- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

SYLLABUS OF DSC-17

UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)
The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)
Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

UNIT-III: Class of Operators and Their Properties (15 hours)
Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

Essential Reading

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

Suggestive Readings

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 18: COMPLEX ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Complex Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2 & 11: Real Analysis, Multivariate Calculus

Learning Objectives: The main objective of this course is to:

- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant practicals.

- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

Learning Outcomes: The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

SYLLABUS OF DSC-18

UNIT – I: Analytic and Elementary Functions (15 hours)

Functions of a complex variable and mappings, Limits, Theorems on limits, Limits involving the point at infinity, Continuity and differentiation, Cauchy-Riemann equations and examples, Sufficient conditions for differentiability, Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

UNIT – II: Complex Integration (15 hours)

Derivatives of functions, Definite integrals of functions; Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals; Antiderivatives; Cauchy-Goursat theorem; Cauchy integral formula and its extension with consequences; Liouville's theorem and the fundamental theorem of algebra.

UNIT – III: Series and Residues (15 hours)

Taylor and Laurent series with examples; Absolute and uniform convergence of power series, Integration, differentiation and uniqueness of power series; Isolated singular points, Residues, Cauchy's residue theorem, Residue at infinity; Types of isolated singular points, Residues at poles and its examples, An application to evaluate definite integrals involving sines and cosines.

Essential Reading

1. Brown, James Ward, & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.
- Zills, Dennis G., & Shanahan, Patrick D. (2003). A First Course in Complex Analysis with Applications. Jones & Bartlett Publishers.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following similar problems using SageMath/Python/Mathematica/Maple/ MATLAB/Maxima/ Scilab etc.

1. Make a geometric plot to show that the nth roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{z : |z| = 1\}$ and form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.
2. Find all the solutions of the equation $z^3 = 8i$ and represent these geometrically.
3. Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units.
   Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.
4. Show that the image of the open disk $D_1(-1-i) = \{z : |z+1+i| < 1\}$ under the linear transformation $w = f(z) = (3-4i)z + 6 + 2i$ is the open disk:
   $$D_5(-1+3i) = \{w: |w+1-3i| < 5\}.$$
5. Show that the image of the right half-plane Re $z = x > 1$ under the linear transformation $w = (-1+i)z - 2 + 3i$ is the half-plane $v > u + 7$, where u = Re(w), etc. Plot the map.
6. Show that the image of the right half-plane A = {z: Re $z \geq \frac{1}{2}$} under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $\overline{D_1(1)} = \{w: |w-1| \leq 1\}$ in the w- plane.
7. Make a plot of the vertical lines $x = a$, for $a = -1, -\frac{1}{2}, \frac{1}{2}, 1$ and the horizontal lines $y = b$, for $b = -1, -\frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $f(z) = \frac{1}{z}$.
8. Find a parametrization of the polygonal path $C = C_1 + C_2 + C_3$ from $-1+i$ to $3-i$, where C_1 is the line from: $-1+i$ to -1, C_2 is the line from: -1 to $1+i$ and C_3 is the line from $1+i$ to $3-i$. Make a plot of this path.
9. Plot the line segment 'L' joining the point $A = 0$ to $B = 2 + \frac{\pi}{4}i$ and give an exact calculation of $\int_L e^z dz$.
10. Evaluate $\int_C \frac{1}{z-2} dz$, where C is the upper semicircle with radius 1 centered at $z = 2$ oriented in a positive direction.
11. Show that $\int_{C_1} z dz = \int_{C_2} z dz = 4 + 2i$, where C_1 is the line segment from $-1-i$ to $3+i$ and C_2 is the portion of the parabola $x = y^2 + 2y$ joining $-1-i$ to $3+i$.
    Make plots of two contours C_1 and C_2 joining $-1-i$ to $3+i$.
12. Use the ML inequality to show that $\left|\int_C \frac{1}{z^2+1} dz\right| \leq \frac{1}{2\sqrt{5}}$, where C is the straight-line segment from 2 to $2+i$. While solving, represent the distance from the point z to the points i and $-i$, respectively, i.e., $|z-i|$ and $|z+i|$ on the complex plane $\mathbb{C}$.
13. Find and plot three different Laurent series representations for the function:
    $f(z) = \frac{3}{2+z-z^2}$, involving powers of z.
14. Locate the poles of $f(z) = \frac{1}{5z^4+26z^2+5}$ and specify their order.
15. Locate the zeros and poles of $g(z) = \frac{\pi \cot(\pi z)}{z^2}$ and determine their order. Also justify that Res (g, 0) = $-\pi^2/3$.

16. Evaluate $\int_{C_1^+(0)} \exp\left(\frac{2}{z}\right) dz$, where $C_1^+(0)$ denotes the circle $\{z: |z| = 1\}$ with positive orientation. Similarly evaluate $\int_{C_1^+(0)} \frac{1}{z^4+z^3-2z^2} dz$.

B.Sc. (Hons) Mathematics, Semester-VI, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(i): MATHEMATICAL FINANCE

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Finance	4	3	0	1	Class XII pass with Mathematics	DSC-3, 11, & 15: Probability and Statistics, Multivariate Calculus, & PDE's

Learning Objectives: The main objective of this course is to:
- Introduce the application of mathematics in the financial world.
- Understand some computational and quantitative techniques required for working in the financial markets and actuarial sciences.

Learning Outcomes: The course will enable the students to:
- Know the basics of financial markets and derivatives including options and futures.
- Learn about pricing and hedging of options.
- Learn the Itô's formula and the Black–Scholes model.
- Understand the concepts of trading strategies.

SYLLABUS OF DSE-4(i)

Unit - I: Interest Rates, Bonds and Derivatives (15 hours)
Interest rates, Types of rates, Measuring interest rates, Zero rates, Bond pricing, Forward rates, Duration, Convexity, Exchange-traded markets and Over-the-counter markets, Derivatives, Forward contracts, Futures contracts, Options, Types of traders, Hedging, Speculation, Arbitrage, No Arbitrage principle, Short selling, Forward price for an investment asset.

Unit - II: Properties of Options and the Binomial Model (15 hours)