Scientific Machine Learning
Final Project

Lovnesh Bhardwaj, Laurynas Varnas

Finite Element Method

Problem definition

Monodomain equation

$$\begin{aligned} \frac{\partial u}{\partial t} - \nabla \cdot \Sigma \nabla u + f(u) &= 0 && \text{in } \Omega \times I, \\ \mathbf{n} \cdot \nabla u &= 0 && \text{on } \partial\Omega \times I, \\ u &= u_0 && \text{in } \Omega \times \{0\}, \end{aligned}$$

IMEX

With $U^n(x) \approx u(x, t_n)$

$$\frac{U^{n+1} - U^n}{\Delta t} = \nabla \cdot \Sigma \nabla U^{n+1} - f(U^n)$$

$$U^{n+1} - \Delta t \nabla \cdot \Sigma \nabla U^{n+1} = U^n - \Delta t\, f(U^n)$$

Algebraic formulation of the FEM discretization

$$(\mathbf{M} + \Delta t\, \mathbf{K})\, \mathbf{u}^{n+1} = \mathbf{M}\, \mathbf{u}^{n} - \Delta t\, \mathbf{f}(U^n),$$

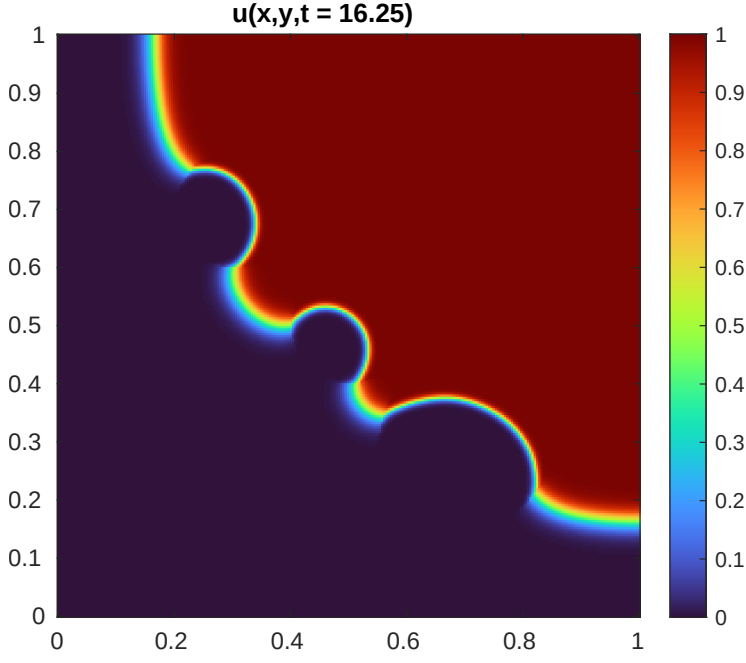
where:

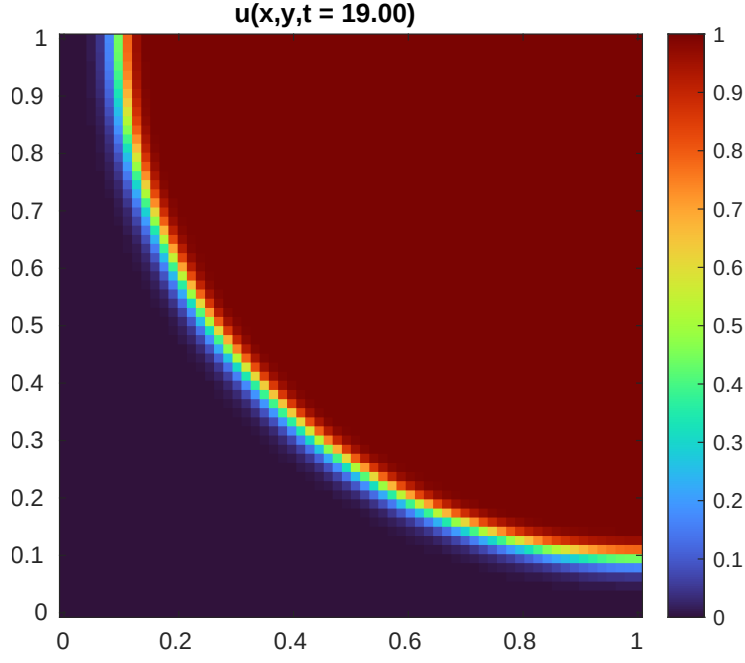
$$\mathbf{M}_{ij} = \int_\Omega \phi_i(x)\phi_j(x)\, \mathrm{d}x \qquad \text{(mass matrix)},$$

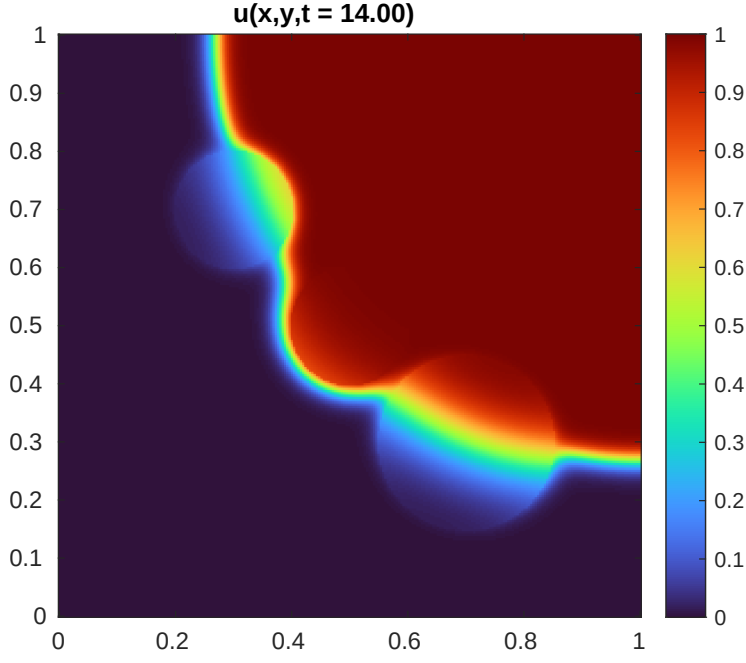
$$\mathbf{K}_{ij} = \int_\Omega \Sigma(x)\nabla\phi_i(x) \cdot \nabla\phi_j(x)\, \mathrm{d}x \qquad \text{(stiffness matrix)},$$

$$\mathbf{f}_i(U^n) = \int_\Omega f(U^n(x))\phi_i(x)\, \mathrm{d}x \qquad \text{(nonlinear load vector)}.$$

FEM Results


u(x,y,t = 16.25)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1


u(x,y,t = 19.00)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1


u(x,y,t = 14.00)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

$\Sigma_d = 10$

Δt	n_e	Activation time	M-matrix?	$u \in [0,1]$
0.1	64	28.60	**true**	false
0.1	128	29.10	**true**	false
0.1	256	29.20	**true**	false
0.05	64	26.60	false	false
0.05	128	27.15	**true**	**true**
0.05	256	27.20	**true**	**true**
0.025	64	25.60	false	false
0.025	128	26.10	**true**	**true**
0.025	256	26.175	**true**	**true**

$\Sigma_d = 1$

Δt	n_e	Activation time	M-matrix?	$u \in [0,1]$
0.1	64	30.20	**true**	false
0.1	128	30.90	**true**	false
0.1	256	31.00	**true**	false
0.05	64	28.10	false	false
0.05	128	28.80	**true**	**true**
0.05	256	28.90	**true**	**true**
0.025	64	27.025	false	false
0.025	128	27.70	**true**	**true**
0.025	256	27.775	**true**	**true**

$\Sigma_d = 0.1$

Δt	n_e	Activation time	M-matrix?	$u \in [0,1]$
0.1	64	31.70	false	false
0.1	128	32.50	false	false
0.1	256	32.60	**true**	false
0.05	64	29.55	false	false
0.05	128	30.25	false	false
0.05	256	30.40	false	**true**
0.025	64	28.40	false	false
0.025	128	29.075	false	false
0.025	256	29.225	false	**true**