TODO

Pranav Tikkawar

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1 Probability Review

Moment Generating Functions

Suppose X is a random variable. The rth moment of X about the origin is defined as

$$\mu'_r := \mathbb{E}(X^r) = \int x^r f(x)dx$$

where $f(x)$ is the PDF.
The first moment is the mean indicated by μ
The rth moment about the mean is defined as

$$\mu_r := \mathbb{E}((X-\mu)^r) = \int (x-\mu)^r f(x)dx$$

μ_2 is the variance of X indicated by σ^2 and is always non-negative
$\mathrm{Var}(x) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$
A random variable X taking values in $\mathbb{R}$ is said to be norm with parameter μ and σ^2 if its PDF is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Case: $\mu = 0$ and $\sigma^2 = 1$ is called the standard normal distribution.
Moment Generating Function
The moment generating function of a random variable X is defined as

$$M_X(t) = \mathbb{E}(e^{tX}) = \int e^{tx} f(x)dx$$

Note $e^{tx} = 1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + ...$
Can also be considered as

$$M_X(t) = \sum_{n=0}^{\infty} \frac{t^n \mu'_n}{n!}$$

where μ_n is the nth moment of X about the origin.

$$M_X(t) = \mathbb{E}(e^{tX}) = \mathbb{E}(1 + tX + \frac{(tX)^2}{2!} + \frac{(tX)^3}{3!} + ...)$$

$$M_X(t)' = \mathbb{E}(Xe^{tX}) = \mathbb{E}(X) + \mathbb{E}(X^2)t + \mathbb{E}(X^3)\frac{t^2}{2!} + ...$$

$$M_X(0)' = \mathbb{E}(X)$$

$$M_X(0)^{(n)} = \mu_n(x) = \mathbb{E}(X^n)$$