Workshop 4: 292

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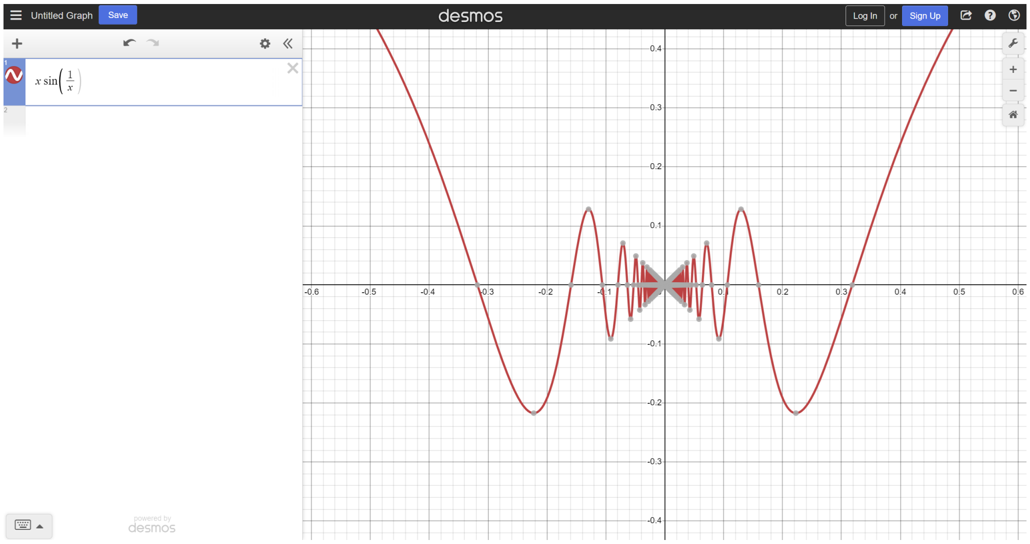
1. -

   a
   - 

   b
   - Since we need to use squeeze theorum to show what the limit of $v(x)$ by bounding it by 2 other functions and show the limit of those function approach the same thing at zero
   - We can notice that $|v(x)| = |xsin(\frac{1}{x})| = |x||sin(\frac{1}{x})|$
   - The values of $|sin(\frac{1}{x})|$ are $0 \leq |sin(\frac{1}{x})| \leq 1$ and $|x|$ are $0 \leq |x| \leq |x|$ so $0 \leq |xsin(\frac{1}{x})| \leq |x|$
   - We can notice than $\lim_{x\to 0} 0 = 0$ and $\lim_{x\to 0} |x| = 0$ so then $\lim_{x\to 0} |xsin(\frac{1}{x})| = 0$

   c
   - The most obvious EQ sol of the DE is $x = 0$ as $v(0) = 0$
   - If $x \neq 0$ then $v(x) = xsin(\frac{1}{x})$
   - If we want $v(x) = 0$ then we need $sin(\frac{1}{x}) = 0$
   - Since $sin(y) = 0$ if $y = n\pi$ where $n \in \mathbb{Z}$
   - So $x = \frac{1}{n\pi}$

d
- There are many maximum intervals but it is important to recognize that as they approach 0 the intvervals become more and more dense
- Let $n \in \mathbb{Z}$ the max intervals for $n > 0$ is $(\frac{1}{(n+1)\pi}, \frac{1}{n\pi})$. It represents all the max intervals for $x > 0$. Also we need to consider the interval $(\frac{1}{\pi}, \infty)$
- The max intervals for $n < 0$ is $(\frac{1}{(n)\pi}, \frac{1}{(n+1)\pi})$ this represents the max intervals for $x < 0$. Also we need to consider the interval $(-\infty, \frac{-1}{\pi})$

e
- Since we know $xsin(1/x)$ is periodic we know that one we find the sign of the $v(x)$ of one interval then we can deduce the rest.
- * look at image*

f
- $\frac{|v(y_n)-v(x_n)|}{|y_n-x_n|}$ will provide an L to test if v is Lipschitz continuous on R, but using the given x_n and y_n we get the ratio $= 4n$
- As $\lim_{n\to\infty} 4n = \infty$ which means that $v(x)$ is not Lipschitz on the interval as for every L we can choose to be the bounding ratio, we choose and n that beats it

g
- To find the maximum interval we need to show that $|v'(x)|$ is bounded by L
- $|v'(x)| = |sin(\frac{1}{x}) - \frac{1}{x}cos(\frac{1}{x})| \leq |sin(\frac{1}{x})| + |\frac{1}{x}||cos(\frac{1}{x})|$ due to the triangle inequality
- Since $0 \leq |sin(\frac{1}{x})| \leq 1$ and $0 \leq |cos(\frac{1}{x})| \leq 1$ and we can suppose an a s.t. $|x| > a > 0$ we get $|v'(x)| \leq 1 + \frac{1}{a}$
- Which gives $L = 1 + \frac{1}{a}$ which indicates that since $Lneq\infty$ then the function $v(x)$ is continuous on that interval

h
- If we suppose an $\epsilon \in \mathbb{R}$ where $(0, \epsilon)$ is the maximum interval, we can find a number smaller than epsilon such that it is a equilibrium point as in the formula for equilibrium points shown above, as n increase the equilibrum points become more dense around 0. Thus making this a contradiction
- Since it is a contradiction and we know that going from one maximum interval to the other in the function, it must change direction, we can see that $x(t) = 0$ is the unique solution to $x(t_0) = 0$

i
- ∞ for $x_0 > \frac{1}{x}$
- 0 for $x_0 = 0$
- $\frac{1}{n\pi} = \frac{1}{n\pi}$
- $\frac{1}{(2n)\pi}$ for $\frac{1}{(2n+1)\pi} < x_0 < \frac{1}{(2n-1)\pi}$
- ∞ for $x_0 < \frac{-1}{\pi}$: