Beyond Classical Search – Local Search

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Search Algorithms So Far

- Designed to explore search space systematically:
  - keep one or more paths in memory
  - record which have been explored and which have not
  - a path to goal represents the solution

Local Search Algorithms

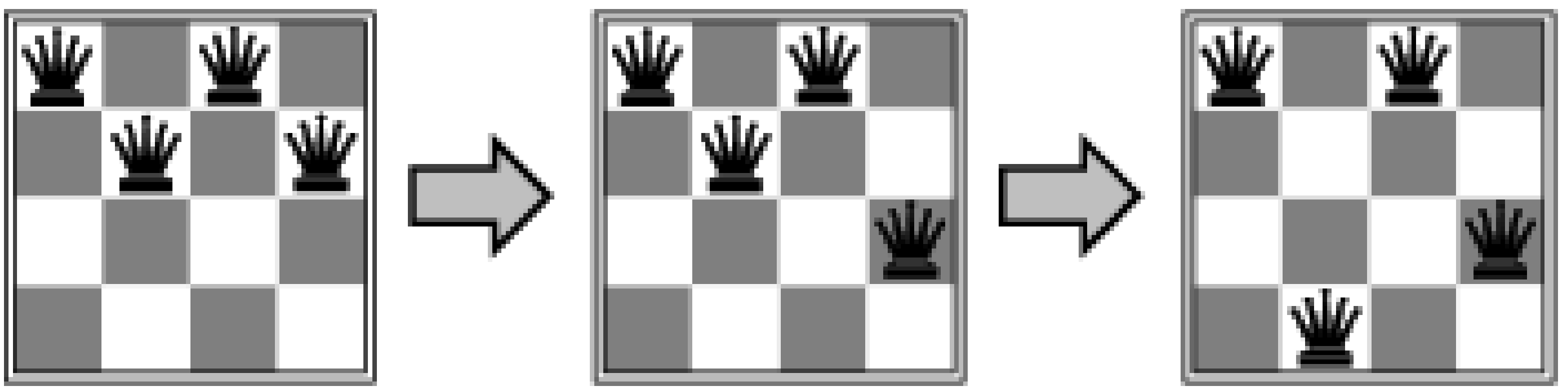
- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms
- keep a single "current" state, try to improve it
  - use very little memory – usually a constant amount
  - find reasonable solutions in large or infinite state spaces for which systematic solutions are unsuitable
  - useful for solving optimization problems, e.g. Darwinian evolution, no “goal test” or “path cost”

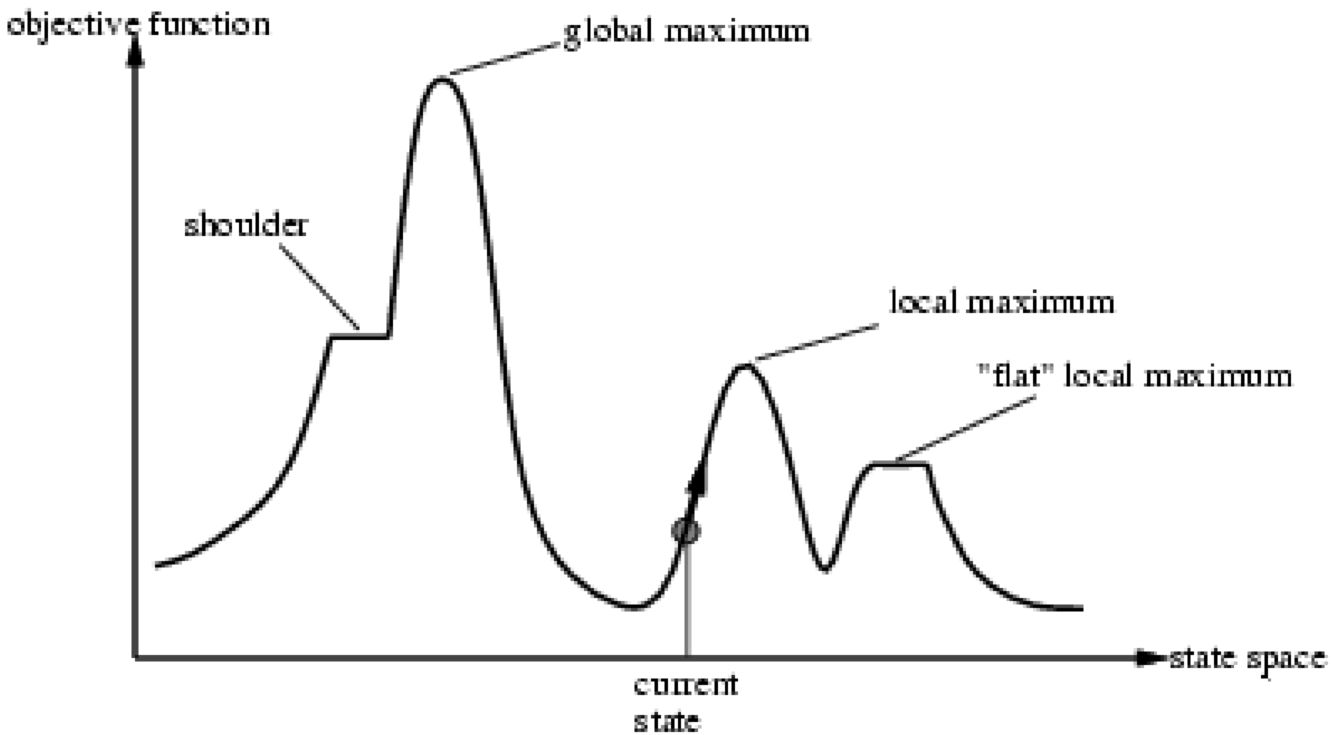
Example: n-Queen Problem

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

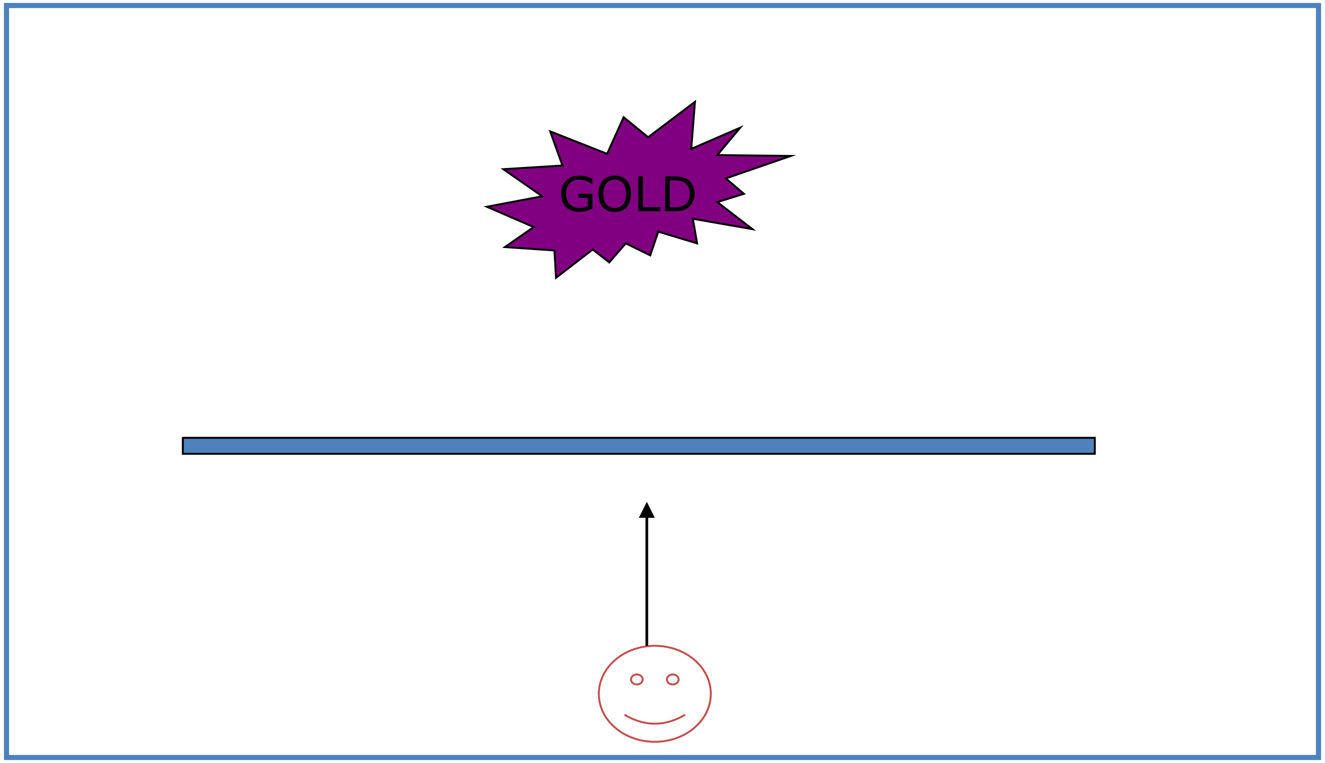


State Space Landscape

- Problem: depending on initial state, can get stuck in local maxima/minima

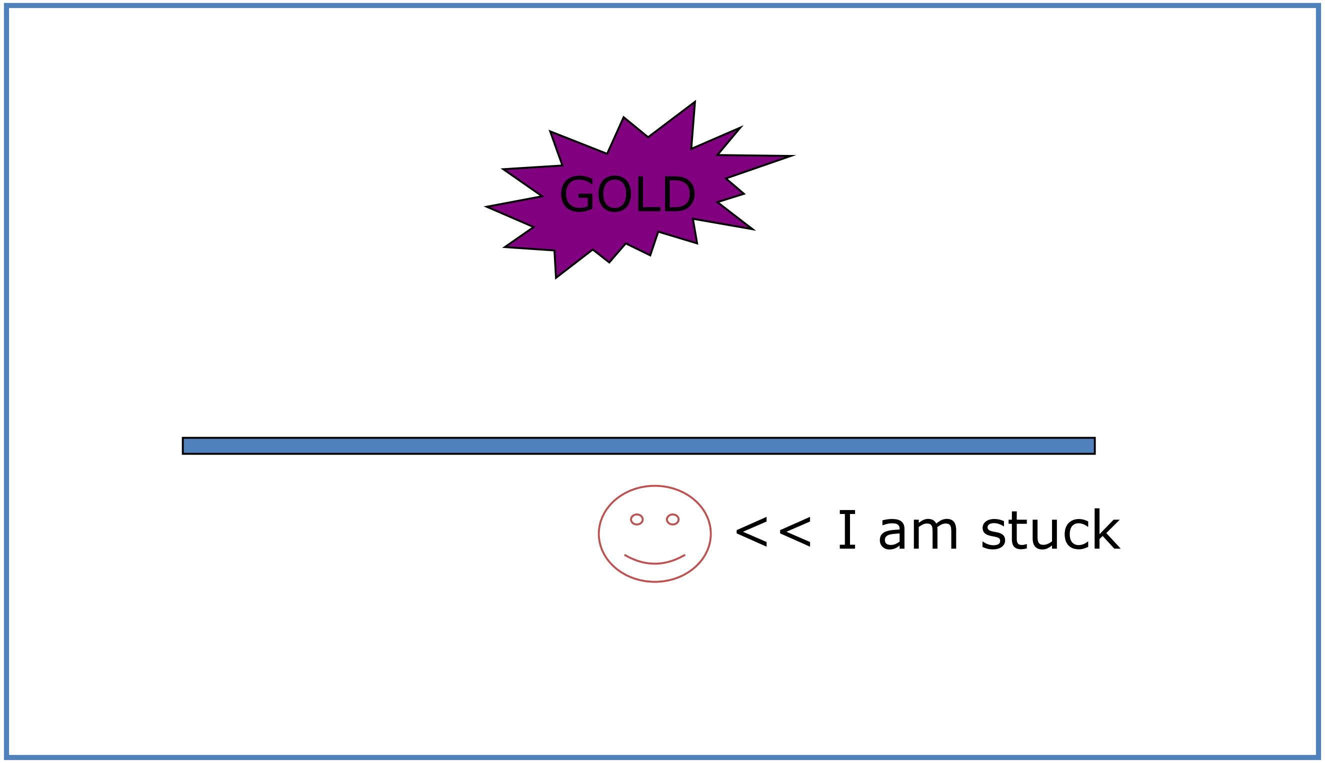


Example: Initial State



Assume the objective function measures the straight-line distance

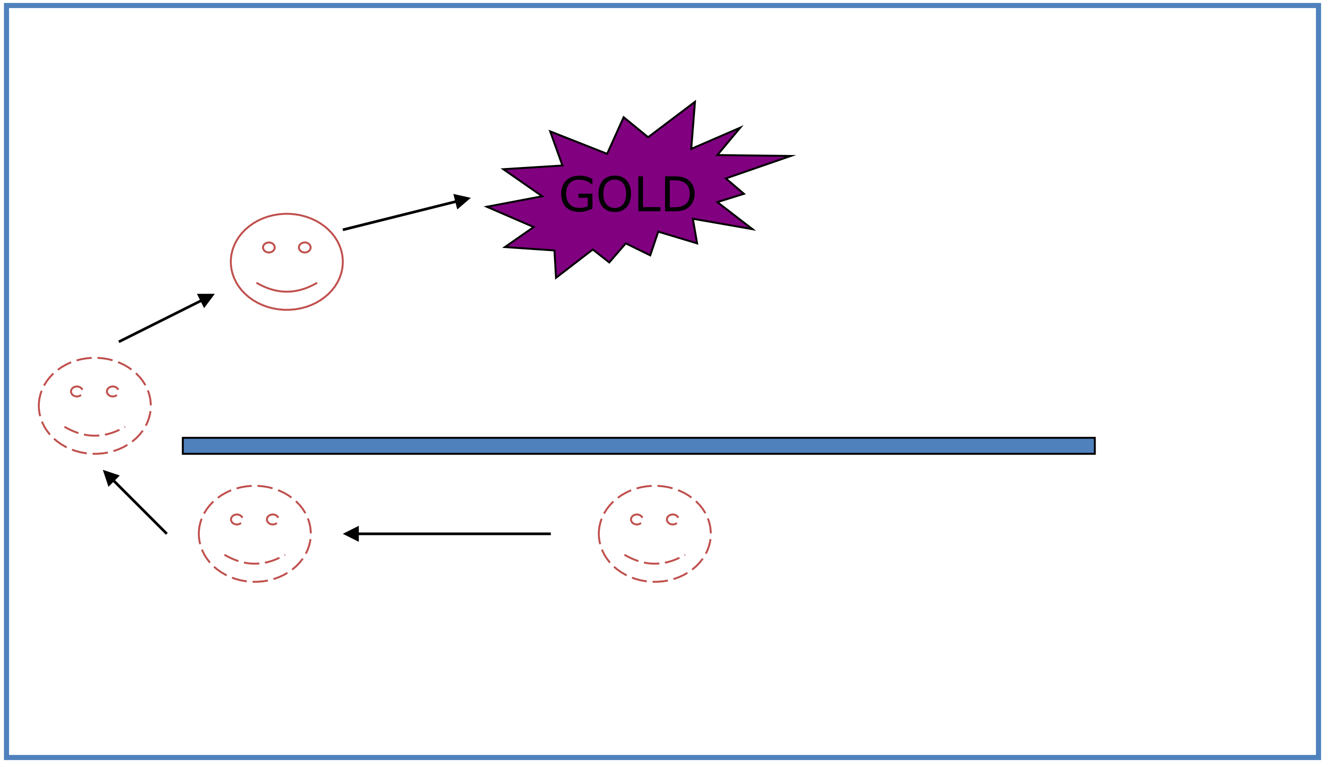
Example: Local Minima



Assume the objective function measures the straight-line distance

Example: A Plausible Solution

Making some “bad” choices is actually not that bad



Assume the objective function measure the straight-line distance

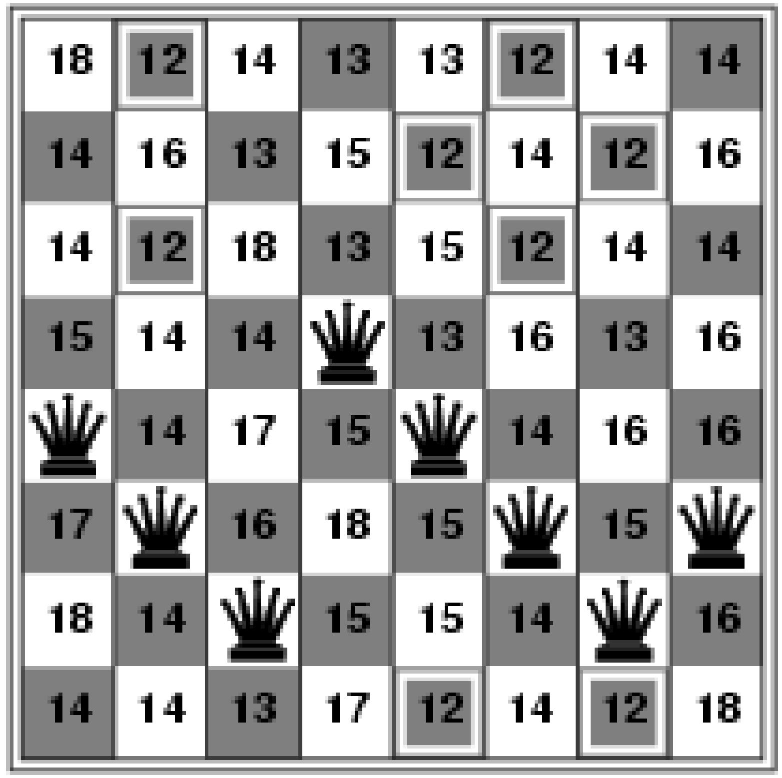
Hill-Climbing Search

- "Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
```

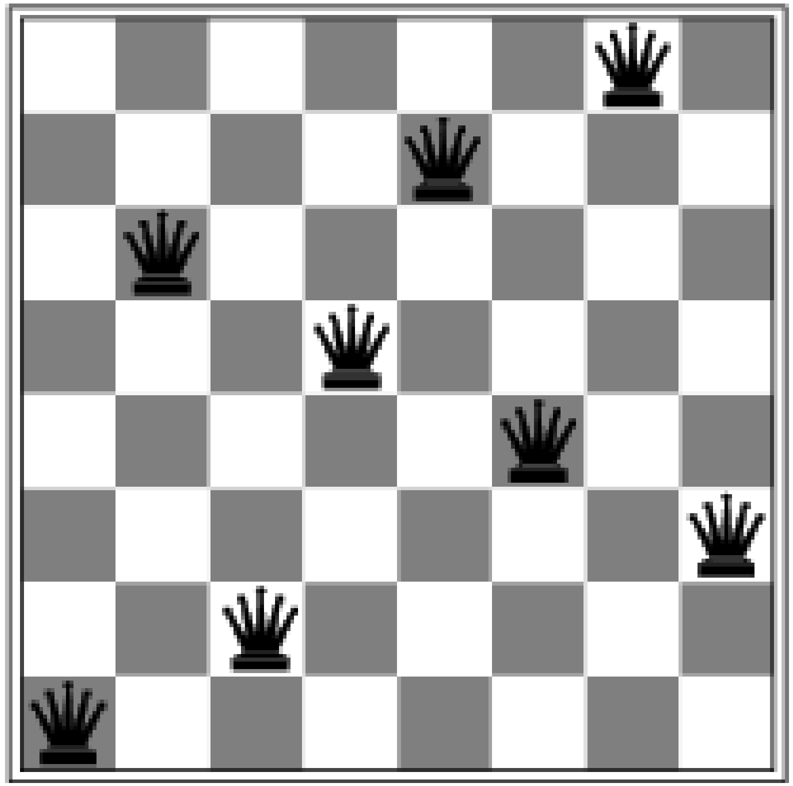
Example: 8-Queen



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly

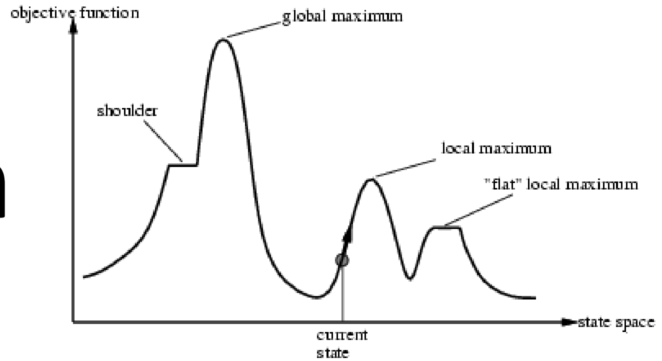
- *h* = 17 for the above state

Example: 8-Queen



- A local minimum with h = 1

More on Hill Clim



- Complete? Optimal?
- Hill climbing is sometimes called greedy local search
- Although greedy algorithms often perform well, hill climbing gets stuck when:
  - Local maxima/minima
  - Ridges
  - Plateau (shoulder or flat local maxima/minima)

- The steepest-ascent hill climbing solves only 14% of the randomly-generated 8-queen problems with an avg. of 4 steps
- Allowing sideways move raises the success rate to 94% with an avg. of 21 steps, and 64 steps for each failure

Variants of Hill Climbing

- Stochastic hill climbing:
  - chooses at random from among uphill moves
  - converges more slowly, but finds better solutions in some landscapes
- First-choice hill climbing:
  - generate successors randomly until one is better than the current
  - good when a state has many successors
- Random-restart hill climbing:
  - conducts a series of hill climbing searches from randomly generated initial states, stops when a goal is found
  - It’s complete with probability approaching 1

More on Random-Restart Hill Climbing

- Assume each hill climbing search has a probability p of success, then the expected number of restarts required is 1/p
- For 8-queen problem, p = 14%, so we need roughly 7 iterations to find a goal
- Expected # of steps = cost_to_success + (1-p)/p * cost_to_failure

- Random-restart hill climbing is very effective for n-queen problem
- 3 million queens can be solved < 1 min

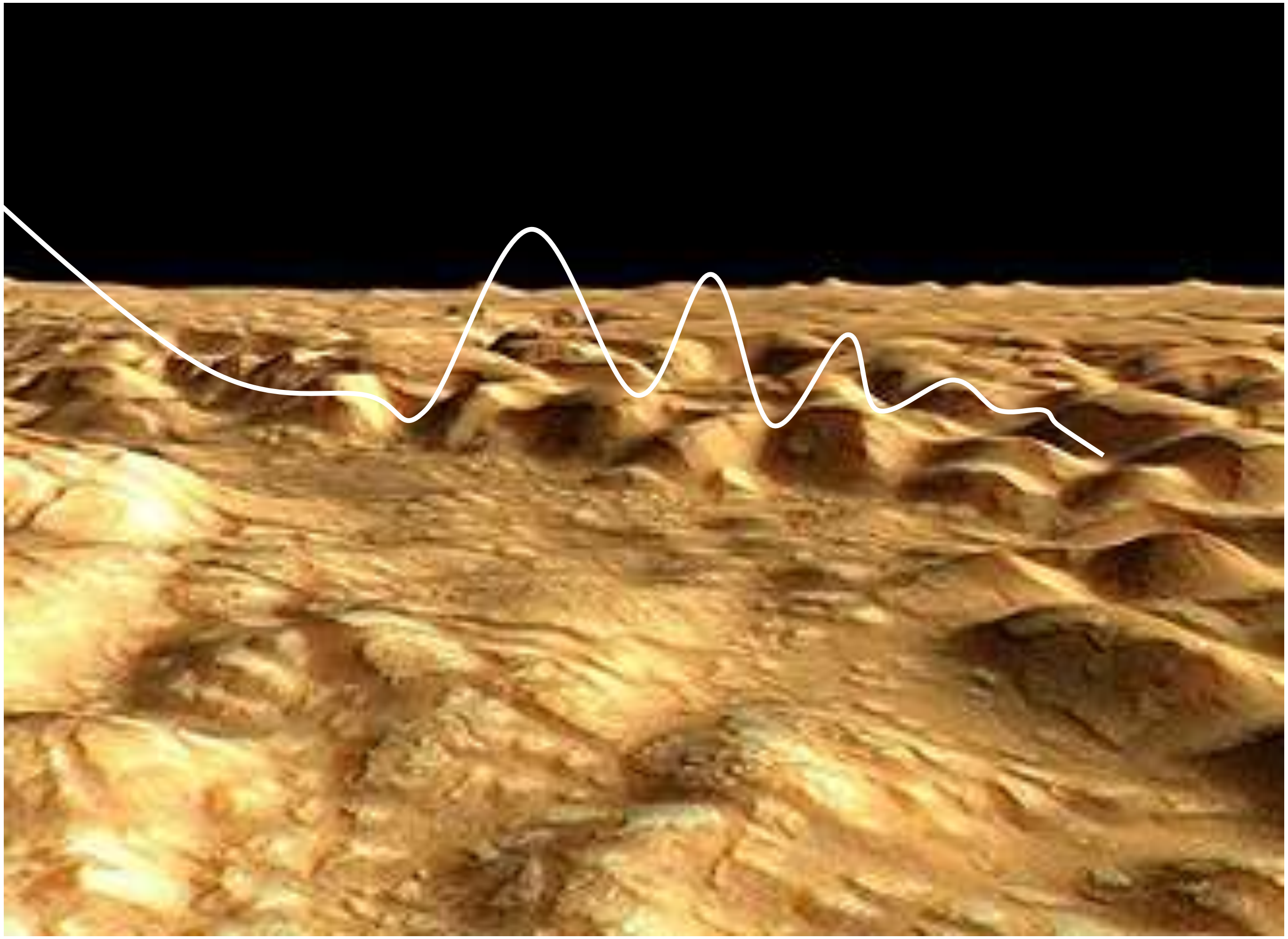
Some Thoughts

- NP-hard problems typically have an exponential number of local maxima/minima to get stuck on
- A hill climbing algorithm that never makes “downhill” (or “uphill”) moves is guaranteed to be incomplete
- A purely random walk – moving to a successor chosen uniformly at random – is complete, but extremely inefficient
- What should we do?
- Simulated annealing

What is Simulated Annealing?

- The process used to harden metals and glass by heating them to a high temperature and then gradually cooling them, thus allowing the material to reach a low-energy crystalline state

Ping-Pong Ball Example



Simulated Annealing

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(ΔE/T)
```

Analysis of Simulated Annealing

- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

- Widely used in VLSI layout, airline scheduling, etc.

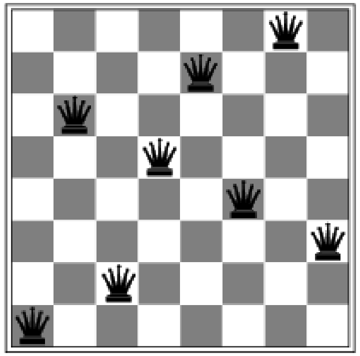
- An example: http://foghorn.cadlab.lafayette.edu/fp/fpIntro.html

Local Beam Search

- Idea:
  - Keep track of *k* states rather than just one
  - Start with *k* randomly generated states
  - At each iteration, all the successors of all *k* states are generated
  - If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat
- Is it the same as running k random-restart searches?
- Useful information is passed among the *k* parallel search threads
- Stochastic beam search: similar to natural selection, offspring of a organism populate the next generation according to its fitness

Genetic Algorithms

- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)



- A state is represented as a string over a finite alphabet (often a string of 0s and 1s or digits)

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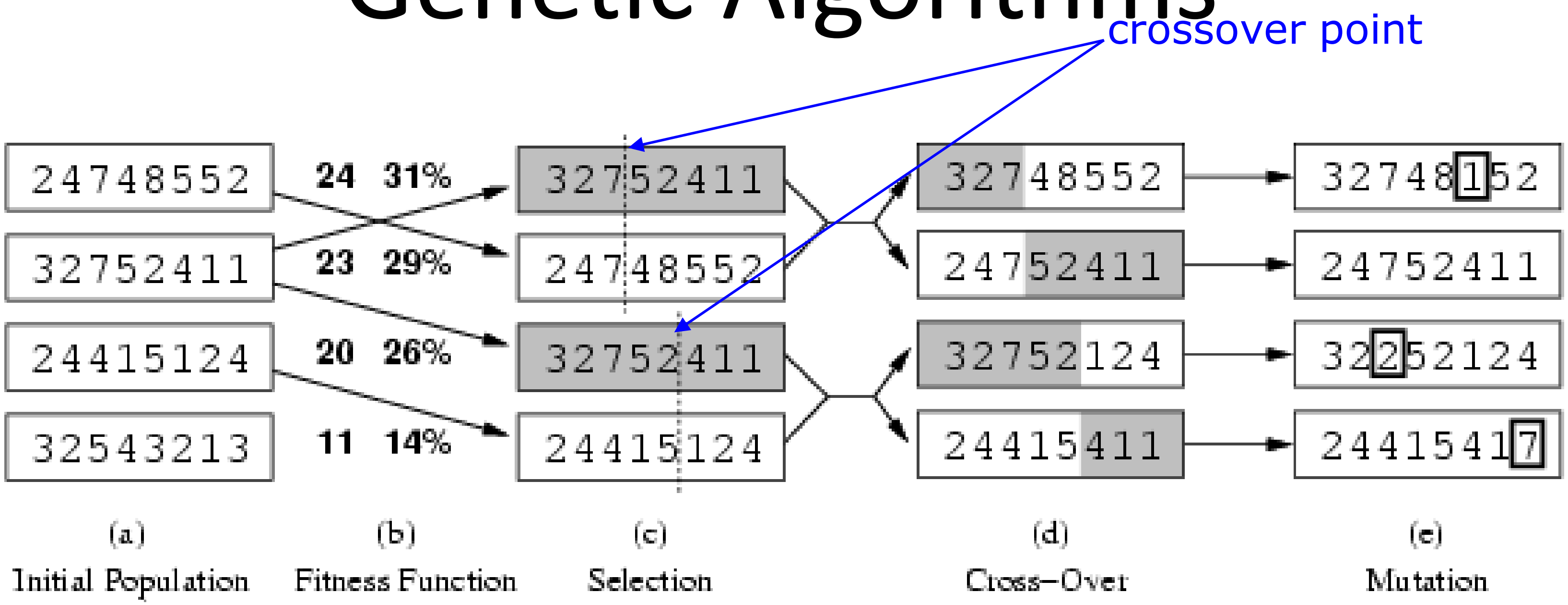
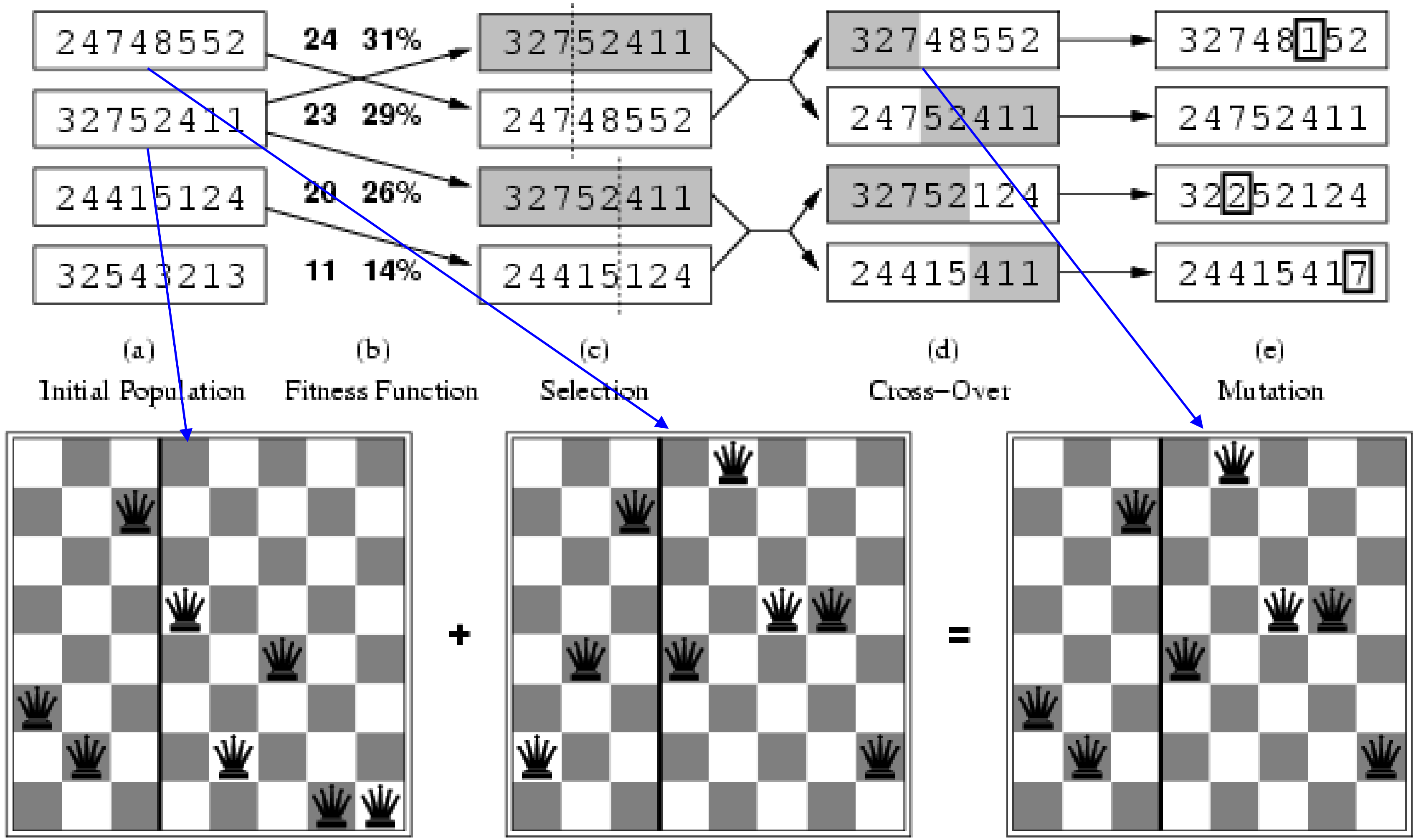
- Evaluation function (fitness function). Higher values for better states
- Produce the next generation of states by selection, crossover, and mutation

Genetic Algorithms



- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Example: 8-Queen



Example: TSA

- http://www.obitko.com/tutorials/genetic-algorithms/

More on Genetic Algorithms

- Genetic algorithms combine an uphill tendency with random exploration and exchange of information among parallel search threads
- Advantages come from “crossover”, which raise the level of granularity

A Genetic Algorithm

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     loop for i from 1 to SIZE(population) do
        x ← RANDOM-SELECTION(population, FITNESS-FN)
        y ← RANDOM-SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

In-Class Exercise #4.1

- Give the name of the algorithm that results from each of the following special cases:
  - Local beam search with $k = 1$
  - Local beam search with one initial state and no limit on the number of states retained
  - Simulated annealing with $T = 0$ at all times (and omitting the termination test)
  - Genetic algorithm with population size $N = 1$

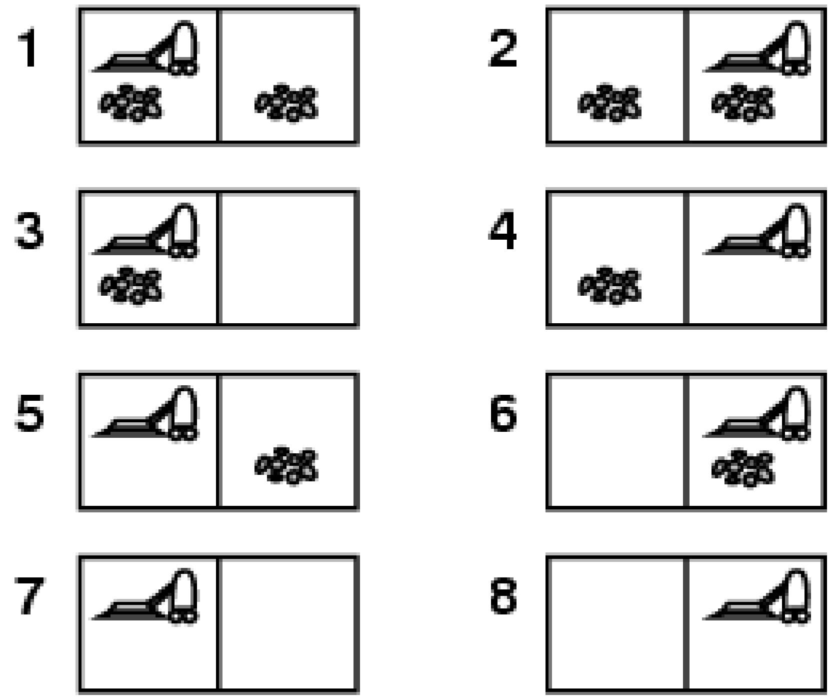
Searching With Partial Information

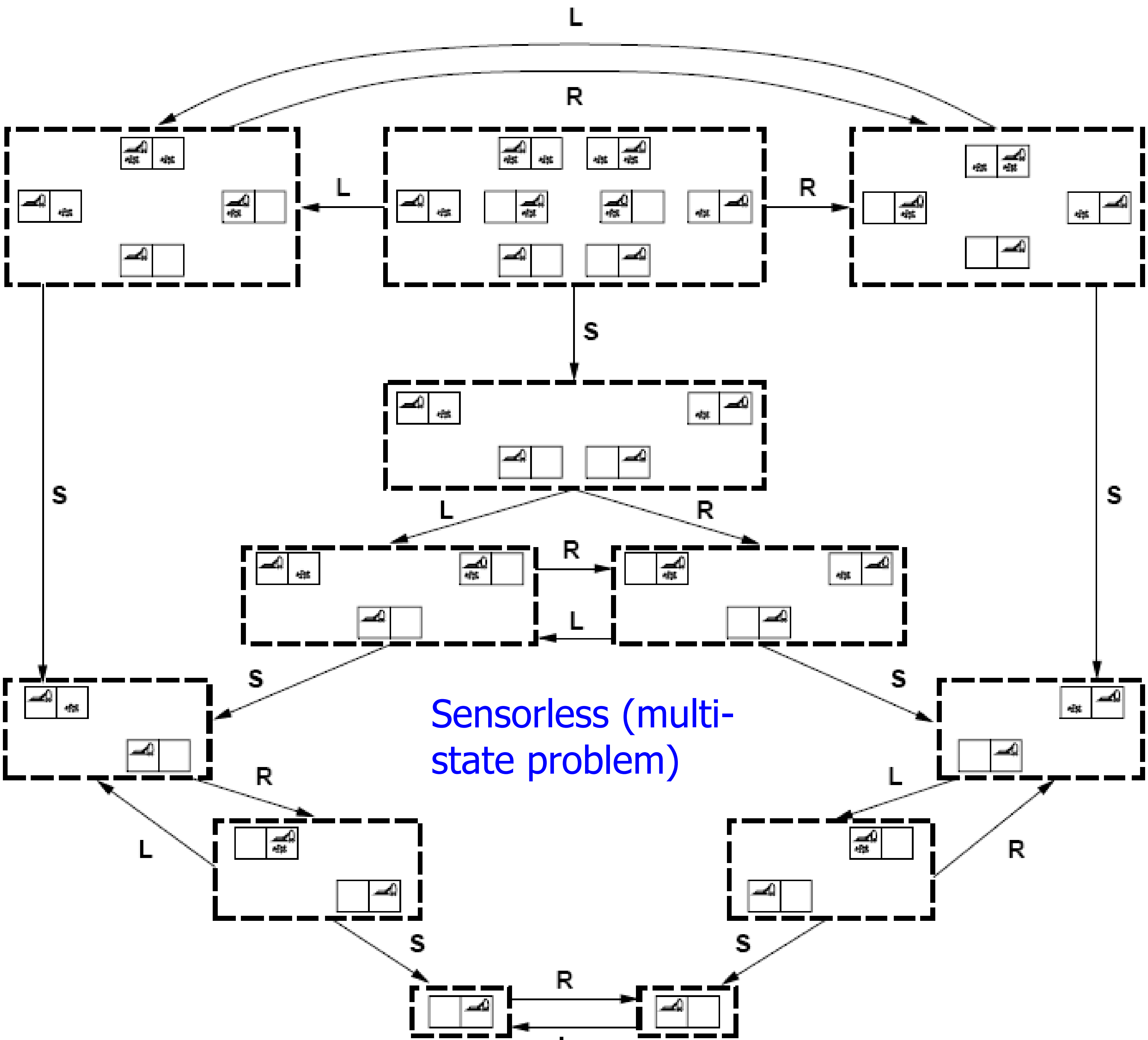
- We have covered: Deterministic, fully observable → single-state problem
  - agent knows exactly which state it will be in
  - solution is a sequence
- Deterministic, non-observable → multi-state problem
  - Also called sensorless problems (conformant problems)
  - agent may have no idea where it is
  - solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave search, execution
  - solution is a tree or policy
- Unknown state space → exploration problem (“online”)
  - states and actions of the environment are unknown

Example: Vacuum World

- Single-state, start in #5. Solution?
  - [Right, Suck]

- Multi-state, start in #[1, 2, ..., 8]. Solution?
  - [Right, Suck, Left, Suck]





Sensorless (multi-state problem)

Contingency Problem

- Contingency, start in #5 & 7.
  - *Nondeterministic*: suck may dirty a clean carpet
  - *local sensing*: dirt, location only at current location
  - [Solution?]
  - Percept: [Left, Clean] → [Right, **if** dirty **then** Suck]