Point out the difference between one-tail and two-tail tests. Briefly explain how a statistical hypothesis is tested. [8,7]

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5539 **J**

Unique Paper Code : 2372012401

Name of the Paper : Sampling Distributions

Name of the Course : **B.Sc. (Hons.), Statistics (NEP-UGCF)**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all by selecting **three questions from each section.**
3. All questions carry equal marks.
4. Use of simple calculator is allowed.

SECTION I

1. (a) If X is a chi-square variate with n d.f., then prove that for large n, $\sqrt{2X} \sim N(\sqrt{2n}, 1)$

   (b) Let t has Student's t-distribution with 2 d.f.. Find the probability $P[t \geq 2]$.

   (c) X is a F-variate with 2 and n ($n \geq 2$) d.f.. Find the probability p = $P[F \geq k]$ and deduce the significance level of F corresponding to the significance level of probability p.

   [5, 5, 5]

2. (a) Let $X_1, X_2, \ldots, X_n$, be a random sample from $N(\mu, \sigma^2)$, and k be a positive integer. Find $E[S^{2k}]$. In particular, find $E[S^2]$ and $V[S^2]$.

   (b) *Let* $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$.

$n\lambda$ if and only if each X_i is exponential with parameter λ. [8,7]

8. (a) Explain the term Sampling distribution and standard error of a statistic. Derive the expression for the standard error of

   (i) the sample mean of a random sample of size n from a population having finite population variance σ^2

   (ii) the difference of means of two independent random samples of size n_1 and n_2 from two populations having finite variances of σ_1^2 and σ_2^2 respectively.

   (b) What are the two types of errors that arise in testing of hypothesis?

(b) Define convergence in law; convergence in probability and convergence with probability one. State the relationship between convergence in probability and convergence with probability one and prove it. [8,7]

7. (a) Let $X_1, X_1, \ldots, X_{2n-1}$ be an odd -size random sample from a $N(\mu, \sigma^2)$ population.

Find p.d.f. of the sample median and show that it is symmetric about μ and has the mean μ.

(b) Let $X_1, X_1, \ldots, X_n$ be a random sample from a population with continuous density. Show that $Y_1 = Min(X_1, X_2, \ldots, X_n)$ is exponential with parameter

Let $\overline{X}$ and S^2 be the sample mean and sum of squares of the deviations from the mean respectively. Let X' be one more observation independent of previous ones. Find the Sampling distribution of

$$U = \frac{X' - \overline{X}}{S}\left[\frac{n(n-1)}{n+1}\right]^{\frac{1}{2}}$$

[8, 7]

3. (a) Define Student's t-statistic and Fisher's t-statistic. Show that Student's t-statistic may be regarded as a particular case of Fisher's t-statistic. Obtain the p.d.f. of Student's t-statistic.

(b) Find the p.d.f. of $\chi_n = +\sqrt{\chi_n^2}$ and $\mu_r' = E[\chi_n^r]$, where χ_n^2 is a χ^2-variate with n d.f.

Hence, establish that for large n,

$E[\chi_n^2] = [E(\chi_n)]^2$

[8, 7]

4. (a) Let $X \sim F_{m,n}$. Find the mean, and mode of X. Also, find the distribution of

$$U = \frac{mX}{n + mX}$$

(b) Show that a χ^2-test involving two sample proportions is equivalent to a large sample significance test of difference in the proportions.

[8,7]

SECTION II

5. (a) State and prove Chebychev's Inequality. Use Chebychev's inequality to show that for n>36, the

probability that in n throws of a fair die, the number of sixes lies between $\frac{n}{6} - \sqrt{n}$ *and* $\frac{n}{6} + \sqrt{n}$ is atleast 31/36.

(b) Let $\{X_n\}$ be a sequence of mutually independent random variables such that: $X_n = \pm 1$ with probability $\frac{1-2^{-n}}{2}$ and $X_n = \pm 2^{-n}$ with probability 2^{-n-1}. Examine whether the W.L.L.N. holds for the sequence $\{X_n\}$. [8,7]

6. (a) Show that the central limit theorem holds for the sequence $\{X_k\}$ of independent random variables defined as $P[X_k = 0] = 1 - k^{1-2\alpha}$, $P[X_k = \pm k^{\alpha}]$ $= \frac{1}{2}k^{-2\alpha}$ if $\alpha < \frac{1}{2}$.

P.T.O.