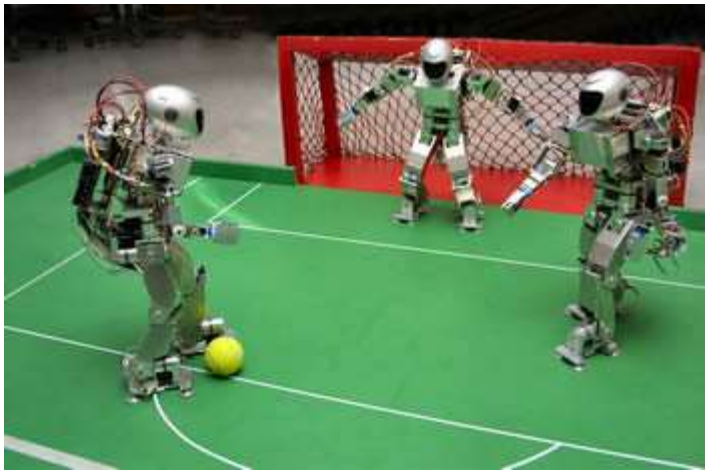


Chapter 5: Adversarial Search

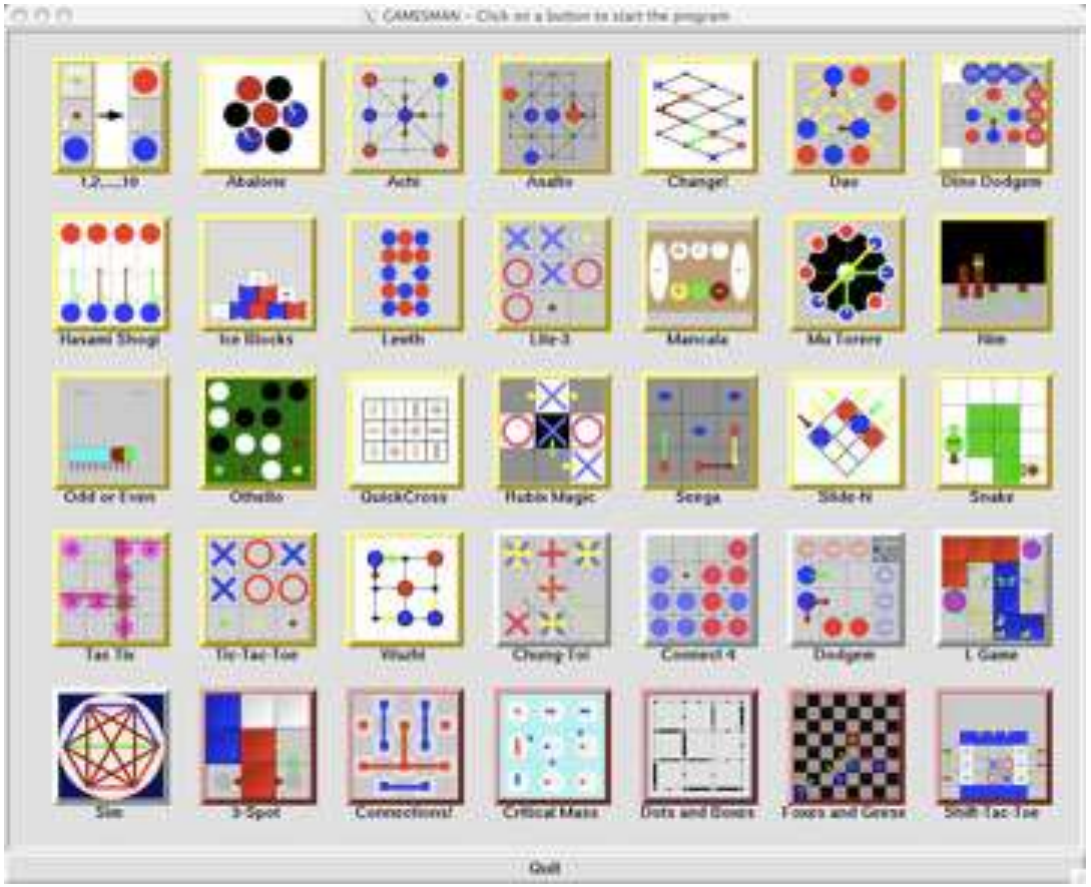
In which we examine the problems that arise when we try to plan ahead in a world where other agents are planning against us.

Adversarial Search

- Multi-agent environment:
  - any given agent needs to consider the actions of other agents and how they affect its own welfare
  - introduce possible contingencies into the agent's problem-solving process
  - cooperative vs. competitive
- Adversarial search problems: agents have conflicting goals -- games

Games vs. Search Problems

- "Unpredictable" opponent
  - specifying a move for every possible opponent reply
- Time limits
  - unlikely to find goal, must approximate



AI and Games

- In AI, "games" have special format:
  - deterministic, turn-taking, 2-player, zero-sum games of perfect information
  - Zero-sum describes a situation in which a participant's gain or loss is exactly balanced by the losses or gains of the other participant(s)
  - Or, the total payoff to all players is the same for every instance of the game (constant sum)

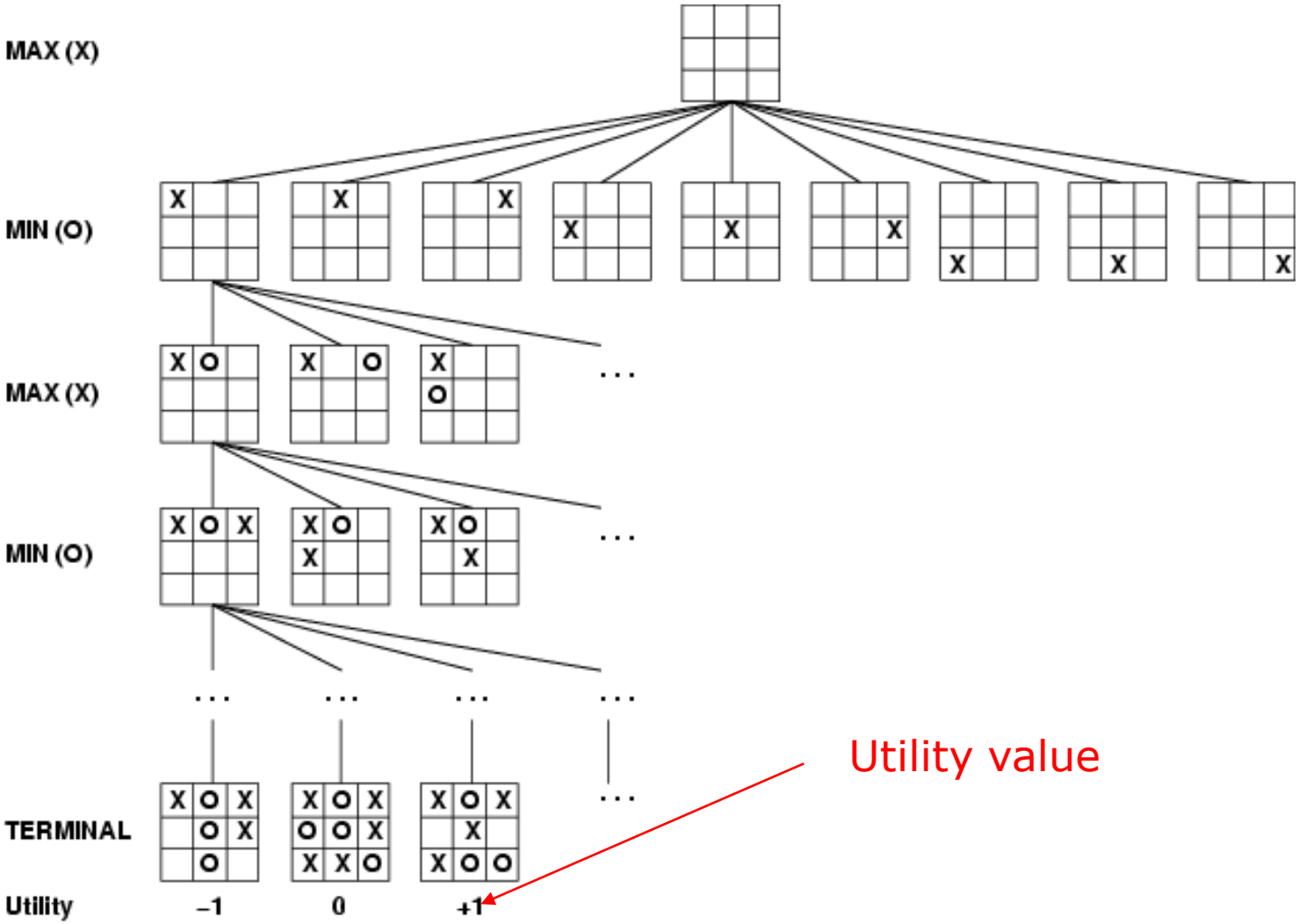
Go! 围棋

Game Problem Formulation

- A game with 2 players (MAX and MIN, MAX moves first, turn-taking) can be defined as a search problem with:
  - initial state: board position
  - player: player to move
  - successor function: a list of legal (move, state) pairs
  - goal test: whether the game is over – terminal states
  - utility function: gives a numeric value for the terminal states (win, loss, draw)

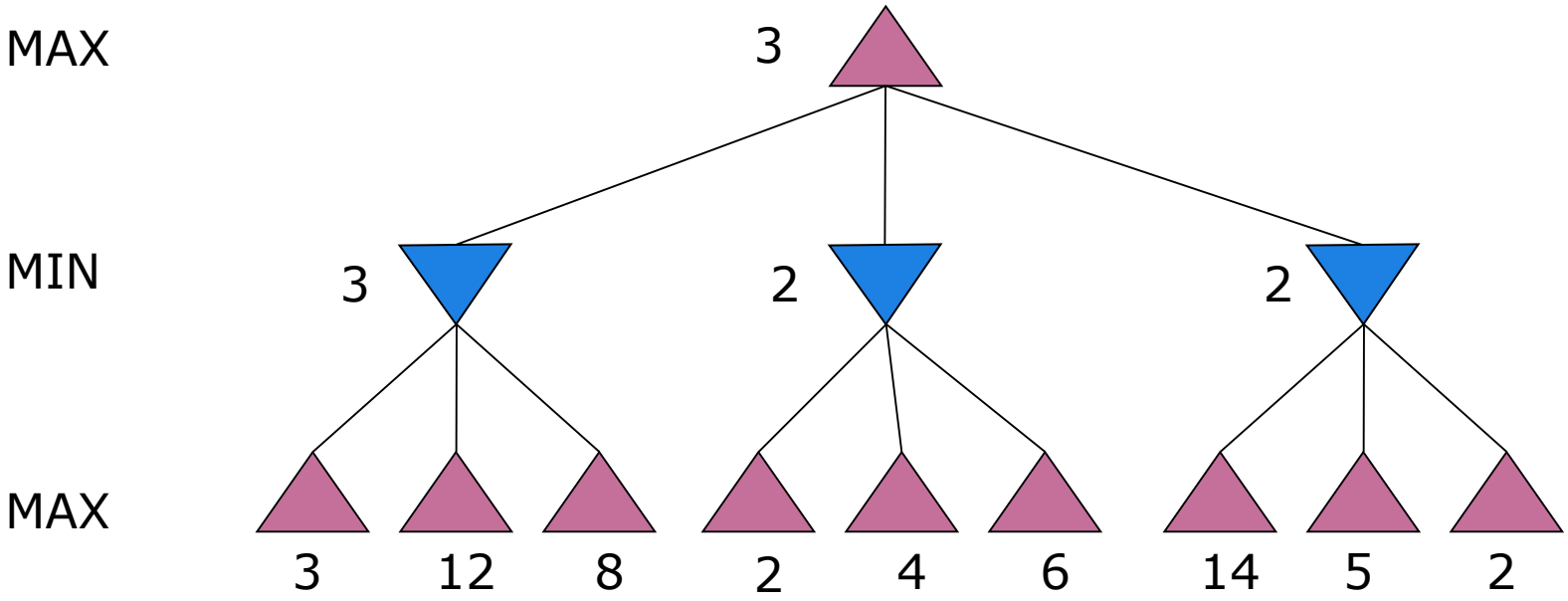
- Game tree = initial state + legal moves

Game Tree (2-player, deterministic)



Optimal Strategies

- MAX must find a contingent strategy, specifying MAX's move in:
  - the initial state
  - the states resulting from every possible response by MIN
- E.g., 2-ply game (the tree is one move deep, consisting of two half-moves, each of which is called a ply):



Minimax Value

- Perfect play for deterministic game, assume both players play optimally
- Idea: choose move to position with highest minimax value = best achievable payoff against best play

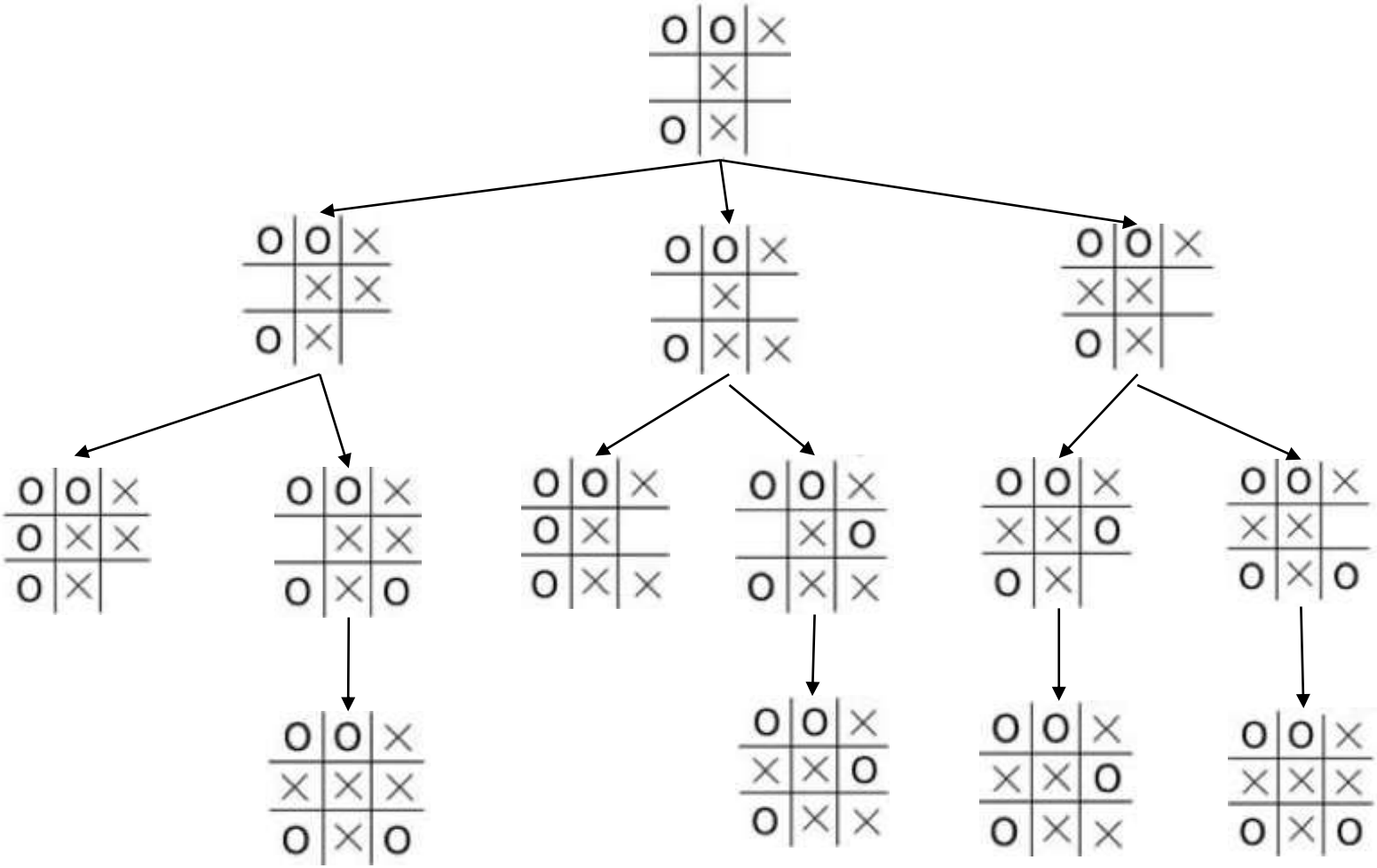
$$MINIMAX-VALUE(n) =$$

$$Utility(n)$$ if n is a terminal state

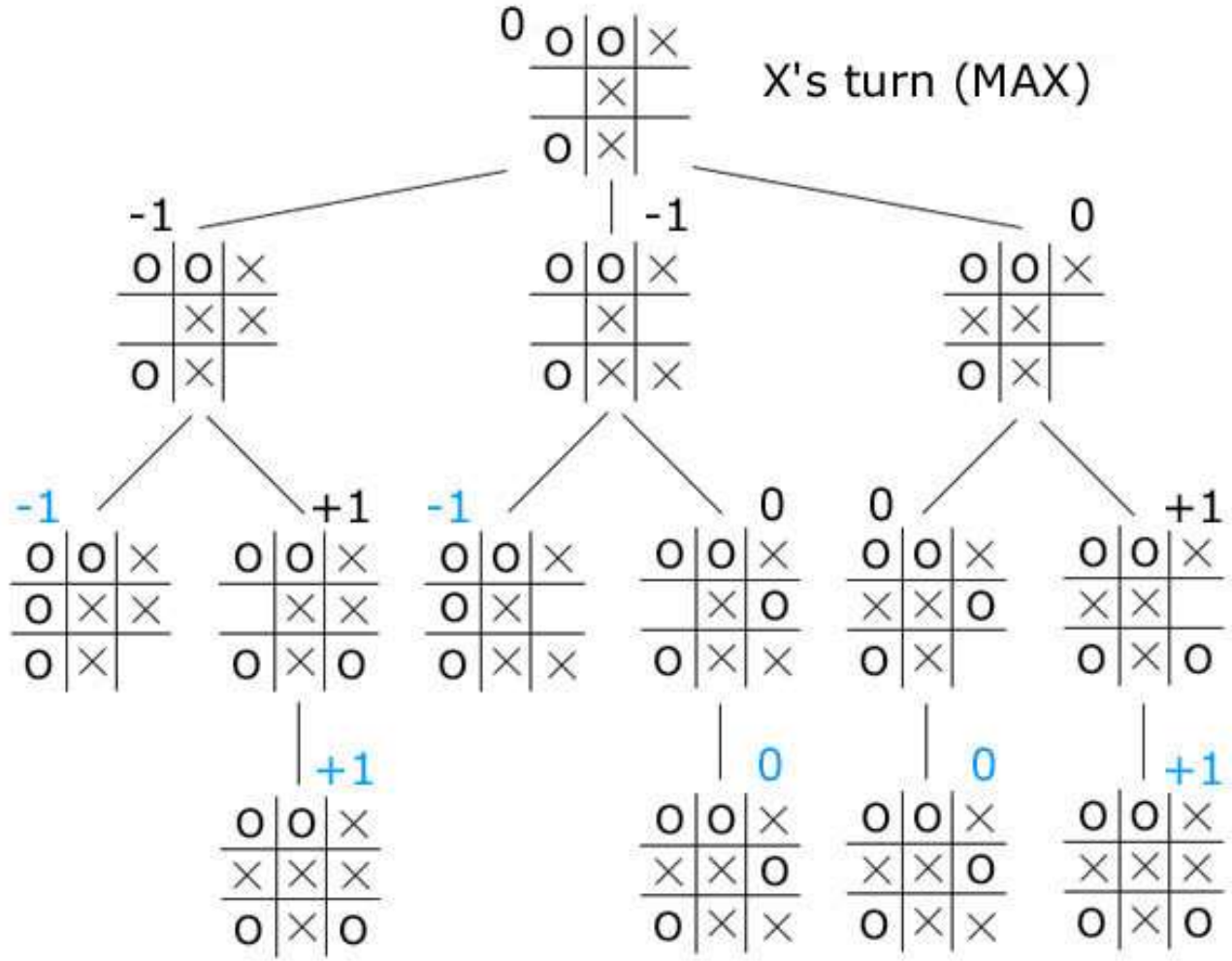
$$\max_{s \in Successors(n)} MINIMAX(s)$$ if n is a MAX node

$$\min_{s \in Successors(n)} MINIMAX(s)$$ if n is a MIN node

A Partial Game Tree for Tic-Tac-Toe



A Partial Game Tree for Tic-Tac-Toe



Minimax Algorithm

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

Minimax with Tic-Tac-Toe

- Work on board.
- Walk through a real tic-tac-toe program.

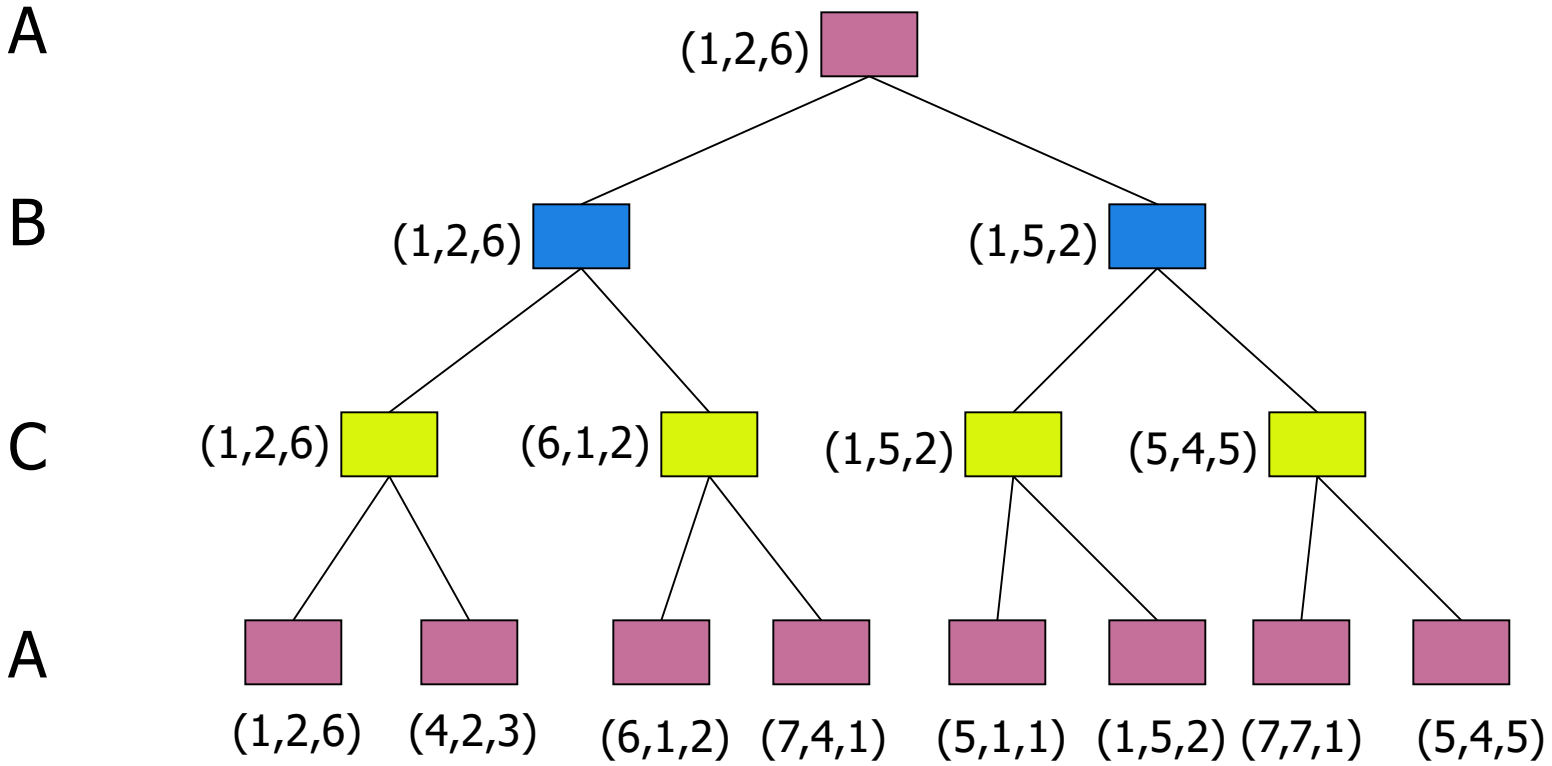
Analysis of Minimax

- Optimal play for MAX assumes that MIN also plays optimally, what if MIN does not play optimally?
- A complete depth-first search?
  - Yes
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - O(bm) (depth-first exploration)

- For chess, b ≈ 35, m ≈ 100 for "reasonable" games
  → exact solution completely infeasible

Optimal Decisions for Multiplayer Games

- Extend minimax idea to multiplayer



Interesting Thoughts

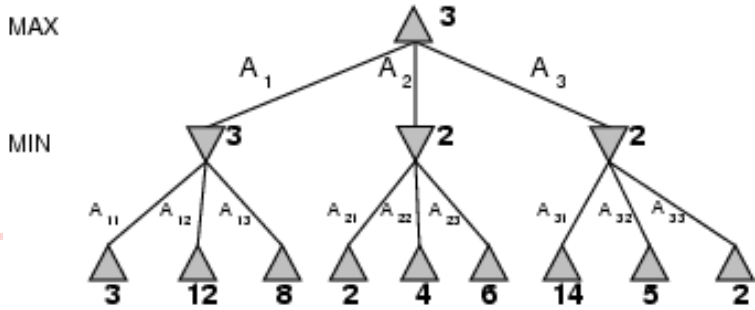
- Multiplayer games usually involve alliances, which can be a natural consequence of optimal strategies

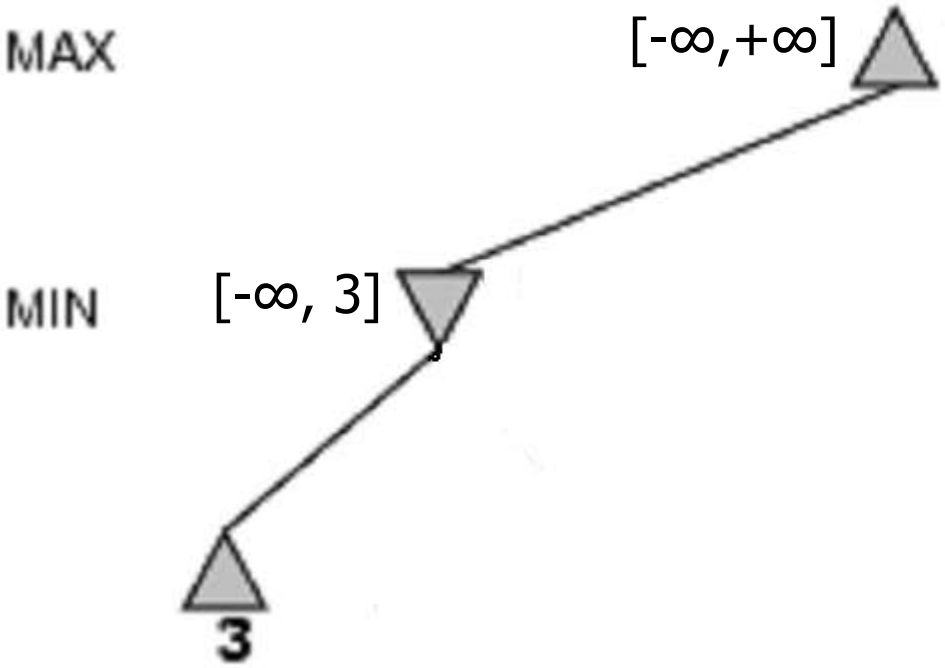
- If the game is non zero-sum, collaboration can also occur
  - For example, a terminal state with utilities <Va = 1000, Vb = 1000>
  - The optimal strategy is for both players to do everything possible to reach this state

α-β Pruning

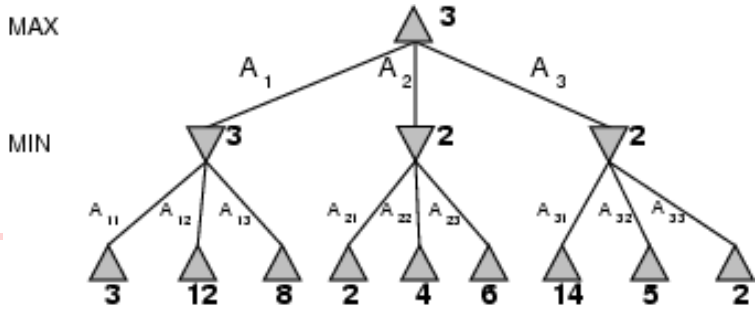
- The number of game states with minimax search is exponential in the # of moves
- Is it possible to compute the correct minimax decision without looking at every node in the game tree?
- Need to prune away branches that cannot possibly influence the final decision

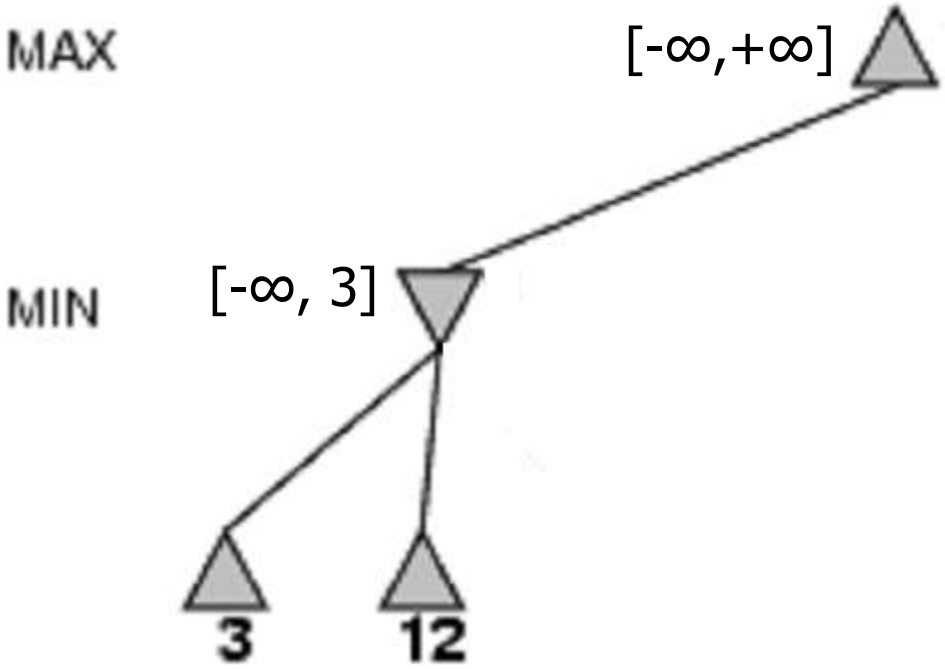
α-β Pruning Example



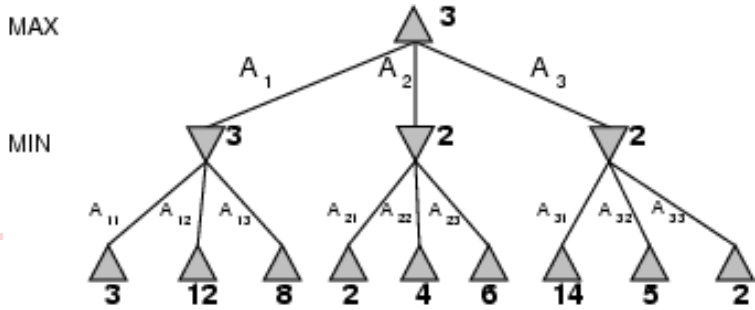


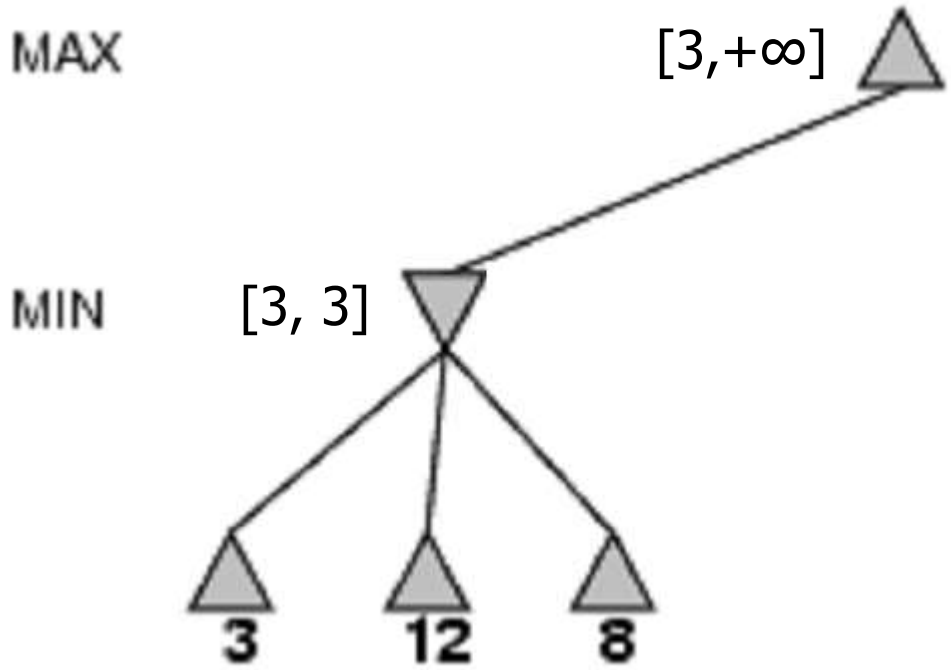
α-β Pruning Example



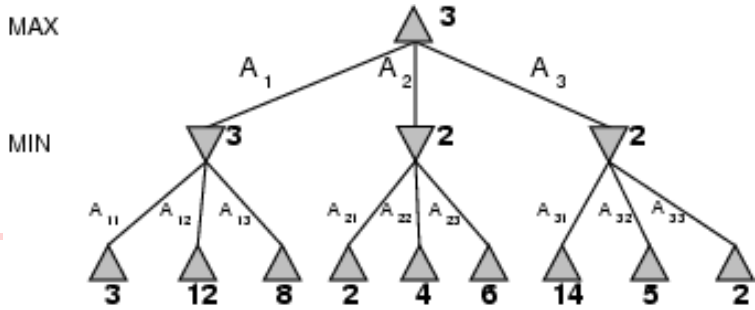


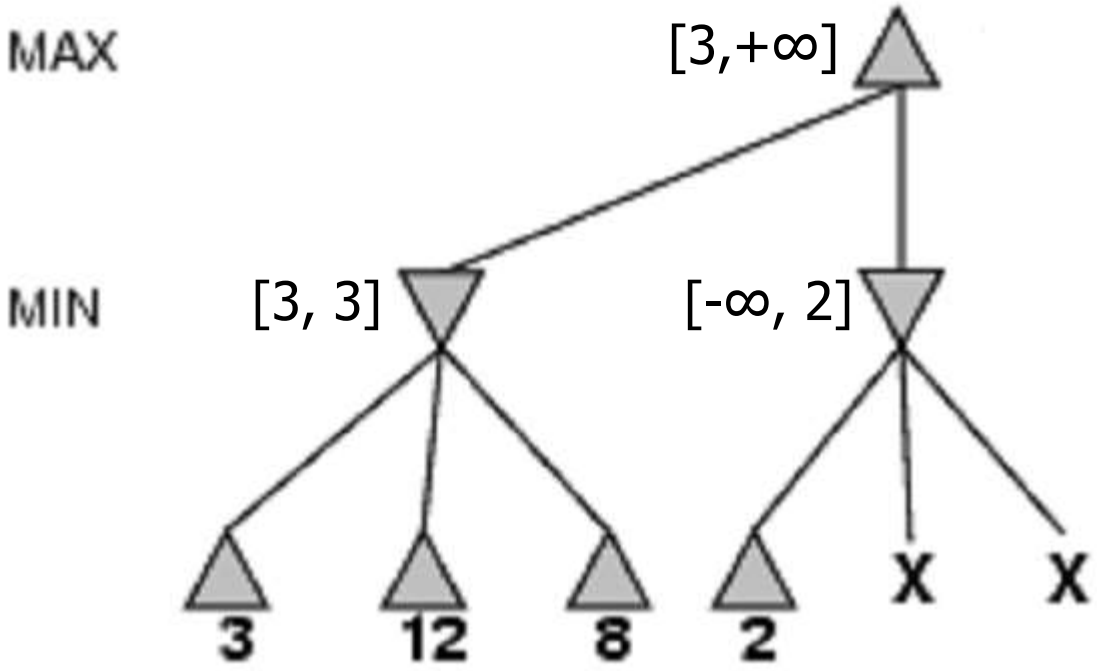
α-β Pruning Example



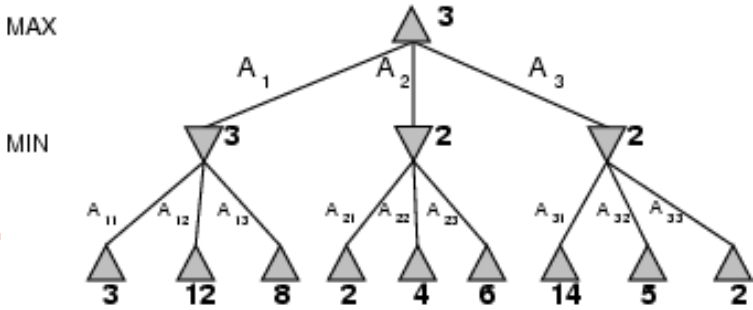


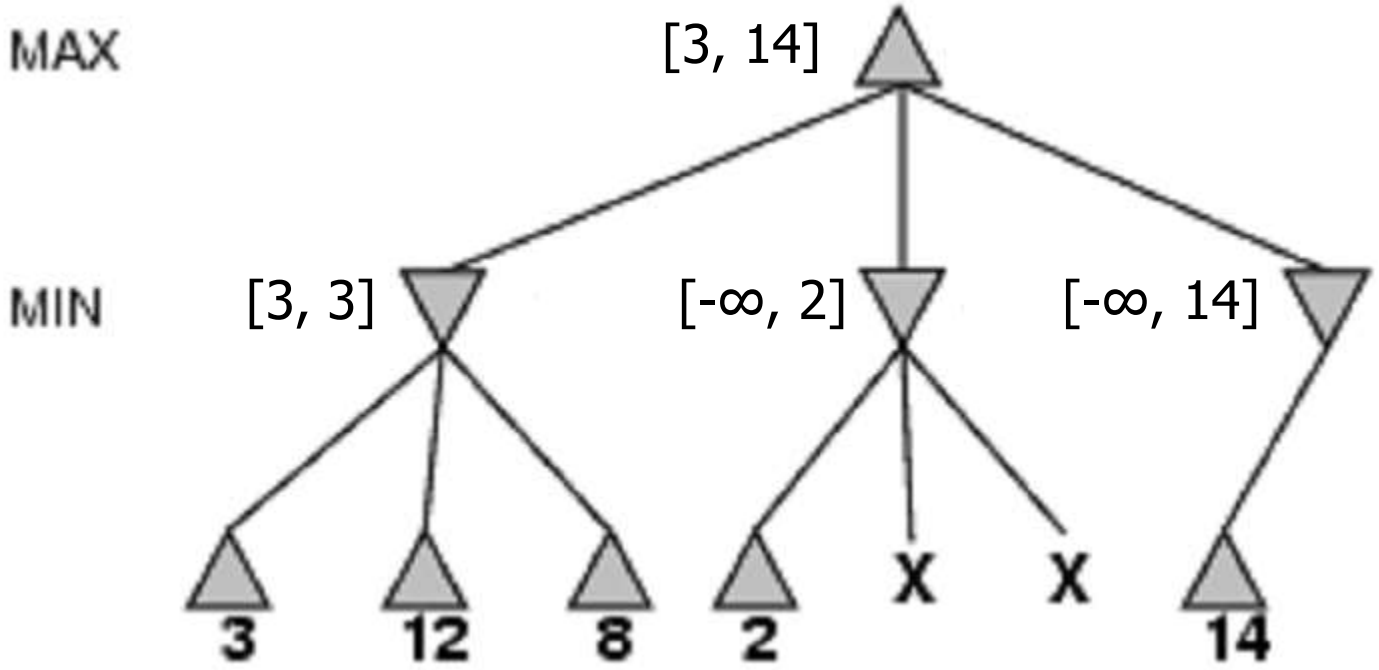
α-β Pruning Example



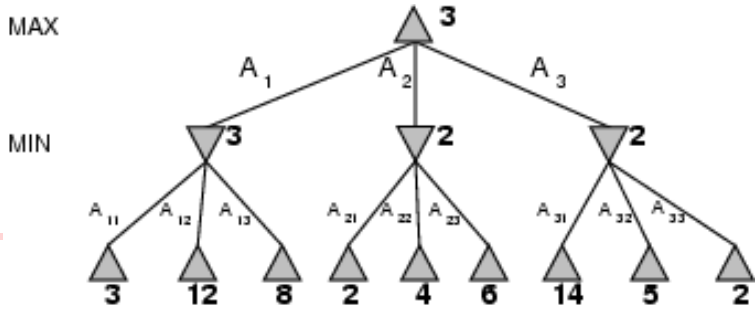


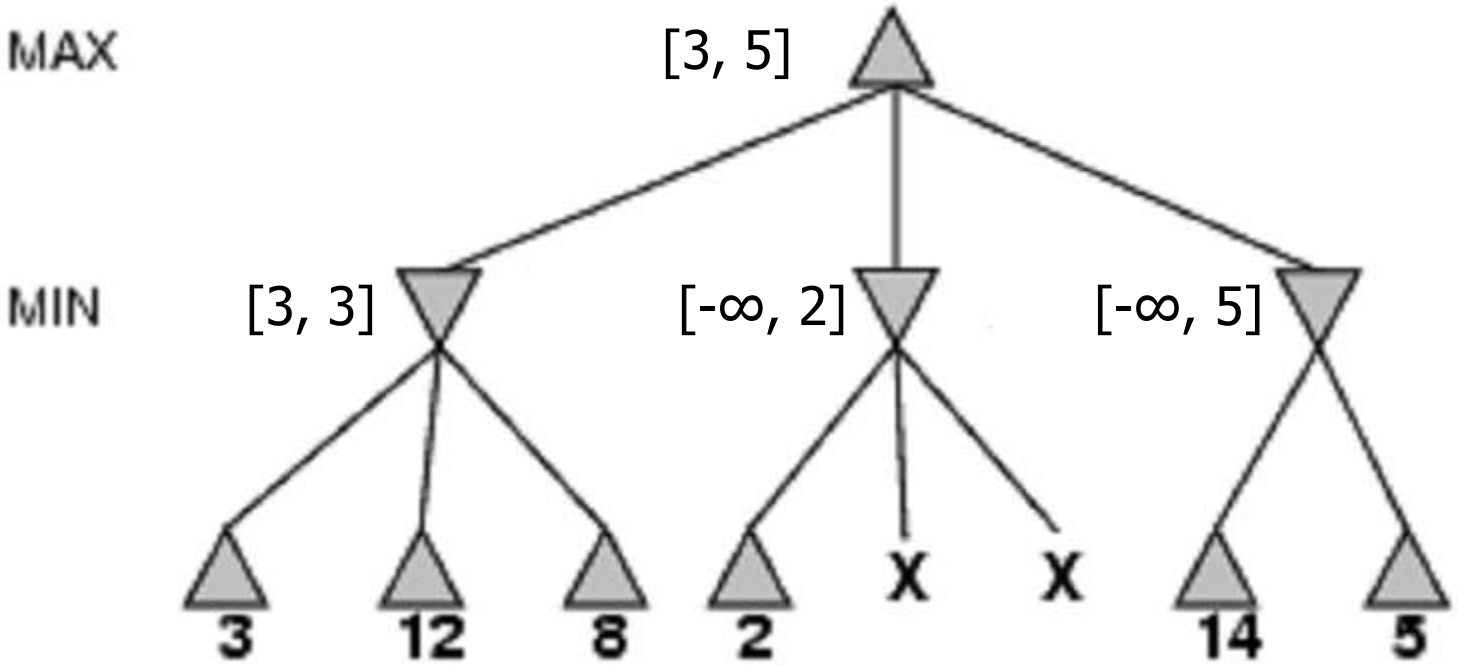
α-β Pruning Example



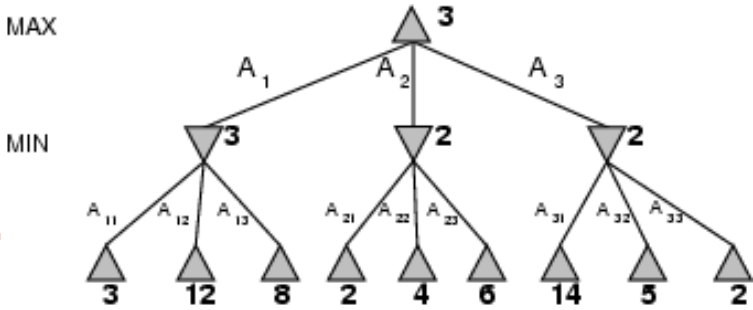


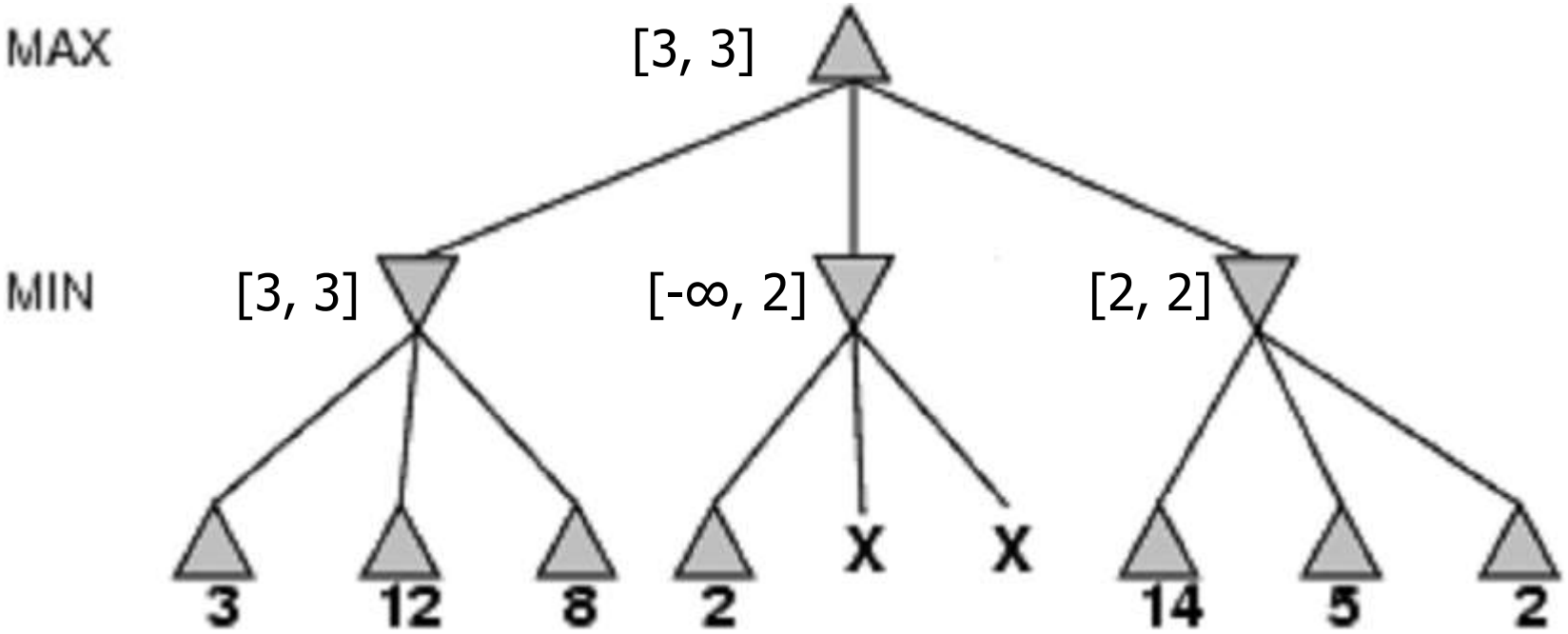
α-β Pruning Example





α-β Pruning Example



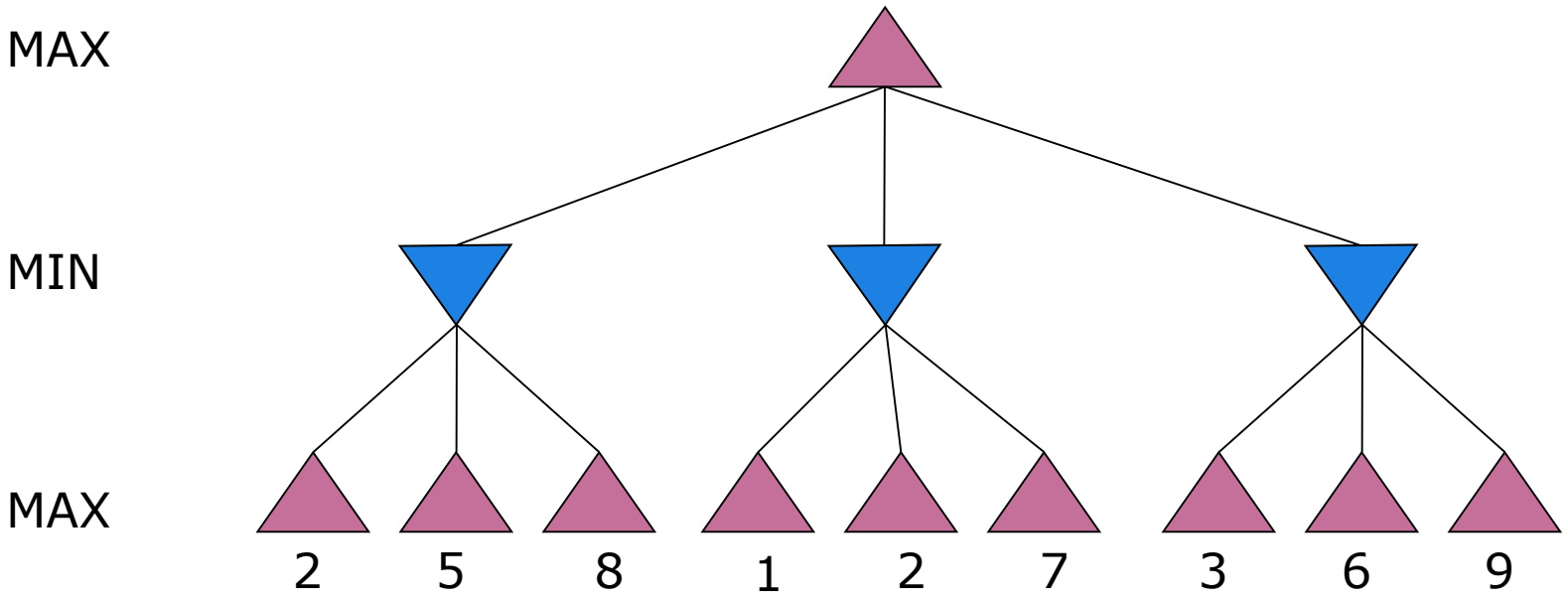


A Simple Formula

$$MINIMAX - VALUE(root)$$
$$= \max(\min(3,12,8), \min(2, x, y), \min(14, 5, 2))$$
$$= \max(3, \min(2, x, y), 2)$$
$$= \max(3, z, 2) \qquad \text{where } z \leq 2$$
$$= 3$$

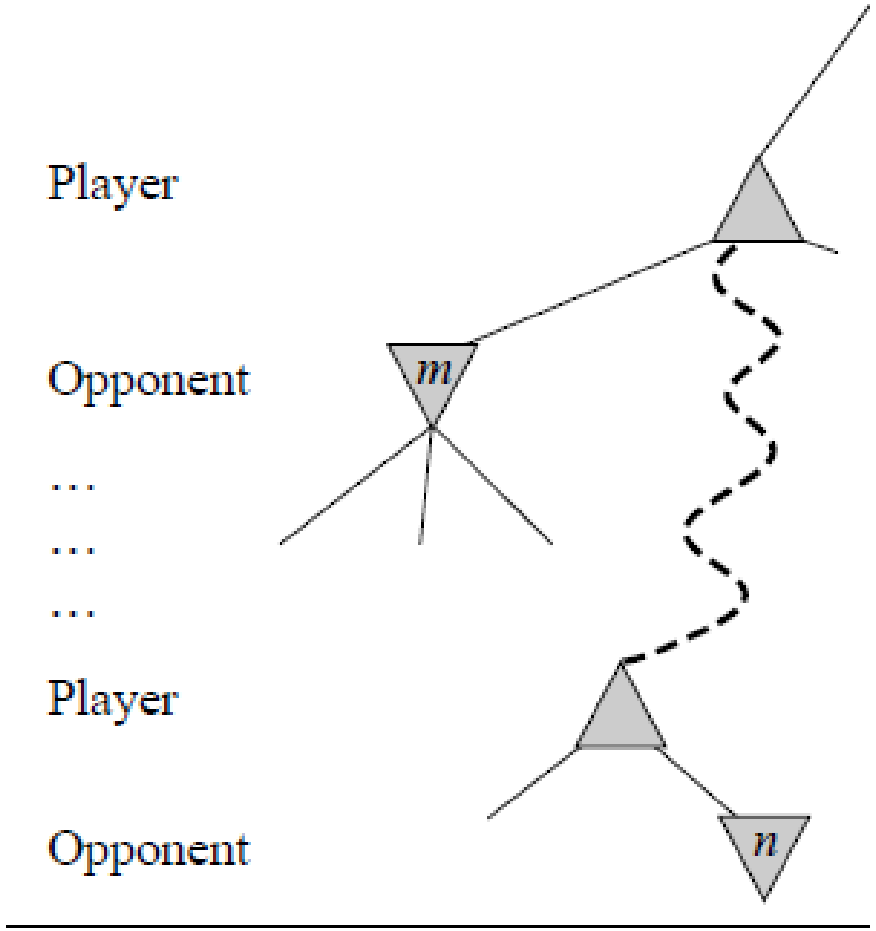
The value of the root and hence the minimax decision are independent of the values of pruned leaves *x* and *y*

α-β Pruning Exercise



Basic Idea of α-β

- Consider a node n such that Player has a choice of moving to
- If Player has a better choice m either at the parent of n or at any choice point further up, then n will never be reached in actual play
- α-β pruning gets it name from the two parameters that describe bounds on the backed-up values



Definitions of α and β

- α = the value of the best (highest-value) choice we have found so far at any choice point along the path for MAX
- β = the value of the best (lowest-value) choice we have found so far at any choice point along the path for MIN
- α-β search updates the values of α and β as it goes along and prunes the remaining branches at a node as soon as the value of the current node is worse than the current α or β for MAX or MIN respectively

```
function ALPHA-BETA-SEARCH(state) returns an action
    inputs: state, current state in game

    v ← MAX-VALUE(state, −∞, +∞)
    return the action in SUCCESSORS(state) with value v
```

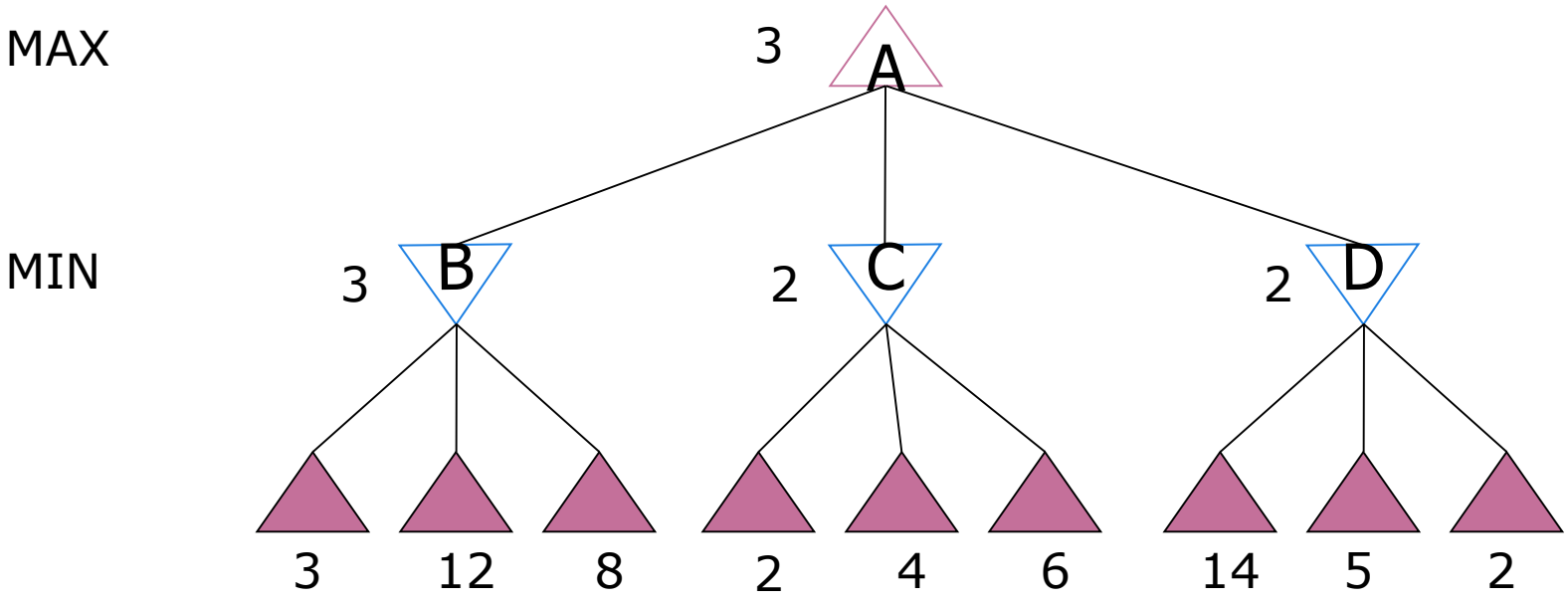
```
function MAX-VALUE(state, α, β) returns a utility value
    inputs: state, current state in game
            α, the value of the best alternative for MAX along the path to state
            β, the value of the best alternative for MIN along the path to state

    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s, α, β))
        if v ≥ β then return v
        α ← MAX(α, v)
    return v
```

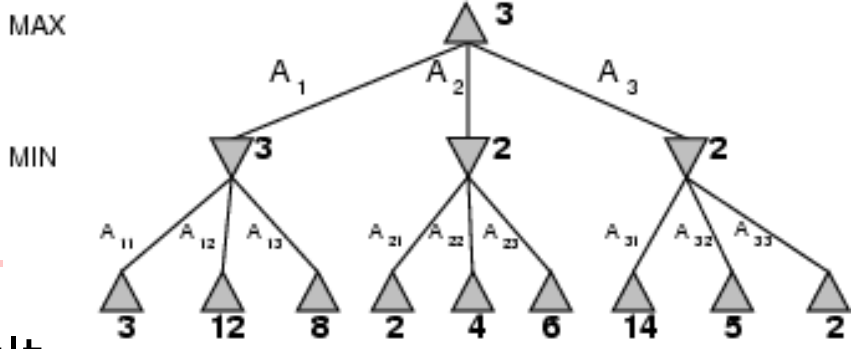
```
function MIN-VALUE(state, α, β) returns a utility value
    inputs: state, current state in game
            α, the value of the best alternative for MAX along the path to state
            β, the value of the best alternative for MIN along the path to state

    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← +∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s, α, β))
        if v ≤ α then return v
        β ← MIN(β, v)
    return v
```

Now, Trace The Behavior



Analysis of α-β



- Pruning does not affect final result
- The effectiveness of alpha-beta pruning is highly dependent on the order of successors
- It might be worthwhile to try to examine first the successors that are likely to be best
- With "perfect ordering," time complexity = $O(b^{m/2})$
  - → effective branching factor becomes $\sqrt{b}$
  - For chess, 6 instead of 35
  - it can look ahead roughly twice as far as minimax in the same amount of time
  - Ordering in chess: captures, threats, forward moves, and then backward moves

Random Ordering?

- If successors are examined in random order rather than best-first, the complexity will be roughly $O(b^{3m/4})$

- Adding dynamic move-ordering schemes, such as trying first the moves that were found to be best last time, brings us close to the theoretical limit

- The best moves are often called killer moves (killer move heuristic)

Dealing with Repeated States

- In games, repeated states occur frequently because of transpositions --
  - different permutations of the move sequence end up in the same position
  - e.g., [a1, b1, a2, b2] vs. [a1, b2, a2, b1]
- It's worthwhile to store the evaluation of this position in a hash table the first time it is encountered
  - similar to the "explored set" in graph-search
- Tradeoff:
  - Transposition table can be too big
  - Which to keep and which to discard

Imperfect, Real-Time Decisions

- Minimax generates the entire game search space
- Alpha-beta prunes large part of it, but still needs to search all the way to terminal states
- However, moves must be made in reasonable amount of time

- Standard approach: turning non-terminal nodes into terminal leaves
  - cutoff test: replaces terminal test, e.g., depth limit
  - heuristic evaluation function = estimated desirability or utility of position

Evaluation Functions

- The performance of a game-playing program is dependent on the quality of its evaluation function
  - Order the terminal states the same way as the true utility function
  - Evaluation of nonterminal states correlate with the actual chance of winning
  - Computation must not take too long!

Playing Chess, Experience is Important

- Most eval. Functions work by calculating various features of the state
- Different features form various categories of states
- Consider the category of two-pawns vs. one pawn and suppose experience suggests that 72% of the states lead to a win (+1), 20% to a loss(0), and 8% to a draw (1/2)
  - Expected value = 0.72*1 + 0.20*0 + 0.08*1/2 = 0.76
- Too many categories and too much experience are required

- Introductory chess books give an approximate material value for each piece:
  - each pawn is worth 1, a knight or bishop is worth 3, a rook 5, and the queen 9
  - Other features such as "good pawn structure" and "king safety" worth half a pawn

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

  - w_i is a weight and f_i is a feature of the position
  - e.g., f is the # of each piece, and w is the value of each piece
  - Linear assumption: contribution of each feature is independent of the values of the other features
  - There are also non-linear combinations of features

Cutting off Search

- Modify alpha-beta search so that:
  - Terminal? is replaced by Cutoff?
  - Utility is replaced by Eval
  - if **Cutoff-Test**(state, depth) then return **Eval**(state)
  - depth is chosen such that the amount of time used will not exceed what the rules of the game allow
  - Iterative deepening search can be applied
    - When time runs out, returns the move selected by the deepest completed search

Result?

- Does it work in practice?
  - Chess problem. Assume we can generate and evaluate a million nodes per second, this will allow us to search roughly 200 million nodes per move under standard time control (3 minutes per move).
  - With minimax, only 5-ply, but with alpha-beta,10-ply

- 4-ply lookahead is a hopeless chess player!
  - 4-ply ≈ human novice
  - 8-ply ≈ typical PC, human master
  - 12-ply ≈ Deep Blue, Kasparov

Deterministic Games in Practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.

- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

- Othello: human champions refuse to compete against computers, who are too good.

- Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Summary

- Games are fun to work on!
- They illustrate several important points about AI
- Perfection is unattainable → must approximate

In-Class Exercise #5.1

- Prove that: for every game tree, the utility obtained by MAX using minimax decisions against a suboptimal MIN will never be lower than the utility obtained playing against an optimal MIN.

In-Class Exercise 5.2

- Consider the following game:
  - Draw the complete game tree, using the following convention:
  - State: (Sa, Sb) where Sa and Sb denote the token locations
  - Identify the terminal state in a square box and assign it a value
  - Put loop states in double square boxes
  - Since it's not clear how to assign values to loop states, annotate each with a "?"