HW 3: Math 300

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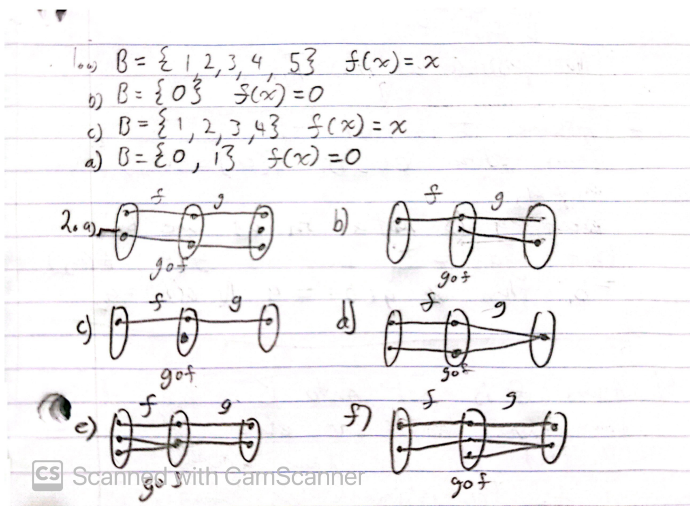
1 -

- a $B = \{1, 2, 3, 4, 5\}$ and $f(x) = x$
- b $B = \{0\}$ and $f(x) = 0$
- c $B = \{1, 2, 3, 4\}$ and $f(x) = x$
- d $B = \{0, 1\}$ and $f(x) = 0$



2 -

3 -

- Proof by contradiction: Assume that f is not one to one.
- Then there exists a_1 and a_2 in A where $a_1 \neq a_2$ but $f(a_1) = f(a_2)$
- This is impossible as we know that there exists a function g where $g \circ f = I_a$. Since I_a is a function which takes in a and returns a, there cannot be a function g which can take in one input and return 2 outputs.
- $f(a_1) = f(a_2) = b$, then it is impossible for $g(b) = a_1$ and $g(b) = a_2$ as we expect $g \circ f = I_a$ but that means g is not a function showing a contradiction

- This means that f must be one to one

4 -

- Proof by contradiction: Assume that f is not onto B
- Then there exists an element in B where it cannot be mapped from an element in A.
- This is impossible as it is given that $f \circ g = I_b$
- I_b is a function that maps b to b and if there is an element in A that doesnt map to B (this is the definiton of something not onto) then g is not defined for all of B, making it not a function and providing a contradiction.