Workshop 3

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1. $x'(t) = [x(t)]^2$

   (a) -

   - If we consider the equation $x'(t) = [x(t)]^2 = v(x(t))$ we know that $x(t_0) = x_0$ is a sol of $x'(t)$ if $v(x_0) = 0$
   - if $x_0 = 0$ then $v(x_0) = 0$ meaning that 0 is the "edge" of the interval
   - The interval then are $(-\infty, 0)$ as well as $(0, \infty)$

   (b) -

   - We can consider this DE as a seperable DE and write it in the form of $F(x(t))x'(t) = G(t)$ we can have $F(x(t)) = \frac{1}{x^2}$ and $G(t) = 1$
   - We can now integrate both sides: $\int \frac{1}{x^2} dx = \int dt$
   - As a result we get $\frac{-1}{x} = t + C$
   - Solving for C we get $C = \frac{-1-t_0x_0}{x_0}$
   - Solving for $x(t)$ we get $x(t) = \frac{1}{-t+1/x_0+t_0}$

   (c) -

   - It will take infinte time both in the past and future to reach $x = 0$ as we can notice that $x(t)$ behaves similar to $-1/t$
   - $\lim_{t\to\infty} x(t) = 0$
   - $\lim_{t\to-\infty} x(t) = 0$

   (d) -

   - We know that 0 is the equilibrium state for this DE as $x_0 = 0$ solves $v(x_0) = 0$ so the other solutions for other starting x_0 cannot cross over 0
   - Since we are given that $x_0 > 0$ for our initial condition we need to see the interval of time that $x(t)$ in our previous equation where $x(t) > 0$
   - That interval is $(-\infty, \frac{1+t_0x_0}{x_0})$

   (e) -

- We can use the FTC to ensure a unique solution to this DE as we are given a initial condition that is on a continous interval ***

(f) -

a Same as (a) in the previous question, $(-\infty, 0)$ as well as $(0, \infty)$

b Same as (b) $x(t) = \frac{1}{-t+1/x_0+t_0}$

c Same as (c) it takes infinite time both past and future

d This is not the same, it is infact the interval where $x(t) < 0$ which is $(\frac{1+t_0x_0}{x_0}, \infty)$

e This is the same as before

(g) -

- Because it takes infinite time to reach $x = 0$ for all the intial values $x_0 \neq 0$ then it stands to reason that $x = 0$ is the unique solution for the DE

(h) -

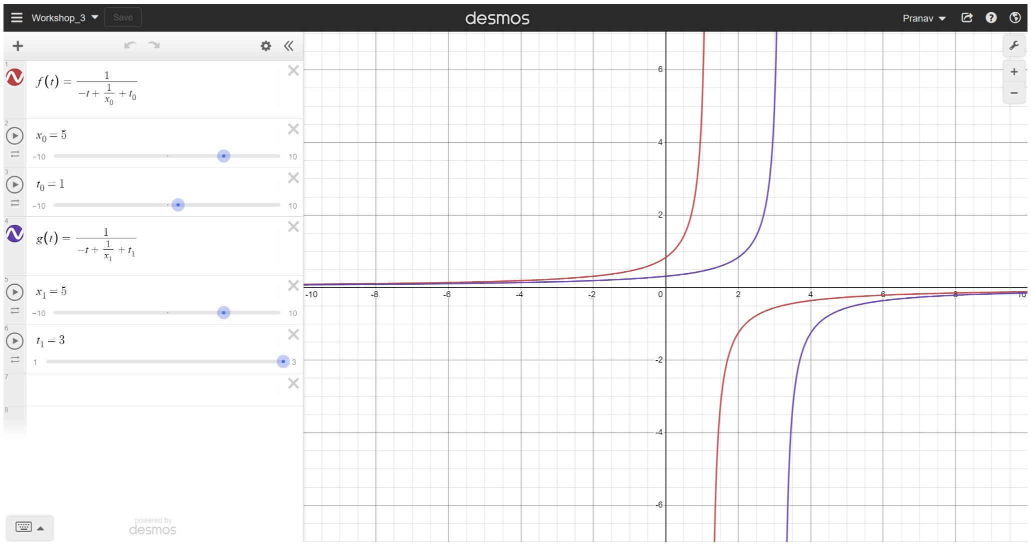
- Graph at bottom
- The relations between the graphs is that they are similar graphs just shifted right by 2. They also have aymptotes that are 2 units away from each other

(i) -

- We can use and example to prove that the function is not well defined on that interval
- We can recognize that the function itself is not well defined on the interval for $x_0 = \frac{1}{t_1-t_0}$
- If we consider $t_1 > t_0$ we can see that x_0 is a positive number, but the flow transomraiton is not well defined which is what we were looking for

(j) -

- For this IVP we can solve it using a guess as we can rewrite $y(t) = u(t) + x(t)$ and $y'(t) = u'(t) + x'(t)$ where we suppose $x(t)$ is a solution to the DE which we guess is $x(t) = 3t$
- We can expand out $y'(t) = 3 + (3t)^2 - 6ty(t) + (y(t))^2$
- Replacing $y(t) = u(t) + x(t)$ we get $u'(t) + x'(t) = 3 + (3t)^2 - 6t(u(t) + x(t)) + (u(t) + x(t))^2$
- We then notice with regrouping the facting some terms cancel as $x(t)$ is a sol to the DE $u'(t)+x'(t) = 3+(3t)^2-6tx(t)+(x(t))^2 - 6tu(t) + 2u(t)x(t) + (u(t))^2$
- Canceling $x'(t)$ and replacing $x(t) = 3t$ we get $u'(t) = (u(t))^2$
- From the previous question we see that $u(t) = \frac{-1}{t+C}$
- $y(t) = \frac{-1}{t+C} + 3t$ where $C = \frac{1}{3t_0-y} - t_0$


Workshop_3
Save
desmos
Pranav
$f(t) = \frac{1}{-t + \frac{1}{x_0} + t_0}$
$x_0 = 5$
$t_0 = 1$
$g(t) = \frac{1}{-t + \frac{1}{x_1} + t_1}$
$x_1 = 5$
$t_1 = 3$
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