01:640:311 - Homework n

Pranav Tikkawar

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This is a set of all of the theorems talked in class and in the book numbered.

Theorem 1 (0.0.0: Theorem Name)**.** *This is a theorem. and a teplate for theorems..*

Proof. This is a proof.

$$\text{This is a proof.} \quad e = mc^2$$

$\square$

Theorem 2 (Nested Interval Property: (s 1.4))**.** *If $I_1 \supseteq I_2 \supseteq I_3 \supseteq \dots$ is a sequence of closed intervals in $\mathbb{R}$ then $\bigcap_{n=1}^{\infty} I_n \neq \emptyset$.*

Proof. Let $A = \{a_1, a_2, a_3, \dots\}$ be the set of left endpoints of the intervals.
Now since the I_ns are nested, $I_n \subseteq I_1$ for all n.
Thus each $a_n \in I_1$ for all n.
so $a_n \leq b_1$.
It follows that b_1 is an upper bound for A so sup A exists.
Now we need to prove that $x \in \bigcap_{n=1}^{\infty} I_n$.
To do thi we need to how that $x \in I_n$ for all n.
This mean that $a_n \leq x \leq b_n$ for all n.
Step 1 $a_n \leq x$ for all n.
Remember that $x = \sup A$.
So $a_n \leq x$ for all n.
Step 2 $x \leq b_n$ for all n.
Since $x = \sup A$, x i less than very upper bound of A so it i enough to show that b_n is an upper bound of A.
$b_n \geq a_m$ for all m.
Case 1 $n > m$.
Then $I_n \subset I_m$ so $b_n \in [a_m, b_m] = I_m$.
Case 2 $n \leq m$.
Then $I_m \subset I_n$ so $a_m \in [a_n, b_n] = I_n$.
so $a_m \leq b_n$.
This b_n is an upper bound of A.
Thus $x \leq b_n$ for all n.
Thus $x \in I_n$ for all n.
Thus $x \in \bigcap_{n=1}^{\infty} I_n$. which means the intersection is not empty. □

Theorem 3 (Archimedan Property)**.** *The set $\mathbb{N}$ is not bounded above.*

Proof. Suppose (by contradiction) $\mathbb{N}$ is bounded above.
Then by the least upper bound property, sup $\mathbb{N}$ exists.
Let us call $\alpha = \sup \mathbb{N}$ and it is a real number.
Thus $\alpha - 1 < \alpha$ so $\alpha - 1$ is not an upper bound of $\mathbb{N}$.
So we can fine an $n \in \mathbb{N}$ such that $\alpha - 1 < n$.
Thus $\alpha < n + 1$.
But $n + 1 \in \mathbb{N}$ so α is not an upper bound of $\mathbb{N}$.

□

Theorem 4 (Density of $\mathbb{Q}$ in $\mathbb{R}$)**.** *$\forall a < b \in \mathbb{R}$ there exists $q \in \mathbb{Q}$ such that $a < q < b$.*