Important Distributions:

Dist	PDF	Mean	Var	MGF
Normal	$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), -\infty < x < \infty$	μ	σ^2	$\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$
Gamma	$\frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}, x > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Chi-square	$\frac{1}{2^{\nu/2}\Gamma(\nu/2)}x^{(\nu-2)/2}e^{-x/2}, x > 0$	ν	2ν	$(1-2t)^{-\nu/2}$
Exponential	$\frac{1}{\lambda}e^{-x/\lambda}, x > 0$	λ	λ^2	$(1-\lambda t)^{-1}$
Uniform	$\frac{1}{\beta-\alpha}, \alpha < x < \beta$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	$\frac{e^{\beta t}-e^{\alpha t}}{t(\beta-\alpha)}$
Bernoulli	$p^x(1-p)^{1-x}, x = 0, 1$	p	$p(1-p)$	$(1-p)+pe^t$
Binomial	$\binom{n}{x}p^x(1-p)^{n-x}, x = 0, 1, 2, \ldots, n$	np	$np(1-p)$	$(1+p(e^t-1))^n$
Poisson	$\frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \ldots$	λ	λ	$e^{\lambda(e^t-1)}$
t-distribution	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$	0	$\frac{\nu}{\nu-2}$	$t \in R$
f-distribution	$g(f) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}} f^{\frac{\nu_1}{2}-1}\left(1+\frac{\nu_1}{\nu_2}f\right)^{-\frac{1}{2}(\nu_1+\nu_2)}$	$f > 0$		

Confidence Intervals: for $1-\alpha$ confidence level

In general, if you repeat experiment N times then $\theta \in\approx (1-\alpha)\%$

$\boldsymbol{\mu}$ **w/ known** $\boldsymbol{\sigma}$: $\mu \in \left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$

$\boldsymbol{\mu}$ **w/ unknown** $\boldsymbol{\sigma}$: $\mu \in \left(\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right)$

$\boldsymbol{\mu_1 - \mu_2}$, **w/known** $\boldsymbol{\sigma_1^2}$ **and** $\boldsymbol{\sigma_2^2}$: $\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}, \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right)$

$\boldsymbol{\mu_1 - \mu_2}$, **w/unknown** $\boldsymbol{\sigma_1^2 = \sigma_2^2 = \sigma^2}$:

$\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}, \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)$

$Z = \frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ Comes from MGF, Add Variance

$S_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ aka Weighted average of S_1 and S_2. $\frac{(n_1+n_2-2)S_p}{\sigma^2} \sim \chi_{n_1+n_2-2}$

$T = \frac{Z}{\sqrt{Y/(\nu_1+\nu_2-2)}} \sim t_{\alpha/2,\nu_1+\nu_2-2}$, where $Z =\sim N(0,1)$ and $Y \sim \chi_{\nu_1+\nu_2-2}$

$\boldsymbol{\sigma^2}$: $\sigma^2 \in \left(\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right)$

$\boldsymbol{\frac{\sigma_1^2}{\sigma_2^2}}$: $\frac{\sigma_1^2}{\sigma_2^2} \in \left(\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha/2,n_1-1,n_2-1}}, \frac{s_1^2}{s_2^2}F_{\alpha/2,n_1-1,n_2-1}\right)$ Remember that $F_{1-\alpha/2,n_1,n_2} = \frac{1}{F_{\alpha/2,n_2,n_1}}$

$F = \frac{U/\nu_1}{V/\nu_2} \sim F_{\nu_1,\nu_2}$, where $U \sim \chi^2_{\nu_1}$ and $V \sim \chi^2_{\nu_2}$

Hypothesis Testing

Type I Error: Rejecting H_0 when it is true. $\alpha = P(\text{Type I Error})$: $\alpha = P(\text{Reject } H_0 | H_0 \text{ is true})$

Type II Error: Failing to reject H_0 when it is false. $\beta = P(\text{Type II Error})$: $\beta = P(\text{Fail to Reject } H_0 | H_0 \text{ is false})$

Critical Region: The set of values of the test statistic that leads to rejection of H_0.

We find the Critical Region by making a plot of $\{x_i\}$ and use our test (usually $\bar{X} > c$) and plot the critical region.

Power: $1-\beta = P(\text{Reject } H_0 | H_0 \text{ is false})$ This is the probability of correctly rejecting H_0 aka how many hits

Transformation Theorems

Transformation of 1 var to 1 var: $Y = u(X)$, $X = u^{-1}(Y) = w(Y)$, $g(y) = f(w(y))|\frac{d}{dy}w(y)|$

Transformation of 2 var to 1 var: $Y = u(X_1, X_2)$, $X_1 = w(Y, X_2)$, $g(y) = \int_R f(w(y, x_2))|\frac{\partial}{\partial y}w(y, x_2)|dx_2$

Method of Moments/Estimators

Method of Moments: $m'_k = \frac{\sum_{i=1}^n x_i^k}{n} = E[X^k]$ is the kth sample moment and by setting $\mu'_k = E[X^k]$ and solving for μ'_k, we get the kth population moment.
Max Likelihood: $\hat{\theta}$ is max of $L(\theta) = \prod_{i=1}^n f(x_i|\theta)$ or $l(\theta) = \sum_{i=1}^n \log f(x_i|\theta)$

Bias and Cramer-Rao

Bias: $B(\hat{\theta}) = \mathbb{E}[\hat{\theta}] - \theta$. We say something is unbasied if $B(\hat{\theta}) = 0$ and asymptotically unbiased if $\lim_{n\to\infty} B(\hat{\theta}) = 0$
Cramer-Rao: $Var(\hat{\theta}) \geq \frac{1}{nI(\theta)}$ where $I(\theta) = -\mathbb{E}\left[\frac{\partial^2}{\partial\theta^2} \log f(x|\theta)\right]$ or $I(\theta) = \mathbb{E}\left[\left(\frac{\partial}{\partial\theta} \log f(x|\theta)\right)^2\right]$

Important Other Information

Gamma function: $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\Gamma(n) = (n-1)!$ and $\Gamma(n) = (n-1)\Gamma(n-1)$
Variance Indentity: $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ and $Var(aX - bY + c) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$
Sum of Squares Identity: $\sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n (X_i - \mu)^2 + n(\bar{X} - \mu)^2$
Chebyshev's: $\mathbb{P}(|X - \mu| < k) \geq 1 - \frac{\sigma^2}{k^2}$ and $\mathbb{P}(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$
Weak Law of large numbers: $P(|\bar{X} - \mu_{pop}| < k) \geq 1 - \frac{\sigma_{pop}^2}{nk^2}$
Central Limit Theorem: if $X_i ... X_n$ are iid from any pop w/(μ, σ^2) $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ as $n \to \infty$
Sum of Normal Squared: If $X_1, X_2 ... X_n$ are iid $N(0, 1)$, then $\sum_{i=1}^n X_i^2 \sim \chi_n^2$
Order Statistics: $X_{(1)} < X_{(2)} < ... < X_{(n)}$. It is the rth item of a sample of n.
$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} F(x)^{r-1} (1 - F(x))^{n-r} f(x)$
Expectation: $\int_{-\infty}^{\infty} x f(x) dx$. Is linear!
Variance: $Var(X) = \mathbb{E}[(X - E[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$
$Var(aX + bY + c) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$
Covariance: $\mathrm{Cov}(X, Y) = \mathbb{E}[(X - \mu_X)(Y - \mu_Y)]$ and $\mathrm{Cov}(X, Y) = \int_R \int_S (x - \mu_X)(y - \mu_Y) f(x, y) dx dy$
MGF $M_X(t) = \mathbb{E}[e^{tX}]$. $M_{aX+bY+c}(t) = e^{ct} M_X(at) M_Y(bt)$ if X,Y are independent.
$\frac{d^r}{dt^r} M_X(t = 0) = \mu'_r$ rth moment of X