that the policy pays 10 units.

For all the parts assume that the insurer will use an annual effective interest rate of i = 0.06.

(b) Express the variance of the loss L associated with n-year endowment insurance in terms of Actuarial present values for fully continuous premiums.

(7,8)



[This question paper contains 12 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 1257** **I**

Unique Paper Code : 2373010005

Name of the Paper : Actuarial Statistics (DSE-3(A))

Name of the Course : **B.Sc. (H) Statistics under CBCS**

Semester : V (NEP-UGCF)

Duration : 3 Hours Maximum Marks : 90

**<u>Instructions for Candidates</u>**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **three** from each section.
3. **All** questions carry equal marks.
4. Use of simple calculator is allowed.

**Section I**

1. (a) (i) Explain what you mean by 'Proportional Reinsurance'. Suppose that an insurer sells this type of reinsurance for a claim amount X. Clearly explain the insurer's and reinsurer's position. What happens in case if $X \sim Gamma(\alpha, \lambda)$? Instead, if $X \sim Lognormal(\mu, \sigma^2)$ how does your answer change?

(ii) Suppose an insurer enters into Excess of Loss Reinsurance Arrangement with retention limit M. Obtain the c.d.f. of the random variable W denoting the amount of non-zero payments made by the reinsurer. For the continuous case, obtain the corresponding p.d.f. Suppose that the insurer incurs claims that follow an Exponential distribution with parameter $\lambda$. Obtain and identify the p.d.f. of the non-zero payments made by the reinsurer. Comment



The policy will pay 1 unit at the end of the year of death in exchange for the payment of a premium P at the beginning of each year, provided the life survives. Find the annual premium, P, as determined by :

(i) *Principle I* : P will be the least annual premium such that the insurer has a probability of positive financial loss of at most 0.25.

(ii) *Principle II* : P will be the annual premium such that the insurer, using a utility of wealth function, $u(x) = x - 0.01x^2$, $x < 50$, with initial wealth W=10 will be able to maximize his expected utility.

(iii) *Principle III* : P will be the annual premium such that the insurer, using a utility of wealth function, $u(x) = -e^{-0.1x}$, will be indifferent between accepting and not accepting the risk. For this case assume

7. (a) A unit is used to purchase a combination benefit consisting of a life income of I per year payable continuously while (x) survives and an insurance of J payable immediately on death of (x). Write the present value random variable for this combination and give its mean and variance.

(b) Prove and interpret the given relations :

(i) $a_{x:\overline{n}|} = {}_1E_x \ddot{a}_{x+1:\overline{n}|}$

(ii) ${}_{n|}a_x = \frac{A_{x:\overline{n}|} - A_x}{d} - {}_nE_x$

where notations have their usual meaning.

(8,7)

8. (a) An insurer is planning to issue a policy to a life age 0 whose curtate future lifetime, K, is governed by the probability function

$${}_{k|}q_0 = 0.2,\ k = 0, 1, 2, 3, 4.$$



on your answer.

(b) Let the random variable X have distribution function F given by

$$F(x) = \begin{cases} 0 & \text{for } x < 20 \\ \frac{x+20}{80} & \text{for } 20 \le x < 40 \\ 1 & \text{for } x \ge 40 \end{cases}$$

Calculate

(i) $P(X \le 30)$

(ii) $P(X = 40)$

(iii) $E[X]$

(iv) $V[X]$. (8,7)

2. (a) What is a Utility Function? Which are the

properties that it must satisfy? How is it linked to the coefficient of risk aversion? Explain in detail the quadratic utility function and obtain the coefficient of risk aversion for this function. An insurer whose wealth is W has a choice between investments 1 and 2, which will result in wealth of $W + X_1$ and $W + X_2$ respectively, offering complete insurance cover against a random loss X such that $E[X_1] = 10$, $V[X_1] = 2$ and $E[X_2] = 10.1$. The individual makes decisions on the basis of a quadratic utility function with parameter $\beta = 0.002$. For what range of values for $V[X_2]$ will the individual choose investment 1, when $W = 200$? Assume that $P(W + X_i < 250) = 1$ for $i = 1, 2$.

(b) State and prove the Jensen's Inequality in context of utility functions. Show using Jensen's Inequality that the minimum acceptable premium II that an insurer whose wealth is W and utility function is v would charge for providing complete insurance



(ii) $F_X(x)$

(iii) $f_X(x)$

(iv) $P(10 < X < 40)$ (7,8)

6. (a) The random variable Z is the present-value random variable for an n-year endowment insurance of a unit amount issued to a life aged (x). If $\delta = 0.05$, $\mu_x(t) = 0.01$ and $n = 20$:

(i) Display the formula for the c.d.f. of Z.

(ii) Graph the c.d.f. of Z.

(iii) Calculate E[Z].

(b) Assume mortality is described by $l_x = 100 - x$ for $0 \le x \le 100$ and $i = 0.05$.

(i) Calculate A40125⌉.

(ii) $(IA)_{40}$ (9,6)

mean $E[X_j]$ would be $(1+\theta)E[X_j]$. The 99th percentile requirement suggests that $\theta>0$. This extra amount $\theta E[X_j]$ is the security loading and $\theta$ is the relative security loading which is same for both the classes. Calculate $\theta$. (7,8)

## Section II

5. (a) Give a detailed explanation of the following terms :

    (i) Ultimate Life Table

    (ii) Select Life Table

   (b) If $s(x)=1-\frac{x}{100}$, $0\le x\le 100$, evaluate the following :

    (i) $\mu(x)$



protection against a random loss X satisfies the inequality $\Pi \ge E[X]$. (7,8)

3. (a) Suppose that an insurance company charges a premium $\Pi_X$ to cover a risk X. List any three principles based on the moments of X that the insurer can use to calculate the premium. For any two of them, check which properties of premium principles are satisfied by these principles? Give detailed working to prove or disprove them.

   (b) With reference to X defined above, calculate premiums using risk adjusted premium principle with risk index $\rho$ if

    (i) X~Exponetial $\lambda$

    (ii) X~Pareto($\alpha$, $\lambda$)

   In each case give detailed working. (8,7)

4. (a) What do you mean by 'Individual risk model'? Clearly explain the connected notations. Consider a portfolio of 32 policies. For each policy, the probability q of a claim is 1/6 and B, the benefit amount given that there is a claim, has a p.d.f.

$$f(y) = \begin{cases} 2(1-y) & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Let S be the total claims for the portfolio. Using a normal approximation, estimate P(S > 4). Explain the underlying random variables.

(b) The policyholders of an automobile insurance company fall into two classes. The claim amount $B_k$ follows a truncated exponential distribution with parameters given below :



| Class k | Number in class $n_k$ | Claim Probability $q_k$ | Parameters β | L |
|---|---|---|---|---|
| 1 | 500 | 0.10 | 1 | 2.5 |
| 2 | 2000 | 0.05 | 2 | 5.0 |

A truncated exponential distribution is defined by the c.d.f.

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\beta x} & 0 \le x < L \\ 1 & x \ge L \end{cases}$$

sketch this c.d.f. The company wants to collect from a population of 2500 individuals, an amount equal to 99 percentile of the distribution of total claims. Moreover, it wants each individual's share of this amount to be proportional to the individual's expected claim. The share for individual j with