Chapter 8: Sample Statistics

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Definition: A random sample of size n from a population with pdf $f(x)$ is a sequence of n independent random variables with pdf $f(x)$.
Thus $X_1, X_2, \ldots, X_n$ are independent random variables with pdf $f(x)$.
Example: $X_i =$ amount of ice cream in the ith scop with the same scoop
Question: What can we infer about the distribution Sample must be diret to the joint pdf
eg: $P(X_1 > X_2 + X_3)$
The jpdf of X_1, X_2, X_3 is $f(x_1, x_2, x_3) = f(x_1)f(x_2)f(x_3)$

$$P(X_1 > X_2 + X_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{x_1=x_2+x_3}^{\infty} f(x_1)f(x_2)f(x_3)dx_1dx_2dx_3$$

Integral over the region $\mathbb{R}^3$ **Definition** A statistic is a random var which is a funtion of the random sample
Example: Sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
Theorem: Suppose $X_1, X_2, \ldots, X_n$ are iid random variables with mean μ and variance σ^2. Then $E[\bar{X}] = \mu$ and $Var(\bar{X}) = \frac{\sigma^2}{n}$
Theorum Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a normal population with distribution $N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$
Proof: Idea get MGF of $\bar{X}$

$$\begin{aligned} M_{\bar{X}}(t) &= M_{1/n\sum X_i}(t) \\ &= M_{\sum x_i}(t/n) \\ &= M_{X_1}(t/n)^n \end{aligned}$$

We know $M_N(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

$$M_{X_1}(t/n)^n = e^{\mu t + \frac{\sigma^2 t^2}{2n}}$$

Suppose X is a rv. Consider $P(|X - \mu_X| < k\sigma_X) \geq 1 - 1/k^2$ **Theorem:** Chebyshev's Inequality
Proof:

$$P(|X - \mu_X|^2 < k^2\sigma_X^2) = \int_{\mu-k\sigma}^{\mu+k\sigma} f(x)dx$$

Application:

$$P(|\bar{X} - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

$$= P(|\bar{X} - \mu| < k\sigma/\sqrt{n}) \geq 1 - \frac{1}{k^2}$$

$$\rightarrow P(|\bar{X} - \mu| < \tilde{k}) \geq 1 - \frac{\sigma_{pop}^2}{n\tilde{k}^2}$$