# Workshop 1 Answers

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1. Level Curves and Differential Equations: a warm up
   (a) The level curves are elipsises centered at the origin
      i. **Refer to LevelCurves.png**
   (b) $2x + 8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
      i. Given $x^2 + 4y^2 = c$ we can differentiate both sides with respect to x
      ii. We then get $2x + 8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
   (c) $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
      i. Given $f(x, y(x)) = c$ we reparamaterize the function in terms of t where $x(t) = t$
      ii. $\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}$
      iii. Now if we replace $t$ with $x$ given by the fact that $x(t) = t$
      iv. $\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}x} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$
      v. since $\frac{\mathrm{d}x}{\mathrm{d}x} = 1$ and $\frac{\mathrm{d}f}{\mathrm{d}x} = 0$ we can simplify the equation
      vi. $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
   (d) No, there will not be a function $f(x, y)$ that satisfies this DE
      i. Based off the prior questions we know that the DE will have the form of $\frac{\partial f}{\partial x} = 2x + 3$ and $\frac{\partial f}{\partial y} = x^2y$
      ii. We can then find 2 possibilities for the function $f(x, y)$: $f(x, y) = x^2 + 3x + g(y)$ when we integrate $\frac{\partial f}{\partial x}$ with respect to $x$ and $f(x, y) = \frac{x^2y^2}{2} + h(x)$ when we integrate $\frac{\partial f}{\partial y}$ with repsect to $y$
      iii. Since they represent the same thing we can set the equal to each other: $x^2 + 3x + g(y) = \frac{x^2y^2}{2} + h(x)$
      iv. Now we can seperate the variables with the $x$ terms on the left and $y$ terms on the right: $x^2 + 3x - h(x) = \frac{x^2y^2}{2} - g(y)$
      v. Differentiating both sides with respect to $y$ yeilds: $0 = x^2y - g'(y)$
      vi. This result creates a "contradiction" as it says that $g'(y)$ which is a function soley of $y$, is written in terms of $x$ and $y$ so there cannot be a function that exists that satisfies this.

2. Line Integrals

   (a) $\int_{C_1} \vec{v} \cdot \mathrm{d}\vec{r} = -2$

      i. $v(x, y, z) =< 1, z, y >$

      ii. $r(t) =< cos(t), sin(t), 0 >$ and $r'(t) =< -sin(t), cos(t), 0 >$ along the interval $0 \leq t \leq \pi$

      iii. $v(x(t), y(t), z(t)) =< 1, 0, sin(t) >$

      iv. $\int_{C_1} \vec{v} \cdot \mathrm{d}\vec{r} = \int_0^\pi \vec{v}(x(t), y(t), z(t)) \cdot r'(t)\mathrm{d}t$

      v. $\vec{v}(x(t), y(t), z(t)) \cdot r'(t) = sin(t)$

      vi. $\int_0^\pi sin(t)\mathrm{d}t = -2$

   (b) $\int_{C_2} \vec{v} \cdot \mathrm{d}\vec{r} = -2$

      i. $v(x, y, z) =< 1, z, y >$

      ii. $r(t) =< -t, 0, 0 >$ and $r'(t) =< -1, 0, 0 >$ along the interval $-1 \leq t \leq 1$

      iii. $v(x(t), y(t), z(t)) =< 1, 0, 0 >$

      iv. $\int_{C_1} \vec{v} \cdot \mathrm{d}\vec{r} = \int_{-1}^1 \vec{v}(x(t), y(t), z(t)) \cdot r'(t)\mathrm{d}t$

      v. $\vec{v}(x(t), y(t), z(t)) \cdot r'(t) = -1$

      vi. $\int_{-1}^1 -1\mathrm{d}t = -2$

   (c) $\nabla \times \vec{v} = 0 \therefore v$ is irroational and has a function $f(x, y)$ which is path independent with integration

3. Flux Integrals

   - ** I tried here but I dont remember it that well, but an attempt was made**
   - ** Refer to FluxIntegralSolutions.png for my solutions** (I was too lazy to type it up and im not sure what im doing)