Math 300

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1 Chapter 1

1.1 Introduction/ 1/17/2024

- Proposition
  - A sentence with a truth value
  - Ask: "Is it true that..."
- Equivalace
  - 2 propositions that have the same truth value
  - is $P \equiv Q$
- Logical Connectives
  - Negation : $\sim P$
    - It is NOT P, it is true only when P is false
  - Conjuction
    - true when P and Q are true
    - $P \wedge Q$
  - Disjunction
    - true when either P and Q are true
    - $P \vee Q$
- DeMorgan's Laws
  - $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$
  - $\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$
  - $\sim (P \Rightarrow Q) \equiv P \wedge (\sim Q)$
- Tautology
  - $P \wedge \sim P$
  - Identically true

- Contradictions
  - $P \vee \sim P$
  - Identically false
- Laws of Excluded Middles
- Conditional Sentences
  - $P \Rightarrow Q$
  - P is Hypothesis and Q is conclusion
- Contrapositive
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  - given $P \Rightarrow Q$ Contrapositive is $\sim Q \Rightarrow \sim P$
- Converse
  - given $P \Rightarrow Q$ Converse is $Q \Rightarrow P$
- Biconditional
  - $P \Leftrightarrow Q$
  - P iff Q
  - Logically equivalent to $P \Rightarrow Q \wedge Q \Rightarrow P$
- Predicates
  - Open Sentences
  - $x < \pi$ and $2x^2 - 1 = 0$ where x is a free variable
  - We have not given x a value so the statement cannot be true or false; only gets a truth value when x is assigned
  - $(\forall x \in \mathbb{R})$ is for all x in $\mathbb{R}$
  - $(\exists x \in \mathbb{R})$ is for all x in $\mathbb{R}$
- DeMorgans for Predicates
  - $\sim (\forall x)p(x) \equiv (\exists x) \sim p(x)$
  - $\sim (\exists x)p(x) \equiv (\forall x) \sim p(x)$