统计机器学习课后作业 7

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2020 年 12 月 13 日

1 问题 (1)

解:

拉格朗日函数:

$$L(w,b,\xi,\alpha,\mu) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^N \xi_i^2 - \sum_{i=1}^N \alpha_i[y_i(w\cdot X_i + b) - 1 + \xi_i] - \sum_{i=1}^N \mu_i\xi_i$$

对偶问题:

$$\max_{\alpha_i\geq 0,\mu_i\geq 0}\min_{w,b,\xi_i} L(w,b,\xi,\alpha,\mu)$$

对 w, b, ξ_i 求导:

$$\begin{cases} \nabla_w L = w - \sum_{i=1}^N \alpha_i y_i X_i = 0 \\ \nabla_b L = \sum_{i=1}^N \alpha_i y_i = 0 \\ \nabla_{\xi_i} L = 2C\xi_i - \alpha_i - \mu_i = 0 \end{cases}$$

$\therefore w = \sum_{i=1}^N \alpha_i y_i X_i,\ \xi_i = \frac{\alpha_i+\mu_i}{2C}$

$$\therefore Q(\alpha,\mu) = \min_{w,b,\xi_i} L(w,b,\xi,\alpha,\mu)$$

$$= -\frac{1}{2}\sum_{i=1}^N\sum_{i=1}^N \alpha_i\alpha_j y_i y_j X_i\cdot X_j - \frac{1}{4C}\sum_{i=1}^N(\alpha_i+\mu_i)^2 + \sum_{i=1}^N \alpha_i$$

则对偶问题为:

$$\begin{aligned} &max\ Q(\alpha,\mu) \\ &s.t.\ \sum_{i=1}^N \alpha_i y_i = 0 \\ &\alpha_i \geq 0 \\ &\mu_i \geq 0 \end{aligned}$$

$i = 1, 2, ..., N$

代入 $Q(\alpha, \mu)$

可得对偶问题等价于:

$min\ \frac{1}{2}\sum_{i=1}^{N}\sum_{i=1}^{N}\alpha_i\alpha_j y_i y_j X_i \cdot X_j + \frac{1}{4C}\sum_{i=1}^{N}(\alpha_i + \mu_i)^2 - \sum_{i=1}^{N}\alpha_i$
$s.t.\ \sum_{i=1}^{N}\alpha_i y_i = 0$
$\alpha_i \geq 0$
$\mu_i \geq 0$
$i = 1, 2, ..., N$

2 问题 (2)

解:

$K(x, z) = (x \cdot z)^p$

$= (x_1 z_1 + x_2 z_2 + ... + x_k z_k)^p$

$= \sum_{i_1+i_2+i_3+...+i_k=p} C_{i_1+i_2+i_3+...+i_k=p}(x_1 z_1)^{i_1}(x_2 z_2)^{i_2}...(x_k z_k)^{i_k}$

由于该式子中，x_i,z_i 次数总是相同

定义 $\phi(x) = \sqrt{C_{i_1+i_2+i_3+...+i_k=p}}x_1^{i_1}x_2^{i_2}...x_k^{i_k}$

同理 $\phi(z) = \sqrt{C_{i_1+i_2+i_3+...+i_k=p}}z_1^{i_1}z_2^{i_2}...z_k^{i_k}$ 实际上与 $\phi_(x)$ 相同, $\phi(z) = \phi(x)$

那么对于每种 $i_1 + i_2 + ...i_k = p$ 的组合，都可以定义 $\phi(x)$ 与 $\phi(z)$, 使得 $C_{i_1+i_2+i_3+...+i_k=p}x_1^{i_1}x_2^{i_2}...x_k^{i_k} = \phi(x)\phi(z)$

我们假设有 n 种 $i_1 + i_2 + ...i_k = p$ 的组合, 对第 j 种组合, 定义 $\phi_j(x) = \sqrt{C_j}x_1^{i_{j1}}x_2^{i_{j2}}...x_k^{i_{jk}}$

那么根据 $K(x, z)$ 的形式，我们可以将 $K(x, z)$ 写成 $K(x, z) = \sum_j \phi_j(x)\phi_j(z) = \sum_j \phi_j(x)^2$ 的形式

因此内积的正整数幂是正定核函数

3 问题 (3)

解:

支持向量有：样本点 2,3,4,6

判断原因：
样本点 2 分类正确，在间隔边界和分类边界之间, $\alpha_i^* = C, 0 < \xi_i < 1$
样本点 3 位于错分类一侧, $\alpha_i^* = C, \xi_i > 1$
同理样本点 4 也是位于错分类一侧
样本点 6 位于间隔边界上, $\alpha_i^* \le C, \xi_i = 0$

4 问题 (4)

解:

1. 读入数据

```
data=read.csv("D:/大数据学院文件资料/2020秋课程/机器学习/assignment/homework7/
library("caTools")
library("pROC")
library("caret")
library("rpart")
```

然后我阅读了文本并了解了自变量的含义, 具体的自变量含义就不在这里再赘述了

2. 划分数据集，用各种模型进行建模，在测试进行预测等

首先划分数据集

```
set.seed(1234)
split=sample(2,nrow(data),replace=T,prob=c(0.7,0.3))
train=data[split==1,]
test=data[split==2,]
```

接下来我们依次用各种模型进行建模，分别是逻辑回归、kNN、决策树、Boosting 模型、随机森林、SVM，并绘制 ROC 曲线以及 AUC 值

(1) 逻辑回归模型

```
# logistic
```

```
# train
train_logis=train
train_logis[1:26]=scale(train_logis[1:26])
reg_log=glm(black~.,data=train_logis,family=binomial(link="logit")
)
summary(reg_log)

# predict
test_logis=test
test_logis[1:26]=scale(test_logis[1:26])
test_logis_pred=predict.glm(reg_log,newdata=test_logis,type="response")

# roc and auc
roc(test$black,as.numeric(test_logis_pred)-1,plot=TRUE,
main="逻辑回归模型测试集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE, print.auc=TRUE, legacy.axes=TRUE, grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1")

test_logis_pred=test_logis_pred>0.5
(sum(test_logis_pred==test_logis$black))/(length(test_logis_pred))
confusionMatrix(as.factor(as.numeric(test_logis_pred)),
as.factor(test_logis$black))
```

具体输出结果如下：

预测准确率:

```
> sum(test_logis_pred==test_logis)/(length(test_logis_pred))
[1] 0.7694257
```

可知逻辑回归模型的预测准确率为 0.7694

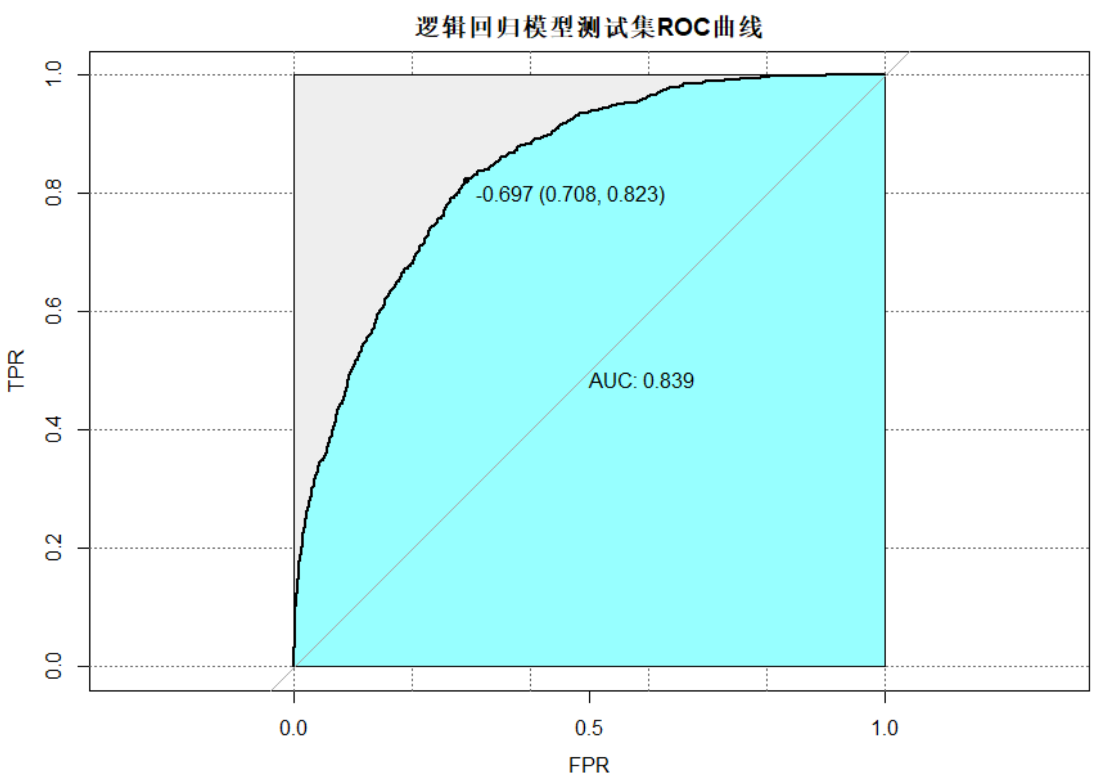
逻辑回归模型的混淆矩阵输出为:

```
                Reference
Prediction       0     1
0             1382   351
1              195   440

Accuracy : 0.7694
```

```
95% CI : (0.7519, 0.7863)
Kappa : 0.455
Sensitivity : 0.8763
Specificity : 0.5563
Pos Pred Value : 0.7975
Neg Pred Value : 0.6929
Prevalence : 0.6660
Detection Rate : 0.5836
Detection Prevalence : 0.7318
Balanced Accuracy : 0.7163
```

逻辑回归模型的 ROC 曲线与 AUC 值为:



(2) KNN 模型

```
# knn
# train
library("kknn")
train_knn=train
```

```
train_knn[1:26]=scale(train_knn[1:26])
test_knn=test
test_knn[1:26]=scale(test_knn[1:26])
model_knn=kknn(black~.,train_knn,test_knn,k=6)

# predict
test_knn_pred=fitted(model_knn)
test_knn_pred=test_knn_pred>0.5
(sum(as.numeric(test_knn_pred)==test_knn$black))/(nrow(test_knn))
confusionMatrix(as.factor(as.numeric(test_knn_pred)),
as.factor(test_knn$black))

# roc and auc
roc(test_knn$black,as.numeric(test_knn_pred)-1,plot=TRUE,
main="KNN模型测试集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE,print.auc=TRUE,legacy.axes=TRUE,grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1")
```

具体输出结果如下：

预测准确率:

```
> (sum(as.numeric(test_knn_pred)==test_knn$black))/(nrow(test_knn))
[1] 0.683277
```

可知 KNN 模型的预测准确率为 0.6833

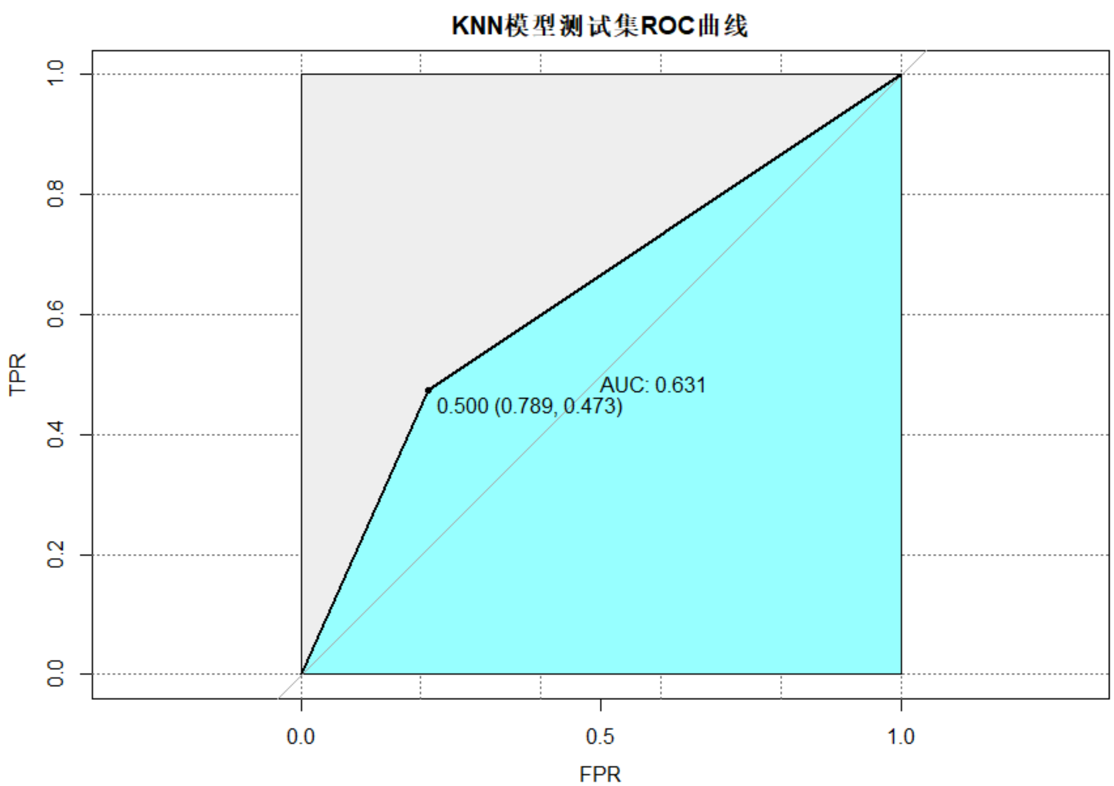
KNN 模型的混淆矩阵输出为:

```
            Reference
Prediction    0    1
0           1244  417
1           333   374

Accuracy : 0.6833
95% CI : (0.6641, 0.702)
Kappa : 0.2688
Sensitivity : 0.7888
Specificity : 0.4728
Pos Pred Value : 0.7489
```

```
Neg Pred Value : 0.5290
Prevalence : 0.6660
Detection Rate : 0.5253
Detection Prevalence : 0.7014
Balanced Accuracy : 0.6308
```

KNN 模型的 ROC 曲线与 AUC 值为:



(3) 决策树模型

```
# decision tree
# train
train_dt=train
# train_dt[1:26]=scale(train_dt[1:26])  the result is better without scale
model_dt=rpart(black~.,data=train_dt,method="class")

# predict
test_dt=test
# test_dt[1:26]=scale(test_dt[1:26])   the result is better without scale
```

```
test_dt_pred=predict(model_dt,test_dt,type="class")
(sum(test_dt_pred==test_dt$black))/nrow(test_dt)
confusionMatrix(as.factor(test_dt_pred), as.factor(test_dt$black))

library(rpart.plot)
rpart.plot(model_dt)

# roc and auc
library(rpart.plot)
rpart.plot(model)

library("pROC")
roc(test_dt$black,as.numeric(test_pred_dt)-1,plot=TRUE,
main="决策树测试集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE, print.auc=TRUE, legacy.axes=TRUE, grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1")
```

具体输出结果如下：

预测准确率:

```
> (sum(test_dt_pred==test_dt$black))/nrow(test_dt)
[1] 0.6967905
```

可知决策树模型的预测准确率为 0.6968

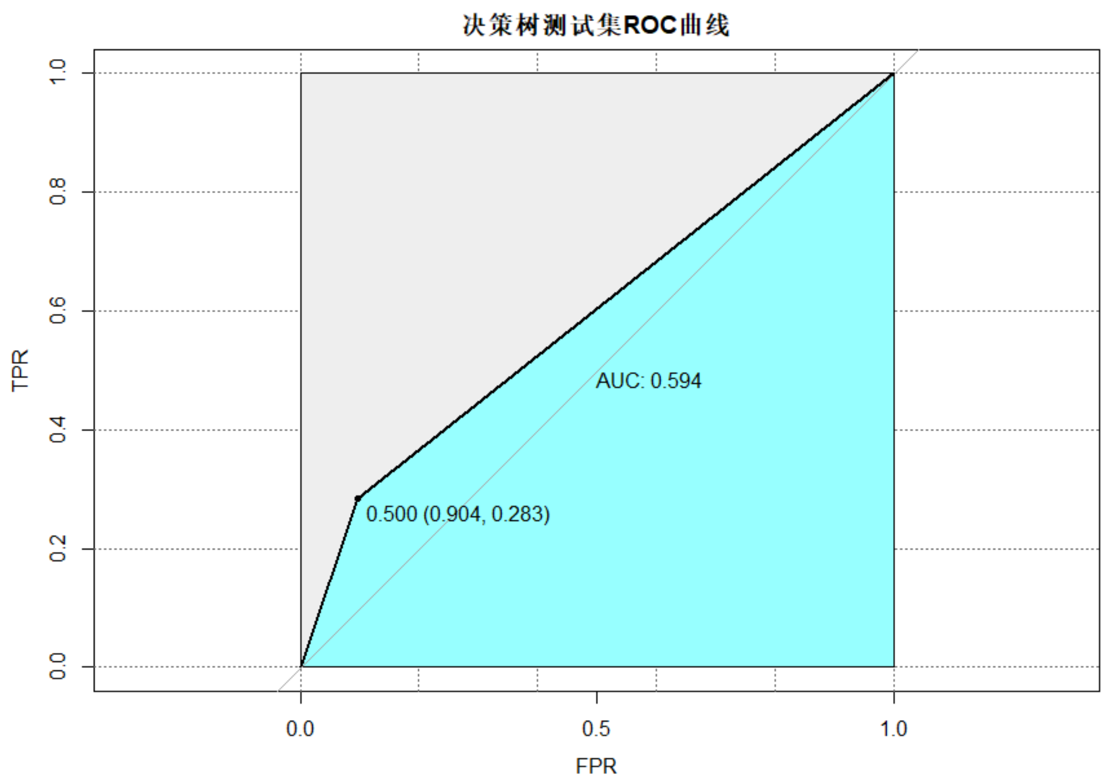
决策树模型的混淆矩阵输出为:

```
              Reference
Prediction      0     1
0            1426   567
1             151   224

Accuracy : 0.6968
95% CI : (0.6778, 0.7153)
Kappa : 0.2157
Sensitivity : 0.9042
Specificity : 0.2832
Pos Pred Value : 0.7155
Neg Pred Value : 0.5973
```

```
Prevalence : 0.6660
Detection Rate : 0.6022
Detection Prevalence : 0.8416
Balanced Accuracy : 0.5937
```

决策树模型的 ROC 曲线与 AUC 值为:



(5) Boosting 模型

```
# boosting
library("adabag")
# train
train_bt=train
train_bt[1:26]=scale(train_bt[1:26])
train_bt$black=as.factor(train_bt$black)
model_bt=boosting(black~.,data=train_bt)

# predict
test_bt=test
```

```
test_bt[1:26]=scale(test_bt[1:26])
test_bt_pred=predict(model_bt,test_bt)
(sum(test_bt_pred$class==test_bt$black))/nrow(test_bt)
confusionMatrix(as.factor(as.numeric(test_bt_pred$class)),
as.factor(test_bt$black))

# roc and auc
roc(test_bt$black,as.numeric(test_bt_pred$class),plot=TRUE,
main="决策树测试集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE, print.auc=TRUE, legacy.axes=TRUE, grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1")
```

具体输出结果如下：

预测准确率:

```
> (sum(test_bt_pred$class==test_bt$black))/nrow(test_bt)
[1] 0.7580236
```

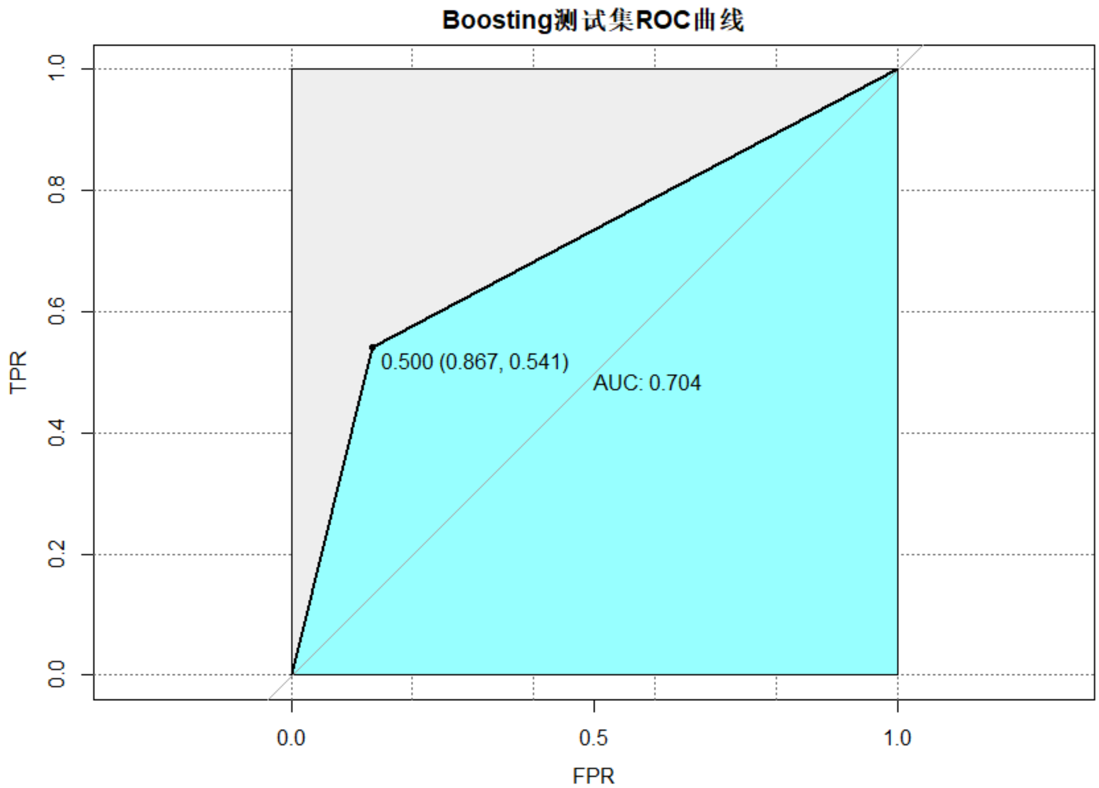
可知 Boosting 模型的预测准确率为 0.7580

Boosting 模型的混淆矩阵输出为:

```
              Reference
Prediction    0     1
0             1367  363
1             210   428

Accuracy : 0.758
95% CI : (0.7402, 0.7752)
Kappa : 0.4286
Sensitivity : 0.8668
Specificity : 0.5411
Pos Pred Value : 0.7902
Neg Pred Value : 0.6708
Prevalence : 0.6660
Detection Rate : 0.5773
Detection Prevalence : 0.7306
Balanced Accuracy : 0.7040
```

Boosting 模型的 ROC 曲线与 AUC 值为:



(5) 随机森林模型

```
# random forest
# train
library("randomForest")
train_rf=train
train_rf[1:26]=scale(train_rf[1:26])
model_rf=randomForest(as.factor(train_rf$black)~.,data=train_rf,importance=T)
importance(model_rf,type=1)

# predict
test_rf=test
test_rf[1:26]=scale(test_rf[1:26])
test_pred_rf=predict(model_rf,test_rf)
(sum(test_pred_rf==test_rf$black))/nrow(test_rf)
confusionMatrix(as.factor(test_pred_rf), as.factor(test_rf$black))
```

```
# roc and auc
roc(test_rf$black ,as.numeric(test_pred_rf)−1,plot=TRUE,
main="随机森林模型测试集ROC曲线",xlab = "FPR", ylab = "TPR",
print.thres=TRUE, print.auc=TRUE, legacy.axes=TRUE, grid=c(0.2,0.2),
grid.col="dimgray",auc.polygon=TRUE,max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1")
```

具体输出结果如下：

预测准确率:

```
> (sum(test_pred_rf==test_rf$black))/nrow(test_rf)
[1] 0.7478885
```

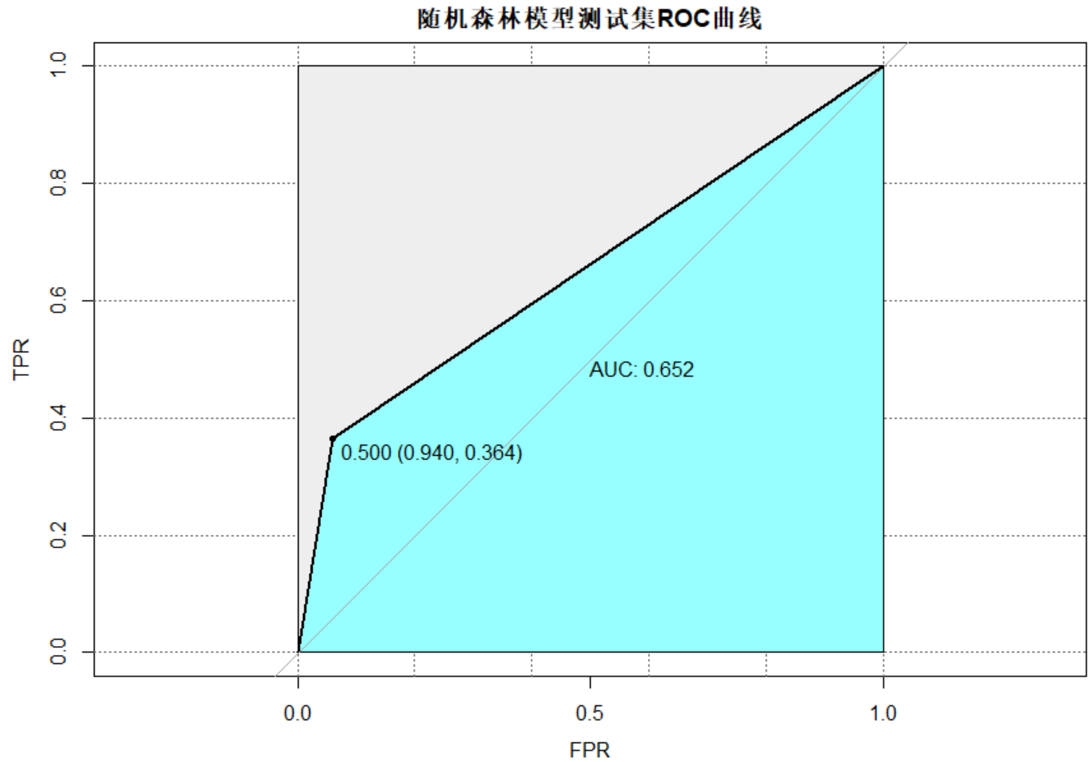
可知随机森林模型的预测准确率为 0.7479

随机森林模型的混淆矩阵输出为:

```
                  Reference
Prediction         0     1
0                 1483   503
1                 94     288

Accuracy : 0.7479
95% CI : (0.7299, 0.7653)
Kappa : 0.3495
Sensitivity : 0.9404
Specificity : 0.3641
Pos Pred Value : 0.7467
Neg Pred Value : 0.7539
Prevalence : 0.6660
Detection Rate : 0.6263
Detection Prevalence : 0.8387
Balanced Accuracy : 0.6522
```

随机森林模型的 ROC 曲线与 AUC 值为:



(6) SVM 模型

```
# svm
# train
library("car")
library("e1071")
train_svm=train
train_svm[1:26]=scale(train_svm[1:26])
model_svm=svm(train_svm$black~.,data=train_svm,type="C-classification")

# predict
test_svm=test
test_svm[1:26]=scale(test_svm[1:26])
test_pred_svm=predict(model_svm,test_svm)
(sum(test_pred_svm==test_svm$black))/nrow(test_svm)
confusionMatrix(as.factor(test_pred_svm), as.factor(test_svm$black))

# roc and auc
```

```
roc(test_svm$black, as.numeric(test_pred_svm)-1, plot=TRUE,
main="SVM模型测试集ROC曲线", xlab = "FPR", ylab = "TPR",
print.thres=TRUE, print.auc=TRUE, legacy.axes=TRUE, grid=c(0.2,0.2),
grid.col="dimgray", auc.polygon=TRUE, max.auc.polygon=TRUE,
auc.polygon.col="darkslategray1")
```

具体输出结果如下：

预测准确率:

```
> (sum(test_pred_svm==test_svm$black))/nrow(test_svm)
[1] 0.7592905
```

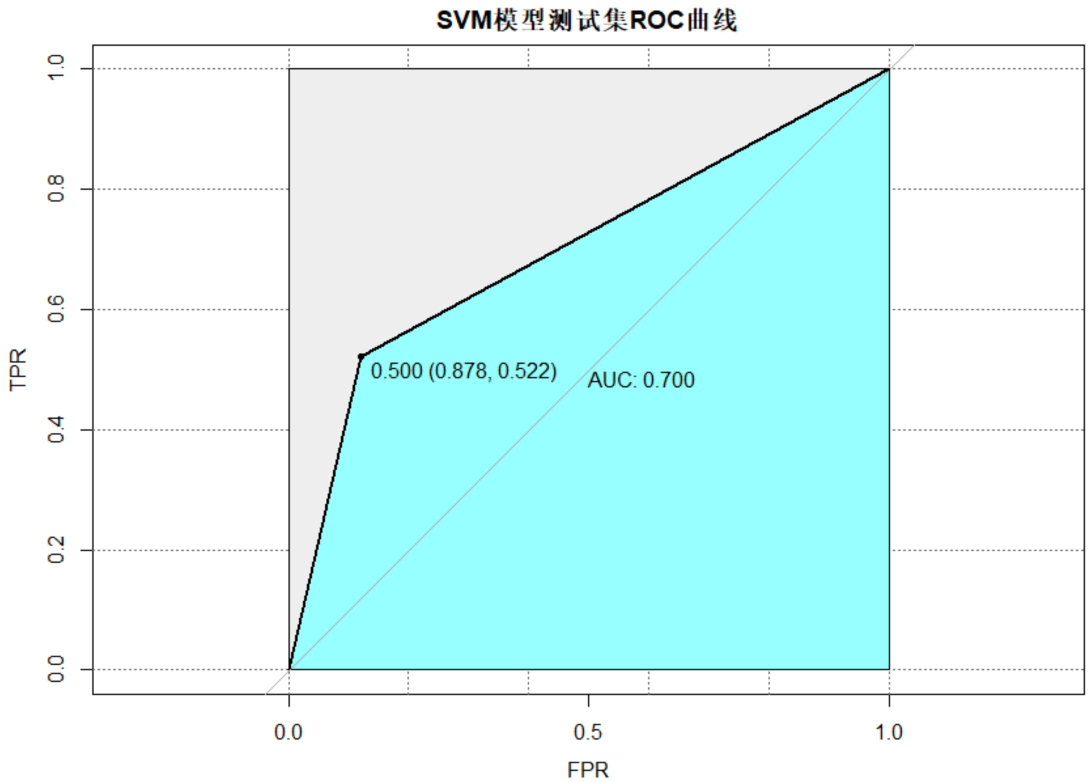
可知 SVM 模型的预测准确率为 0.7593

SVM 模型的混淆矩阵输出为:

```
                    Reference
Prediction       0      1
0             1385    378
1              192    413

Accuracy : 0.7593
95% CI : (0.7415, 0.7764)
Kappa : 0.4253
Sensitivity : 0.8782
Specificity : 0.5221
Pos Pred Value : 0.7856
Neg Pred Value : 0.6826
Prevalence : 0.6660
Detection Rate : 0.5849
Detection Prevalence : 0.7445
Balanced Accuracy : 0.7002
```

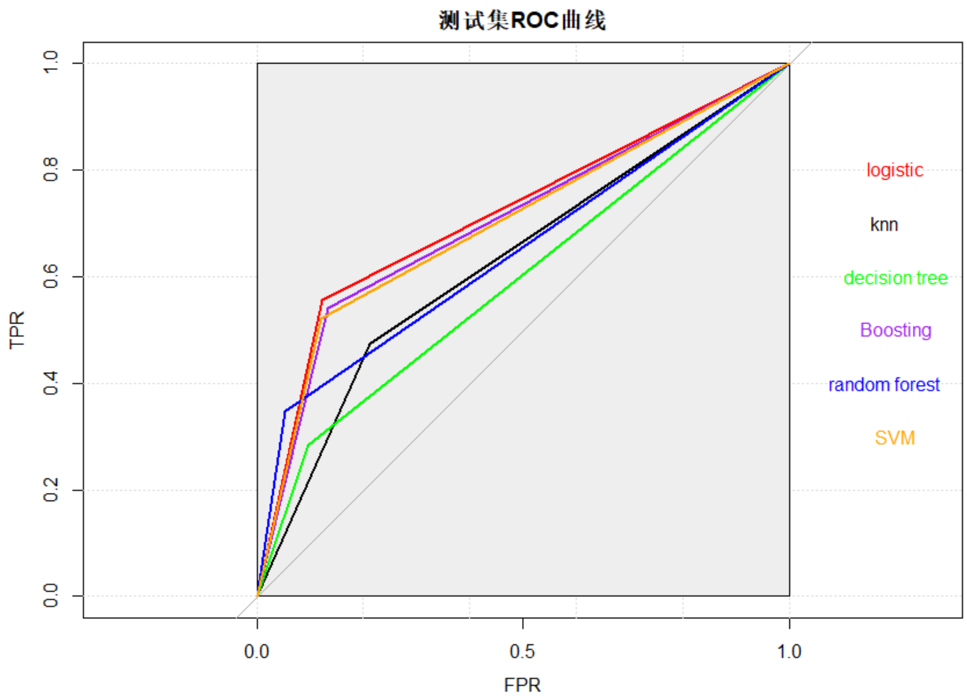
SVM 模型的 ROC 曲线与 AUC 值为:



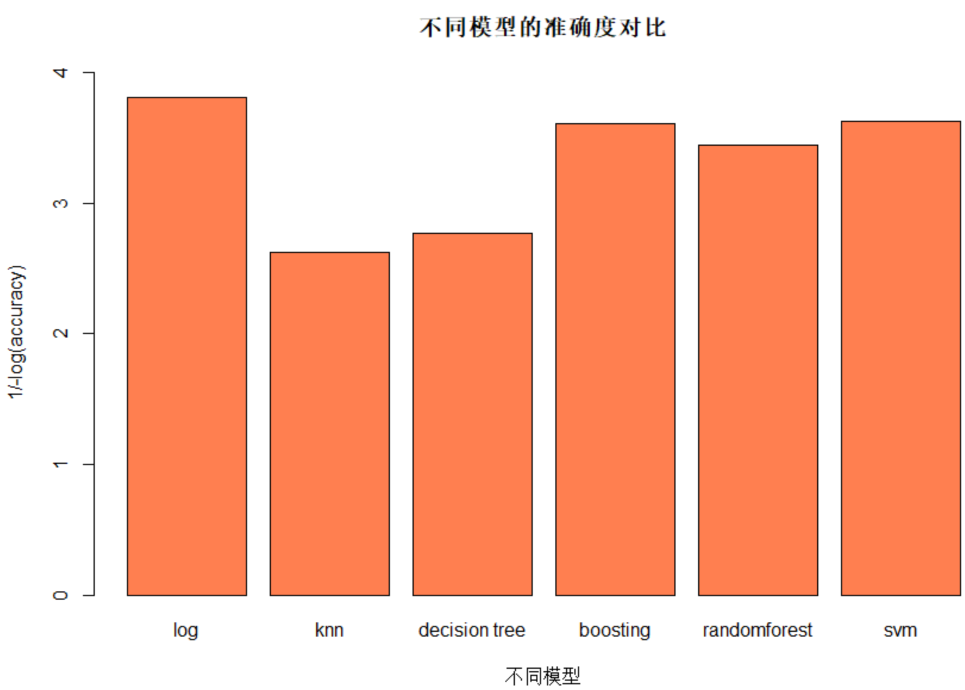
3. 对比模型并选择最优模型

模型对比如下

ROC 曲线对比:(均取 p>0.5)



准确度对比:(为了使对比更明显, 对准确度做了 $\frac{1}{-log(accuracy)}$ 的操作)



根据 ROC 曲线, AUC 值以及准确度对比，逻辑回归模型获得了最大的 AUC 值 (0.839) 和最高准确度 (0.7694)，因此选择逻辑回归模型作为最优的模型