Math Theory of Probability

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Contents

1 Chapter 1: Combinatorial Analysis

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Basic Principle of Counting.
Suppose that 2 experiments are to be preformed. Then if exp 1 can result in any one of n_1 possible outcomes and for each of these outcomes, exp 2 can result in any one of n_2 possible outcomes, then the total number of possible outcomes for the 2 experiments is $n_1 \cdot n_2$.

Permutations.
How many ways are there of arranging n distinct things?
There are n ways to choose the first thing, $n-1$ ways to choose the second thing, $n-2$ ways to choose the third thing, and so on.
Thus, the total number of ways of arranging n distinct things is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 = n!$

Permutations with repeats.

$$\frac{n!}{n_1! \cdot n_2! \cdot \ldots \cdot n_r!}$$

different permutation of n objects which any arbitrary n_i are alike.

Combinations.

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

How many ways are there of choosing r things from n distinct things?

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Example 4c: n items m are dysfunctional and $n-m$ are functional. What is the probability that no two dysfunctional items are adjacent?
Sol: There are $\binom{n-m+1}{m}$ ways. If we think of the functional (plus one for the before spot) we can put the dysfunctional items in. Thus resulting in $\binom{n-m+1}{m}$ ways.

Question Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$
It is shown is pascal's triangle.
Since the left is the number of Combinations of n things taken r at a time, and the right is the number of Combinations of $n-1$ things taken $r-1$ at a time and r at a time.
Thus the right side is the number of Combinations in which A is included and the number of Combinations in which A is not included.

Binomial Theorum

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

This gives the coefficients of the expansion of $(x+y)^n$

Multinomial Theorum

$$(x_1 + x_2 + \ldots + x_k)^n = \sum_{r_1+r_2+\ldots+r_k=n} \frac{n!}{r_1!r_2!\ldots r_k!} x_1^{r_1} x_2^{r_2} \ldots x_k^{r_k}$$

This gives the coefficients of the expansion of $(x_1 + x_2 + \ldots + x_k)^n$

Something cool $\binom{n}{r}\binom{r}{k} = \frac{n!}{r!k!(n-r-k)!}$

Example 5: 8 players, 4 matches (identitcal) played of 2 players. How many ways can the matches be played?
Sol: There are $\frac{8!}{2!2!2!2!4!} = 105$ ways.
How many ways can people win?
16 ways.

Class Activity: Consider the equation $x_1 + x_2 + \ldots + x_r = n$ where each x_i is non-negative. How many possible solutions are there to this equation.

2 Chapter 2: Axioms of Probability

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Axioms of probability
The probability of Something happening is the number of ways the thing happens dived by the possible outcomes.
Axiom 1: $0 \leq P(A) \leq 1$
Axiom 2: $P(S) = 1$ where S is whole same space
Axiom 3: If $A_1, A_2, \ldots$ are mutually exclusive events, then $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

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$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

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n ppl who throw the hats in what is the probablity that no one gets their own hat?
$\frac{1}{e}$ as $n \to \infty$
We want probability of $A_1 \cap A_2 \cap \ldots \cap A_n$
$P(A_1 \cap A_2 \cap \ldots \cap A_n) = 1 - P(A_1^c \cup A_2^c \cup \ldots \cup A_n^c)$
$P(A_1^c \cup A_2^c \cup \ldots \cup A_n^c) = 1 - P(A_1^c) - P(A_2^c) - \ldots - P(A_n^c) + P(A_1^c \cap A_2^c) + \ldots$
With a buch of work we get:

$$1 - \sum_{i=1}^{n} (-1)^{i+1} \frac{(x-i)!}{x!} \binom{x}{i}$$

This results in $1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots$

Example 50 on page 42 of 8th edition.

3 Chapter 3: Conditional Probability and Independence

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$$P(A \cap B) = P(A)P(B|A) = P(A)P(B)$$

iff E and F are independent.

4 Chapter 4: Random Variables

5 Chapter 5: Continuous Random Variables