TODO

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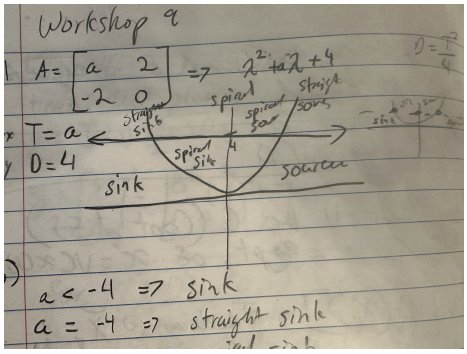
March 31, 2024

Question 1

a)



b)

Clearly the Line on the T-D graph has 7 distinct features:

1. $a < -4$ sink
2. $a = -4$ straight sink
3. $-4 < a < 0$ spiral sink
4. $a = 0$ spiral
5. $0 < a < 4$ spiral source
6. $a = 4$ straight sources
7. $a > 4$ source

Question 2

$x'(t) = x(1 - (1/2)y)$ and $y'(t) = y(-3/4 + (1/4)x)$

a)

Equilibrum points: $(0,0)$ and $(3,2)$

b)

The jacobian matrix is:

$$\begin{bmatrix} 1-y/2 & -x/2 \\ y/4 & -3/4+x/4 \end{bmatrix}$$

At $(0,0)$ the jacobian matrix is:

$$\begin{bmatrix} 1 & 0 \\ 0 & -3/4 \end{bmatrix}$$

At $(3,2)$ the jacobian matrix is:

$$\begin{bmatrix} 0 & -3/2 \\ 1/2 & 0 \end{bmatrix}$$

c)

The eigenvalues at $(3,2)$ are $i\sqrt{3}/2, -i\sqrt{3}/2$. They have no real part

d)

To prove the stability of the equilibrium points we need to observe that that the level curves of the system stay within a boundary. to do this we can solve for $\frac{dy}{dx}$ and find the level curves.

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{y(-3/4+(1/4)x)}{x(1-(1/2)y)}$$

$$\int \frac{1-(1/2)y}{y}dy = \int \frac{-3/4+(1/4)x}{x}dx$$

$$\ln|y| - \frac{1}{2}y = -\frac{3}{4}\ln|x| + \frac{1}{4}x + C$$

$$\ln|y| - \frac{1}{2}y + \frac{3}{4}\ln|x| - \frac{1}{4}x = C$$

e)

By setting C to .023 we can see that ϵ is a boundary for the level curves.