TODO

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1.1

1

$W = X * \mathbb{E}[Y]$

2

$U = \frac{Z}{\mathbb{E}[Y]}$

3

$V = Y$

1.2

1

$E[X^2 + 4XY + 4Y^2|X] = E[X^2|X] + 4E[XY|X] + 4E[Y^2|X]$
$= X^2 + 4X\mathbb{E}[Y] + 4\mathbb{E}[Y^2]$

2

$E[X^2 + 4XY + 4Y^2|X, Z] = E[X^2|X, Z] + 4E[XY|X, Z] + 4E[Y^2|X, Z]$
$= X^2 + 4XY + 4Y$

3

$E[X + 2Y|Z] = E[X|Z] + 2E[Y|Z]$
$= \mathbb{E}[X] + 2\mathbb{E}[Y]$

4

1.3

1

$$
\begin{aligned}
\mathbb{E}[S_n] &= \mathbb{E}[X_1 + X_2 + ... + X_n] \\
&= \mathbb{E}[X_1] + \mathbb{E}[X_2] + ... + \mathbb{E}[X_n] \\
&= n * \mathbb{E}[X_j] \\
&= 0 \\
\mathbb{E}[S_n^2] &= \mathbb{E}[(X_1 + X_2 + ... + X_n)^2] \\
&= \mathbb{E}[X_1^2 + X_2^2 + ... + X_n^2 + 2X_1X_2 + 2X_1X_3 + ... + 2X_{n-1}X_n] = n\mathbb{E}[X_j^2] + n(n-1)\mathbb{E}[X_jX_k] \\
&= n\mathbb{E}[X_j^2] \\
&= 2n \\
\mathbb{E}[S_n^3] &= \mathbb{E}[(X_1 + X_2 + ... + X_n)^3] \\
&= \mathbb{E}[X_1^3 + X_2^3 + ... + X_n^3 + 3X_1^2X_2 + 3X_1^2X_3 + ... + 3X_{n-1}^2X_n + 6X_1X_2X_3 + 6X_1X_2X_4 + ... + 6X_{n-2}X_n \\
&= n\mathbb{E}[X_j^3] + 3n(n-1)\mathbb{E}[X_j^2X_k]/2 + n(n-1)(n-2)\mathbb{E}[X_jX_kX_l] \\
&= \frac{7n}{3}
\end{aligned}
$$

2

$$
\begin{aligned}
E[S_n|F_m] = S_m + \mu(n-m) = S_m \\
E[S_n^2|F_m] = S_m^2 + \sigma^2(n-m) = S_m^2 + 2(n-m) \\
E[S_n^3|F_m] =
\end{aligned}
$$

3

$E[X_m|S_n] = \frac{S_n}{n}$

1.4

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1.5

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1.6

1

I believe it is a submartingale as the expected value of the next value is greater than the current value.

2

3

4

5