

Assignment 2 is made up of the problems taken from [Bertsimas & Tsiliklis, 1997] Textbook.

- Exercise 1.4 costs 1 point. You need to provide a CVX solution.
- Exercise 1.8 costs 2 points. You need to write a CVX solution (you can initialize  $I^*$  and  $a$  to random values).
- Exercise 1.15 costs 4 points. You need to solve all cases in CVX.
- Exercise 3.12 costs 6 points (all pen-and-paper, you can scan/photo your drawing for 3. ).

Best of luck!

## 1 Exercise 1.4

Consider the problem

$$\begin{aligned} & \text{minimize } 4x_1 + 5|x_2 - 1| \\ & \text{subject to } |2x_1| + |x_2 - 3| \leq 5, \end{aligned}$$

and reformulate it as a linear programming problem.

## 2 Exercise 1.8 (Road lighting)

Consider a road divided into  $n$  segments and illuminated by  $m$  lamps. Let  $p_j$  be the power of the  $j$ th lamp. The illumination  $I_i$  of the  $i$ th segment is assumed to be  $\sum_{j=1}^m a_{ij}p_j$ , where  $a_{ij}$  are known coefficients. Let  $I_i^*$  be the desired illumination of road  $i$ .

We are interested in choosing the lamp powers  $p_j$  so that the illuminations  $I_i$  are close to the desired illuminations  $I_i^*$ . Provide a reasonable linear programming formulation of this problem. Note that the wording of the problem is loose and there is more than one possible formulation.

### 3 Exercise 1.15

A company produces two kinds of products. A product of the first type requires  $1/4$  hours of assembly labor,  $1/8$  hours of testing, and \$1.2 worth of raw materials. A product of the second type requires  $1/3$  hours of assembly,  $1/3$  hours of testing, and \$0.9 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing, each day. Products of the first and second type have a market value of \$9 and \$8, respectively.

1. Formulate a linear programming problem that can be used to maximize the daily profit of the company.
2. Consider the following two modifications to the original problem:
  - (a) Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of \$7 per hour.
  - (b) Suppose that the raw material supplier provides a 10% discount if the daily bill is above \$300.

Which of the above two elements can be easily incorporated into the linear programming formulation and how? If one or both are not easy to incorporate, indicate how you might nevertheless solve the problem.

### 4 Exercise 3.12

Consider the problem

$$\begin{aligned} &\text{minimize} && -x_1 - 2x_2 \\ &\text{subject to} && -x_1 + 3x_2 \leq 3 \\ &&& x_1 + x_2 \leq 5 \\ &&& x_1, x_2 \geq 0. \end{aligned} \tag{1}$$

1. Convert the problem into standard form and construct a basic feasible solution at which  $(x_1, x_2) = (0, 0)$ .
2. Carry out the full tableau implementation of the simplex method, starting with the basic feasible solution of part 1.

3. Draw a graphical representation of the problem in terms of the original variables  $x_1$ ,  $x_2$ , and indicate the path taken by the simplex algorithm.