Foundations of Calculus & Numerical Applications

Exploring the mathematical foundations that power modern computation and analysis

Shafia Ameeruddin

Limits & Derivatives

Limit Definition

$$\lim_{x \to a} f(x) = L$$

Function approaches a specific value as input approaches a point

Derivative Concept

Measures instantaneous rate of change at any point

Example: $f(x) = x^2 \to f'(x) = 2x$

Integrals & Fundamental Theorem

Integration Types

- **Indefinite:** $\int f(x)dx = F(x) + C$
- **Definite:** $\int_a^b f(x)dx = F(b) - F(a)$

Fundamental Theorem: Derivative and integral are inverse processes - they undo each other

Visual Concept

Integral represents the area under a curve between two points

Multivariate Gradients

1

Gradient Formula

For $f(x, y)$:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

2

Example

$f(x, y) = x^2 + y^2$

$\nabla f = (2x, 2y)$

3

Application

Powers gradient descent optimization in machine learning algorithms

Numerical Methods with NumPy

```
import numpy as np

f = lambda x: np.sin(x)

# Derivative at pi/4
x, dx = np.pi/4, 1e-5
derivative = (f(x+dx)-f(x-dx))/(2*dx)

# Integral from 0 to pi
xs = np.linspace(0, np.pi, 1000)
integral = np.trapz(np.sin(xs), xs)

print("f'(pi/4):", derivative)  # ≈ 0.707 (√2/2)
print("∫0^π sin(x) dx:", integral)  # ≈ 2
```

Practical computation bridges theory with real-world applications

Worked Problem: Derivative of e^x

> Find the derivative of $f(x) = e^x$ at $x = 0$ using limit definition

01

Apply Definition

$$f'(0) = \lim_{h \to 0} \frac{e^{0+h} - e^0}{h}$$

02

Simplify

$$= \lim_{h \to 0} \frac{e^h - 1}{h}$$

03

Evaluate Limit

Result: 1

Key Takeaways

Limits

Foundation of all calculus concepts

Derivatives

Measure instantaneous rate of change

Integrals

Calculate accumulation and area

Gradients

Enable multivariable optimization

NumPy Methods

Bridge theory to practical computation