SAMPLE WORK

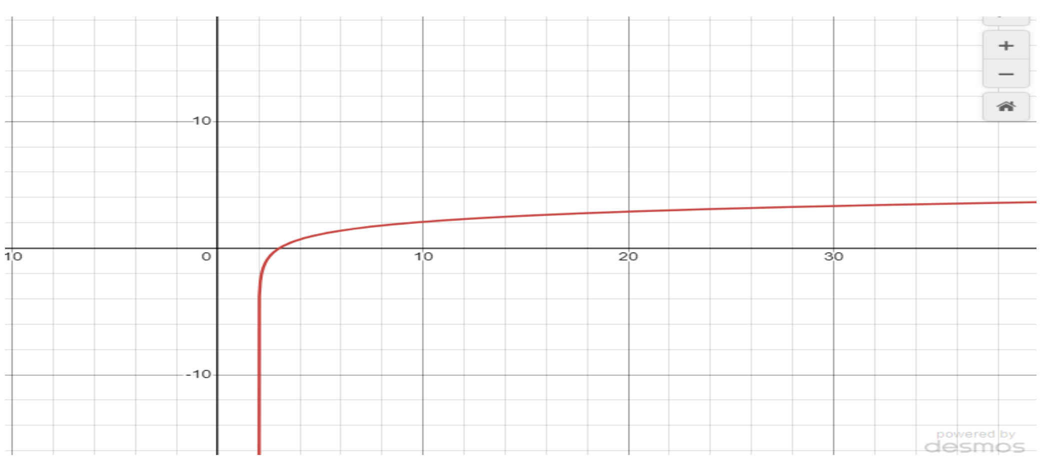
Calculus

QUESTION: 01

$$y = \lim_{x \to \infty} \ln(x - 2)$$

Solution:

As x approaches to infinity ‘y’ approaches to infinity, we cannot apply L’Hospital rule because it is not in the form of $\frac{0}{0}$ and $\frac{\infty}{\infty}$. Furthermore we can see it graphically.



Thus,

$$\lim_{x\to\infty} \ln(x-2) = \infty$$

QUESTION: 02

$$y = \lim_{x\to\infty} \frac{(2x^3 + 9000)}{x^4}$$

Solution:

$$y = \lim_{x\to\infty} \frac{(2x^3 + 9000)}{x^4}$$

$$y = \lim_{x\to\infty} \left(\frac{2x^3}{x^4} + \frac{9000}{x^4}\right)$$

$$y = \lim_{x\to\infty} \left(\frac{2}{x} + \frac{9000}{x^4}\right)$$

Using limit property, we have,

$$\therefore \lim_{x\to a}\big(f(x) + g(x)\big) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$y = \lim_{x\to\infty} \left(\frac{2}{x}\right) + \lim_{x\to\infty} \frac{9000}{x^4}$$

Using limit property, we have,

$$\therefore \lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$$

$$y = 2\lim_{x\to\infty}\left(\frac{1}{x}\right) + 9000\lim_{x\to\infty}\left(\frac{1}{x^4}\right)$$

$$\therefore \lim_{x\to\infty}\frac{1}{x^n} = 0,\ \ where\ \ n > 0$$

$$y = 2(0) + 9000(0)^4$$

$$y = 0$$

Hence,

$$\lim_{x\to\infty}\frac{(2x^3 + 9000)}{x^4} = 0$$

Statistics

QUESTION: 03

The amount of cold drink was measured (in liters, L) in 20 randomly selected 1.5L bottles of a company as given below. Construct a discrete frequency table of the measured amount of cold drink. Also compute relative and percentage frequencies.

1.48	**1.51**	**1.50**	**1.49**	**1.49**	**1.49**	**1.51**
1.48	**1.49**	**1.52**	**1.51**	**1.49**	**1.51**	**1.50**
1.50	**1.50**	**1.51**	**1.49**	**1.49**	**1.50**	

Solution:

The data are quantitative, and in particular continuous. The distinct numbers: 1.48, 1.49, 1.50, 1.51 and 1.52 (in ascending order) are five classes with Tally marks and frequencies, we get the discrete frequency table the relative and percentage frequencies are also computed in 4^{th} and 5^{th} columns.

Class limits "amounts of cold drink (L)"	Tally Marks	Frequency numbers of bottles	Relative frequency $\frac{f}{n}$	Percentage frequency (%) $\frac{f}{n} \times 100$
1.48	$\vert\vert$	**2**	**0.1**	**10**
1.49	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	**7**	**0.35**	**35**
1.50	$\cancel{\vert\vert\vert\vert}$	**5**	**0.25**	**25**
1.51	$\cancel{\vert\vert\vert\vert}$	**5**	**0.25**	**25**
1.52	$\vert$	**1**	**0.05**	**5**
Total	…	**n=20**	**1**	**100**

NOTE: $\cancel{\vert\vert\vert\vert}$=5

Linear Algebra

Basis Representation:

Defined transformation T

$$T: R^3 \to R^2, \qquad T(x, y, z) = (x + y, 2z - x)$$

Solution:

$$V = R^3$$

$$\beta = \{v_1 = (1, 0, 0), v_2 = (0, 1, 0), v_3 = (0, 0, 1)\}$$

$$W = R^2$$

$$\gamma = \{w_1 = (1, 0), w_2 = (0, 1)\}$$

$$T(1, 0, 0) = (1 + 0, 2(0) - 1) = (1, -1)$$

$$T(0, 1, 0) = (0 + 1, 2(0) - 0) = (1, 0)$$

$$T(0, 0, 1) = (0 + 0, 2(1) - 0) = (0, 2)$$

Compute inner product, $< Tv_{i,} w_j >$

$$Tv_1 = (1, -1)$$

$$< Tv_{1,} w_1 >= (1, -1).(1, 0) = 1 - 0 = 1$$

$$< Tv_{1,} w_2 >= (1, -1).(0, 1) = 0 - 1 = 1$$

$$Tv_2 = (1,0)$$

$$< Tv_{2,}\, w_1 >= (1,0).(1,0) = 1 + 0 = 1$$

$$< Tv_{2,}\, w_2 >= (1,0).(0,1) = 0 + 0 = 0$$

$$Tv_3 = (0,2)$$

$$< Tv_{3,}\, w_1 >= (0,2).(1,0) = 0 + 0 = 0$$

$$< Tv_{3,}\, w_2 >= (0,2).(0,1) = 0 + 2 = 2$$

$$[T] = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

Interpretation:

- The first row represents the projection of Tv_1, Tv_2, Tv_3 onto w_1.
- The second row represents the projection of Tv_1, Tv_2, Tv_3 onto w_2.