Chapter 4

Pranav Tikkawar

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Chapter 4

Markov Property

If the probability of the nest state only depends on the current state, it satisfies the "Markov Property".
Drunkards walk example

$$\mathbb{P}(x_{i+1} = x_i \pm 1) = \frac{1}{2}\mathbb{P}(x_{i+1} \neq x_i \pm 1) = 0$$

$$\mathbb{P}(x_{i+1} = x + 1 | x_i = x) = 1/2$$

$$\mathbb{P}(x_{i+1} = x - 1 | x_i = x) = 1/2$$

Formal Definition

Let $\{X_n, n \in \mathbb{N}\}$ be a stochastic process that takes discrete time values. Suppose $\mathbb{P}(X_{n+1} = j | X_n = i_n ... X_0 = i_0) = P_{i,j}$ Such a stochastic process is called a Markov Chain. P_{ij} is the transition probability from state i to state j.

Transition Probability Matrix

Let $i, j \in \mathbb{N}$ be possible states of the Markov Chain. The matrix $P = [P_{ij}]$ is called the transition probability matrix of the Markov Chain. Where $P_{ij} = \mathbb{P}(x_{n+1} = j | x_n = i)$. **Ex 4.1**

$$\mathbb{P}(\text{rain tomorrow}|\text{rain today}) = \alpha$$

$$\mathbb{P}(\text{rain tomorrow}|\text{no rain today}) = \beta$$

$$\text{Let} \begin{cases} 0 = \text{rain} \\ 1 = \text{no rain} \end{cases}$$

$$P = \begin{bmatrix} \alpha & 1-\alpha \\ \beta & 1-\beta \end{bmatrix}$$

Ex 4.4 Suppose whether it rains tomorrow or not depends on both todays and yesterdays weather.

Today's Weather	Yesterdays's Weather	Value
Rain	Rain	0
Rain	No Rain	1
No Rain	Rain	2
No Rain	No Rain	3

Suppose:
$\mathbb{P}$(rain tomorrow|rain today, rain yesterday) = .7
$\mathbb{P}$(rain tomorrow|rain today, no rain yesterday) = .5
$\mathbb{P}$(rain tomorrow|no rain today, rain yesterday) = .4
$\mathbb{P}$(rain tomorrow|no rain today, no rain yesterday) = .2

$$P = \begin{bmatrix} .7 & 0 & .3 & 0 \\ .5 & 0 & .5 & 0 \\ 0 & .4 & 0 & .6 \\ 0 & .2 & 0 & .8 \end{bmatrix}$$

4.2 Chapman-Kolmogorov Theorem

P_{ij} = probability of going from state i to state j
$P_{ij}^{(n)}$ = probability of going from state i to state j in n steps.
$P_{ij}^{(n+m)} = \sum_k P_{ik}^{(n)} P_{kj}^{(m)}$ (pg.197)
Look at example 4.10 for next class

Martingale

Martingale Convergence Theorem A martingale is defined as