HW5 Solution

Q1

Let H be the hypothesis that 'behind door 1 has a car', and E be the evidence that behind door 3 has nothing.

$P(H)$: prior probability that door 1 has car is $\frac{1}{3}$

$P(E/H)$: probability that door 3 has nothing, given that door 1 has car.

$P(E)$: sum of intersect $E \cap H$ and $E \cap H_c$, $P(H_c) = 1 - P(H)$

$$P(H/E) = \frac{P(E/H)\,P(H)}{P(E)} = \frac{P(E/H)\,P(H)}{P(E|H)P(H) + P(E|H_c)P(H_c)}$$

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + 1 \times \frac{2}{3}} = \frac{1}{3}$$

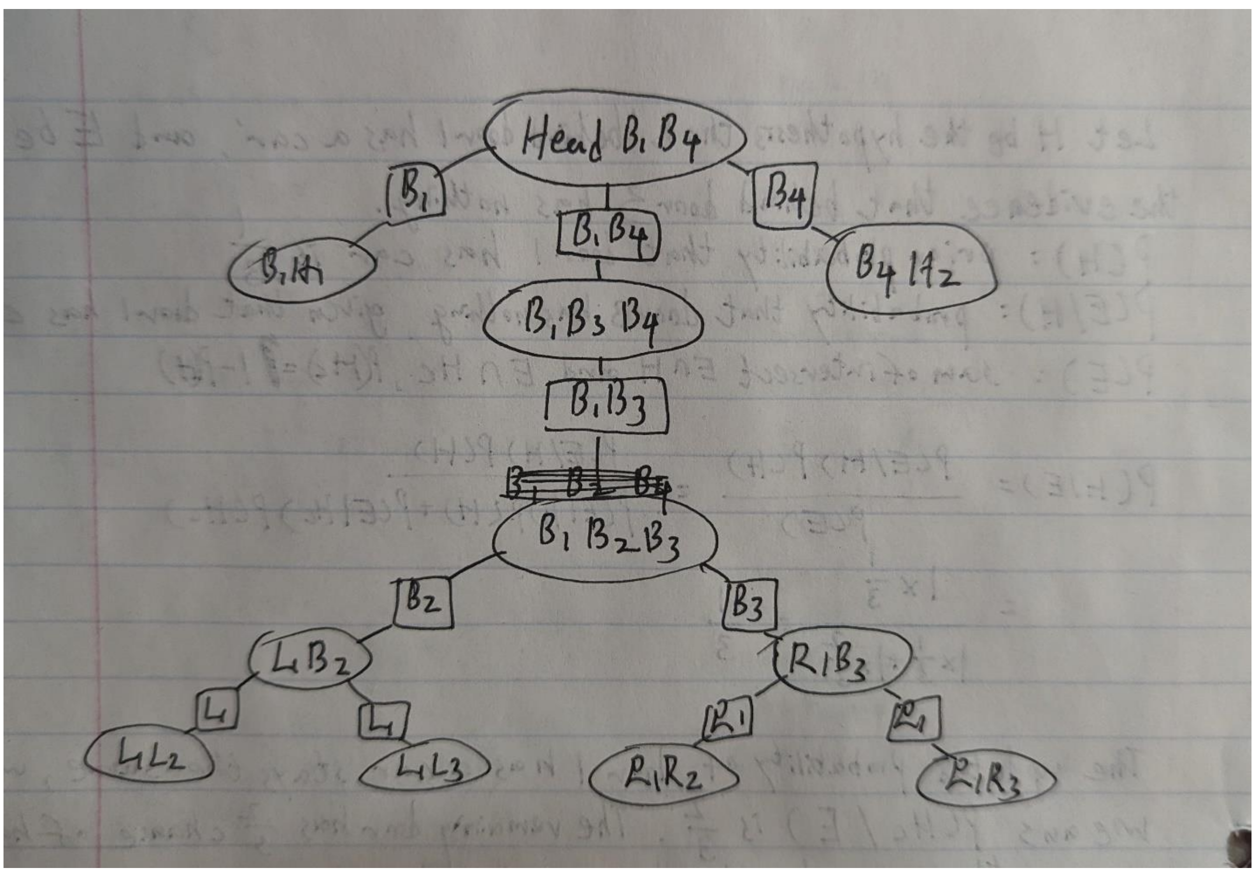
The updated probability of door 1 has a car stays the same, which means $P(H_c / E)$ is $\frac{2}{3}$. The remaining door has $\frac{2}{3}$ chance of having a car, thus we should change the door.

Q2

$P(X_1, X_2 \ldots X_5) = P(X_1)P(X_2|X_1)\ P(X_3)\ P(X_4|X_1, X_3)\ P(X_5|X_2, X_4)$

1. False
2. False
3. True
4. False
5. True
6. False
7. True
8. True
9. False
10. False

Q3



Q4

Build junction tree:

$$(X_1X_2) - \boxed{X_2} - (X_2X_3) - \boxed{X_3} - (X_3X_4) - \boxed{X_4} - (X_4X_5)$$

Result:

$\psi(x_1, x_2) =$

	$x_2=0$	$x_2=1$
$x_1=0$	0.0405	0.4451
$x_1=1$	0.3237	0.1908

$\psi(x_4, x_5) =$

	$x_5=0$	$x_5=1$
$x_4=0$	0.5690	0.1897
$x_4=1$	0.0603	0.1819

$\psi(x_2, x_3) =$

	$x_3=0$	$x_3=1$
$x_2=0$	0.2601	0.1041
$x_2=1$	0.0578	0.5780

$\psi(x_3, x_4) =$

	$x_4=0$	$x_4=1$
$x_3=0$	0.1192	0.1987
$x_3=1$	0.6395	0.0426

$\phi(x_2) =$

$x_2=0$	$x_2=1$
0.3642	0.6358

$\phi(x_3) =$

$x_3=0$	$x_3=1$
0.3179	0.6821

$\phi(x_4) =$

$x_4=0$	$x_4=1$
0.7587	0.2413

CODE

```
x1x2 = [0.1,0.7;0.8,0.3];
x2x3 = [0.5,0.1;0.1,0.5];
x3x4 = [0.1,0.5;0.5,0.1];
x4x5 = [0.9,0.3;0.1,0.3];

n=5;
psis = cell(n-1,1);
psis{1} = x1x2;
psis{2} = x2x3;
psis{3} = x3x4;
psis{4} = x4x5;


s = cell(n-2,1);
for i=1:n-2
s{i} = [1,1];
end


for t = 1:n-2
    temp = s{t} ;
    s{t} = sum( psis{t},1 );
    psis{t+1} =repmat( (s{t} ./temp)',1,2) .* psis{t+1};
end

for k = n-2:-1:1
    temp = s{k} ;
    s{k} = sum( psis{k+1}' );
    psis{k} = repmat(s{k} ./temp,2,1) .* psis{k};
end

for i=1:n-2
    s{i} = s{i}./sum(s{i});
end

for i=1:n-1
    psis{i} = psis{i} ./ sum(sum(psis{i})) ;
end
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