Wiley Plus 5.4

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1

Explanation:
The integral of acceleration $a(t)$ with respect to time t over a certain interval gives the net change in velocity during that time period. This is because velocity is the time integral of acceleration.
In the given formula, the integral represents the net change in velocity from $t = 0$ to $t = 11$. The acceleration $a(t)$ is given in km/hr^2 and time t is in hours. when integrating acceleration with respect to time, the unit will be km/hr corresponds to the unit of velocity.
Answer: The integral represents the net change in velocity from $t = 0$ to $t = 11$. Unit: km/hr

Insights:
Students might confuse acceleration with velocity and think the integral gives the total distance traveled rather than the change in velocity.

2

Explanation:

(a) The statement $f(10) = 400$ means on day 10, pollution is removed from the lake at a rate of 400 kg/day. This value represents the rate at which pollution is removed on that specific day.

(b) This integral represents the total amount of pollution removed between days 5 and 17.
Units of 5 and 17: These are days because they represent specific points in time (days 5 and 17).
Units of 3800: The units of an integral is a rate (pollution removed per day) over time (days). Since $f(t)$ is in kg per day, integrating it over time gives kilograms. Therefore, the units of 3800 are kg.

(c) The statement $\int_5^{13} f(t)\,dt$ means that during the 8 days from day 5 to day 13, a total of 3800 kg were removed from the lake.

Insights:
Students might confuse $f(14) = 600$ as representing the total amount of pollution removed by day 14, rather than just the rate at which pollution is being removed on day 14. However, $f(t)$ represents the rate so $f(14) = 600$ indicates how fast pollution is being removed on that specific day.

3

Explanation:

(a) - Gas produced in 2004: Since t represents the number of years since 2004, when $t = 0$, the year is 2004. Substitute $t = 0$ into the given N formula: $N = 2711 + 77(0) = 2711$ billion cubic meters.
- Gas produced in 2015: For year 2015, $t = 2015 - 2024 = 11$ (2015 is 11 years after the starting year). Substitute $t = 11$ into the given N formula: $N = 2711 + 77(11) = 3558$ billion cubic meters.

(b) To find the total amount of natural gas produced during 6-year period from 2004 to 2010, we need to integrate the production function over the interval from 0 to 6:
$\int_0^6 (2711 + 77t)dt$
$= \left[2711t + \frac{77t^2}{2}\right]_0^6$
$= 2711(6) + \frac{77t^2}{2}(6^2) - 2711(0) - \frac{77t^2}{2}(0^2) = 17652$

Insights

1. For part a, students might mistakenly plug in the actual year (2004 or 2015) directly into the formula, rather than converting it to the correct t value (the number of years since 2004).

2. For part b, students might not understand that the integral represents the total production over a period and may mistakenly try to compute just the production at $t = 6$.

4

Explanation:

- Change in position: Change in position is the net change in the particle's position, considering the direction of motion. For a velocity vs. time graph, this is equivalent to finding the area under the curve. To find the change in position in this graph, we need to calculate the net area between $t = 0$ and $t = 5$ under the curve (for this situation, under t-axis). The area is below the t-axis represents movement to the left.
Answer: The change in position is $8 \cdot 9 = 72$ cm to the left.

- Total distance traveled: Total distance traveled is the sum of the absolute values of the areas under the velocity graph, irrespective of direction (positive or negative velocity). Answer: The total distance traveled is $8 \cdot 9 = 72$ cm

Insights:

1. Students might mistake displacement for total distance traveled. They might calculate the area under the curve (net change in position) and assume it's the total distance, not accounting for the fact that distance cannot be negative. The total distance must include the absolute values of areas, while displacement takes the sign into account.

2. Students might not interpret the negative velocity correctly. When the velocity graph is below the t-axis, this means the particle is moving left (negative direction).

3. Students might confuse between change in position and total distance traveled. It is important to notice that the change in position takes direction into account while the total distance traveled does not.

5

Explanation:
This is a rate of change problem where water is being removed from a tank. To find how much water remains after one hour, we need to integrate the rate of change over time from $t = 0$ to $t = 60$ to determine the total amount of water pumped out during that period.
Total water pumped out: $\int_0^{60}(6 - 6e^{-0.11t})dt = [6t]_0^{60}$ - $\left[\frac{-600e^{-0.11t}}{11}\right]_0^{60}$
$= 305.5$
Water remaining: The tank started with 1000 liters, so the amount of water remaining after one hour is: $1000 - 305.5 = 694.5$
Answer: 694.5
Insights:

1. Students might mistakenly try to plug in $t = 60$ into the rate equation but we actually need to integrate the rate over time to get the total amount of water removed.

2. After finding the total water pumped out, students might forget to subtract it from the initial 1000 liters to find how much water remains.