Phy 112 Assignment

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Section : 03

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2.1|

(a) $\vec{r}_{qp} = \vec{r}_p - \vec{r}_q$

Now, $\vec{r}_p = 0\hat{i} + 0\hat{j} + 21\hat{k}\ m$

$\vec{r}_q = 41\hat{i} + 0\hat{j} + 0\hat{k}\ m$

$\therefore \vec{r}_{qp} = 21\hat{k} - 41\hat{i}$

(Ans)

(b) $\vec{E}_q(P) = \dfrac{kq}{(r_{qp})^2} \times \hat{r}_{qp}$

$= \dfrac{kq}{(r_{qp})^3} \times \vec{r}_{qp}$

$q = 31\mu C = 31\times10^{-6} C$

$r_{qp} = \sqrt{(21)^2 + (-41)^2}$

$= 46.065\ m$

$= \dfrac{8.987\times10^9 \times 31\times10^{-6}}{(46.065)^3} \times (21\hat{k} - 41\hat{i})$

$= 2.85 \times (21\hat{k} - 41\hat{i})$

$= 59.85\hat{k} - 116.85\hat{i}$

(Ans)

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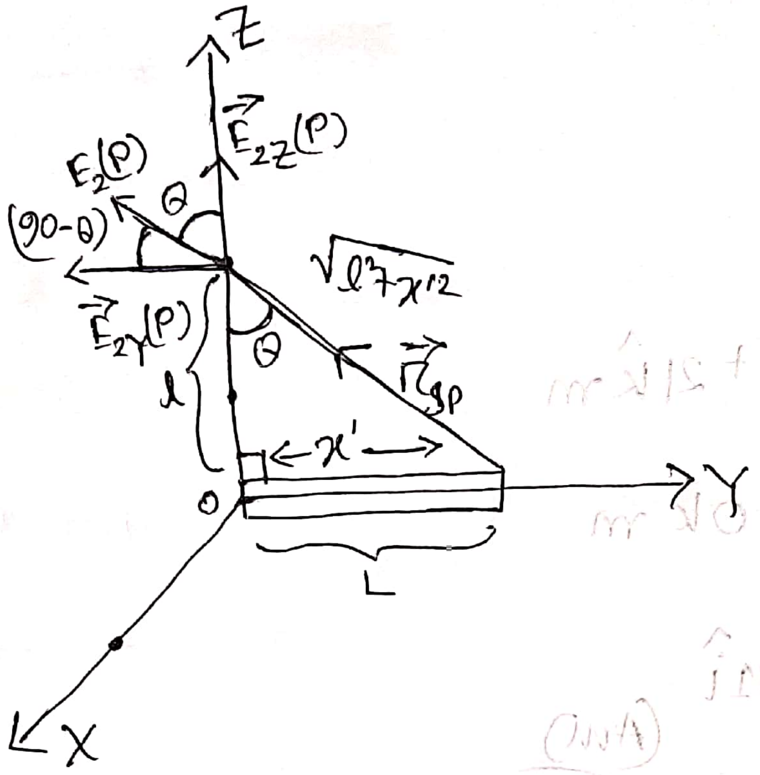
(c)



Here,

$L = 49\ m$, $\lambda = 43\ \mu C = 43 \times 10^{-6}\ C$, $l = 21\ m$

x' is the integration variable.

$$Q = \lambda L\ ; \quad dq = \frac{Q}{L} dx'$$

$$= \lambda dx'$$

We know,

$$\vec{E}(P) = \int_{source} d\vec{E} = \int_{source} \frac{k\, dq}{(r_{sp})^2} \times \hat{r}_{sp}$$

$$\hat{r}_{sp} = -\cos(90-\theta)\hat{j} + \sin(90-\theta)\hat{k}$$

$$= -\hat{j}\sin\theta + \hat{k}\cos\theta$$

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$r_{3P} = \sqrt{l^2 + x'^2}$, $\cos\theta = \dfrac{l}{\sqrt{l^2 + x'^2}}$, $\sin\theta = \dfrac{x'}{\sqrt{x'^2 + l^2}}$

Now,

$$\vec{E}_2(P) = \vec{E}_{2x}(P) + \vec{E}_{2y}(P) + \vec{E}_{2z}(P)$$

But $\vec{E}_{2x}(P) = 0$

$$\vec{E}_{2y}(P) = \int_0^L \frac{k\lambda\, dx'}{(r_{3P})^2} \times \hat{r}_{3P}$$

$$= \int_0^L \frac{k\lambda\, dx'}{(r_{3P})^2} \times (-\hat{j}\sin\theta + \hat{k}\cos\theta)$$

$$= \int_0^L \frac{-k\lambda\, dx'}{\left(\sqrt{l^2 + x'^2}\right)^2} \times \frac{x'}{\sqrt{l^2 + x'^2}} \times \hat{j} + \int_0^L k\lambda \frac{dx'}{(x'^2 + l^2)^2} \times \frac{l}{\sqrt{x'^2 + l^2}} \times \hat{k}$$

$$= -k\lambda\hat{j} \int_0^L \frac{x'}{(x'^2 + l^2)^{3/2}} dx' + k\lambda l\hat{k} \int_0^L \frac{dx'}{(x'^2 + l^2)^{3/2}} \quad \text{(i)}$$

Now,

$$\int_0^L \frac{x'}{(x'^2 + l^2)^{3/2}} dx'$$

$$= \int_0^L \frac{x'}{u^{3/2}} \times \frac{du}{2x'}$$

Let, $u = x'^2 + l^2$

$\Rightarrow du = 2x'dx'$

$\Rightarrow dx' = du \times \frac{1}{2x'}$

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$$= \frac{1}{2}\int_0^L \frac{1}{u^{3/2}}\, du$$

$$= \frac{1}{2}\int_0^L u^{-3/2}\, du$$

$$= \frac{1}{2}\left[\frac{u^{-1/2}}{-1/2}\right]_0^L$$

$$= \frac{1}{2}\left[\frac{-2}{\sqrt{u}}\right]_0^L$$

$$= \left[-\frac{1}{\sqrt{x'^2+l^2}}\right]_0^L$$

$$= -\frac{1}{\sqrt{L^2+l^2}} + \frac{1}{\sqrt{l^2}} = \frac{1}{l} - \frac{1}{\sqrt{L^2+l^2}} \quad \text{— (ii)}$$

Now,

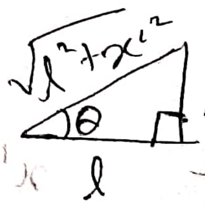
(i) $\displaystyle\int_0^L \frac{dx'}{(x'^2+l^2)^{3/2}}$

Let, $x' = l\tan\theta$

$dx' = l\sec^2\theta\, d\theta$

$\theta = \tan^{-1}\frac{x'}{l}$

$\sin\theta = \frac{x'}{\sqrt{l^2+x'^2}}$



$$= \int_0^L \frac{l\sec^2\theta\, d\theta}{(l^2\tan^2\theta + l^2)^{3/2}}$$

$$= \int_0^L \frac{l\sec^2\theta\, d\theta}{(l^2\sec^2\theta)^{3/2}}$$

$$= \int_0^L \frac{l \sec^2\theta \, d\theta}{l^3 \sec^3\theta}$$

$$= \frac{1}{l^2}\int_0^L \cos\theta \, d\theta$$

$$= \frac{1}{l^2}\left[\sin\theta\right]_0^L$$

$$= \frac{1}{l^2}\left[\frac{x'}{\sqrt{x'^2+l^2}}\right]_0^L$$

$$= \frac{1}{l^2} \times \frac{L}{\sqrt{l^2+L^2}} \quad \text{— (iii)}$$

Putting (ii), (iii) in (i),

$$\vec{E}_2(P) = -k\lambda\hat{j}\left[\frac{1}{l} - \frac{1}{\sqrt{l^2+L^2}}\right] + k\lambda l \hat{k} \times \frac{1}{l^2} \times \frac{L}{\sqrt{l^2+L^2}}$$

$$\cancel{= -\hat{j}\,k\lambda}$$

$$= (-8.987\times10^9 \times 43\times10^{-6})\left[\frac{1}{21} - \frac{1}{\sqrt{(21)^2+(49)^2}}\right]\hat{j}$$

$$+ 8.987\times10^9 \times 43\times10^{-6} \times 21 \times \frac{1}{(21)^2} \times \frac{49}{\sqrt{(21)^2+(49)^2}}\hat{k}$$

$$= -11153.068\hat{j} + 16914.063\hat{k}.$$

(Ans)

d) Net electric field,

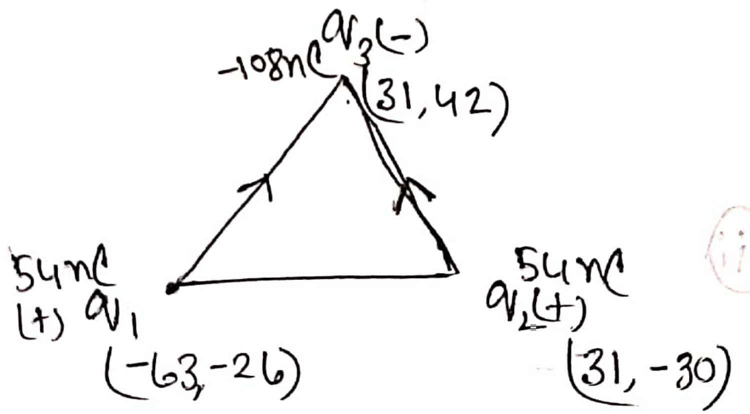
$$\vec{E}(P) = \vec{E}_{q}(P) + \vec{E}_2(P)$$

$$= 59.85\hat{k} - 116.85\hat{i} - 11153.068\hat{j} + 16914.063\hat{k}$$

$$= -116.85\hat{i} - 11153.068\hat{j} + 16973.913\hat{k}$$

(Ans).

2·21 (a)



$$\vec{P} = q_1\vec{r_1} + q_2\vec{r_2} + q_3\vec{r_3}$$

$$= 54\times10^{-9}\times\left(-63\times10^{-9}\hat{i} - 26\times10^{-9}\hat{j}\right) + 54\times10^{-9}\left(31\times10^{-9}\hat{i} - 30\times10^{-9}\hat{j}\right.$$

$$-108\times10^{-9}\times\left(31\times10^{-9}\hat{i} + 42\times10^{-9}\hat{j}\right)$$

$$= -5.076\times10^{-15}\hat{i} - 7.56\times10^{-15}\hat{j}$$

(Ans)

(b) In region II,

$$\vec{E}_x(II) = 0$$

$$\vec{\sigma}_B = 49\ \mu C/m^2$$

$$\therefore \vec{E}_Y(II) = \vec{E}(II) = \frac{\sigma_B}{2\times\epsilon_0}\hat{j}$$

$$= \frac{49\times10^{-6}}{2\times 8.854\times10^{-12}}\hat{j}$$

$$= 2767110.91\,\hat{j}\ N/C$$

(Ans)

(c) $\vec{\tau} = \vec{P} \times \vec{E}(II)$

$= \cancel{\left(-5.076\times10^{-15}\hat{i} - 7.56\times10^{-15}\right)\times}$

$= \left(-5.076\times10^{-15}\hat{i} - 7.56\times10^{-15}\hat{j}\right)\times\left(2767110.01\,\hat{j}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5.076\times10^{-15} & -7.56\times10^{-15} & 0 \\ 0 & 2767110.01 & 0 \end{vmatrix}$$

$= -1.403\times10^{-8}\,\hat{k}$ Nm

$\therefore |\tau| = 1.403\times10^{-8}$ Nm (Ans)

(d) For region III,

$\vec{E}_x(III) = 0$

$\therefore \vec{E}_y(III) = \vec{E}(III) = \dfrac{\sigma_c}{2\times\epsilon_0}\hat{j}$

$= \dfrac{-49\times10^{-6}}{2\times8.854\times10^{-12}}\hat{j}$

$= -2767110.01\,\hat{j}$ N/C

$\sigma_c = -49\ \mu C/m^2$

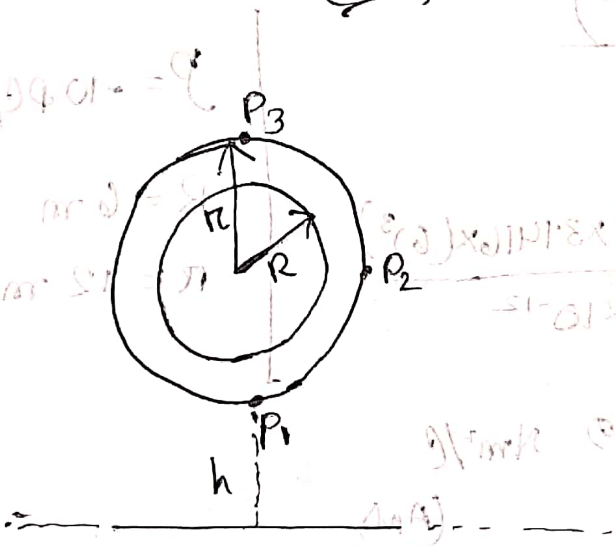
(e) $\vec{\tau} = \vec{p} \times \vec{E}$ (III)

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5.076\times10^{-15} & -7.56\times10^{-15} & 0 \\ 0 & -2767110.91 & 0 \end{vmatrix}$$

$= 1.403\times10^{-8}\,\hat{k}$ Nm

(Ans)

2.31 (a)



$$2EA = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

$$\therefore E_{P1} = E_{P2} = \frac{-16\times10^{-12}}{2\times8.854\times10^{-12}}$$

$\sigma = -16$ pC/m²

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$= -0.9035 \ N/C$

$\therefore |E_{P1}| = |\vec{E}_{P2}| = 0.9035 \ N/C$

(Ans)

(b) Amount of charge enclosed of infinite sheet in gaussian surface, $Q_{enc} = 0$.

$$\therefore \Phi_l = \frac{Q_{enc}}{\epsilon_0} = 0 \ Nm^2C^{-1}$$

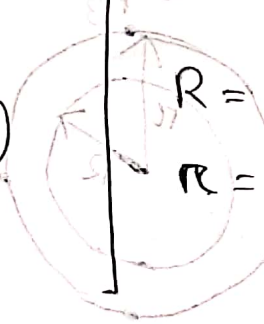
(Ans)

(c) $$\Phi_r = \frac{\rho\left(\frac{4}{3}\pi R^3\right)}{\epsilon_0}$$

$\rho = -10 \ pC/m^3$

$R = 6 \ m$

$r = 12 \ m$



$$= \frac{-10\times10^{-12}\left(\frac{4}{3}\times3.1416\times(6)^3\right)}{8.854\times10^{-12}}$$

$= -1021.889 \ Nm^2/C$

(Ans)

(d) We know,

$$E_{P_2} A = \Phi$$

$$\Rightarrow E_{P_2} = \frac{\Phi_R}{4\pi R^2} \qquad R = 12\ m$$

$$= \frac{-1021.889}{4 \times 3.1416 \times (12)^2}$$

$$= -0.5647\ N/C$$

$$\therefore |E_{P2}| = |E_{P3}| = 0.5647\ N/C$$

(Ans)

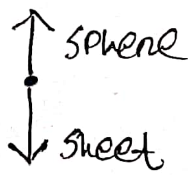
(e) Net flux

$$\Phi_{net} = \Phi_1 + \Phi_R$$

$$= -1021.889\ Nm^2/C$$

(Ans)

(f) $\vec{E}_{P1,net} = \vec{E}_{x,net}\hat{i} + \vec{E}_{y,net}\hat{j}$



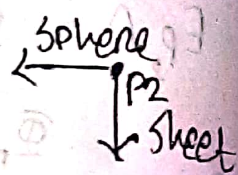
But, $\vec{E}_{x,net}\hat{i} = 0$.

$$\therefore \vec{E}_{P1,net} = \vec{E}_{y,net}\hat{j} = 0.5647\hat{j} - 0.9035\hat{j}$$

$$= -0.3388\hat{j}$$

(Ans)

(g) $\vec{E}_{P2,net} = \vec{E}_{x,net}\,\hat{i} + \vec{E}_{y,net}\,\hat{j}$



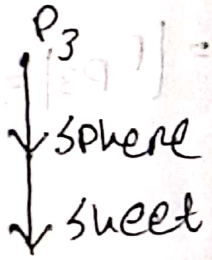
$\vec{E}_{x,net} = -0.5647\,\hat{i}$

$\vec{E}_{y,net} = -0.9035\,\hat{j}$

$\therefore \vec{E}_{2,net} = -0.5647\,\hat{i} - 0.9035\,\hat{j}$

(Ans)

(h) $\vec{E}_{P3,net} = \vec{E}_{x,net}\,\hat{i} + \vec{E}_{y,net}\,\hat{j}$



$\vec{E}_{y,net} = -0.5647\,\hat{j} - 0.9035\,\hat{j}$

$= -1.4682\,\hat{j}$

$\vec{E}_{x,net} = 0$

$\therefore \vec{E}_{P3,net} = -1.4682\,\hat{j}$

(Ans)