

Problem-2

$$\underline{1} \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 1 \end{pmatrix} \quad u_3 = \begin{pmatrix} 0 \\ 16 \\ 1 \\ 1 \end{pmatrix}$$

$$\underline{2} \quad P_1 = u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad |P_1| = 2$$

$$q_1 = \frac{P_1}{|P_1|} = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$P_2 = u_2 - (u_2^T q_1) q_1$$

$$(u_2^T q_1) = \frac{1}{2}(0 \ 4 \ -1 \ 1) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 2$$

$$P_2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 1 \end{pmatrix} - 2 \cdot \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$|P_2| = \sqrt{1+9+4} = \sqrt{14}$$

$$q_2 = \frac{P_2}{|P_2|} = \frac{1}{\sqrt{14}} \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$P_3 = u_3 - \left((u_3^T q_1) q_1 + (u_3^T q_2) q_2 \right)$$

$$(u_3^T q_1) = \frac{1}{2} (0 \ 16 \ 1 \ 1) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{18}{2} = 9$$

$$u_3^T q_2 = \frac{1}{\sqrt{14}} (0 \ 16 \ 1 \ 1) \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix} = \frac{46}{\sqrt{14}}$$

$$(u_3^T q_1) q_1 = \frac{9}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad (u_3^T q_2) q_2 = \frac{46}{14} \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 \\ 16 \\ 1 \\ 1 \end{pmatrix} - \frac{9}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{23}{7} \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} -17 \\ 23 \\ 43 \\ -49 \end{pmatrix}$$

~~q₃~~ $q_3 = \frac{P_3}{|P_3|} \quad |P_3| = 5.08$

$$q_3 = \begin{pmatrix} -0.239 \\ 0.323 \\ 0.604 \\ -0.688 \end{pmatrix}$$

$$\stackrel{3}{R} = \begin{pmatrix} u_1^T q_1 & u_2^T q_1 & u_3^T q_1 \\ 0 & u_2^T q_2 & u_3^T q_2 \\ 0 & 0 & u_3^T q_3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 9 \\ 0 & 3.74 & 12.3 \\ 0 & 0 & 5.08 \end{pmatrix}$$

$$\stackrel{4}{R} x = \begin{pmatrix} 2 & 2 & 9 \\ 0 & 3.74 & 12.3 \\ 0 & 0 & 5.08 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$= \begin{pmatrix} 2a_0 + 2a_1 + 9a_2 \\ 3.74a_1 + 12.3a_2 \\ 5.08a_2 \end{pmatrix}$$

$$Q^T b = \begin{pmatrix} 2 \\ -3.475 \\ -0.843 \end{pmatrix} \quad \therefore$$

$$\stackrel{5}{\begin{aligned} a_0 &= 2.129 \\ a_1 &= -0.3833 \\ a_2 &= -0.1659 \end{aligned}} \quad P_2(x) = 2.129 - 0.3833x - 0.1659x^2$$