Math 300H: HW 2

Pranav Tikkawar

February 15, 2024

1. -

   a :

   - R is a function with domain and codomain A (it is on A) as:
     * if $x = 1$ then $y = 2$ as $3(1) + 2 = 5$ (prime)
     * if $x = 2$ then $y = 1$ as $3(2) + 1 = 7$ (prime)
     * if $x = 3$ then $y = 2$ as $3(3) + 2 = 11$ (prime)
     * All the elements in the domain are defined and they are defined as elements in the codomain, which in this case is both A.

   b :

   - R is a function with domain $\mathbb{Z}$.
   - We can define a function $f(x) = 2 - x^2$ to satisfy $x^2 + y = 2$ as $f(x) = y$
   - Since for all x in $\mathbb{Z}$ there exists a y in $\mathbb{Z}$ we can say R is a function with domain $\mathbb{Z}$
   - $(\forall x \in \mathbb{Z})(\exists! y \in \mathbb{Z})(f(x) = y)$

   c :

   - R is a not function with domain $\mathbb{Z}$.
   - We can define a function $f(x) = (2 - x^2)/2$ to satisfy $x^2 + 2y = 2$ as $f(x) = y$
   - For odd values of x we cannot have a $y \in \mathbb{Z}$. Therefore it is not a function.

   d :

   - R is not a function with domain $\mathbb{Z}$
   - This is true as there exists at least 1 x in the domain such that there are more than one y that staisfies the relation R
   - Example: $x = 1$ means $y = 1$ or $y = -1$

2. -

   a : $f * g$ is odd as $f(-x)g(-x) = -f(x)g(x)$ this is an odd property b : $f+g$ depends on f and g as $f(-x)+g(-x) = -f(x)+g(x)$ the only way $f+g$ is even or odd depends if f or g are 0

c : $f \circ g$ is even as $f(g(-x)) = f(-g(x)) = f(g(x))$

d : $g \circ f$ is even as $g(f(-x)) = g(f(x))$