Abridged Important Notes for Math 350

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1 Speer's Notes

2 Lecture 1

2.1 Vector Space Axioms

1. Commutative property of addition
2. Associative property of addition
3. Additive identity
4. additive inverse
5. multiplicative identity
6. Associativity of scalar multiplication
7. distributivity of 1 vector to 2 scalars
8. distributivity of 2 vectors to 1 scalar

3 Lecture 2

3.1 Theorem 1.1

Let V be a vector space over $\mathbb{F}$, let $x, y, z \in V$, and assume $x + z = y + z$. Then $y = z$.
This is cancellation from the right

3.2 Theorem 1.1 '

Let $x, y, z \in V$ If $z + x = z + y$, then $x = y$.
This is cancellation from the left

3.3 Theorem 1.1 Corollary 1

The vector $\underline{0}$ (VS 3) is unique.

3.4 Theorem 1.1 Corollary 2

The vector y or $-x$ in (VS 4) is unique.

4 Lecture 3

4.1 Theorem 1.2(a)

$\forall x \in V, 0 \cdot x = \underline{0}$

4.2 Theorem 1.2(b)

$\forall a \in \mathbb{F}$ and $x \in V$, $(-a)c = (-ax)$

4.3 Definition of a subspace

Let V be a vector space over $\mathbb{F}$. A subset W of V is called a subspace of V if W is a vector space over $\mathbb{F}$ when equiped with the same operations of addition and scalar multiplication as in V.

4.4 Theorem 1.3

Let $W \subset V$ Then W is a subspace of V iff

- $\underline{0} \in W$
- W is closed under addition, i.e. $\forall x, y \in W, x + y \in W$
- W is closed under scalar multiplication, i.e. $\forall a \in \mathbb{F}, x \in W, ax \in W$

4.5 Linear combination

Let V be a vector space over $\mathbb{F}$ and let S be a nonempty subset of V. A vector $v \in V$ is called linear combination of vectors of D is $\exists$ finitely many vectors $u_1, \ldots, u_n \in S$ and scalars $a_1, \ldots, a_n \in \mathbb{F}$ such that $v = a_1 u_1 + \cdots + a_n u_n$

4.6 Elementary Row Operations

1. Interchange two rows
2. Multiply a row by a nonzero scalar
3. Add a multiple of one row to another row

Make sure to denote an operation as $\xrightarrow{r_1+2r_2\to r_1}$

4.7 RREF

A matrix is in RREF if:

1. The leading entry of each nonzero row is 1
2. The leading 1 in each row is to the right of the leading 1 in the row above it
3. All entries in the column above and below a leading 1 are 0

The process of converting a matrix to RREF is called Gaussian Elimination

5 Lecture 4

5.1 Span and Theorem 1.5

The span of any subset S of a vector space V is a subspace of V that contains S. Moreover, any subspace of V that conains S also contains the span of S. It is also the smallest subspace of V that contains S.

5.2 Linear Dependence/Independence

A subset S of a vector space V is linearly dependent if $\exists$ finitely many distinct vectors $u_1, \ldots, u_n \in S$ and scalars $a_1, \ldots, a_n \in \mathbb{F}$, not all zero, such that $a_1u_1 + \cdots + a_nu_n = \underline{0}$. Otherwise, S is linearly independent.
To calculate this we can solve the homogeneous system of equations $a_1u_1 + \cdots + a_nu_n = \underline{0}$ or $Ax = \underline{0}$ where A is the matrix with columns $u_1, \ldots, u_n$ and x is the column vector with entries $a_1, \ldots, a_n$

5.3 Bases

A basis β for a vector space V is a linearly independent subset of V that spans V. If β is a basis for V, , we also say that the vectors of β form a basis for V.

6 Lecture 5

6.1 Theorem 1.8

Let V be a vector space and let $u_1, \ldots, u_n$ b distinct vectors in V. Then $\beta = \{u_1, \ldots, u_n\}$ is basis for V iff every $v \in V$ can bexpressed uniquely as a linear combinations of the vectors of β.