# Birthday Paradox: Making the Unintuitive Intuitive

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# About Me

- Hi! My name is Pranav Tikkawar!
- Quadruple major in Math, Computer Science, Statistics, and Data Science
- Researching in Applied Math Modeling and Machine Learning
- Active in Clubs like Data Science Club, Quantitative Finance Club, and RUCATS
- Enthusiastic about reformulating math education into engaging experiences

# Why Math?

## Math is the Language of Modeling

Math provides the tools to abstract away from complex real-world problems and create simplified models that can be analyzed and understood across various disciplines.

For every personal interest, hobby, or career path, there exists a branch of mathematics that can enhance understanding and problem-solving abilities in that area, ultimately making your life easier!

# The Birthday Paradox: Questions

Let's say you are an aspiring mathematician who loves hosting birthday parties for your friends and wants to know the following:

What do you think is the probability that someone in this call shares your birthday?

When is each person's birthday?

How many people do you think are needed in a room to have a 50% chance that two people share a birthday?

# The Birthday Paradox: Surprise!

- The Birthday Paradox refers to the counterintuitive probability that in a group of just 23 people, there is a better than 50% chance that two people share the same birthday.
- This paradox highlights how human intuition about probability can often be misleading.
- It is a classic example used to show the power of combinatorial mathematics and probability theory that also has real-world applications.

# Birthday Paradox: Explanation

- Assume there are 365 days in a year and each birthday is equally likely and independent of others.
- To find the probability that at least two people share a birthday, it's easier to calculate the complement: the probability that no one shares a birthday.

$$P(\text{at least one shared birthday}) = 1 - P(\text{no shared birthdays})$$

- This is known as the complement rule in probability as generally when considering two complementary events $A$ and $B$, $P(A) = 1 - P(B)$.

# Birthday Paradox: Explanation (Continued)

- For the first person, there are 365 choices for their birthday. For the second person, there are 364 choices (to avoid matching the first), for the third person, 363 choices, and so on.
- The probability that no two people share a birthday in a group of $n$ people is given by:

$$P(\text{no shared birthdays}) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{365-n+1}{365}$$

- Now we can substitute this into the complement rule:

$$P(\text{at least one shared birthday}) = 1 - P(\text{no shared birthdays})$$

# Birthday Paradox: Explanation (Continued)

- For $n = 23$:

$$P(\text{no shared birthdays}) \approx 0.4927$$

- Therefore:

$$P(\text{at least one shared birthday}) = 1 - 0.4927 \approx 0.5073$$

- This means there is approximately a 50.73% chance that in a group of 23 people, at least two will share a birthday.

# Birthday Paradox: Explanation (Continued)

- As the number of people increases, the probability of shared birthdays increases rapidly.
- For example, with 57 people, the probability exceeds 99%.
- When considering a room and pairs of people, each new person "multiplies" the number of potential connections, increasing the likelihood of shared birthdays.
  - With 23 people, there are $\binom{23}{2} = 253$ unique pairs of people.
  - $\binom{n}{2} = \frac{n(n-1)}{2}$ gives the number of ways to choose 2 people from $n$ people.

# Birthday Paradox: Visualization

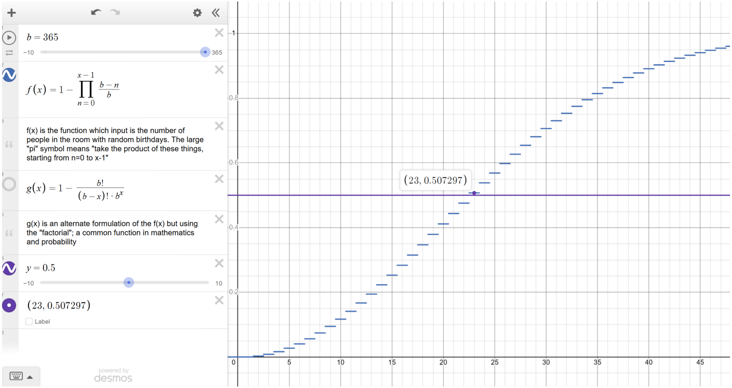


Figure: Plot of the probability of at least one shared birthday versus the number of people in the room.

# Making the Unintuitive Intuitive

- By breaking down complex problems into simpler components, mathematics allows us to analyze and understand situations that may initially seem counterintuitive.
- Mathematical models and visualizations can help illustrate concepts that are difficult to grasp through intuition alone.
- Especially with the advancements in AI, mathematics remains the building block to understanding and solving complex problems.

# Real World Application of Mathematical Modeling

- The Birthday Paradox has practical applications in fields such as cryptography, where it informs the design of hash functions and digital signatures.
- It also has implications in data security, particularly in understanding collision probabilities in hash tables.
- Additionally, the principles behind the Birthday Paradox can be applied to network security, epidemiology, and social network analysis.

# Next Steps for Aspiring Mathematicians (or Party Planners)

- Continue exploring various branches of mathematics to find your area of interest.
- Engage in clubs and extracurricular activities related to mathematics to build a community to share and grow your mathematical interests.
- Take advantage of the Rutgers resources such as tutoring centers, research opportunities, and faculty office hours to deepen your understanding.
- Stay curious and keep learning, as mathematics is a vast and ever-evolving field.