01:XXX:XXX - Homework n

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Theorem 1. *Suppose $X_t \in \mathbb{R}(t \in \mathbb{Z})$ is a weakly stationary stochastic process*
Then

$$\begin{aligned}\exists a_0, a_1, \dots, \textit{such that} a_0 = 1, \sum_{j=1}^{\infty} a_j^2 < \infty \\ \exists \epsilon_t, \mu_t (t \in \mathbb{Z}) \textit{ such that } \forall s, t \in \mathbb{Z} : \\ \epsilon_t \in L_t, \mu_t \in L_{-\infty} \\ E(\epsilon_t) = 0, Cov(\epsilon_t, \epsilon_s) = \sigma_\epsilon^2 \delta_{t,s} < \infty, Cov(\epsilon_s, \mu_t) = 0 \\ X_{t_{a.sL^2(\Omega)}} = \mu_t + \sum_{j=0}^{\infty} a_j \epsilon_{t-j} (t \in \mathbb{Z})\end{aligned}$$

*Where L_t is the infinite linear past of X_t.****

Definition (Weakly Stationary)**.** A process X_t is weakly stationary if the mean and covariance are time invariant. That is, $\mu_X(t) = \mu_X$ and $\gamma_X(t_1, t_2) = \gamma_X(t_1 - t_2)$ for all $t_1, t_2 \in Z$.

Definition (Linear Past)**.**

Definition.

- Time Series data vs IID Data
  - Glivenko-Cantelli theorem
- Ergodic Property with a Cosntant Limit
  - L^2 EPCL
  - E1, E2, E3 (pg 6-7)
  - Strict and Weak Stationary
  - Covariance, Correlation, Autocovariance, Autocorrelation
- Weak Stationary and Hilbert Space
  - Measure Theory
    - σ-algebra
    - L^2 space
    - Completeness
    - Hilbert space
  - K-step mean squared error
    - What method to minimize k-step MSE?
    - Closed Convex Hilbert space
    - Optimal Forecast
    - L^2-projection
- Conditional Expectation
- Linear Past
- Deterministic process
- Wold Decomposition
  - Karhunan Lovuve theorem
  - Wold's theorem
  - Tie Everythig Together
- Purely stochastic process
- Why we care now
  - AR Model
  - MA Model
  - ARMA Model
  - ARIMA Model
  - SV Model