Workshop 5: Math 292

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1. Model:

   (a) Equilibrium points:

   - Equilibrium points: (0,0) and (K,0)
   - For $\gamma = \delta k$ then (K, c) for all positive reals c.

   (b) Particular Solution:

   - $x'(t) = rx(1 - \frac{x}{K})$. We notice this is similar to the logistic curve from class: $x' = x(r - ax)$
   - If we have $r = r$ and $a = \frac{r}{k}$ we have the same equation and then the same Solution
   - $x = \frac{x_0 k}{x_0 + e^{-rt}(k - x_0)}$
   - Plugging into y' we get $y' = (\delta[\frac{x_0 k}{x_0 + e^{-rt}(k - x_0)}] - \gamma)y$
   - We know the sol for y is in the form of $y = y_0 e^{\int_{t_0}^{t} pds}$ with $\int_{t_0}^{t} pds = \int_0^t (\delta[\frac{x_0 k}{x_0 + e^{-rt}(k - x_0)}] - \gamma)$
   - The integral evaluates to $\frac{\delta K}{r} ln[\frac{x_0 e^{rt} + k - x_0}{k}] - \gamma t$
   - the sol is $y = y_0 e^{\frac{\delta K}{r} ln[\frac{x_0 e^{rt} + k - x_0}{k}] - \gamma t}$

   (c) Limit

   - $\gamma < \delta k$ then $\lim_{t\to\infty} y(t) = \infty$
   - $\gamma > \delta k$ then $\lim_{t\to\infty} y(t) = 0$
   - $\gamma = \delta k$ then $\lim_{t\to\infty} y(t) = y_0 \frac{x_0}{K}^{\frac{\delta k}{r}}$

2. Linear Algebra

   (a) -

   - $X(t) = e^{\lambda k}\vec{V}$ is a sol as $X'(t) = \lambda e^{\lambda k}\vec{V}$ and $X'(t) = \lambda X(t)$ and $X'(t) = AX(t)$ since $AV = \lambda V$
   - $\lambda = 0$ then $X = \vec{V}$ for all time
   - $\lambda > 0$ then $\lim_{x\to\infty} X(t) = \infty$ and $\lim_{x\to-\infty} X(t) = 0$
   - $\lambda < 0$ then $\lim_{x\to\infty} X(t) = 0$ and $\lim_{x\to-\infty} X(t) = \infty$

   (b) -

- Proof by induction:
- Base Case $p(1)$: It is clear that a set of 1 vector is linearly indepdendant as the only c that satisfies $c\vec{V} = 0$ where $\vec{V} \neq 0$ is 0
- Inductive Steps: Assume $p(k)$ holds. Prove $p(k+1)$:
- $p(k+1)$ is $c_1\vec{V^{(1)}} + c_2\vec{V^{(2)}} + ... + c_k\vec{V^{(k)}} + c_{k+1}V^{(\vec{k+1})} = 0$ since we know that $p(k)$ holds: $c_1\vec{V^{(1)}} + c_2\vec{V^{(2)}} + ... + c_k\vec{V^{(k)}} = 0$ we can sub it into $p(k+1)$ to get $0 + c_{k+1}V^{(\vec{k+1})} = 0$
- since $V^{(\vec{k+1})} \neq 0$ then $c_{k+1} = 0$ thus proving by induction that any set of eigenvectors with distinct eigenvalues is linearly indepdendant

(c) -

- Since we know from the previous part that the columns of $M(t)$ are linearly indepdendant, we can see that it is inveratble.
- By plugging in 0 for t in the solution we can see $\vec{X}(0) = M(0)\vec{C}$
- Solving for C we get $C = M^{-1}(0)\vec{X}(0)$
- Plugging back gets $X(t) = M(t)M^{-1}(0)\vec{X}(0)$

(d) -