

[This question paper contains 8 printed pages.]

Library

Your Roll No....Dec'23

Sr. No. of Question Paper : 1528

G

Unique Paper Code : 2512012301

Name of the Paper : Engineering Mathematics

Name of the Course : **B.Sc. (H) Electronics**

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions in all, including, Question number **1**, which is compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

P.T.O

1. (a) Solve the following differential equation:

$$y' = \sin^2(x - y + 1) \quad (3)$$

(b) Prove $\Gamma(p + 1) = p\Gamma(p)$. (3)

(c) In the differential equation

$$2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (1 - x^2)y = 0$$

is $x = 0$ an ordinary or regular singular point?

(3)

(d) Show that

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

is Skew-Hermitian and also unitary.

(3)

(e) Test for the convergence of the following series :

$$\sum_{n=1}^{\infty} \frac{n^2 - n}{2n^2 + n} \quad (3)$$

(f) Separate into real and imaginary :

(i) $e^{2 \pm 3\pi i}$

(ii) $e^{5 + i\frac{\pi}{2}}$ (3)

2. (a) Solve the following system of equations by Gaussian elimination method :

$$2x_1 + x_2 + 4x_3 = 12$$

$$8x_1 - 3x_2 + 2x_3 = 20$$

$$4x_1 + 11x_2 - x_3 = 33 \quad (6)$$

(b) Find the eigen values and eigen vectors of :

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} \quad (6)$$

- (c) Determine the algebraic and geometric multiplicity for the following matrix :

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} \quad (6)$$

3. (a) Solve

$$x(1 - 4y)dx - (x^2 + 1)dy = 0$$

$$\text{with } y(2) = 1. \quad (6)$$

- (b) Solve

$$y'' - 3y' + 2y = 0$$

$$\text{with } y(0) = -1, y'(0) = 0. \quad (4)$$

- (c) Using Frobenius method, obtain two linearly independent solutions about $x_0 = 0$, for the following differential equation :

$$8x^2y'' + 10xy' - (1 + x)y = 0 \quad (8)$$

4. (a) Solve the following differential equation :

$$\left(3x^2y + \frac{y}{x}\right)dx + (x^3 + \ln x)dy = 0 \quad (6)$$

(b) Solve the following system of equations by Gauss Seidel method :

$$x_1 - 8x_2 + 3x_3 = -4$$

$$2x_1 + x_2 + 9x_3 = 12$$

$$8x_1 + 2x_2 - 2x_3 = 8 \quad (6)$$

(c) Evaluate

$$\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta \, d\theta \quad (6)$$

5. (a) Use D' Alembert's test to test the convergence of the following series whose nth term is :

$$(i) \frac{n^3 + a}{2^n + a}$$

$$(ii) \frac{n!}{(2n)!} \quad (6)$$

(b) Using Cauchy's integral test, determine the convergence of the following series :

$$(i) \frac{1}{2} + \frac{4}{9} + \frac{9}{28} \dots$$

$$(ii) \sum_1^{\infty} \frac{1}{n^2 + 1} \quad (6)$$

(c) Test for the convergence of the following series :

$$(i) 1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \dots \infty \text{ with } x > 0$$

$$(ii) \sum \frac{x^{2n}}{2^n} \text{ with } x > 0 \quad (6)$$

6. (a) Show that the function

$$u = 3xy^2 - x^3$$

is harmonic and find its conjugate harmonic. (6)

- (b) Evaluate the integral

$$\oint \frac{e^z}{z-2} dz$$

Where c is (a) the circle $|z| = 3$; (b) the circle $|z| = 1$. (6)

- (c) Find Taylor's series expansion of

$$f(z) = e^z \sin z \text{ about } z = 0 \quad (6)$$

7. (a) Determine where the Cauchy Riemann equations are satisfied for the following function :

$$f(z) = e^z = e^x(\cos y + i \sin y)$$

Determine the region of analyticity. (6)

(b) Test for the convergence of the following series :

$$(i) \quad 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \dots$$

$$(ii) \quad \frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} + \dots \quad (6)$$

(c) Expand $\operatorname{erf}(x)$ in ascending powers of x . Find

$$\frac{d}{dx} [\operatorname{erf}(ax)]. \quad (6)$$