

Math 300

Pranav Tikkawar

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1 Chapter 1

1.1 Introduction/ 1/17/2024

- Proposition
 - A sentence with a truth value
 - Ask: "Is it true that..."
- Equivalence
 - 2 propositions that have the same truth value
 - is $P \equiv Q$
- Logical Connectives
 - Negation : $\sim P$
 - * It is NOT P, it is true only when P is false
 - Conjunction
 - * true when P and Q are true
 - * $P \wedge Q$
 - Disjunction
 - * true when either P and Q are true
 - * $P \vee Q$
- DeMorgan's Laws
 - $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$
 - $\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$
 - $\sim (P \Rightarrow Q) \equiv P \wedge (\sim Q)$
- Tautology
 - $P \wedge \sim P$
 - Identically true

- Contradictions
 - $P \vee \sim P$
 - Identically false
- Laws of Excluded Middles
- Conditional Sentences
 - $P \Rightarrow Q$
 - P is Hypothesis and Q is conclusion
- Contrapositive
 - given $P \Rightarrow Q$ Contrapositive is $\sim Q \Rightarrow \sim P$
- Converse
 - given $P \Rightarrow Q$ Converse is $Q \Rightarrow P$
- Biconditional
 - $P \Leftrightarrow Q$
 - P iff Q
 - Logically equivalent to $P \Rightarrow Q \wedge Q \Rightarrow P$
- Predicates
 - Open Sentences
 - $x < \pi$ and $2x^2 - 1 = 0$ where x is a free variable
 - We have not given x a value so the statement cannot be true or false; only gets a truth value when x is assigned
 - $(\forall x \in \mathbb{R})$ is for all x in \mathbb{R}
 - $(\exists x \in \mathbb{R})$ is for all x in \mathbb{R}
- DeMorgans for Predicates
 - $\sim (\forall x)p(x) \equiv (\exists x) \sim p(x)$
 - $\sim (\exists x)p(x) \equiv (\forall x) \sim p(x)$