7. Write short notes on any of the three:

(a) Usefulness and limitations of SPRT in testing a hypothesis.

(b) Type I and Type II errors and power of test.

(c) Single sample sign test

(d) Kruskal-Wallis test. (5,5,5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5519 **J**

Unique Paper Code : 2372013601

Name of the Paper : Testing of Hypothesis (NEP)

Name of the Course : **B.Sc.(H) Statistics**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. **Question 1 is compulsory.**

3. Attempt **four** more questions from choosing **two** from **Section A** and **Section B each.**

4. Use of simple **scientific calculator (non-programmable)** is allowed.

5. Use of **statistical tables** is permitted.

1. Attempt any FIVE parts:

(a) In order to test the null hypothesis that the mean of a normal population with $\sigma^2 = 1$ is μ_0 against the alternate that it is μ_1 ($>\mu_0$), derive the value of K such that $\overline{x} > K$ provides a x critical region of size $\alpha = 0.05$ for a random sample of size n.

(b) A manufacturer of pharmaceutical products claims that the recovery rate using a new medication is 90%. A competitor claims that the recovery rate is just 60%. In order to reach at a decision the drug is tested on 20 patients and the number of successes is observed. The claim of the producer will be accepted if the number of successes is at least 14.Set the null and the alternate hypotheses and calculate the size of the critical region and the power of the test.

(c) The number of successes in n trials is to be used to test the null hypothesis that the parameter θ of a binomial population equals $\frac{1}{2}$ against the alternate that it does not. Show that the critical region of the LRT can be written as

value 6 or more. Find the critical region, acceptance region and size of two types of errors. (8,7)

Section B

5. (a) If X ~ $N(\theta, \sigma^2)$ and the hypothesis is: $H_0 : \theta = \theta_0$ vs $H_0 : \theta = \theta_1$, and the strength is (α_0, β_0 and σ^2 is known then find the ASN and OC function.

(b) Name a non-parametric test which is an alternative to the one way analysis of variance for testing that k-independent sample are drawn from different population. Described its methods and function in details. (8,7)

6. (a) State and prove the theorem that SPRT terminates with probability 1.

(b) If a person gets seven heads and three tails in 10 tosses of a fair coin, then find the probabilities of 2,5,6 runs. (9,6)

$f(x,\theta) = \theta e^{-\theta x}, x > 0$

(iii) Show that there exists no UMP test for $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. ?. (8,7)

4. (a) Discuss the method of construction of Likelihood Ratio Test(LRT) and states its important properties. Consider n Bernoulli trials with probability of success p for each trial. Derive the LRT for testing $H_0 = p=p_0$ against $H_1 = p>p_0$.

(b) A random variable X follows the following exponential distribution

$$f(x,\theta) = {}^{1}/_{\theta}\, e^{-x/\theta};\ 0 \le x < \infty, \theta > 0$$
$$= 0, \quad \text{elsewhere.}$$

The null hypothesis $H_0 : \theta = 2$ is rejected and alternative hypothesis $H_1 : \theta = 4$ is accepted. If the observation selected at random takes the

$$x * \ln(x) + (n-x) * \ln(n-x) \ge K,$$

where x is the observed number of successes and K is a suitable chosen constant.

(d) State the Wald's equation and show that if E(z) = 0, then $E(S_N^2) = E(N) \times Var(z)$ where $S_N = z_1+z_2+\ldots+z_N$, and z_i's are i.i.d random variables.

(e) If there are $n_1 = 6$ letters of one kind and $n_2 = 5$ letters of another kind, then for how many runs would we reject the null hypothesis of randomness at 1% level of significance?

(f)In order to judge the effectiveness of foot position in discus throw, a team of 16 players was taught a new foot position. The following scores (in meters) were obtained before and after the completion of training:

Before :	16.3	10.1	10.7	13.5	14.9	11.8	14.3	10.2
After:	21.3	23.8	15.4	19.6	12.0	13.9	18.8	19.2
Before:	12.0	14.7	23.6	15.1	14.5	18.4	13.2	14.0
After:	15.3	20.1	14.8	18.9	20.7	21.1	15.8	16.2

Use a rank sum test to examine the effectiveness of the new position at 1% level of significance.

(g) A large industrial plant emits a lot of poisonous gases. The median emission level has been found to be 160. After being warned by the authorities, a new technology was adopted by the plant. The data was collected for 19 days after the implementation of the technology to find if there is any impact of the technology on emission reduction. The day wise emission is as follows:

163	165	160	189	161	171	158	151	169	162
163	139	172	165	148	166	172	163	187	173

Is there any evidence to suggest a reduction in the emission levels at 5% level of significance? (5×6)

Section A

2. (a) What are simple and composite hypothesis? State and prove Neyman-Pearson Lemma for testing a simple hypothesis against a simple alternative.

(b) Let X have a p.d.f. of the form

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}e^{-\frac{x}{\theta}}; & 0<x<\infty, \theta>0\\ 0; & \text{otherwise}\end{cases}$$

In order to test $H_0: \theta = 2$ against $H_1: \theta = 1$, use a random sample X_1, X_2 of size 2 and define a critical region $C=\{(x_1,x_2): 9.5 \leq x_1 + x_2\}$. Find

(i) Power function of the test

(ii) Significance level of the test. (8,7)

3. (a) Define UMPU critical region. Prove that if W is an MP region for testing $H_0 : \theta = \theta_0$ = against $H_1 : \theta = \theta_1$,, then it is necessarily unbiased. Also, prove that the same holds good if W is an UMP region.

(b) Given a random sample x_1, x_2, ..., x_n from the distribution with p.d.f.