统计机器学习课后作业 4

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2020 年 11 月 9 日

1　问题 1

解:

我们知道:

$p(x_i;\beta) = P(Y=1|X=x_i) = \frac{exp(x_i^T\beta)}{1+exp(x_i^T\beta)}$

所以有:

$\iota(\beta) = \sum_{i=1}^{N}\{y_i log p(x_i;\beta) + (1-y_i)log(1-p(x_i;\beta))\} = \sum_i\{y_i log\frac{exp(x_i^T\beta)}{1+exp(x_i^T\beta)} + (1-y_i)log(1-\frac{exp(x_i^T\beta)}{1+exp(x_i^T\beta)})\}$

$= \sum_i\{y_i x_i^T\beta - y_i log(1+exp(x_i^T\beta)) + (1-y_i)log(\frac{1}{1+exp(x_i^T\beta)})\}$

$= \sum_i\{y_i x_i^T\beta - y_i log(1+exp(x_i^T\beta)) + (y_i-1)(log(1+exp(x_i^T\beta))\}$

$= \sum_i\{y_i x_i\beta - log(1+exp(x_i^T\beta))\}$

那么我们就求出了蓝色部分

2　问题 2

解:

我们先求二阶导数表达式:

课上已经求完了一阶导数的表达式:

$\frac{\partial l(\beta)}{\partial\beta} = \sum_i(y_i x_i - \frac{exp(x_i^T\beta)x_i}{1+exp(x_i^T\beta)})$

$\therefore \frac{\partial^2 l(\beta)}{\partial\beta\partial\beta^T} = \sum_i \frac{\partial(\frac{x_i}{1+exp(x_i^T\beta)})}{\partial\beta^T} = \sum_i \frac{-exp(x_i^T\beta)x_ix_i^T}{(1+exp(x_i^T\beta))^2}$

把 $p(x_i;\beta) = \frac{exp(x_i^T\beta)}{1+exp(x_i^T\beta)}$ 带到等式里

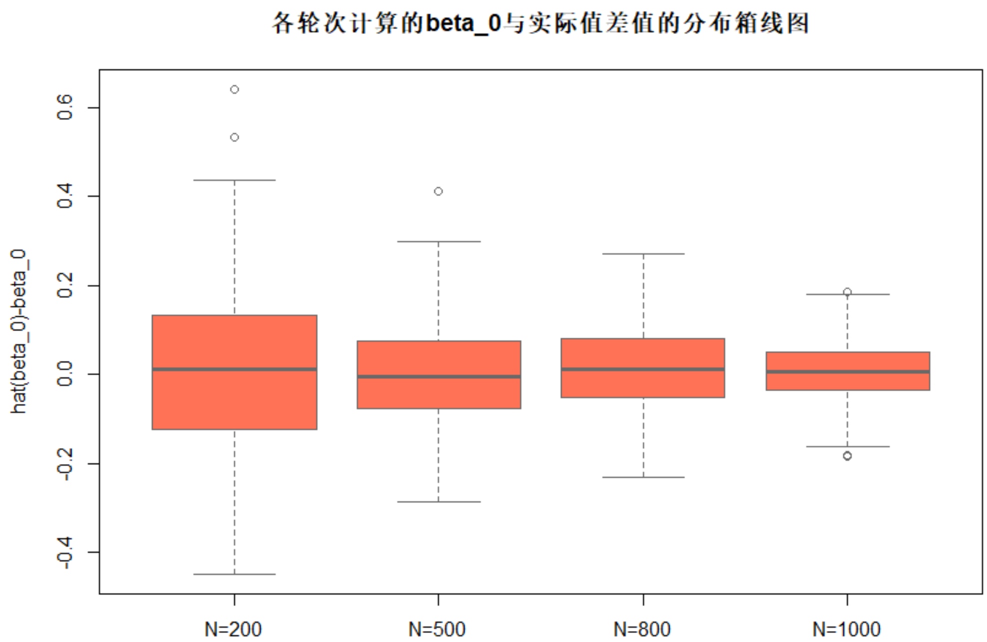
$\frac{\partial^2 l(\beta)}{\partial\beta\partial\beta^T} = -\sum_i x_ix_i^T p(x_i;\beta)(1-p(x_i;\beta))$

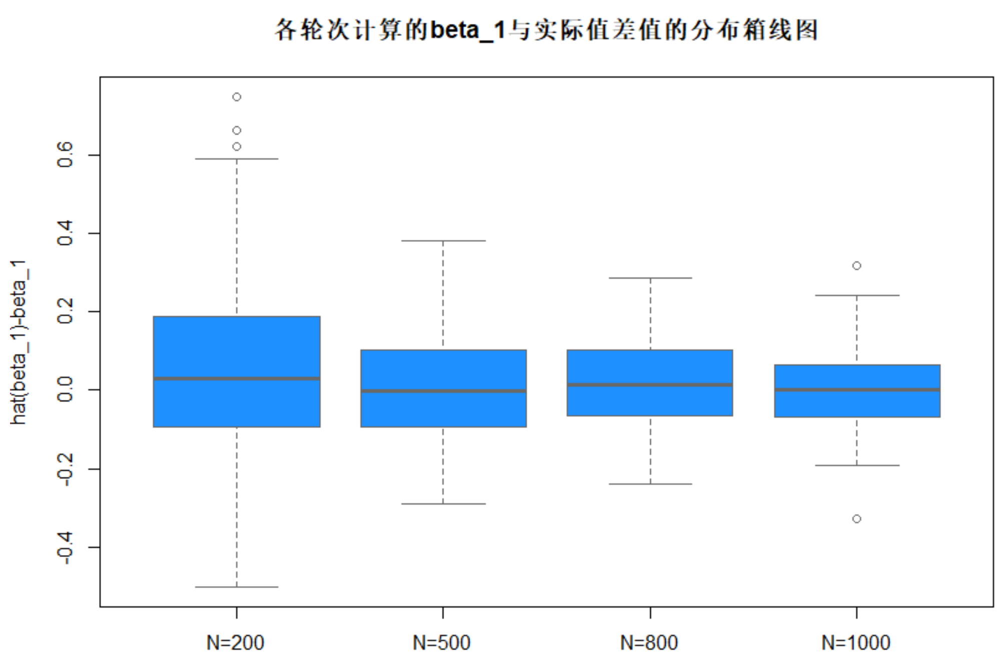
The R codes:

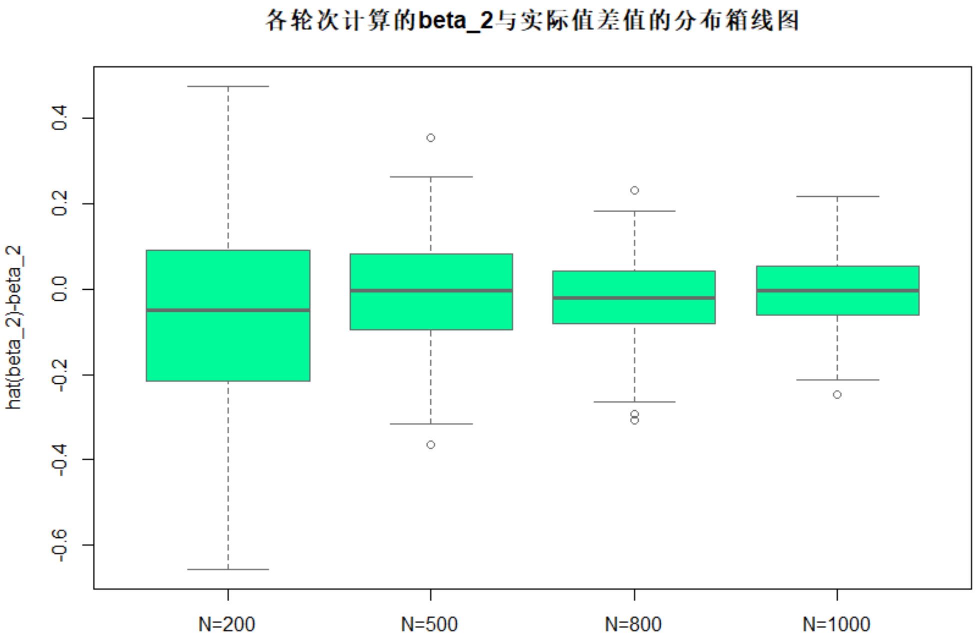
```
# Problem 2

# N-R algorithm for R=200 rounds
NR = function(N){
        beta = c(0.5, 1.2, -1)
        R = 200
        result = matrix(0,0,3)
        for (i in 1:R){
                X1 = rnorm(N)
                X2 = rnorm(N)
                X = cbind(1, X1, X2)
                Y = exp(X%*%beta)/(1+exp(X%*%beta))
                for (j in 1: length(Y)){
                        judge=runif(1,min=0,max=1)
                        if (judge>Y[j]){
                                Y[j]=0
                        }
                        if (judge<=Y[j]){
                                Y[j]=1
                        }
                }
                beta_old = c(-1, -1, -1)
                beta_new = c(0.5, 0.5, 0.5)
                while(max(abs(beta_old-beta_new))>1e-5)
                {
                        P = as.numeric(exp(X %*% beta_new)/(1+exp(X %*% beta_new)))
                        W = diag(P*(1-P))
                        partial1 = t(X)%*%(Y-P)
                        partial2 = t(X)%*%W%*%X
```

```
                beta_old = beta_new
                beta_new = beta_old+solve(partial2)%*%partial1
            }
            result=rbind(result,t(beta_new))
        }
        return(result)
}

# calculate result for N=200,500,800,1000
NL=c(200,500,800,1000)
result1 = NR(NL[1])
result2 = NR(NL[2])
result3 = NR(NL[3])
result4 = NR(NL[4])

# do boxplot (recall that beta=c(0.5,1.2,-1))
boxplot(result1[,1]-0.5, result2[,1]-0.5, result3[,1]-0.5, result4[,1]-0.5,
col="coral1", border="dimgray",
main="各轮次计算的beta_0与实际值差值的分布箱线图", ylab="hat(beta_0)-beta_0",
names=c('N=200','N=500','N=800','N=1000'))

boxplot(result1[,2]-1.2, result2[,2]-1.2, result3[,2]-1.2, result4[,2]-1.2,
col="dodgerblue", border="dimgray",
main="各轮次计算的beta_1与实际值差值的分布箱线图", ylab="hat(beta_1)-beta_1",
names=c('N=200','N=500','N=800','N=1000'))

boxplot(result1[,3]+1, result2[,3]+1, result3[,3]+1, result4[,3]+1,
col="mediumspringgreen",border="dimgray",
main="各轮次计算的beta_2与实际值差值的分布箱线图", ylab="hat(beta_2)-beta_2",
names=c('N=200','N=500','N=800','N=1000'))
```

The results are:


各轮次计算的beta_0与实际值差值的分布箱线图
hat(beta_0)-beta_0
-0.4
-0.2
0.0
0.2
0.4
0.6
N=200
N=500
N=800
N=1000


各轮次计算的beta_1与实际值差值的分布箱线图
hat(beta_1)-beta_1
-0.4
-0.2
0.0
0.2
0.4
0.6
N=200
N=500
N=800
N=1000



从箱线图中我们可以看到 $\hat{\beta}$ 通过 NR 算法，很好地往 β 收敛了，在 200 轮模拟中得到的 $\hat{\beta}$ 大部分都比较靠近 β

此外随着 N 的增大，$\hat{\beta}$ 的分布更为集中和收缩，分布范围缩小了很多

3 问题 3

解:

从 ROC 曲线的定义我们可以知道：

ROC 曲线上方的面积等同于 m^- 个底为 $\frac{1}{m^-}$, 高为 $\frac{1}{m^+}\times$ 剩余正例数的长方形的面积之和

$\therefore$ S(area above ROC)=$\sum_{x^+\in D^+}\sum_{x^-\in D^-}(\frac{1}{m^-}\times\frac{1}{m^+}\times I(f(x^+)<f(x^-)))$
$=\frac{1}{m^+m^-}\sum_{x^+\in D^+}\sum_{x^-\in D^-}(I(f(x^+)<f(x^-)))$=$l_{rank}$

$\therefore$ S(area under ROC)=AUC=1-S(area above ROC)=1-l_{rank}

4 问题 4

解:

原始状态下，在训练集中有 48393 个观测样本，在测试集中有 47900 个测试样本

根据”expense”, ”tightness”, ”degree” 变量的定义，它们都是大于 0 的正值

但是根据初步观察，发现数据集中有一些观测样本的这些变量值是小于 0 的负值，说明这些样本存在问题，但数量不是特别多。为了避免这些观测对后续分析的干扰，我们把他们筛除

筛除后，训练集中剩余 48285 个观测样本，测试集中剩余 47771 个观测样本

Problem(1) 读入数据:

```
# (1) read data
train=read.csv(”D:/大数据学院文件资料/2020秋课程/机器学习/sampledata.csv”)
```

Results:

Data	
train	48285 obs. of 9 variables

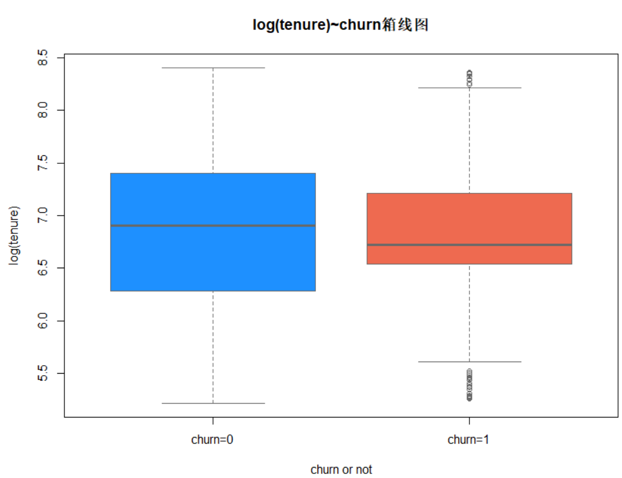
Problem(2) 作箱线图:

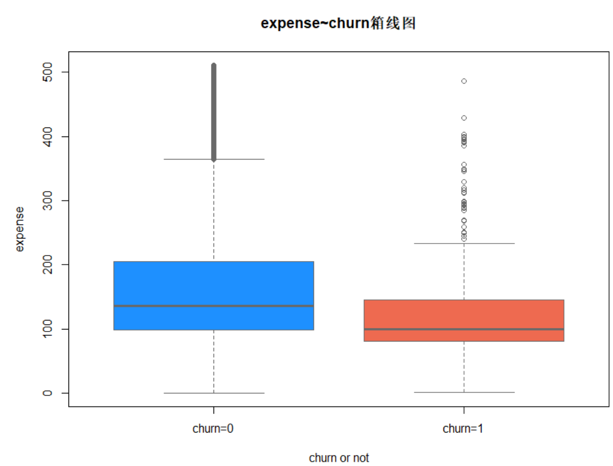
为作图简洁，箱线图中的变量用英文表示；(备注：churn=1 表示流失，churn=2 表示不流失)

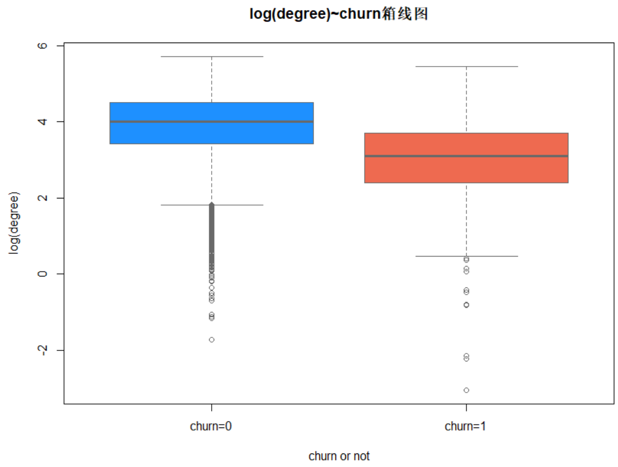
```
# (2) box plot
boxplot(log(tenure)~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main='log(tenure)~churn箱线图',names=c(”churn=0”,”churn=1”),
xlab=”churn␣or␣not”,ylab=”log(tenure)”,plot=T)

boxplot(expense~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main='expense~churn箱线图',names=c(”churn=0”,”churn=1”),
xlab=”churn␣or␣not”,ylab=”expense”,plot=T)

boxplot(log(degree)~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main='log(degree)~churn箱线图',names=c(”churn=0”,”churn=1”),
```

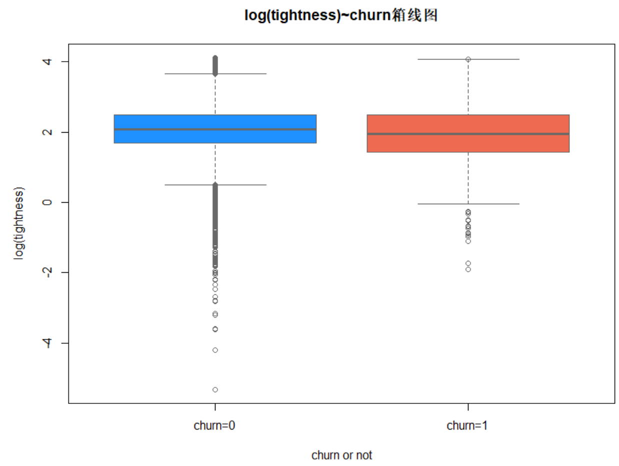
```
xlab=”churn␣or␣not”,ylab=”log(degree)”,plot=T)

boxplot(log(tightness)~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main=’log(tightness)~churn箱线图’,names=c(”churn=0”,”churn=1”),
xlab=”churn␣or␣not”,ylab=”log(tightness)”,plot=T)

boxplot(entropy~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main=’entropy~churn箱线图’,names=c(”churn=0”,”churn=1”),
xlab=”churn␣or␣not”,ylab=”entropy”,plot=T)

boxplot(chgdegree~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main=’chgdegree~churn箱线图’,names=c(”churn=0”,”churn=1”),
xlab=”churn␣or␣not”,ylab=”chgdegree”,plot=T)

boxplot(chgexpense~churn,data=train,col=c(”dodgerblue”,”coral2”),
border=c(”dimgray”,”dimgrey”),
main=’chgexpense~churn箱线图’,names=c(”churn=0”,”churn=1”),
xlab=”churn␣or␣not”,ylab=”chgexpense”,plot=T)
```

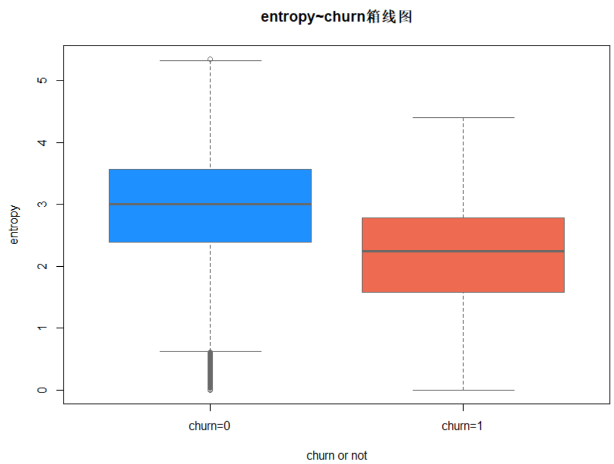
Results are:

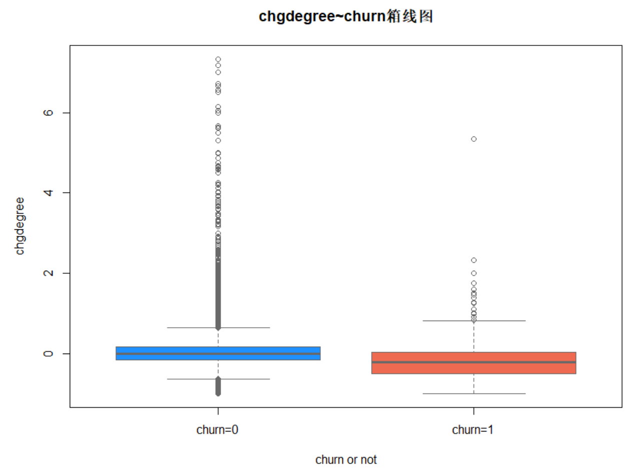


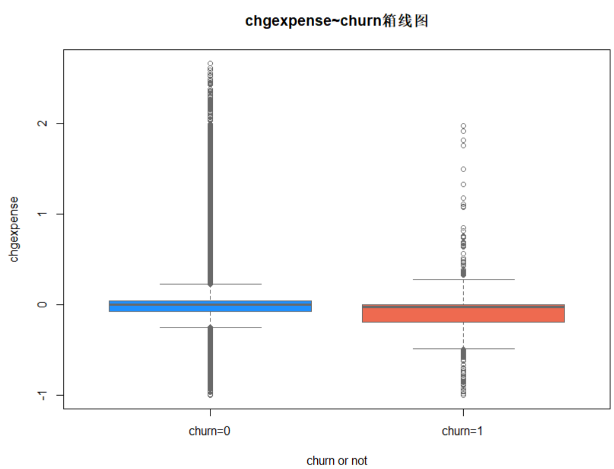












从箱线图 churn=1 和 churn=0 的分布中我们可以看到：

1、在网时长相对较短的客户，更可能流失
2、当月话费较少的客户，更可能流失
3、和客户通话的总人数越少的客户，越可能流失
4、联系强度越少的客户流失的可能性相对较大，但不明显
5、个体信息熵越低的客户流失的可能性越大
6、个体度变化越低的客户，越可能流失
7、话费变化越低的客户，越可能流失

Problem(3) 标准化与逻辑回归:

```
# (3)
train[2:8]=scale(train[2:8])
reg_log=glm(churn~tenure+expense+degree+tightness+entropy+chgexpense+chgdegree,
data=train,family=binomial(link="logit")
)
summary(reg_log)
```

逻辑回归的参数结果是:

```
Call:
glm(formula = churn ~ tenure + expense + degree + tightness +
entropy + chgexpense + chgdegree, family = binomial(link = "logit"),
data = train)

Deviance Residuals:
Min        1Q     Median      3Q      Max
-0.8909   -0.1748   -0.1198   -0.0748   4.2072

Coefficients:
Estimate Std. Error z value Pr(>|z|)
(Intercept) -5.05338    0.07212 -70.067  < 2e-16 ***
tenure      -0.24767    0.06070  -4.080 4.50e-05 ***
expense     -0.29229    0.05904  -4.951 7.39e-07 ***
degree      -0.73751    0.13172  -5.599 2.15e-08 ***
tightness   -0.22660    0.04254  -5.327 9.99e-08 ***
entropy     -0.35176    0.07208  -4.880 1.06e-06 ***
chgexpense  -0.16071    0.04893  -3.284  0.00102 **
chgdegree   -0.38279    0.05208  -7.349 1.99e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

Null deviance: 6475.5  on 48284  degrees of freedom
Residual deviance: 5757.6  on 48277  degrees of freedom
AIC: 5773.6

Number of Fisher Scoring iterations: 8
```

解读：

1、从结果上看，对因变量“是否流失”而言，所有的这些变量在逻辑回归模型中都有较高的显著性水平 (0.01 和 0.001)
2、模型截距项参数为-5.0534；自变量的系数估计分别为：
在网时长的系数-0.2476
当月话费系数-0.2923
个体的度系数-0.7375
联系强度系数-0.2266
个体信息熵系数-0.3518
个体度的变化系数-0.1607
花费的变化系数-0.3828
可以看到这些系数均小于 0，这与箱线图中展示出的分布差异与初步结论也比较一致

Problem(4) 逻辑回归预测

```
# (4)
train_pred=predict.glm(reg_log,newdata=train,type="response")
head(train_pred)
test=read.csv("D:/大数据学院文件资料/2020秋课程/机器学习/preddata.csv")
test[2:8]=scale(test[2:8])
test_pred=predict.glm(reg_log,newdata=test,type="response")
head(test_pred)
```

预测结果 (开头的一部分):

```
> head(train_pred)
1           2           3           4           5           6
0.004999719 0.010796015 0.003467837 0.008023648 0.002977289 0.001469916

> head(test_pred)
1           2           3           4           5           6
```

```
0.0332009167  0.0062862747  0.0085627283  0.0006695617  0.0073670728  0.0037128759
```

train_pred 和 test_pred 就是预测流失概率值的向量

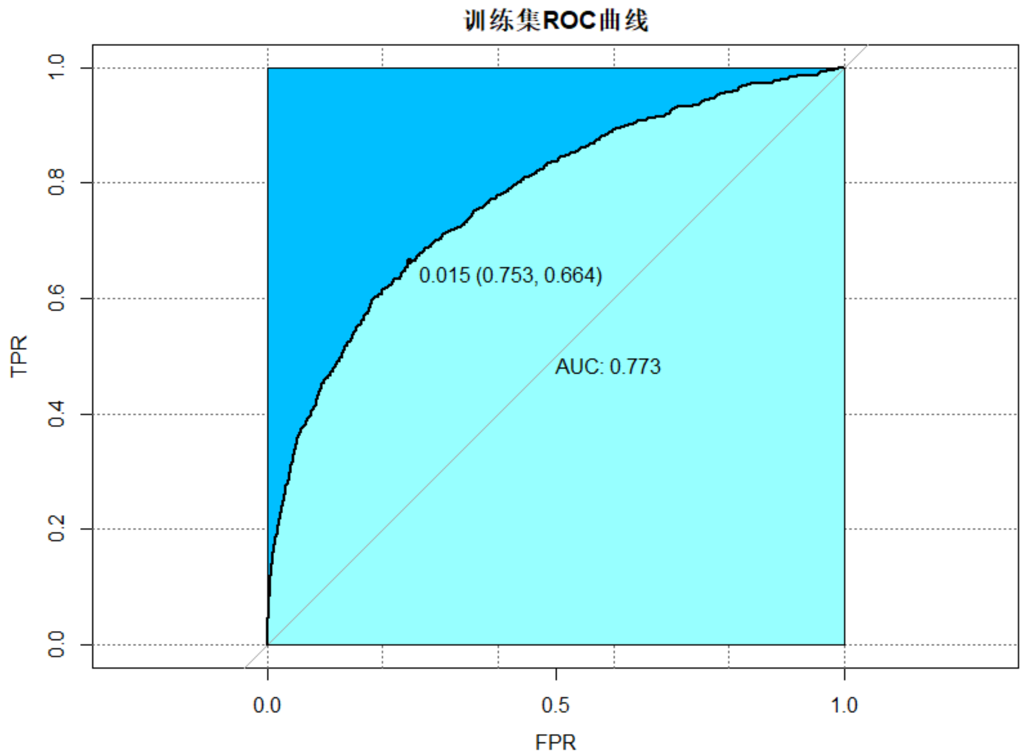
Problem(5) 绘制 ROC 曲线与 AUC 计算 R codes:

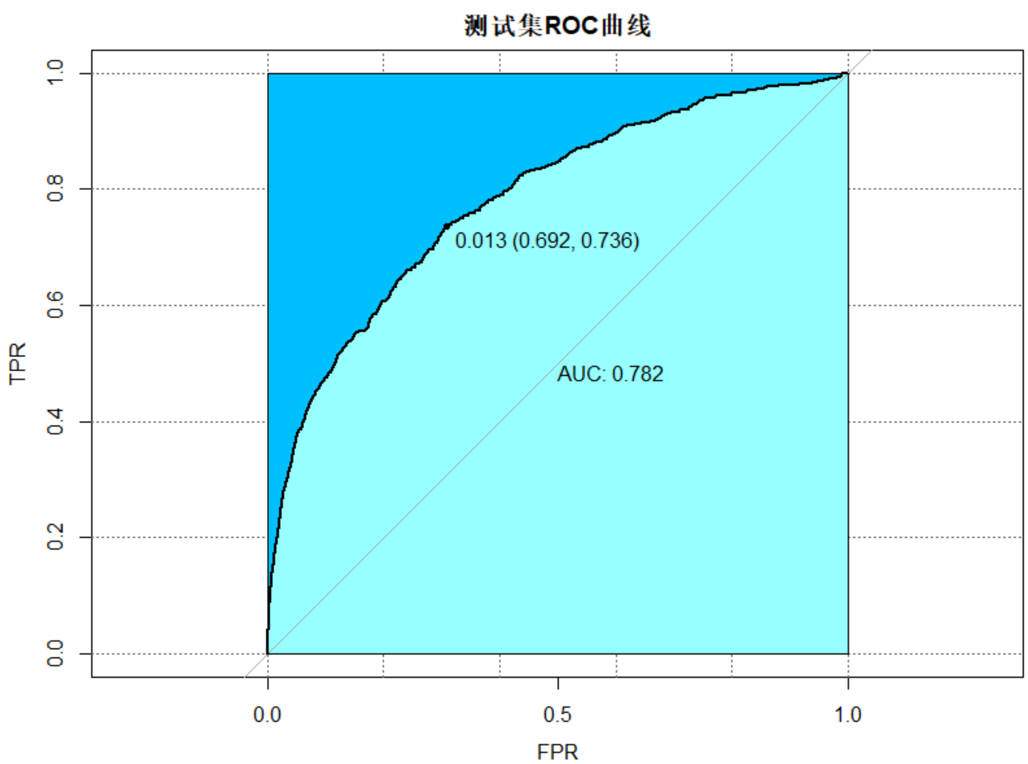
```
# (5)
library("pROC")

roc(train$churn,train_pred,plot=TRUE,main="训练集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE,print.auc=TRUE,legacy.axes=TRUE,grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1",max.auc.polygon.col="deepskyblue")

roc(test$churn,test_pred,plot=TRUE,main="测试集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE,print.auc=TRUE,legacy.axes=TRUE,grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1",max.auc.polygon.col="deepskyblue")
```

ROC 曲线绘制的结果报告:

```
Call:
roc.default(response = train$churn, predictor = train_pred, plot = TRUE,
main = "训练集ROC曲线", xlab = "FPR", ylab = "TPR", print.thres = TRUE,
print.auc = TRUE, legacy.axes = TRUE, grid = c(0.2, 0.2),     grid.col = "dimgray", au
auc.polygon.col = "darkslategray1", max.auc.polygon.col = "deepskyblue")

Data: train_pred in 47683 controls (train$churn 0) < 602 cases (train$churn 1).
Area under the curve: 0.7728

Call:
roc.default(response = test$churn, predictor = test_pred, plot = TRUE,
main = "测试集ROC曲线", xlab = "FPR", ylab = "TPR", print.thres = TRUE,
print.auc = TRUE, legacy.axes = TRUE, grid = c(0.2, 0.2),     grid.col = "dimgray", au
auc.polygon.col = "darkslategray1", max.auc.polygon.col = "deepskyblue")

Data: test_pred in 47111 controls (test$churn 0) < 660 cases (test$churn 1).
Area under the curve: 0.7818
```

对应的训练集上和测试集上的 ROC 图:


训练集ROC曲线
0.015 (0.753, 0.664)
AUC: 0.773
TPR
FPR
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0


测试集ROC曲线
0.013 (0.692, 0.736)
AUC: 0.782
TPR
FPR
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0

计算 AUC:

```
AUC_c = function(TP, FP){
        lrank = 0
        for (i in 1:length(TP))
        {
                lrank = lrank+1*sum(FP>TP[i]) + 0.5*sum(FP == TP[i])
        }
        lrank = 1 - lrank/ (length(TP) * length(FP))
        return(lrank)
}
train_TP = train_pred[(train$churn==1)]
train_FP= train_pred[(train$churn==0)]
AUC_c(train_TP,train_FP)

test_TP = test_pred[(test$churn==1)]
test_FP= test_pred[(test$churn==0)]
AUC_c(test_TP,test_FP)
```

```
> AUC_c(train_TP,train_FP)
[1] 0.7728054
> AUC_c(test_TP,test_FP)
[1] 0.7818464
```

计算出的 AUC 与 roc 函数自动计算出的 AUC 吻合

根据 ROC 曲线和 AUC 值，我们看到模型在训练集上表现出了较好的预测效果 (AUC=0.773)
此外模型在数据规模与训练集差不多的测试集上也一样表现出恶较好的预测效果 (AUC=0.782)，说明该模型的泛化能力还比较不错