

Q4

### Derivation

$$\text{Let } h_{\theta}(x_i) = f(x_i; \theta)$$

$$L(\theta) = - \sum_{i=1}^n y_i \log(h_{\theta}(x_i)) + (1-y_i) \log(1-h_{\theta}(x_i))$$

$$\textcircled{1} \frac{\partial}{\partial \theta_j} (y \log(h_{\theta}(x)) + (1-y) \log(1-h_{\theta}(x))) = \left( y \frac{1}{h_{\theta}(x)} - (1-y) \frac{1}{1-h_{\theta}(x)} \right) \frac{\partial}{\partial \theta_j} (h_{\theta}(x))$$

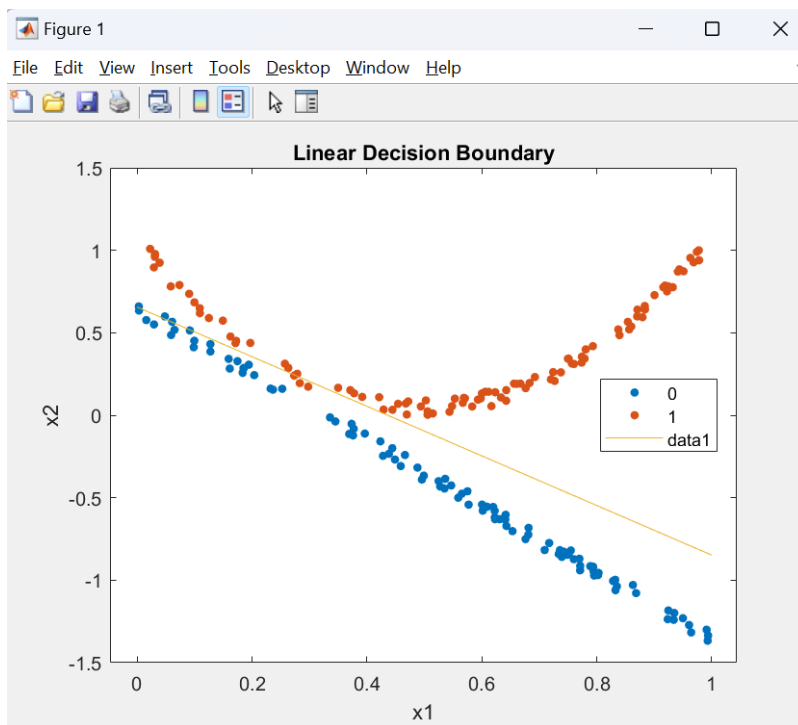
$$\frac{\partial}{\partial \theta_j} (h_{\theta}(x))$$

$$\textcircled{2} \frac{\partial}{\partial \theta_j} (h_{\theta}(x)) = h_{\theta}(x)(1-h_{\theta}(x)) x^j$$

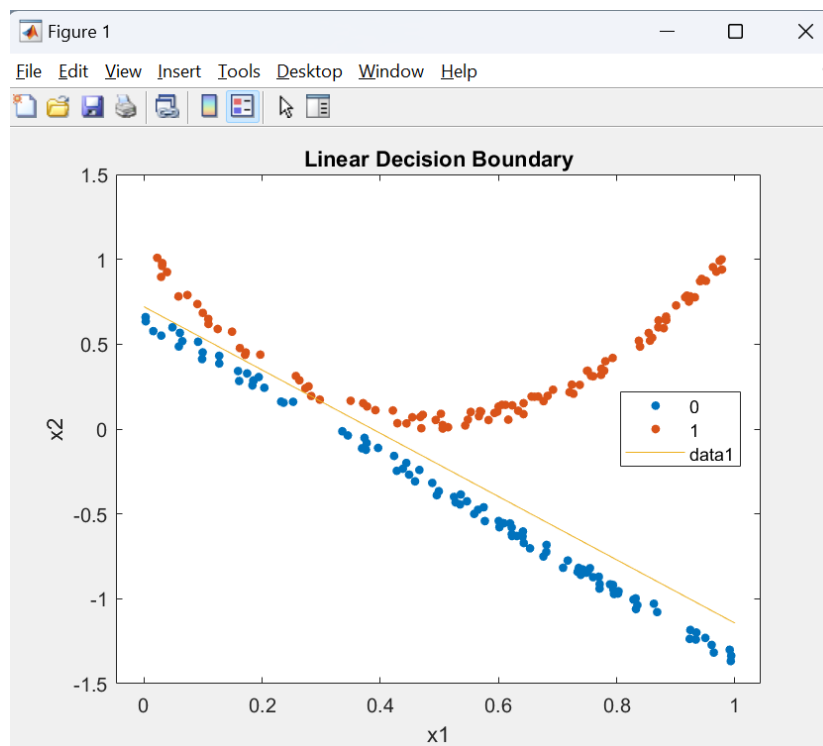
$$\textcircled{1} \textcircled{2} \Rightarrow \frac{y(1-h_{\theta}(x)) - (1-y)h_{\theta}(x)}{h_{\theta}(x)(1-h_{\theta}(x))} h_{\theta}(x)(1-h_{\theta}(x)) x^j$$
$$= (y - h_{\theta}(x)) x^j$$

$$\frac{\partial L(\theta)}{\partial \theta_j} = \sum_{i=1}^n (h_{\theta}(x_i) - y_i) x_i^{(j)}$$

**$e = 0.01$  and  $\eta = 0.01$ , total 796 iterations, errors 6, empirical risk 1.2565**



**$e = 0.01$  and  $\eta = 0.1$ , total 2231 iterations, errors 0, empirical risk 2.3638**



**$\epsilon = 0.001$  and  $\eta = 0.01$ , total 22480 iterations, errors 0, empirical risk 2.3637**

