

01:XXX:XXX - Homework n

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Theorem 1. Suppose $X_t \in \mathbb{R}(t \in \mathbb{Z})$ is a weakly stationary stochastic process
Then

$$\begin{aligned} \exists a_0, a_1, \dots, \text{ such that } a_0 = 1, \sum_{j=1}^{\infty} a_j^2 < \infty \\ \exists \epsilon_t, \mu_t (t \in \mathbb{Z}) \text{ such that } \forall s, t \in \mathbb{Z} : \\ \epsilon_t \in L_t, \mu_t \in L_{-\infty} \\ E(\epsilon_t) = 0, \text{Cov}(\epsilon_t, \epsilon_s) = \sigma_\epsilon^2 \delta_{t,s} < \infty, \text{Cov}(\epsilon_s, \mu_t) = 0 \\ X_{t_{a.s.} L^2(\Omega)} = \mu_t + \sum_{j=0}^{\infty} a_j \epsilon_{t-j} (t \in \mathbb{Z}) \end{aligned}$$

Where L_t is the infinite linear past of X_t .***

Definition (Weakly Stationary). A process X_t is weakly stationary if the mean and covariance are time invariant. That is, $\mu_X(t) = \mu_X$ and $\gamma_X(t_1, t_2) = \gamma_X(t_1 - t_2)$ for all $t_1, t_2 \in \mathbb{Z}$.

Definition (Linear Past).

Definition.

- Time Series data vs IID Data
 - Glivenko-Cantelli theorem
- Ergodic Property with a Constant Limit
 - L^2 EPCL
 - E1, E2, E3 (pg 6-7)
 - Strict and Weak Stationary
 - Covariance, Correlation, Autocovariance, Autocorrelation
- Weak Stationary and Hilbert Space
 - Measure Theory
 - * σ -algebra
 - * L^2 space
 - * Completeness
 - * Hilbert space
 - K-step mean squared error
 - * What method to minimize k-step MSE?
 - * Closed Convex Hilbert space
 - * Optimal Forecast
 - * L^2 -projection
- Conditional Expectation
- Linear Past
- Deterministic process
- Wold Decomposition
 - Karhunen Loeve theorem
 - Wold's theorem
 - Tie Everything Together
- Purely stochastic process
- Why we care now
 - AR Model
 - MA Model
 - ARMA Model

- ARIMA Model
- SV Model

Note: Talk about a deterministic process and how they are talk about infinite learning past and Deterministic process and what it would be equivalent to