HW 4: Math 300

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For the entierty of this assigment: Suppose $x \in A$ and $y \in B$ in the context for each function's A and B

a Find two nonempty sets A, B and a function $f_0 : A \to B$ such that f_0 has no left inverse

- Let $f_0(x) = x^2$, $A = \mathbb{R}$, $B = \mathbb{R}$
- We know that if a function is left invertible then the function is one to one. Taking the contrapositive we get if a function is not one to one then it is not left invertible.
- Since the function $f_0 = x^2$ is not one to one as there is at least an element $y \in B$ that has more than one preimage in A. Ex: $f_0(1) = 1, f_1(-1) = 1$
- Therefore the function f_0 is not left invertible

b Find two nonempty sets A, B and a function $f_1 : A \to B$ such that f_1 has exactly one left inverse

- Let $f_1(x) = x$, $A = \mathbb{R}$, $B = \mathbb{R}$
- We know that due to the goldilocks criterion that if a function is bijective then it will have a unique inverse. This is due to the fact that for every element $y \in B$, there will exist one and only preimage in A meaning there is one and only one way to take the inverse of it.
- This function is bijective as it is 1-1 and it is onto its codomain, in this case it is onto its range.
- Due to the function f_1 having one and only one inverse, that means it will have one and only one left inverse

c Find two nonempty sets A, B and a function $f_2 : A \to B$ such that f_2 has more than one left inverse

- Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$, and $f_2(x) = x$
- The function is 1 to 1 as for every element in B there is at most 1 element in A that is it's preimage. But the function is not onto.

- We can let $g_0(1) = 1, g_0(2) = 2, g_0(3) = 1$ as a left inverse function of f_0 as it satisfies $g \circ f = I_A$
- We can also let $g_1(1) = 1, g_1(2) = 2, g_1(3) = 2$ as another left inverse function of f_2 as it satisfies $g \circ f = I_A$
- Since there are are multiple functions g that satisfy $g \circ f_2 = I_A$ that means that f_2 has more than 1 inverse

d Find two nonempty sets A, B and a function $h_0 : A \to B$ such that h_0 has no right inverse

- Let $h_0(x) = x$, $A = \mathbb{R}$, $B = \mathbb{C}$
- We know that if a function is right invertible then it is onto its codomain. Taking the contrapositive, we get that if a function is not onto its codomain then it is not right invertible.
- Every element in A can be mapped to B in this function, but there is at least on element in B that doesnt have a preimage in A. For example 1+i is in B but doesnt have a preimage in A.
- Since the function h_0 is not onto it's codomain then h_0 is not right invertible.

e Find two nonempty sets A, B and a function $h_1 : A \to B$ such that h_1 has exactly one right inverse

- Let $h_1(x) = x$, $A = \mathbb{R}$, $B = \mathbb{R}$
- We know that due to the goldilocks criterion that if a function is bijective then it will have a unique inverse. This is due to the fact that for every element $y \in B$, there will exist one and only preimage in A meaning there is one and only one way to take inverse of it.
- This function is bijective as it is clearly 1-1 and onto its codomain.
- Due to the function having one and only one inverse, that means it will have one and only one right inverse.

f Find two nonempty sets A, B and a function $h_2 : A \to B$ such that h_2 has more than one right inverse

- Let $h_2(x) = x^2$ $A = \mathbb{R}$, $B = \mathbb{R}_{\geq 0}$
- This function has more than one right inverse as we can find 2 functions that satisfy $(h_0 \circ g)(y) = y$ where $y \in B$:
- $g_0 : B \to A$ $g_0 = \sqrt{y}$
- $g_1 : B \to A.g_1 = -\sqrt{y}$
- Suppose $y \in B$. Both functions satisfy $(f \circ g)(y) = I_B = y$. where f is h_2 and g is a right inverse of h_2
- $(h_2 \circ g_0)(y) = (\sqrt{y})^2 = y$
- $(h_2 \circ g_1)(y) = (-\sqrt{y})^2 = y$
- This shows that the function h_2 has more than one right inverse.