

Workshop 1 Answers

Pranav Tikkawar

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1. Level Curves and Differential Equations: a warm up

- (a) The level curves are ellipses centered at the origin
 - i. **Refer to LevelCurves.png**
- (b) $2x + 8y \frac{dy}{dx} = 0$
 - i. Given $x^2 + 4y^2 = c$ we can differentiate both sides with respect to x
 - ii. We then get $2x + 8y \frac{dy}{dx} = 0$
- (c) $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx} = 0$
 - i. Given $f(x, y(x)) = c$ we reparamaterize the function in terms of t where $x(t) = t$
 - ii. $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$
 - iii. Now if we replace t with x given by the fact that $x(t) = t$
 - iv. $\frac{df}{dx} = \frac{\partial f}{\partial x} \frac{dx}{dx} + \frac{\partial f}{\partial y} \frac{dy}{dx}$
 - v. since $\frac{dx}{dx} = 1$ and $\frac{df}{dx} = 0$ we can simplify the equation
 - vi. $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx} = 0$
- (d) No, there will not be a function $f(x, y)$ that satisfies this DE
 - i. Based off the prior questions we know that the DE will have the form of $\frac{\partial f}{\partial x} = 2x + 3$ and $\frac{\partial f}{\partial y} = x^2 y$
 - ii. We can then find 2 possibilities for the function $f(x, y)$: $f(x, y) = x^2 + 3x + g(y)$ when we integrate $\frac{\partial f}{\partial x}$ with respect to x and $f(x, y) = \frac{x^2 y^2}{2} + h(x)$ when we integrate $\frac{\partial f}{\partial y}$ with respect to y
 - iii. Since they represent the same thing we can set the equal to each other: $x^2 + 3x + g(y) = \frac{x^2 y^2}{2} + h(x)$
 - iv. Now we can separate the variables with the x terms on the left and y terms on the right: $x^2 + 3x - h(x) = \frac{x^2 y^2}{2} - g(y)$
 - v. Differentiating both sides with respect to y yields: $0 = x^2 y - g'(y)$
 - vi. This result creates a "contradiction" as it says that $g'(y)$ which is a function solely of y , is written in terms of x and y so there cannot be a function that exists that satisfies this.

2. Line Integrals

(a) $\int_{C_1} \vec{v} \cdot d\vec{r} = -2$

i. $v(x, y, z) = \langle 1, z, y \rangle$

ii. $r(t) = \langle \cos(t), \sin(t), 0 \rangle$ and $r'(t) = \langle -\sin(t), \cos(t), 0 \rangle$ along the interval $0 \leq t \leq \pi$

iii. $v(x(t), y(t), z(t)) = \langle 1, 0, \sin(t) \rangle$

iv. $\int_{C_1} \vec{v} \cdot d\vec{r} = \int_0^\pi \vec{v}(x(t), y(t), z(t)) \cdot r'(t) dt$

v. $\vec{v}(x(t), y(t), z(t)) \cdot r'(t) = \sin(t)$

vi. $\int_0^\pi \sin(t) dt = -2$

(b) $\int_{C_2} \vec{v} \cdot d\vec{r} = -2$

i. $v(x, y, z) = \langle 1, z, y \rangle$

ii. $r(t) = \langle -t, 0, 0 \rangle$ and $r'(t) = \langle -1, 0, 0 \rangle$ along the interval $-1 \leq t \leq 1$

iii. $v(x(t), y(t), z(t)) = \langle 1, 0, 0 \rangle$

iv. $\int_{C_1} \vec{v} \cdot d\vec{r} = \int_{-1}^1 \vec{v}(x(t), y(t), z(t)) \cdot r'(t) dt$

v. $\vec{v}(x(t), y(t), z(t)) \cdot r'(t) = -1$

vi. $\int_{-1}^1 -1 dt = -2$

(c) $\nabla \times \vec{v} = 0 \therefore v$ is irroational and has a function $f(x, y)$ which is path independent with integration

3. Flux Integrals

- ** I tried here but I dont remember it that well, but an attempt was made**
- ** Refer to FluxIntegralSolutions.png for my solutions** (I was too lazy to type it up and im not sure what im doing)