Wiley Plus 2.4

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Explanation:
The exercise requires us to compare $f''(1)$ and $f''(3)$ using the graph of $f(x)$. The second derivative of a function $f''(x)$ provides information about the concavity of a function. Specifically,
If $f''(x) > 0$, the function is concave up at that point ("U" shape)
If $f''(x) < 0$, the function is concave down at that point ($\cap$ shape)
Looking at the provided graph:
at $x = 1$, the graph is bending downwards ($\cap$ shape) $\rightarrow$ $f''(1)$ is negative.
at $x = 3$, the graph is bending upwards ("U" shape) $\rightarrow$ $f''(3)$ is positive.
Since a positive number is greater than a negative number, $f''(3) > f''(1)$.
The correct answer is $f''(3) > f''(1)$

Insights:

(1) Students may mistakenly use the first derivative to determine whether $f''(1)$ or $f''(3)$ is greater.

(2) Students might look at the height of the function $f''(x)$ at $x = 3$ and $x = 1$ to determine the value of $f''(1)$ and $f''(3)$.

(3) Students may incorrectly assume that $f''(1)$ or $f''(3)$ could be zero if they see a point of inflection without checking for changes in concavity.

2

Explanation:

(a) The first derivative $w^{'}(t)$ represents the rate of change of $w(t)$, tells us whether $w(t)$ is increasing or decreasing over an interval.
If $w^{'}(t) > 0$, $w(t)$ is increasing. Otherwise, if $w^{'}(t) < 0$, $w(t)$ is decreasing.
As we can see from the table, $w(t)$ increases from 10.4 to 15.9 as t increases from 100 to 140, which means $w(t)$ is increasing over the given interval.
Therefore, the derivative of the function appears to be positive over the given interval,

(b) The second derivative $w^{''}(t)$ represents the rate of change of $w^{'}(t)$, tells us whether $w(t)$ is concave up or concave down.
If $w^{''}(t) > 0$, $w(t)$ is concave up. Otherwise, if $w^{''}(t) < 0$, $w(t)$ is concave down.
We can calculate the difference between consecutive values of $w(t)$:
$w(110) - w(100) = 12.9 - 10.4 = 2.5$
$w(120) - w(110) = 14.1 - 12.9 = 1.2$
$w(130) - w(120) = 15.2 - 14.1 = 1.1$
$w(140) - w(130) = 15.9 - 15.2 = 0.7$
From these differences, we can see that the rate of increase in $w(t)$ is decreasing, which means $w^{'}(t)$ is becoming smaller and $w(t)$ is concave down. Therefore, $w^{''}(t)$ is negative.
The second derivative of the function appears to be negative over the given interval

Insights:

1. Students might confuse the increase or decrease of $w(t)$ with $w^{'}(t)$. It is important to remember that the first derivative measures the change in $w(t)$.

2. Students might interpret $w(t)$ increasing as the second derivative is positive. It is important to notice that to determine whether a second derivative is positive or negative, we need to look at the concavity.

3

Explanation

(a) First derivative: We can see from the table that $f(t)$ increases as t increases from 0 to 10. This means $f^{'}(t)$ is positive over the given interval.
Second derivative: Calculate the differences between consecutive values of $f(t)$:
t = 0 to t = 2: $f(t)$ increases by 25.
t = 2 to t = 4: $f(t)$ increases by 21.

t = 4 to t = 6: $f(t)$ increases by 17.
t = 6 to t = 8: $f(t)$ increases by 14.
t = 8 to t = 10: $f(t)$ increases by 10.
Since the rate of increase is decreasing, the second derivative is negative.

(b) Difference Quotient formula:
$f^{'}(t) \approx \frac{f(t+h)-f(t)}{h}$
$f^{'}(2)$: $f^{'}(2) \approx \frac{f(4)-f(2)}{4-2} = \frac{61-40}{2} = 10.5$
$f^{'}(8)$: $f^{'}(8) \approx \frac{f(10)-f(8)}{10-8} = \frac{102-92}{2} = 5$

Insights:

(1) Students may mix up the signs of the first and second derivatives. For example, thinking that a decreasing $f(t)$ means $f^{'}(t) < 0$ without checking the slope.

(2) Students may choose incorrect intervals for h or misunderstand which points to use for $f^{'}(2)$ and $f^{'}(8)$. For example, they might use $t = 2$ and $t = 6$ instead of $t = 2$ and $t = 4$.

4

This question asks you to find the signs of f, $f^{'}$, $f^{''}$ at four points on the graph.
Context knowledge:
- f: The value of the function at a given point represents its height on the graph.
- $f^{'}$: The first derivative indicates the slope of the function. If the slope is positive (function is increasing), $f^{'} > 0$, if the slope is negative (function is decreasing), $f^{'} < 0$. A zero slope $f^{'} = 0$ indicates a horizontal tangent, usually at local maxima or minima.
- $f^{''}$: The second derivative measures the concavity of the function. If the graph is concave up, $f^{''} > 0$, if concave down, $f^{''} < 0$.

Looking at our graph:

1. Point A:
   - f: - (below the x-axis)
   - $f^{'}$: 0 (horizontal tangent, a local minimum)
   - $f^{''}$: + (concave up)

2. Point B:
   - f: + (above the x-axis)

- f': 0 (horizontal tangent, a local maximum)
- f'': - (concave down)

3. Point C:
   - f: + (above the x-axis)
   - f': - (slope is downward)
   - f'': - (concave down)

4. Point D:
   - f: - (below the x-axis)
   - f': + (slope is upward)
   - f'': + (concave up)

Insights:

1. Confusing where the function is increasing or decreasing with the derivative values being positive or negative. For example, students might think that if a function is above the x-axis, f' must be positive

2. Misunderstanding concavity as being related to the slope instead of the rate of change of the slope. This can lead to wrong conclusions about the sign of f''

3. Assuming that if $f' = 0$, f must be zero or the function must be at the x-axis.

5

Analysis of the given information "For three minutes the temperature of a feverish person has had negative first derivative and positive second derivative."
- Negative first derivative: Temperature is falling over the three minutes.
- Positive second derivative: The rate of fall is slowing down, meaning the temperature is decreasing at a slower pace over time.

The most suitable statement would be "The temperature fell in the last minute, but less than it fell in the minute before." Since the first derivative is negative, the temperature is falling. The second derivative is positive, the rate of the fall is decreasing, meaning it fell less in the last minute than it did earlier.

Insights

1. The statement "The temperature rose two minutes ago but fell in the last minute." is incorrect because The first derivative is negative, so the temperature has been falling for the entire period. It never rose at any point in the last three minutes.

2. The statement "The temperature rose in the last minute, but less than it rose in the minute before." is incorrect because the first derivative is negative, meaning the temperature has been falling, not rising.

3. The statement "The temperature rose in the last minute more than it rose in the minute before." is incorrect because this suggests that the temperature is rising at an increasing rate, which is the opposite of the given situation.

6

Explanation:

(a) A graph showing constant velocity means the position vs. time graph will be a straight line (with a non-changing slope). In Graph (I), the position decreases linearly with time, which implies constant velocity.
Answer: (I)

(b) Velocity corresponds to the slope of the position-time graph. The initial velocity is determined by the slope of the graph at $t = 0$. In Graph (IV), the curve is steepest at the start and increasing, meaning it has the largest positive initial velocity.
Answer: (IV)

(c) Average velocity is calculated as $\frac{f(5)-f(0)}{5-0}$. For Graph (III), the position increases significantly between $t = 0$ and $t = 5$, leading to the largest positive displacement. The denominator (time changes) are the same for every graphs so the graph with largest positive displacement will has the largest average velocity
Answer: (III)

(d) Using the average velocity formula again, we have particle has zero velocity when $\frac{f(5)-f(0)}{5-0} = 0$. In graph (III), the particle starts and ends at the same position, so the displacement (numerator) is zero. Therefore, the average velocity is zero.
Answer: (II)

(e) Zero acceleration means means constant velocity, and the position graph is a straight line. In Graph (I), the velocity is constant with no change in slope means no acceleration.
Answer: (I)

(f) Positive acceleration means the graph is concave up. The curve of Graph (III) is always concave up, indicating positive acceleration.
Answer: (III)

Insights

1. Confusing about "greatest initial velocity": Students might mistake highest starting value on y-axis as the greatest initial velocity and choose graph (I). However, when we want to look for the velocity, we need to focus on the slope of line at the starting point rather than the initial height of the function.
2. Students may misinterpret zero velocity as applying to moments when the graph flattens, like in Graph (IV). However, zero velocity here refers to the entire interval's net displacement, not just specific moments.
3. Students might confuse zero velocity (Graph II) with zero acceleration. Graph II has changing velocity, so the acceleration is not zero.

7

Explanation:

(a) The average rate of change between two points t_1 and t_2 is calculated by this formula: $\frac{P(t_2)-P(t_1)}{t_2-t_1}$
Answer:
$0-2$: 9.7
$2-4$: 21.05
$4-6$: 51.75
$6-8$: 88.05
$8-10$: 81.3
$10-12$: 40.75
$12-14$: 23
$14-16$: 7.55
$16-18$: 2.95

(b) We need to determine the second derivative $\frac{d^2P}{dt^2}$. The sign of the second derivative is positive when the rates of change are increasing or negative when the rates of change are decreasing.
Answer:
The sign of $\frac{d^2P}{dt^2}$ appears to be positive from $t = 0$ to $t = 8$
The sign of $\frac{d^2P}{dt^2}$ appears to be negative from $t = 8$ to $t = 18$

Insights In part (b), students might misunderstand the sign of the second derivative and think that because the population continues to increase, the second derivative remains positive. The second derivative refers to the rate of change of the rate of change. Even though the population is increasing, if the rate of increase is slowing down, the second derivative becomes negative.

8

(a) $f^{'}(x)$: negative. $f(x)$ is decreasing as x increases.
$f^{''}(x)$: positive. The decrease is slowing down (the differences are getting smaller), which means the function is concave up and $f^{''}(x)$.

(b) $f^{'}(x)$: positive. $f(x)$ is increasing as x increases.
$f^{''}(x)$: negative. The increase is slowing down (the differences are getting smaller), which means the function is concave down

(c) $f^{'}(x)$: positive. $f(x)$ is increasing as x increases.
$f^{''}(x)$: negative. The increase is slowing down (the differences are getting smaller), which means the function is concave down

Insights

1. Students might think that if the function is increasing or decreasing, this directly affects concavity. However, concavity is determined by how the rate of change (the slope) is behaving, not just whether the function is increasing or decreasing.
2. Some students might miss how the change in values is slowing down or speeding up and mistakenly conclude that the second derivative is the same as the first derivative in sign. They may assume that a negative first derivative automatically means a negative second derivative.
3. Instead of checking how the differences between the function values are evolving (whether they are increasing or decreasing), students might focus too much on raw values. It's essential to look at the pattern in changes

9

Explanation:

(a) The position is positive
Position refers to the y-value of the graph at each point in time. A positive position occurs when the graph is above the x-axis.
At t_1, t_4, t_5, $f(x)$ is above the x-axis, indicating positive position values.

(b) The velocity is positive
Velocity is the slope of the tangent line at each point
At t_2, t_3, the slope of line is rising, indicating positive velocity.

(c) The acceleration is positive
Acceleration is the rate of change of velocity, or the concavity of the graph
At t_1, t_2, t_5, the graph is concave upwards, indicating positive acceleration.

(d) The position is decreasing
Decreasing position means that the y-values of the graph are getting lower (the graph is going down). In this case, "decreasing position" refers to the slope of the graph, which tells us whether the position is decreasing or increasing over time.
At t_1, t_4, t_5, the graph is sloping downwards, indicating that the position is decreasing.

(e) The velocity is decreasing
Decreasing velocity refers to the concavity of the graph.
At t_3, t_4, the graph is concave down, indicating the velocity is decreasing

Insights:

1. Students may confuse "decreasing position" with velocity decreasing. In this case, decreasing position refers to the slope, not the concavity.
2. Students might confuse "decreasing velocity" with position decreasing. Here, decreasing velocity refers to the concavity of the graph, not the slope. They might incorrectly choose points where the graph is sloping downwards but not concave down.

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Explanation:
Looking at the position graph, the first part of the graph is concave down $\rightarrow$ negative acceleration; the second part of the graph is concave up, $\rightarrow$ positive acceleration.
Therefore, the acceleration graph starts negative (due to concave down) and increases to positive (due to concave up), which matches the shape of graph (d).

Insights:

1. Students might miss the fact that the graph transitions from concave down to concave up and focus only on one region and think acceleration is constant, which could lead to choosing a graph like (b).
2. Graph (a) shows consistently decreasing acceleration, which could confuse students who don't recognize the change from concave down to concave up in the position graph.

3. Graph (c) shows a decreasing curve, which might make students think it represents deceleration. Since the position graph is moving downward, students could mistakenly assume that the velocity or acceleration is continuously decreasing. However, this is incorrect because we need to focus on concavity to analyze acceleration, not just the direction.