Optimization

Name: Shafayat Alam

Instructor: Foluso Ladiende

TA: Taiwo Alare and Xueting Deng

Date: 4/25/2025

Table of Contents

Introduction

Effective thermal management is crucial for maintaining the system performance of many industrial and engineering applications. A multi-state air compression system requires such cooling to improve efficiency and prevent damage/failure between the stages of the event when compressed air inside it heat up significantly. The system under consideration utilizes a precooler, refrigeration unit, and a cooling tower to revise temperature from 95℃ to 10℃. All three of theses systems described by Fig. 1 work together to transfer heat and form the compressed air to the water, and then eventually to the atmosphere. The capital investment for such a system can be significant and the initial investestment cost is a primary engineering concern.
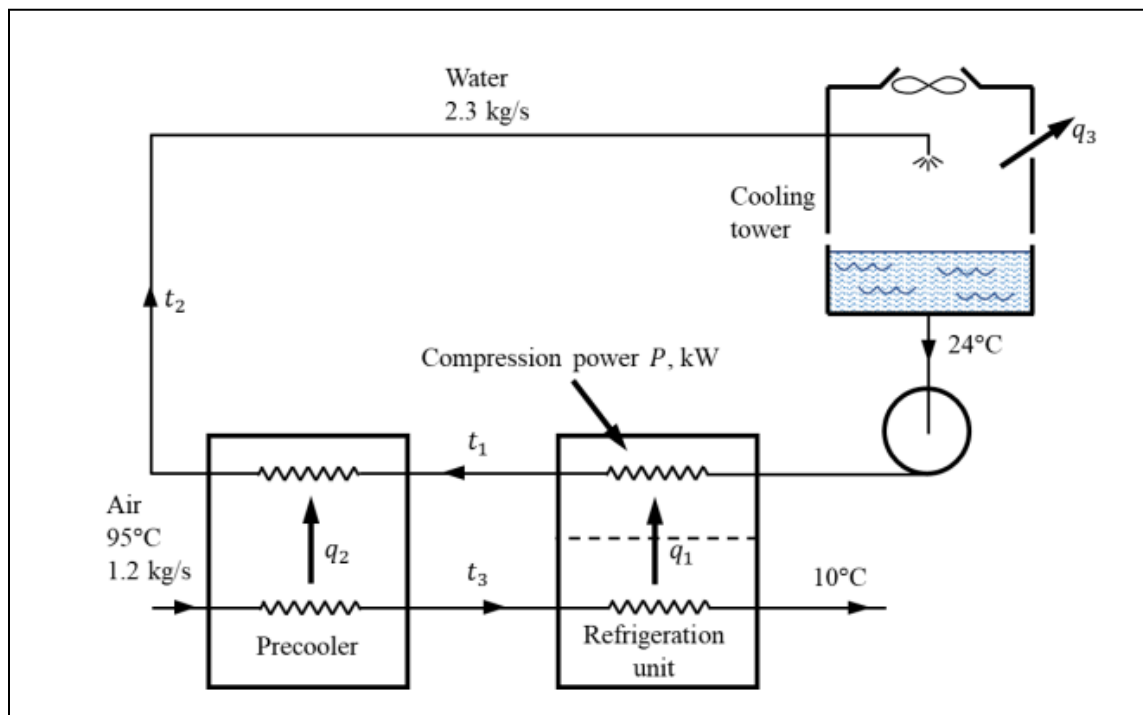

FIG 1. Schematic of the Air-cooling system

Source: MEC 320 Programming Assignment 2 Instructions

Objective

The objective of this report is to translate a formulated mathematical model with constraints considering the system's cost functions and energy balances, and creating an optimization model to determine minimum cost values. To achieve such a task, a numerical method must be implemented in MATLAB to solve for the design variables that minimize the total cost described by the objective function and its constraints shown on Fig. 2.

The constraint equations above can be transformed into the following set:

$$q_1 = (1.2)(1.0)(t_3 - 10) \qquad (1)$$

$$q_2 = (1.2)(1.0)(95 - t_3) \qquad (2)$$

$$P = 0.25q_1 \qquad (3)$$

$$x_1 = 48q_1 \qquad (4)$$

$$x_2 = \frac{50q_2}{t_3 - t_1} \qquad (5)$$

$$x_3 = 25q_3 \qquad (6)$$

$$q_1 + P = (2.3)(4.19)(t_1 - 24) \qquad (7)$$

$$(1.2)(1.0)(95 - t_3) = (2.3)(4.19)(t_2 - t_1) \qquad (8)$$

$$(2.3)(4.19)(t_2 - 24) = q_3 \qquad (9)$$

The optimization problem can then be summarized as follows:

Minimize $\quad z = a_1 x_1 + a_2 x_2 + a_3 x_3$

subject to $\quad a_4 x_1 x_2 + a_5 x_2 + a_6 x_1 = a_7$

$\qquad\qquad a_8 x_3 + a_9 x_1 = a_{10}$

$a_1 = a_2 = a_3 = 1$

$a_4 = 0.0146 \quad a_5 = -14 \quad a_6 = 1,040 \quad a_7 = 5092$

$a_8 = 7.68 \quad a_9 = -1 \quad a_{10} = 19585.253$

$Z = X_1 + X_2 + X_3 \rightarrow Z = X_1 + \dfrac{5092 - 1.040 x_1}{0.0146 x_1 - 14} + \dfrac{19585.253 + X_1}{7.68}$

$0.0146 x_1 x_2 - 14 x_2 + 1.040 x_1 = 5092 \rightarrow X_2 = \dfrac{5092 - 1.040 x_1}{0.0146 x_1 - 14}$

$7.68 x_3 - x_1 = 19585.253 \rightarrow X_3 = \dfrac{19585.253 + x_1}{7.68}$

FIG 2. Describes the optimization model

2

Mathematical/Numerical Approach

FIG 3. Describes the derivation of the constraint equations

Utilizing the set of 9 constraint equations, two constraint equations were derived to be

$$0.0146x_1x_2 - 14x_2 + 1.040x_1 = 5092 \tag{1}$$

and

$$7.68x_3 - 1x_2 = 19585.253 \tag{2}$$

Therefore, the objective function was derived to be

$$z = x_1 + \frac{5092 - 1.040x_1}{0.046x_1 - 14} + \frac{19585.253 + x_1}{7.68} \tag{3}$$

Having used direct substitution of constraints into the objective function to derive the final form of objective function Eq. (3), allowed for the use of steepest descent method (gradient method) to minimize the function. Other methods such as solving a system of equations using Newton-Raphson that are formulated by taking the partial derivatives of all variables of the lagrangian were considered. However, solutions of the system of equations of the lagrangian using Newton-Raphson does not promise a minimization result.

There is likelihood of resulting in minimization using the lagrangian multiplier method which can be verified by utilizing the Hessian, it leaves room for the likelihood of there also to result in maximization. The use of lagrange multipliers in combination with Newton-Raphson yield correct extrema points, however they do not promize a minimization and does not iteratively get closer to determining an acceptable minimum extrema. Specific to the objective of this analysis, the lagrangian in combination with Newton-Raphson proposes an inefficient and inaccurate numerical approach. On the other hand, the capability of utilizing direct substitution allows for the use of the steepest gradient method that proposes an efficient and accurate numerical approach as the steepest descent gradient method guarantees to iteratively get closer to a minimum extrema.

Algorithm (Pseudocode)

1. Decide on an initial guess for x_1, the step size, the convergence tolerance, the maximum number of iterations.

2. Repeat the following steps until convergence is achieved or the maximum number of iterations

   a. Compute the current objective function

   b. Approximate the gradient using finite difference

   c. Update x_1 by moving opposite the direction of gradient

   d. Check for convergence

3. Output the final values of x_1, x_2, x_3 and z.

MATLAB code

```
1. clear; clc;
2.
3. x1 = 5000; %initial guess of x1
4. alpha = 0.01;
5. tol = 1e-5;
6. max_iter = 1000;
7. x1_history = x1;
8. z_history = compute_z(x1);
9.
10.   %steepest method iteration
```

```matlab
11.   for iter = 1:max_iter
12.       grad = compute_dzdx1(x1);
13.       x1_new = x1 - alpha * grad;
14.       x1_history(end+1) = x1_new;
15.       z_history(end+1) = compute_z(x1_new);
16.       if abs(x1_new - x1) < tol
17.           break;
18.       end
19.       x1 = x1_new;
20.   end
21.
22.   %Output of results
23.   fprintf('x1: %.4f\n', x1);
24.   x2 = (5092 - 1.040 * x1) / (0.0146 * x1 - 14);
25.   x3 = (x1 + 19585.253) / 7.68;
26.   fprintf('x2: %.4f\n', x2);
27.   fprintf('x3: %.4f\n', x3);
28.   fprintf('z: %.4f\n', compute_z(x1));
29.
30.   %z_value computation
31.   function val = compute_z(x1)
32.       x2 = (5092 - 1.040 * x1) / (0.0146 * x1 - 14);
33.       x3 = (x1 + 19585.253) / 7.68;
34.       val = x1 + x2 + x3;
```

```
35.   end

36.

37.   %steepest method computation

38.   function grad = compute_dzdx1(x1)

39.       h = 1e-5;

40.       grad = (compute_z(x1 + h) - compute_z(x1 - h)) / (2 *
   h);

41.   end
```

Results

Initial Guess of x_1 = 5000

Maximum number of Iteration = 1000

Tolerance = 1 x 10^{-5}

Table 1. Illustrates the Minimized Resulting Values

Variable	Numerical Value
z	8186.99
x_1	4988.87
x_2	-1.64
x_3	3199.76

Discussion

The objective function was successfully minimized to the total cost shown by $z = 8186.99$, with corresponding design variables $x_1 = 4988.87$, $x_2 =- 1.64$, and $x_3 = 3199.76$. However, this is one minimization solution as the numerical approach is a local minimization method not a global minimization method. The advantage of this approach is flexibility of deciding on the magnitude of a cost of the Refrigeration unit design variable x_1. This approach gives the ability to control minimization based on an decided x_1 value and also the other two design variables. This approach also proposes an efficient methodology of optimization as with each iteration it moves towards an extrema of choice compared to methods that might require a higher computational cost or random search methodology. Nevertheless, with a decided Refrigeration unit cost of $5000, the methodology implemented of choice minimized the cost of the other two components of the system resulting in a total cost of $8186.99 for the whole system.

Conclusion

It was challenging to optimize an objective function that has a nonlinear constraint, however utilizing gradient method made the process feasible. The ability to make the objective function a function of one of the design variables by the use of direct substitution gave the ability to use steepest descent method. In addition to producing acceptable minimization results, the methodology provides flexibility and control over minimization because of the ability of choosing an initial guess of one of the design variables. However, an improvement to propose is

8

better implementation of MATLAB code that prompts the user with which design variable to decide on an initial guess on and set gradient method parameters such as step size, convergence tolerance, and maximum number of iterations. Nevertheless, the choice of methodology and its implementation in this report produce feasible optimization results to achieve the objective of minimization of the cost of capital investment.