1. (a) (i) Prove that $\int_{-\infty}^{\infty} x\,(t)\delta(t)dt = x(0)$.

- **Proof of the Sifting Property of the Dirac Delta Function:**
  - The Dirac delta function, $\delta(t)$, has the property that it is zero everywhere except at $t = 0$, where it is infinitely large such that its integral over all time is one.
    - $\delta(t) = 0$ for $t \neq 0$
    - $\int_{-\infty}^{\infty} \delta\,(t)dt = 1$
  - Consider the integral $\int_{-\infty}^{\infty} x\,(t)\delta(t)dt$.
  - Since $\delta(t)$ is zero for all $t \neq 0$, the product $x(t)\delta(t)$ will also be zero for all $t \neq 0$. The only point where the product is non-zero is at $t = 0$.
  - Therefore, we can replace $x(t)$ with its value at $t = 0$, which is $x(0)$, within the integral, as it is the only point that contributes to the integral.
  - $\int_{-\infty}^{\infty} x\,(t)\delta(t)dt = \int_{-\infty}^{\infty} x\,(0)\delta(t)dt$
  - Since $x(0)$ is a constant with respect to the integration variable t, we can take it out of the integral:
  - $\int_{-\infty}^{\infty} x\,(0)\delta(t)dt = x(0) \int_{-\infty}^{\infty} \delta\,(t)dt$

- From the definition of the delta function, we know that $\int_{-\infty}^{\infty} \delta\,(t)dt = 1$.
- Thus, $\int_{-\infty}^{\infty} x\,(t)\delta(t)dt = x(0) \times 1 = x(0)$.
- This proves the sifting property.

(ii) Evaluate $\int_{-\infty}^{\infty} \delta\,(t)\sin(2\pi t)dt$.

- **Evaluation:**
  - Using the sifting property proved above, $\int_{-\infty}^{\infty} x\,(t)\delta(t)dt = x(0)$.
  - In this case, $x(t) = \sin(2\pi t)$.
  - Therefore, we need to evaluate $x(t)$ at $t = 0$.
  - $x(0) = \sin(2\pi \times 0) = \sin(0) = 0$.
  - So, $\int_{-\infty}^{\infty} \delta\,(t)\sin(2\pi t)dt = 0$.

(b) Differentiate between Time-invariant and time variant systems. Determine whether or not the following system is time-invariant. $y[n] = x^2[n-1]$.

- **Differentiation between Time-Invariant and Time-Variant Systems:**
  - **Time-Invariant System:** A system is time-invariant if its input-output relationship does not change with time. This means that if an input signal is delayed, the

output signal will be delayed by the exact same amount, without any change in its shape or characteristics.

  - Mathematically, if $y(t)$ is the output for an input $x(t)$, then for a time-invariant system, the output for $x(t - t_0)$ is $y(t - t_0)$ for any arbitrary delay t_0.

- **Time-Variant System:** A system is time-variant if its input-output relationship changes with time. This implies that a delayed input signal does not necessarily produce a simply delayed version of the original output signal; its characteristics might change.
  - Mathematically, if $y(t)$ is the output for an input $x(t)$, then for a time-variant system, the output for $x(t - t_0)$ is not equal to $y(t - t_0)$.

- **Determining if $y[n] = x^2[n-1]$ is time-invariant:**
  - **Step 1: Find the output for a delayed input.** Let the input be $x[n - n_0]$. The output $y_1[n]$ for this delayed input is: $y_1[n] = (x[n - n_0])^2 = x^2[n - n_0]$
  - **Step 2: Find the delayed output of the original system.** The original output is $y[n] = x^2[n-1]$. Now, delay this output by n_0: $y[n - n_0] = x^2[(n - n_0) - 1] = x^2[n - n_0 - 1]$

- **Step 3: Compare $y_1[n]$ and $y[n-n_0]$.** We have $y_1[n] = x^2[n-n_0]$ and $y[n-n_0] = x^2[n-n_0-1]$. Since $x^2[n-n_0] \neq x^2[n-n_0-1]$ (unless $n_0 = -1$ which is not general), the output for a delayed input is not simply a delayed version of the original output.
- **Conclusion:** The system $y[n] = x^2[n-1]$ is **not time-invariant**, it is a **time-variant** system.
  - *(Correction based on standard definition: $x[n-1]$ inherently implies a delay. The square is an amplitude scaling. Let's re-evaluate more carefully)*
  - **Re-evaluation for Time-Invariance:**
    - Let $y[n]$ be the output for input $x[n]$: $y[n] = x^2[n-1]$.
    - Consider a delayed input $x_d[n] = x[n-n_0]$.
    - The output $y_d[n]$ due to $x_d[n]$ is: $y_d[n] = (x_d[n-1])^2 = (x[n-1-n_0])^2 = x^2[n-1-n_0]$.
    - Now consider the original output $y[n]$ delayed by n_0: $y[n-n_0] = x^2[(n-n_0)-1] = x^2[n-n_0-1]$.

- Since $y_d[n] = y[n - n_0]$, the system is **time-invariant**. My previous conclusion was incorrect. The presence of $n - 1$ alone does not make it time-variant; it's the function of n that would be a multiplier for $x[n]$ or a changing coefficient.

(c) Find Fourier transform of the signal $x(t) = t \cdot e^{-at}u(t)$.

- **Fourier Transform Definition:** The Fourier transform of a signal $x(t)$ is given by $X(j\omega) = \int_{-\infty}^{\infty} x\,(t)e^{-j\omega t}dt$.
- **Given signal:** $x(t) = t \cdot e^{-at}u(t)$. Since $u(t) = 0$ for $t < 0$ and $u(t) = 1$ for $t \geq 0$, the integral limits become from 0 to ∞. $X(j\omega) = \int_0^{\infty} t \cdot e^{-at}e^{-j\omega t}dt$ $X(j\omega) = \int_0^{\infty} t \cdot e^{-(a+j\omega)t}dt$
- **Using Integration by Parts ($\int udv = uv - \int vdu$):** Let $u = t \Longrightarrow du = dt$ Let $dv = e^{-(a+j\omega)t}dt \Longrightarrow v = \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}$

  $X(j\omega) = \left[t \cdot \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}dt$

  - Evaluate the first term: As $t \to \infty$, $t \cdot e^{-(a+j\omega)t} \to 0$ (assuming $a > 0$ for convergence). As $t \to 0$, $0 \cdot \frac{e^0}{-(a+j\omega)} = 0$. So the first term is $0 - 0 = 0$.

- Evaluate the second term: $X(j\omega) = -\int_0^\infty \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} dt = \frac{1}{(a+j\omega)} \int_0^\infty e^{-(a+j\omega)t}\, dt$ $X(j\omega) = \frac{1}{(a+j\omega)} \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^\infty$ $X(j\omega) = \frac{1}{(a+j\omega)} \left[(0) - \left(\frac{e^0}{-(a+j\omega)}\right)\right]$ $X(j\omega) = \frac{1}{(a+j\omega)} \left[-\frac{1}{-(a+j\omega)}\right] = \frac{1}{(a+j\omega)} \left[\frac{1}{(a+j\omega)}\right]$ $X(j\omega) = \frac{1}{(a+j\omega)^2}$

- **Alternatively, using the differentiation property of Fourier Transform:** We know that if $x(t) \leftrightarrow X(j\omega)$, then $tx(t) \leftrightarrow j\frac{d}{d\omega}X(j\omega)$. Also, $e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}$. Here, $x(t) = t \cdot (e^{-at}u(t))$. So $X(j\omega) = j\frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right)$. $\frac{d}{d\omega}((a + j\omega)^{-1}) = -1(a + j\omega)^{-2} \cdot (j) = -j(a + j\omega)^{-2} = \frac{-j}{(a+j\omega)^2}$ So, $X(j\omega) = j \cdot \frac{-j}{(a+j\omega)^2} = \frac{-j^2}{(a+j\omega)^2} = \frac{-(-1)}{(a+j\omega)^2} = \frac{1}{(a+j\omega)^2}$.
- **Fourier Transform:** $X(j\omega) = \frac{1}{(a+j\omega)^2}$.

(d) Find the inverse Laplace transform of $X(s) = \frac{1}{(s+4)^2}$.

- **Inverse Laplace Transform Definition:** We need to find $x(t)$ such that $\mathcal{L}\{x(t)\} = X(s)$.
- **Using Standard Laplace Transform Pairs:** We know the standard Laplace transform pair: $\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}$ And

the differentiation in s-domain property (multiplication by t in time domain): $\mathcal{L}\{t \cdot x(t)\} = -\frac{dX(s)}{ds}$

Let's use the property: $\mathcal{L}\{t^n e^{-at} u(t)\} = \frac{n!}{(s+a)^{n+1}}$. Here, comparing $X(s) = \frac{1}{(s+4)^2}$ with $\frac{n!}{(s+a)^{n+1}}$: $n + 1 = 2 \Rightarrow n = 1$ $a = 4$ So, $n! = 1! = 1$. Therefore, $x(t) = t^1 e^{-4t} u(t) = te^{-4t} u(t)$.

- **Alternatively, using the derivative in the s-domain property:** We know $\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\} = e^{-4t} u(t)$. Let $F(s) = \frac{1}{s+4}$. Then $X(s) = -\frac{dF(s)}{ds}$. This is incorrect, $X(s)$ is not the derivative of $F(s)$. The property is $\mathcal{L}\{tx(t)\} = -\frac{d}{ds}X(s)$. So, if $\mathcal{L}\{x(t)\} = \frac{1}{s+4}$, then $\mathcal{L}\{tx(t)\} = \mathcal{L}\{te^{-4t}u(t)\} = -\frac{d}{ds}\left(\frac{1}{s+4}\right) = -(-(s+4)^{-2}) = \frac{1}{(s+4)^2}$. Thus, the inverse Laplace transform of $X(s) = \frac{1}{(s+4)^2}$ is $x(t) = te^{-4t} u(t)$.
- **Inverse Laplace Transform:** $x(t) = te^{-4t} u(t)$.

(e) Give the impulse response of discrete time memoryless LTI systems.

- **Discrete-Time Memoryless System:** A discrete-time system is memoryless if its output at any time instant n depends only on the input at the same time instant n. It does not depend on past or future input values.

- **LTI System:** Linear Time-Invariant system.
- **Impulse Response ($h[n]$):** The impulse response of an LTI system is its output when the input is a discrete-time unit impulse $\delta[n]$.
- **Derivation of Impulse Response for Memoryless LTI System:**
    - For a general discrete-time LTI system, the output $y[n]$ is given by the convolution sum: $y[n] = \sum_{k=-\infty}^{\infty} h\,[k]x[n-k]$
    - For a memoryless system, $y[n]$ depends only on $x[n]$. This means that in the convolution sum, $h[k]$ must be zero for all $k \neq 0$.
    - Therefore, the only non-zero term in the sum is when $k = 0$: $y[n] = h[0]x[n-0] = h[0]x[n]$
    - So, for a memoryless LTI system, the input-output relationship is $y[n] = C \cdot x[n]$, where $C = h[0]$ is a constant gain.
    - To find the impulse response, set $x[n] = \delta[n]$: $h[n] = C \cdot \delta[n]$
- **Impulse Response:** The impulse response of a discrete-time memoryless LTI system is of the form $h[n] = C\delta[n]$, where C is a constant. This means the impulse response is a scaled version of the unit impulse occurring at $n = 0$.

(f) Sketch the waveform of discrete time unit step signal $u[n]$ and $u[-2n+2]$.

- **Discrete-Time Unit Step Signal $u[n]$:**
  - Definition: $u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$
  - **Sketch:**
    - At $n = 0$, amplitude is 1.
    - At $n = 1$, amplitude is 1.
    - At $n = 2$, amplitude is 1.
    - ...and so on for all positive integers.
    - For $n = -1, -2, ...$, the amplitude is 0.
    - The sketch would show a sequence of impulses of amplitude 1 starting from $n = 0$ and extending to the right, and zeros to the left of $n = 0$.
- **Discrete-Time Signal $u[-2n+2]$:**
  - First, find the argument where the step becomes 1. $-2n + 2 \geq 0$ $-2n \geq -2$ $n \leq 1$
  - So, $u[-2n+2] = \begin{cases} 1 & \text{for } n \leq 1 \\ 0 & \text{for } n > 1 \end{cases}$
  - **Sketch:**
    - At $n = 1$, amplitude is 1.

- At $n = 0$, amplitude is 1.
- At $n = -1$, amplitude is 1.
- ...and so on for all negative integers.
- At $n = 2$, amplitude is 0.
- At $n = 3$, amplitude is 0.
- ...and so on for all integers greater than 1.
- The sketch would show a sequence of impulses of amplitude 1 starting from $n = 1$ and extending to the left, and zeros to the right of $n = 1$.

○ **(Cannot make schematic diagrams as per user instructions)**

2. (a) Determine whether $x(t) = tu(t)$ is energy signal or power signal.

○ **Energy Signal:** A signal is an energy signal if its total energy E_x is finite and non-zero ($0 < E_x < \infty$). $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

○ **Power Signal:** A signal is a power signal if its average power P_x is finite and non-zero ($0 < P_x < \infty$). For a periodic signal, $P_x = \frac{1}{T}\int_T |x(t)|^2 dt$. For a non-periodic signal, $P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$.

- A signal cannot be both an energy and a power signal, but it can be neither.

- **Analysis of $x(t) = tu(t)$:**
  - **Calculate Energy (E_x):** $E_x = \int_{-\infty}^{\infty} |tu(t)|^2 dt = \int_0^{\infty} t^2\, dt$ $E_x = \left[\frac{t^3}{3}\right]_0^{\infty} = \lim_{T\to\infty}\left(\frac{T^3}{3} - 0\right) = \infty$ Since the energy is infinite, $x(t)$ is not an energy signal.
  - **Calculate Power (P_x):** $P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |tu(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_0^{T} t^2\, dt$ $P_x = \lim_{T\to\infty} \frac{1}{2T}\left[\frac{t^3}{3}\right]_0^{T}$ $P_x = \lim_{T\to\infty} \frac{1}{2T}\left(\frac{T^3}{3}\right) = \lim_{T\to\infty} \frac{T^2}{6} = \infty$ Since the average power is infinite, $x(t)$ is not a power signal.

- **Conclusion:** The signal $x(t) = tu(t)$ is **neither an energy signal nor a power signal**. It is an infinitely growing signal, and such signals typically have infinite energy and infinite power.

(b) A continuous-time signal $x(t)$ is as shown below. Sketch and label the following signals.

- (i) $x(t)[u(t) - u(t-1)]$
  - **Analyze $u(t) - u(t-1)$:**
    - $u(t)$ is 1 for $t \geq 0$ and 0 for $t < 0$.

        - $u(t-1)$ is 1 for $t \geq 1$ and 0 for $t < 1$.
        - Therefore, $u(t) - u(t-1)$ is:
            - 1 for $0 \leq t < 1$
            - 0 for $t < 0$ and $t \geq 1$.
        - This is a rectangular pulse of amplitude 1 from $t = 0$ to $t = 1$ (excluding $t = 1$).
    - **Sketch** $x(t)[u(t) - u(t-1)]$**:**
        - The product $x(t)[u(t) - u(t-1)]$ will only be non-zero for the interval $0 \leq t < 1$.
        - In this interval, the value of the signal will be exactly $x(t)$ because $u(t) - u(t-1) = 1$.
        - Outside this interval, the value will be 0.
        - **(Cannot make schematic diagrams as per user instructions)**
        - Imagine a graph of $x(t)$. Take only the portion of $x(t)$ that exists between $t = 0$ and $t = 1$ (excluding $t = 1$). All other parts of $x(t)$ are multiplied by 0 and thus become 0.
- (ii) signal. (a) $x(0)$ (This part is fragmented in the source, appearing as "signal. (a) x(0)" with an incomplete (ii) section.)

- This part seems to be an instruction to find the value of $x(t)$ at $t = 0$. Without the actual waveform of $x(t)$, its value at $t = 0$ cannot be determined.
- If it was a continuation of (b) and (b) was supposed to describe a sketch, then $x(0)$ would be the amplitude of the signal at $t = 0$ from the given sketch.

(c) Show that the product of two even signals or two odd signals is an even signal and that the product of an even and an odd signal is an odd.

- **Definitions:**
  - **Even Signal:** A signal $x(t)$ is even if $x(t) = x(-t)$.
  - **Odd Signal:** A signal $x(t)$ is odd if $x(t) = -x(-t)$.
- **Case 1: Product of two even signals.**
  - Let $x_e(t)$ and $y_e(t)$ be two even signals.
    - $x_e(t) = x_e(-t)$
    - $y_e(t) = y_e(-t)$
  - Let $z(t) = x_e(t) \cdot y_e(t)$ be their product.
  - Now, let's check $z(-t)$:
    - $z(-t) = x_e(-t) \cdot y_e(-t)$
    - Since $x_e(-t) = x_e(t)$ and $y_e(-t) = y_e(t)$,
    - $z(-t) = x_e(t) \cdot y_e(t) = z(t)$

- Therefore, the product of two even signals is an **even signal**.

- **Case 2: Product of two odd signals.**
  - Let $x_o(t)$ and $y_o(t)$ be two odd signals.
    - $x_o(t) = -x_o(-t) \Longrightarrow x_o(-t) = -x_o(t)$
    - $y_o(t) = -y_o(-t) \Longrightarrow y_o(-t) = -y_o(t)$
  - Let $z(t) = x_o(t) \cdot y_o(t)$ be their product.
  - Now, let's check $z(-t)$:
    - $z(-t) = x_o(-t) \cdot y_o(-t)$
    - Substitute $x_o(-t) = -x_o(t)$ and $y_o(-t) = -y_o(t)$:
    - $z(-t) = (-x_o(t)) \cdot (-y_o(t)) = x_o(t) \cdot y_o(t) = z(t)$
  - Therefore, the product of two odd signals is an **even signal**.

- **Case 3: Product of an even and an odd signal.**
  - Let $x_e(t)$ be an even signal and $y_o(t)$ be an odd signal.
    - $x_e(t) = x_e(-t)$
    - $y_o(t) = -y_o(-t) \Longrightarrow y_o(-t) = -y_o(t)$

- Let $z(t) = x_e(t) \cdot y_o(t)$ be their product.
- Now, let's check $z(-t)$:
    - $z(-t) = x_e(-t) \cdot y_o(-t)$
    - Substitute $x_e(-t) = x_e(t)$ and $y_o(-t) = -y_o(t)$:
    - $z(-t) = x_e(t) \cdot (-y_o(t)) = -(x_e(t) \cdot y_o(t)) = -z(t)$
- Therefore, the product of an even and an odd signal is an **odd signal**.

3. (a) Consider the system shown in figure below. Determine whether it is (a) memoryless, (b) causal, (c) linear, (d) time-invariant, or (e) stable system.
    - **Without the figure, specific properties cannot be determined.** A system diagram is crucial for this analysis.
    - **General Definitions of System Properties:**
        - **(a) Memoryless:** Output at any time t (or n) depends only on the input at the same time t (or n). (e.g., $y(t) = 2x(t)$)
        - **(b) Causal:** Output at any time t (or n) depends only on the present and past values of the input. It does not depend on future input values. (e.g.,

$y(t) = x(t) + x(t-1)$ is causal, $y(t) = x(t+1)$ is non-causal)

- **(c) Linear:** Satisfies superposition and homogeneity.
    - Additivity: $T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\}$
    - Homogeneity: $T\{ax(t)\} = aT\{x(t)\}$ for any constant a.
    - (e.g., $y(t) = 2x(t)$ is linear, $y(t) = x^2(t)$ is non-linear)
- **(d) Time-Invariant:** As proved in 1(b), a time shift in the input results in an identical time shift in the output. (e.g., $y(t) = x(t-1)$ is time-invariant, $y(t) = tx(t)$ is time-variant)
- **(e) Stable (BIBO Stable - Bounded Input Bounded Output):** For every bounded input, the output is also bounded. If $|x(t)| < M_x < \infty$, then $|y(t)| < M_y < \infty$.
    - For an LTI system, BIBO stability requires that the impulse response $h(t)$ be absolutely integrable: $\int_{-\infty}^{\infty} |h(t)| dt < \infty$.
    - For a discrete-time LTI system, $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$.

- **Please provide the system diagram or equation for a specific answer.**

(b) Determine the convolution Sum of two sequences $x[n] = \{1,4,3,2\}$; $h[n] = \{1,3,2,1\}$ the sequence $x[n]$ starts at $n = -1$ and $h[n]$ starts at $n = 0$.

- **Given Sequences:**
  - $x[n] = \{\ldots,0,1,4,3,2,0,\ldots\}$ starting at $n = -1$. So, $x[-1] = 1$, $x[0] = 4$, $x[1] = 3$, $x[2] = 2$.
  - $h[n] = \{\ldots,0,1,3,2,1,0,\ldots\}$ starting at $n = 0$. So, $h[0] = 1$, $h[1] = 3$, $h[2] = 2$, $h[3] = 1$.
- **Convolution Sum Formula:** $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x\,[k]h[n-k]$
- **Length of Output Sequence:** Length of $x[n]$ is $L_x = 4$. Length of $h[n]$ is $L_h = 4$. Length of $y[n]$ is $L_y = L_x + L_h - 1 = 4 + 4 - 1 = 7$.
- **Starting Index of Output Sequence:** Starting index of $y[n]$ is $n_{start,y} = n_{start,x} + n_{start,h} = -1 + 0 = -1$. So $y[n]$ will span from $n = -1$ to $n = -1 + 7 - 1 = 5$.
- **Calculation using the formula:**
  - $y[-1] = x[-1]h[0] = 1 \times 1 = 1$
  - $y[0] = x[-1]h[1] + x[0]h[0] = (1 \times 3) + (4 \times 1) = 3 + 4 = 7$

- $y[1] = x[-1]h[2] + x[0]h[1] + x[1]h[0] = (1 \times 2) + (4 \times 3) + (3 \times 1) = 2 + 12 + 3 = 17$
- $y[2] = x[-1]h[3] + x[0]h[2] + x[1]h[1] + x[2]h[0] = (1 \times 1) + (4 \times 2) + (3 \times 3) + (2 \times 1) = 1 + 8 + 9 + 2 = 20$
- $y[3] = x[0]h[3] + x[1]h[2] + x[2]h[1] = (4 \times 1) + (3 \times 2) + (2 \times 3) = 4 + 6 + 6 = 16$
- $y[4] = x[1]h[3] + x[2]h[2] = (3 \times 1) + (2 \times 2) = 3 + 4 = 7$
- $y[5] = x[2]h[3] = 2 \times 1 = 2$

- **Convolution Sum:** $y[n] = \{1,7,17,20,16,7,2\}$ starting at $n = -1$.

(c) The LTI system shown in Figure(a) is formed by connecting two Systems in cascade. The impulse response of the system are given by $h_1(t)$ and $h_2(t)$ respectively and $h_1(t) = e^{-t}u(t)$, $h_2(t) = 2e^{-t}u(t)$.

- (i) Find the impulse response $h(t)$ of the overall system shown in figure (b).
- (ii) Determine if the overall system is BIBO stable.
- **System in Cascade:**
  - When two LTI systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade, the overall

impulse response $h(t)$ is the convolution of the individual impulse responses: $h(t) = h_1(t) * h_2(t)$.

- **(i) Find the impulse response $h(t)$ of the overall system:**
  - Given $h_1(t) = e^{-t}u(t)$ and $h_2(t) = 2e^{-t}u(t)$.
  - $h(t) = \int_{-\infty}^{\infty} h_1(\tau)h_2(t-\tau)d\tau$
  - $h(t) = \int_{-\infty}^{\infty} e^{-\tau}\, u(\tau) \cdot 2e^{-(t-\tau)}u(t-\tau)d\tau$
  - The terms $u(\tau)$ and $u(t-\tau)$ limit the integration range:
    - $u(\tau) = 1$ for $\tau \geq 0$.
    - $u(t-\tau) = 1$ for $t-\tau \geq 0 \Rightarrow \tau \leq t$.
  - So, the integral is non-zero only for $0 \leq \tau \leq t$.
  - Also, the output $h(t)$ will be zero for $t < 0$, so $h(t)$ will also have a $u(t)$ term.
  - For $t \geq 0$: $h(t) = \int_0^t e^{-\tau} \cdot 2e^{-t}e^{\tau}d\tau$ $h(t) = 2e^{-t}\int_0^t e^{-\tau}\, e^{\tau}d\tau$ $h(t) = 2e^{-t}\int_0^t e^0\, d\tau$ $h(t) = 2e^{-t}\int_0^t 1\, d\tau$ $h(t) = 2e^{-t}[\tau]_0^t$ $h(t) = 2e^{-t}(t-0)$ $h(t) = 2te^{-t}$
  - Including the unit step function for causality: $h(t) = 2te^{-t}u(t)$
- **(ii) Determine if the overall system is BIBO stable.**

- An LTI system is BIBO stable if its impulse response is absolutely integrable: $\int_{-\infty}^{\infty} |h(t)|dt < \infty$
- For $h(t) = 2te^{-t}u(t)$, we need to evaluate: $\int_0^{\infty} |2te^{-t}|dt = \int_0^{\infty} 2\,te^{-t}dt$ (since $t \geq 0$, $2te^{-t}$ is positive).
- Use integration by parts ($\int udv = uv - \int vdu$): Let $u = t \Rightarrow du = dt$ Let $dv = 2e^{-t}dt \Rightarrow v = -2e^{-t}$

  $\int_0^{\infty} 2\,te^{-t}dt = [t(-2e^{-t})]_0^{\infty} - \int_0^{\infty}(-2e^{-t})dt = [-2te^{-t}]_0^{\infty} + 2\int_0^{\infty} e^{-t}\,dt$

  - Evaluate the first term: $\lim_{t\to\infty}(-2te^{-t}) = 0$ (since exponential decays faster than t grows). At $t = 0$: $-2(0)e^0 = 0$. So, the first term is $0 - 0 = 0$.
  - Evaluate the second term: $2\int_0^{\infty} e^{-t}\,dt = 2[-e^{-t}]_0^{\infty} = 2[(0) - (-e^0)] = 2[0 - (-1)] = 2 \times 1 = 2$.
- The integral evaluates to 2, which is a finite value.
- Since $\int_{-\infty}^{\infty} |h(t)|dt = 2 < \infty$, the overall system is **BIBO stable**.

(a) Find the convolution of the following $x_1(t) = u(t)$; $x_2(t) = u(t)$ (Note: This (a) sub-part appears after the (c) part of question 3 in the source, but is formatted as a sub-part of 5. It is likely a question related to convolution.)

- **Convolution Integral Formula:** $y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$
- **Given signals:** $x_1(t) = u(t)$ $x_2(t) = u(t)$ So, $x_1(\tau) = u(\tau)$ and $x_2(t-\tau) = u(t-\tau)$.
- **Determine limits of integration:**
  - $u(\tau) = 1$ for $\tau \geq 0$, and 0 otherwise.
  - $u(t-\tau) = 1$ for $t - \tau \geq 0 \Longrightarrow \tau \leq t$, and 0 otherwise.
  - For the integral to be non-zero, both conditions must be met: $\tau \geq 0$ and $\tau \leq t$.
  - This means the integration limits are from 0 to t, but only if $t \geq 0$. If $t < 0$, there is no overlap, so the integral is 0.
  - Therefore, the result will have a $u(t)$ term.
- **Perform the integration for $t \geq 0$:** $y(t) = \int_0^t 1 \cdot 1 d\tau$ $y(t) = \int_0^t d\,\tau$ $y(t) = [\tau]_0^t$ $y(t) = t - 0 = t$
- **Combine with the unit step function:** $y(t) = tu(t)$
- **Convolution Result:** The convolution of $u(t)$ with $u(t)$ is $tu(t)$.

4. (b) State and prove in continuous time LTI system the distributive property of the convolution.

- **Distributive Property of Convolution:**
    - **Statement:** The distributive property of convolution states that convolution distributes over addition. That is, if a signal $x(t)$ is convolved with the sum of two other signals, $(h_1(t) + h_2(t))$, the result is the same as convolving $x(t)$ with $h_1(t)$ and $x(t)$ with $h_2(t)$ separately, and then adding the results.
    - Mathematically: $x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$
- **Proof:**
    - Start with the left-hand side (LHS) of the equation, using the definition of convolution: $x(t) * [h_1(t) + h_2(t)] = \int_{-\infty}^{\infty} x\,(\tau)[h_1(t-\tau) + h_2(t-\tau)]d\tau$
    - Due to the linearity property of integration (the integral of a sum is the sum of the integrals), we can split the integral: $= \int_{-\infty}^{\infty} x\,(\tau)h_1(t-\tau)d\tau + \int_{-\infty}^{\infty} x\,(\tau)h_2(t-\tau)d\tau$
    - Recognize that each integral on the right-hand side corresponds to the definition of convolution:
        - $\int_{-\infty}^{\infty} x\,(\tau)h_1(t-\tau)d\tau = x(t) * h_1(t)$

- $\int_{-\infty}^{\infty} x(\tau) h_2(t-\tau) d\tau = x(t) * h_2(t)$

- Therefore, we have: $x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$
- This proves the distributive property of convolution.

(c) Give the block diagram representation for the causal continuous time system described by the first order differential equation. $\frac{dy(t)}{dt} + 2y(t) = 5x(t)$.

- **Rearrange the differential equation to solve for the highest derivative of the output:** $\frac{dy(t)}{dt} = 5x(t) - 2y(t)$
- **Integrate both sides to obtain** $y(t)$**:** $y(t) = \int (5x(t) - 2y(t))dt$
- **Block Diagram Components:**
  - **Integrator:** The operation $\int \alpha(t)dt$ can be represented by an integrator block. If the input is $Z(t)$, the output is $\int Z(t)dt$.
  - **Summing Junction:** To sum or subtract signals.
  - **Gain Blocks:** To multiply signals by constants (e.g., 5x(t), -2y(t)).
- **Construction of the Block Diagram (Conceptual):**

a. The output of the integrator will be $y(t)$. So, the input to the integrator must be $\frac{dy(t)}{dt}$.

b. From the rearranged equation, $\frac{dy(t)}{dt}$ is the sum of $5x(t)$ and $-2y(t)$.

c. Create a summing junction.

d. Input to the summing junction will be $5x(t)$ (from an input $x(t)$ and a gain block of 5).

e. The other input to the summing junction will be $-2y(t)$ (from the output $y(t)$ fed back through a gain block of -2).

- **(Cannot make schematic diagrams as per user instructions)**
- **Description of the Block Diagram:**
    - An input signal $x(t)$ enters a **gain block** with a gain of 5. The output of this block is $5x(t)$.
    - The output signal $y(t)$ is fed back into a **gain block** with a gain of -2. The output of this block is $-2y(t)$.
    - The signals $5x(t)$ and $-2y(t)$ are fed into a **summing junction**. The output of the summing junction is $5x(t) - 2y(t)$, which represents $\frac{dy(t)}{dt}$.

- The output of the summing junction ($\frac{dy(t)}{dt}$) is fed into an **integrator block**. The output of the integrator block is $y(t)$.
- This represents a closed-loop system with feedback.

5. (a) Obtain the trigonometric Fourier series for the waveform shown in figure 4a).
   - **Without Figure 4a), the waveform cannot be analyzed, and its Fourier series cannot be obtained.**
   - **General Approach for Trigonometric Fourier Series:** For a periodic signal $x(t)$ with period T_0, the trigonometric Fourier series is given by: $x(t) = a_0 + \sum_{k=1}^{\infty}(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t))$ Where $\omega_0 = \frac{2\pi}{T_0}$ is the fundamental frequency. The coefficients are calculated as:
     - $a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$ (Average value)
     - $a_k = \frac{2}{T_0}\int_{T_0} x(t)\cos(k\omega_0 t)dt$
     - $b_k = \frac{2}{T_0}\int_{T_0} x(t)\sin(k\omega_0 t)dt$
   - **Steps to follow once waveform is known:**
     i. Determine the period T_0 from the waveform.

ii. Determine the fundamental angular frequency $\omega_0 = 2\pi/T_0$.

iii. Define the mathematical expression for $x(t)$ over one period.

iv. Calculate a_0 by integrating $x(t)$ over one period and dividing by T_0.

v. Calculate a_k by integrating $x(t)\cos(k\omega_0 t)$ over one period and multiplying by $2/T_0$.

vi. Calculate b_k by integrating $x(t)\sin(k\omega_0 t)$ over one period and multiplying by $2/T_0$.

vii. Substitute the calculated coefficients into the Fourier series formula.

viii. Utilize symmetry properties (even, odd, half-wave symmetric) if applicable, as they can simplify the calculation of coefficients (e.g., for an even signal, all $b_k = 0$; for an odd signal, all $a_k = 0$ and $a_0 = 0$).

(b) Determine the discrete Fourier Transform representation of the given sequence $x[n] = \cos(\frac{\pi n}{4})$.

- **Discrete Fourier Transform (DFT):** The DFT is typically used for finite-length sequences. The given sequence $x[n] = \cos(\frac{\pi n}{4})$ is an infinite-length discrete-time signal (a sinusoid).

- **It's likely that the question intends to ask for the Discrete-Time Fourier Transform (DTFT) or implies a finite number of samples for DFT.**
  - **If DTFT is intended:** The DTFT of a discrete-time signal $x[n]$ is given by: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x\,[n]e^{-j\omega n}$ We know Euler's formula: $\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$. So, $x[n] = \frac{e^{j(\frac{\pi}{4})n}+e^{-j(\frac{\pi}{4})n}}{2}$. The DTFT of $e^{j\omega_0 n}$ is $2\pi \sum_{k=-\infty}^{\infty} \delta\,(\omega - \omega_0 - 2\pi k)$. Applying this property: $X(e^{j\omega}) = \frac{1}{2}\left[2\pi \sum_{k=-\infty}^{\infty} \delta\,(\omega - \frac{\pi}{4} - 2\pi k) + 2\pi \sum_{k=-\infty}^{\infty} \delta\,(\omega + \frac{\pi}{4} - 2\pi k)\right]$ $X(e^{j\omega}) = \pi \sum_{k=-\infty}^{\infty} \left[\delta(\omega - \frac{\pi}{4} - 2\pi k) + \delta(\omega + \frac{\pi}{4} - 2\pi k)\right]$ This represents impulses at $\omega = \pm\frac{\pi}{4}, \pm\frac{\pi}{4} \pm 2\pi, \ldots$ with magnitude π.
  - **If DFT of a finite sequence is intended:** We need the length N of the sequence. If $x[n] = \cos(\frac{2\pi kn}{N})$ then the DFT will have non-zero values at specific indices. For $x[n] = \cos(\frac{\pi n}{4})$, this implies $\frac{2\pi k}{N} = \frac{\pi}{4}$. So, $k/N = 1/8$. This means the sequence must be periodic with period $N = 8$. So if $N = 8$, then $x[n] = \cos(\frac{2\pi n}{8})$. This is for $k = 1$. The DFT of $x[n]$ for a length-N sequence $X[k]$ is: $X[k] = \sum_{n=0}^{N-1} x\,[n]e^{-j\frac{2\pi}{N}kn}$ If

$x[n] = \cos(\frac{2\pi f_0 n}{N})$ over N samples, then $X[k]$ will have non-zero values at $k = f_0$ and $k = N - f_0$. Here $f_0 = 1$ for $\cos(\frac{2\pi n}{8})$. So $X[1]$ and $X[7]$ will be non-zero. $X[k] = \frac{N}{2}$ at $k = 1$ and $k = N - 1$. (for a cosine, if amplitude is 1) So for $N = 8$, $X[1] = 4$ and $X[7] = 4$. All other $X[k]$ are 0 for $k = 0, \dots, 7$.

- **Assuming the question asks for the Discrete-Time Fourier Transform (DTFT) since "representation" is asked for an infinite sequence:** The DTFT of $x[n] = \cos(\frac{\pi n}{4})$ is $X(e^{j\omega}) = \pi \sum_{k=-\infty}^{\infty} \left[\delta(\omega - \frac{\pi}{4} - 2\pi k) + \delta(\omega + \frac{\pi}{4} - 2\pi k)\right]$.

6. (a) State and prove the convolution property of the Fourier series of the RL circuit. (Note: The wording "of the RL circuit" seems to be an addition that may or may not directly relate to the convolution property of Fourier series, as written).

   - **Convolution Property of the Fourier Series:**
     - **Statement:** If $x(t)$ and $y(t)$ are two periodic signals with the same period T_0, and their Fourier series coefficients are $X[k]$ and $Y[k]$ respectively, then the Fourier series coefficients of their convolution (which is also periodic with

T_0) are proportional to the product of their individual Fourier series coefficients.

- Specifically, if $z(t) = x(t) * y(t)$ (periodic convolution), then $Z[k] = T_0 X[k] Y[k]$.
- *(Note: The phrase "of the RL circuit" is unusual in this context. Convolution property is a fundamental property of Fourier Series applicable to any periodic signals, not specific to RL circuits, unless it implies convolution in the time domain corresponds to multiplication in the frequency domain for circuits analysis.)*

- **Proof:**
  - Let $x(t)$ and $y(t)$ be periodic signals with period T_0.
  - Their Fourier series coefficients are given by: $X[k] = \frac{1}{T_0} \int_{T_0} x\,(t) e^{-jk\omega_0 t} dt$ $Y[k] = \frac{1}{T_0} \int_{T_0} y\,(t) e^{-jk\omega_0 t} dt$
  - Let $z(t) = x(t) * y(t)$ be their periodic convolution, which is defined as: $z(t) = \int_{T_0} x\,(\tau) y(t-\tau) d\tau$ (where the integration is over one period).

- Now, let's find the Fourier series coefficients of $z(t)$, denoted as $Z[k]$: $Z[k] = \frac{1}{T_0}\int_{T_0} z(t)e^{-jk\omega_0 t}dt$ $Z[k] = \frac{1}{T_0}\int_{T_0} \left(\int_{T_0} x(\tau)y(t-\tau)d\tau\right)e^{-jk\omega_0 t}dt$
- Change the order of integration: $Z[k] = \frac{1}{T_0}\int_{T_0} x(\tau)\left(\int_{T_0} y(t-\tau)e^{-jk\omega_0 t}dt\right)d\tau$
- Consider the inner integral: $\int_{T_0} y(t-\tau)e^{-jk\omega_0 t}dt$. Let $\lambda = t-\tau$. Then $t = \lambda+\tau$, and $dt = d\lambda$. When t goes over one period, λ also goes over one period. $\int_{T_0} y(\lambda)e^{-jk\omega_0(\lambda+\tau)}d\lambda = \int_{T_0} y(\lambda)e^{-jk\omega_0\lambda}e^{-jk\omega_0\tau}d\lambda = e^{-jk\omega_0\tau}\int_{T_0} y(\lambda)e^{-jk\omega_0\lambda}d\lambda$ We know that $\int_{T_0} y(\lambda)e^{-jk\omega_0\lambda}d\lambda = T_0Y[k]$. So, the inner integral is $e^{-jk\omega_0\tau}T_0Y[k]$.
- Substitute this back into the expression for $Z[k]$: $Z[k] = \frac{1}{T_0}\int_{T_0} x(\tau)(e^{-jk\omega_0\tau}T_0Y[k])d\tau$ $Z[k] = Y[k]\int_{T_0} x(\tau)e^{-jk\omega_0\tau}d\tau$ We know that $\int_{T_0} x(\tau)e^{-jk\omega_0\tau}d\tau = T_0X[k]$. Therefore: $Z[k] = Y[k](T_0X[k])$ $Z[k] = T_0X[k]Y[k]$

- This proves the convolution property for Fourier series.

(b) Find the phase response and magnitude response $X(s) = \frac{s+4}{s^2+3s+5}$.

- **Understanding $X(s)$:** $X(s)$ is a Laplace transform, not directly a frequency response unless $s = j\omega$. If $X(s)$ represents the transfer function $H(s)$, then we can find the frequency response.
- **Assuming $X(s)$ is the Transfer Function $H(s)$:**
  - Let $H(s) = \frac{s+4}{s^2+3s+5}$.
  - To find the frequency response, substitute $s = j\omega$: $H(j\omega) = \frac{j\omega+4}{(j\omega)^2+3(j\omega)+5}$ $H(j\omega) = \frac{4+j\omega}{-\omega^2+j3\omega+5}$ $H(j\omega) = \frac{4+j\omega}{(5-\omega^2)+j3\omega}$
- **Magnitude Response ($|H(j\omega)|$):** The magnitude of a complex number $a + jb$ is $\sqrt{a^2 + b^2}$. $|H(j\omega)| = \frac{|4+j\omega|}{|(5-\omega^2)+j3\omega|}$ $|H(j\omega)| = \frac{\sqrt{4^2+\omega^2}}{\sqrt{(5-\omega^2)^2+(3\omega)^2}}$ $|H(j\omega)| = \frac{\sqrt{16+\omega^2}}{\sqrt{25-10\omega^2+\omega^4+9\omega^2}}$ $|H(j\omega)| = \frac{\sqrt{16+\omega^2}}{\sqrt{\omega^4-\omega^2+25}}$
- **Phase Response ($\angle H(j\omega)$):** The phase of a complex number $a + jb$ is $\arctan(b/a)$. The phase of a quotient is the phase of the numerator minus the phase of the

denominator. $\angle H(j\omega) = \angle(4 + j\omega) - \angle((5 - \omega^2) + j3\omega)$
$\angle H(j\omega) = \arctan\left(\frac{\omega}{4}\right) - \arctan\left(\frac{3\omega}{5-\omega^2}\right)$

- **Phase Response:** $\angle H(j\omega) = \arctan\left(\frac{\omega}{4}\right) - \arctan\left(\frac{3\omega}{5-\omega^2}\right)$
- **Magnitude Response:** $|H(j\omega)| = \frac{\sqrt{16+\omega^2}}{\sqrt{\omega^4-\omega^2+25}}$

7. (a) Find initial value $x(0)$ of the signal $x(t)$ if $X(s) = \frac{s+4}{s^2+3s+5}$.
   - **Initial Value Theorem:** For a signal $x(t)$ whose Laplace Transform is $X(s)$, the initial value $x(0^+)$ can be found using the Initial Value Theorem, provided that the limit exists: $x(0^+) = \lim_{s\to\infty} sX(s)$
   - **Applying the theorem to** $X(s) = \frac{s+4}{s^2+3s+5}$**:** $x(0^+) = \lim_{s\to\infty} s \cdot \frac{s+4}{s^2+3s+5}$ $x(0^+) = \lim_{s\to\infty} \frac{s(s+4)}{s^2+3s+5}$ $x(0^+) = \lim_{s\to\infty} \frac{s^2+4s}{s^2+3s+5}$
   - To evaluate this limit, divide both numerator and denominator by the highest power of s (which is s^2): $x(0^+) = \lim_{s\to\infty} \frac{\frac{s^2}{s^2}+\frac{4s}{s^2}}{\frac{s^2}{s^2}+\frac{3s}{s^2}+\frac{5}{s^2}}$ $x(0^+) = \lim_{s\to\infty} \frac{1+\frac{4}{s}}{1+\frac{3}{s}+\frac{5}{s^2}}$
   - As $s \to \infty$, terms like $\frac{4}{s}$, $\frac{3}{s}$, and $\frac{5}{s^2}$ all approach 0. $x(0^+) = \frac{1+0}{1+0+0} = 1$

- **Initial Value:** $x(0) = 1$.

(b) Find the delta response of the LTI system described by transfer function $H(s) = \frac{s+4s+5}{s^2+6}$.

- **Understanding Delta Response:**
  - The delta response of an LTI system is its output when the input is a Dirac delta function $\delta(t)$.
  - In the Laplace domain, the Laplace transform of $\delta(t)$ is 1.
  - If the input is $X(s) = \mathcal{L}\{\delta(t)\} = 1$, then the output $Y(s)$ is given by $Y(s) = H(s)X(s) = H(s) \cdot 1 = H(s)$.
  - Therefore, the delta response (which is the impulse response $h(t)$) is simply the inverse Laplace transform of the transfer function $H(s)$.
- **Given Transfer Function (assuming correct interpretation of fragments):** $H(s) = \frac{s^2+4s+5}{s^2+6}$
- **Find Inverse Laplace Transform of $H(s)$:**
  - Perform polynomial long division or rewrite $H(s)$ to simplify: $H(s) = \frac{s^2+6+4s-1}{s^2+6} = \frac{s^2+6}{s^2+6} + \frac{4s-1}{s^2+6}\, H(s) = 1 + \frac{4s-1}{s^2+6}\, H(s) = 1 + \frac{4s}{s^2+6} - \frac{1}{s^2+6}$

- Now, use standard inverse Laplace transform pairs:
  - $\mathcal{L}^{-1}\{1\} = \delta(t)$
  - $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos(at)u(t)$
  - $\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin(at)u(t)$
- For the term $\frac{4s}{s^2+6}$: Here $a^2 = 6 \Longrightarrow a = \sqrt{6}$. $\mathcal{L}^{-1}\left\{\frac{4s}{s^2+6}\right\} = 4\cos(\sqrt{6}t)u(t)$
- For the term $-\frac{1}{s^2+6}$: Here $a^2 = 6 \Longrightarrow a = \sqrt{6}$. We need 'a' in the numerator. $-\frac{1}{s^2+6} = -\frac{1}{\sqrt{6}} \cdot \frac{\sqrt{6}}{s^2+6}$ $\mathcal{L}^{-1}\left\{-\frac{1}{s^2+6}\right\} = -\frac{1}{\sqrt{6}}\sin(\sqrt{6}t)u(t)$
- Combine all terms to find $h(t)$: $h(t) = \delta(t) + 4\cos(\sqrt{6}t)u(t) - \frac{1}{\sqrt{6}}\sin(\sqrt{6}t)u(t)$

- **Delta Response:** $h(t) = \delta(t) + \left(4\cos(\sqrt{6}t) - \frac{1}{\sqrt{6}}\sin(\sqrt{6}t)\right)u(t)$.

(c) A LTI system is described by the differential equation $\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} = \frac{dx(t)}{dt} + x(t)$. For initial conditions $y(0) = 3$, $y'(0) = 1$ and $x(t) = u(t)$, Find the transfer function and the output signal $y(t)$.

- **1. Find the Transfer Function $H(s)$:**
    - Take the Laplace Transform of both sides of the differential equation, assuming zero initial conditions for the transfer function derivation (input is $\delta(t)$): $\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} + 8\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = \mathcal{L}\left\{\frac{dx(t)}{dt}\right\} + \mathcal{L}\{x(t)\}$
    $s^2Y(s) + 8sY(s) = sX(s) + X(s)$
    - Factor out $Y(s)$ and $X(s)$: $Y(s)(s^2 + 8s) = X(s)(s + 1)$
    - The transfer function $H(s) = \frac{Y(s)}{X(s)}$ is: $H(s) = \frac{s+1}{s^2+8s} = \frac{s+1}{s(s+8)}$
- **Transfer Function:** $H(s) = \frac{s+1}{s(s+8)}$.
- **2. Find the Output Signal $y(t)$:**
    - Take the Laplace Transform of the differential equation, including initial conditions: $\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0)$ $\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = sY(s) - y(0)$
    $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0)$
    - Given: $y(0) = 3$, $y'(0) = 1$, and $x(t) = u(t)$. From $x(t) = u(t)$, $X(s) = \frac{1}{s}$ and $x(0) = u(0) = 1$.

- Substitute these into the transformed equation: $(s^2Y(s) - s(3) - 1) + 8(sY(s) - 3) = (s(\frac{1}{s}) - 1) + \frac{1}{s}$ $s^2Y(s) - 3s - 1 + 8sY(s) - 24 = 1 - 1 + \frac{1}{s}$ $Y(s)(s^2 + 8s) - 3s - 25 = \frac{1}{s}$ $Y(s)(s^2 + 8s) = \frac{1}{s} + 3s + 25$ $Y(s)(s^2 + 8s) = \frac{1+3s^2+25s}{s}$ $Y(s) = \frac{3s^2+25s+1}{s(s^2+8s)} = \frac{3s^2+25s+1}{s^2(s+8)}$
- **Partial Fraction Expansion of** $Y(s)$**:** $Y(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+8}$ $3s^2 + 25s + 1 = A(s + 8) + Bs(s + 8) + Cs^2$
- To find A: Set $s = 0$: $1 = A(8) \Longrightarrow A = \frac{1}{8}$
- To find C: Set $s = -8$: $3(-8)^2 + 25(-8) + 1 = C(-8)^2$ $3(64) - 200 + 1 = 64C$ $192 - 200 + 1 = 64C$ $-7 = 64C \Longrightarrow C = -\frac{7}{64}$
- To find B: Equate coefficients of s^2 (or pick another value for s, e.g., $s = 1$): Coeff of s^2: $3 = B + C$ $3 = B - \frac{7}{64}$ $B = 3 + \frac{7}{64} = \frac{192+7}{64} = \frac{199}{64}$
- So, $Y(s) = \frac{1/8}{s^2} + \frac{199/64}{s} - \frac{7/64}{s+8}$
- **Find Inverse Laplace Transform of** $Y(s)$**:**

- $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = tu(t)$
- $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$
- $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}u(t)$

$y(t) = \frac{1}{8}tu(t) + \frac{199}{64}u(t) - \frac{7}{64}e^{-8t}u(t)$ $y(t) = \left(\frac{1}{8}t + \frac{199}{64} - \frac{7}{64}e^{-8t}\right)u(t)$

- **Output Signal:** $y(t) = \left(\frac{1}{8}t + \frac{199}{64} - \frac{7}{64}e^{-8t}\right)u(t).$