Question 1: Attempt any five :(a) What is the velocity of n-mesons whose observed mean life is 2.5×10^{-7} sec. The proper mean life of these n-mesons is 2.5×10^{-8} sec. (b) Find the value of a if the vector force field F = $(y^2z^3-\alpha xz^2)$ i + $2xyz^3$ j + $(3xy^2z^2 - 6x^2z)$ k is conservative. (c) Write the formula for Lorentz and inverse Lorentz transformations. (d) State the law of gravitational attraction and hence define the gravitational constant G. Also write its dimensions. (e) A light and heavy body has equal kinetic energies of translation. Which one has the larger momentum? (f) State Newton's Laws of motion. Show that Newton's first law of motion is a special case of second law.

(a) What is the velocity of n-mesons whose observed mean life is 2.5×10^{-7} sec. The proper mean life of these n-mesons is 2.5×10^{-8} sec.

- The observed mean life (τ) = 2.5×10^{-7} sec
- The proper mean life (τ_0) = 2.5×10^{-8} sec
- According to time dilation, $\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}$* Squaring both sides:$\tau^2 = \frac{\tau_0^2}{1-\frac{v^2}{c^2}}$*

  Rearranging the terms:$1 - \frac{v^2}{c^2} = \frac{\tau_0^2}{\tau^2}$* $\frac{v^2}{c^2} = 1 - \frac{\tau_0^2}{\tau^2}$* $v^2 = c^2\left(1 - \frac{\tau_0^2}{\tau^2}\right)$* $v = c\sqrt{1-\left(\frac{\tau_0}{\tau}\right)^2}$* Substitute the given values:$v = c\sqrt{1-\left(\frac{2.5\times10^{-8}}{2.5\times10^{-7}}\right)^2}$* $v = c\sqrt{1-\left(\frac{1}{10}\right)^2}$* $v = c\sqrt{1-0.01}$* $v = c\sqrt{0.99}$* $v \approx 0.995c$
- Therefore, the velocity of n-mesons is approximately 0.995 times the speed of light.

(b) Find the value of a if the vector force field F = $(y^2z^3-\alpha xz^2)$ i + $2xyz^3$ j + $(3xy^2z^2 - 6x^2z)$ k is conservative.

- For a vector force field F to be conservative, its curl must be zero, i.e., $\nabla \times F = 0$.
- Alternatively, we can use the condition that for a conservative field, the partial derivatives must satisfy:

- $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} * \frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x} * \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$* Given$F_x = y^2z^3 - \alpha xz^2$* Given$F_y = 2xyz^3$* Given$F_z = 3xy^2z^2 - 6x^2z$* Consider the first condition:$\frac{\partial F_x}{\partial y} = 2yz^3and\frac{\partial F_y}{\partial x} = 2yz^3$. This condition is satisfied.

- Consider the second condition: $\frac{\partial F_x}{\partial z} = 3y^2z^2 - 2\alpha xzand\frac{\partial F_z}{\partial x} = 3y^2z^2 - 12xz$.
- For these to be equal: $3y^2z^2 - 2\alpha xz = 3y^2z^2 - 12xz$* $-2\alpha xz = -12xz$* $-2\alpha = -12$* $\alpha = 6$* Consider the third condition to verify:$\frac{\partial F_y}{\partial z} = 6xyz^2and\frac{\partial F_z}{\partial y} = 6xyz^2$. This condition is also satisfied.
- Therefore, the value of α is 6.

(c) Write the formula for Lorentz and inverse Lorentz transformations.

- Lorentz Transformation equations describe how measurements of space and time by two observers are related, when one observer is moving at a constant velocity with respect to the other.
- **Lorentz Transformations (for motion along the x-axis):**
    - $x' = \gamma(x - vt)$ * $y' = y$ * $z' = z$ * $t' = \gamma\left(t - \frac{vx}{c^2}\right)$ * Where$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$is the Lorentz factor,$v$is the relative velocity between the frames, and$c$ is the speed of light.
- **Inverse Lorentz Transformations:**
    - $x = \gamma(x' + vt')$ * $y = y'$ * $z = z'$ * $t = \gamma\left(t' + \frac{vx'}{c^2}\right)$ * These equations can be obtained by simply replacing$v$with$-v$ and swapping primed and unprimed coordinates in the Lorentz transformation equations.

(d) State the law of gravitational attraction and hence define the gravitational constant G. Also write its dimensions.

- **Law of Gravitational Attraction (Newton's Law of Universal Gravitation):**
  - Every particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.
  - Mathematically, the force (F) between two masses m_1 and m_2 separated by a distance r is given by: $F = G\frac{m_1 m_2}{r^2}$
- **Definition of Gravitational Constant G:**
  - The gravitational constant G is a constant of proportionality in Newton's Law of Universal Gravitation.
  - It is numerically equal to the force of attraction between two bodies of unit mass (1 kg each) placed at a unit distance (1 meter) apart from each other.
- **Dimensions of G:**
  - From the formula $F = G\frac{m_1 m_2}{r^2}$, we can write $G = \frac{Fr^2}{m_1 m_2}$.
  - Dimensions of Force (F) = $[M^1 L^1 T^{-2}]$ * Dimensions of Distance (r) =$[L^1]$ * Dimensions of Mass (m) =$[M^1]$ * Therefore, dimensions of G =$\frac{[M^1 L^1 T^{-2}][L^1]^2}{[M^1][M^1]}$ * Dimensions of G =$\frac{[M^1 L^3 T^{-2}]}{[M^2]}$ * Dimensions of G =$[M^{-1} L^3 T^{-2}]$

(e) A light and heavy body has equal kinetic energies of translation. Which one has the larger momentum?

- Let the mass of the light body be m_L and its momentum be p_L.
- Let the mass of the heavy body be m_H and its momentum be p_H.
- We know that $m_L < m_H$.

- The kinetic energy (K.E.) of translation is given by $K.E. = \frac{1}{2}mv^2$.
- Momentum (p) is given by $p = mv$.
- We can express kinetic energy in terms of momentum: $K.E. = \frac{p^2}{2m}$.
- Given that the kinetic energies are equal: $K.E._L = K.E._H$* $\frac{p_L^2}{2m_L} = \frac{p_H^2}{2m_H}$*

  $p_L^2 m_H = p_H^2 m_L$* $\frac{p_L^2}{p_H^2} = \frac{m_L}{m_H}$* $\left(\frac{p_L}{p_H}\right)^2 = \frac{m_L}{m_H}$* Since$m_L < m_H$, the ratio $\frac{m_L}{m_H} < 1$.
- This implies $\left(\frac{p_L}{p_H}\right)^2 < 1$, which means $\frac{p_L}{p_H} < 1$.
- Therefore, $p_L < p_H$.
- The heavy body has the larger momentum.

(f) State Newton's Laws of motion. Show that Newton's first law of motion is a special case of second law.

- **Newton's Laws of Motion:**
  - **First Law (Law of Inertia):** An object at rest stays at rest, and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.
  - **Second Law:** The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object. Mathematically, it is expressed as $F_{net} = ma$, where $F_{net}$is the net force,$m$is the mass, and$a$ is the acceleration.
  - **Third Law:** For every action, there is an equal and opposite reaction. When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.
- **Newton's First Law as a Special Case of the Second Law:**

- Newton's Second Law states $F_{net} = ma$.
- Newton's First Law describes the behavior of an object when no net force acts on it.
- If there is no net external force acting on a body, then $F_{net} = 0$.
- According to the second law, if $F_{net} = 0$, then $ma = 0$.
- Since the mass (m) of a body cannot be zero (for a physical object), it implies that the acceleration (a) must be zero ($a = 0$).
- If acceleration is zero, it means that the velocity of the object is constant.
- A constant velocity implies two scenarios:
  - If the object is initially at rest ($v = 0$), it will remain at rest.
  - If the object is initially in motion ($v \neq 0$), it will continue to move with the same constant velocity in a straight line.
- These two scenarios are precisely what Newton's First Law describes. Therefore, Newton's First Law is a special case of the Second Law where the net external force is zero.

Question 2: (a) Prove that ∇^2 (ln r) = 0. (b) Solve : $(D^2+4)y = \sin 3x$.

(a) Prove that ∇^2 (ln r) = 0.

- The Laplacian operator in Cartesian coordinates is given by: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$* We are given the function $f = \ln r$.
- We know that $r = \sqrt{x^2 + y^2 + z^2}$.
- Therefore, $\ln r = \frac{1}{2}\ln(x^2 + y^2 + z^2)$.

- First, let's find the first partial derivative with respect to x: $\frac{\partial}{\partial x}(\ln r) = \frac{\partial}{\partial x}\left(\frac{1}{2}\ln(x^2+y^2+z^2)\right) = \frac{1}{2}\frac{1}{(x^2+y^2+z^2)}(2x) = \frac{x}{x^2+y^2+z^2} = \frac{x}{r^2}$
- Now, find the second partial derivative with respect to x: $\frac{\partial^2}{\partial x^2}(\ln r) = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2+z^2}\right)$ *Using the quotient rule,* $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2} = \frac{1\cdot(x^2+y^2+z^2)-x\cdot(2x)}{(x^2+y^2+z^2)^2} = \frac{x^2+y^2+z^2-2x^2}{(x^2+y^2+z^2)^2} = \frac{y^2+z^2-x^2}{(r^2)^2} = \frac{y^2+z^2-x^2}{r^4}$
- By symmetry, we can find the second partial derivatives with respect to y and z: $\frac{\partial^2}{\partial y^2}(\ln r) = \frac{x^2+z^2-y^2}{r^4}$ $\frac{\partial^2}{\partial z^2}(\ln r) = \frac{x^2+y^2-z^2}{r^4}$* Now, sum these second partial derivatives to find$\nabla^2(\ln r)$: $\nabla^2(\ln r) = \frac{y^2+z^2-x^2}{r^4} + \frac{x^2+z^2-y^2}{r^4} + \frac{x^2+y^2-z^2}{r^4} = \frac{(y^2+z^2-x^2)+(x^2+z^2-y^2)+(x^2+y^2-z^2)}{r^4} = \frac{x^2+y^2+z^2}{r^4}$ $Since x^2+y^2+z^2 = r^2 = \frac{r^2}{r^4} = \frac{1}{r^2}$* However, the question asks to prove$\nabla^2(\ln r) = 0$. This is true only for 2-D case.
- Let's consider the 2-D case where $r = \sqrt{x^2+y^2}$.
- $\ln r = \frac{1}{2}\ln(x^2+y^2)$* $\frac{\partial}{\partial x}(\ln r) = \frac{1}{2}\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2}$* $\frac{\partial^2}{\partial x^2}(\ln r) = \frac{(x^2+y^2)\cdot 1-x\cdot(2x)}{(x^2+y^2)^2} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$* By symmetry,$\frac{\partial^2}{\partial y^2}(\ln r) = \frac{x^2-y^2}{(x^2+y^2)^2}$* Then,$\nabla^2(\ln r) = \frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2} = \frac{y^2-x^2+x^2-y^2}{(x^2+y^2)^2} = \frac{0}{(x^2+y^2)^2} = 0$* So,$\nabla^2(\ln r) = 0$is true in 2-D. In 3-D,$\nabla^2(\ln r) = \frac{1}{r^2}$. Assuming the context implies 2-D or a specific interpretation where r is distance from origin for 2-D.

(b) Solve : $(D^2+4)y = \sin 3x$.

- This is a second-order non-homogeneous linear differential equation.
- **Step 1: Find the Complementary Function (y_c)**

- The auxiliary equation is obtained by setting the left side to zero: $m^2 + 4 = 0$ * $m^2 = -4$ * $m = \pm\sqrt{-4} = \pm 2i$ * Since the roots are complex conjugate ($\alpha \pm i\beta$ where$\alpha = 0$and$\beta = 2$), the complementary function is: $y_c = e^{\alpha x}(C_1\cos(\beta x) + C_2\sin(\beta x))$ $y_c = e^{0x}(C_1\cos(2x) + C_2\sin(2x))$ $y_c = C_1\cos(2x) + C_2\sin(2x)$

- **Step 2: Find the Particular Integral (y_p)**
  - The particular integral is given by $y_p = \frac{1}{D^2+4}\sin 3x$.
  - For terms of the form $\sin(ax)$or$\cos(ax)$, we replace $D^2$with$-a^2$. Here, $a = 3$, so $D^2 = -(3)^2 = -9$.
  - $y_p = \frac{1}{-9+4}\sin 3x$ * $y_p = \frac{1}{-5}\sin 3x$ * $y_p = -\frac{1}{5}\sin 3x$
- **Step 3: General Solution**
  - The general solution is the sum of the complementary function and the particular integral: $y = y_c + y_p$ $y = C_1\cos(2x) + C_2\sin(2x) - \frac{1}{5}\sin 3x$

Question 3: (a) What are central forces? Show that angular momentum of particle moving under the influence of central forces is always conserved. (b) A neutron moving with a velocity of 106 m/s collides with a deuteron at rest. After collision, the combined mass (titron) moves with a certain velocity. Calculate the velocity, if the mass of neutron is 1.67×10^{-27} kg and the mass of the deuteron is 3.34×10^{-27} kg.

(a) What are central forces? Show that angular momentum of particle moving under the influence of central forces is always conserved.

- **Central Forces:**
  - A force is called a central force if it is always directed towards or away from a fixed point (called the center of force) and its magnitude depends only on the distance from that point.

- Mathematically, a central force can be expressed as $\vec{F} = f(r)\hat{r}$, where $f(r)$is a scalar function of the distance$r$from the center of force, and$\hat{r}$ is the unit vector in the radial direction.
- Examples include gravitational force and electrostatic force between two point charges.

- **Conservation of Angular Momentum under Central Forces:**
  - Angular momentum of a particle is defined as $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{r}$is the position vector and$\vec{p}$ is the linear momentum.
  - The rate of change of angular momentum is equal to the net torque acting on the particle: $\frac{d\vec{L}}{dt} = \vec{\tau}$ * The torque is given by$\vec{\tau} = \vec{r} \times \vec{F}$.
  - For a central force, the force vector $\vec{F}$is always parallel or anti-parallel to the position vector$\vec{r}$(i.e.,$\vec{F} = f(r)\hat{r}$).
  - Therefore, the cross product $\vec{r} \times \vec{F}$ will be zero because the two vectors are collinear.
  - $\vec{\tau} = \vec{r} \times (f(r)\hat{r})$ * Since$\vec{r}$and$\hat{r}$are in the same direction,$\vec{r} \times \vec{F} = 0$.
  - This implies that $\frac{d\vec{L}}{dt} = 0$.
  - If the rate of change of angular momentum is zero, it means that the angular momentum vector $\vec{L}$ is constant both in magnitude and direction.
  - Thus, the angular momentum of a particle moving under the influence of a central force is always conserved.

(b) A neutron moving with a velocity of 106 m/s collides with a deuteron at rest. After collision, the combined mass (titron) moves with a certain velocity. Calculate the velocity, if the mass of neutron is 1.67×10^{-27} kg and the mass of the deuteron is 3.34×10^{-27} kg.

- This is a problem involving an inelastic collision where the two particles stick together after the collision (forming a combined mass, often called a perfectly inelastic collision).
- **Given:**
  - Mass of neutron (m_n) = 1.67×10^{-27} kg
  - Initial velocity of neutron (v_n) = 106 m/s
  - Mass of deuteron (m_d) = 3.34×10^{-27} kg
  - Initial velocity of deuteron (v_d) = 0 m/s (at rest)
- **To find:**
  - Final velocity of the combined mass (v_f)
- **Principle:** Conservation of linear momentum.
  - Total momentum before collision = Total momentum after collision
  - $m_n v_n + m_d v_d = (m_n + m_d) v_f$
- **Calculation:**
  - $(1.67 \times 10^{-27}\ \text{kg}) \times (106\ \text{m/s}) + (3.34 \times 10^{-27}\ \text{kg}) \times (0\ \text{m/s}) = (1.67 \times 10^{-27}\ \text{kg} + 3.34 \times 10^{-27}\ \text{kg}) v_f$ * $1.67 \times 10^{-27} \times 106 = (1.67 + 3.34) \times 10^{-27} v_f$ * $177.02 \times 10^{-27} = 5.01 \times 10^{-27} v_f$ * $v_f = \frac{177.02 \times 10^{-27}}{5.01 \times 10^{-27}}$ * $v_f = \frac{177.02}{5.01}$ * $v_f \approx 35.33$ m/s
- The velocity of the combined mass (titron) after collision is approximately 35.33 m/s.

Question 4: (a) Derive a general differential equation of motion of a simple harmonic oscillator and obtain its solution. (b) State Kepler's laws of planetary motion. Show that areal velocity of a planet around the sun is constant.

(a) Derive a general differential equation of motion of a simple harmonic oscillator and obtain its solution.

- **Derivation of the Differential Equation:**
  - A simple harmonic oscillator (SHO) is a system that, when displaced from its equilibrium position, experiences a restoring force that is directly proportional to the displacement and acts in the opposite direction.
  - Consider a mass $m$attached to a spring with spring constant$k$. Let x be the displacement from the equilibrium position.
  - According to Hooke's Law, the restoring force exerted by the spring is $F = -kx$. The negative sign indicates that the force is always directed opposite to the displacement.
  - According to Newton's Second Law of Motion, $F = ma$, where a is the acceleration.
  - We know that acceleration is the second derivative of displacement with respect to time: $a = \frac{d^2x}{dt^2}$.
  - Equating the two expressions for force: $m\frac{d^2x}{dt^2} = -kx$
  - Rearranging the terms, we get the general differential equation for a simple harmonic oscillator: $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ * Let$\omega_0^2 = \frac{k}{m}$, where ω_0 is the angular frequency of the oscillation.
  - The differential equation becomes: $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$
- **Solution of the Differential Equation:**
  - The equation $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$ is a second-order, linear, homogeneous differential equation with constant coefficients.
  - The auxiliary equation is $r^2 + \omega_0^2 = 0$.
  - $r^2 = -\omega_0^2$ * $r = \pm i\omega_0$ * Since the roots are purely imaginary complex conjugates ($\alpha \pm i\beta$ where$\alpha = 0$and$\beta = \omega_0$), the general

solution for $x(t)$ is: $x(t) = e^{\alpha t}(A\cos(\beta t) + B\sin(\beta t))$ $x(t) = e^{0t}(A\cos(\omega_0 t) + B\sin(\omega_0 t))$ $x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

- Alternatively, this solution can be expressed in a more compact form: $x(t) = C\cos(\omega_0 t + \phi)$ or $x(t) = C\sin(\omega_0 t + \phi')$ * Where A, B, C, ϕ, and ϕ' are constants determined by the initial conditions of the system (e.g., initial position and initial velocity).
- $C = \sqrt{A^2 + B^2}$ and $\phi = \arctan\left(-\frac{B}{A}\right)$.

(b) State Kepler's laws of planetary motion. Show that areal velocity of a planet around the sun is constant.

- **Kepler's Laws of Planetary Motion:**
  - **First Law (Law of Orbits):** All planets move in elliptical orbits with the Sun at one of the two foci.
  - **Second Law (Law of Areas):** A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time. This implies that the areal velocity of the planet is constant.
  - **Third Law (Law of Periods):** The square of the orbital period (T) of a planet is directly proportional to the cube of the semi-major axis (a) of its orbit. Mathematically, $T^2 \propto a^3$, or $\frac{T^2}{a^3} =$ constant.
- **Show that areal velocity of a planet around the sun is constant:**
  - Consider a planet of mass m orbiting the Sun of mass M. The force acting on the planet is the gravitational force, which is a central force directed towards the Sun.
  - The angular momentum $\vec{L}$ of the planet about the Sun is given by $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{r}$ is the position vector from the Sun to the planet, and $\vec{p} = m\vec{v}$ is the linear momentum.
  - As shown in Q3(a), for a central force, the torque $\vec{\tau} = \vec{r} \times \vec{F}$ is zero because $\vec{F}$ is always parallel to $\vec{r}$.

- Since $\vec{\tau} = \frac{d\vec{L}}{dt}$, if $\vec{\tau} = 0$, then $\frac{d\vec{L}}{dt} = 0$.
- This means that the angular momentum $\vec{L}$ of the planet about the Sun is conserved (constant).
- Now, let's relate angular momentum to areal velocity. Consider the area swept by the radius vector in a small time interval dt.
- In a small time dt, the planet moves a distance vdt. The area dAswept by the radius vector is approximately the area of a triangle with base$\vec{r}$and height$vdt\sin\theta$(where$\theta$is the angle between$\vec{r}$and$\vec{v}$).
- $dA = \frac{1}{2}|\vec{r} \times d\vec{r}|$ * Since$d\vec{r} = \vec{v}dt$,
- $dA = \frac{1}{2}|\vec{r} \times \vec{v}dt|$ * $dA = \frac{1}{2}|\vec{r} \times \vec{v}|dt$ * The areal velocity is$\frac{dA}{dt}$.
- $\frac{dA}{dt} = \frac{1}{2}|\vec{r} \times \vec{v}|$ * We know that angular momentum$|\vec{L}| = |\vec{r} \times \vec{p}| = |\vec{r} \times m\vec{v}| = m|\vec{r} \times \vec{v}|$.
- Therefore, $|\vec{r} \times \vec{v}| = \frac{|\vec{L}|}{m}$.
- Substituting this into the areal velocity equation: $\frac{dA}{dt} = \frac{1}{2}\frac{|\vec{L}|}{m}$ * Since both the angular momentum$\vec{L}$and the mass$m$of the planet are constant (due to conservation of angular momentum and constant mass), the areal velocity$\frac{dA}{dt}$ is constant.
- This proves Kepler's Second Law.

Question 5: (a) Describe Michelson-Morley experiment with suitable mathematical expression. (b) Atomic particles in the form of a beam have a velocity of 95% speed of light. What is their relativistic mass compared with their rest mass?

(a) Describe Michelson-Morley experiment with suitable mathematical expression.

- **Michelson-Morley Experiment:**

- The Michelson-Morley experiment, performed in 1887 by Albert Michelson and Edward Morley, was a crucial experiment in the history of physics. Its primary aim was to detect the existence of a hypothetical medium called "luminiferous aether," which was believed to permeate all space and act as the medium for the propagation of light waves. It was thought that if Earth was moving through this aether, then the speed of light would be different depending on the direction of travel relative to the aether.

- **Experimental Setup:**
  - The experiment used a Michelson interferometer. A beam of light from a source is split into two perpendicular paths by a semi-silvered mirror (beam splitter).
  - One beam (let's say along the x-axis) travels to a mirror and reflects back to the beam splitter.
  - The other beam (along the y-axis) travels to another mirror and reflects back to the beam splitter.
  - Upon recombination at the beam splitter, the two beams interfere, and the resulting interference pattern is observed on a telescope.
  - The entire apparatus was mounted on a heavy stone slab floating in mercury, allowing it to be rotated smoothly to observe any change in the interference pattern.

- **Hypothesis and Expected Outcome:**
  - If the Earth was moving through the aether with a velocity v, then the speed of light would be different in different arms of the interferometer.
  - Let c be the speed of light in the stationary aether.
  - **Time for light to travel along the arm parallel to Earth's motion (length L_1):**

- Light travels from beam splitter to mirror: The effective speed is $(c - v)$(against the aether wind). Time$t_1 = \frac{L_1}{c-v}$.
- Light travels from mirror back to beam splitter: The effective speed is $(c + v)$(with the aether wind). Time$t_2 = \frac{L_1}{c+v}$.
- Total time for the parallel arm: $T_{parallel} = t_1 + t_2 = \frac{L_1}{c-v} + \frac{L_1}{c+v} = \frac{L_1(c+v)+L_1(c-v)}{c^2-v^2} = \frac{2L_1 c}{c^2-v^2} = \frac{2L_1}{c(1-\frac{v^2}{c^2})}$ * **Time for light to travel along the arm perpendicular to Earth's motion (lengthL_2):**
- In this arm, light has to travel across the aether wind. To reach the mirror and return, light must follow a diagonal path relative to the aether.
- The actual path length in the aether for one way is such that the velocity components satisfy $c^2 = v^2 + v_p^2$, where $v_p$is the perpendicular component of velocity. So,$v_p = \sqrt{c^2 - v^2}$.
- Time for one way: $t_3 = \frac{L_2}{\sqrt{c^2-v^2}}$ * Total time for the perpendicular arm:$T_{perpendicular} = 2t_3 = \frac{2L_2}{\sqrt{c^2-v^2}} = \frac{2L_2}{c\sqrt{1-\frac{v^2}{c^2}}}$

- **Expected Time Difference (if aether existed):**
  - If $L_1 = L_2 = L$, then the expected time difference is: $\Delta T = T_{parallel} - T_{perpendicular} = \frac{2L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ * Using binomial approximation$(1-x)^{-n} \approx 1 + nx$for small$x$(since$v \ll c$): $\frac{1}{1-\frac{v^2}{c^2}} \approx 1 + \frac{v^2}{c^2}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = (1 - \frac{v^2}{c^2})^{-1/2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$ * $\Delta T \approx \frac{2L}{c}\left((1 + \frac{v^2}{c^2}) - (1 + \frac{1}{2}\frac{v^2}{c^2})\right) = \frac{2L}{c}\left(\frac{1}{2}\frac{v^2}{c^2}\right) = \frac{Lv^2}{c^3}$ * This time difference would lead to a phase shift and thus a shift in

the interference pattern (fringe shift) when the apparatus was rotated by 90 degrees. The expected fringe shift was calculated to be $N = \frac{\Delta T}{\text{period of light}} \approx \frac{2Lv^2}{\lambda c^2}$, where λ is the wavelength of light.

- **Results and Conclusion:**
  - Despite the high precision of the experiment, Michelson and Morley found no detectable fringe shift. They repeated the experiment at different times of the year and at different orientations, always with a null result.
  - This null result directly contradicted the aether hypothesis. It indicated that the speed of light is constant for all observers, regardless of their motion or the motion of the source.
  - This groundbreaking result was a key piece of evidence that led to the development of Albert Einstein's Special Theory of Relativity in 1905, which postulated that the speed of light in a vacuum is invariant and that the concept of a luminiferous aether is unnecessary.

(b) Atomic particles in the form of a beam have a velocity of 95% speed of light. What is their relativistic mass compared with their rest mass?

- **Given:**
  - Velocity of the particles (v) = 95% of the speed of light (c) = $0.95c$
- **To find:**
  - Relativistic mass (m) compared with their rest mass (m_0), i.e., find the ratio $\frac{m}{m_0}$.
- **Relativistic Mass Formula:**
  - The relativistic mass m of a particle moving with velocity v is given by: $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ * Where m_0 is the rest mass and c is the speed of light.
- **Calculation:**

- Substitute $v = 0.95c$ into the formula: $m = \frac{m_0}{\sqrt{1-\frac{(0.95c)^2}{c^2}}}$ $m = \frac{m_0}{\sqrt{1-\frac{0.95^2 c^2}{c^2}}}$ $m = \frac{m_0}{\sqrt{1-0.95^2}}$ $m = \frac{m_0}{\sqrt{1-0.9025}}$ $m = \frac{m_0}{\sqrt{0.0975}}$ $m \approx \frac{m_0}{0.31225}$ $m \approx 3.202m_0$

* **Conclusion:**
  - The relativistic mass of the atomic particles is approximately 3.202 times their rest mass. This demonstrates the effect of relativistic mass increase at speeds approaching the speed of light.