Reactive Power (Q):** Reactive power is given by $Q = V_{rms}I_{rms}\sin(\theta_v - \theta_i)$. $Q = \frac{112}{\sqrt{2}} \times \frac{4}{\sqrt{2}} \times \sin(60°)$

  $Q = \frac{112\times4}{2} \times \frac{\sqrt{3}}{2}$ $Q = 224 \times \frac{\sqrt{3}}{2} = 112\sqrt{3}VAR$ $Q \approx 112 \times 1.732 \approx 194.0VAR$

  Therefore:
  - Average Power $(P) = 112W$
  - Reactive Power $(Q) \approx 194.0VAR$

(c) Obtain the power factor for the following circuit. Specify whether the power factor is leading or lagging. (A circuit diagram is provided with inductors, capacitors and resistors: $-j1\Omega$, 4Ω, 1Ω, $j2\Omega$, $j1\Omega$.)

To obtain the power factor and determine if it's leading or lagging:

- **Find the total equivalent impedance (Z_{eq}) of the circuit.** Without a diagram, I will assume a series-parallel combination. Let's assume the following:
  - 4Ω resistor, $j2\Omega$ inductor, and $j1\Omega$ inductor are in series. (Let this be Branch 1)
  - 1Ω resistor and $-j1\Omega$ capacitor are in series. (Let this be Branch 2)
  - Branch 1 and Branch 2 are in parallel.
  - **Impedance of Branch 1 (Z_1):** $Z_1 = 4\Omega + j2\Omega + j1\Omega = 4 + j3\Omega$
  - **Impedance of Branch 2 (Z_2):** $Z_2 = 1\Omega - j1\Omega$
  - **Equivalent impedance (Z_{eq}) of parallel combination:** $Z_{eq} = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$ $Z_{eq} = \frac{(4+j3)(1-j1)}{(4+j3)+(1-j1)}$ $Z_{eq} = \frac{4-j4+j3-j^{23}}{5+j2}$ $Z_{eq} = \frac{4-j1+3}{5+j2} = \frac{7-j1}{5+j2}$

- **Rationalize the denominator:** $Z_{eq} = \frac{7-j1}{5+j2} \times \frac{5-j2}{5-j2}$ $Z_{eq} = \frac{(7)(5)+(7)(-j2)+(-j1)(5)+(-j1)(-j2)}{5^2+2^2}$ $Z_{eq} = \frac{35-j14-j5+j^{22}}{25+4}$ $Z_{eq} = \frac{35-j19-2}{29}$ $Z_{eq} = \frac{33-j19}{29} = \frac{33}{29} - j\frac{19}{29}\Omega$ $Z_{eq} \approx 1.138 - j0.655\Omega$

- **Calculate the Power Factor (PF):** The power factor is $\cos(\phi)$, where ϕ is the phase angle of the equivalent impedance. From $Z_{eq} = R_{eq} + jX_{eq}$, the phase angle $\phi = \arctan(\frac{X_{eq}}{R_{eq}})$. Here, $R_{eq} = \frac{33}{29}$ and $X_{eq} = -\frac{19}{29}$.

  $\phi = \arctan\left(\frac{-19/29}{33/29}\right) = \arctan\left(-\frac{19}{33}\right)$ $\phi \approx \arctan(-0.5757) \approx -29.9°$

  Power Factor (PF) $= \cos(\phi) = \cos(-29.9°) \approx 0.866$

- **Specify whether leading or lagging:** Since the phase angle ϕ is negative (capacitive), the current leads the voltage. Therefore, the power factor is **leading**.

  Therefore:

  - Power Factor (PF) ≈ 0.866
  - The power factor is **leading**.

5.(a) Determine the Norton's equivalent circuit across terminals AB. Also find I_{AB} if a 6Ω resistance is connected across terminals AB. (A circuit diagram is provided with a 2A current source, 12V voltage source, and resistors of 8Ω, 4Ω, 5Ω, 8Ω.)

To determine the Norton's equivalent circuit (I_N, R_N) and I_{AB}:

- **1. Find Norton Current (I_N or I_{sc}):**
  - Short-circuit the terminals AB.
  - Calculate the current flowing through the short circuit.
  - Let's assume the standard configuration.
    - 2A current source, 12V voltage source.
    - Resistors: 8Ω, 4Ω, 5Ω, 8Ω.
  - Assume a common circuit where AB are terminals across some components.
  - Apply nodal or mesh analysis.

Let's assume a specific configuration:

- Left branch: 2A current source in parallel with 8Ω resistor.
- Middle branch: 4Ω resistor.
- Right branch: 5Ω resistor in series with 12V source.
- The 8Ω (fourth resistor) is connected such that it influences current I_{sc}. Perhaps between the middle and right branch, or defining terminal A.

- This is very difficult without a diagram. Let's assume a common setup where AB are across a load, and the sources/resistors are feeding it.
- **Assume:** 2A source and 8Ω are in parallel. This is connected to a node 'X'. Node 'X' connects to 4Ω (to ground), and to a node 'Y'. Node 'Y' connects to 5Ω (to ground), and to a node 'Z'. Node 'Z' connects to 8Ω and 12V source in series, leading to ground. And AB are across a specific branch. This is getting too complex without a diagram.

Simplified Common Structure for Example:

- Assume a circuit with a node V.
- Branch 1: 2A current source going into V.
- Branch 2: 8Ω resistor from V to ground.
- Branch 3: 4Ω resistor from V to a new node V_A.
- Branch 4: 5Ω resistor from V_A to a new node V_B.
- Branch 5: 8Ω resistor from V_B to ground.
- Branch 6: 12V voltage source from V_B to ground (or in series with one of the 8Ω or 5Ω).

Let's use a simpler common configuration to demonstrate the steps:

- Let 2A source in parallel with 8Ω and then 4Ω in series.
- Let 12V source in series with 5Ω and 8Ω.
- And these two main branches are connected in parallel, with AB across one of the resistors.

Due to the lack of a circuit diagram, let's assume a straightforward parallel/series connection for a simple demonstration of the Norton method:

- Let terminals AB be across the 8Ω resistor that is not connected to the current source.
- **Short AB.** Now the 8Ω resistor is shorted.
- **Apply superposition:**
  - **With 2A source ON, 12V source OFF (shorted):**
    - 2A source, 8Ω, 4Ω, 5Ω, 8Ω (shorted).
    - The current will flow.
  - **With 12V source ON, 2A source OFF (open):**
    - 12V source, 8Ω, 4Ω, 5Ω, 8Ω (shorted).

This approach is complex without the visual. A more practical approach in the absence of a drawing is to calculate R_N first, then V_{oc} (open-circuit voltage) and find $I_N = V_{oc}/R_N$.

- **1. Find Norton Resistance (R_N or R_{Th}):**
  - Deactivate all independent sources:
    - Voltage source (12V) becomes a short circuit.
    - Current source (2A) becomes an open circuit.
  - Calculate the equivalent resistance seen from terminals AB.
  - Assuming the 8Ω (first one), 4Ω, 5Ω, 8Ω (second one) are interconnected.
  - If the 8Ω in parallel with 2A source is now just 8Ω.
  - If the 5Ω in series with 12V is now just 5Ω.

Let's assume the 8Ω is in parallel with 2A, and 4Ω is next, then 5Ω with 12V, and the last 8Ω is between the 4 and 5 Ohm. And AB across the last 8Ω.

- If 2A source is open, 8Ω resistor is still there.
- If 12V source is shorted, 5Ω resistor is still there.
- Then, we'd have 8Ω in series with 4Ω, in parallel with 5Ω, and then in series with the 8Ω at AB. This is a complete guess.

General approach for R_{Th}/R_N:

h. Turn off all independent sources (voltage sources short, current sources open).

i. Look into the terminals AB and calculate the equivalent resistance.

- Let's assume a simple case: 8Ω and 4Ω are in series, this combination is parallel with 5Ω. And the 8Ω (last one) is in series with this parallel combo, with AB across it. This doesn't make sense for R_{Th} across AB.

Consider a standard T or Pi network for simplification: Let's assume the circuit is a simple series/parallel arrangement that results in a calculable R_N. Assume the 8Ω (from 2A source) is R_1, 4Ω is R_2, 5Ω is R_3, and 8Ω (last) is R_4. If terminals AB are across R_4:

- Open 2A source. $R_1 = 8\Omega$.
- Short 12V source. $R_3 = 5\Omega$.
- The network might be: R_1 in parallel with (R_2 in series with R_3). And R_4 is the load.
- R_N would be the equivalent resistance looking back from AB.
- $R_N = R_4 + (R_1||(R_2 + R_3))$ (This is a guess about the topology).
- $R_N = 8 + (8||(4 + 5)) = 8 + (8||9) = 8 + \frac{8\times9}{8+9} = 8 + \frac{72}{17} = 8 + 4.235 \approx 12.235\Omega$

- **2. Find Norton Current (I_N):**

- Place a short circuit across terminals AB.
- Calculate the current flowing through this short. This requires solving the full circuit.
- This is where a diagram is indispensable.

Due to the lack of a diagram, I cannot provide a numerical solution for I_N or I_{AB}. The steps above are the general procedure.

(b) Find the value of R_L for maximum power transfer. Also, find the maximum power. (A circuit diagram is provided on Source 7, showing a 20A current source, and resistors of 2Ω, 4Ω, R_L.)

To find R_L for maximum power transfer and the maximum power:

- **1. Find Thevenin's equivalent resistance (R_{Th}) across R_L terminals.**
  - Deactivate the independent current source (20A) by open-circuiting it.
  - Look into the terminals where R_L is connected.
  - Assuming the 2Ω and 4Ω resistors are in the circuit and become part of R_{Th}.
  - **Common configuration:** If the 20A source is in parallel with the 2Ω resistor, and this combination is

in series with the 4Ω resistor, and R_L is connected after the 4Ω.

- In this case, when the 20A source is open-circuited, the 2Ω and 4Ω resistors are in series.
- $R_{Th} = 2\Omega + 4\Omega = 6\Omega$.
- For maximum power transfer, $R_L = R_{Th}$.
- So, $R_L = 6\Omega$.

- **2. Find Thevenin's equivalent voltage (V_{Th} or V_{oc}) across R_L terminals.**
  - With the 20A source active.
  - The 20A current source will flow through the 2Ω and 4Ω resistors to create voltage.
  - If the 20A source is in parallel with the 2Ω resistor, and that branch is in series with 4Ω.
  - The 20A current flows into the 2Ω resistor.
  - The voltage across the 2Ω resistor is $V_{2\Omega} = 20A \times 2\Omega = 40V$.
  - This 40V is effectively the voltage across the combination that includes the current source and the 2Ω resistor.
  - This current (20A) then flows through the 4Ω resistor.

- The voltage across the 4Ω resistor is $V_{4\Omega} = 20A \times 4\Omega = 80V$.
- $V_{Th} = V_{oc} = V_{2\Omega} + V_{4\Omega} = 40V + 80V = 120V$. (Assuming a series connection of 2Ω and 4Ω after the current source).
- **Alternative interpretation:** If 20A source is only parallel to 2Ω. And 4Ω is elsewhere.
- **Let's assume the current source is in parallel with 2Ω, and this combination is connected to the 4Ω resistor. The terminals for R_L are across the 4Ω resistor.** This is a more common configuration.
- Current divides between 2Ω and R_L if R_L is directly across 2Ω.
- **If the 20A source is connected in parallel with the 2Ω resistor, and the 4Ω resistor is in series with this parallel combination, and R_L is connected across the 4Ω resistor, then for V_{Th}, we open R_L.**
  - The 20A source current goes through the 2Ω resistor. $V_{2\Omega} = 20A \times 2\Omega = 40V$.
  - This voltage $40V$ is then applied across the 4Ω resistor (if it's in series with the source-resistor combo).

- V_{Th} is the open-circuit voltage across R_L. If R_L is across the 4Ω resistor, then V_{Th} is the voltage across the 4Ω resistor.
- The total current flows through the 4Ω resistor.
- $V_{Th} = 20A \times 4\Omega = 80V$. (This assumes the 2Ω is irrelevant for the Thevenin voltage, which is only true if it's in parallel to the source, and then the whole combination is in series with 4Ω).

○ **Let's use the most common configuration for such problems: The 20A source and 2Ω resistor are in parallel, and this entire sub-circuit is in series with the 4Ω resistor, with R_L as the load.**

- To find V_{Th}, calculate the voltage across the load terminals when R_L is open.
- The 20A current source flows through the 2Ω resistor.
- Then this current (20A) flows into the 4Ω resistor.
- $V_{Th} = 20A \times 4\Omega = 80V$.

- **3. Calculate Maximum Power (P_{max}):** $P_{max} = \frac{V_{Th}^2}{4R_{Th}}$

$P_{max} = \frac{(80V)^2}{4\times6\Omega} = \frac{6400}{24} = 266.67W$

Therefore:

- Value of R_L for maximum power transfer $= 6\Omega$
- Maximum Power (P_{max}) $\approx 266.67W$

(c) State and prove the Superposition Theorem.

- **Superposition Theorem:**
  - **Statement:** The superposition theorem states that in any linear, bilateral circuit containing multiple independent sources, the current through or voltage across any element can be determined by finding the algebraic sum of the currents or voltages produced by each independent source acting alone, while all other independent sources are turned off (deactivated).
  - **Deactivation rules:**
    - Independent voltage sources are replaced by a short circuit (zero voltage).
    - Independent current sources are replaced by an open circuit (zero current).
    - Dependent sources are **not** turned off; they remain in the circuit as their values depend on other circuit variables.
- **Proof (Conceptual/Illustrative):**

- Consider a linear circuit with two independent sources, V_1 and I_2, and a resistor R through which we want to find the current I.
- **Step 1: Consider Source V_1 acting alone.**
  - Turn off I_2 (replace with an open circuit).
  - The circuit simplifies. Calculate the current through R due to V_1 alone. Let this be I'.
  - Since the circuit is linear, $I' = V_1/R_{eq1}$ where R_{eq1} is the equivalent resistance seen by V_1 in this simplified circuit, or using KVL/KCL, I' is proportional to V_1.
- **Step 2: Consider Source I_2 acting alone.**
  - Turn off V_1 (replace with a short circuit).
  - The circuit simplifies again. Calculate the current through R due to I_2 alone. Let this be I''.
  - Since the circuit is linear, $I'' = I_2 \times (R_{eq2}/(R_{eq2} + R_x))$ using current division, or using KVL/KCL, I'' is proportional to I_2.
- **Step 3: Combine the results.**
  - According to the superposition theorem, the total current I through R is the algebraic sum of the individual currents: $I = I' + I''$.

- **Reasoning (why it works for linear circuits):**
  - Linear circuits are governed by linear differential equations (or algebraic equations in DC/AC steady state).
  - The principle of superposition applies to linear systems. If a system's output is a linear combination of its inputs, then the response to multiple inputs is the sum of the responses to each input individually.
  - In circuit terms, Ohm's Law ($V = IR$) and Kirchhoff's Laws (sum of currents at a node is zero, sum of voltages around a loop is zero) are linear relationships. These fundamental laws ensure that the current or voltage in any part of the circuit is a linear function of the applied sources.
  - Therefore, if we sum the individual effects (currents or voltages) produced by each source acting independently, we get the total effect when all sources are acting simultaneously.

6.(a) Find the Thevenin's equivalent circuit across terminals AB. Also, calculate the voltage across 10Ω resistance if it is connected across the terminals AB. (A circuit diagram is provided with a $5\angle 0° V$ voltage source, and resistors/inductor of 5Ω, 5Ω, $5j\Omega$.)

To find Thevenin's equivalent circuit (V_{Th}, Z_{Th}) and voltage across 10Ω:

- **1. Find Thevenin's Impedance (Z_{Th}):**
  - Deactivate the independent voltage source ($5\angle 0° V$) by short-circuiting it.
  - Assume the configuration of 5Ω, 5Ω, and $5j\Omega$.
  - Let's assume the 5Ω (first) is in series with the $5\angle 0° V$ source.
  - Let the other 5Ω be in parallel with the $5j\Omega$.
  - And the terminals AB are across the parallel combination.
  - When the $5\angle 0° V$ source is shorted, the first 5Ω resistor is effectively in parallel with the rest of the circuit as seen from AB.
  - This is a common configuration: a voltage source in series with a resistor, then a parallel branch.
  - **Assumption:** The $5\angle 0° V$ source is in series with the first 5Ω resistor. This combination is in parallel with the other 5Ω resistor. And this whole setup is then in series with the $5j\Omega$ inductor. And AB are across the $5j\Omega$ inductor and the parallel 5Ω resistor.

- **Revised Assumption (more common for Z_{Th} calculation):**
  - Let the $5\angle0°V$ source be in series with a 5Ω resistor (let's call it R_1).
  - Let the other 5Ω resistor be R_2.
  - Let the $5j\Omega$ inductor be L_1.
  - And assume R_2 and L_1 are in parallel, and AB are across this parallel combination. And the series $V_{source} - R_1$ is connected across this parallel combo.
- When the $5\angle0°V$ source is shorted, R_1 is in parallel with the $R_2||L_1$ combination. This is not the Thevenin equivalent looking back into AB if AB is across $R_2||L_1$.
- **Let's assume the circuit looks like this:** A voltage source, then a 5Ω resistor (R_1). From that node, another 5Ω resistor (R_2) goes to ground, and a $5j\Omega$ inductor (L_1) goes to ground. And AB are across L_1. This is also not standard.

Most likely interpretation for Z_{Th} calculation:

- The circuit has a $5\angle0°V$ source.
- One 5Ω resistor (R_1) is in series with the source.

- The other 5Ω resistor (R_2) and the $5j\Omega$ inductor (X_L) are in parallel, and this parallel combination is connected to the node after R_1. And terminals AB are across this parallel combination ($R_2||X_L$).
- To find Z_{Th}: Short the voltage source. Now R_1 is in parallel with ($R_2||X_L$). This would be the equivalent impedance. No, this is incorrect for Z_{Th} across AB if AB are the load terminals.
- Looking into terminals AB:
  - Short the $5\angle 0°V$ source.
  - The 5Ω resistor (in series with the source) will be in parallel with the other 5Ω resistor. This parallel combination will then be in series with the $5j\Omega$ inductor.
  - This is a plausible configuration for the components.
  - $Z_{Th} = (5\Omega \,||\, 5\Omega) + 5j\Omega$
  - $5\Omega \,||\, 5\Omega = \frac{5\times5}{5+5} = \frac{25}{10} = 2.5\Omega$
  - $Z_{Th} = 2.5 + j5\Omega$

- **2. Find Thevenin's Voltage (V_{Th} or V_{oc}):**
  - Remove the load (open terminals AB).
  - Calculate the voltage across the open terminals AB.

- With the $5\angle 0°V$ source, and series 5Ω and another parallel 5Ω and $5j\Omega$.
- Assume the source is in series with one 5Ω resistor. Let this be R_1.
- The other 5Ω (R_2) and $5j\Omega$ (X_L) are in parallel, and the voltage across this parallel combination is V_{Th}.
- The current flowing through R_1 is given by $I = \frac{V_{source}}{R_1+(R_2||X_L)}$. $R_2||X_L = \frac{5\times j5}{5+j5} = \frac{j25}{5(1+j)} = \frac{j5}{1+j}\frac{j5}{1+j} \times \frac{1-j}{1-j} = \frac{j5-j^{25}}{1^2+1^2} = \frac{5+j5}{2} = 2.5 + j2.5\Omega$
- Total impedance seen by the source: $Z_{total} = R_1 + (R_2||X_L) = 5 + (2.5 + j2.5) = 7.5 + j2.5\Omega$
- Current from source: $I_{source} = \frac{5\angle 0°}{7.5+j2.5} = \frac{5}{7.5+j2.5}$

  $7.5 + j2.5 = \sqrt{7.5^2 + 2.5^2}\angle\arctan(2.5/7.5) = \sqrt{56.25 + 6.25}\angle\arctan(1/3) = \sqrt{62.5}\angle 18.43° \approx 7.906\angle 18.43°$ $I_{source} = \frac{5}{7.906\angle 18.43°} \approx 0.632\angle -18.43°A$
- V_{Th} is the voltage across the parallel combination $(R_2||X_L)$: $V_{Th} = I_{source} \times (R_2||X_L)$ $V_{Th} = (0.632\angle -18.43°) \times (2.5 + j2.5)$ $2.5 + j2.5 = \sqrt{2.5^2 + 2.5^2}\angle\arctan(2.5/2.5) = \sqrt{6.25 + 6.25}\angle 45° = \sqrt{12.5}\angle 45° \approx 3.535\angle 45°$

$V_{Th} = (0.632\angle -18.43°) \times (3.535\angle 45°)$ $V_{Th} \approx (0.632 \times 3.535)\angle(-18.43° + 45°)$ $V_{Th} \approx 2.233\angle 26.57°V$

Therefore, Thevenin's equivalent circuit:

- $Z_{Th} = 2.5 + j5\Omega$
- $V_{Th} \approx 2.233\angle 26.57°V$

- **3. Calculate the voltage across 10Ω resistance if it is connected across terminals AB.**
  - Connect $R_L = 10\Omega$ across Z_{Th} and V_{Th}.
  - The current through the load is $I_L = \frac{V_{Th}}{Z_{Th}+R_L}$ $I_L = \frac{2.233\angle 26.57°}{(2.5+j5)+10} = \frac{2.233\angle 26.57°}{12.5+j5}$ $12.5 + j5 = \sqrt{12.5^2 + 5^2}\angle\arctan(5/12.5) = \sqrt{156.25 + 25}\angle\arctan(0.4) = \sqrt{181.25}\angle 21.8° \approx 13.46\angle 21.8°$ $I_L = \frac{2.233\angle 26.57°}{13.46\angle 21.8°} \approx 0.1659\angle(26.57° - 21.8°) = 0.1659\angle 4.77°A$
  - Voltage across 10Ω resistance (V_L) is $I_L \times R_L$: $V_L = (0.1659\angle 4.77°) \times 10$ $V_L \approx 1.659\angle 4.77°V$

Therefore:

- Voltage across 10Ω resistance $\approx 1.659\angle 4.77°V$

(b) Determine voltage 'V' using superposition theorem (A circuit diagram is provided with a 15V voltage source, 2.5A current source, and resistors of 40Ω, 10Ω, 4Ω.)

To determine voltage 'V' using superposition theorem:

- **Identify sources:** 15V voltage source, 2.5A current source.
- **Identify resistors:** 40Ω, 10Ω, 4Ω.
- **Assume 'V' is the voltage across the 4Ω resistor.** This is a common definition for 'V'.
- **Case 1: Only 15V Voltage Source Active.**
  - Turn off the 2.5A current source (replace with an open circuit).
  - Let's assume the 15V source is in series with the 40Ω resistor.
  - Let the 10Ω and 4Ω resistors be in parallel, and this parallel combination is in series with the 40Ω resistor and 15V source. 'V' is across the 4Ω resistor.
  - The 10Ω and 4Ω resistors are in parallel. $R_{parallel} = \frac{10\times4}{10+4} = \frac{40}{14} = \frac{20}{7} \approx 2.857\Omega$
  - The total resistance in the circuit is $R_{total} = 40\Omega + R_{parallel} = 40 + \frac{20}{7} = \frac{280+20}{7} = \frac{300}{7}\Omega$.

- Current from 15V source: $I' = \frac{15V}{300/7\Omega} = \frac{15\times7}{300} = \frac{105}{300} = \frac{7}{20} = 0.35A.$
- This current I' flows through the parallel combination of 10Ω and 4Ω.
- Use current division to find current through 4Ω: $I_{4\Omega'} = I' \times \frac{10\Omega}{10\Omega+4\Omega} = 0.35A \times \frac{10}{14} = 0.35 \times \frac{5}{7} = 0.05 \times 5 = 0.25A.$
- Voltage across 4Ω due to 15V source (V'): $V' = I_{4\Omega'} \times 4\Omega = 0.25A \times 4\Omega = 1V.$

- **Case 2: Only 2.5A Current Source Active.**
  - Turn off the 15V voltage source (replace with a short circuit). So 40Ω is in parallel with 10Ω.
  - Let's assume the 2.5A current source is connected to the node between the 10Ω and 4Ω resistors.
  - Let the 2.5A current source be in parallel with the 4Ω resistor, and then this branch is in series with 10Ω, and 40Ω is in series with shorted 15V source.
  - **Assumption:** The 2.5A current source is connected such that it flows into the node where 10Ω and 4Ω are connected, and V is across 4Ω.

- The 40Ω and 10Ω resistors are now in parallel (since 15V source is shorted). $R_{parallel2} = \frac{40\times10}{40+10} = \frac{400}{50} = 8\Omega$.
- Now, the 2.5A current source sees $R_{parallel2}$ (8Ω) in parallel with the 4Ω resistor.
- Current from 2.5A source divides between $R_{parallel2}$ (8Ω) and 4Ω.
- Current through 4Ω due to 2.5A source ($I_{4\Omega''}$): $I_{4\Omega''} = 2.5A \times \frac{R_{parallel2}}{R_{parallel2}+4\Omega} = 2.5A \times \frac{8}{8+4} = 2.5A \times \frac{8}{12} = 2.5A \times \frac{2}{3} = \frac{5}{3} \approx 1.667A$.
- Voltage across 4Ω due to 2.5A source (V''): $V'' = I_{4\Omega''} \times 4\Omega = \frac{5}{3}A \times 4\Omega = \frac{20}{3} \approx 6.667V$.

* **Total Voltage 'V':** $V = V' + V'' = 1V + 6.667V = 7.667V$.

  Therefore:

  - Voltage 'V' using superposition theorem $\approx 7.667V$.

(c) Verify reciprocity theorem between 10V source and current I (A circuit diagram is provided with a 10V voltage source, and resistors of 5Ω, 10Ω, 7Ω, with current I indicated.)

To verify the reciprocity theorem:

- **Reciprocity Theorem Statement:** In any linear, bilateral network, if a single voltage source (or current source) in branch 'a' causes a current (or voltage) response in branch 'b', then if the same source is moved to branch 'b', it will cause the same current (or voltage) response in branch 'a'.
- **Step 1: Original Circuit (Source in Branch 'a', Response in Branch 'b').**
    - **Circuit description based on common setups:**
        - Let the 10V source be in series with the 5Ω resistor.
        - Let this combination be in parallel with the 10Ω resistor.
        - Let this whole setup be in series with the 7Ω resistor.
        - Let current I be the current flowing through the 7Ω resistor. (This is a common configuration.)
    - **Calculate current I in the original circuit:**
        - The 5Ω and 10Ω resistors are in parallel (effectively, the 10V source is across the 5Ω and the 10Ω). No, that's not how it works.
        - Let's assume the 10V source is in series with 5Ω. This branch is in parallel with 10Ω. This

combination is in series with 7Ω. And current I is through 7Ω.

- **Redefining based on a common scenario for reciprocity:**
  - Branch 'a': 10V source in series with 5Ω resistor.
  - Branch 'b': 7Ω resistor, and I is the current through it.
  - A 10Ω resistor connects between these branches.
  - Let's assume Node A after the 5 Ohm, Node B after the 10 Ohm, and the 7 Ohm from Node B to ground, with current I flowing through 7 Ohm.
  - **Original Setup:**
    - 10V source - 5Ω - Node A.
    - Node A - 10Ω - Node B.
    - Node B - 7Ω - Ground.
    - Current I is flowing through 7Ω.
  - **Apply node analysis at Node A and Node B.**

- Node A: $\frac{V_A - 10}{5} + \frac{V_A - V_B}{10} = 0$ Multiply by 10: $2(V_A - 10) + (V_A - V_B) = 0$ $2V_A - 20 + V_A - V_B = 0$ $3V_A - V_B = 20$ (Equation 1)
- Node B: $\frac{V_B - V_A}{10} + \frac{V_B}{7} = 0$ Multiply by 70: $7(V_B - V_A) + 10V_B = 0$ $7V_B - 7V_A + 10V_B = 0$ $-7V_A + 17V_B = 0$ (Equation 2)
- From Eq 2: $V_B = \frac{7}{17} V_A$
- Substitute into Eq 1: $3V_A - \frac{7}{17} V_A = 20$ $\frac{51V_A - 7V_A}{17} = 20$ $\frac{44V_A}{17} = 20$ $V_A = \frac{20 \times 17}{44} = \frac{340}{44} = \frac{85}{11} \approx 7.727V$
- $V_B = \frac{7}{17} \times \frac{85}{11} = \frac{7 \times 5}{11} = \frac{35}{11} \approx 3.182V$
- Current $I = \frac{V_B}{7} = \frac{35/11}{7} = \frac{5}{11} \approx 0.4545A$
- So, for a 10V source in branch 'a', current I in branch 'b' is $0.4545A$.

- **Step 2: Modified Circuit (Same Source in Branch 'b', Response in Branch 'a').**
  - Move the 10V source to where the 7Ω resistor was.

- Replace the original 10V source in branch 'a' with a short circuit.
- Now, we need to find the current flowing through the 5Ω resistor (branch 'a'). Let this be I_{rec}.
- **Modified Setup:**
  - Short circuit where 10V source was, now just 5Ω - Node A'.
  - Node A' - 10Ω - Node B'.
  - Node B' - 10V source (positive towards Node B') - 7Ω - Ground. (Assuming 10V source is in series with 7Ω).
  - We want to find current through 5Ω resistor.
- **Apply node analysis at Node A' and Node B'.**
  - Node A': $\frac{V_{A'}}{5} + \frac{V_{A'} - V_{B'}}{10} = 0$ Multiply by 10: $2V_{A'} + V_{A'} - V_{B'} = 0$ $3V_{A'} - V_{B'} = 0 \Longrightarrow V_{B'} = 3V_{A'}$ (Equation 3)
  - Node B': $\frac{V_{B'} - V_{A'}}{10} + \frac{V_{B'} - 10}{7} = 0$ (Assuming 10V source is connected such that it raises the voltage at B' relative to the 7 Ohm) Multiply by 70: $7(V_{B'} - V_{A'}) + 10(V_{B'} - 10) = 0$ $7V_{B'} - 7V_{A'} + 10V_{B'} - 100 = 0$ $-7V_{A'} + 17V_{B'} = 100$ (Equation 4)

- Substitute Eq 3 into Eq 4: $-7V_{A'} + 17(3V_{A'}) = 100$ $-7V_{A'} + 51V_{A'} = 100$ $44V_{A'} = 100$ $V_{A'} = \frac{100}{44} = \frac{25}{11} \approx 2.273V$
- Current through 5Ω (branch 'a') is $I_{rec} = \frac{V_{A'}}{5}$ (since the other side is grounded due to source replacement). $I_{rec} = \frac{25/11}{5} = \frac{5}{11} \approx 0.4545A$

- **Conclusion:**
  - In the original circuit, the current I in branch 'b' due to the 10V source in branch 'a' was $\approx 0.4545A$.
  - In the modified circuit, the current I_{rec} in branch 'a' due to the 10V source in branch 'b' is also $\approx 0.4545A$.
  - Since $I = I_{rec}$, the reciprocity theorem is verified.

7.(a) Can a Bandstop Filter be made using a Low Pass Filter and a High Pass Filter? Explain your answer briefly.

- Yes, a Bandstop Filter (also known as a Band-Reject Filter or Notch Filter) can be made using a combination of a Low Pass Filter (LPF) and a High Pass Filter (HPF).
- **Explanation:**
  - A Bandstop Filter is designed to attenuate or block signals within a specific range of frequencies (the

stopband) while allowing frequencies outside this range to pass through.

- To achieve this, you connect a Low Pass Filter and a High Pass Filter in **parallel**, and then sum their outputs.
- The LPF allows frequencies below its cutoff frequency ($f_{c,LP}$) to pass.
- The HPF allows frequencies above its cutoff frequency ($f_{c,HP}$) to pass.
- For a bandstop characteristic, the cutoff frequency of the High Pass Filter ($f_{c,HP}$) must be **higher** than the cutoff frequency of the Low Pass Filter ($f_{c,LP}$).
- Frequencies below $f_{c,LP}$ will pass through the LPF.
- Frequencies above $f_{c,HP}$ will pass through the HPF.
- The frequencies between $f_{c,LP}$ and $f_{c,HP}$ (i.e., the stopband) will be attenuated by both filters. The LPF will block them because they are above its cutoff, and the HPF will block them because they are below its cutoff.
- The combined output, therefore, effectively "rejects" the frequencies in the band ($f_{c,LP}$ to $f_{c,HP}$), while passing frequencies outside this range.

(b) (i) Derive an expression for the cutoff frequency of a high pass RC filter. (ii) Plot the frequency response of an ideal high pass filter.

(i) Derive an expression for the cutoff frequency of a high pass RC filter.

- **Circuit Diagram:** A high pass RC filter consists of a resistor (R) and a capacitor (C) connected in series, with the output taken across the resistor.
  - Input voltage is applied across the series combination.
  - Output voltage is measured across the resistor.
- **Derivation:**
  - The impedance of the capacitor is $X_C = \frac{1}{j\omega C}$.
  - The total impedance of the series circuit is $Z_{total} = R + \frac{1}{j\omega C}$.
  - Using the voltage divider rule, the output voltage V_{out} across the resistor R is: $V_{out} = V_{in} \frac{R}{R+\frac{1}{j\omega C}}$
  - The transfer function (gain) of the filter is $H(j\omega) = \frac{V_{out}}{V_{in}}$: $H(j\omega) = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega CR}{j\omega CR+1}$

- The magnitude of the transfer function is: $|H(j\omega)| = \frac{|j\omega CR|}{|1+j\omega CR|} = \frac{\omega CR}{\sqrt{1^2+(\omega CR)^2}} = \frac{\omega CR}{\sqrt{1+(\omega CR)^2}}$
- **Cutoff Frequency (ω_c or f_c):** The cutoff frequency (also known as the -3dB frequency or half-power frequency) is the frequency at which the magnitude of the output voltage is $1/\sqrt{2}$ (or 0.707) times the maximum (input) voltage. At this point, the output power is half of the maximum power.
- Set $|H(j\omega)| = \frac{1}{\sqrt{2}}$: $\frac{\omega_c CR}{\sqrt{1+(\omega_c CR)^2}} = \frac{1}{\sqrt{2}}$
- Square both sides: $\frac{(\omega_c CR)^2}{1+(\omega_c CR)^2} = \frac{1}{2}$
- Cross-multiply: $2(\omega_c CR)^2 = 1 + (\omega_c CR)^2$ $2(\omega_c CR)^2 - (\omega_c CR)^2 = 1$ $(\omega_c CR)^2 = 1$ $\omega_c CR = 1$
- Therefore, the angular cutoff frequency is: $\omega_c = \frac{1}{RC}$ (radians/second)
- In terms of linear frequency $f_c = \frac{\omega_c}{2\pi}$: $f_c = \frac{1}{2\pi RC}$ (Hertz)

(ii) Plot the frequency response of an ideal high pass filter.

- **Ideal High Pass Filter Characteristics:**

- **Passband:** For frequencies above the cutoff frequency ($f > f_c$), the gain is unity (or 0 dB). This means all signals in this range pass through without attenuation.
- **Stopband:** For frequencies below the cutoff frequency ($f < f_c$), the gain is zero (or -infinity dB). This means all signals in this range are completely blocked or attenuated.
- **Sharp Transition:** The transition from the stopband to the passband at the cutoff frequency is instantaneous, forming a perfectly vertical line on the frequency response plot. This is not achievable in practice but is an ideal representation.

- **Plot Description:**
  - **X-axis:** Frequency (f) on a logarithmic scale.
  - **Y-axis:** Gain (Magnitude of H(jω)) in dB, or simply as a ratio from 0 to 1.
  - The plot would show a horizontal line at 0 dB (or gain = 1) for all frequencies from f_c to infinity.
  - It would show a vertical line dropping to -infinity dB (or gain = 0) at the cutoff frequency f_c.
  - It would show a horizontal line at -infinity dB (or gain = 0) for all frequencies from 0 to f_c.

- **(Cannot make a schematic diagram as per user instructions)**
- Imagine a graph:
  - From frequency 0 up to f_c, the gain is 0 (or -infinity dB).
  - At f_c, there is a sudden, vertical jump in gain.
  - From f_c to very high frequencies, the gain is 1 (or 0 dB).

(c) Find the resonant frequency ω_0, the quality factor Q, and the bandwidth B for the following RLC circuit: (A circuit diagram is provided with an inductor 20mH, resistor 2kΩ, and capacitors 3μF, 6μF.)

To find ω_0, Q, and B for the RLC circuit:

- **1. Determine the equivalent capacitance (C_{eq}):**
  - Assume the capacitors $3\mu F$ and $6\mu F$ are in parallel. (This is a common configuration in RLC circuits for simplicity if not specified otherwise.)
  - For parallel capacitors, $C_{eq} = C_1 + C_2 + \cdots$
  - $C_{eq} = 3\mu F + 6\mu F = 9\mu F = 9 \times 10^{-6} F$
- **2. Identify the other components:**
  - Inductance (L) = 20mH = $20 \times 10^{-3} H$

- Resistance (R) = 2kΩ = $2 \times 10^3 \Omega$

- **3. Calculate the Resonant Frequency (ω_0):**
  - For a series or parallel RLC circuit, the resonant frequency is given by: $\omega_0 = \frac{1}{\sqrt{LC_{eq}}}$
  - $\omega_0 = \frac{1}{\sqrt{(20\times10^{-3}H)\times(9\times10^{-6}F)}}$
  - $\omega_0 = \frac{1}{\sqrt{180\times10^{-9}}} = \frac{1}{\sqrt{18\times10^{-8}}} = \frac{1}{10^{-4}\sqrt{18}} = \frac{1}{10^{-4}\times3\sqrt{2}}$
  - $\omega_0 = \frac{10^4}{3\sqrt{2}} = \frac{10000}{3\times1.414} = \frac{10000}{4.242} \approx 2357.4$ rad/s

- **4. Calculate the Quality Factor (Q):**
  - The formula for Q depends on whether it's a series or parallel resonant circuit. Without a diagram, let's assume a series RLC circuit for Q calculation, as it's often the base case. If the resistor is in parallel with L and C, the formula is different.
  - **For a series RLC circuit:** $Q = \frac{\omega_0 L}{R}$ $Q = \frac{2357.4 \text{ rad/s}\times(20\times10^{-3}H)}{2\times10^3\Omega}$ $Q = \frac{2357.4\times0.02}{2000} = \frac{47.148}{2000} = 0.023574$
  - This Q value is very low, which usually suggests a parallel RLC circuit if it were designed for filtering.

- **For a parallel RLC circuit:** $Q = \frac{R}{\omega_0 L}$ (where R is in parallel with L and C) $Q = \frac{2\times10^3\Omega}{2357.4 \text{ rad/s}\times(20\times10^{-3}H)}$ $Q = \frac{2000}{47.148} \approx 42.426$
- Given the high resistance value (2kΩ), it's more likely to be a parallel RLC circuit for a meaningful quality factor. Let's proceed with the parallel RLC assumption for Q, which usually means the resistor is in parallel with the LC tank.
- **Assuming Parallel RLC circuit Q:** $Q = 42.426$

- **5. Calculate the Bandwidth (B):**
  - The bandwidth is given by $B = \frac{\omega_0}{Q}$.
  - $B = \frac{2357.4 \text{ rad/s}}{42.426}$
  - $B \approx 55.56$ rad/s

Therefore:

- Resonant frequency (ω_0) ≈ 2357.4 rad/s
- Quality factor (Q) ≈ 42.426 (assuming parallel RLC)
- Bandwidth (B) ≈ 55.56 rad/s