[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1363 **I**

Unique Paper Code : 2352011101

Name of the Paper : Algebra (DSC-1)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting **two** parts from each question.

3. **All** questions carry equal marks.

1. (a) (i) Find a cubic equation with real coefficients two of whose roots are 1 and 3+2i. Also state the result being used.

(ii) Find an upper limit (using both the theorems) to the roots of

$$x^7 + 2x^5 + 4x^4 - 8x^2 - 32 = 0. \qquad (3.5+4)$$

(b) Solve $3x^3 + 11x^2 + 12x + 4 = 0$, being given that roots are in Harmonic progression. (7.5)

(c) Find all the rational roots of $6y^3 - 11y^2 + 6y - 1 = 0$. (7.5)

2. (a) Compute $z^n + \frac{1}{z^n}$ if $z + \frac{1}{z} = \sqrt{3}$. (7.5)

(b) Find $|z|$, arg z, Arg z, arg(-z) for $z = (7-7\sqrt{3}\ i)(-1\ -i)$. (7.5)

(c) Solve the equation $z^7 - 2iz^4 - iz^3\ 2 = 0$. (7.5)

3. (a) Solve $28x^3 + 9x^2 - 1 = 0$ by Cardan's method. (7.5)

(b) If a, b and c are non-zero integers with a and c relatively prime, prove that gcd (a, bc) = gcd (a, b) (7.5)

(c) (i) Find gcd of 1800 and 756 and express it in the form ma + nb for some integers m and n.

(ii)If a and b are relatively prime integers, prove that gcd (a + b, a - b) = 1 or 2. (4+3.5)

4. (a) Solve the following pair of congruences, if possible. If no solution exists, explain why not.

$$2x + y \equiv 1\ (mod\ 6)$$

$$x + 3y \equiv 3\ (mod\ 6)$$

(7.5)

(b) If $a \equiv x\ (mod\ n)$ and $b \equiv y\ (mod\ n\ )$, then

(i) $a + b \equiv x + y\ (mod\ n)$

(ii) $ab \equiv xy\ \ (mod\ n)$ (3.5+4)

(c) State fundamental theorem of arithmetic. Suppose a and b are integers and p is a prime such that plab. Prove that $p|a\ or\ p|b.$ (2.5+5)

5. (a) Describe symmetries of a non-square rectangle with diagrams. Also, construct the corresponding Cayley table. (3.5+4)

(b) Define an Abelian group. Show that in a group G if ab = ac then b = c (called left cancellation property). Further, show that in a group G if ab =

ca implies b = c for all a, b, c in G then G is Abelian (that is, left-right cancellation property implies Abelian). (2+2.5+3)

(c) Show that the set $G = \{\begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R}, a \neq 0\}$

is a group under matrix multiplication. (7.5)

6. (a) State two-step subgroup test. Let G be an Abelian group and H, K be subgroups of G then show that

$HK = \{hk \mid h \in H, k \in K\}$

is a subgroup of G. (2+5.5)

(b) Define order of an element 'a', $O(a)$,, in a group G. Prove that in any group G, $O(bab^{-1}) = O(a)$ for all $a, b \in$ G.. (2+5.5)

(c) Write all the generators of the cyclic group Z_{24}. Further describe all the subgroups of Z_{24} and find all generators of the subgroup of order 8 in Z_{24}. (3+3+1.5)