[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1068 **I**

Unique Paper Code : 2372013501

Name of the Paper : Theory of Estimation

Name of the Course : **B. Sc. (Hons.), Statistics under NEP-UGCF**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt six questions in all, selecting three questions from **section I** and three questions from **section II.**

3. Use of simple calculator is allowed.

P.T.O.

SECTION I

1. (a) State and prove Invariance property of consistent estimators. Using this property, obtain consistent estimator of $\theta^2 + \theta - \sqrt{\theta}$, when X follows Bernoulli distribution with parameter θ.

(b) Let $X_1, X_2, \ldots, X_n$, be a random sample from population having p.d.f. $f(x, \theta) = (\theta+1)x^{\theta}$, $0 < x < 1$, $\theta > -1$. Show that $\left[\frac{-(n-1)}{\sum_{i=1}^{n} \log x_i} - 1\right]$ is an unbiased estimator of θ (8, 7)

2. (a) Show that the estimator of the form a $a\overline{X}$ for in random sampling from $N(\theta,\sigma^2)$, has the minimum

mean-square error, when $a = \dfrac{\theta^2}{(\theta^2 + \frac{\sigma^2}{n})}$, Which $\to 1$ as $n \to \infty$ but < 1 when n is finite.

(b) State Cramer-Rao Inequality. Let $X_1, X_2, \ldots, X_n$, be a random sample from Population having p.d.f.

$$f(x, \theta) = \frac{1}{\pi[1+(x-\theta)^2]}, -\infty < x < \infty, -\infty < \theta < \infty.$$

Find Cramer-Rao Lower bound for variance of an unbiased estimator of θ. Also, examine whether MVB estimator exists for θ. [7, 8]

3. (a) Let X and Y be two random variables having joint probability density function $f(x, y) = \frac{2}{\theta^2} \exp[-(x+y)/\theta], 0 < x < y < \infty$.

(i) Show that the mean and variance of Y are respectively $\frac{3\theta}{2}$ and $\frac{5\theta^2}{4}$.

(ii) Show that $E[Y]X = x] = \phi(x) = x + \theta$ and expected value of $X + \theta$ is that of Y.

(iii) Show that the variance of $\phi(X)$ is less than that of Y.

(iv) Comment on the result.

(b) Define Minimum Variance Unbiased (MVU) Estimator. Let T_0 be an MVU estimator, while T_1 is an unbiased estimator with efficiency e_θ. If ρ_θ, be the correlation coefficient between T_0 and T_1, then prove that $\rho_\theta = \sqrt{e_\theta}$. (8, 7)

4. (a) Let $X_1, X_2, \ldots, X_n$, be a random sample from $U(0,\theta)$ population. Show that the largest order statistic $X_{(n)}$ is a complete sufficient statistic for θ. Using Lehman Scheffe's theorem, find UMVUE of θ.

(b) Let $X_1, X_2, \ldots, X_n$, be a random sample from the distribution having p.d.f. $f(x,\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ $0 \le x < \infty$, $\theta > 0$. Show that $\frac{n-1}{n\overline{X}}$ is the only unbiased estimator of $\frac{1}{\theta}$ based on $\overline{X}$. [8,7]

SECTION II

5. (a) Describe the Method of Moments. Let $X_1, X_2, \ldots, X_n$, be a random sample from the distribution with p.d.f. $f(x;\alpha,\beta) = \frac{\beta^n}{\Gamma\alpha} x^{n-1} e^{-\beta x}, x \ge 0, \alpha > 0, \beta > 0$.

Estimate the parameters α and β by the Method of Moments.

(b) Let X_1, $X_2,\ldots,X_n$, be a random sample from a distribution with p.d.f. $f(x;\theta)=\frac{1}{\theta}.x^{\frac{1-\theta}{\theta}}, 0<x<1,\ 0<\theta<\infty$. Find the Maximum likelihood estimator of θ. Also, examine the estimator for unbiasedness property. (8,7)

6. (a) What is failure-censored sample in life testing experiment? Obtain the Maximum likelihood estimator of the expected life time and reliability function in case of failure censored sample from the life time distribution:

$$f(x;\sigma)=\frac{1}{\sigma}e^{-\frac{x}{\sigma}}, x>0, \sigma>0.$$

(b) Show by means of an example that

(i) MLE may not be unbiased

(ii) MLE may not be unique (8, 7)

7. (a) Let X_1, X_2,...,X_n, be a random sample from a distribution having p.d.f. $f(x, \theta) = e^{-(x-\theta)}$, $\theta \leq x < \infty$, $-\infty > \theta < \infty$. Obtain 100 (1-$\alpha$)% confidence interval for θ.

(b) Describe a method of constructing confidence intervals. Find 100 (1-α)% confidence interval for binomial proportion based on a random sample of size n for large samples. (7, 8)

8. Describe any three of the following:

(a) Minimum Chi-square method of estimation

(b) Rao-Blackwell theorem

(c) Fisher-Neyman criterion

(d) Sufficient conditions for consistency (15)