Available online at www.sciencedirect.com
ScienceDirect
Automatica 43 (2007) 290–300
automatica
www.elsevier.com/locate/automatica

Brief paper

Freeway traffic estimation within particle filtering framework[☆]

Lyudmila Mihaylova[a,*], René Boel[b], Andreas Hegyi[c]

[a]Department of Communication Systems, Lancaster University, South Drive, Lancaster LA1 4WA, UK
[b]SYSTeMS Research Group, University of Ghent, B-9052 Zwijnaarde, Belgium
[c]Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands

Received 13 May 2005; received in revised form 25 July 2006; accepted 24 August 2006

Abstract

This paper formulates the problem of real-time estimation of traffic state in freeway networks by means of the particle filtering framework. A particle filter (PF) is developed based on a recently proposed speed-extended cell-transmission model of freeway traffic. The freeway is considered as a network of components representing different freeway stretches called *segments*. The evolution of the traffic in a segment is modelled as a *dynamic stochastic system*, influenced by states of neighbour segments. Measurements are received only at boundaries between some segments and averaged within possibly irregular time intervals. This limits the measurement update in the PF to only these time instants when a new measurement arrives, while in between measurement updates any simulation model can be used to describe the evolution of the particles. The PF performance is validated and evaluated using synthetic and real traffic data from a Belgian freeway. An unscented Kalman filter is also presented. A comparison of the PF with the unscented Kalman filter is performed with respect to accuracy and complexity.

Keywords: Bayesian estimation; Particle filtering; Macroscopic traffic models; Stochastic systems; Unscented Kalman filter

1. Motivation

Traffic state estimation and prediction is of paramount importance for on-line road traffic management, efficiency and safety. Vehicular traffic is characterized by highly nonlinear behaviour (Helbing, 2002), with many complex interactions between vehicles, making this problem challenging. This behaviour can be described by *macroscopic* models (Helbing, 2002; Hoogendoorn & Bovy, 2001; Kotsialos, Papageorgiou, Diakaki, Pavlis, & Middelham, 2002) that are suitable for real-time problems in view of the fact that they represent the *average* traffic behaviour in terms of aggregated variables (flow, density and speed at different locations). Most papers dealing with recursive traffic state estimation apply the extended Kalman filter (EKF) to such macroscopic models. For example, Wang and Papageorgiou (2005) propose an EKF to estimate the unknown parameters and states of a stochastic version of the macroscopic freeway traffic flow model METANET (Papageorgiou & Blosseville, 1989) of freeway traffic. These estimators have all the advantages and disadvantages of the EKF technique: presumably computationally cheap, but relying on a linearization of the state and measurement models which can cause problems.

A powerful and scalable approach has recently been developed, known under different names such as *particle filters* (PFs) (Chen, 2003; Doucet, Freitas, & Gordon, 2001; Ristic, Arulampalam, & Gordon, 2004) and *bootstrap method* (Gordon, Salmond, & Smith, 1993). All information about the states of interest can be obtained from the conditional distribution of the state given the past observations and the dynamics of the system. It approximates the posterior density function of the state by an empirical histogram obtained from samples generated by a Monte Carlo simulation. Particle filtering allows to cope with uncertainties and nonlinearities of any kind and hence is suitable for the traffic estimation problem. Another motivation for suggesting the PF approach to the traffic flow estimation is its modularity. It is a compositional approach to filtering that

[☆] This paper was not presented at any IFAC meeting. The paper was recommended for publication in revised form by Associate Editor Ioannis Paschalidis under the direction of Editor Ian Petersen.

* Corresponding author. Tel.: +44 1524 510388; fax: +44 1524 510493.
E-mail addresses: mila.mihaylova@lancaster.ac.uk (L. Mihaylova), rene.boel@ugent.be (R. Boel), a.hegyi@dcsc.tudelft.nl (A. Hegyi).

0005-1098/$ - see front matter
doi:10.1016/j.automatica.2006.08.023

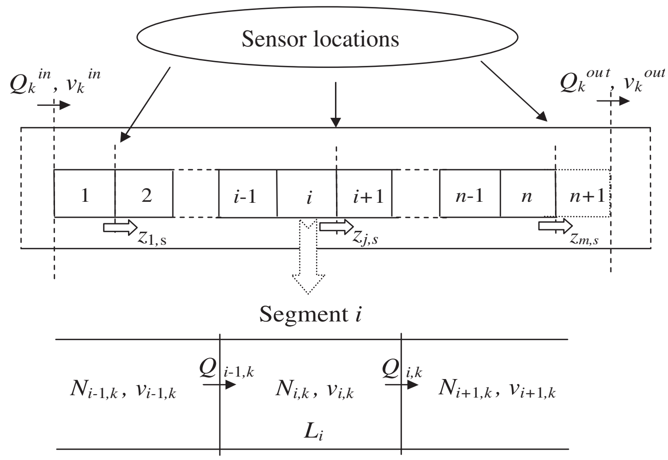

Fig. 1. Freeway segments and measurement points. Q_i is the average number of vehicles crossing the boundary between segments i and $i+1$, N_i and v_i are, respectively, the average number of vehicles and speed within segment i.

is easily adapted to changes in the model of large traffic networks. In the present paper we formulate the traffic estimation problem within this Bayesian framework and we develop a PF for freeway traffic flow estimation. This is an extension and generalization of the results reported in Mihaylova and Boel (2004). The structure of the PF fits well to the compositional traffic networks, and it allows for parallelization for different segments.

In Sun, Muñoz, and Horowitz (2003), a solution to highway traffic estimation is proposed by a sequential Monte Carlo algorithm, the so-called mixture Kalman filtering. First-order traffic models represent the network, i.e., only the traffic density is modelled, distinguishing between the *free-flow mode* and *congestion mode*. In contrast to Sun et al. (2003), the traffic in the present paper is described by a second-order macroscopic model, and we develop a PF that estimates both traffic density and speed. The traffic is described by a recently proposed model (Boel & Mihaylova, 2006) that is an extension to the cell-transmission model (Daganzo, 1994). The freeway network is modelled as a sequence of segments (Fig. 1). Sensors are available only at some boundaries between segments. Technological limitations, such as limited bandwidth of communication channels, force one to average the measurements over regular or irregular time intervals before they are transmitted to the agent where the Bayesian update of conditional densities is carried out.

We investigate the PF performance with respect to accuracy and computational complexity and we compare it with another traffic flow estimator, the unscented Kalman filter (UKF) (Julier & Uhlmann, 2004; Julier, Uhlmann, & Durrant-Whyte, 2000; Wan & van der Merwe, 2001). The UKF is a derivative-free estimation method. The UKF can be viewed as a method to approximate the first two conditional moments of the state: the mean and the covariance. Unlike the EKF, the UKF does not require calculation of Jacobians and Hessians, which for the traffic problem with interconnected components is quite complicated. Deterministic sampling is used to calculate the mean and covariance, by the so-called sigma points. Compared to the EKF's first-order accuracy, the estimation accuracy of the UKF is to the third order (Taylor series expansion) for any nonlinearity. The UKF is much easier to implement and more accurate than the EKF. However, the UKF can encounter the ill-conditioned problem of the covariance matrix (though theoretically it is positive semi-definite). Methods for enhancing the numerical properties of the UKF (e.g., based on singular-value decomposition) can overcome these numerical instabilities (Chen, 2003). The EKF and UKF performance was investigated in Hegyi, Girimonte, Babuška, and De Schutter (2006) for traffic state estimation using METANET to model the freeway traffic. It is shown there that the UKF accuracy is nearly equal in most considered traffic situation or slightly better in others than the EKF accuracy. However, both the EKF and UKF cannot track multi-modal distributions, which is one of the advantages of particle filtering.

The added values and innovative aspects of this paper as compared to previous investigations include:

1. We demonstrate that particle filtering can be efficiently and easily implemented for large compositional models and sparse measurements. The developed approach is general and applicable to freeway networks with different topologies. In particular we show that a recently developed traffic flow model leads to an easily implementable PF with acceptable accuracy.
2. We compare the PF performance with respect to an UKF. We show that the PF estimates are more accurate than those of the UKF, nevertheless the PF is more computationally expensive. However, both algorithms have real-time capabilities. The implemented PF works well with a relatively small number of particles. The reason is that for light traffic different segments provide approximately independent information while for dense traffic the high correlation between segments reduces the effective size of the state space.

The outline of the paper is as follows. Section 2 presents a stochastic macroscopic traffic model for freeway stretches and a model for real-time traffic measurements. Bayesian formulation of the traffic estimation problem is given in Section 3. A PF framework for traffic state estimation is developed in Section 4 which takes advantage of the compositional traffic model. Section 5 describes the UKF for traffic estimation that is compared with the developed PF. The PF performance is evaluated in Section 6. Conclusions and future research issues are highlighted in Section 7.

2. Freeway traffic flow model

2.1. Compositional macroscopic traffic model

Traffic states are estimated consecutively at discrete time instants $t_1, t_2, \ldots, t_k, \ldots,$ possibly *asynchronously*, based on all the incoming information up to the current time transmitted by sensors to the filter. The overall state vector $\boldsymbol{x}_k = (\boldsymbol{x}_{1,k}^T, \boldsymbol{x}_{2,k}^T, \ldots, \boldsymbol{x}_{n,k}^T)^T$ at time t_k consists of local state vectors

$\boldsymbol{x}_{i,k} = (N_{i,k}, v_{i,k})^{\mathrm{T}}$, where $N_{i,k}$, veh, is the number of vehicles counted in segment $i \in \mathscr{I} = \{1, 2, \ldots, n\}$, and $v_{i,k}$, km/h, is their average speed. The traffic state evolution is described by the system of equations

$$\boldsymbol{x}_{1,k+1} = f_1(Q_k^{\mathrm{in}}, v_k^{\mathrm{in}}, \boldsymbol{x}_{1,k}, \boldsymbol{x}_{2,k}, \boldsymbol{\eta}_{1,k}), \tag{1}$$

$$\boldsymbol{x}_{i,k+1} = f_i(\boldsymbol{x}_{i-1,k}, \boldsymbol{x}_{i,k}, \boldsymbol{x}_{i+1,k}, \boldsymbol{\eta}_{i,k}), \tag{2}$$

$$\boldsymbol{x}_{n,k+1} = f_n(\boldsymbol{x}_{n-1,k}, \boldsymbol{x}_{n,k}, Q_k^{\mathrm{out}}, v_k^{\mathrm{out}}, \boldsymbol{\eta}_{n,k}), \tag{3}$$

where f_i is specified by the traffic model, Q_k^{in} is the number of vehicles entering segment 1 during the interval $\Delta t_k = t_{k+1} - t_k$ with average speed v_k^{in}, Q_k^{out} is the outflow leaving a "fictitious" segment $n+1$, with an average speed v_k^{out}. $\boldsymbol{\eta}_k$ is a disturbance vector, reflecting random fluctuations and the effect of modelling errors in the state evolution. Note that Q_k^{in}, v_k^{in}, and Q_k^{out}, v_k^{out} are, respectively, inflow and outflow *boundary variables*. They *are not* traffic states and are not estimated. They are supplied by the traffic detectors as boundary conditions for the chain of interconnected segments.

In this paper the general state-space description (1)–(3) takes a particular form of the recently developed compositional stochastic macroscopic traffic model (Boel & Mihaylova, 2006). This speed-extended cell-transmission model describes the complex traffic behaviour with *forward* and *backward* propagation of traffic perturbations and is suitable for large networks and for distributed processing. The forward and backward traffic perturbations were characterized by Daganzo (1994) through deterministic sending and receiving functions, piecewise affine relations between flow and density. In Boel and Mihaylova (2004, 2006) *speed-dependent* random sending and receiving functions are introduced that represent also the evolution of the average speed in each segment. The model is given in concise form as Algorithm 1.

The *sending function* $S_{i,k}$ for each segment i, having length L_i, is calculated by (4). $S_{i,k}$ represents the vehicles that "intend to leave" segment i within Δt_k. The *receiving function* $R_{i,k}$ (6) expresses the maximum number of vehicles that are allowed to enter segment $i+1$. In (6) $N_{i+1,k}^{\max}$ characterizes the *maximum number of vehicles* that can simultaneously be present in segment $i+1$ at time t_k. $N_{i+1,k}^{\max}$ depends on the available space, L_{i+1} time the number of lanes $\ell_{i+1,k}$, in segment $i+1$, on the average length A_ℓ of vehicles, the average speed $v_{i+1,k}$ and the safe time distance t_{d} between two vehicles.

The evolution of $N_{i,k+1}$ is governed by the principle of conservation of vehicles (9). The traffic density $\rho_{i,k+1}$, veh/km/lane, is given by (10). The anticipated density $\rho_{i,k+1}^{\mathrm{antic}}$ is then obtained as a weighed average between the density of segment i and segment $i+1$ (11). This corresponds to the drivers' tendency usually to look ahead when they change their speed. The average vehicle speed $v_{i,k+1}$ is a function of the 'intermediate' speed $v_{i,k+1}^{\mathrm{interm}}$, calculated in step 5 of Algorithm 1, and of the equilibrium speed satisfying some speed–density relation $v^e(\rho_{i,k+1}^{\mathrm{antic}})$, e.g., Kotsialos et al. (2002).

Design traffic parameters are: the free-flow speed v_{free}, the critical density ρ_{crit} (density below which the interactions between vehicles will be negligible), the density in jam, ρ_{jam}, above which the vehicles do not move, and the minimum vehicle speed $v_{\min}$, the parameters $\alpha \in (0, 1]$, $0 < \beta^I < \beta^{II}$, the threshold density value $\rho_{\mathrm{threshold}}$ and the function $v^e(\rho_{i,k+1}^{\mathrm{antic}})$. Details for the model can be found in Boel and Mihaylova (2004, 2006) where this extended cell-transmission model has been validated both against the well established METANET model (Kotsialos et al., 2002; Papageorgiou & Blosseville, 1989), and over real traffic data.

Algorithm 1. The compositional traffic model.

1. *Forward wave*: for $i = 1, 2, \ldots, n$,

$$S_{i,k} = \max\left(N_{i,k}\frac{v_{i,k}.\Delta t_k}{L_i} + \eta_{S_i,k},\ N_{i,k}\frac{v_{\min}.\Delta t_k}{L_i}\right) \tag{4}$$

and set $Q_{i,k} = S_{i,k}$. (5)

2. *Backward wave*: for $i = n, n-1, \ldots, 1$,

$$R_{i,k} = N_{i+1,k}^{\max} - N_{i+1,k} + Q_{i+1,k}, \tag{6}$$

where $N_{i+1,k}^{\max} = (L_{i+1}\ell_{i+1,k})/(A_\ell + v_{i+1,k}t_{\mathrm{d}})$,

$$\textit{if } S_{i,k} < R_{i,k}, \quad Q_{i,k} = S_{i,k}, \tag{7}$$

$$\textit{else } Q_{i,k} = R_{i,k}, \quad v_{i,k} = Q_{i,k}L_i/(N_{i,k}\Delta t_k). \tag{8}$$

3. Update the number of vehicles inside segments,

for $i = 1, 2, \ldots, n$,

$$N_{i,k+1} = N_{i,k} + Q_{i-1,k} - Q_{i,k}. \tag{9}$$

4. Update the density, for $i = 1, 2, \ldots, n$,

$$\rho_{i,k+1} = N_{i,k+1}/(L_i\ell_{i,k+1}), \tag{10}$$

$$\rho_{i,k+1}^{\mathrm{antic}} = \alpha\rho_{i,k+1} + (1-\alpha)\rho_{i+1,k+1}. \tag{11}$$

5. Update of the speed, for $i = 1, 2, \ldots, n$,

$$v_{i,k+1}^{\mathrm{interm}} = \begin{cases} \dfrac{v_{i-1,k}Q_{i-1,k} + v_{i,k}(N_{i,k} - Q_{i,k})}{N_{i,k+1}} & \text{for } N_{i,k+1} \neq 0, \\ v_f & \text{otherwise}, \end{cases}$$

$$v_{i,k+1}^{\mathrm{interm}} = \max(v_{i,k+1}^{\mathrm{interm}}, v_{\min}),$$

$$v_{i,k+1} = \beta_{k+1}v_{i,k+1}^{\mathrm{interm}} + (1-\beta_{k+1})v^e(\rho_{i,k+1}^{\mathrm{antic}}) + \eta_{v_i,k+1},$$

where

$$\beta_{k+1} = \begin{cases} \beta^I & \text{if } |\rho_{i+1,k+1}^{\mathrm{antic}} - \rho_{i,k+1}^{\mathrm{antic}}| \geqslant \rho_{\mathrm{threshold}}, \\ \beta^{II} & \text{otherwise}. \end{cases}$$

2.2. Measurement model

Sensors (magnetic loops, video cameras, radar detectors) are located in boundaries between *some* segments. Usually, measurements are collected at the entrance and at the exit of the considered stretch of the road, providing boundary conditions Q^{in}, Q^{out}, v^{in} and v^{out}, at the on-ramps and off-ramps.

Traffic behaviour is measured at discrete time instants t_s, $s = 1, 2, \ldots$. The overall measurement vector $z_s = (z_{1,s}^{\mathrm{T}}, z_{2,s}^{\mathrm{T}}, \ldots, z_{m,s}^{\mathrm{T}})^{\mathrm{T}}$ at time t_s consists of local measurement vectors $z_{j,s} = (\bar{Q}_{j,s}, \bar{v}_{j,s})^{\mathrm{T}}$, where $j \in \mathscr{J} = \{1, 2, \ldots, m\}$. $\bar{Q}_{j,s}$ is the number of vehicles crossing the boundaries between the corresponding segment i and segment $i+1$ during the time interval $\Delta t_s = t_{s+1} - t_s$, and $\bar{v}_{j,s}$ is the mean speed of these vehicles. The intervals Δt_s are typically several times longer than the intervals Δt_k between q successive state update steps, i.e., $\Delta t_s = q\Delta t_k$.

Given the measurement equation

$$z_s = \boldsymbol{h}(\boldsymbol{x}_s, \boldsymbol{\xi}_s), \tag{12}$$

the distribution $p(\boldsymbol{x}_0)$ of the initial state vector $\boldsymbol{x}_0$, the state update model (1)–(3) with noises $\boldsymbol{\eta}$, $\boldsymbol{\xi}$, the traffic estimation problem can be formulated within the recursive Bayesian framework. We consider the following particular form for equation (12):

$$z_{j,s} = \begin{pmatrix} \bar{Q}_{j,s} \\ \bar{v}_{j,s} \end{pmatrix} + \boldsymbol{\xi}_{j,s}. \tag{13}$$

Although Gaussian distributions of the noise vector $\boldsymbol{\xi}_{j,s} = (\xi_{Q_{j,s}}, \xi_{v_{j,s}})^{\mathrm{T}}$ in (13) have been used previously in the literature (Wang & Papageorgiou, 2005), we propose a more realistic noise model. This is another advantage of the PF: the knowledge of the noise distributions can be easily utilized for a better state tracking. Based on statistical analysis of different sets of traffic data we found that there are two kinds of measurement errors in the count vehicle errors due to *false detections*, $\xi_{Q_{j,s}^{\text{false}}}$, and errors due to *missed vehicles*, $\xi_{Q_{j,s}^{\text{missed}}}$. Hence, the measurement error in the observation equation (12), resp., (13) is of the form

$$\xi_{j,s} = Q_{j,s}^{\text{err}} = Q_{j,s}^{\text{false}} - Q_{j,s}^{\text{missed}}, \tag{14}$$

where the number of the vehicles that a detector j missed is denoted by $Q_{j,s}^{\text{missed}}$, and the number of the false detections by $Q_{j,s}^{\text{false}}$. Analysis of data from video cameras has shown (Rasschaert, 2003) that $Q_{j,s}^{\text{false}}$ and $Q_{j,s}^{\text{missed}}$ can both be considered independent Poisson random variables with parameters λ_1 and λ_2. Under typical conditions we estimated $\lambda_1 + \lambda_2 = 2$, $\lambda_1 = \frac{4}{3}$, $\lambda_2 = \frac{2}{3}$. Then the PDF of the measurement noise $\xi_{Q_i,s}$ is a convolution of the form

$$p(Q_{j,s}^{\text{err}} = v_{i,s}^{\text{err}}) = \sum_{v_{i,s}^{\text{missed}} = \max(0, -v_{i,s}^{\text{err}})}^{\infty} \frac{\lambda_1^{(v_{i,s}^{\text{err}} + v_{i,s}^{\text{missed}})} \mathrm{e}^{-\lambda_1}}{(v_{i,s}^{\text{err}} + v_{i,s}^{\text{missed}})!} \cdot \frac{\lambda_2^{v_{i,s}^{\text{missed}}} \mathrm{e}^{-\lambda_2}}{v_{i,s}^{\text{missed}}!}. \tag{15}$$

Eq. (15) represents the PF likelihood function of the observations over the counted number of vehicles. We assume that the speed noises $\xi_{v_j,s}$ are Gaussian. Under the assumption that the vehicle counts are statistically independent from the speed measurements, the entire likelihood $p(z_s|\boldsymbol{x}_s)$ given the state $\boldsymbol{x}_s$ is the product of the likelihood of the measured counts with the likelihood of the measured speeds.

3. Bayesian estimation of traffic flows

Bayesian estimation evaluates the *posterior probability density function* (PDF) $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ of the state vector $\boldsymbol{x}_k$ up to time instant t_k given a set $\boldsymbol{Z}^k = \{z_1, \ldots, z_k\}$ of sensor measurements available at time t_k. Within the recursive Bayesian framework (Ristic et al., 2004), the conditional density function $p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1})$ of the state $\boldsymbol{x}_k$ given a set of measurements $\boldsymbol{Z}^{k-1}$ is recursively updated according to

$$p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1}) = \int_{\mathbb{R}^{n_x}} p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) p(\boldsymbol{x}_{k-1}|\boldsymbol{Z}^{k-1}) \, \mathrm{d}\boldsymbol{x}_{k-1}, \tag{16}$$

$$p(\boldsymbol{x}_k|\boldsymbol{Z}^k) = \frac{p(z_k|\boldsymbol{x}_k) p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1})}{p(z_k|\boldsymbol{Z}^{k-1})}, \tag{17}$$

where $p(z_k|\boldsymbol{Z}^{k-1})$ is a normalizing constant. Therefore, the recursive update of $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ is proportional to

$$p(\boldsymbol{x}_k|\boldsymbol{Z}^k) \propto p(z_k|\boldsymbol{x}_k) p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1}). \tag{18}$$

The state prediction step (16) and the measurement update step (17) use, respectively, the conditional density functions $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ and $p(z_k|\boldsymbol{x}^k)$, defined by the model from Section 2.1, by Eqs. (1)–(3) and (17).

4. Particle filtering for freeway traffic

The evaluation of (16)–(18) is computationally expensive. The PF technique (Doucet et al., 2001; Gordon et al., 1993) provides an approximate solution to (16)–(18) by a discrete-time recursive update of the *posterior* PDF $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ of the state given the measurements. The PF approximates $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ by the empirical histogram corresponding to a collection of M *particles* (*samples*) $\{\boldsymbol{x}_k^{(l)}\}_{l=1}^M$ of a state trajectory. To each particle l a weight $w_k^{(l)}$ is assigned at time t_k (the sum of these weights must be normalized to 1). The weight and the value of all particles together define a histogram that approximates the conditional density function of the state vector $\boldsymbol{x}_k$. After the arrival of a new observation vector z_s, the PF updates the weights according to (18). The cloud of particles evolves with time and depending on the observations, so that the particles represent with sufficient accuracy the true PDF of the state (Doucet et al., 2001). A *resampling* procedure introduces variety in the particles, by eliminating the particles with small weights and by replicating particles with larger weights.

The traffic estimation problem has particularities distinguishing it from other estimation problems: (i) The *limited* amount of available data from traffic detectors. The number of traffic variables to be estimated is much larger than the number of the traffic variables that are directly observed, and this “interpolation” is an essential contribution to the freeway traffic estimation

Algorithm 2. A particle filter for traffic estimation.

I. Initialization: $k=0$

For $l=1,\ldots,M$, generate samples $\{\boldsymbol{x}_0^{(l)}\}$ from the initial distribution $p(\boldsymbol{x}_0)$ and initial weights $w_0^{(l)}=1/M$.

End For

II. For $k=1,2,\ldots,$

(1) Prediction step:

For $l=1,\ldots,M$, sample $\boldsymbol{x}_k^{(l)} \sim p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^{(l)})$ according to (4)–(11) for segments between two boundaries where measurements arrive

End For

(2) Measurement processing step (only for $t_k \equiv t_s$, on boundaries between the segments where measurements are available) compute the weights

For $l=1,\ldots,M$

$w_s^{(l)}=w_{s-1}^{(l)}p(\boldsymbol{z}_s|\boldsymbol{x}_s^{(l)})$,

End For

where the likelihood $p(\boldsymbol{z}_s|\boldsymbol{x}_s^{(l)})$ is calculated by the model (13) from Section 2.2.

For $l=1,\ldots,M$

Normalize the weights: $\hat{w}_s^{(l)}=w_s^{(l)}/\sum_{l=1}^{M}w_s^{(l)}$.

End For

(3) Output: $\hat{\boldsymbol{x}}_s=\sum_{l=1}^{M}\hat{w}_s^{(l)}\boldsymbol{x}_s^{(l)}$,

(4) Selection step (resampling) only for $t_k \equiv t_s$:

Multiply/suppress samples $\boldsymbol{x}_s^{(l)}$ with high/low importance weights $\hat{w}_s^{(l)}$, in order to obtain M random samples approximately distributed according to $p(\boldsymbol{x}_s^{(l)}|\boldsymbol{Z}^s)$, e.g. by residual resampling.

* For $l=1,\ldots,M$, set $w_s^{(l)}=\hat{w}_s^{(l)}=1/M$, End For

(5) $k \leftarrow k+1$ and return to step (1).

task. (ii) The state estimates are highly dependent on the inflow Q^{in}, v^{in} and outflow random Q^{out}, v^{out} boundary variables.

The likelihood function $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is calculated from (13) only when a measurement arrives, using the predicted state values and the known measurement noise density function $p(\boldsymbol{\xi}_s)$. The cloud of weighted particles representing the posterior conditional PDF, is used to map integrals to discrete sums: $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ is approximated by

$$\hat{p}(\boldsymbol{x}_k|\boldsymbol{Z}^k) \approx \sum_{l=1}^{M} \tilde{w}_k^{(l)}\delta(\boldsymbol{x}_k-\boldsymbol{x}_k^{(l)}), \tag{19}$$

where δ is the delta-Dirac function and $\tilde{w}_k^{(l)}$ are the normalized weights of the posterior conditional PDF. New weights are calculated putting more weight on particles that are important according to the posterior PDF (19). The random samples $\{\boldsymbol{x}_k^{(l)}, l=1,2,\ldots,M\}$ are drawn from $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$. It is often impossible to sample from the posterior density function $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$. However, this difficulty is circumvented by making use of the importance sampling from a known *proposal distribution* $\pi(\boldsymbol{x}_k|\boldsymbol{Z}^k)$. The transition prior is the most popular choice of the proposal distribution (Wan & van der Merwe, 2001): $\pi(\boldsymbol{x}_k|\boldsymbol{Z}^k)=p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$, which in our solution to the traffic problem is the traffic state model. Algorithm 2 presents the PF developed in this paper.

5. An UKF for traffic flow estimation

Other techniques for recursive approximation of the posterior state PDF have been introduced. As explained in the Introduction it is very difficult to obtain all the Jacobians and Hessians of Eqs. (1)–(12) needed for an EKF implementation. Hence, we compare the PF results with the results from a UKF. The UKF relies on the unscented transformation (Julier & Uhlmann, 2004; Wan & van der Merwe, 2001). Consider the propagation of a random variable $\boldsymbol{x}$ (having dimension n_x), with mean $\hat{\boldsymbol{x}}$ and covariance matrix $\boldsymbol{P}$, through a nonlinear transformation $\boldsymbol{y}=\boldsymbol{f}(\boldsymbol{x})$. To calculate the statistics of $\boldsymbol{y}$, a matrix $\mathscr{X}$ of $2.n_x+1$ sigma points $\mathscr{X}_i$ is formed. These sigma points $\mathscr{X}_i$ are propagated through the measurement function $\boldsymbol{h}$ in the measurement update step. Transformed points $\mathscr{Z}_{i,k/k-1}$ are obtained that form the matrix $\mathscr{Z}_{k/k-1}$. Similarly to the Kalman filter, the Kalman gain $\boldsymbol{K}$, the state estimate $\hat{\boldsymbol{x}}$ and the corresponding covariance matrix $\boldsymbol{P}$ are updated by (20)–(22). The UKF equations are given as Algorithm 3. We implemented the UKF using an augmented state vector concatenating the original state and the noise variables: $\boldsymbol{x}_k^a=(\boldsymbol{x}_k^{\mathrm{T}},\ \boldsymbol{\eta}_k^{\mathrm{T}},\ \boldsymbol{\xi}_k^{\mathrm{T}})^{\mathrm{T}}$ (Wan & van der Merwe, 2001). The corresponding matrix with sigma points is $\mathscr{X}^a=((\mathscr{X}^x)^{\mathrm{T}},\ (\mathscr{X}^\eta)^{\mathrm{T}},\ (\mathscr{X}^\xi)^{\mathrm{T}})^{\mathrm{T}}$. Unlike the PF, the sigma points of the UKF are deterministically chosen so that they exhibit certain properties, e.g., they have a given mean and covariance. For a comparison between the UKF and EKF see Hegyi et al. (2006) but note that a different traffic model is used.

6. Performance evaluation

6.1. Investigations with synthetic data

The PF performance is evaluated versus the UKF over of a freeway stretch of 4 km, consisting of eight segments each with three lanes, during 3 h. This experiment comprises smooth changes in the inflow and outflow that happen often on freeways. The traffic congestion is modelled through respective changes of the inflow Q_k^{in} (increase and decrease) within the time intervals [1.12–1.17) h and [1.70, 1.82) h, and decrease of the outflow speed v_k^{out} within the interval [2.40–2.65] h (shown in Fig. 2). In order to avoid the stepwise increase of the traffic flow, the data were generated by taking a stretch of 19 segments. We observed that the backward wave is well pronounced from the 13th segment to the sixth segment. This is why the performance of the PF and UKF is validated based on measurements (and inflow at the same time) from segment 4 and, respectively, segment 11 (it is also the outflow). Traffic states of segments 4–11 are the eight estimated state segments by the filters.

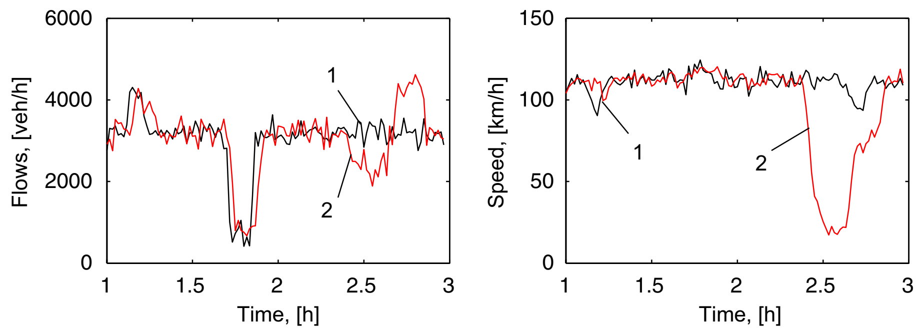

Fig. 2. Boundary conditions: 1—in, 2—out.

Algorithm 3. Unscented Kalman filter equations.

I. Initialize with:

$\hat{\boldsymbol{x}}_0 = E[\boldsymbol{x}_0], \boldsymbol{P}_0 = E[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)^{\mathrm{T}}], (\hat{\boldsymbol{x}}_0^a = E[\boldsymbol{x}_0^a],$

$\boldsymbol{P}_0^a = E[(\boldsymbol{x}_0^a - \hat{\boldsymbol{x}}_0^a)(\boldsymbol{x}_0^a - \hat{\boldsymbol{x}}_0^a)^{\mathrm{T}}] = \mathrm{diag}\,\{\boldsymbol{P}_0, \boldsymbol{P}_{\boldsymbol{\eta}}, \boldsymbol{P}_{\boldsymbol{\xi}}\}.$

For $k = 1, 2, \ldots,$

II. Calculate sigma points:

$\mathscr{X}_{k-1}^a = [\hat{\boldsymbol{x}}_{k-1}^a, \hat{\boldsymbol{x}}_{k-1}^a + \gamma\sqrt{\boldsymbol{P}_{k-1}^a}, \hat{\boldsymbol{x}}_{k-1}^a - \gamma\sqrt{\boldsymbol{P}_{k-1}^a}],$

where $\sqrt{\boldsymbol{P}_{k-1}^a}$ is a Cholesky factor, $\gamma = \sqrt{n_x + \lambda}$,

$\lambda = \alpha^2(n_x + \kappa) - n_x, 1 \leqslant \alpha \leqslant 1e-4, \kappa = 3 - n_x.$

III. Time update:

$$\mathscr{X}_{k/k-1}^x = \boldsymbol{f}(\mathscr{X}_{k-1}^x, \mathscr{X}_{k-1}^\eta),$$

$$\hat{\boldsymbol{x}}_{k/k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathscr{X}_{i,k/k-1}^x,$$

$$\boldsymbol{P}_{k/k-1} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathscr{X}_{i,k/k-1}^x - \hat{\boldsymbol{x}}_{k/k-1}] \times [\mathscr{X}_{i,k/k-1}^x - \hat{\boldsymbol{x}}_{k/k-1}]^{\mathrm{T}},$$

$$\mathscr{Z}_{k/k-1} = \boldsymbol{h}(\mathscr{X}_{k/k-1}^x, \mathscr{X}_{k-1}^\xi),$$

$$\hat{z}_{k/k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathscr{Z}_{i,k/k-1}.$$

IV. Measurement update equations:

$$\boldsymbol{P}_{z_k z_k} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathscr{Z}_{i,k/k-1} - \hat{z}_{k/k-1}][\mathscr{Z}_{i,k/k-1} - \hat{z}_{k/k-1}]^{\mathrm{T}},$$

$$\boldsymbol{P}_{\boldsymbol{x}_k z_k} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathscr{X}_{i,k/k-1}^x - \hat{\boldsymbol{x}}_{k/k-1}][\mathscr{Z}_{i,k/k-1} - \hat{z}_{k/k-1}]^{\mathrm{T}},$$

$$\mathscr{K}_k = \boldsymbol{P}_{\boldsymbol{x}_k z_k} \boldsymbol{P}_{z_k z_k}^{-1}, \tag{20}$$

$$\hat{\boldsymbol{x}}_{k/k} = \hat{\boldsymbol{x}}_{k/k-1} + \mathscr{K}_k(z_k - \hat{z}_{k/k-1}), \tag{21}$$

$$\boldsymbol{P}_{k/k} = \boldsymbol{P}_{k/k-1} - \mathscr{K}_k \boldsymbol{P}_{z_k z_k} \mathscr{K}_k^{\mathrm{T}}, \tag{22}$$

where the weights are: $W_0^{(m)} = \lambda/(n_x + \lambda)$,

$W_0^{(c)} = \lambda/(n_x + \lambda) + (1 - \alpha^2 + \beta),$

$W_i^{(m)} = W_i^{(c)} = \frac{1}{2}(n_x + \lambda), \ i = 1, \ldots, 2n_x.$

The initial conditions for all segments are: $N_{i,0} = 14$ veh, $v_{i,0} = 100$ km/h, $i = 1, \ldots, 19$. For having a smooth change of the number of vehicles crossing the boundaries, and their corresponding speed, the inflow data were changed according to

$$Q_k^{\mathrm{in}} = c \exp\left\{-\left[\frac{\rho_{1,k-1}}{\rho_{\mathrm{crit}}}\right]\right\}, \tag{23}$$

$$v_k^{\mathrm{in}} = v_f \exp\left\{-\frac{1}{a_m}\left[\frac{\rho_{1,k}}{\rho_{\mathrm{crit}}}\right]^{a_m}\right\} + 3.15 n_{v,k} = v^e(\rho_{1,k}) + 3.15 n_{v,k}, \tag{24}$$

where c is a constant, having different values (resp., $105 + 1.25 n_{N,k}$ in [1.12, 1.17) h, $2 + 0.50 n_{N,k}$ in [1.70, 1.82) h, and $14 + 1.25 n_{N,k}$ in [2.40, 2.65]). $n_{v,k}$ is a white Gaussian noise sequence (introducing fluctuations in the speed) with a standard deviation 3.15, $n_{N,k}$ is a white Gaussian noise sequence (introducing fluctuations in the inflow) with standard deviations, respectively, 1.25, 0.50 and 1.25 in the three intervals. Eq. (23) accounts for the density $\rho_{1,k}$ in the next cell. In the interval [2.40, 2.65] h the speed in segment 13 is suddenly decreased to $v_{13,k} = 2v_{\min} = 14.80$ km/h.

The data from segments 4 and 11 have additionally added measurement noises. These measurements are used in the filters also as inflow/outflow boundary conditions (for the state model). The augmented state vector is $\boldsymbol{x}_k = (\boldsymbol{x}_{4,k}^{\mathrm{T}}, \boldsymbol{x}_{5,k}^{\mathrm{T}}, \ldots, \boldsymbol{x}_{11,k}^{\mathrm{T}})^{\mathrm{T}}$, i.e., $i = 4, 5, \ldots, 11$, and the measurement vector $z_s = (z_{4,s}^{\mathrm{T}}, z_{11,s}^{\mathrm{T}})^{\mathrm{T}}$. The per minute aggregated measurements are supplied to the PF and UKF as would be the case with real data. The state prediction is performed also at each intermediate state update time step. We are estimating the states of all segments between two measurements as one augmented state vector.

Table 1
Parameters of the PF and UKF

$v_{\text{free}} = 120\,\text{km/h},\ v_{\min} = 7.4\,\text{km/h}$
$\rho_{\text{crit}} = 20.89\,\text{veh/km/lane},\ \rho_{\text{jam}} = 180\,\text{veh/km}$
$\alpha = 0.65,\ \beta_{k+1} = \begin{cases} 0.25 & \text{if } \lvert\rho^{\text{antic}}_{i+1,k+1} - \rho_{i,k+1}\rvert \geqslant 2, \\ 0.75 & \text{otherwise.} \end{cases}$
$\Delta t_i = 10\,\text{s},\ t_{\text{d}} = 2\,\text{s},\ L_i = 0.5\,\text{km},\ i = 1, \ldots, 8,$
$M = 200$ particles, $t_{\text{d}} = 2\,\text{s},\ A_\ell = 0.01\,\text{km},\ \ell_i = 3$
$cov\{\eta_{S_{i,k}}\} = (0.03 N_{i,k} v_{i,k} \Delta t_k / L_i)^2\ (\text{veh})^2$
$cov\{\eta_{Q_i}\} = 1^2\ (\text{veh})^2,\ cov\{\eta_{v_i}\} = 3.5^2\ (\text{km/h})^2$
$cov\{\xi_{Q_i}\} = 1^2\ (\text{veh})^2,\ cov\{\xi_{v_i}\} = 5^2\ (\text{km/h})^2$

The filters' performance is evaluated by *root mean square errors* (RMSEs) $\varepsilon(\hat{x}_{i,k}) = [\frac{1}{r}\sum_{i=1}^{r}(\varepsilon_{i,k})^{\mathrm{T}}(\varepsilon_{i,k})]^{1/2}$, for the state errors, $\varepsilon_{i,k} = x_{i,k} - \hat{x}_{i,k}$, over $r = 100$ independent Monte Carlo runs, with respect to density, speed and flow. The initial particles for the PF are randomly generated by adding Gaussian noise to the actual states. Table 1 gives the parameters of the state model.

The evolution of the flow and speed in time (for one realization) are given in Figs. 3 and 4, where the presence of a backward wave is well pronounced in time. The flow-density and the speed–flow diagrams have the typical bell-shaped forms. RMSEs calculated with $M = 200$ for segments 4, 8, and 11 are presented in Fig. 5. We see the influence of the backward wave on these RMSEs. We observe that the RMSE values in segment 4 are smaller than their values in the intermediate segment 8 for which no sensor data are available. According to these results the PF estimates are more accurate than the UKF estimates. However, the PF complexity is computationally more expensive than the UKF. The complexity of the PF is proportional to the number of particles, times the dimension of the overall state vector, $M.n_x$, whilst the complexity of the UKF is proportional to the number $2.n_x + 1$ of sigma points. Note that n_x is equal to the number of segments n times the number of states 2 in a segment. We calculated the ratio between the PF and UKF computational time and it is 2.8 (with $M = 100$ particles), 5.45 (with $M = 200$), 15 (with $M = 500$).

In general, the number of necessary particles is increasing with the increased number of states for reaching a certain accuracy, but not very much. It is difficult to characterize in general the PF accuracy and complexity because they highly depend on the road structure and the traffic conditions. We tested the PF on 12.5 km freeway stretch (25 segments) with 350 particles and we obtained accuracy comparable to the accuracy with 4 km (with 200 particles). The approach proposed in Vaswani, Yezzi, Rathi, and Tannenbaum (2006) shows also that the particle filtering is able to cope with large dimensional state spaces. Other PF variants such as Rao–Blackwellization (Doucet et al., 2001; Ristic et al., 2004) can be applied to reduce the computational load. These PF approaches, however, require splitting the state variables into linear and nonlinear parts, applying a PF to the nonlinear part, and an optimal linear estimator to the linear relations.

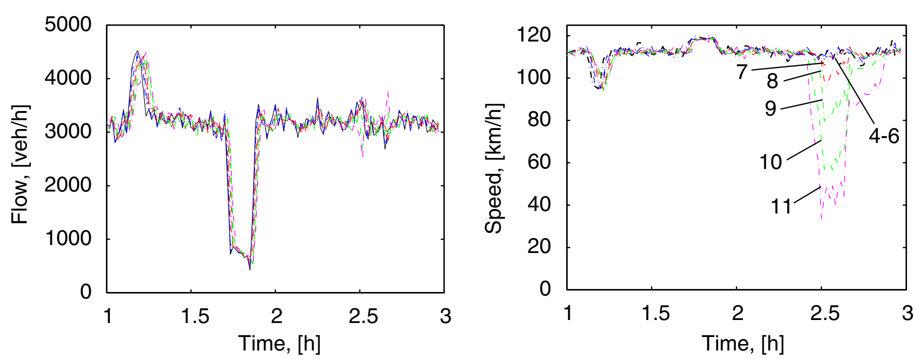

Fig. 3. PF estimated states of segments 4–11: the backward wave is well visible in segments 7–11.

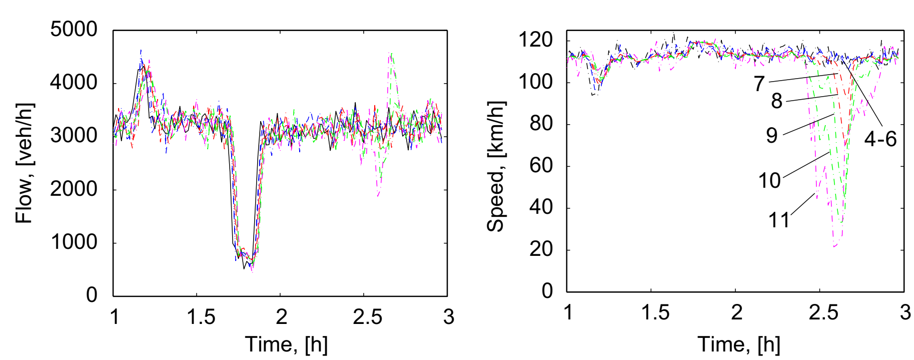

Fig. 4. UKF estimated states of segments 4–11: the backward wave is well visible in segments 7–11.

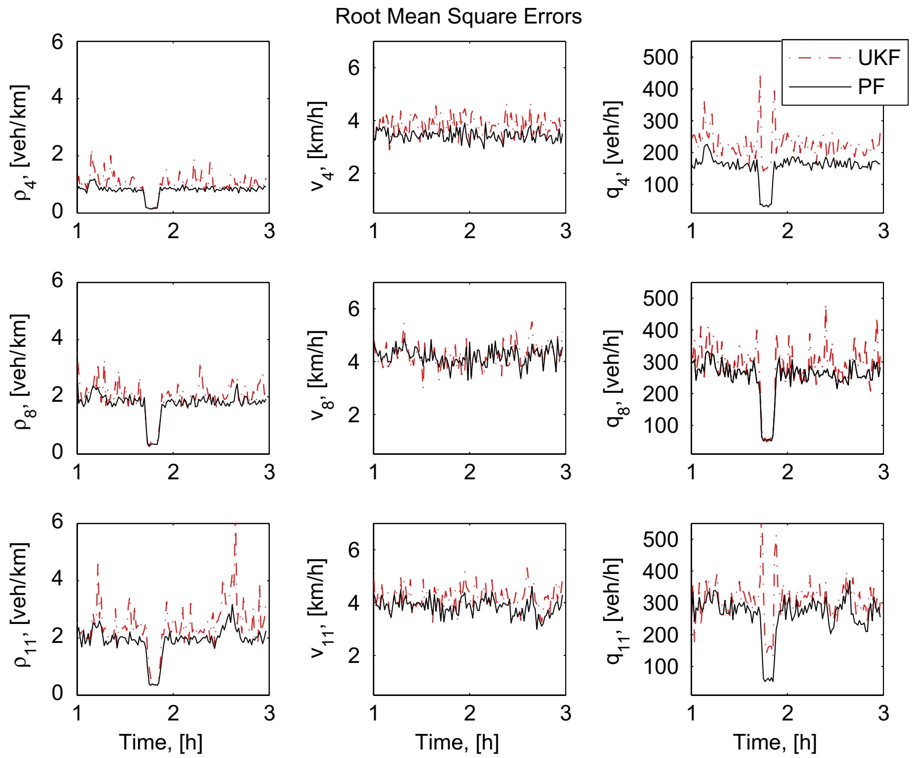

Fig. 5. PF and UKF RMSEs of the density (for all lanes), speed and flow of segments 4, 8 and 11 (with $M = 200$ for the PF).

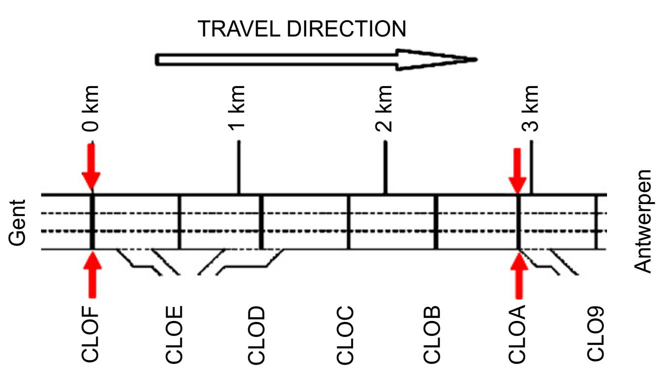

Fig. 6. Schematic representation of the segmentation of the E17 case study freeway. The labels CLOF to CLO9 indicate the locations of the traffic measurement cameras. The vertical arrows indicate the location of the used measurements.

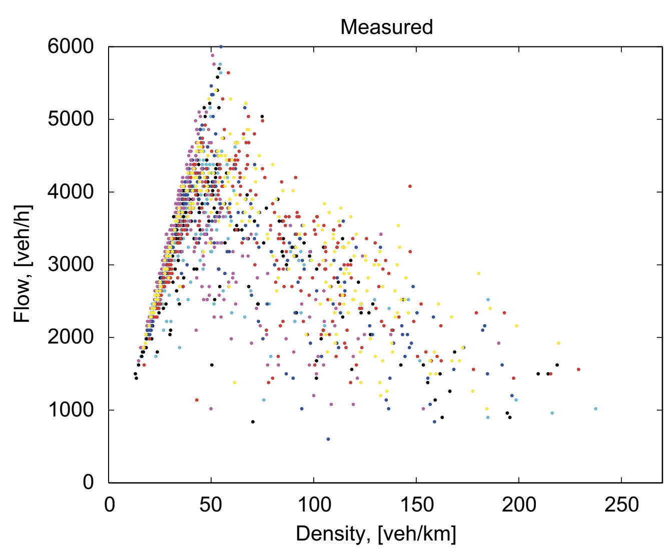

Fig. 7. Diagram from measurements in CLOF, CLOE, CLOD, CLOC, CLOB, CLOA.

The more accurate performance of the PF compared to the UKF performance can be explained by the fact that the PF approximates the state PDF function, whereas the UKF propagates only the first two moments.

6.2. Investigations with traffic data from E17 freeway in Belgium

The PF has also been evaluated with real data, over a frequently congested stretch of the E17 (between CLOF and CLOA in Fig. 6) freeway between the cities of Ghent and Antwerp. Video cameras installed at location CLOA, CLOB, CLOD, CLOE, and CLOF measure the total number of vehicles that cross the sensor location during each 1 min interval, and the average speed of these vehicles during that 1 min interval. We tested the PF and UKF using data measured from September, 2001 from 6.4 h a.m. till 10.6 h a.m. (Fig. 7). This period includes heavy congestions. The PF uses data from the

Table 2
Parameters of the PF and UKF

$v_{\text{free}} = 120\,\text{km/h}$, $v_{\min} = 7.4\,\text{km/h}$
$\rho_{\text{crit}} = 20.89\,\text{veh/km/lane}$, $\rho_{\text{jam}} = 180\,\text{veh/km}$
$\alpha = 0.65$, $\beta_{k+1} = \begin{cases} 0.3 & \text{if } \lvert \rho^{\text{antic}}_{i+1,k+1} - \rho_{i,k+1} \rvert \geqslant 2, \\ 0.7 & \text{otherwise.} \end{cases}$
$\Delta t_i = 10\,\text{s}$, $t_{\text{d}} = 1.5\,\text{s}$, $A_\ell = 0.01\,\text{km}$, $\ell_i = 3$
$L_1 = L_2 = L_3 = 0.6\,\text{km}$, $L_4 = L_5 = 0.5\,\text{km}$
$M = 100$ particles
Gaussian noises $\eta_{S_{i,k}}$, $\eta_{v_{i,k}}$ with covariances:
$cov\{\eta_{S_{i,k}}\} = (0.035 N_{i,k} v_{i,k} \Delta t_k / L_i)^2$ (veh)^2
$cov\{\eta_{Q_i}\} = 1^2$ (veh)^2, $cov\{\eta_{v_i}\} = 3.5^2$ (km/h)^2
$cov\{\xi_{Q_i}\} = 1^2$ (veh)^2, $cov\{\xi_{v_i}\} = 5^2$ (km/h)^2

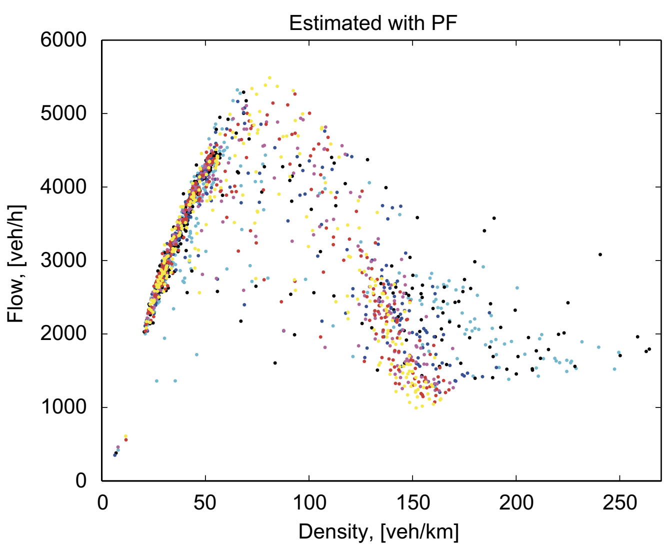

Fig. 8. Diagram based on the PF estimated states.

two sensors installed at CLOF and CLOA (Fig. 6). The link CLOF to CLOA contains an off-ramp towards and an on-ramp from a parking lot, but we assume that the vehicles passing through this parking lot is negligible so that the conservation (9) remains valid in the state prediction step. The parameters of the models and of the filters are given in Table 2. The filters generate estimates of the state of each segment in a link, and also of the speed and density (and hence also of the flow) at each boundary between segments. Figs. 8 and 9 present flow–density diagrams plotted based on the estimates. The bell-shaped diagram shows nicely that the estimated states indeed have properties as one can expect for traffic data. These estimates of the density, speed, and flow at the boundaries are compared with the measured data in the intermediate segment boundary CLOD (Figs. 10 and 11).

7. Conclusions and open issues

This paper formulates the freeway traffic flow estimation within Bayesian recursive framework. A PF is developed us-

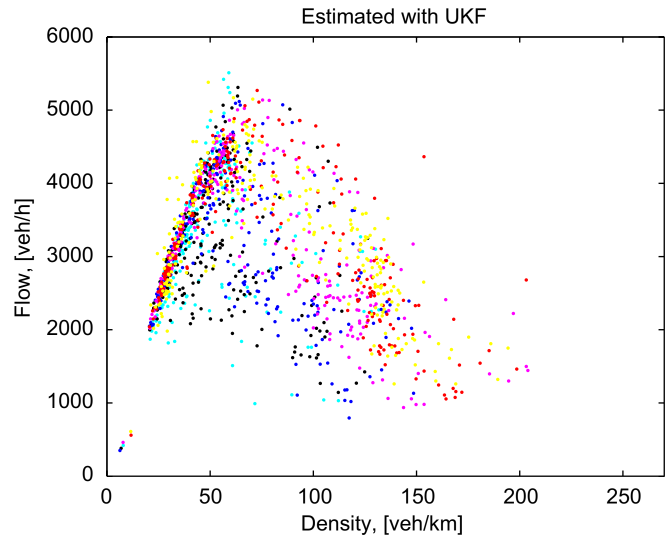

Fig. 9. Diagram based on the UKF estimated states.

ing traffic and observation models with aggregated variables. The traffic is modelled by a recently developed stochastic compositional traffic model with interconnected states of neighbour segments. The PF and UKF performance is investigated and validated by simulated data and by real traffic data from a Belgian freeway. Both the results with simulated and real traffic data confirm that the PF provides accurate tracking performance, better than the UKF. Both the PF and the UKF are suitable methods for real-time traffic estimation, and both are easy to implement because of the fact that they do not require linearization. The estimation approach presented is straightforward, general, easily executable to freeway and urban networks, with different topologies, with any number of sensors, with regularly or irregularly received data in space and in time. Both methods are suitable for distributed realization and parts of them—for parallel computations.

The PF and UKF can be used for on-line traffic control strategies, e.g., within the model predictive control framework (Hegyi, 2004; Sun et al., 2003). One could interpret the results of this paper as follows. Particle filtering can successfully estimate and predict the state of all segments of a road link using only observations on the inflow and the outflow of the link. This suggests that it will be possible to obtain efficient filters in large networks if a few intermediate measurements of the flow are available, and moreover it suggests that these filters for a large network will be nicely decomposable.

Acknowledgements

Financial support by the project DWTC-CP/40 "*Sustainability effects of traffic management*", sponsored by the Belgian government is gratefully acknowledged, as well as by the Belgian Programme on Inter-University Poles of Attraction initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture, and partially by project MI-1506/05

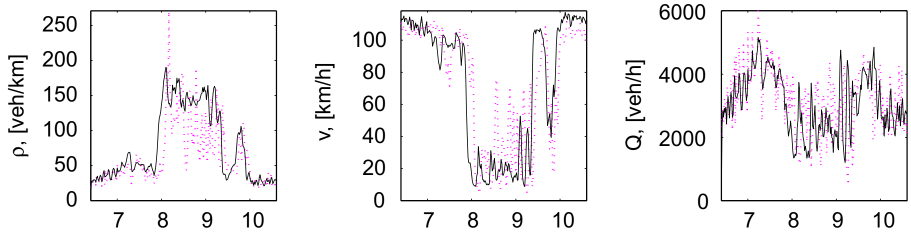

Fig. 10. PF estimated states (solid line) versus measured states in CLOD (dashed line).

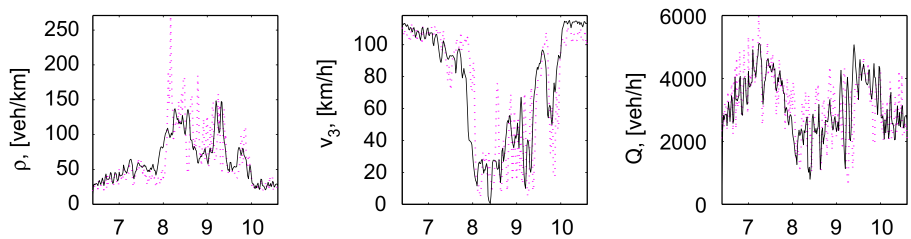

Fig. 11. UKF estimated states (solid line) versus measured states in CLOD (dashed line).

with the Bulgarian Science Fund. We also thank the "*Vlaams Verkeerscentrum Antwerpen*", Antwerp, Belgium for providing the data used in this study. The authors appreciate the useful comments and suggestions of the reviewers and associate editor, helping to improve this paper.

References

Boel, R., & Mihaylova, L. (2004). Modelling freeway networks by hybrid stochastic Models. In *Proceedings of the IEEE intelligent vehicle symposium* (pp. 182–187).

Boel, R., & Mihaylova, L. (2006). A compositional stochastic model for real-time freeway traffic simulation. *Transportation Research B*, *40*(4), 319–334.

Chen, Z. (2003). Bayesian filterig: From Kalman filters to particle filters, and beyond. *Technical Report*, McMaster University, Canada.

Daganzo, C. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research B*, *28B*(4), 269–287.

Doucet, A., Freitas, N., & Gordon, N. (2001). *Sequential Monte Carlo methods in practice*. New York: Springer.

Gordon, N., Salmond, D., & Smith, A. (1993). A novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings on Radar and Signal Processing*, *40*, 107–113.

Hegyi, A. (2004). *Model predictive control for integrating traffic control measures*. Ph.D. thesis, Delft University of Technology, The Netherlands.

Hegyi, A., Girimonte, D., Babuška, R., & De Schutter, B. (2006). A comparison of filter configurations for freeway traffic state estimation. In *Proceedings of the IEEE ITSC conference on intelligent transportation systems* (pp. 1029–1034), 17–20 September 2006, Toronto, Cannada (on CD).

Helbing, D. (2002). Traffic and related self-driven many-particle systems. *Reviews of Modern Physics*, *73*, 1067–1141.

Hoogendoorn, S., & Bovy, P. (2001). State-of-the-art of vehicular traffic flow modelling. *Journal of Systems Control Engineer—Proceedings of the Institution of Mechanical Engineers, Part I*, *215*(14), 283–303.

Julier, S., & Uhlmann, J. (2004). Unscented filtering and nonlinear estimation. *Proceedings of the IEEE*, *92*(3), 401–422.

Julier, S., Uhlmann, J., & Durrant-Whyte, H. (2000). A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transactions on Automatic Control*, *45*(3), 477–482.

Kotsialos, A., Papageorgiou, M., Diakaki, C., Pavlis, Y., & Middelham, F. (2002). Traffic flow modeling of large-scale motorway using the macroscopic modeling tool METANET. *IEEE Transactions on Intelligent Transportation Systems*, *3*(4), 282–292.

Mihaylova, L., & Boel, R. (2004). A particle filter for freeway traffic estimation. In *Proceedings of the 43rd IEEE conference on decision and control* (pp. 2106–2111), Bahamas.

Papageorgiou, M., & Blosseville, J.-M. (1989). Macroscopic modelling of traffic flow on the boulevard Périphérique in Paris. *Transportation Research B*, *23*(1), 29–47.

Rasschaert, G. (2003). *Analysis of traffic data*. M.Sc. thesis, Department of Electrical Energy, Systems and Automation, SYSTeMS, University of Ghent, Belgium.

Ristic, B., Arulampalam, S., & Gordon, N. (2004). *Beyond the Kalman filter: Particle filters for tracking applications*. Artech House.

Sun, X., Muñoz, L., & Horowitz, R. (2003). Highway traffic state estimation using improved mixture Kalman filters for effective ramp metering control. In *Proceedings of the 42th IEEE conference on decision and control* (pp. 6333–6338), Hawaii, USA.

Vaswani, N., Yezzi, A., Rathi, Y., & Tannenbaum, A. (2006). Particle filters for infinite (or large) dimensional state spaces—part 1. In *Proceedings of the IEEE intl. conference on acoustics, speech and signal processing*.

Wan, E., & van der Merwe, R. (2001). In S. Haykin (Ed.), *The unscented Kalman filter*, *Chapter 7: Kalman filtering and neural networks* (pp. 221–280), New York: Wiley.

Wang, Y., & Papageorgiou, M. (2005). Real-time freeway traffic state estimation based on extended Kalman filter: A general approach. *Transportation Research B*, *39*(2), 141–167.

Lyudmila Mihaylova received an M.Sc. degree in Systems and Control Engineering in 1989, an M.Sc. degree in Applied Mathematics and Informatics in 1991, and a Ph.D. degree in Systems and Control Engineering in 1996, all from the Technical University of Sofia, Bulgaria. From 1996 to 2000 she was a Research Assistant at the Bulgarian Academy of Science and a part-time Assistant Professor at the Technical University of Sofia, Bulgaria. Dr. Mihaylova is currently a

Lecturer at the Department of Communication Systems, Lancaster University, UK. Previously she was a Research Fellow with the Signal and Image Processing group at the University of Bristol, UK (2004–2006) and had several visiting positions: with the SYSTEMS group, at the University of Ghent, Belgium (2002–2004) and with the Robotics group at the Katholieke Universiteit Leuven, Belgium (2000–2002). Her interests are in the area of nonlinear filtering, statistical signal and image processing, autonomous intelligent systems, decision making, and sensor data fusion. Dr. Mihaylova is a member of the IEEE, Signal Processing Society and of the International Society of Information Fusion.

René K. Boel received a degree in Electromechanical Engineering (1969) and in Nuclear Engineering (1970) from Ghent University, an M.Sc. (1972) and a Ph.D. degree (1974) in Electrical Engineering and Computer Sciences from the University of California, Berkeley. He is currently a professor at the School of Engineering, Ghent University, Ghent (Belgium). He was a research fellow of the F.W.O.-Vlaanderen (Flemish Foundation for Scientific research) until September 2000. He has held temporary appointments at the Department of Electrical Engineering and Computer Sciences, University of California Berkeley, at the Department of Technical Cybernetics NTH-Trondheim Norway, at the Mathematics Department of the Katholieke Universiteit Leuven, at the Mathematics and Statistics Department of Bell Laboratories, Murray Hill (NJ), at the Department of Computing and Control of Imperial College London, at the Department of Systems Engineering, Australian National University, Canberra, at the Department of Electrical and Computer Engineering, University of Newcastle, Australia, and at the Electrical and Electronic Engineering Department, Melbourne University. His current research interests are in distributed control and estimation for stochastic, discrete event and hybrid systems, with applications to traffic and power systems.

Andreas Hegyi received his M.Sc. degree in Electrical Engineering in 1998 and the doctoral degree in 2004, both at Delft University of Technology, The Netherlands. In 2005, he visited the Ghent University as a postdoctoral researcher where he worked on traffic state estimation with particle filters. Currently, he is a postdoctoral researcher at the Delft Center for Systems and Control of Delft University of Technology, The Netherlands. His current research interests include intelligent transportation systems, integrated/coordinated control of freeway and urban traffic networks, model predictive control, and traffic state estimation with data fusion and filtering techniques.