Lecture 2 Worksheet

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The Exponential distrubtion is specifice by one parameter, $\lambda > 0$. It has pdf:

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda}$$

1: Compute the MGF of the exponential distribution.

$$\begin{aligned} M_x(t) &= E[e^{tx}] \\ &= \int_0^\infty e^{tx} \frac{1}{\lambda} e^{-x/\lambda} dx \\ &= \frac{1}{\lambda} \int_0^\infty e^{x(t-1/\lambda)} dx \end{aligned}$$

This integral converges if $t < 1/\lambda$. If $t < 1/\lambda$, then:

$$\begin{aligned} M_x(t) &= \frac{1}{\lambda} \cdot \frac{1}{t - 1/\lambda} e^{x(t-1/\lambda)} \Big|_0^\infty \\ &= \frac{1}{\lambda t - 1} [0 - 1] \\ &= \frac{1}{1 - \lambda t} \end{aligned}$$

2 Use the MGF to determine the mean and variance of the exponential distribution with parameter $\lambda = 3$.

Mean

$$\begin{aligned} \mathbb{E} &= M_x'(0) \\ M_x(t) &= \frac{1}{1-3t} \\ M_x'(t) &= \frac{3}{(1-3t)^2} \\ M_x'(0) &= 3 \end{aligned}$$

Variance

$$\begin{aligned} \text{Var} &= M_x''(0) - M_x(0)^2 \\ M_x''(t) &= \frac{18}{(1-3t)^3} \\ M_x''(0) - M_x(0)^2 &= 9 \end{aligned}$$

3: If X is an exponential random variable is $Y = 3X$ also exponential?