Workshop 2

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1. How Much to fish?

(a)

- The IVP that models this relation is$y'(t) = ky(t) - c$
- Where $y(t_0) = y_0$ is the intial condition

(b)

- The process of solving this IVP is to use an integrating factor $h(t)$ to multiply through the DE and then "group" the left side as the derivative of the product of 2 functions
- $y' - ky = -c$
- $h(t) = e^{-kt}$
- $h'(t) = -ke^{-kt} = -kh(t)$
- $hy' - khy = -ch$
- $[h(t)y(t)]' = -ch(t)$
- $h(t)y(t) - h(t_0)y_0 = -c\int_{t_0}^{t} h(t)dt$
- After simplifying and replacing $h(t)$, $h'(t)$ we get:
- $y(t) = y_0e^{kt} + c(\frac{1}{k} - \frac{e^{kt}}{k})$
- k is $\frac{ln(2)}{5}$ as we know the populaiton doubles every 5 weeks.
- so $y(t) = y_0e^{\frac{ln(2)}{5}t} + c(\frac{5}{ln(2)} - \frac{5e^{\frac{ln(2)}{5}t}}{ln(2)})$

(c)

- as $t \to \infty; y(t) \to -\infty$
- See image at bottom

(d)

- c would be ln(2)/5 as that would lead the the funciton y(t) = 1 for all values of t

(e)

- $\Phi_{t,0}(y) = ye^{\frac{ln(2)}{5}t} + c(\frac{5}{ln(2)} - \frac{5e^{\frac{ln(2)}{5}t}}{ln(2)})$
- We replace the y_0 for y