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[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1394 I

Unique Paper Code : 2342011103

Name of the Paper : Mathematics for Computing

Name of the Course : B.Sc (H) Computer Science

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. **Question No. 1** is compulsory.

3. Attempt any four of Question nos. **2 to 7**

4. Parts of a Question must be answered together.

1. (a) Find the dot product, cross product and angle between the vector $\vec{a} = \hat{\imath} + \ 5\ \ 4\hat{\jmath} - 2\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + 3\hat{k}$ (5)

P.T.O.

(b) Is $Q(x) = 6x_1^2 + 3x_2^2 + x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$ is positive definite ?

(5)

(c) Define bases of vector space. Check whether A={[1,0,0], [0,1,1],[1,1,1]} is a bases of vector space $\mathbb{R}^3$ or not? (5)

(d) Find Rank of the following matrix using reduced row echelon form

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$ (5)

(e) Determine whether $f: \mathbb{R}^3 \to \mathbb{R}^3$ such that $f([x_1, x_2, x_3]) = [x_2, x_3, x_1]$ is linear transformation or not. (5)

(f) Find the directional derivative of $F(x, y, z) = 4x^2 + y^2 + 3z^2$ at P(3,2,4) in the direction $5\hat{\imath} + 6\hat{k}$. (5)

2 (a) The set $\mathbb{R}^2$ defined with the addition operation $[x, y] \oplus [w, z] = [x + w - 2, y + z + 3]$ and scalar multiplication

$$a \odot [x, y] = [ax - 2a + 2, ay + 3a - 3].$$

Show that $\mathbb{R}^2$ is a vector space over addition and scalar multiplication. (8)

(b) Define inner product space. Consider a real vector space $\mathbb{R}^2$, which is defined as $<x, y> = x_1 y_1 - x_1 y_2 - x_2 y_1 + 2x_2 y_2$. Show that it is inner product space. (7)

3. (a) Solve using the Gauss Jordan Method

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 16 \\ 3x_1 + 2x_2 + x_4 &= 16 \\ 2x_1 + 12x_2 - 5x_4 &= 5 \end{aligned}$$

(8)

(b) Find the bases of row space and null space of the following matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

(7)

4. (a) Solve the following set of equations using Gauss Elimination method.

$$\begin{aligned} 5x - 5y - 15z &= 40 \\ 4x - 2y - 6z &= 19 \\ 3x - 6y - 17z &= 41 \end{aligned}$$

(8)

(b) Find value(s) of λ for which following system of equations is consistent.

$$\begin{aligned} 2x + 3y &= 4 \\ x + y + z &= 4 \\ x + 2y - z &= \lambda \end{aligned}$$

(7)

5. (a) Diagonalize the following matrix (8)

$$\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

(b) Define Cayley-Hamilton theorem and verify it for the following matrix

$$\begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}$$

(7)

6. (a) Apply Gram Schmidt orthonormalization process to obtain an orthonormal bases for given bases of $\mathbb{R}^3 : \{[1,0,-1],[-1,4,-1],[2,1,2]\}$ (8)

(b) Find inverse of the following matrix using row echelon form.

$$\begin{bmatrix} 2 & -6 & 5 \\ -4 & 12 & -9 \\ 2 & -9 & 8 \end{bmatrix}$$ (7)

7. (a) Calculate grad(div(curl $\vec{F}$)) of the following vector field

$$\vec{F} = x^3y^3z\hat{\imath} + x^2y^3z^4\hat{\jmath} + xyz\hat{k}$$ (8)

(b) A weather model uses a Markov chain to predict daily weather based on the states Sunny

(S), Rainy (R) and Cloudy (C) with transition matrix (1+3+3)

$$\begin{array}{c} S \\ R \\ C \end{array}\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$
$$\begin{array}{ccc} S & R & C \end{array}$$

(i) If today is **Sunny**, what is the probability that it will be **Cloudy** tomorrow?

If today is **Rainy**, what is the probability that it will be **Sunny** after two days?

(iii) If the initial state vector is:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

What is the state probability vector after 2 days?