driftsim: Simulation of radial drift in PPDs

Omkar H. Ramachandran

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Mathematical Formulation

Lagrangian formulation without dissipation

For a dust particle with mass m at a position (r, θ) from the central star of mass M encountering no dissipation, the Lagrangian can be written as follows:

$$L = T - U = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{GMm}{r} \tag{1}$$

We know from the Euler-Lagrange equation that

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i} \tag{2}$$

where q_i refers to the ith generalized coordinate. Substituting equation 1 into 2, we get the following:

$$m\ddot{r} = -\frac{GMm}{r^2} + mr\dot{\theta}^2 \tag{3}$$

$$mr^2\ddot{\theta} = 0 \tag{4}$$

Naive implementation of dissipation due to gas

Assuming that no energy dissipation occurs along r, the only damping factor is caused by the difference in relative velocities between gas and dust. This leaves equation 3 unchanged while altering 4 as follows:

$$mr^2\ddot{\theta} = -br\dot{\theta_{rel}}$$

which implies that

$$\ddot{\theta} = -\frac{\alpha}{mr}\dot{\theta} \tag{5}$$

where $\alpha = b(1-\beta)$ where $\beta \in [0, 1]$ is the ratio of velocities between gas and dust. Note that equation 5 depends on the mass of the dust object, thus implying that more massive dust particles will drift slower than less massive ones at the same r.