Assignment 8
Computational methods in simulation 2012

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1 Building our own 2D simulator

This report deals with the exercise of building our own 2D simulator. The simulator will be simulating hyperelastic materials using the Finite Volume Method (FEM).

Due to time pressure I will adhere from deriving the model since they are already given in the slides handed out. I have however implemented the steps for the simulation which leads to the following final update rules

Computing the forces:

$$m_i \ddot{\boldsymbol{x}}_i = \underbrace{\boldsymbol{f}_i^{ext} + \boldsymbol{f}_t + \sum_e f_i^e}_{=\boldsymbol{f}^{total}} \tag{1}$$

The final finite difference approximations are given by

$$\boldsymbol{v}_i^{t+\Delta t} = \boldsymbol{v}_i^t + \frac{\Delta t}{m_i} \boldsymbol{f}_i^{total} \tag{2}$$

$$\boldsymbol{x}_i^{t+\Delta t} = \boldsymbol{x}_i^{t+\Delta t} + \Delta t \boldsymbol{v}_i^{t+\Delta t} \tag{3}$$

2 Simulations

I have performed two simulations: One in which we see a beam being affected by gravity (Figure 2) and one in which we see the same beam being affected solely by a pull in the right border (Figure 3). The green arrows show the total forces in each of the vertices and the blue arrows show the velocities. In both simulations the left border of the beam is fixed and hence we see the elasticity forces counteracting the forces we apply.

In the gravity example we see the beam bending down and moving back up to the starting point again caused by the elastic forces. Likewise we see the beam contracting after having been pulled out a bit for the 2nd simulation.

3 Volume

In one of the previous assignments we saw a beam being pushed down by a nodal force on the right-hand side of the beam. In this simulation we saw a somewhat noticeable volume loss. In order to find out whether this simulator has volume conservation we compute the volume in each frame of each of the two simulations described above. The results are seen in Figures 1a and 1b for the gravity and pull simulations respectively.

We see that there is some compression but it doesn't have the same noticeable compression as the beam in the previous assignment.

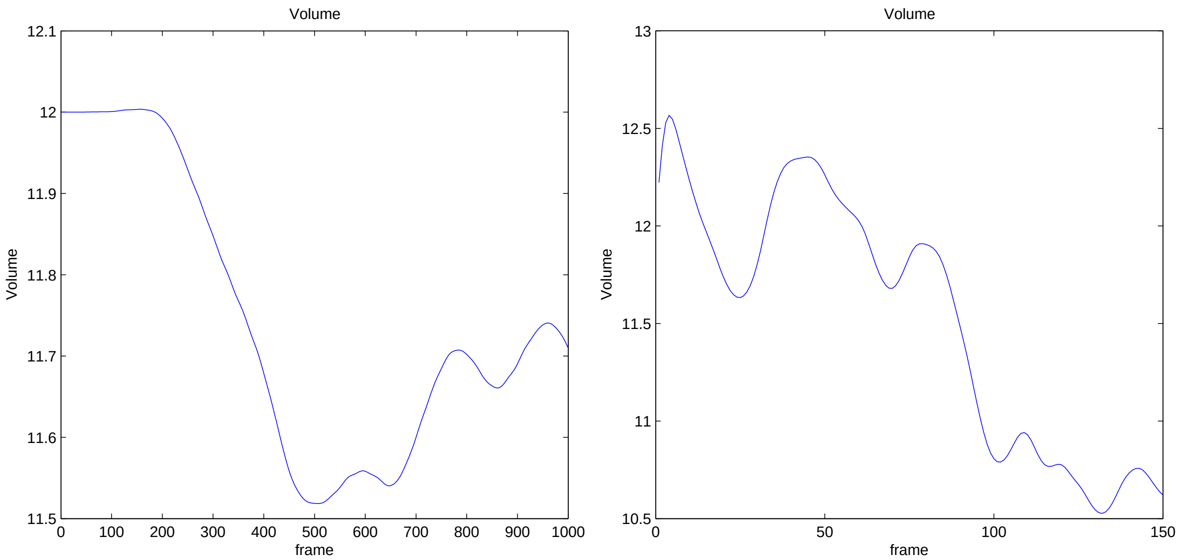
4 Real world parameters

In order for us to be able to model real world hyperelastic materials we need the two parameters Lamé's first parameter λ and shear modulus μ. We have however not been able to find λ directly. By finding the Young's modulus E and μ we are however able to compute λ[1].

$$\lambda = \frac{\mu(E - 2\mu)}{3\mu - E}$$

We have found the parameters above here[2].

Due to time pressure I have however not had the time to play around with the real world parameters.



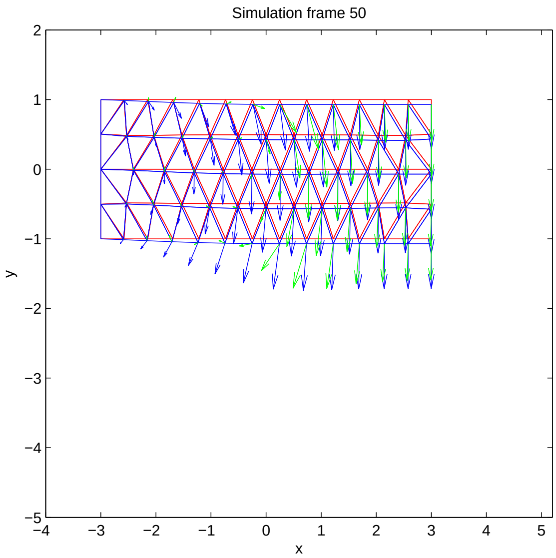
(a) Simulation of "gravity"

(b) Simulation of pull in the right edge

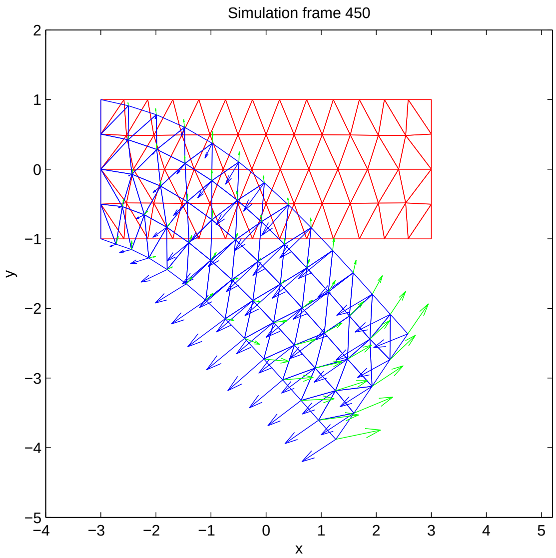
Figure 1: Volume of the grid during the simulations

[1] Wikipedia. "Lamé parameters", `http://en.wikipedia.org/wiki/Lam%C3%A9_parameters`

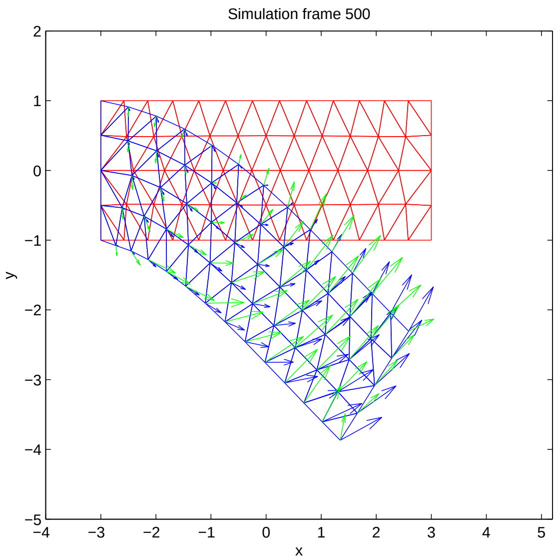
[2] `http://homepages.which.net/~paul.hills/Materials/MaterialsBody.html`



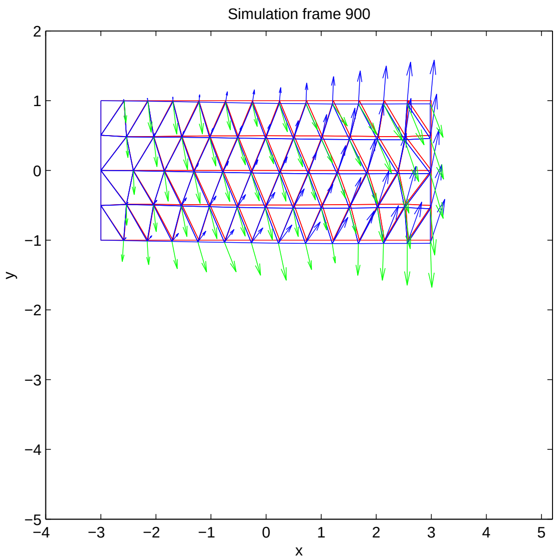
(a) Frame 50



(b) Frame 450

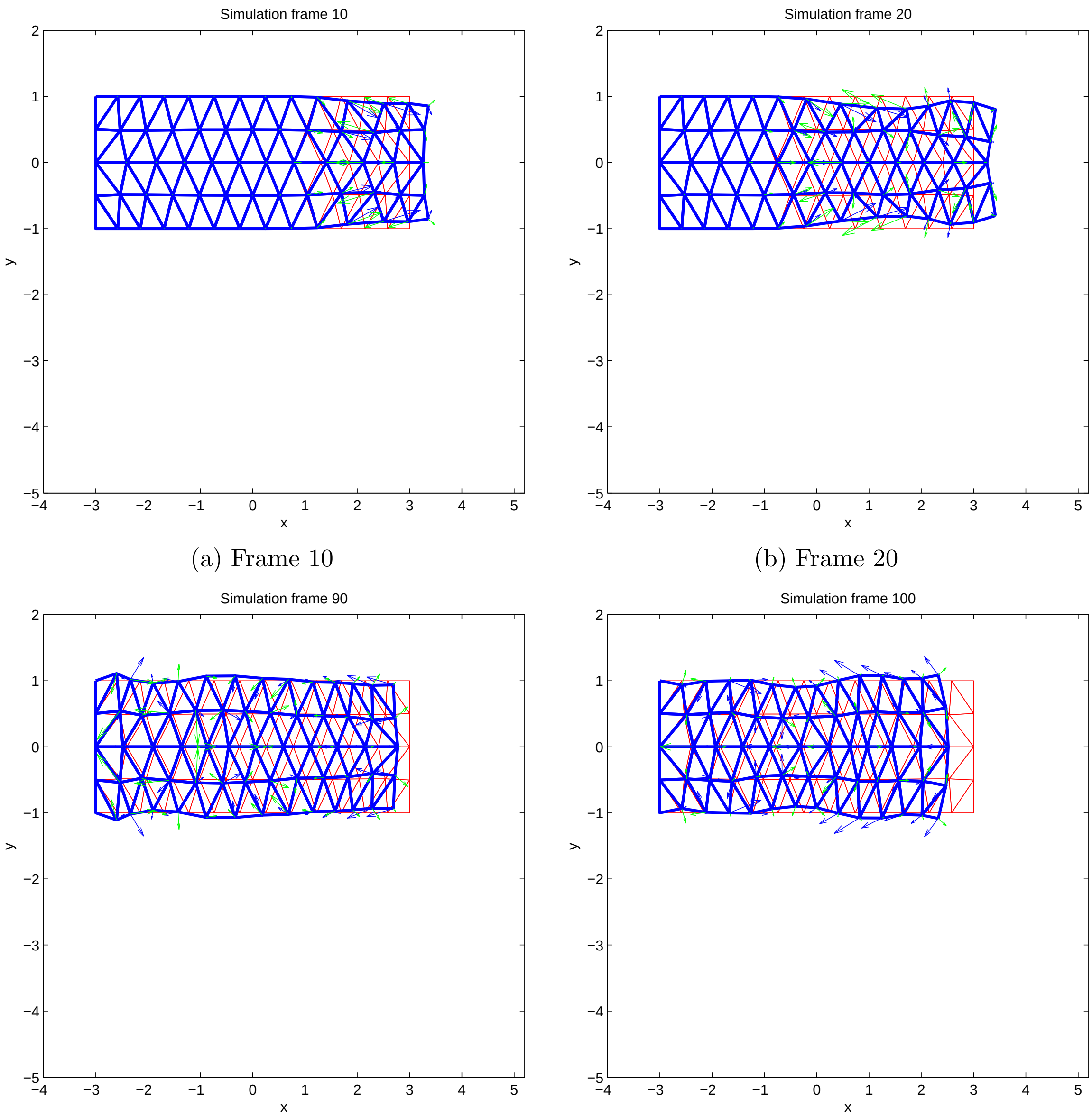


(c) Frame 500



(d) Frame 900

Figure 2: Simulation of “gravity”



(a) Frame 10

(b) Frame 20

(c) Frame 90

(d) Frame 100

Figure 3: Simulation of “gravity”