

# Lecture 3 Worksheet

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The Gamma distribution is specific by two parameters positive  $\alpha$  and  $\beta$ . Its MGF is given by  $M_X(t) = \frac{1}{(1-\beta t)^\alpha}$

## Question 1

Suppose  $X$  is a random variable with Gamma distribution with parameters  $\alpha = 3$  and  $\beta = 2$ . Use the MGF to compute  $E(X)$  and  $Var(X)$ .

$\mathbb{E}[X]$

$$\begin{aligned}\mathbb{E}[X] &= M'_X(0) \\ &= \frac{d}{dt} \frac{1}{(1-2t)^3} \Big|_{t=0} \\ &= \frac{d}{dt} (1-2t)^{-3} \Big|_{t=0} \\ &= -3(1-2t)^{-4}(-2) \Big|_{t=0} \\ &= 6\end{aligned}$$

$\text{Var}[X]$

$$\begin{aligned}\text{Var}[X] &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\ &= M''_X(0) - \mathbb{E}[X]^2 \\ &= \frac{d^2}{dt^2} \frac{1}{(1-2t)^3} \Big|_{t=0} - 6^2 \\ &= \frac{d^2}{dt^2} (1-2t)^{-3} \Big|_{t=0} - 36 \\ &= 12(1-2t)^{-5}(-2)^2 \Big|_{t=0} - 36 \\ &= 48 - 36 \\ &= 12\end{aligned}$$

Question 2

Question 3