Distributions

Normal: **Pars:** μ (mean), σ^2 (variance) **PDF:** $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ **MGF:** $M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$ **Mean:** $\mathbb{E}[X] = \mu$ **Var:** $\text{Var}(X) = \sigma^2$
Chi-Squared: **Pars:** k (degrees of freedom) **PDF:** $f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} \exp\left(-\frac{x}{2}\right)$ **MGF:** $M_X(t) = (1-2t)^{-k/2}$ **Mean:** $\mathbb{E}[X] = k$ **Var:** $\text{Var}(X) = 2k$ **IMP** Is Gamma with $\alpha = \nu/2, \beta = 2$. If $X \sim N$, $X^2 \sim \chi^2$
Gamma: **Pars:** α, β **PDF**: **MGF:** $M_X(t) = (1-\beta t)^{-\alpha}$ **Mean:** $\mathbb{E}[X] = \frac{\alpha}{\beta}$ **Var:** $\text{Var}(X) = \frac{\alpha}{\beta^2}$
Exponential: **Pars:** λ (rate) **PDF:** $f(x) = \lambda \exp(-\lambda x)$ **MGF:** $M_X(t) = \frac{\lambda}{\lambda - t}$ **Mean:** $\mathbb{E}[X] = \frac{1}{\lambda}$ **Var:** $\text{Var}(X) = \frac{1}{\lambda^2}$

Samples

Sample Mean: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ **Mean:** $\mathbb{E}[\bar{X}] = \mu$ **Var:** $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$ **Dist:** $\bar{X} \sim N(\mu, \sigma^2/n)$
Sample Variance: $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ **Mean:** $\mathbb{E}[S^2] = \sigma^2$ **Var:** $\text{Var}(S^2) = \frac{2\sigma^4}{n-1}$ **Dist:** $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1}$
Chebyshev's $\mathbb{P}(|X-\mu| < k) \geq 1 - \frac{\sigma^2}{k^2}$ and $\mathbb{P}(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$
Law of Large Numbers: $\bar{X} \in (\mu \pm c) \geq 1 - \frac{\sigma^2}{nc^2}$
Central Limit Theorem: $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ as $n \to \infty$
Order Statistics: The rth smallest value of an n sample. $f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} F(x)^{r-1}(1-F(x))^{n-r} f(x)$ or $= \frac{n!}{(r-1)!(n-r)!} f(x) \int_{-\infty}^{x} f(y)dy^{r-1} \int_{x}^{\infty} f(y)dy^{n-r}$
Max Likelihood $\hat{\theta}$ is max of $L(\theta) = \prod_{i=1}^{n} f(x_i|\theta)$ or $l(\theta) = \sum_{i=1}^{n} \log f(x_i|\theta)$