Question 1:

(i) What is Compton effect? Write the expression for the Compton wavelength of scattering particle.

- **Compton Effect:** The Compton effect is a phenomenon observed when X-rays or gamma rays scatter off free electrons. It involves a change in the wavelength of the scattered radiation, indicating a loss of energy by the photon and a gain of kinetic energy by the electron. This effect provides strong evidence for the particle nature of light.

- **Expression for Compton wavelength of scattering particle:** The Compton wavelength (λ_c) of a particle is given by: $\lambda_c = h/(m_0 c)$ where h is Planck's constant, m_0 is the rest mass of the particle, and c is the speed of light. The expression for the change in wavelength (Compton shift, $\Delta\lambda$) of the scattered photon is: $\Delta\lambda = \lambda' - \lambda = (h/(m_e c)) * (1 - \cos\theta)$ where λ' is the wavelength of the scattered photon, λ is the wavelength of the incident photon, m_e is the rest mass of the electron, and θ is the scattering angle. The term $h/(m_e c)$ is the Compton wavelength of the electron.

(ii) What are the conditions for observing sustained interference pattern?

- **Conditions for observing sustained interference pattern:**
  - **Coherent Sources:** The two sources of light must be coherent, meaning they must emit waves with a constant phase difference and the same frequency and wavelength.
  - **Monochromatic Light:** The light used should be monochromatic, i.g., having a single wavelength. If polychromatic light is used, the interference patterns for different wavelengths will overlap, leading to a loss of distinct fringes.
  - **Sources close to each other:** The two sources must be very close to each other to produce a wide and observable interference pattern.
  - **Small source size:** The sources must be narrow or point-like to ensure a distinct and sharp interference pattern.

- **Constant phase difference:** The phase difference between the waves from the two sources must remain constant over time.
- **Equal amplitude:** For maximum contrast (darkest dark and brightest bright fringes), the amplitudes of the waves from the two sources should be nearly equal.
- **Opposite polarization:** If the light waves are polarized, their planes of polarization must be the same or have components in the same plane to produce interference.

(iii) Distinguish between Haidinger and Fizeau fringes. What kind of fringes are seen in a Newton's rings setup?

- **Distinction between Haidinger and Fizeau fringes:**
  - **Haidinger Fringes (Fringes of Equal Inclination):**
    - Formed by light rays interfering after reflection from parallel or nearly parallel surfaces.
    - Localized at infinity or in the focal plane of a lens.
    - Circular in shape, concentric with the normal to the surfaces.
    - Observed when the thickness of the film is uniform, and interference depends on the angle of inclination of the incident rays.
    - Examples: Michelson interferometer fringes.
  - **Fizeau Fringes (Fringes of Equal Thickness):**
    - Formed by light rays interfering after reflection from surfaces of varying thickness (wedge-shaped film).
    - Localized in the plane of the thin film.
    - Shape follows the contours of constant thickness of the film.

- Observed when the thickness of the film varies, and interference depends on the thickness of the film at a given point.
- Examples: Newton's rings.

- **Fringes seen in a Newton's rings setup:** In a Newton's rings setup, **Fizeau fringes (fringes of equal thickness)** are seen. These fringes are concentric dark and bright rings produced due to interference in the air wedge formed between the plano-convex lens and the plane glass plate.

(iv) 'An exceedingly thin film appears to be perfectly black when seen by reflected light. Why?'

- **Reason for exceedingly thin film appearing black:** When an exceedingly thin film is seen by reflected light, it appears perfectly black because of the phenomenon of interference.
  - For reflection from the upper surface of the film, there is a phase change of π (or 180°).
  - For reflection from the lower surface of the film, there is no additional phase change if the film is denser than the surrounding medium (or vice versa, depending on the indices of refraction).
  - When the film is exceedingly thin (thickness t approaches zero, $t \approx 0$), the path difference between the two reflected rays (one from the top surface and one from the bottom surface) becomes negligible.
  - However, due to the π phase change at one of the reflections, the two reflected rays are effectively out of phase by π.
  - This results in destructive interference for all wavelengths of visible light, causing the film to appear black.

(v) Distinguish between Fresnel and Fraunhoffer class of diffraction. The diffraction of star light in a telescope is an example of what kind of diffraction?

- **Distinction between Fresnel and Fraunhoffer class of diffraction:**

- **Fresnel Diffraction:**
  - Occurs when either the source or the screen (or both) are at a finite distance from the diffracting obstacle/aperture.
  - The wavefronts incident on the obstacle/aperture are spherical or cylindrical.
  - No lenses are required to observe the pattern.
  - The diffraction pattern is a complicated interplay of geometry and path differences, and the fringes are not clearly defined or parallel.
  - Examples: Diffraction by a straight edge, slit, or circular aperture when the source or screen is close.
- **Fraunhoffer Diffraction:**
  - Occurs when both the source and the screen are effectively at infinite distances from the diffracting obstacle/aperture. This is usually achieved by using lenses to make the incident wavefront plane and to focus the diffracted light onto the screen.
  - The wavefronts incident on the obstacle/aperture are plane.
  - Lenses are typically used to achieve the infinite distances (collimating lens for incident light, converging lens for diffracted light).
  - The diffraction pattern consists of well-defined, parallel bright and dark fringes.
  - Examples: Diffraction by a single slit, double slit, or diffraction grating.

- **Diffraction of star light in a telescope:** The diffraction of star light in a telescope is an example of **Fraunhoffer diffraction**. This is because the star light, coming from a distant source, can be considered as a plane wavefront incident on the telescope's objective lens (which acts as an aperture). The eyepiece then focuses this diffracted light, similar to how a lens is used in Fraunhoffer diffraction setups to bring the pattern to a focus.

(vi) Compare the double slit diffraction pattern observed in reality with the theory of Young's double slit experiment in terms of slit width and slit spacing.

- **Comparison of double slit diffraction pattern with Young's double slit theory:**
    - **Young's Double Slit Experiment (Ideal Theory):**
        - **Slit Width:** Assumes infinitesimally narrow slits (point sources). This implies that each slit produces a spherical wave without any diffraction spread of its own.
        - **Slit Spacing:** Considers the interference of waves from two ideal point sources separated by a distance 'd'. The intensity pattern is solely due to interference, resulting in equally bright fringes with uniform intensity.
        - **Pattern:** Uniformly bright interference fringes with equal spacing. The intensity is given by $I = 4I_0\cos^2(\pi d\sin\theta/\lambda)$.
    - **Observed Double Slit Diffraction Pattern (Reality):**
        - **Slit Width (finite 'a'):** In reality, the slits have a finite width ('a'). Each slit itself acts as a single slit, producing its own Fraunhoffer diffraction pattern (a central maximum with decreasing intensity side maxima).
        - **Slit Spacing ('d'):** The actual observed pattern is a combination of the interference pattern due to the slit spacing ('d') and the diffraction pattern due to the finite slit width ('a').
        - **Pattern:** The overall pattern is an interference pattern (due to 'd') whose intensity is modulated by the single-slit diffraction envelope (due to 'a'). This means the interference fringes are brightest at the center and their intensity gradually decreases as

one moves away from the center, governed by the single-slit diffraction pattern. The maxima of the interference pattern fall at the minima of the single-slit diffraction pattern, leading to "missing orders."

- **Intensity Formula:** The intensity distribution is given by $I = I_0(\sin^2\alpha/\alpha^2)\cos^2\beta$, where $\alpha = (\pi a \sin\theta)/\lambda$ (due to diffraction) and $\beta = (\pi d \sin\theta)/\lambda$ (due to interference).

Question 2:

(a) What is Photoelectric Effect? What are the observations that cannot be explained by the wave theory of light? How did Einstein explain photoelectric effect?

- **Photoelectric Effect:** The photoelectric effect is the phenomenon of emission of electrons from a metal surface when light of suitable frequency (or wavelength) falls on it. The emitted electrons are called photoelectrons, and the current produced is called photoelectric current.
- **Observations that cannot be explained by the wave theory of light:**
  - **Threshold Frequency:** For a given metal, there exists a minimum frequency (called threshold frequency, v_0) of incident light, below which no photoelectrons are emitted, no matter how intense the incident light is or how long it falls on the surface. Wave theory predicts that any frequency of light, given sufficient intensity and time, should be able to eject electrons.
  - **Instantaneous Emission:** The emission of photoelectrons is almost instantaneous (within 10^{-9} seconds) after the light strikes the surface, even for very low intensities. Wave theory predicts a time lag for electrons to accumulate enough energy from the incident wave.
  - **Kinetic Energy Dependence on Frequency:** The maximum kinetic energy of the emitted photoelectrons is found to be directly proportional to the frequency of the incident light and is independent

of its intensity. Wave theory predicts that the kinetic energy should depend on the intensity of light.

- **Independence of Emission on Intensity (above threshold):** For a given frequency above the threshold frequency, the number of emitted photoelectrons (and thus the photoelectric current) is directly proportional to the intensity of incident light, but the maximum kinetic energy remains unchanged. Wave theory predicts that increasing intensity should increase the kinetic energy of the electrons.

- **How Einstein explained photoelectric effect:**
    - Einstein explained the photoelectric effect based on Planck's quantum theory of radiation.
    - He proposed that light consists of discrete packets of energy called photons (or quanta).
    - The energy of each photon is directly proportional to its frequency: $E = hv$, where h is Planck's constant and v is the frequency of light.
    - He suggested that when a photon strikes an electron in the metal, it transfers its entire energy to the electron in an all-or-nothing fashion.
    - A part of this energy, called the work function (ϕ or W_0), is used by the electron to escape the metal surface.
    - The remaining energy is converted into the kinetic energy of the emitted electron.
    - This can be expressed by Einstein's photoelectric equation: $hv = \phi + K_{max}$, where K_{max} is the maximum kinetic energy of the emitted electron.
    - **Explanation of observations based on Einstein's theory:**
        - **Threshold Frequency:** An electron can only be emitted if the photon energy (hv) is greater than or equal to the work function

(ϕ). If $hv < \phi$, no emission occurs, explaining the threshold frequency ($v_0 = \phi/h$).

- **Instantaneous Emission:** Since the energy transfer is a single-event collision between a photon and an electron, the emission is instantaneous.
- **Kinetic Energy Dependence on Frequency:** As $hv = \phi + K_{max}$, then $K_{max} = hv - \phi$. This shows that K_{max} is linearly dependent on frequency (v) and independent of intensity.
- **Independence of Emission on Intensity:** A higher intensity means more photons are incident per unit time. Each photon, if its energy is above the work function, can eject one electron. Therefore, more photons lead to more electrons, explaining the proportionality of photoelectric current to intensity.

(b) Show that the group velocity of De-Broglie waves associated with a moving particle is equal to the particle velocity.

- Let's consider a moving particle with velocity v.
- According to De-Broglie hypothesis, every moving particle has a wave associated with it, called De-Broglie wave, whose wavelength is given by $\lambda = h/p = h/(mv)$, where p is the momentum, m is the mass, and h is Planck's constant.
- The frequency of the De-Broglie wave is related to the energy of the particle by $E = hv_{wave}$.
- Also, the relativistic energy of a particle is given by $E = mc^2$, where c is the speed of light.
- Therefore, $hv_{wave} = mc^2$, which implies $v_{wave} = mc^2/h$.
- The phase velocity (v_p) of the De-Broglie wave is given by $v_p = \lambda v_{wave}$.
- Substituting the values of λ and v_{wave}: $v_p = (h/(mv)) * (mc^2/h) = c^2/v$.

- Since the particle velocity v is always less than c, the phase velocity v_p is always greater than c. This implies that the De-Broglie wave is not a single monochromatic wave but rather a wave packet or group of waves.
- The group velocity (v_g) of a wave packet is given by the relation: $v_g = d\omega/dk$ where $\omega = 2\pi v_{wave}$ is the angular frequency and $k = 2\pi/\lambda$ is the wave number.
- From $E = hv_{wave}$, we have $\omega = 2\pi E/h$.
- From $\lambda = h/p$, we have $k = 2\pi p/h$.
- So, $\omega = E/\hbar$ and $k = p/\hbar$, where $\hbar = h/(2\pi)$.
- Therefore, $v_g = d(E/\hbar)/d(p/\hbar) = dE/dp$.
- Now, let's use the relativistic energy-momentum relation: $E^2 = (pc)^2 + (m_0c^2)^2$, where m_0 is the rest mass.
- Differentiating with respect to p: $2E(dE/dp) = 2pc^2$.
- So, $dE/dp = pc^2/E$.
- We know that $p = mv$ and $E = mc^2$ (where m is the relativistic mass).
- Substituting these into the expression for dE/dp: $v_g = (mvc^2)/(mc^2) = v$.
- Thus, the group velocity of the De-Broglie waves associated with a moving particle is equal to the particle velocity.

(c) Ultraviolet light of wavelength 3000 Å is falling on a surface whose work function is 2.28 eV. What is the maximum possible speed of the emitted electrons in m/s?

- Given:
  - Wavelength of ultraviolet light, $\lambda = 3000$ Å $= 3000 * 10^{-10}$ m $=$ $3 * 10^{-7}$ m.
  - Work function, $\phi = 2.28$ eV.

- Constants:
  - Planck's constant, $h = 6.626 * 10^{-34}$ J s.
  - Speed of light, $c = 3 * 10^8$ m/s.
  - Charge of an electron, $e = 1.602 * 10^{-19}$ C.
  - Mass of an electron, $m_e = 9.109 * 10^{-31}$ kg.
- First, convert the work function from eV to Joules: $\phi = 2.28$ eV $* (1.602 * 10^{-19}$ J/eV$) = 3.65256 * 10^{-19}$ J.
- Calculate the energy of the incident photon (E_{photon}): $E_{photon} = hc/\lambda$ $E_{photon} = (6.626 * 10^{-34}$ J s $* 3 * 10^8$ m/s$)/(3 * 10^{-7}$ m$)$ $E_{photon} = (1.9878 * 10^{-25})/(3 * 10^{-7})$ J $E_{photon} = 6.626 * 10^{-19}$ J.
- Using Einstein's photoelectric equation: $E_{photon} = \phi + K_{max}$ Where K_{max} is the maximum kinetic energy of the emitted electrons. $K_{max} = E_{photon} - \phi$ $K_{max} = (6.626 * 10^{-19}$ J$) - (3.65256 * 10^{-19}$ J$)$ $K_{max} = 2.97344 * 10^{-19}$ J.
- The maximum kinetic energy is also given by $K_{max} = (1/2)m_e v_{max}^2$, where v_{max} is the maximum speed of the emitted electrons. $v_{max}^2 = (2 * K_{max})/m_e$ $v_{max}^2 = (2 * 2.97344 * 10^{-19}$ J$)/(9.109 * 10^{-31}$ kg$)$ $v_{max}^2 = (5.94688 * 10^{-19})/(9.109 * 10^{-31})$ $v_{max}^2 = 0.65287 * 10^{12}$ m^2/s^2 $v_{max}^2 = 6.5287 * 10^{11}$ m^2/s^2.
- Now, calculate v_{max}: $v_{max} = \sqrt{6.5287 * 10^{11}\ \text{m}^2/\text{s}^2}$ $v_{max} = \sqrt{65.287 * 10^{10}\ \text{m}^2/\text{s}^2}$ $v_{max} \approx 8.079 * 10^5$ m/s.
- Therefore, the maximum possible speed of the emitted electrons is approximately $8.08 * 10^5$ m/s.

Question 3:

(a) Derive an expression for the diameter of the nth bright ring in Newton's rings apparatus.

- **Derivation for the diameter of the nth bright ring in Newton's rings apparatus:**
    - **Setup:** Newton's rings are formed by the interference of light reflected from the upper and lower surfaces of an air film of varying thickness, created between a plano-convex lens and a plane glass plate.
    - **Path Difference:** Consider a ray of light incident normally on the plano-convex lens. It undergoes reflection from the upper surface of the air film (lower surface of the lens) and the lower surface of the air film (upper surface of the glass plate).
    - The path difference between the two reflected rays is $2\mu t$, where μ is the refractive index of the medium in the film (for air, $\mu = 1$) and t is the thickness of the film at that point.
    - **Phase Change on Reflection:** A phase change of π (or 180°) occurs when light reflects from an optically denser medium. In this case, reflection from the lower surface of the lens (air to glass) does not involve a phase change, but reflection from the upper surface of the glass plate (air to glass) does involve a phase change of π.
    - Therefore, the effective path difference is $2\mu t + \lambda/2$.
    - **Condition for Bright Rings:** For bright rings (constructive interference), the effective path difference must be an integral multiple of the wavelength λ: $2\mu t + \lambda/2 = n\lambda$, where $n = 1,2,3,...$ (for bright rings starting from the first bright ring, not the central dark spot). $2\mu t = (n - 1/2)\lambda$ $2\mu t = (2n - 1)\lambda/2$ For air film, $\mu = 1$, so $2t = (2n - 1)\lambda/2$.
    - **Relation between thickness (t) and radius of curvature (R):** Let R be the radius of curvature of the plano-convex lens and r_n be the radius of the nth ring. From the geometry of the lens (using the property of a circle): $r_n^2 = (2R - t)t$ Since t is very small compared to R, we can approximate $(2R - t)$ as $2R$. So, $r_n^2 \approx 2Rt$. Therefore, $t = r_n^2/(2R)$.

- **Substituting t into the bright ring condition:** $2 * (r_n^2/(2R)) = (2n-1)\lambda/2 \; r_n^2/R = (2n-1)\lambda/2 \; r_n^2 = (2n-1)R\lambda/2 \; r_n = \sqrt{(2n-1)R\lambda/2}$.
- **Diameter of the nth bright ring (D_n):** $D_n = 2r_n = 2 * \sqrt{(2n-1)R\lambda/2} \; D_n = \sqrt{4 * (2n-1)R\lambda/2} \; D_n = \sqrt{2(2n-1)R\lambda}$.
- This expression gives the diameter of the nth bright ring. Note that for the first bright ring, $n = 1$, for the second bright ring, $n = 2$, and so on.

(b) Explain how the refractive index of a liquid can be determined by Newton's rings method.

- **Determination of Refractive Index of a Liquid by Newton's Rings Method:**
  - **Setup Modification:** The standard Newton's rings setup is modified. Instead of an air film between the plano-convex lens and the glass plate, a few drops of the transparent liquid whose refractive index (μ_L) is to be determined are introduced between them. This creates a liquid film.
  - **Observation with Air Film (Initial Measurement):**
    - First, conduct the experiment with an air film. Measure the diameters of a certain number of dark rings (say, D_m and D_k for the m^{th} and k^{th} dark rings, respectively).
    - The condition for dark rings for an air film is $2t = n\lambda$ (since the $\lambda/2$ phase change on reflection makes the total path difference for destructive interference $2t + \lambda/2 = (n + 1/2)\lambda$).
    - From the geometry, $t = r^2/(2R)$. So, $2r^2/(2R) = n\lambda$, which gives $r^2 = nR\lambda$.
    - For the k^{th} dark ring, $D_k^2 = (2r_k)^2 = 4kR\lambda$.

- For the m^{th} dark ring, $D_m^2 = (2r_m)^2 = 4mR\lambda$.
- The difference in squares of diameters: $D_m^2 - D_k^2 = 4(m - k)R\lambda$.
- From this, the wavelength of light (λ) can be determined if R is known, or $R\lambda$ can be determined.

- **Observation with Liquid Film (Second Measurement):**
  - Now, introduce the liquid between the lens and the glass plate. The new path difference will be $2\mu_L t$.
  - The condition for dark rings in the liquid film is $2\mu_L t = n\lambda$.
  - Using $t = r^2/(2R)$, we get $2\mu_L({r'}^2/(2R)) = n\lambda$, where r' is the radius of the rings in the liquid.
  - So, $\mu_L {r'}^2/R = n\lambda$, which implies ${r'}^2 = nR\lambda/\mu_L$.
  - For the k^{th} dark ring in liquid, $D_{k'}^2 = 4kR\lambda/\mu_L$.
  - For the m^{th} dark ring in liquid, $D_{m'}^2 = 4mR\lambda/\mu_L$.
  - The difference in squares of diameters for liquid: $D_{m'}^2 - D_{k'}^2 = 4(m - k)R\lambda/\mu_L$.
- **Calculating Refractive Index:** Let $\Delta D_{air}^2 = D_m^2 - D_k^2$ and $\Delta D_{liquid}^2 = D_{m'}^2 - D_{k'}^2$. We have: $\Delta D_{air}^2 = 4(m - k)R\lambda$ (Equation 1) $\Delta D_{liquid}^2 = 4(m - k)R\lambda/\mu_L$ (Equation 2) Dividing Equation 1 by Equation 2: $\Delta D_{air}^2/\Delta D_{liquid}^2 = (4(m - k)R\lambda)/(4(m - k)R\lambda/\mu_L) = \mu_L$. Therefore, $\mu_L = (D_m^2 - D_k^2)_{air}/(D_m^2 - D_k^2)_{liquid}$.
- By measuring the diameters of corresponding dark rings with air and then with the liquid, the refractive index of the liquid can be accurately determined.

(c) In a Newton's rings experiment, the diameter of 15th ring was found to be 0.59 cm and that of the 5th ring was 0.336 cm. If the radius of plano-convex lens is 100 cm, calculate the wavelength of the light used.

- Given:
  - Diameter of the 15th dark ring, $D_{15} = 0.59 \text{ cm} = 0.59 * 10^{-2}$ m.
  - Diameter of the 5th dark ring, $D_5 = 0.336 \text{ cm} = 0.336 * 10^{-2}$ m.
  - Radius of plano-convex lens, $R = 100 \text{ cm} = 1 \text{ m}$.
- The formula for the diameter of the nth dark ring in Newton's rings for an air film is $D_n^2 = 4nR\lambda$.
- For the 15th dark ring: $D_{15}^2 = 4 * 15 * R * \lambda = 60R\lambda$.
- For the 5th dark ring: $D_5^2 = 4 * 5 * R * \lambda = 20R\lambda$.
- Subtracting the two equations: $D_{15}^2 - D_5^2 = 60R\lambda - 20R\lambda = 40R\lambda$.
- Now, substitute the given values: $(0.59 * 10^{-2} \text{ m})^2 - (0.336 * 10^{-2} \text{ m})^2 = 40 * (1 \text{ m}) * \lambda$. $(0.0059)^2 - (0.00336)^2 = 40\lambda$. $0.00003481 - 0.0000112896 = 40\lambda$. $0.0000235204 = 40\lambda$.
- Solve for λ: $\lambda = 0.0000235204/40$ $\lambda = 5.8801 * 10^{-7}$ m.
- Convert to Angstroms (Å): $1 \text{ m} = 10^{10}$ Å. $\lambda = 5.8801 * 10^{-7} * 10^{10}$ Å $\lambda = 5880.1$ Å.
- Therefore, the wavelength of the light used is approximately 5880 Å.

Question 4:

(a) Derive an expression for the focal length of a zone plate.

- **Derivation of Focal Length of a Zone Plate:**
  - **Concept of Zone Plate:** A zone plate is a device that functions like a lens, focusing light by diffraction rather than refraction. It consists of

a series of concentric dark and transparent (or alternating phase-shifting) rings, called Fresnel zones.

- **Construction Principle:** For a point source of light S on the axis and a point P on the axis where light is focused, the transparent zones are constructed such that the path difference between a wave arriving at P from the edge of the m^{th} transparent zone and a wave arriving directly at P from the center of the zone plate is an integral multiple of $\lambda/2$. For constructive interference, this path difference should be an even multiple of $\lambda/2$.
- Let r_m be the radius of the m^{th} Fresnel zone (i.e., the radius of the outer edge of the m^{th} zone).
- Let a be the distance of the point source from the zone plate, and b be the distance of the focal point P from the zone plate.
- From the geometry, for the m^{th} transparent zone, the path length from the source to the edge of the zone and then to the focal point is $Sr_mP = \sqrt{a^2 + r_m^2} + \sqrt{b^2 + r_m^2}$.
- The path length from the source to the center and then to the focal point is $SOP = a + b$.
- The condition for constructive interference at point P due to the m^{th} zone (assuming it's a transparent zone designed for constructive interference) is that the path difference from the center should be $m(\lambda/2)$ or $(m + \delta)(\lambda/2)$, where δ depends on whether it's the m^{th} transparent or opaque zone boundary.
- Let's consider the general condition for the radius of the m^{th} Fresnel zone based on the condition that the wave from the edge of the m^{th} zone is $\lambda/2$ out of phase (or in phase, depending on construction) with the wave from the previous zone boundary.
- A simpler approach for derivation of focal length is to consider the condition that the radius r_m of the m^{th} zone is such that the path

difference from the edges of the m^{th} transparent zone to the focal point f (for plane incident wave, $a = \infty$) is $m(\lambda/2)$ relative to the path from the center.

- So, $\sqrt{f^2 + r_m^2} - f = m\lambda/2$.
- $\sqrt{f^2 + r_m^2} = f + m\lambda/2$.
- Squaring both sides: $f^2 + r_m^2 = f^2 + (m\lambda/2)^2 + 2f(m\lambda/2)$.
- $f^2 + r_m^2 = f^2 + m^2\lambda^2/4 + fm\lambda$.
- Since r_m is typically much smaller than f, and $m^2\lambda^2/4$ is much smaller than $fm\lambda$, we can neglect the $m^2\lambda^2/4$ term.
- $r_m^2 \approx fm\lambda$.
- Therefore, $f = r_m^2/(m\lambda)$.
- This expression indicates that a zone plate has multiple foci. For a given zone plate, r_m is fixed for a particular zone, and m represents the zone number.
- The primary focal length (for $m = 1$) is $f_1 = r_1^2/\lambda$.
- In general, for various values of m, the focal length can be expressed as: $f_m = r_m^2/(m\lambda)$.
- It's important to note that a zone plate produces multiple focal points, both real and virtual, for odd and even values of m respectively, due to the nature of diffraction. The brightest focal point corresponds to $m = 1$.

(b) Explain multiple focii of a zone plate.

- **Multiple Foci of a Zone Plate:**
  - Unlike a conventional lens which has a single focal length for monochromatic light, a zone plate exhibits multiple focal points, both real and virtual, for a given wavelength.

- **Mechanism:** This arises from the way a zone plate functions. It doesn't converge light by refraction but rather by constructive interference of diffracted light. The condition for constructive interference at a point (a focal point) is that the path difference from different zones to that point must be an integral multiple of the wavelength λ.
- **Formula:** The focal length f_m for the m^{th} order of diffraction is given by $f_m = r_m^2/(m\lambda)$, where r_m is the radius of the m^{th} zone boundary and λ is the wavelength of light.
- **Real Foci (Positive Foci):** When light is converged to a real point on the other side of the zone plate, the path difference contribution from consecutive transparent zones to this point is $m\lambda$. These are the "positive" or real foci.
  - The primary (brightest) real focus occurs for $m = 1$, so $f_1 = r_1^2/\lambda$.
  - Other real foci occur for $m = 3,5,7, \ldots$ (odd integers).
  - Thus, $f_3 = f_1/3$, $f_5 = f_1/5$, and so on. These higher-order foci are successively weaker in intensity because less light energy contributes to them.
- **Virtual Foci (Negative Foci):** A zone plate also produces virtual focal points on the same side as the incident light (for a plane wave). This happens when the light diffracts outwards, appearing to originate from a virtual point. These are the "negative" or virtual foci.
  - These correspond to $m = -1, -3, -5, \ldots$ (odd negative integers).
  - For example, $f_{-1} = -r_1^2/\lambda$.
- **Absence of Even Orders:** For a typical zone plate designed with alternating opaque and transparent zones, the even orders of diffraction ($m = 2,4, \ldots$) are often missing or very weak. This is

because the construction of the zones, usually with equal areas, tends to cancel out these even orders. However, if the opaque and transparent zones do not have exactly equal areas, or if the phase relationship is different, some even orders might be present.

- **Chromatic Aberration:** Since the focal length ($f_m = r_m^2/(m\lambda)$) depends on the wavelength λ, a zone plate suffers from severe chromatic aberration. Different colors (wavelengths) will focus at different points, making it unsuitable for imaging with white light unless monochromatic light is used.
- In summary, the multiple foci arise because the zone plate is a diffraction device, and constructive interference can occur at various distances from the plate, corresponding to different integer multiples of the fundamental path difference.

(c) The diameter of the first ring of a zone plate is 1.2 mm. If a plane wave of wavelength 6000Å is incident on the plate normally, where should the screen be placed so that the light is focused to the brightest point?

- Given:
  - Diameter of the first ring, $D_1 = 1.2 \text{ mm} = 1.2 * 10^{-3}$ m.
  - Radius of the first ring, $r_1 = D_1/2 = (1.2 * 10^{-3} \text{ m})/2 = 0.6 * 10^{-3}$ m.
  - Wavelength of plane wave, $\lambda = 6000 \text{ Å} = 6000 * 10^{-10} \text{ m} = 6 * 10^{-7}$ m.
- We need to find the position of the screen for the brightest point, which corresponds to the primary focal length (f_1).
- The formula for the focal length of a zone plate is $f_m = r_m^2/(m\lambda)$.
- For the brightest point, we consider the primary focal length, which corresponds to $m = 1$ and the radius of the first transparent zone r_1.
- $f_1 = r_1^2/(1 * \lambda)$

- $f_1 = (0.6 * 10^{-3}\ \text{m})^2/(6 * 10^{-7}\ \text{m})$
- $f_1 = (0.36 * 10^{-6}\ \text{m}^2)/(6 * 10^{-7}\ \text{m})$
- $f_1 = (0.36/6) * 10^{(-6-(-7))}\ \text{m}$
- $f_1 = 0.06 * 10^1\ \text{m}$
- $f_1 = 0.6\ \text{m}.$
- Therefore, the screen should be placed at a distance of 0.6 m from the zone plate for the light to be focused to the brightest point.

Question 5:

(a) Derive the intensity distribution formula for Fraunhoffer diffraction by a grating of N slits given below: $I = I_0(sin^2\alpha sin^2(N\gamma))/(\alpha^2 sin^2\gamma)$ Where $\alpha = (\pi a sin\theta)/\lambda$ And $\gamma = (\pi d sin\theta)/\lambda$. Here the grating element $d = a + b$ and other symbols have their usual meaning.

- **Derivation of Intensity Distribution for Fraunhoffer Diffraction by N Slits (Grating):**
  - **Assumptions:**
    - Plane wave incident normally on the grating.
    - All slits are identical, having width 'a' and separated by opaque spaces of width 'b'.
    - The grating element (period) is $d = a + b$.
    - A converging lens is placed after the grating to focus the diffracted light onto a screen.
  - **Step 1: Diffraction due to a single slit:**
    - Consider Fraunhoffer diffraction from a single slit of width 'a'. The amplitude of the diffracted light in a direction θ (angle with the normal to the grating) is proportional to $(\sin\alpha)/\alpha$, where $\alpha = (\pi a \sin\theta)/\lambda$.

- The intensity due to a single slit is $I_{single} = I_0(\sin^2\alpha/\alpha^2)$, where I_0 is the maximum intensity at $\theta = 0$.

- **Step 2: Interference due to N slits (phasor method):**
  - Each slit acts as a source of diffracted light. We now consider the interference of these N diffracted waves.
  - The phase difference between the waves coming from corresponding points of two consecutive slits, diffracted at an angle θ, is $\phi = (2\pi/\lambda) *$ path difference.
  - The path difference between rays from consecutive slits is $d\sin\theta$.
  - So, the phase difference is $\phi = (2\pi/\lambda) * d\sin\theta = 2\gamma$, where $\gamma = (\pi d\sin\theta)/\lambda$.
  - We can use the phasor method to find the resultant amplitude (A_R) of N waves, each having an amplitude $A_1 = A_0(\sin\alpha/\alpha)$ (amplitude from single slit) and a constant phase difference 2γ with the preceding wave.
  - The resultant amplitude for N waves with a common amplitude A_1 and progressive phase difference ϕ is given by: $A_R = A_1 * (\sin(N\phi/2)/\sin(\phi/2))$ Substituting $\phi = 2\gamma$: $A_R = A_1 * (\sin(N\gamma)/\sin(\gamma))$.
  - Now, substitute $A_1 = A_0(\sin\alpha/\alpha)$: $A_R = A_0(\sin\alpha/\alpha) * (\sin(N\gamma)/\sin(\gamma))$.

- **Step 3: Resultant Intensity:**
  - The resultant intensity I is proportional to the square of the resultant amplitude, $I = kA_R^2$.
  - $I = k[A_0(\sin\alpha/\alpha) * (\sin(N\gamma)/\sin(\gamma))]^2$
  - Let $I_0 = kA_0^2$ (maximum intensity).

- Therefore, the intensity distribution formula for Fraunhoffer diffraction by a grating of N slits is: $I = I_0 * (\sin^2\alpha/\alpha^2) * (\sin^2(N\gamma)/\sin^2\gamma)$. Or, as given in the question: $I = I_0(sin^2\alpha sin^2(N\gamma))/(\alpha^2 sin^2\gamma)$.

- **Interpretation of Terms:**
  - The term $(\sin^2\alpha/\alpha^2)$ represents the single-slit diffraction pattern (envelope). It determines the intensity distribution for a single slit.
  - The term $(\sin^2(N\gamma)/\sin^2\gamma)$ represents the interference pattern produced by N coherent sources. It gives the principal maxima and minima due to interference.
  - The observed grating pattern is a combination of these two effects.

(b) How many orders will be visible if the light of wavelength 5000 Å is normally incident on a grating having 2620 lines per inch.(1 inch = 2.54 cm).

- Given:
  - Wavelength, $\lambda = 5000$ Å $= 5000 * 10^{-10}$ m $= 5 * 10^{-7}$ m.
  - Number of lines per inch = 2620 lines/inch.
  - 1 inch = 2.54 cm = 0.0254 m.
- First, calculate the grating element 'd' (distance between two consecutive lines). $d = 1$ inch$/2620$ lines $d = 0.0254$ m$/2620 = 9.6946 * 10^{-6}$ m.
- For a diffraction grating, the condition for principal maxima (bright fringes) is $d\sin\theta = n\lambda$, where n is the order of diffraction (an integer: 0, 1, 2, ...).
- The maximum possible value of $\sin\theta$ is 1 (when $\theta = 90°$).
- So, $d * 1 \geq n\lambda$.
- Therefore, the maximum possible order n_{max} is given by $n_{max} \leq d/\lambda$.

- $n_{max} = d/\lambda\, n_{max} = (9.6946 * 10^{-6}$ m$)/(5 * 10^{-7}$ m$)\, n_{max} = 9.6946/5 * 10^1\, n_{max} = 1.93892 * 10^1 = 19.3892$.
- Since n must be an integer, the maximum visible order will be $n = 19$.
- The orders visible are $n = 0$ (central maximum), and $n = \pm1, \pm2, \ldots, \pm19$.
- Total number of visible orders = 2 * (maximum order) + 1 (for the central maximum).
- Total visible orders = $2 * 19 + 1 = 38 + 1 = 39$.
- So, there will be 19 orders on each side of the central maximum, plus the central maximum itself.
- Therefore, 39 orders will be visible.

(c) Discuss the concept of missing orders. What orders will be missing if b = a.

- **Concept of Missing Orders:**
  - In a diffraction grating, the observed intensity pattern is a product of two factors: the diffraction pattern of a single slit and the interference pattern of N slits.
  - $I = I_0 * (\sin^2\alpha/\alpha^2) * (\sin^2(N\gamma)/\sin^2\gamma)$.
  - The principal maxima of the interference term occur when $\sin\gamma = 0$, which means $\gamma = n\pi$, where $n = 0, \pm1, \pm2, \ldots$.
    - Substituting $\gamma = (\pi d\sin\theta)/\lambda$, we get $(\pi d\sin\theta)/\lambda = n\pi$, so $d\sin\theta = n\lambda$. This is the grating equation.
  - The minima of the single-slit diffraction term (the envelope) occur when $\sin\alpha = 0$, which means $\alpha = m\pi$, where $m = \pm1, \pm2, \ldots$.
    - Substituting $\alpha = (\pi a\sin\theta)/\lambda$, we get $(\pi a\sin\theta)/\lambda = m\pi$, so $a\sin\theta = m\lambda$.

- **Missing Orders occur when a principal maximum of the interference pattern coincides with a minimum of the single-slit diffraction pattern at the same angle θ.**
- If $d\sin\theta = n\lambda$ and $a\sin\theta = m\lambda$ simultaneously for the same θ, then dividing the two equations: $(d\sin\theta)/(a\sin\theta) = (n\lambda)/(m\lambda)$ $d/a = n/m$. So, if $n = (d/a) * m$, then the n^{th} order principal maximum will be missing from the overall diffraction pattern.
- This implies that certain orders predicted by the interference condition ($d\sin\theta = n\lambda$) will not be observed because the intensity at those angles is suppressed to zero by the diffraction envelope ($a\sin\theta = m\lambda$).

- **What orders will be missing if b = a:**
  - If $b = a$, then the grating element $d = a + b = a + a = 2a$.
  - Now, apply the condition for missing orders: $d/a = n/m$.
  - Substitute $d = 2a$: $(2a)/a = n/m$.
  - $2 = n/m$.
  - This means $n = 2m$.
  - So, any principal maximum whose order n is an even multiple of m (where m represents the minimum of the single-slit pattern) will be missing.
  - For $m = 1$, $n = 2 * 1 = 2$. (The 2nd order will be missing).
  - For $m = 2$, $n = 2 * 2 = 4$. (The 4th order will be missing).
  - For $m = 3$, $n = 2 * 3 = 6$. (The 6th order will be missing).
  - And so on.
  - Therefore, if $b = a$, all even orders ($n = \pm 2, \pm 4, \pm 6, ...$) will be missing from the diffraction pattern.