Gradient Computation

ss2571

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This document contains the gradient formula for the linear and nonlinear lasso problem.

1 Gradient computation

The normalized forward solution reads:

$$Est(\boldsymbol{Q}) = \hat{\Phi}(\mu)(\boldsymbol{Q}) = \sum_{i=1}^{N} \frac{I_i}{G(\boldsymbol{P_i})} \phi_{\boldsymbol{P_i}}(\boldsymbol{Q}),\ \boldsymbol{Q} \in \partial\Omega. \tag{1}$$

Define the source parameter vector as follows:

$$\boldsymbol{V} = \{I_1, I_2, \cdots, I_N, \boldsymbol{P}_1, \boldsymbol{P}_2, \cdots, \boldsymbol{P}_N\}, \tag{2}$$

$$I_{tot} = \{I_1, I_2, \cdots, I_N\} \tag{3}$$

For the linear lasso problem, we compute the pseudo gradient of the term:

$$Ojb = \frac{1}{2}||Est(\boldsymbol{Q}) - \phi^d(\boldsymbol{Q})||_2^2 + \lambda||I_{tot}||_1, \tag{4}$$

$$= \int_{\partial\Omega} [Est(\boldsymbol{Q}) - \phi^d(\boldsymbol{Q})]^2 d\boldsymbol{Q} + \lambda \sum_i |I_i|. \tag{5}$$

For convenience, we introduce the following two terms:

$$Descrepency(\boldsymbol{Q}) := \frac{1}{2}||Est(\boldsymbol{Q}) - \phi^d(\boldsymbol{Q})||_2^2 = \frac{1}{2}\int_{\partial\Omega} [Est(\boldsymbol{Q}) - \phi^d(\boldsymbol{Q})]^2 d\boldsymbol{Q}, \tag{6}$$

$$Regularization := \lambda||I_{tot}||_1. \tag{7}$$

1.1 Linear FBS for the lasso problem

For the FBS solver, only the gradient of the descrepency term is required, and locations of the source are given. Therefore we only compute:

$$Grad_i = \frac{\partial Descrepency(\boldsymbol{Q})}{\partial I_i} = \int_{\partial\Omega} (Est(\boldsymbol{Q}) - \phi^d(\boldsymbol{Q})) \frac{\partial Est(\boldsymbol{Q})}{\partial I_i} d\boldsymbol{Q}, \tag{8}$$

$$= \int_{\partial\Omega} (Est(\boldsymbol{Q}) - \phi^d(\boldsymbol{Q})) \frac{1}{G(\boldsymbol{P_i})} \phi_{\boldsymbol{P}_i}(\boldsymbol{Q}) d\boldsymbol{Q} \tag{9}$$