GA 1.1

Here,

$K = 8.987 \times 10^9 \; Nm^2/c^2$

$q_1 = -44e$

$q_2 = 41e$

$q_3 = 27e$

$\vec{r}_{21} = -a\hat{i}$

$\vec{r}_{31} = -\frac{a}{2}\hat{i} - \frac{\sqrt{3}a}{2}\hat{j}$

$a = 20 \times 10^{-9} m$

$e = 1.6 \times 10^{-19} c$

We knew

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$

$$= \frac{kq_1q_2}{a^2}\hat{r}_{21} + \frac{kq_1q_3}{a^2}\hat{r}_{31}$$

$$\frac{kq_1q_2}{a^3}(-a\hat{i}) + \frac{kq_1q_3}{a^3}\left(-\frac{a}{2}\hat{i}\right) + \frac{kq_1q_3}{a^3}\left(-\frac{\sqrt{3}a}{2}\hat{j}\right)$$

$$\cancel{= \frac{kq_1q_2}{a^2}(-q_2}$$

$$= \frac{kq_1q_2}{a^2}\hat{i} - \frac{kq_1q_3}{2a^2}\hat{i} - \frac{\sqrt{3}kq_1q_3}{2a^2}\hat{j}$$

$$= \frac{kq_1}{a^2}\left(-q_2 - \frac{q_3}{2}\right)\hat{i} - \frac{\sqrt{3}kq_1q_3}{2a^2}\hat{j}$$

$$= \frac{8.987 \times 10^9 \times (-44e)}{(20 \times 10^{-9})^2}\left(41e - \frac{27e}{2}\right)\hat{i} - \frac{\sqrt{3} \times 8.987 \times 10^9 \times (-44e)(27e)}{2 \times (20 \times 10^{-9})^2}$$

$$\cancel{= 1.0775 \times 10^{21}\hat{i}} = 1.39 \times 10^{-9}\hat{i}$$

$$\cancel{= 5.387 \times 10^{28}} - 5.92 \times 10^{-10}\hat{j}$$

b) $\vec{F}_2 = \vec{F}_{12} + \vec{F}_{13}$

Here,

$\vec{r}_{12} = a\,\hat{i}$

$\vec{r}_{13} = \frac{a}{2}\hat{i} - \frac{\sqrt{3}}{2}a\,\hat{j}$

$$= \frac{k q_1 q_2}{a^3}\vec{r}_{12} + \frac{k q_2 q_3}{a^3}\vec{r}_{13}$$

$$= \frac{k q_2}{a^2}\left(q_1 + \frac{q_3}{2}\right)\hat{i} - \frac{\sqrt{3}\,k q_2 q_3}{2a^2}\hat{j}$$

$$= \frac{8.987\times10^9 \times 41e}{(20\times10^{-9})^2}\left(-44e + \frac{27e}{2}\right)\hat{i} - \frac{\sqrt{3}\times 8.987\times10^9 \times (41e)(27e)}{2(20\times10^{-9})^2}\hat{j}$$

$$= -7.21\times10^{-10}\,\hat{i} - 5.529\times10^{-10}\,\hat{j}$$

c) $\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$

$$= \frac{k q_1 q_3}{a^3}\left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j}\right) + \frac{k q_2 q_3}{a^3}\left(-\frac{a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j}\right)$$

$$= \frac{k q_3}{2a^2}(q_1 - q_2)\hat{i} + \frac{\sqrt{3}\, k q_3}{2a^2}(q_1 + q_2)\hat{j}$$

$$= \frac{8.987\times 10^9 \times 27e}{2(20\times 10^{-9})^2}(-44e - 41e)\hat{i} + \frac{\sqrt{3}\ 8.987\times 10^9 \times 27e}{2(20\times 10^{-9})^2}(-44e + 41e)\hat{j}$$

$$= -6.618\times 10^{-10}\,\hat{i} - 4.0456\times 10^{-11}$$

GA 1.2

a) $q_1 = q_5 = 14\,\mu C = 14\times10^{-6}\ C$

$q_2 = q_4 = 38\,\mu C = 38\times10^{-6}\ C$

$q_3 = q_6 = -36\,\mu C = -36\times10^{-6}\ C$

$R = 48\ cm = 0.48\ m$

$K = 8.987\times10^{9}\ Nm^2/C^2$

$\vec{r}_{1,6} = \vec{r}_6 - \vec{r}_1 = 0\hat{i} + R\hat{j})$

$= -R\hat{j} = -0.48\hat{j}$

b) We know

$$\vec{F} = k\,\frac{q_1 q_2}{r^3} \times \vec{r}$$

$$\vec{F}_{16} = \frac{8.987\times10^{9}\times14\times10^{-6}\times38\times10^{-6}}{\left(\sqrt{(-0.48)^2}\right)^3}\,(-0.48\hat{j})$$

$= 20.75\,\hat{j}\ N$

c) $\vec{r}_{3,6} = \vec{r}_6 - \vec{r}_3 = 0\hat{i} + 0\hat{j} - (-R\hat{i} + 0\hat{j})$

$$= R\hat{i} = 0.48\hat{i}$$

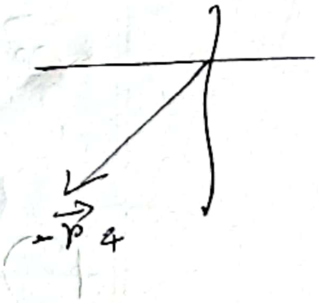
(Ans)

d) $\vec{F}_{3,6} = k \dfrac{q_3 q_6}{(r_{3,6})^3} \times \vec{r}_{3,6}$

$$= 8.987\times10^{9} \frac{(-32\times10^{-6})^2}{(0.48)^3}(0.48\hat{i})$$

$$= 50.55\hat{i}\,N$$

e) $\vec{r}_{4,6} = \vec{r}_6 - \vec{r}_4 = 0\hat{i} + 0\hat{j} - \vec{r}_4$



$$= -\vec{r}_4$$

Since, the 5 charg q_1 to q_5 are dis-tributed among a semi-circle maintain equal distance the angle θ between q_5 and q_4 45°

Therefore the vector $\vec{r}_{4,6}$ create

225° withe $+x$ axis.

we know,

$$|\vec{r}| = 0.48\,m = R$$

So, $\vec{r}_4 = R\cos 225^\circ \hat{i} + R\sin 225^\circ \hat{j}$

$$= -0.339\hat{i} - 0.339\hat{j}$$

$$\vec{r}_{46} = -\vec{r}_4 = 0.339\hat{i} + 0.339\hat{j}$$

f) $$\vec{F}_{4,6} = k\frac{q_4 q_6}{(r_{4,6})^3} \times \vec{r}_{4,6}$$

$$= 8.987\times10^9 \times \frac{(38\times10^{-6})(-36\times10^{-6})}{\left(\sqrt{(0.339)^2+(0.339)^2}\right)^3} \times (0.339\hat{i}+0.339\hat{j})$$

$$= -37.823\hat{i} - 37.823\hat{j}$$

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g) $\vec{F}_{net} = \vec{F}_{1,6} + \vec{F}_{2,6} + \vec{F}_{3,6} + \vec{F}_{4,6} + \vec{F}_{5,6} \dots (i)$

For $\vec{F}_{2,6}$,

$\vec{r}_{2,6} = \vec{r}_6 - \vec{r}_2 = -\vec{r}$

q_2 ~~c~~ creates an angle of $90^\circ + 45^\circ = 135^\circ$

with $+x$ axis.

So, $\vec{r}_2 = R\cos 135^\circ \hat{i} + R\sin 135^\circ \hat{j}$

$= -0.339\hat{i} + 0.339\hat{j}$

$\vec{r}_{2,6} = -\vec{r}_2 = 0.339\hat{i} - 0.339\hat{j}$

$\vec{F}_{2,6} = K \dfrac{q_2 q_6}{(r_{26})^3} \times \vec{r}_{2,6}$

$= 8.987 \times 10^9 \times \dfrac{(38\times10^{-2})(-36\times10^{-2})}{\left(\sqrt{(0.339)^2+(-0.339)^2}\right)^3} \times (0.339\hat{i} - 0.339\hat{j})$

$= -37.823\hat{i} + 37.823\hat{j}$

For, $\vec{F}_{5,6}$

$$\vec{r}_{5,6} = \vec{r}_6 - \vec{r}_5 = -\vec{r}_5 = (0\hat{i} - R\hat{j})$$

$$= 0.48\hat{j}\ m$$

$$\therefore \vec{F}_{5,6} = K\frac{q_5 q_6}{(r_{5,6})^3} \times \vec{r}_{56}$$

$$= 8.987 \times 10^9 \times \frac{(24\times10^{-6})(-36\times10^{-6})}{(0.48)^3} \times 0.48$$

$$= -19.65\hat{j}\ N$$

$$\vec{F}_{net} = 20.75\hat{j} - 37.823\hat{i} + 37.823\hat{j} + 50.55\hat{i} - 37.823\hat{i} - 37.823\hat{i} - 19.65\hat{j}$$

$$= -25.096 - 1.1\hat{j}$$

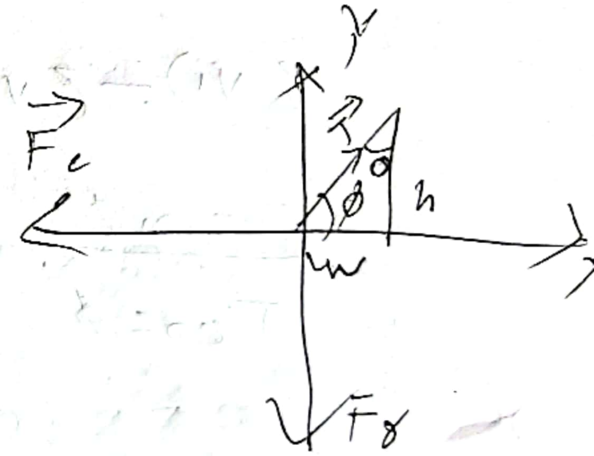
G 1-3

i) Mass, $m = m_1 = m_2$

charge, $q = q_1 = q_2 = 20\,nC = 20 \times 10^{-9}\,C$

length, $L = 23\cancel{5}\,cm = 2.35\,m$

Separation, $d = 17\,cm = 0.17\,m$



$\vec{F}_c = -F_c\,\hat{i} \quad \dots (i)$

$\vec{F}_g = -F_g\,\hat{j} \quad \dots (ii)$

$\vec{T} = T\cos\theta\,\hat{i} + T\sin\theta \quad \dots (iii)$

Since, the spheres are in equilibrium state,

$\vec{F}_{Net} = 0 \quad \dots (iv)$

Using equation (i), (ii), (iii), (iv)

$$-F_c\hat{i} - F_g\hat{j} + T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j} = 0$$

$$\Rightarrow -F_c\hat{i} + T\cos\phi\,\hat{i} - F_g\hat{j} + T\sin\phi\,\hat{j} = 0\hat{i} + 0\hat{j}$$

$$\Rightarrow T\cos\phi = F_c \dots (v) \quad ; \quad T\sin\phi = F_g \dots (vi)$$

$(vi) \div (v) \Rightarrow$

$$\frac{T\sin\phi}{T\cos\phi} = \frac{F_g}{F_c}$$

$$\Rightarrow \tan\phi = \frac{mg}{Kq^2} \times x^2$$

$$\tan\phi = \frac{h}{x/2}$$

$$h = \sqrt{l^2 - (x/2)^2}$$
$$= 2.35\ m$$
$$= L$$
$$L > x$$

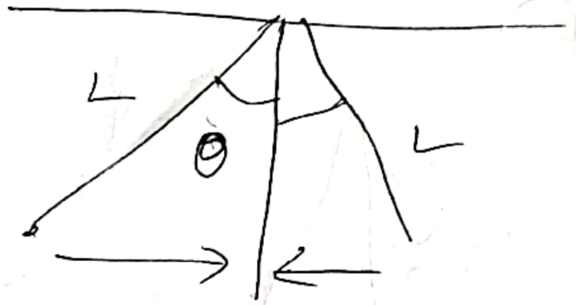
$$\Rightarrow \frac{L}{x/2} = \frac{mgx^2}{Kq^2}$$

$$\Rightarrow \frac{K \cdot L \cdot q^2}{x/2 \cdot g x^2} = m$$

$$\Rightarrow m = \frac{8.987\times10^9 \times 2.35 \times (22\times10^{-9})^2}{0.085 \times 9.8 \times (0.72)^2}$$

$$= 3.309 \times 10^{-4}\ kg$$

b)



$$q_1' = q_2' = q' = \frac{q}{2} = \frac{20}{2} nC = 10\,nC = 10 \times 10^{-9}$$

(Ans)

c) From a,

$$\frac{L}{x/2} = \frac{mgx^2}{kq_2^2}$$

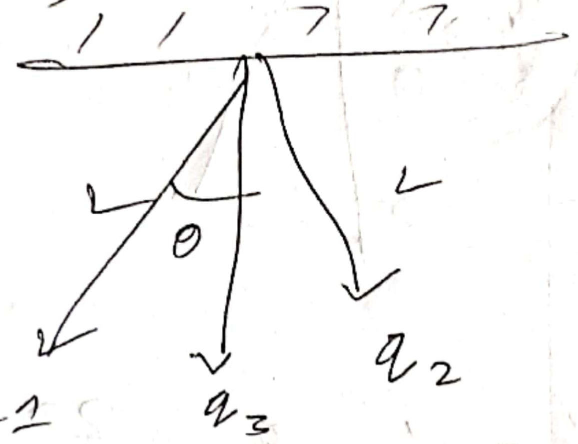
~~Here,~~

$$x' = \left(\frac{2Lkq'^2}{mg}\right)^{1/3}$$

$$= \left\{\frac{2 \times 2.35 \times 8.987 \times 10^9 \times (10 \times 10^{-9})^2}{3.509 \times 10^{-4} \times 9.8}\right\}^{1/3}$$

$$= 0.107\ m$$

(d, e, f)



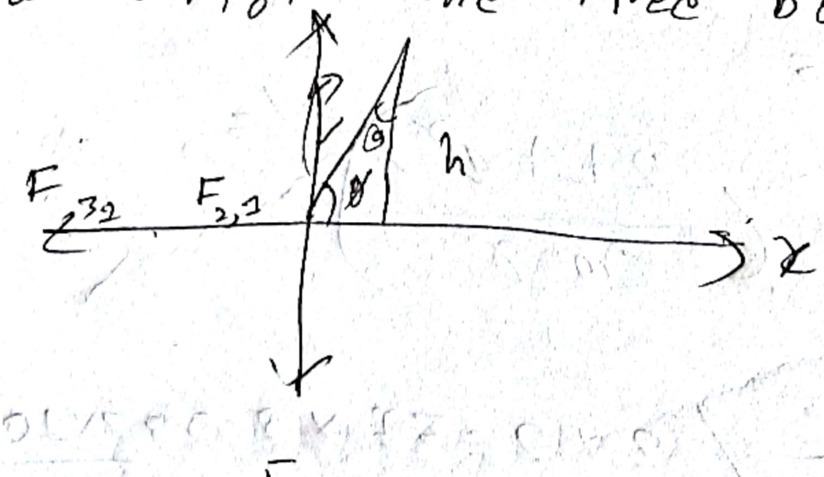
$$q_1 = q_2 = 10 \times 10^{-9}\,C$$

$$q_3 = 2.5 \times 10^{-19}\,C$$

For small angle approximation $h \to L$.

$h \neq L$

Using q_1 as origin the free body diagram



we get,

$$\vec{F}_{2,1} = -F_{2,1}\,\hat{i}$$

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$$\vec{F}_{3,1} = -F_{3,1}\hat{i}$$

$$\vec{F}_g = -F_g\hat{j}$$

$$\vec{T} = T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j}$$

In equilibrium state,

$$\vec{F}_{net} = 0$$

There,

$$\cancel{T\cos\phi}$$

$$-F_{21}\hat{i} - F_{3,1}\hat{i} - F_g\hat{j} + T\cos\phi\,\hat{i} + T\sin\phi\,\hat{j} = 0$$

$$\Rightarrow T\cos\phi = F_{21} + F_{31} \quad \ldots(vii) \quad ; \quad T\sin\phi = F_g \quad \ldots(viii)$$

$(viii) \div (viii) \Rightarrow$

$$\tan\theta = \frac{F_a}{F_{21} + F_{31}}$$

$$F_{21} = k\frac{q_1 q}{(x')^2}$$

$$F_{31} = k\frac{q_1 q_3}{\left(\frac{x'}{2}\right)^2}$$

$$\tan\theta = h/\frac{x}{2}$$

$$h \rightarrow L$$

$$\Rightarrow \tan\theta = \frac{mg}{k\frac{q_1 q_2}{(x')^2} + k\frac{q_1 q_3}{\left(\frac{x'}{2}\right)^2}}$$

$$\Rightarrow \frac{L}{\frac{x'}{2}} = \frac{mg}{k q_1 \left\{\frac{q_2}{(x')^2} + q_3 \times \frac{4}{(x')^2}\right\}}$$

$$\Rightarrow \frac{2L}{x'} \times k q \times \frac{1}{mg} = \frac{1}{\frac{q_2 + 4q_3}{(x')^2}}$$

$$\Rightarrow \frac{2Lkq_1}{mg} = x' \times \frac{(x')^2}{q_2 + 4q_3}$$

$$\Rightarrow x' = \left\{\frac{2Lkq_1(q_2 + 4q_3)}{mg}\right\}^{1/3}$$

$$\Rightarrow x' = \left\{ \frac{2 \times 2.35 \times 8.987 \times 10^{9} \times 10 \times 10^{-9} (10 \times 10^{-9} + 4 \times 2.5 \times 10^{-9})}{3.509 \times 10^{-4} \times 9.8} \right\}^{1/3}$$

$$= 0.13\cancel{5}\ m$$

(Ans)

Now,

$$F_{2,1} = k \frac{q_1 q_2}{(x')^2} = \cancel{m}\, k \frac{(10 \times 10^{-9})^2}{(0.135)^2}$$

$$= 4.9341 \times 10^{-5}$$

(Ans)

$$F_{3,1} = k \frac{q_1 q_3}{(x/2)^2} = 8.987 \times 10^{9} \frac{(10 \times 10^{-9} \times 2.5 \times 10^{-9})}{\left(\frac{0.135}{2}\right)^2}$$

$$= 4.9341 \times 10^{-5}$$

(Ans)