Stat 958:587 Homework No. 1
Due 9/30/2025

Note: you are only allowed to use random number generator for a uniform distribution.
Programming: R, Python or JAVA

Problem 1: (a) Use Monte-Carlo method to directly approximate π ("random hit" method). To get a single estimation, use N=10000 samples. Generate 1000 estimated $\hat{\pi}$.
(b) Use Monte-Carlo method to calculate an integral which leads to an approximation of π (Hint: find an integral that equals to some value involving π). To do the integration, use N=10000 samples. Generate 1000 estimated $\hat{\pi}$.
(c) Compare the two methods in (a) and (b) in terms of variance and running time. Which one do you prefer?

Problem 2: Write a program to calculate the integration of $\int_{-5}^{5}(x^3 - x^2)e^{-x^2/2}dx$ using following methods:

(1) Monte Carlo simulation with N samples from a uniform distribution
(2) Numerical integration with N partitions, for $N = 10,\ 100,\ 1000$.

For method (1) and each choice of N, repeat the experiment for 500 times, compute the variance and visualize the relationship between the variance and N.

(3) For method (1), can you find a way to reduce the variance derived above without additional draws of sample? Compare the results. (Hint: symmetry)

Problem 3: Re-write the integral:

$$I = \int_{1}^{+\infty}\int_{-\infty}^{+\infty}(1 + x^2 + \sin(x))^{-|y|^3-2}dydx$$

as some expected value, and estimate the integral using Monte-Carlo simulation.

Problem 4: Write your own code to simulate 100 samples from

(1) Exponential distribution: $Exp(\lambda)$ with $\lambda = 2.8$.
(2) Normal distribution using BOTH Box-Muller transformation AND central limit theorem: $N(\mu, \sigma^2)$ with (a) $(\mu, \sigma^2) = (0,1)$ and (b) $(\mu, \sigma^2) = (3.5,2)$
(3) Log-normal distribution $LnN(\mu, \sigma^2)$ with (a) $(\mu, \sigma^2) = (0,1)$ and (b) $(\mu, \sigma^2) = (-4,2)$
(4) Binomial Distribution: Binomial(n, p) with $n = 10, p = 0.24$.

In each case, plot the density of sample sets to illustrate (validate) your simulated samples.

Problem 5: Write a program to simulate a set of 500 samples from the following distribution with density

$$f(x) = \begin{cases} \frac{1}{c} f_{t_5}(x) \left\{ 1 - \frac{\sin(20x)}{4} \right\} & if\ |x| < 3 \\ 0 & if\ |x| > 3 \end{cases}$$

where $f_{t_5}(x)$ is the density function of the t-distribution with 5 degrees of freedom and

$$c = \int_{-3}^{3} f_{t_5}(x) \left\{ 1 - \frac{\sin(20x)}{4} \right\} dx$$

Plot the density of your simulated samples.