

CSC510 - Set and Set Operations: Example Question

Scenario: A Neighborhood Community Survey

Imagine a small, friendly neighborhood where a community survey was conducted to understand the demographics of households with children. For simplicity, let's say the survey focused on children whose ages are positive integers and are 15 years old or younger.

So, our *Universal Set (U)* represents all possible ages of children surveyed in this neighborhood:

$U = \{ \text{child's age} \in \mathbb{Z}^+ \mid \text{child's age} \leq 15 \} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$.

Based on the survey data, we've categorized children into the following groups:

* *Set A: "Game Night Enthusiasts"* - Children whose ages are even numbers, making them ideal for playing board games that often have even-numbered age recommendations.

* *Set B: "Young Innovators"* - Children whose ages are prime numbers, often associated with early stages of significant cognitive development and curiosity.

* *Set C: "Team Players"* - Children whose ages are multiples of 3, making them suitable for forming small teams or groups (e.g., for sports, reading clubs) where groups of three are common.

Questions:

Representing Sets:

1. a) List the members of Set A.

a) Set A: Even integers in U.

$A = \{2, 4, 6, 8, 10, 12, 14\}$

2. b) List the members of Set B.

b) Set B: Prime numbers in U. (A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.)

$B = \{2, 3, 5, 7, 11, 13\}$

3. c) List the members of Set C.

c) Set C: Multiples of 3 in U.

$$C = \{3, 6, 9, 12, 15\}$$

Cardinality:

4. a) What is the cardinality of Set A, denoted as $|A|$?

a) $|A| = 7$

5. b) What is the cardinality of Set B, denoted as $|B|$?

b) $|B| = 6$

6. c) What is the cardinality of Set C, denoted as $|C|$?

c) $|C| = 5$

Set Operations (Intersection):

7. a) Find $A \cap B$. What does this set represent in words in the context of our scenario?

a) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$$A \cap B = \{2\}$$

In words: This set represents children who are both "Game Night Enthusiasts" (even-aged) and "Young Innovators" (prime-aged). The only child age fitting this description is 2 years old.

8. b) Find $A \cap C$. What does this set represent in words in the context of our scenario?

b) $A \cap C = \{x \mid x \in A \text{ and } x \in C\}$

$$A \cap C = \{6, 12\}$$

In words: This set represents children who are both "Game Night Enthusiasts" (even-aged) and "Team Players" (multiples of 3). These are children aged 6 and 12, suitable for both board games and group activities.

9. c) Find $B \cap C$. What does this set represent in words in the context of our scenario?

c) $B \cap C = \{x \mid x \in B \text{ and } x \in C\}$

$$B \cap C = \{3\}$$

In words: This set represents children who are both "Young Innovators" (prime-aged) and "Team Players" (multiples of 3). The only child fitting this is 3 years old.

Set Operations (Union):

10. a) Find $A \cup B$.

a) $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14\}$$

11. b) Find $B \cup C$.

b) $B \cup C = \{x \mid x \in B \text{ or } x \in C\}$

$$B \cup C = \{2, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$$

Complement:

12. a) Find the complement of Set A, denoted as A^c or \bar{A} . List its members and describe what this set represents in the scenario.

a) $A^c = U - A = \{x \mid x \in U \text{ and } x \notin A\}$

$$A^c = \{1, 3, 5, 7, 9, 11, 13, 15\}$$

In words: This set represents children whose ages are not even numbers; i.e., they are children whose ages are odd. In the scenario, these are the children who are not "Game Night Enthusiasts".

13. b) What is $|A^c|$?

b) $|A^c| = 8$

Disjoint Sets:

14. Are Set A and Set B disjoint? Explain your answer based on the definition of disjoint sets. (In the scenario: Does any child's age fall into both "Game Night Enthusiasts" and "Young Innovators"?)

Answer: No, Set A and Set B are not disjoint.

***Explanation:** Disjoint sets are sets that have no elements in common, meaning their intersection is the empty set ($A \cap B = \emptyset$). In this case, $A \cap B = \{2\}$, which is not an empty set. Since they share the element '2', they are not disjoint. In the scenario, a 2-year-old child is both a "Game Night Enthusiast" and a "Young Innovator."

***Subset and Proper Subset:**

Consider a new set $D = \{2, 4, 6\}$, representing children who are eligible for the "Junior Robotics Club".

15. a) Is D a subset of A? Explain why or why not in the context of the scenario.

***Answer:** Yes, D is a subset of A.

***Explanation:** A set D is a subset of A ($D \subseteq A$) if every element of D is also an element of A. Here, all members of D ($\{2, 4, 6\}$) are also members of A ($\{2, 4, 6, 8, 10, 12, 14\}$). In the scenario, all children eligible for the "Junior Robotics Club" (ages 2, 4, 6) are also "Game Night Enthusiasts".

16. b) Is D a proper subset of A? Explain why or why not in the context of the scenario.

***Answer:** Yes, D is a proper subset of A.

***Explanation:** A set D is a proper subset of A ($D \subset A$) if D is a subset of A, and D is not equal to A (meaning A contains at least one element not in D). Here, D is a subset of A, and A contains elements like 8, 10, 12, 14 that are not in D. So, D is strictly "smaller" than A. In the scenario, while all "Junior Robotics Club" members are "Game Night Enthusiasts", there are "Game Night Enthusiasts" who are not in the "Junior Robotics Club" (e.g., an 8-year-old).

***Power Set:**

17. a) Let $S = \{1, 2\}$, representing the possible ages for a "Toddler Playgroup". List all the members of the power set of S, denoted as $P(S)$.

$P(S) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

18. b) What is the cardinality of $P(S)$? Verify this with the formula given in the lecture.

***Answer:** The cardinality of $P(S)$ is 4.

***Verification:** The formula for the cardinality of a power set is $2^{|S|}$.

Here, $|S| = 2$.

So, $|P(S)| = 2^2 = 4$. This matches the number of members listed.