1.8 HW

Question 1:

If $f(x) = 3\,x - 2$ and $g(x) = 3\,x + 9$, find the following.

(a) $f(g(x)) =$

(b) $g(f(x)) =$

(c) $f(f(x)) =$

Solution: Remember how, for example, to find $f(9)$ from some given function $f(x)$, we have to replace all instances of x with 9 and evaluate. For these problems, instead of replacing x in $f(x)$ with a number, we are replacing it with *another function*.

So, for (a), $f(g(x))$ is equal to $f(3x + 9)$ (since $g(x)$ is given as $3x + 9$). So, plugging in $3x + 9$ for all instances of x will give us the following:

$$f(g(x)) = f(3x + 9) = 3(3x + 9) - 2 = 9x + 25$$

Repeating a similar process for (b):

$$g(f(x)) = g(3x - 2) = 3(3x - 2) + 9 = 9x + 3$$

Part (c) is a little trickier, but all you need to do is plug $f(x)$ in as the input of $f(x)$! Don't worry about there being two instances of f here, this will still make a valid composite function:

$$f(f(x)) = f(3x - 2) = 3(3x - 2) - 2 = 9x - 8$$

Question 2:

Let $f(x) = x^2$ and $g(x) = 6x - 16$.

Find the following:

(a) $f(3) + g(3) =$

(b) $f(3) \cdot g(3) =$

(c) $f(g(3)) =$

(d) $g(f(3)) =$

Solution: We know that to evaluate a function at a specific value of x, we can plug in that value for all instances of x in order to find an answer.

For part (a), we need to find $f(3)$ and $g(3)$, and add them together:

$$f(3) = (3)^2 = 9$$
$$g(3) = 6(3) - 16 = 2$$
$$f(3) + g(3) = 9 + 2 = 11$$

Now, we can easily solve part (b) since we already found $f(3)$ and $g(3)$ in the previous part:

$$f(3) \cdot g(3) = 9 \cdot 2 = 18$$

Part (c) takes an approach similar to that of Question 1. We found that $g(3) = 2$ already, meaning that this part is really asking us to find $f(2)$:

$$f(g(3)) = f(2) = (2)^2 = 4$$

Similarly, for part (d):

$$g(f(3)) = g(9) = 6(9) - 16 = 38$$

Question 3:

For $g(x) = x^2 + 8\,x + 7$, find and simplify fully $g(7+h) - g(7)$.

(a) $g(7+h) =$

(b) $g(7) =$

(c) $g(7+h) - g(7) =$

Solution: In order to find $g(7 + h) - g(7)$, we need to find both $g(7 + h)$ and $g(7)$, following similar steps as the previous questions. Do not let this h fool you, our final answer will have an h in it, as $g(7 + h)$ will output a function of h:

$$g(7 + h) = (7 + h)^2 + 8(7 + h) + 7 = 49 + 14h + h^2 + 56 + 8h + 7 = h^2 + 22h + 112$$

$$g(7) = (7)^2 + 8(7) + 7 = 112$$

Adding them together:

$$g(7 + h) - g(7) = h^2 + 22h + 112 - 112 = h^2 + 22h$$

Question 4:

If $f(x) = x^2 + 5$, find and simplify fully:

(a) $f(t+1) =$

(b) $f(t^2+1) =$

(c) $f(2) =$

(d) $2f(t) =$

(e) $(f(t))^2 + 1 =$

Solution: Using the skills used in the previous questions, we can easily solve parts (a) through (c), just like how in Question 3, we were not concerned with there being a variable other than x inside the parentheses, we are not concerned that there is a t here:

$$f(t + 1) = (t + 1)^2 + 5 = t^2 + 2t + 6$$
$$f(t^2 + 1) = (t^2 + 1)^2 + 5 = t^4 + 2t^2 + 6$$
$$f(2) = (2)^2 + 5 = 9$$

Parts (d) and (e) are a little trickier. Starting with (d): first, notice that instead of an x in the parentheses of the function, there is a t, meaning we need to replace all instances of x with t in the given $f(x)$:

$$f(t) = t^2 + 5$$

Now, we can find $2f(t)$ by multiplying this result by 2. Be careful to not find $f(2t)$; since the 2 is not inside the parentheses for the function, we are not plugging it into it. $2f(t) \neq f(2t)$. In doing this we find:

$$2f(t) = 2(t^2 + 5) = 2t^2 + 10$$

Now for part (e), notice that once again, the only thing in the parentheses is t. This means we have to use $f(t)$, which we already found in part (d). All other operations are outside the function input, so we can simplify from there:

$$(f(t))^2 + 1 = (t^2 + 5)^2 + 1 = t^4 + 10t^2 + 2$$

Question 5:

Use the table below to find the following values.

x	1	2	3	4	5	6
$f(x)$	5	3	3	4	5	6
$g(x)$	6	5	4	4	1	1

(a) $f(g(1)) =$

(b) $g(f(1)) =$

(c) $f(g(4)) =$

(d) $g(f(4)) =$

(e) $f(g(6)) =$

(f) $g(f(6)) =$

Solution: For this question, the best approach is to use an "inside-out" strategy. What I mean by this is, for example, notice how part (a) asks us to find $f(g(1))$. By working inside out, we can first find $g(1)$ from the table, which equals 6. Then, replacing $g(1)$ with 6 in $f(g(1)$, we can now see that we just need to find $f(6)$, which equals 6. Repeating a similar process for the remaining parts, we find:

$$\begin{aligned} g(f(1)) &= g(5) = 1 \\ f(g(4)) &= g(4) = 4 \\ g(f(4)) &= g(4) = 4 \\ f(g(6)) &= f(1) = 5 \\ g(f(6)) &= g(6) = 1 \end{aligned}$$

Question 6:

Use the variable u for the inside function to express each of the following as a composite function:

(a) $y = 5^{6x-1}$ **(b)** $P = \sqrt{8t^2 + 16}$ **(c)** $w = 11\ln(13r + 14)$

Solution: We know that a composite function usually takes the form $f(g(x))$. In other words, f is a function of $g(x)$. For example, part (a) is asking what combination of $f(x)$ and $g(x)$ we can use in order to create a function 5^{6x-1}. Here, u is that inside function g.

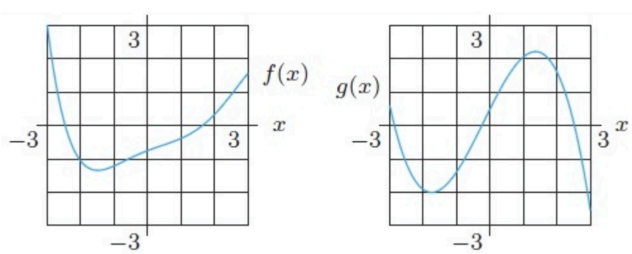
By looking at it, we can see that the power that 5 is raised to, $6x - 1$, kind of looks like another function. If we try setting $u = 6x - 1$, we can rewrite the given y as $y = 5^u$. This is one of the answer choices! Since $u = 6x - 1$ and $y = 5^u$, plugging in $6x - 1$ for u will just give us the given equation from the problem.

Repeating a similar process for part (b), we find that $u = 8t^2 + 16$, so P can be written as $P = \sqrt{u}$. This is one of the answer choices.

Again for part (c), $13r + 14$ looks like a function itself, so we can make that equal to u, making $w = 11\ln(u)$. This is one of the answer choices.

Question 7:

Use the following graphs to estimate $f(g(0))$.



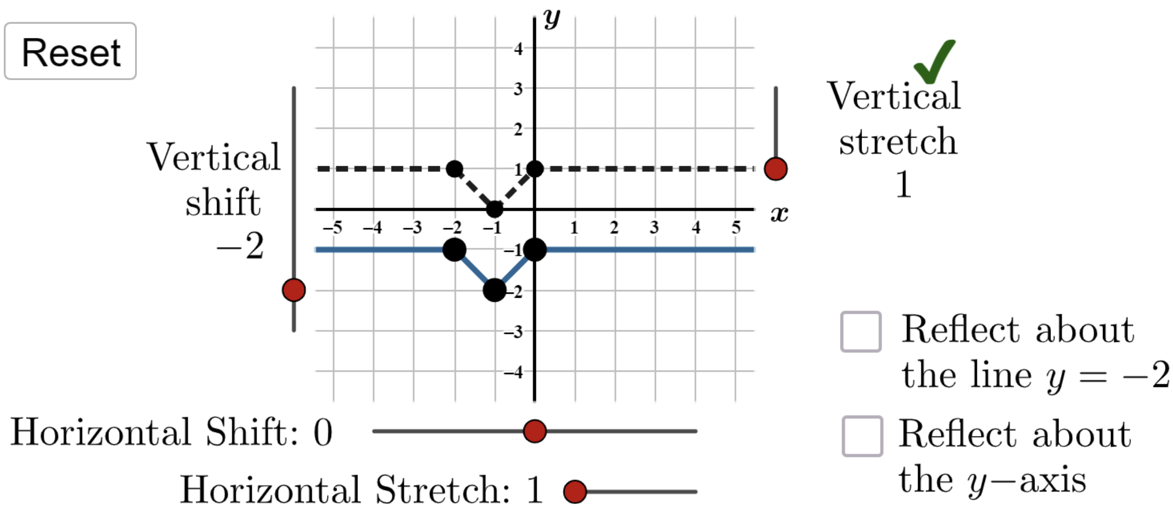
Solution: This is similar to question 5, except this time we are looking at a graph instead. Looking at the graph for $g(x)$, we can see that at $x = 0$, $g \approx 0.5$. Now, we just need to find $f(0.5)$, which by looking at the graph, looks to be ≈ -0.5.

Question 8:

The function $f(x)$ is shown in the figure. Use the controls to transform $f(x)$ to graph $y = f(x) - 2$.

NOTE: Use the sliders below to transform the graph.

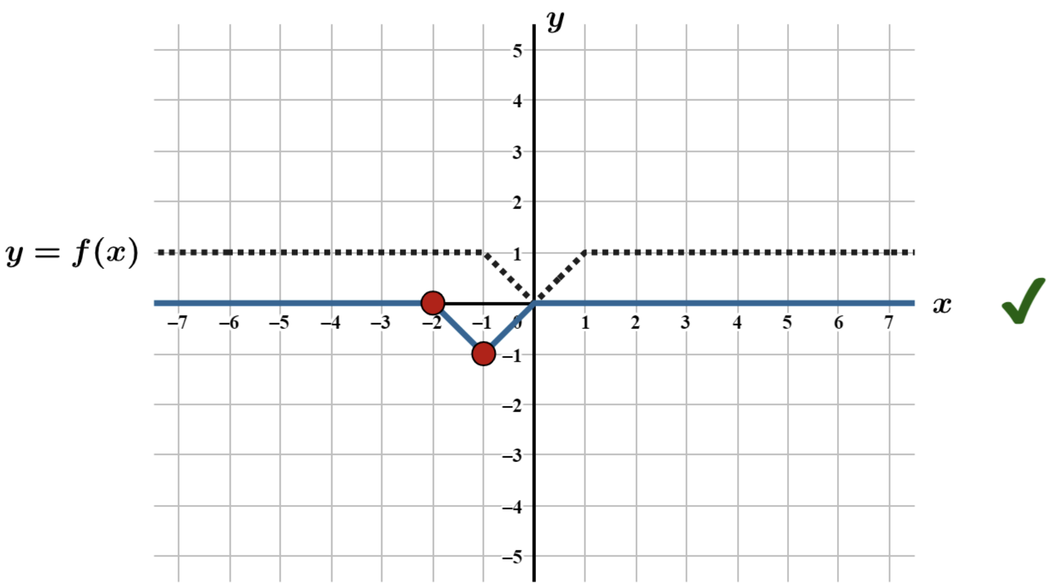
Solution: The only modification of $y = f(x)$ in this question is subtraction by 2, indicating a vertical shift down by 2. So, let's move the entire function down 2 units.



Question 9:

Use the graph of $y = f(x)$ below to graph the function $y = f(x+1) - 1$

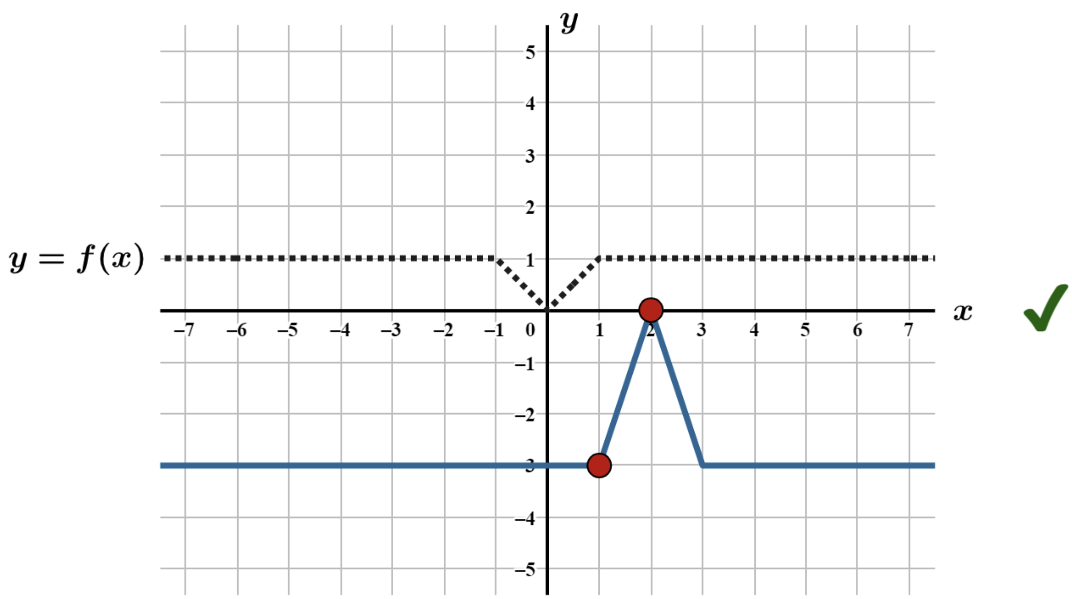
Solution: Like the previous question, we have a subtraction by 1 that indicates a vertical shift, down 1 unit. However in addition to that, we have addition by 1 *inside* the parentheses for $f(x)$. This indicates a horizontal shift *left* by 1 unit. Be careful to remember that when dealing with horizontal shifts, addition indicates a shift to the left, and subtraction indicates a shift to the right.



Question 10:

Use the graph of $y = f(x)$ below to graph the function $y = -3f(x-2)$

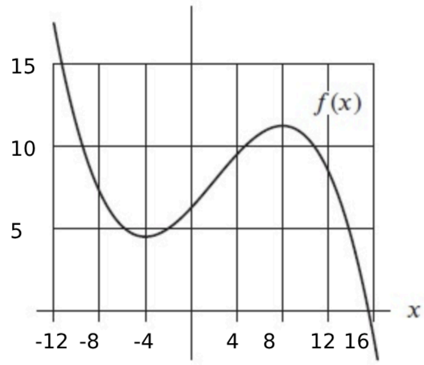
Solution: First, notice that we have subtraction by 2 inside the parentheses, indicating a horizontal shift right 2 units. Then, notice the negative sign in front of the whole function, meaning we flip it about the x-axis. Finally, a vertical stretch by a factor of 3 means that the y-value of the point we are allowed to modify in Wiley, $(1, -1)$, is multiplied by 3, moving it to $(1, -3)$.



4.3 HW

Question 1:

For $f(x)$ in the figure below, find the x-values of the global maximum and global minimum on the domain $-8 \leq x \leq 4$.



Solution: In order to solve this, we just need to look at the graph and find the x value of the points where the maximum and minimum values of $f(x)$ are, ONLY from $x = -8$ to $x = 4$. By looking, we can see that the maximum on this interval is at $x = 4$, and the minimum is at $x = -4$.

Question 2:

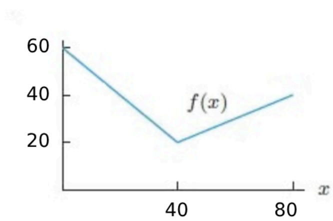
For each interval, use the figure below to choose the statement that gives the location of the global maximum and global minimum of f on the interval.

(I) Maximum at right endpoint, minimum at left endpoint.

(II) Maximum at right endpoint, minimum at critical point.

(III) Maximum at left endpoint, minimum at right endpoint.

(IV) Maximum at left endpoint, minimum at critical point.



(a) The statement that gives the location of the global maximum and global minimum of f on the interval $19 \leq x \leq 42$ is

(b) The statement that gives the location of the global maximum and global minimum of f on the interval $41 \leq x \leq 61$ is

(c) The statement that gives the location of the global maximum and global minimum of f on the interval $19 \leq x \leq 39$ is

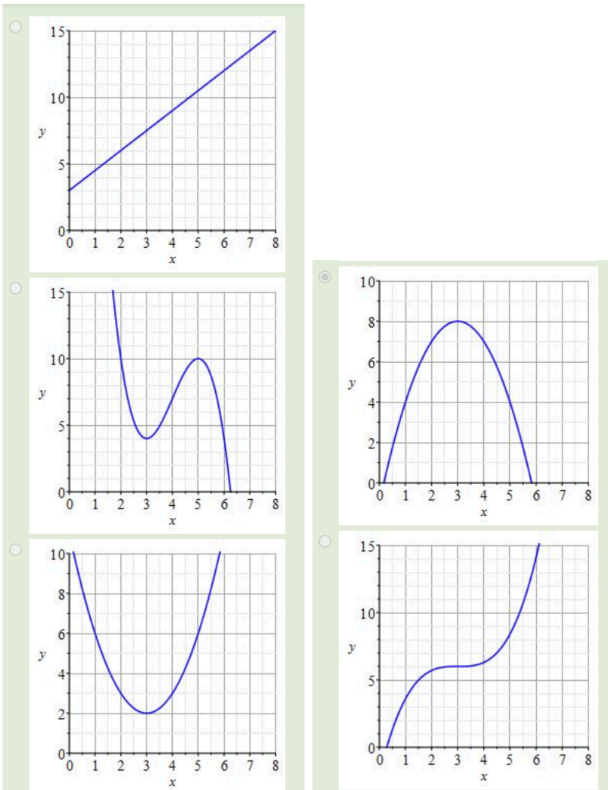
(d) The statement that gives the location of the global maximum and global minimum of f on the interval $38 \leq x \leq 78$ is

Solution: Like the previous question, we need to make sure we are ONLY looking at the given interval when trying to find a solution for each part. Note that the *critical point* refers to the point where the function changes from decreasing to increasing, at $x = 40$. Looking at the graph in the given intervals, we can find that statement (IV) describes part (a), statement (I) describes part (b), statement (III) describes part (c), and statement (II) describes part (b).

Question 3:

Select the correct graph of a function that has a local maximum and global maximum at $x = 3$ but no local or global minimum.

Assume that each graph continues beyond the window shown.



Solution: We are looking for the graph of a function that has a local AND global maximum at $x = 3$. We can eliminate the first graph, as the function takes values both greater than and less than $f(3)$. We can then eliminate the second graph, since there is a local minimum at $x = 3$. The third graph has a local and global MINIMUM at $x = 3$, but not a maximum. The fourth graph is our answer. Since we can assume that these functions continue beyond the window shown, it can be concluded that the maximum value the function takes, both locally and globally, is at $x = 3$. The fifth graph can also be eliminated as it is in a similar situation to the first, the function takes values both greater and less than $f(3)$ at other points.

Question 4:

Given the function $f(x) = x^3 + 3x^2 - 24x + 12$ over the interval $-8 \leq x \leq 4$, answer the following questions.

Solution: For part (a), to find both $f'(x)$ and $f''(x)$, we can use the power rule for each term:

$$f'(x) = \frac{d}{dx}(f(x)) = 3x^2 + 6x - 24$$
$$f''(x) = \frac{d}{dx}(f'(x)) = 6x + 6$$

Now, for part (b), to find critical points of f, we can set the first derivative f' equal to zero, and solve for x using the quadratic formula:

$$3x^2 + 6x - 24 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-24)}}{2(3)}$$
$$x = -4,\ x = 2$$

So, f has critical points at $x = -4$ and $x = 2$. Now for part (c), we can find inflection points of f by setting the second derivative f'' equal to zero:

$$6x + 6 = 0$$
$$6x = -6$$
$$x = -1$$

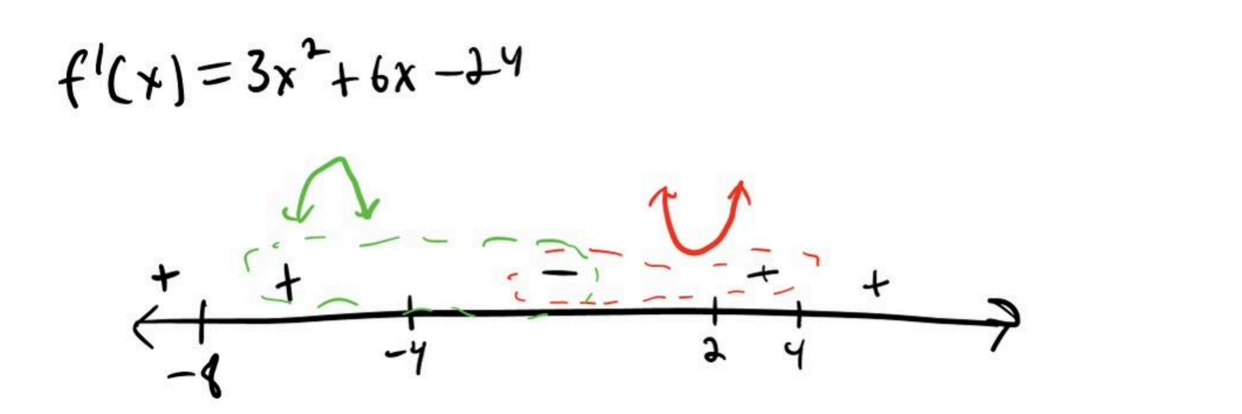
So, we can conclude that f has an inflection point at $x = -1$. Be careful with the difference between a critical point and an inflection point; critical points correspond to the first derivative while inflection points correspond to the second derivative.

Now for part (d), first we can evaluate the function at each of its critical points and the endpoints of the given interval. Plugging values into a calculator will give the following:

$$f(-8) = -116$$
$$f(-4) = 92$$
$$f(2) = -16$$

$$f(4) =- 28$$

Now we have to figure out if each of these points is a local or global maximum or minimum. We can already tell which ones are global extrema: $x =- 4$ is the global maximum and $x =- 8$ is the global minimum. As for the local extrema, we can create a number line, mark each critical / endpoint, and find out whether the first derivative is positive or negative *between* these points. This will tell us if the function is increasing or decreasing, which we can use to conclude the status of each point:



Thus, we have determined that $x = 2$ is a local minimum. Note that $x =- 4$ is also a local maximum since the values of f close to it on the interval are smaller.

Question 5:

Given the function $f(x) = x^3 - 3x^2 - 24x + 11$ over the interval $-6 \leq x \leq 5$, answer the following questions.

Solution: Following the same process from question 4:

$$f'(x) = 3x^2 - 6x - 24$$
$$f''(x) = 6x - 6$$

Finding critical points:

$$3x^2 - 6x - 24 = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-24)}}{2(3)}$$
$$x = -2,\ x = 4$$

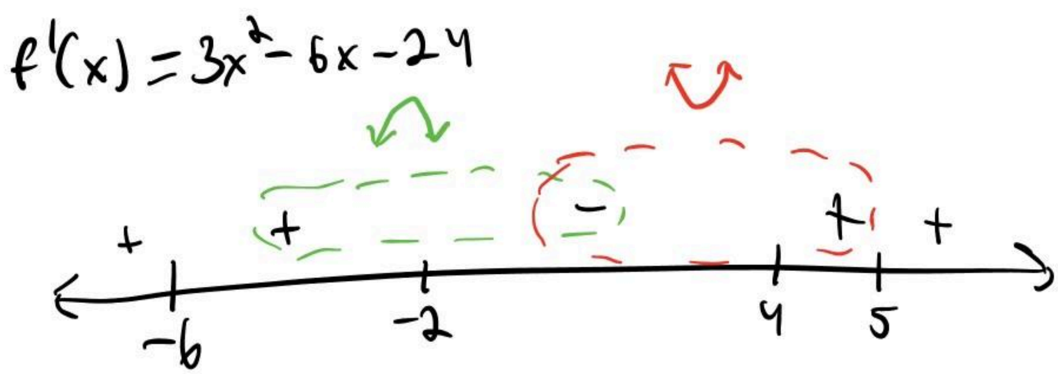
Finding inflection point:

$$6x - 6 = 0$$
$$x = 1$$

Evaluating f at endpoints & critical points:

$$f(-6) = -169$$
$$f(-2) = 39$$
$$f(4) = -69$$
$$f(5) = -59$$

Using the number line method:



Thus, $x = -6$ is a local & global minimum, $x = -2$ is a local & global maximum, $x = 4$ is a local minimum, and $x = 5$ is a local maximum.

Question 6:

Find the values of x that give critical points at $y = ax^2 + bx + c$, where a, b, c are constants. Under what conditions on a, b, c is the critical value a maximum? A minimum?

$x =$ [ ]

Use the drop-down menus to indicate whether the critical value is a maximum or a minimum under certain conditions.

The critical value is a [Choose one ▾] if $a < 0$ and a [Choose one ▾] if $a > 0$.

Solution: Since we are told that $a,\ b,\ c$ are constants, we can still take the first derivative of y in order to find the critical point for part (a). Differentiating y with respect to x gives us:

$$\frac{dy}{dx} = 2ax + b$$

Now, we can find the critical point of y by setting $\frac{dy}{dx} = 0$:

$$2ax + b = 0$$
$$2ax = -b$$
$$x = -\frac{b}{2a}$$

We know that $y = ax^2 + bx + c$ is a parabola since it has a degree of 2. The leading coefficient determines which way the parabola opens up: when it is positive, it opens upward, and when it is negative, it opens downward. In knowing this, we can conclude that when $a < 0$ the critical point is a maximum and when $a > 0$ it is a minimum.

Question 7:

The continuous function $h(x)$ has exactly one critical point. Find the x-values at which the global maximum and the global minimum occur in the interval $3 \le x \le 12$.

$h'(6)$ is undefined, $h'(x) = -1$ for $x < 6$ and $h'(x) = 1$ for $x > 6$.

Enter the exact answers.

The global maximum occurs when $x =$ [ ] .

The global minimum occurs when $x =$ [ ] .

Solution: We are told that $h'(6)$ is undefined, meaning this function $h(x)$ has a critical point at $x = 6$. We are also told that to the left of $x = 6$, the first derivative is equal to -1, and to the left of $x = 6$, the first derivative is equal to 1. This means that from the left endpoint to $x = 6$, the graph is going down, then starts traveling back up after $x = 6$. It starts at $x = 3$, goes down three units by $x = 6$, then goes up 6 units by $x = 12$. This means that the global minimum takes place at the critical point $x = 6$, and the global maximum takes place at the right endpoint $x = 12$.

Question 8:

A grapefruit is tossed straight up with an initial velocity of 50 ft/sec. The grapefruit is 5 feet above the ground when it is released. Its height at time t is given by

$y = -16t^2 + 50t + 5.$

How high does it go before returning to the ground?

Round your answer to one decimal place.

The maximum height of the grapefruit is [ ] feet.

Solution: When this grapefruit reaches its maximum height, it will stop moving for a brief moment. Since we have a function for its height, we can take the first derivative in order to figure out at what time t the grapefruit stops moving:

$$y = -16t^2 + 50t + 5$$
$$y' = -32t + 50$$

Finding critical point:

$$-32t + 50 = 0$$
$$-32t = -50$$
$$t = \frac{-50}{-32} = \frac{50}{32}$$

Now, we can plug this critical value of t into the given equation for the grapefruit's height to find its maximum height:

$$y_{max} = -16\left(\frac{50}{32}\right)^2 + 50\left(\frac{50}{32}\right) + 5 \approx 44.1 \text{ feet}$$

Question 9:

Consider the following function.

$h(z) = \frac{1}{z} + 9z^2$ for $z > 0$.

Select the exact global maximum and minimum values of the function.

- The global maximum of $h(z)$ on $z > 0$ does not exist, the global minimum is $\sqrt[3]{18} + \sqrt[3]{\frac{9}{4}}$
- The global maximum of $h(z)$ on $z > 0$ is $\frac{1}{18} + 729$, the global minimum is $\sqrt[3]{9} + \sqrt[3]{\frac{9}{4}}$
- The global maximum of $h(z)$ on $z > 0$ is $\frac{1}{18} + 729$, the global minimum is $\sqrt[3]{18} + \sqrt[3]{\frac{9}{4}}$
- The global maximum of $h(z)$ on $z > 0$ does not exist, the global minimum is $\sqrt[3]{18} + \sqrt[3]{\frac{9}{2}}$
- The global maximum of $h(z)$ on $z > 0$ is $\frac{1}{9} + 729$, the global minimum is $\sqrt[3]{9} + \sqrt[3]{\frac{9}{4}}$

Solution: Since we know we are looking for global extrema for this function h, we know we will need the first derivative:

$$h'(z) = -z^{-2} + 18z$$

Equating this to zero will give us the critical point of h:

$$\begin{aligned}
-z^{-2} + 18z &= 0 \\
-\frac{1}{z^2} + 18z &= 0 \\
18z &= \frac{1}{z^2} \\
18z^3 &= 1 \\
z^3 &= \frac{1}{18} \\
z &= \sqrt[3]{\frac{1}{18}}
\end{aligned}$$

Since the question is asking in part whether or not global extrema exist, we can use the second derivative to figure out the concavity of $h(z)$ for $z > 0$:

$$h''(z) = 2z^{-3} + 18 = \frac{2}{z^3} + 18$$

Notice how for any $z > 0$, the second derivative $h''(z)$ is positive. This means that h is concave up for all $z > 0$, moreover, $\lim_{z \to 0} h(z) = \lim_{z \to \infty} h(z) = \infty$. This means that there is no global maximum on the interval since the function keeps increasing and increasing as z approaches both zero and infinity. However, there will be a local minimum at the critical point:

$$h(\sqrt[3]{\frac{1}{18}}) = \frac{1}{\sqrt[3]{\frac{1}{18}}} + 9(\sqrt[3]{\frac{1}{18}})^2 = \sqrt[3]{18} + \sqrt[3]{\frac{9}{4}}$$

Question 10:

For $f(x) = x - \ln x$, and $0.1 \leq x \leq 3$, find the following.

Solution: We are asked to find values of x such that f local or maximum extrema. Our first step, like previous questions, is to take the first derivative:

$$f'(x) = 1 - \frac{1}{x}$$

Equating this to zero will give us the critical point of f:

$$1 - \frac{1}{x} = 0$$
$$1 = \frac{1}{x}$$
$$x = 1$$

Now, we can test f when x is equal to both endpoints and its critical point to find extrema:

x	$f(x)$	Type?
0.1	2.4	Local/Global Max
1	1	Local/Global Min
3	1.9	Local Max

Note that $x = 0.1$ and $x = 3$ are both local maximums since the function dips down between them, as shown at $x = 1$.