[This question paper contains 8 printed pages]

Your Roll No. : .....

Sl. No. of Q. Paper : **1230** **I**

Unique Paper Code : 2353012001

Name of the Paper : Graph Theory-DSE

Name of the Course : **B.Sc.(H) Mathematics**

Semester : V

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions has **three** parts **(a)**, **(b)** and **(c)**. You have to attempt any **two** parts of each question.

(c) **All** questions carry equal marks.

(d) Parts of each question to be attempted together.

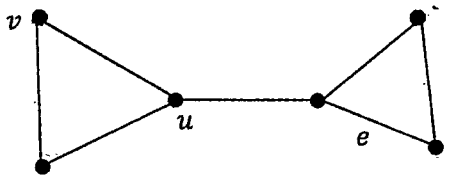
(e) Use of Calculator **not** allowed.

1. (a) (i) Draw a graph with 6 vertices and as many edges as possible. How many edges does your graph contain. What is the name of this graph and how is it denoted ? 4.5

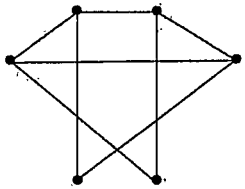
(ii) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 6. Justify your answer. 3

P.T.O.

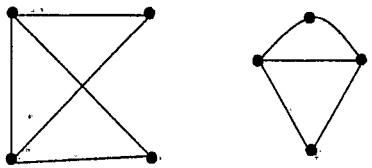
(b) (i) Define sub-graph of a graph. Draw pictures of the sub-graphs G\{e}, G\{v} and G\{u} of the following graph G : 4



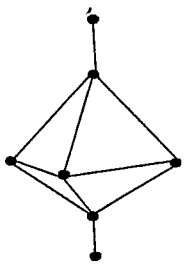
(ii) What is bipartite graph ? Determine whether graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite. 3.5



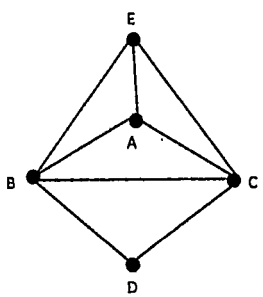
(c) (i) Define the term Isomorphic graphs. For the below pair of graphs, either label the graphs so as to exhibit an isomorphism or explain why graphs are not isomorphic. 4



(ii) Solve the Chinese Postman Problem for the graph below (All edges are of equal weight) : 3.5



2. (a) Define Eulerian graph and Hamiltonian graph. Consider the following Graph : 7.5



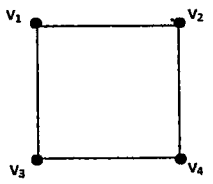
(i) Is it Hamiltonian ?

(ii) Is there a Hamiltonian Path ?

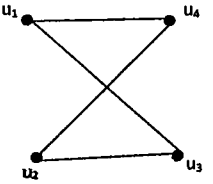
(iii) Is it Eulerian ?

(iv) Is there an Eulerian trail ? Explain your answer.

(b) Define adjacency matrix of a graph. Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$. 7.5

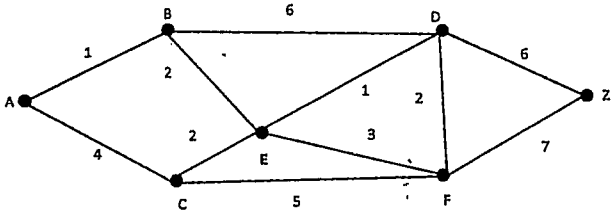


G_1

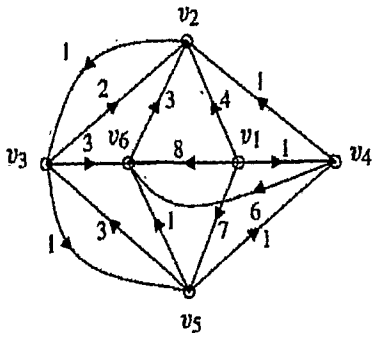


G_2

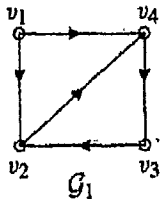
(c) Apply the improved version of Dijkstra's Algorithm to find a shortest path from A to Z in the graph shown below. Label all vertices and write steps. 7.5

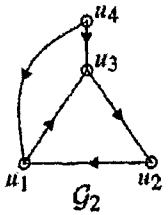


3. (a) Apply the Bellman-Ford algorithm to find the shortest distance from v_1 to all other vertices in the following Weighted Digraph : 7.5

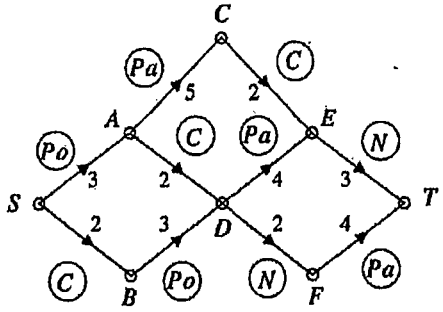


(b) When are two Digraphs said to be isomorphic ? Are the following Digraphs G_1 and G_2 isomorphic ? Explain your answer. 7.5





(c) The construction of a fence involves four tasks : setting posts (Po), cutting wood (C), painting (Pa) and nailing (W). Setting posts must precede painting and nailing, and cutting must precede nailing. Suppose that setting posts takes 3 units of time, cutting wood takes 2 units of time, painting takes 5 units of time for uncut wood and 4 units of time otherwise, and nailing takes 2 units of time for unpainted wood and 3 units of time otherwise.



(i) What type of scheduling problem is this (type **I** or **II**) ? Why ? 2

(ii) What is the shortest time required for this job ? Describe the critical path. 5.5

4. (a) (i) Define spanning tree and find the number of spanning trees in K_4. 3

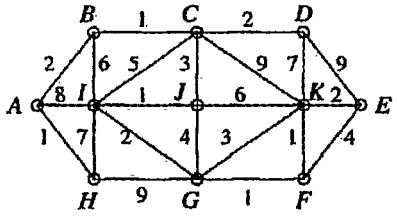
(ii) Let G be a graph. Then the following statements are equivalent. 4.5

(1) G is a tree.

(2) G is connected and acyclic, that is, without cycles.

(3) Between any two vertices of G there is precisely one path.

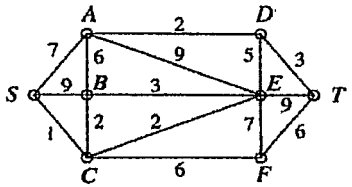
(b) Find a minimum spanning tree for the graph using Prim's algorithm. Give the weight of your minimum tree and show your steps. 7.5



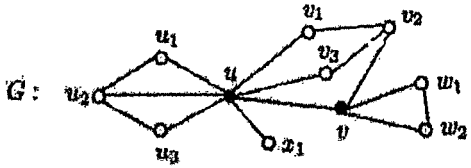
(c) (i) Find a Hamiltonian cycle of lowest weight. What is the weight ? 2.5

(ii) By removing vertex A, find a lower bound for the weight of any Hamiltonian cycle. 2.5

(iii) By removing vertex B, find a lower bound for the weight of any Hamiltonian cycle. 2.5



5. (a) (i) Define a block of graph G and determine all the blocks of the graph G given below. 4.5

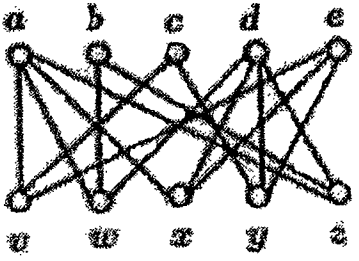


(ii) Check whether the graph K_7 is non-separable or not. Explain. 3

(b) Let G be a nontrivial connected graph and let $u \in V(G)$. If v is a vertex that is farthest from u in G, then show that v is not a cut-vertex of G. Also show that every connected graph contains at least two vertices that are not cut-vertices. 7.5

(c) Let $V \geq 3$ for a planar graph G with E edges, then show that $E \leq 3V = 6$. Is K_5 a Planar graph, explain. 7.5

6. (a) Define Matching in a graph G, and for a graph G with partite sets U = {v, w, x, y, z} and W = {a, b, c, d, e}, Can U matched to W ? 7.5



(b) (i) Find the chromatic number of K_n, $K_{m,n}$ with explanations. 4.5

(ii) Suppose T is a tree with n-vertices, then what is the chromatic number of T and why. 3

(c) Write the definition of maximum matching & perfect matching and prove that every tree has at most one perfect matching. 7.5

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