01:XXX:XXX - Homework n

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Info on paper

Renewal reward processes

Example (7.3 (b))**.**

$$\frac{E(R(t))}{t} = \frac{E[R]}{E[X]} \text{ w/ p} = 1$$

Rate at which rewards are earned $= \frac{\text{Expected reward in a cycle}}{\text{Expected time in a cycle}}$ Go over examples 7.14, 7.15, 7.16

Example (1)**.** A policement cruses on average 10 mts before stopping a speeder. 90% of the time the speeder is fined \$80 taking 5 mins. 10% of the time the speeder is fined \$300 taking 30 mins. What is the average amount of money the police officer makes in 1 hour?

Solution: Average time in a cycle $= 10 + 5(0.9) + 30(0.1) = 10 + 4.5 + 3 = 17.5$ mins
Average reward in a cycle $= 80(0.9) + 300(0.1) = 72 + 30 = 102$
Thus the average rate is $\frac{102}{17.5} = 5.28$ dollars per minute

Example (2)**.** A truck drivers drives from Atlanta to Boston and back. Each from A to B drive is a fixed speed from a uniform distribution from $(40, 60)$ mph. The return trip is a 40 or 60 mph with equal probability.
What is the long run proportion of driving at 40 mph?

Solution: We can utilize a rewards process, where the reward is earned at a rate of 1 per unit time when in the 40 mph state. Let the distance between the two cites be $d = d_{AB} = d_{BA}$ and let X be the speed of the truck.

$$E[\text{Cycle length}] = E[T_{AB}] + E[T_{BA}] = \frac{1}{20}(\int_{40}^{60} \frac{d}{s} ds)_{T_{AB}} + (\frac{d}{80} + \frac{d}{120})_{T_{BA}} = \frac{d}{20} ln\left(\frac{3}{2}\right) + \frac{d}{48}$$

$$E[\text{Reward in a cycle}] = \frac{1}{2}\frac{d}{40} \cdot (1) + 0 * \frac{d}{40}(1)$$

$$\frac{E[R]}{E[X]} = \frac{\frac{1}{80}}{\frac{1}{48} + \frac{1}{20} ln\left(\frac{3}{2}\right)}$$

4/13 is the proportion of driving at 40 mph