01:640:350H - Quiz 2

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Definition. Ordered Basis
An ordered basis for V (finite diminsional vector space) is a basis for V endowed with a specific order.

Example. In F^3, an ordered basis is $\beta = \{e_1, e_2, e_3\}$ but so is $\beta' = \{e_3, e_2, e_1\}$. but $\beta \neq \beta'$.

Definition. Standard Ordered Basis
The (notice the article) standard ordered basis is the basis $\beta = \{e_1, e_2, \ldots, e_n\}$ for F^n.

Definition. Coordinate Vector of a Basis
Let $\beta = \{u_1, u_2, \ldots, u_n\}$ be an ordered basis for V. Then for any $x \in V$ let $a_1 ... a_n$ be the unique scalars such that $x = \sum_{i=1}^{n} a_i u_i$.

We then define the coordinate vector of x with respect to β as $[x]_\beta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

Think of it like the vector in the basis of the basis vectors. In other words, the basis vectors are our axes and our $[x]_\beta$ is the vector in that space.

Definition. Matrix Representation of a Linear Transformation
Let $T : V \to W$ be a linear transformation between finite dimensional vector spaces V and W. Let $\beta = \{u_1, u_2, \ldots, u_n\}$ be an ordered basis for V and $\gamma = \{v_1, v_2, \ldots, v_m\}$ be an ordered basis for W.
Then the matrix representation of T with respect to β and γ is the $m \times n$ matrix $[T]_\beta^\gamma$ such that for any $x \in V$, $[T(x)]_\gamma = [T]_\beta^\gamma [x]_\beta$.
If $V = W$ and $\beta = \gamma$, then we write $[T]_\beta = [T]_\beta^\beta$.
Notice that the jth column of A is simply $[T(u_j)]_\gamma$. In other words. each column of T_β^γ is the coordinate vector of $T(u_j)$ where u_j is the jth basis vector of β in the coordinate system of γ.

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