HW 5: 300H

Pranav Tikkawar

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Question 1

Let $A = \{1, 2, 3\}$. Give a relation on A that is For all these relations, consider that $R \subset A \times A$.

a

Reflexive, symmetric, and transitive.
Solution:
Let $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1)\}$.

b

Reflexive, symmetric, but not transitive.
Solution:
Let $R = \{(1, 1), (2, 2), (3, 3)\}$.

c

Reflexive, not symmetric, and transitive.
Solution:
Let $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$.

d

Reflexive, not symmetric, and not transitive.
Solution:
Let $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$.

e

Not reflexive, symmetric, and transitive.
Solution:
Let $R = \emptyset$.

f

Not reflexive, symmetric, and not transitive.
Solution:
Let $R = \{(1,2),(2,1)\}$.

g

Not reflexive, not symmetric, and transitive.
Solution:
Let $R = \{(1,2),(2,3),(1,3)\}$.

h

Not reflexive, not symmetric, and not transitive.
Solution:
Let $R = \{(1,2),(2,3)\}$.

Question 2

a

Let $A = \{1,2\}$. All the relations on A which are symmetric and transitive, but not reflexive **Solution:**
$R = \emptyset$

b

Let $A = \{1,2,3,4,5\}$. How many relations which are both symmetric and antisymmetric
Solution:
There are 32 such relations. If we consider the powerset of A then see that every single subset of A can be a relation that is symetric and antisymmetric if the relation is the identy relation. So there are $2^5 = 32$ such relations.

Question 3

Let $A = \{1,2,3\}$ For each of the following relations on A, determine whether it is reflexive, symmetric, antisymmetric, and/or transitive.

a

$R = \{(1,2)\}$
Solution:
Reflexive: No. $(1,1)$ is not in R.
Symmetric: No. $(2,1)$ is not in R.

Antisymmetric: Yes.
Transitive: Yes.

b

$S = \{(1,2),(1,3)\}$ **Solution:**
Reflexive: No. $(1,1)$ is not in S.
Symmetric: No. $(2,1)$ is not in S.
Antisymmetric: Yes.
Transitive: Yes.

c

$T = \{(1,2),(2,1),(1,1)\}$ **Solution:**
Reflexive: No. $(2,2)$ is not in T.
Symmetric: Yes Antisymmetric: No. $(1,2)$ and $(2,1)$ are in T but $1 \neq 2$.
Transitive: No. $(1,2)$ and $(2,1)$ are in T but $(2,2)$ is not in T.

Question 4

Let $A = \{1,2,3\}$. Size of relations:

- Min Reflexive: 3
- Min symmetric: 0
- Min antisymmetric: 0
- Min transitive: 0
- Min equivalence: 3
- Min partial order: 3
- Max symmetric: 9
- Max antisymmetric: 6
- Max equivalence: 9
- Max partial: 3

Question 5

Let S be the relation on $\mathbb{R}$ defined by $xSy : x < y + 1$. Determine whether S is reflexive, symmetric, antisymmetric, transitive.

Reflexive:

$xSx : x < x + 1$ which is true for all $x \in \mathbb{R}$. So S is reflexive.

Symmetric:

if $xSy : x < y + 1$ then $ySx : y < x + 1$. This would be impossible if $x \neq y$. So S is not symmetric.

Antisymmetric:

if $xSy : x < y + 1$ and $ySx : y < x + 1$ then $x = y$. This would be impossible if $x \neq y$. So S is antisymmetric.

Transitive:

if $xSy : x < y + 1$ and $ySz : y < z + 1$. Then xSz would be $x < z + 1$. If we consider xRy and yRz then we can rewite the comination as the statement $x < y + 1$ and $y < z + 1$ to $x < z + 2$. which is also true. So S is transitive.

Question 6

Let $E \subset \mathbb{N} \times \mathbb{N}$ be the relation defined as $xEy : xy \leq x + y$. Determine whether E is reflexive, symmetric, antisymmetric, transitive.

Reflexive:

$xEx : x \cdot x \leq x + x$. This is not true for values of 3 or greater. So E is not reflexive.

Symmetric:

if $xEy : xy \leq x + y$ then $yEx : yx \leq y + x$. This is true as multiplication and addition is commutative. So E is symmetric.

Antisymmetric:

if $xEy : xy \leq x + y$ and $yEx : yx \leq y + x$ then $x = y$. This is not true as $x = 2$ and $y = 3$ is a counterexample. So E is not antisymmetric.

Transitive:

if $xEy : xy \leq x + y$ and $yEz : yz \leq y + z$ then xEz would be $xz \leq x + z$. This is not true for $x = 2$, $y = 1$, and $z = 3$. So E is not transitive.