[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1405 **I**

Unique Paper Code : 2372011103

Name of the Paper : Calculus

Name of the Course : **B.Sc. (Hons) Statistics (NEP-UGCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all.

3. **All** questions/parts carry equal marks.

1. (a) Show that the function f, defined by

   $f(x) = (1 + 3\chi)^{1/x}$ when $x \neq 0$, $f(0) = e^3$, is continuous for $x = 0$.

   (b) If $y = x \log \frac{x-1}{x+1}$, prove that

   $$\frac{d^n y}{dx^n} = (-1)^n (n-2)! \left[\frac{(x-n)}{(x-1)^n} - \frac{(x+n)}{(x+1)^n}\right]$$

2. (a) Determine $\lim_{x \to 0} \left(\frac{1}{x^2} - cot^2 x\right)$.

   (b) If A, B, C are the angles of a triangle such that

   $\sin^2 A + \sin^2 B + \sin^2 C =$ Constant, prove that

   $$\frac{dA}{dB} = \frac{\tan A - \tan B}{\tan C - \tan A}.$$

3. (a) Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$.

   (b) Solve $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$.

4. (a) Solve the partial differential equation

$$p(q^2 + 1) + (b - z)q = 0.$$

(b) Show that $\int_0^{\pi/2} \sqrt{tan\theta}\, d\theta = \frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right) = 2\int_0^{\infty} \frac{x^2}{1+x^4} dx.$

5. (a) Change the order of integration in the integral

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{ax}} f(x,y)\, dx\, dy.$$

(b) Find $y_n(0)$ when $y = \log\left(x + \sqrt{1+x^2}\right)$.

6. (a) Assuming the validity of differentiation under integral sign, show that

$$\int_0^{\infty} e^{-x^2} \cos ax\, dx = \frac{1}{2}\sqrt{\pi}\, e^{\frac{-1}{4}\alpha^2}.$$

(b) Solve the partial differential equation,

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y.$$

7. (a) Solve $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2$.

(b) Show that $\int \cos^3 x \, dx = \frac{3}{4} \sin x + \frac{1}{12} \sin 3x$.

8. (a) Discuss the derivability of the function :

$$f(x) = \begin{cases} x & , \; x < 1 \\ 2 - x & , \; 1 \leq x \leq 2 \\ -2 + 3x - x^2 & , \; x > 2 \end{cases}$$

at x = 1 and 2.

(b) By the elimination of the constants h and k, find the differential equation for which $(x - h)^2 + (y - k)^2 = a^2$, is a solution.