HW 1

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1. -
    - Suppose A, B, and C are sets.
    - Assume $(A \subseteq B)(B \subseteq C)(C \subseteq A)$
    - Need $(B \subseteq A)(C \subseteq B)(A \subseteq C)$
    - Since $(B \subseteq C)\&(C \subseteq A)$ Then $(B \subseteq A)$
    - Since $(B \subseteq A)\&(A \subseteq B)$ Then $A = B$
    - Since $(C \subseteq A)\&(A \subseteq B)$ Then $(C \subseteq B)$
    - Since $(C \subseteq B)\&(B \subseteq C)$ Then $B = C$
2. -
    - Suppose $x \in (A \cup B) \cap C$
    - Need $x \in A \cup (B \cap C)$
    - Since $x \in (A \cup B) \cap C$ Then $\{x \in (A \cup B) : x \in C\}$
    - Since $x \in (A \cup B)$ Then $\{x \in A \text{ or } x \in B\}$
    - Case 1: $(x \in A)\&(x \in C)$
        – Since $x \in A$ then $x \in A \cup (B \cap C)$
    - Case 2 $(x \in B)\&(x \in C)$
        – Since $x \in B \& x \in C$ Then $x \in B \cup C$
        – Since $x \in B \cup C$ Then $x \in A \cup (B \cap C)$
    - An Example of this is A = [1,2,3] , B = [2,3] , C = [3]
3. -
    - Suppose $x \in A^c - B^c$
    - Need $x \in (A - B)^c$
    - Need $x \notin (A - B)$
    - Need $x \notin A : x \in B$
    - Since $x \in A^c - B^c$ Then $x \in A^c : x \notin B^c$

- Since $x \in A^c : x \notin B^c$ Then $x \notin A : x \in B$ as desired

4. -

- Prove $Q_{\leq a} \subseteq Q_{\leq b}$ iff $a \leq b$
- Part 1: Proving if $Q_{\leq a} \subseteq Q_{\leq b}$ then $a \leq b$
- Assume $Q_{\leq a} \subseteq Q_{\leq b}$
- Need $a \leq b$
- Suppose $x \in Q_{\leq a}$
- Since a is a rational number and $\exists x : x = a$ Then $a \in Q_{\leq a}$
- Since $a \in Q_{\leq a}$ and $Q_{\leq a} \subseteq Q_{\leq b}$ Then $a \in Q_{\leq b}$
- Since $a \in Q_{\leq b}$ and the definiton of $Q_{\leq b}$ is $\{r \in Q : r \leq b\}$ Then $a \leq b$
- Part 2: Proving if $a \leq b$ then $Q_{\leq a} \subseteq Q_{\leq b}$
- Assume $a \leq b$
- Need $Q_{\leq a} \subseteq Q_{\leq b}$
- Since a and b are rational then $a \in Q_{\leq a}$ and $b \in Q_{\leq b}$
- Since a is a rational number and $a \leq b$ then $a \in \mathbb{Q} : a \leq b$ and $a \in Q_{\leq b}$
- Suppose $x \in Q_{\leq a}$ Then $x \leq a$
- since $x \leq a$ and $a \in Q_{\leq b}$ Then $x \in Q_{\leq b}$
- Since $x \in Q_{\leq b}$ Then $Q_{\leq a} \subset Q + \leq b$