Important Distributions:

Dist	PDF	Mean	Var	MGF
Normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), -\infty < x < \infty$	μ	σ^2	$\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$
Gamma	$\frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, x > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Chi-square	$\frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{(\nu-2)/2} e^{-x/2}, x > 0$	ν	2ν	$(1-2t)^{-\nu/2}$
Exponential	$\frac{1}{\lambda} e^{-x/\lambda}, x > 0$	λ	λ^2	$(1-\lambda t)^{-1}$
Uniform	$\frac{1}{\beta-\alpha}, \alpha < x < \beta$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	$\frac{e^{\beta t}-e^{\alpha t}}{t(\beta-\alpha)}$
Bernoulli	$p^x(1-p)^{1-x}, x = 0, 1$	p	$p(1-p)$	$(1-p) + pe^t$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, 2, \ldots, n$	np	$np(1-p)$	$(1 + p(e^t - 1))^n$

Information

Gamma: $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\Gamma(n) = (n-1)!$ and $\Gamma(n) = (n-1)\Gamma(n-1)$
Standard normal: If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$
CLT of Bionomial: If $X \sim B(n,p)$, then $\frac{X-np}{\sqrt{np(1-p)}} \sim N(0,1)$
Sample Mean:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Mean: μ **Var:** $\frac{\sigma^2}{n}$ **Dist:** $\bar{X} \sim N(\mu, \sigma^2/n)$

Sample Variance:

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

Mean: σ^2 **Var:** $\frac{2\sigma^4}{n-1}$ **Dist:** $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1}$
Note: $\bar{X}$ and S^2 are independent.
Imp Identity:

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n}(X_i - \mu)^2 - n(\bar{X} - \mu)^2$$

Important Theorems

Chebyshev's

$$\mathbb{P}(|X-\mu| < k) \geq 1 - \frac{\sigma^2}{k^2} \text{ and } \mathbb{P}(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

Weak Law of large numbers: $P(|\bar{X} - \mu_{pop}| < k) \geq 1 - \frac{\sigma^2_{pop}}{nk^2}$
Central Limit Theorem: if $X_i ... X_n$ are iid from any pop w/(μ, σ^2) $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ as $n \to \infty$

S. Normal squared: If $X \sim N(0,1)$, then $X^2 \sim \chi^2_1$
Sum S. Normal Squared: If $X_1, X_2 ... X_n$ are iid $N(0,1)$, then $\sum_{i=1}^n X_i^2 \sim \chi^2_n$
Order Statistics: $X_{(1)} < X_{(2)} < ... < X_{(n)}$. It is the rth item of a sample of n.

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} F(x)^{r-1}(1-F(x))^{n-r} f(x)$$

$$= \frac{n!}{(r-1)!(n-r)!} f(x) \int_{-\infty}^{x} f(y)dy^{r-1} \int_{x}^{\infty} f(y)dy^{n-r}$$

Unbiased Estimator: $\mathbb{E}[\hat{\theta}] = \theta$
Asymtotically unbiased: $\lim_{n\to\infty} \mathbb{E}[\hat{\theta}] = \theta$
Max Likelihood: $\hat{\theta}$ is max of $L(\theta) = \prod_{i=1}^n f(x_i|\theta)$ or $l(\theta) = \sum_{i=1}^n \log f(x_i|\theta)$

Important identities:

Expectation: $\int_{-\infty}^{\infty} xf(x)dx$

$$\mathbb{E}[aX + bY + c] = a\mathbb{E}[X] + b\mathbb{E}[Y] + c$$

Variance: $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$$\text{Var}(aX + bY + c) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X,Y)$$

We can remove Cov if X,Y are independent
Covariance: $\text{Cov}(X,Y) = \mathbb{E}[(X-\mu_X)(Y-\mu_Y)]$

$$\text{Cov}(X,Y) = \int_R \int_S (x-\mu_X)(y-\mu_Y)f(x,y)dxdy$$

$$Y = \sum a_i X_i \text{ then } Var[Y] = \sum a_i^2 Var[X_i] + 2\sum_{i<j} a_i a_j Cov[X_i, X_j]$$

$$Y = \sum a_i X_i, Z = \sum b_i X_i \text{ then } Cov[Y,Z] = \sum a_i b_i Var[X_i] + \sum\sum_{i<j} (a_i b_j + a_j b_i) Cov[X_i, X_j]$$

MGF:

$$M_X(t) = \mathbb{E}[e^{tX}]$$

$$M_{aX+bY+c}(t) = e^{ct} M_X(at) M_Y(bt)$$

if Y and X are independent

$$\frac{d^r}{dt^r} M_X(t=0) = \mu_r'$$

rth moment of X