

Dist	PDF	Mean	Var	MGF
Normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), -\infty < x < \infty$	μ	σ^2	$\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$
Gamma	$\frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, x > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$\frac{1}{2^{\nu/2} \Gamma(\nu/2)} x^{(\nu-2)/2} e^{-x/2}, x > 0$	ν	2ν	$(1 - 2t)^{-\nu/2}$
Exponential	$\frac{1}{\lambda} e^{-x/\lambda}, x > 0$	λ	λ^2	$(1 - \lambda t)^{-1}$
Uniform	$\frac{1}{\beta - \alpha}, \alpha < x < \beta$	$\frac{\alpha + \beta}{2}$	$\frac{(\beta - \alpha)^2}{12}$	$\frac{e^{\beta t} - e^{\alpha t}}{t(\beta - \alpha)}$
Bernoulli	$p^x (1 - p)^{1-x}, x = 0, 1$	p	$p(1 - p)$	$(1 - p) + pe^t$
Geometric	$p(1 - p)^{x-1}, x = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1 - (1-p)e^t}$
Binomial	$\binom{n}{x} p^x (1 - p)^{n-x}, x = 0, 1, 2, \dots, n$	np	$np(1 - p)$	$(1 + p(e^t - 1))^n$
Poisson	$\frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots$	λ	λ	$e^{\lambda(e^t - 1)}$
t-distribution	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}\Gamma(\frac{\nu}{2})} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$	0	$\frac{\nu}{\nu-2}$	$t \in R$
f-distribution	$g(f) = \frac{\Gamma(\frac{\nu_1+\nu_2}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}} f^{\frac{\nu_1}{2}-1} \left(1 + \frac{\nu_1}{\nu_2} f\right)^{-\frac{1}{2}(\nu_1+\nu_2)}$	$f > 0$		

Gamma function: $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\Gamma(n) = (n-1)!$ and $\Gamma(n) = (n-1)\Gamma(n-1)$

Variance Identity: $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ and $Var(aX + bY + c) = a^2 Var(X) + b^2 Var(Y) + 2ab Cov(X, Y)$

Sum of Squares Identity: $\sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n (X_i - \mu)^2 + n(\bar{X} - \mu)^2$

Chebyshev's: $\mathbb{P}(|X - \mu| < k) \geq 1 - \frac{\sigma^2}{k^2}$ and $\mathbb{P}(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

Weak Law of large numbers: $P(|\bar{X} - \mu_{pop}| < k) \geq 1 - \frac{\sigma_{pop}^2}{nk^2}$

Central Limit Theorem: if $X_1 \dots X_n$ are iid from any pop w/ (μ, σ^2) $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ as $n \rightarrow \infty$

Sum of Normal Squared: If $X_1, X_2 \dots X_n$ are iid $N(0, 1)$, then $\sum_{i=1}^n X_i^2 \sim \chi_n^2$

Order Statistics: $X_{(1)} < X_{(2)} < \dots < X_{(n)}$. It is the r th item of a sample of n .

$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!} F(x)^{r-1} (1 - F(x))^{n-r} f(x)$

In general, if you repeat experiment N times then $\theta \approx (1 - \alpha)\%$

μ w/ known σ : $\mu \in \left(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$

μ w/ unknown σ : $\mu \in \left(\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}\right)$

$\mu_1 - \mu_2$, w/known σ_1^2 and σ_2^2 : $\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, \bar{x}_1 - \bar{x}_2 + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$

$\mu_1 - \mu_2$, w/unknown $\sigma_1^2 = \sigma_2^2 = \sigma^2$:

$\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 - t_{\alpha/2, n_1+n_2-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \bar{x}_1 - \bar{x}_2 + t_{\alpha/2, n_1+n_2-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$

$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$ Comes from MGF, Add Variance

$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$ aka Weighted average of S_1 and S_2 . $\frac{(n_1+n_2-2)S_p}{\sigma^2} \sim \chi_{n_1+n_2-2}$

$T = \frac{Z}{\sqrt{Y/(n_1+n_2-2)}} \sim t_{\alpha/2, \nu_1+\nu_2-2}$, where $Z \sim N(0, 1)$ and $Y \sim \chi_{\nu_1+\nu_2-2}$

σ^2 : $\sigma^2 \in \left(\frac{(n-1)s^2}{\chi_{\alpha/2, n-1}^2}, \frac{(n-1)s^2}{\chi_{1-\alpha/2, n-1}^2}\right)$

$\frac{\sigma_1^2}{\sigma_2^2}$: $\frac{\sigma_1^2}{\sigma_2^2} \in \left(\frac{s_1^2}{s_2^2} \frac{1}{F_{\alpha/2, n_1-1, n_2-1}}, \frac{s_1^2}{s_2^2} F_{\alpha/2, n_1-1, n_2-1}\right)$ Remember that $F_{1-\alpha/2, n_1, n_2} = \frac{1}{F_{\alpha/2, n_2, n_1}}$

$F = \frac{U/\nu_1}{V/\nu_2} \sim F_{\nu_1, \nu_2}$, where $U \sim \chi_{\nu_1}^2$ and $V \sim \chi_{\nu_2}^2$

Type I Error: Rejecting H_0 when it is true. $\alpha = P(\text{Type I Error})$: $\alpha = P(\text{Reject } H_0 | H_0 \text{ is true})$ False Positive

Type II Error: Failing to reject H_0 when it is false. $\beta = P(\text{Type II Error})$: $\beta = P(\text{Fail to Reject } H_0 | H_0 \text{ is false})$

False Negative

Critical Region: The set of values of the test statistic that leads to rejection of H_0 .

We find the Critical Region by making a plot of $\{x_i\}$ and use our test (usually $\bar{X} > c$) and plot the critical region.

Power: $1 - \beta = P(\text{Reject } H_0 | H_0 \text{ is false})$ This is the probability of correctly rejecting H_0 aka how many hits

Transformation of 1 var to 1 var: $Y = u(X)$, $X = u^{-1}(Y) = w(Y)$, $g(y) = f(w(y)) \left| \frac{d}{dy} w(y) \right|$

Transformation of 2 var to 1 var: $Y = u(X_1, X_2)$, $X_1 = w(Y, X_2)$, $g(y) = \int_R f(w(y, x_2)) \left| \frac{\partial}{\partial y} w(y, x_2) \right| dx_2$

Method of Moments: $m'_k = \frac{\sum_{i=1}^n x_i^k}{n} = E[X^k]$ is the kth sample moment and by setting $\mu'_k = E[X^k]$ and solving for μ'_k , we get the kth population moment.

Max Likelihood: $\hat{\theta}$ is max of $L(\theta) = \prod_{i=1}^n f(x_i|\theta)$ or $l(\theta) = \sum_{i=1}^n \log f(x_i|\theta)$

Bias: $B(\hat{\theta}) = \mathbb{E}[\hat{\theta}] - \theta$. We say something is unbiased if $B(\hat{\theta}) = 0$ and asymptotically unbiased if $\lim_{n \rightarrow \infty} B(\hat{\theta}) = 0$

Cramer-Rao: $Var(\hat{\theta}) \geq \frac{1}{nI(\theta)}$ where $I(\theta) = -\mathbb{E} \left[\frac{\partial^2}{\partial \theta^2} \log f(x|\theta) \right]$ or $I(\theta) = \mathbb{E} \left[\left(\frac{\partial}{\partial \theta} \log f(x|\theta) \right)^2 \right]$

Neyman-Pearson Lemma: $\frac{L_0}{L_1} \leq k$, $\forall C$ of size α is the most powerful test. Convert to nice form for test

Likelihood Ratio Test: $\Lambda = \frac{L_\omega}{L_\Omega}$ and solve similarly for nice form. large n , $-2\ln(\Lambda) \sim \chi_1^2$

Steps of a Test: 1. State Hypothesis 2. Choose Test Statistic 3. Find Critical Region 4. Make Decision

REMEMBER CLT for $n > 30$

μ w/ σ ; or $n \geq 30$: $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$, $H_1 : \mu < \mu_0$, $H_1 : \mu > \mu_0$

for $|z| \geq z_{\alpha/2}$, $z \geq z_\alpha$, $z \leq -z_\alpha$

μ w/ unknown σ ; $n < 30$: $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$ $H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$, $H_1 : \mu < \mu_0$, $H_1 : \mu > \mu_0$

for $|t| \geq t_{\alpha/2, n-1}$, $t \geq t_{\alpha, n-1}$, $t \leq -t_{\alpha, n-1}$

$\mu_1 - \mu_2$, w/known σ_1^2 and σ_2^2 : $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$

$H_0 : \mu_1 - \mu_2 = \delta$ $H_1 : \mu_1 - \mu_2 \neq \delta$, $H_1 : \mu_1 - \mu_2 < \delta$, $H_1 : \mu_1 - \mu_2 > \delta$

for $|z| \geq z_{\alpha/2}$, $z \geq z_\alpha$, $z \leq -z_\alpha$

$\mu_1 - \mu_2$, w/unknown $\sigma_1^2 = \sigma_2^2 = \sigma^2$: $T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$

$H_0 : \mu_1 - \mu_2 = \delta$ $H_1 : \mu_1 - \mu_2 \neq \delta$, $H_1 : \mu_1 - \mu_2 < \delta$, $H_1 : \mu_1 - \mu_2 > \delta$

for $|t| \geq t_{\alpha/2, n_1+n_2-2}$, $t \geq t_{\alpha, n_1+n_2-2}$, $t \leq -t_{\alpha, n_1+n_2-2}$

σ^2 : $\chi^2 = \frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}^2$

$H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 \neq \sigma_0^2$, $H_1 : \sigma^2 < \sigma_0^2$, $H_1 : \sigma^2 > \sigma_0^2$

for $\chi^2 \geq \chi_{\alpha/2, n-1}^2$, $\chi^2 \geq \chi_{1-\alpha/2, n-1}^2$, $\chi^2 \leq \chi_{\alpha/2, n-1}^2$, $\chi^2 \leq \chi_{1-\alpha/2, n-1}^2$

$\frac{\sigma_1^2}{\sigma_2^2}$: $F = \frac{s_1^2}{s_2^2} \sim F_{n_1-1, n_2-1}$

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$ $H_1 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$, $H_1 : \frac{\sigma_1^2}{\sigma_2^2} < 1$, $H_1 : \frac{\sigma_1^2}{\sigma_2^2} > 1$

for $F \geq F_{\alpha/2, n_1-1, n_2-1}$, $F \geq F_{1-\alpha/2, n_1-1, n_2-1}$

Note that is like if statistic $\theta \notin$ Confidence Interval, reject H_0

Expectation: $\int_{-\infty}^{\infty} xf(x)dx$. Is linear!

Variance: $Var(X) = \mathbb{E}[(X - E[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$Var(aX + bY + c) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$

Covariance: $Cov(X, Y) = \mathbb{E}[(X - \mu_X)(Y - \mu_Y)]$ and $Cov(X, Y) = \int_R \int_S (x - \mu_X)(y - \mu_Y)f(x, y)dxdy$

MGF $M_X(t) = \mathbb{E}[e^{tX}]$. $M_{aX+bY+c}(t) = e^{ct}M_X(at)M_Y(bt)$ if X, Y are independent.

$\frac{d^r}{dt^r} M_X(t=0) = \mu'_r$ rth moment of X