HW 2: 292H

Pranav Tikkawar

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1. $-4x'(t) + tx(t) = 2t(x(t))^3$
   - Divide out the terms by $(x(t))^3$: $\frac{-4x'}{x^3} + \frac{t}{x^2} = 2t$
   - Set $y = \frac{1}{x^2}$ and $-2y' = \frac{x'}{x^3}$ and get $8y' + ty = 2t$ and then $y' + \frac{ty}{8} = \frac{t}{4}$
   - Then we can multiply through by $h(t) = e^{\frac{t^2}{16}}$ to get $hy' + \frac{t}{8}hy = \frac{t}{4}h$
   - Then we get $[hy]' = \frac{t}{4}h$
   - Then we get $y = 2 + Ce^{\frac{-t^2}{16}}$
   - Since $x = \frac{1}{\sqrt{y}}$ then $x = \frac{1}{\sqrt{2+Ce^{\frac{-t^2}{16}}}}$
2. $x' = \frac{-1}{t^2} - \frac{x}{t} + x^2$
   - Suppose x is a solution of the DE above that we guess and Y is also a solution to the DE given by $y = u + x$ where u is also a function
   - We get $u' + x' = \frac{-1}{t^2} - \frac{x}{t} + x^2 - \frac{u}{t} + 2ux + u^2$
   - Notice that we can cancel x' from both sides to get $u' = \frac{u}{t} + u^2$
   - Now this is a Bernoulli and we can set $h = \frac{1}{u}$ and $-h' = \frac{u'}{u^2}$
   - By dividing and subbing in we get $-h' = \frac{1}{t}h + 1$ and then $h' + \frac{1}{t}h = -1$
   - Then we introduce $j = \frac{t^2}{2}$ and mulitply through to get $jh' + \frac{1}{t}jh = -j$
   - Then we notice that $[jh]' = -j$
   - Replacing J and integrating we get $h = \frac{-t^3}{3} + \frac{c}{t}$
   - Since $u = \frac{1}{h}$ then $u = \frac{3t}{-t^3+C}$
   - Since $y = u + x$ and y is the original solution we desite then we get $y = \frac{3t}{-t^3+C} + \frac{1}{t}$
3. $x'(t) = sin(x(t)), x(0) = x_0$
   - From the DE we get $\int_{x_0}^{x} \frac{dx}{sinx} = \int_0^t dt$
   - Evaluating the integral using some algebra magic we get $ln(csc(x_0) - cot(x_0)) - ln(csc(x) - cot(x)) = t - 0$

- Since we know that $\frac{1-cos(x)}{sin(x)} = tan(\frac{x}{2}) = csc(x) - cot(x)$ then we get $ln(tan(\frac{x}{2}))/(tan(\frac{x_0}{2})) = t$
- Then we get that $tan(x/2) = e^t tan(x_0/2)$
- Then we get $x = 2arctan(e^t tan(x_0/2))$
- For where x is a valid solution we get that $x = 0, x_0 = 0, t \in (-\infty, \infty)$ and $x = 2arctan(e^t tan(x_0/2)), x_0 \in (0, \pi), t \in (-\infty, \infty)$ and $x = \pi, x_0 = \pi, t \in (-\infty, \infty)$