Phy 112 Assignment

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Section : 03

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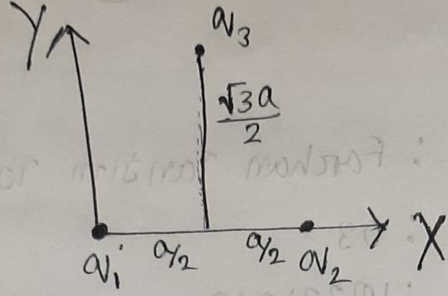
1.11

a) Drawing the three charges in X-Y plane,



$q_1 = -19e = -19 \times -1.6\times10^{-19}C = 3.04\times10^{-18}C$

$q_2 = 49e = 49\times -1.6\times10^{-19}C = -7.84\times10^{-18}C$

$q_3 = 15e = 15\times -1.6\times10^{-19}C = -2.4\times10^{-18}C$

$a = 38\times10^{-9}m$; $k = 8.987\times10^{9}\,Nm^2C^{-2}$

We know,

$$\vec{F} = k\frac{q_1 q_2}{r^2}\times\hat{r} = k\frac{q_1 q_2}{r^3}\times\vec{r}$$

Net ~~charge~~ force on q_1,

$$\vec{F}_{1} = \vec{F}_{21} + \vec{F}_{31} = k\frac{q_1 q_2}{r_{21}^3}\times\vec{r}_{21} + k\frac{q_1 q_3}{r_{31}^3}\times\vec{r}_{31} \quad \text{(i)}$$

For. $\vec{F}_{21}$,

$$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_2 = -38\times10^{-9}\hat{i}\ m$$

$$r_{21} = \sqrt{(-38\times10^{-9})^2} = 3.8\times10^{-8}\ m$$

For $\vec{F}_{31}$,

$$\vec{r}_{31} = \vec{r}_1 - \vec{r}_3 = -\frac{a}{2}\hat{i} - \frac{\sqrt{3}a}{2}\hat{j}\ m$$

$$= -1.9\times10^{-8}\,\hat{i} - 3.29\times10^{-8}\,\hat{j}\ m$$

$$r_{31} = \sqrt{(-1.9\times10^{-8})^2 + (-3.29\times10^{-8})^2}$$

$$= 3.799\times10^{-8}\ m$$

Putting the values in equation (i),

$$\vec{F}_1 = k\,q_1\left(\frac{q_2}{r_{21}^3}\times\vec{r}_{21} + \frac{q_3}{r_{31}^3}\times\vec{r}_{31}\right)$$

$$= 8.987\times10^{9}\times3.04\times10^{-18}\left(\frac{-7.84\times10^{-18}}{(3.8\times10^{-8})^3}\times(-38\times10^{-9}\,\hat{i}) + \frac{-2.4\times10^{-18}}{(3.799\times10^{-8})^3}\times(-1.9\times10^{-8}\,\hat{i} - 3.29\times10^{-8}\,\hat{j})\right)$$

$$= 2.73\times10^{-8}\left(5.43\times10^{-3}\,\hat{i} + 8.317\times10^{-4}\,\hat{i} + 1.44\times10^{-3}\right)$$

$$= 1.7\times10^{-10}\,\hat{i} + 3.93\times10^{-11}\,\hat{j}\ N$$

(Ans)

(b) from a,

$q_1 = 3.04 \times 10^{-18} C$

$q_2 = -7.84 \times 10^{-18} C$

$q_3 = -2.4 \times 10^{-18} C$

$a = 38 \times 10^{-9} m$

$k = 8.987 \times 10^{9} Nm^2C^{-2}$

Net ~~charge~~ force on q_2,

$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{32} = kq_2\left(\frac{q_1}{r_{12}^3} \times \vec{r}_{12} + \frac{q_3}{r_{32}^3} \times \vec{r}_{32}\right) \quad \text{(ii)}$$

For $\vec{F}_{12}$,

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = 38 \times 10^{-9} \hat{i}\ m$

$r_{12} = \sqrt{(38 \times 10^{-9})^2} = 3.8 \times 10^{-8}\ m$

For $\vec{F}_{32}$,

$\vec{r}_{32} = \vec{r}_2 - \vec{r}_3 = 3.8 \times 10^{-8} \hat{i} - 1.9 \times 10^{-8} \hat{i} - 3.29 \times 10^{-8} \hat{j}$

$= 1.9 \times 10^{-8} \hat{i} - 3.29 \times 10^{-8} \hat{j}\ m$

$r_{32} = \sqrt{(1.9 \times 10^{-8})^2 + (-3.29 \times 10^{-8})^2}$

$= 3.799 \times 10^{-8}\ m$

Putting the values in equation (ii),

$$\vec{F_2} = 8.987\times10^{9}\times(-7.84\times10^{-18})\left(\frac{3.04\times10^{-18}}{(3.8\times10^{-8})^3}\times(3.8\times10^{-8}\hat{i}) + \frac{-2.4\times10^{-18}}{(3.799\times10^{-8})^3}\times(1.9\times10^{-8}\hat{i}-3.29\times10^{-8}\hat{j})\right)$$

$$= -7.04\times10^{-8}\left(2.1\times10^{-3}\hat{i} - 8.32\times10^{-4}\hat{i} + 1.44\times10^{-3}\hat{j}\right)$$

$$= -8.93\times10^{-11}\hat{i} - 1.01\times10^{-10}\hat{j}\ N$$

(Ans)

(c) From a,

$q_1 = 3.04\times10^{-18}\ C$

$q_2 = -7.84\times10^{-18} C$

$q_3 = -2.4\times10^{-18} C$

$a = 38\times10^{-9} m$; $K = 8.987\times10^{9}\ Nm^2C^{-2}$

Net force on q_3,

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = Kq_3\left(\frac{q_1}{r_{13}^3}\times\vec{r}_{13} + \frac{q_2}{r_{23}^3}\times\vec{r'}_{23}\right) \quad \text{(iii)}$$

For $\vec{F}_{13}$,

$$\vec{r}_{13} = \vec{r}_3 - \vec{r}_1 = 1.9\times10^{-8}\,\hat{i} + 3.29\times10^{-8}\,\hat{j}\ m$$

$$r_{13} = \sqrt{(1.9\times10^{-8})^2 + (3.29\times10^{-8})^2}$$

$$= 3.799\times10^{-8}\ m$$

For $\vec{F}_{23}$,

$$\vec{r}_{23} = \vec{r}_3 - \vec{r}_2 = 1.9\times10^{-8}\,\hat{i} + 3.29\times10^{-8}\,\hat{j} - 3.8\times10^{-8}\,\hat{i}$$

$$= -1.9\times10^{-8}\,\hat{i} + 3.29\times10^{-8}\,\hat{j}\ m$$

$$r_{23} = \sqrt{(-1.9\times10^{-8})^2 + (3.29\times10^{-8})^2}$$

$$= 3.799\times10^{-8}\ m$$

Putting the values in equation (iii),

$$\vec{F}_3 = 8.987\times10^{9}\times(-2.4\times10^{-18})\left(\frac{3.04\times10^{-18}}{(3.799\times10^{-8})^3}\times(1.9\times10^{-8}\,\hat{i} + 3.29\times10^{-8}\,\hat{j}) + \frac{-7.84\times10^{-18}}{(3.799\times10^{-8})^3}\times(-1.9\times10^{-8}\,\hat{i} + 3.29\times10^{-8}\,\hat{j}\right)$$

$$= -2.16\times10^{-8}\left(1.05\times10^{-3}\,\hat{i} + 1.82\times10^{-3}\,\hat{j} + 2.71\times10^{-3}\,\hat{i} - 4.7\times10^{-3}\,\hat{j}\right)$$

$= -8.12\times10^{-11}\hat{i} + 6.22\times10^{-11}\hat{j}$ N

(Ans)

1.21

(a) $q_1 = q_5 = 49\,\mu C = 49\times10^{-6}$ C

$q_2 = q_4 = 10\,\mu C = 10\times10^{-6}$ C

$q_3 = q_6 = -40\,\mu C = -40\times10^{-6}$ C

$R = 16$ cm $= 0.16$ m ; $K = 8.987\times10^{9}$ Nm^{2}C^{-2}

$$\vec{r}_{1,6} = \vec{r}_6 - \vec{r}_1 = 0\hat{i} + 0\hat{j} - (0\hat{i} + R\hat{j})$$

$$= -R\hat{j} = -0.16\hat{j}\ \text{m}$$

(Ans)

(b) We know,

$$\vec{F} = k\frac{q_1 q_2}{r^3}\times\vec{r}$$

$$\therefore \vec{F}_{1,6} = k\frac{49\times10^{-6}\times -40\times10^{-6}}{\left\{\sqrt{(-0.16)^2}\right\}^3}(-0.16\hat{j})$$

$$= 688.067\hat{j}\ \text{N}$$

(Ans)

(c) $\vec{r}_{3,6} = \vec{r}_6 - \vec{r}_3 = 0\hat{i} + 0\hat{j} - (-R\hat{i} + 0\hat{j})$

$= R\hat{i} = 0.16\hat{i} + 0\hat{j}$ m

(Ans)

(d) $\vec{F}_{3,6} = k \dfrac{q_3 q_6}{(r_{3,6})^3} \times \vec{r}_{3,6}$

$= k \times \dfrac{(-40\times10^{-6})^2}{\left\{\sqrt{(0.16)^2}\right\}^3} \times (0.16\,\hat{i})$

$= 561.6875\,\hat{i}$ N

(e) $\vec{r}_{4,6} = \vec{r}_6 - \vec{r}_4 = 0\hat{i} + 0\hat{j} - \vec{r}_4$

$= -\vec{r}_4$

Now, since the 5 charges q_1 to q_5 are distributed along a semi-circle maintaining equal distance, the angle, θ between q_3 and q_4 is 45°. Therefore, the vector $\vec{r}_{4,6}$ creates an angle ϕ of ~~45°~~ 225° with positive x axis.

we also know,

$|\vec{r}| = 0.16\text{ m} = R$

So, $\vec{r}_4 = R\cos 225^\circ \hat{i} + R\sin 225^\circ \hat{j}$

$= -0.113\hat{i} - 0.113\hat{j}$ m

$\therefore \vec{r}_{4,6} = -\vec{r}_4 = 0.113\hat{i} + 0.113\hat{j}$ m

(Ans)

(f) $\vec{F}_{4,6} = k\dfrac{q_4 q_6}{(r_{4,6})^3} \times \vec{r}_{4,6}$

$= k\dfrac{10\times10^{-6} \times -40\times10^{-6}}{\left(\sqrt{(0.113)^2+(0.113)^2}\right)^3} \times (0.113\hat{i} + 0.113\hat{j})$

$= -99.534\hat{i} - 99.534\hat{j}$ N

(Ans)

(g) $\vec{F}_{Net} = \vec{F}_{1,6} + \vec{F}_{2,6} + \vec{F}_{3,6} + \vec{F}_{4,6} + \vec{F}_{5,6}$ — (i)

For $\vec{F}_{2,6}$,

$\vec{r}_{2,6} = \vec{r}_6 - \vec{r}_2 = -\vec{r}_2$

q_2 creates an angle of $90^\circ + 45^\circ = 135^\circ$ with positive x axis.

So, $\vec{r}_2 = R\cos 135^\circ \hat{i} + R\sin 135^\circ \hat{j}$

$= -0.113\hat{i} + 0.113\hat{j}$ m

$$\therefore \vec{r}_{2,6} = -\vec{r}_2 = 0.113\hat{i} - 0.113\hat{j} \text{ m}$$

$$\therefore \vec{F}_{2,6} = k\frac{q_2 q_6}{(r_{2,6})^3} \times \vec{r}_{2,6}$$

$$= k\frac{10\times10^{-6} \times -40\times10^{-6}}{\left(\sqrt{(0.113)^2 + (-0.113)^2}\right)^3} \times (0.113\hat{i} - 0.113\hat{j})$$

$$= -99.56\hat{i} + 99.56\hat{j} \text{ N}$$

For, $\vec{F}_{5,6}$

$$\vec{r}_{5,6} = \vec{r}_6 - \vec{r}_5 = -\vec{r}_5 = -(0\hat{i} - R\hat{j})$$

$$= 0.16\hat{j} \text{ m}$$

$$\therefore \vec{F}_{5,6} = k\frac{q_5 q_6}{(r_{5,6})^3} \times \vec{r}_{5,6}$$

$$= k\frac{49\times10^{-6} \times -40\times10^{-6}}{\left(\sqrt{(0.16)^2}\right)^3} \times (0.16\hat{j})$$

$$= -688.07\hat{j} \text{ N}$$

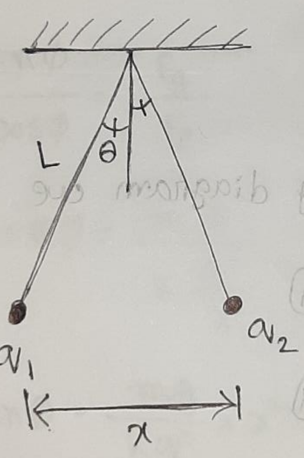
Putting the values in equation (i),

$$\vec{F}_{Net} = \cancel{688.067}\hat{j} - 99.56\hat{i} + \cancel{99.56}\hat{j} + 561.6875\hat{i} - 99.534\hat{i} - \cancel{99.534}\hat{j} - \cancel{688.067}\hat{j}$$

$$= 362.5935\hat{i} + 0\hat{j} \text{ N}$$

(Ans)

1.3 (a)



The given two spheres are of equal mass 'm' and equal charge q.

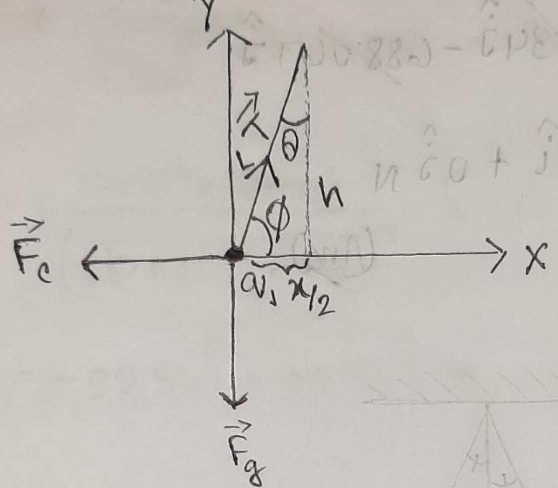
So, mass, $m = m_1 = m_2$

Charge, $q = q_1 = q_2 = 28 nC = 28 \times 10^{-9} C$

Length of the thread, $L = 234 cm = 2.34 m$

horizontal-seperation, $x = 17 cm = 0.17 m$

Now, using q_1 as the origin of our coordinate system we draw the free body diagram.



From this free body diagram we get,

$$\vec{F}_c = -F_c \hat{i} \quad \text{—} (i)$$

$$\vec{F}_g = -F_g \hat{j} \quad \text{—} (ii)$$

$$\vec{T} = T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j} \quad \text{—} (iii)$$

Since. the spheres are in equilibrium state, net force on q_1,

$$\vec{F}_{Net\, q_1} = 0 \quad \text{—} (iv)$$

using equation (i), (ii), (iii), (iv) we get,

$$-F_c\hat{i} - F_g\hat{j} + T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j} = 0$$

$$\Rightarrow -F_c\hat{i} + T\cos\phi\,\hat{i} - F_g\hat{j} + T\,\cancel{\sin\phi}\,\sin\phi\,\hat{j} = 0\hat{i} + 0\hat{j}$$

$$\Rightarrow -F_c + T\cos\phi = 0 \quad ; \quad -F_g + T\sin\phi = 0$$

$$\Rightarrow T\cos\phi = F_c \text{ — (v)} \quad ; \quad T\sin\phi = F_g \text{ — (vi)}$$

Now, (vi) $\div$ (v) $\Rightarrow$

$$\frac{T\sin\phi}{T\cos\phi} = \frac{F_g}{F_c}$$

$$\Rightarrow \tan\phi = \frac{mg}{k\frac{q^2}{x^2}}$$

$$\Rightarrow \tan\phi = \frac{mg}{kq^2} \times x^2$$

$$\Rightarrow \frac{h}{x/2} = \frac{mgx^2}{kq^2}$$

$$\Rightarrow \frac{L}{x/2} = \frac{mgx^2}{kq^2}$$

$$\Rightarrow \frac{K.L.q^2}{x/2 \cdot g \cdot x^2} = m$$

$$\Rightarrow m = \frac{8.987\times10^9 \times 2.34 \times (28\times10^{-9})^2}{0.085 \times 9.8 \times (0.17)^2}$$

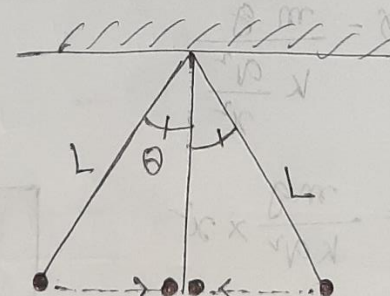
$$\tan\phi = \frac{h}{x/2}$$

$$h = \sqrt{L^2 - (x/2)^2}$$
$$= 2.34\ m$$
$$= L$$
$$\therefore L \gg x$$

$\Rightarrow m = 6'849 \times 10^{-4}$ Kg

(Ans)

(b) If we discharge sphere 2 then, there will be no repulsion force, only gravitational ~~force~~ force and tension of the threads. Due to the gravitational force that acts downward vertically the two spheres will meet at the center point of the previous distance.



When they get in contact with each other, the sphere 1 transfers some of it's charge to sphere 2. The amount of charge transferred is half of it's charge so that there is a uniform distribution of charge. So, both the spheres (will) have the same amount of charge that is $q/2$.

$$\therefore q_1' = q_2' = q' = \frac{q}{2} = \frac{28}{2}\,nC = 14\,nC = 14\times10^{-9}\,C.$$

(Ans)

(c) From a,

$$\frac{L}{x/2} = \frac{mgx^2}{kq^2}$$

Here, $L = 2{\cdot}34\,m$

$m = 6{\cdot}849\times10^{-4}\,kg$

$g = 9{\cdot}8\,ms^{-2}$

$K = 8{\cdot}987\times10^{9}\,Nm^2C^{-2}$

$q = q' = 14\times10^{-9}\,C.$

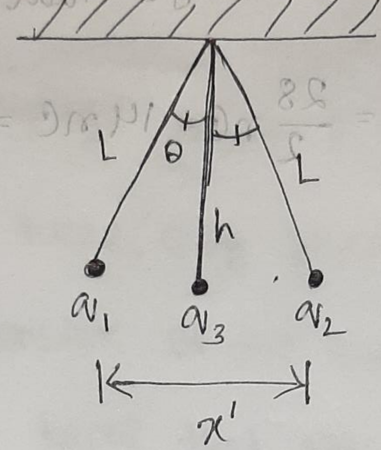
$$\therefore x' = \left(\frac{2LKq^2}{mg}\right)^{1/3}$$

$$= \left\{\frac{2\times2{\cdot}34\times8{\cdot}987\times10^{9}\times(14\times10^{-9})^2}{6{\cdot}849\times10^{-4}\times9{\cdot}8}\right\}^{1/3}$$

$\cancel{= 2{\cdot}307\times10^{-10}}$ $\cancel{= 2{\cdot}307\times10^{-8}\,m}$

$= 0{\cdot}109\,m$

$\boxed{(d, e, f)}$



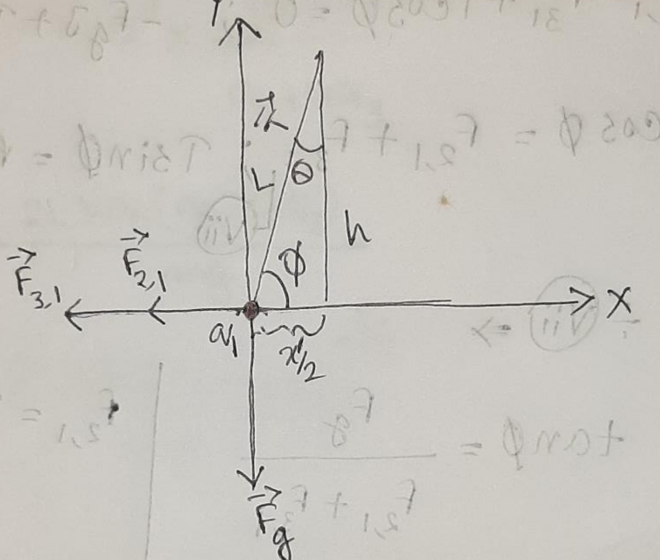
$$q_1 = q_2 = 14\times10^{-9}\,C$$

$$q_3 = 3.5\,nC = 3.5\times10^{-9}\,C$$

For small angle approximation,

$$h \to L$$

Using a_1 as the origin of our coordinate system let us draw the free body diagram.



From this we get,

$$\vec{F}_{2,1} = -F_{2,1}\hat{i}$$

$$\vec{F}_{3,1} = -F_{3,1}\hat{i}$$

$$\vec{F}_g = -F_g\hat{j}$$

$$\vec{T} = T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j}$$

In equilibrium state,

$$\vec{F}_{net} = 0$$

Therefore,

$$-F_{2,1}\hat{i} - F_{3,1}\hat{i} - F_g\hat{j} + T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j} = 0$$

$$\Rightarrow -F_{2,1} - F_{3,1} + T\cos\phi = 0 \;;\quad -F_g\hat{j} + T\sin\phi\,\hat{j} = 0$$

$$\Rightarrow T\cos\phi = F_{2,1} + F_{3,1} \text{ --- (vii)} \;;\quad T\sin\phi = F_g \text{ --- (viii)}$$

(viii) $\div$ (vii) $\Rightarrow$

$$\tan\phi = \frac{F_g}{F_{2,1} + F_{3,1}}$$

$$\Rightarrow \tan\phi = \frac{mg}{k\dfrac{q_1q_2}{(x')^2} + k\dfrac{q_1q_3}{\left(\frac{x'}{2}\right)^2}}$$

$$\Rightarrow \tan\phi = \frac{mg}{kq_1\left\{\dfrac{q_2}{(x')^2} + \dfrac{q_3}{\left(\frac{x'}{2}\right)^2}\right\}}$$

$$\Rightarrow \frac{L}{\frac{x'}{2}} = \frac{mg}{kq_1\left\{\dfrac{q_2}{(x')^2} + q_3 \times \dfrac{4}{(x')^2}\right\}}$$

$$F_{2,1} = k\frac{q_1q_2}{(r_{2,1})^2} = k\frac{q_1q_2}{(x')^2}$$

$$F_{3,1} = k\frac{q_1q_3}{\left(\frac{x'}{2}\right)^2}$$

$$\tan\phi = \frac{h}{x'/2}$$

$$h \to L$$

$$\Rightarrow \frac{2L}{x'} \times k q_1 \times \frac{1}{mg} = \frac{1}{\frac{q_2 + 4q_3}{(x')^2}}$$

$$\Rightarrow \frac{2Lkq_1}{mg} = x' \times \frac{(x')^2}{q_2 + 4q_3}$$

$$\Rightarrow (x')^3 = \frac{2Lkq_1(q_2 + 4q_3)}{mg}$$

$$\Rightarrow x' = \left\{\frac{2Lkq_1(q_2 + 4q_3)}{mg}\right\}^{1/3}$$

$$\Rightarrow x' = \left\{\frac{2 \times 2.43 \times 8.987 \times 10^9 \times 14 \times 10^{-9}(14 \times 10^{-9} + 3.5 \times 10^{-9} \times 4)}{6.849 \times 10^{-4} \times 9.8}\right\}^{1/3}$$

$$= 0.137 \text{ m.}$$

Now, $F_{2,1} = k\frac{q_1 q_2}{(x')^2} = k\frac{(14 \times 10^{-9})^2}{(0.137)^2}$

$$= 9.385 \times 10^{-5} \text{ N}$$

$$F_{3,1} = k\frac{q_1 q_3}{\left(\frac{x'}{2}\right)^2} = k\frac{14 \times 10^{-9} \times 3.5 \times 10^{-9}}{\left(\frac{0.137}{2}\right)^2} = 9.385 \times 10^{-5} \text{ N}$$

(Ans)