

# TODO

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## 1.1

### 1

$$W = X * \mathbb{E}[Y]$$

### 2

$$U = \frac{Z}{\mathbb{E}[Y]}$$

### 3

$$V = Y$$

## 1.2

### 1

$$\begin{aligned} E[X^2 + 4XY + 4Y^2|X] &= E[X^2|X] + 4E[XY|X] + 4E[Y^2|X] \\ &= X^2 + 4X\mathbb{E}[Y] + 4\mathbb{E}[Y^2] \end{aligned}$$

### 2

$$\begin{aligned} E[X^2 + 4XY + 4Y^2|X, Z] &= E[X^2|X, Z] + 4E[XY|X, Z] + 4E[Y^2|X, Z] \\ &= X^2 + 4XY + 4Y \end{aligned}$$

### 3

$$\begin{aligned} E[X + 2Y|Z] &= E[X|Z] + 2E[Y|Z] \\ &= \mathbb{E}[X] + 2\mathbb{E}[Y] \end{aligned}$$

4

### 1.3

1

$$\begin{aligned}\mathbb{E}[S_n] &= \mathbb{E}[X_1 + X_2 + \dots + X_n] \\ &= \mathbb{E}[X_1] + \mathbb{E}[X_2] + \dots + \mathbb{E}[X_n] \\ &= n * \mathbb{E}[X_j] \\ &= 0\end{aligned}$$

$$\begin{aligned}\mathbb{E}[S_n^2] &= \mathbb{E}[(X_1 + X_2 + \dots + X_n)^2] \\ &= \mathbb{E}[X_1^2 + X_2^2 + \dots + X_n^2 + 2X_1X_2 + 2X_1X_3 + \dots + 2X_{n-1}X_n] = n\mathbb{E}[X_j^2] + n(n-1)\mathbb{E}[X_jX_k] \\ &= n\mathbb{E}[X_j^2] \\ &= 2n\end{aligned}$$

$$\begin{aligned}\mathbb{E}[S_n^3] &= \mathbb{E}[(X_1 + X_2 + \dots + X_n)^3] \\ &= \mathbb{E}[X_1^3 + X_2^3 + \dots + X_n^3 + 3X_1^2X_2 + 3X_1^2X_3 + \dots + 3X_{n-1}^2X_n + 6X_1X_2X_3 + 6X_1X_2X_4 + \dots + 6X_{n-2}X_{n-1}X_n] \\ &= n\mathbb{E}[X_j^3] + 3n(n-1)\mathbb{E}[X_j^2X_k]/2 + n(n-1)(n-2)\mathbb{E}[X_jX_kX_l] \\ &= \frac{7n}{3}\end{aligned}$$

2

$$\begin{aligned}E[S_n|F_m] &= S_m + \mu(n-m) = S_m \\ E[S_n^2|F_m] &= S_m^2 + \sigma^2(n-m) = S_m^2 + 2(n-m) \\ E[S_n^3|F_m] &= \end{aligned}$$

3

$$E[X_m|S_n] = \frac{S_n}{n}$$

### 1.4

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### 1.5

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## 1.6

### 1

I believe it is a submartingale as the expected value of the next value is greater than the current value.

### 2

### 3

### 4

### 5