DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics Semester-V

DISCIPLINE SPECIFIC CORE COURSE – 13: METRIC SPACES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Metric Spaces	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

Learning Outcomes: This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities. Become aware one such formulations leading to metric spaces.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.
- Know about Banach fixed point theorem, whose far-reaching consequences have resulted into an independent branch of study in analysis, known as fixed point theory.

SYLLABUS OF DSC-13

UNIT – I: Topology of Metric Spaces (18 hours)

Definition, examples, sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem, Subspaces.

UNIT – II: Continuity and Uniform Continuity in Metric Spaces (15 hours)

Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity; Homeomorphism, Isometry and equivalent metrics, Contraction mapping, Banach fixed point theorem.

UNIT – III: Connectedness and Compactness (12 hours)

Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings, Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

Essential Reading

3. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

Suggestive Readings

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.
- Rudin, Walter. Principles of mathematical Analysis (3rd ed.).
- Simmons, George F. (2004). Introduction to Topology and Modern Analysis. McGraw-Hill Education. New Delhi.

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

DISCIPLINE SPECIFIC CORE COURSE – 14: RING THEORY

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Ring Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The primary objective of this course is to:

- Introduce the fundamental theory of rings, and their homomorphisms.
- Develop the basic concepts of polynomial rings and irreducibility tests for polynomials over the ring of integers, and rational numbers.
- Introduce polynomial analog of a prime number.
- Describe polynomial rings, principal ideal domains, Euclidean domains and unique factorization domains, and their relationships.

Learning Outcomes: This course will enable the students to:

- Learn about the fundamental concept of rings, integral domains, and fields.
- Know about ring homomorphisms and isomorphisms theorems of rings, and construct quotient fields for integral domains.
- Appreciate the significance of unique factorization in rings and integral domains.
- Apply several criteria for determining when polynomials with integer coefficients have rational roots or are irreducible over the field of rational numbers.

SYLLABUS OF DSC-14

UNIT – I: Introduction to Rings and Ideals (18 hours)

Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, Characteristic of a ring; Ideals, operations on ideals, ideal generated by a set and properties, Factor rings, Prime ideals and maximal ideals, Principal ideal domains.

UNIT – II: Ring Homomorphisms and Polynomial Rings (15 hours)

Definition, examples and properties of ring homomorphisms; First, second and third