CSE291
Topics in Computer Graphics
Mesh Animation

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Fall 2006

Today

- Review
- *Elastically deformable models,* Terzopoulos et al.
- Presentation by Erik

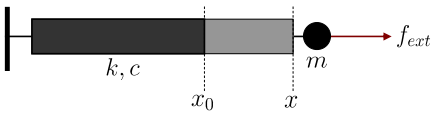
Overview

- Motion equation for 1D spring
- Explicit Euler integration
- Implicit Euler integration
- Review meshless deformation based on shape matching

Motion equation for 1D spring

- Mass m, damping c, stiffness k, external force f_{ext}

$$mx'' + cx' + k(x - x_0) = f_{ext}$$



Motion equation for 1D spring

- Mass m, damping c, stiffness k, external force f_{ext}

$$mx'' + cx' + k(x - x_0) = f_{ext}$$

- System of first order ODEs

$$mv' + cv + k(x - x_0) = f_{ext}, x' = v$$

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- Finite difference approximation

$$v' \approx \frac{v(t + \Delta t) - v(t)}{\Delta t} \qquad x' \approx \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

Explicit Euler integration

- Use finite differences

$$m\frac{v(t+\Delta t)-v(t)}{\Delta t}+cv(t)+k(x(t)-x_0)=f_{ext}$$

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- Use finite differences

$$m\frac{v(t+\Delta t)-v(t)}{\Delta t}+cv(t)+k(x(t)-x_0)=f_{ext}$$

1. Velocity update

$$v(t+\Delta t)=v(t)+\frac{\Delta t}{m}(f_{ext}-cv(t)-k(x(t)-x_0))$$

2. Position update

$$x(t+\Delta t)=x(t)+\Delta t v(t+\Delta t)$$

- Unstable, system gains energy for large time steps

Implicit Euler integration

- Use unknown position at $x(t+\Delta t)$

$$m\frac{v(t+\Delta t)-v(t)}{\Delta t}+cv(t)+k(\underbrace{x(t)+\Delta t v(t+\Delta t)}_{x(t+\Delta t)}-x_0)=f_{ext}$$

Implicit Euler integration

- Use unknown position at $x(t+\Delta t)$

$$m\frac{v(t+\Delta t)-v(t)}{\Delta t}+cv(t)+k(x(t)+\Delta t v(t+\Delta t)-x_0)=f_{ext}$$

1. Velocity update

$$(1-\frac{\Delta t^2}{m}k)v(t+\Delta t)=$$
$$v(t)+\frac{\Delta t}{m}(f_{ext}-cv(t)-k(x(t)-x_0))$$

2. Position update

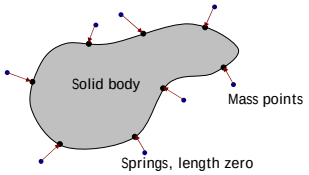
$$x(t+\Delta t)=x(t)+\Delta t v(t+\Delta t)$$

Implicit Euler integration

- Velocity update requires solution of system of linear equations
- More stable than explicit scheme
- Allows larger time steps

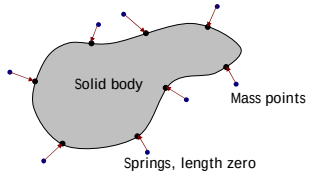
Meshless deformation

- Springs attached to solid body



Meshless deformation

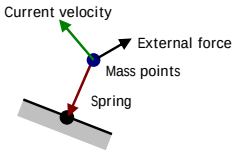
- Springs attached to solid body



- Algorithm
  1. Update springs
  2. Shape matching

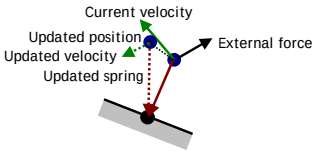
Update springs

- Modified, unconditionally stable springs
- Explicit integration scheme



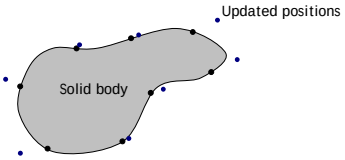
Update springs

- Modified, unconditionally stable springs
- Explicit integration scheme



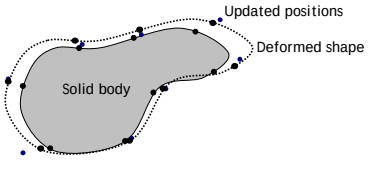
Shape matching

- Deform shape to best match updated positions



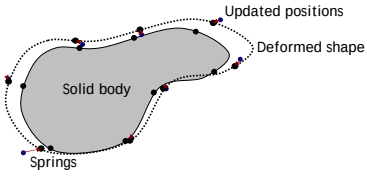
Shape matching

- Deform shape to best match updated positions



Shape matching

- Deform shape to best match updated positions



- Go back to step 1, update springs

Shape matching

Types of deformations

- Rigid
- Linear
- Quadratic
- Clustered