01:XXX:XXX - Homework n

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February 3, 2025

1 Fair Pricing of a random payment

Theme Exchange a fixed payment for a random payment.
Q: What criterion can we apply to justify fairness in the exchange?
Point: There are more than one criteria. Under which condition the criterion apply, and which do not?
1) Suppose X is a random payment, then $E(X)$ is the fair price of X.
Justified by the LLN.
So the criteria must be that the payment is repeated many times and independent.
Call Option on a fixed day the holder can choose to buy a share of stock for fixed price K.
Let S be the price of the stock on that day and our profit is $X = (S - K)_+$.
Clealry this cannot follow LLN because we only have one day.
Pricing by Risk Prefrence: Utility function: U : cash $\to$ happiness
Indifference pricing: ϕ is our utility function, then our indeffrence price is $\phi^{-1}(\mathbb{E}(\phi(X)))$
Jensen's Inequality: ϕ convex $\implies \mathbb{E}(\phi(X)) \geq \phi(\mathbb{E}(X))$
Risk Aversion: ϕ concave $\implies \mathbb{E}(\phi(X)) \leq \phi(\mathbb{E}(X))$