Math 300 Midterm 4

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Definitions
Section 1: Functions

One to One

A one to one function is a function $f : A \to B$ such that for all $a_1, a_2 \in A$, if $f(a_1) = f(a_2)$, then $a_1 = a_2$.
This is also called a injective function.
This also means that there exists a left inverse g such that $g \circ f = I_A$.

Onto

An onto function is a function $f : A \to B$ such that for all $b \in B$, there exists an $a \in A$ such that $f(a) = b$.
This is also called a surjective function.
This also means that there exists a right inverse g such that $f \circ g = I_B$.

Bijection

Inverse Function

Proofs:

Proof: One to One $\to g(f(x)) = x$
Suppose A, B are non empty sets and $f : A \to B$ Assume f is one to one.
In other words $(\forall a_1, a_2 \in A)(f(a_1) = f(a_2) \to a_1 = a_2)$
Need $g : B \to A$ such that $g \circ f = I_A$
Let $y \in B$
If $y \in range(f)$ and $f(x)$ is one to one, then define $g(y) = x$ such that $f(x) = y$
If $y \notin range(f)$, then define $g(y) = a_0$ for an arbitrary $a_0 \in A$
Need to show $g \circ f = I_A$
Let $x \in A$
Need to show $g(f(x)) = x$
Let $y = f(x)$
Clearly $y \in range(f)$
due to the fact that $f(x) \in range(f)$

Then $g(f(x)) = x$