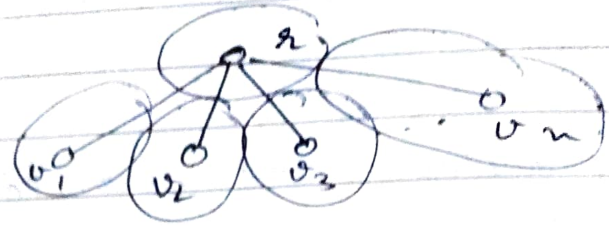
won't be able to proof (ii) for ...
Here, we will be able to ~~solve~~ show an average
vertices will satisfy (ii).



18.11.22

Steiner Forest (recap)

$$\min \sum c_e x_e \qquad\qquad \max \sum y_S$$

$$\sum_{e \in \partial(S)} x_e \geq 1 \qquad\qquad \sum_{S: e \in \partial(S)} y_S \leq c_e \quad \forall e$$

$$x \geq 0 \qquad\qquad y \geq 0$$

$F' \leftarrow \phi$, $y_S = 0 \ \forall S$, $x_e = 0 \ \forall e$

1. while ($\exists$ (s_i, t_i) not connected in F')

   (a) Increase all active duals simultaneously

   (b) when some edge e is tight $x_e = 1$, make the corresponding moats inactive.

2. Prune all unnecessary edges - F is the final.

Suppose some s_i is disconnected from its t_i, means there is some cut separating s_i and t_i. Some edges which is not tight.

$$c(F) \leq \sum_{e \in F^*} c_e \cdot \hat{x}_e$$

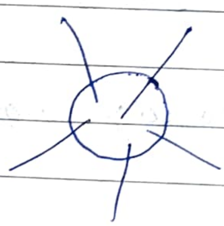
$\hat{x}_e$: our algo. dual values

$$= \sum_{e \in F'} \left(\sum_{S : e \in \delta(S)} \hat{y}_S \right) \cdot 1.$$

For the ones for which $\hat{x}_e = 1$, we replace by the C.S condition.
$\downarrow$
we followed in our algo (1.b) step

$$= \sum_{S : \hat{y}_S > 0} \hat{y}_S \cdot \deg_F(S)$$

$$\left[(?) \quad \leq 2 \cdot \sum_{S : \hat{y}_S > 0} \hat{y}_S \right]$$



one dual may pay towards lot of edges.

Suppose at some iteration, every active moat grows by δ.

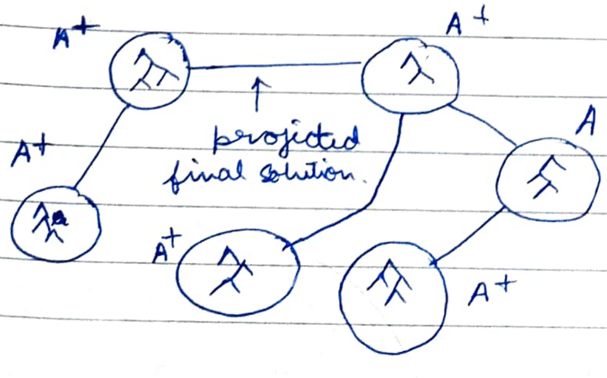
L.H.S increases by $\delta \cdot \sum_{S : S \text{ is active}} \deg_F(S)$ [∧ ?]

R.H.S increases by $2\delta \cdot$ (#active moats)

To show : avg. deg of active moat ≤ 2

(A^-) inactive moat $\rightarrow$ all s_i, t_i ~~pl~~ are in pairs in that moat.
The leafs must be active.

at some iteration



Lemma :-

Inactive ~~moat~~ moat
$\Rightarrow$ internal supernode

Lemma 2

Average degree of a tree :- $\#\text{edges} = (n-1)$

$\Sigma \deg = 2(n-1) = 2n-2$

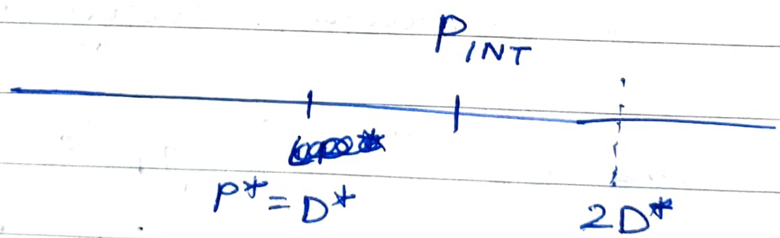
$\Rightarrow \dfrac{\Sigma \deg}{n} \leq 2$

Final Lemma :-

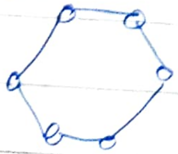
Average degree of inactive moats ≥ 2.

[inactive moats are internal moats, that is, have at least 2 edges, so avg ≥ 2]

$\therefore$ Average degree of active moats ≤ 2



One example



Other example

P_{INT} close to $p^* = D^*$,

far away from $2D^*$

FACILITY LOCATION (Metric)

$\mathcal{C}$: clients ; $\mathcal{F}$: facilities

LP $\Rightarrow$

$$\min \sum_{i \in \mathcal{F}} f_i \cdot y_i + \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{F}} d_{ij} \cdot x_{ij}$$

$\alpha_j \Rightarrow$ s.t. $\sum_{i \in \mathcal{F}} x_{ij} = 1 \quad \forall\, j \in \mathcal{C}$

β_{ij} $\quad x_{ij} \le y_i \quad \forall\, i \in \mathcal{F}$ and $j \in \mathcal{C}$

$\Rightarrow y_i - x_{ij} \geq 0$

$x, y \geq 0$

Dual

For each client, we will have a dual variable.
For each pair (i,j), " " another " " .

Objective function

$$\max \sum_{j \in \mathcal{C}} \alpha_j$$

s.t. $\sum_{j \in \mathcal{C}} \beta_{ij} \le f_i \quad \forall\, i \in \mathcal{F}$

$\alpha_j - \beta_{ij} \le d_{ij} \quad \forall\, i \in \mathcal{F}, j \in \mathcal{C}$

$\Rightarrow \alpha_j \le \beta_{ij} + d_{ij}$

$\beta_{ij} \geq 0, \quad \underbrace{\alpha_j}_{\text{free variable}} \in \mathbb{R}$

$\alpha_j \leq \beta_{ij} + d_{ij}$ → connection cost.

- α_j: Tot. money client paying
- β_{ij}: Part of that going to opening facility

$\sum \beta_{ij} \leq f_i$

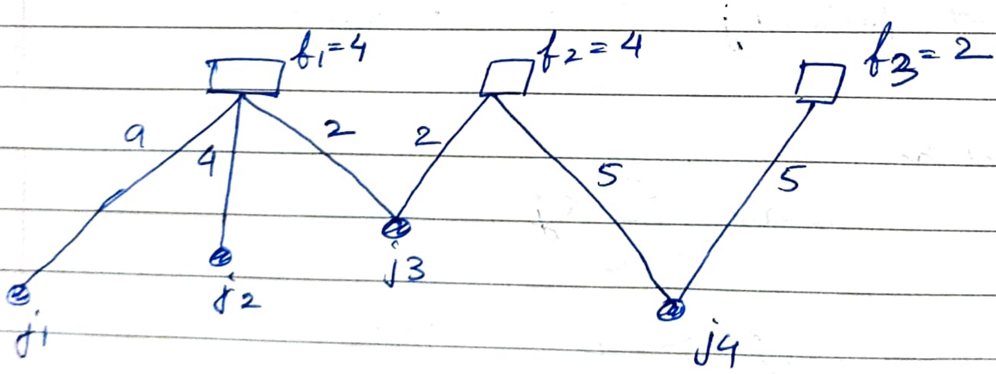
- Total opening facility cost ~~should be~~ contribution

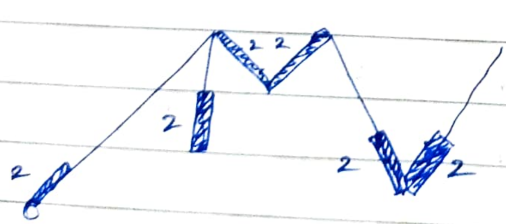
(1) Construct feasible (integral) primal and feasible dual

(2) Argue : Primal $\leq \lambda$ dual.

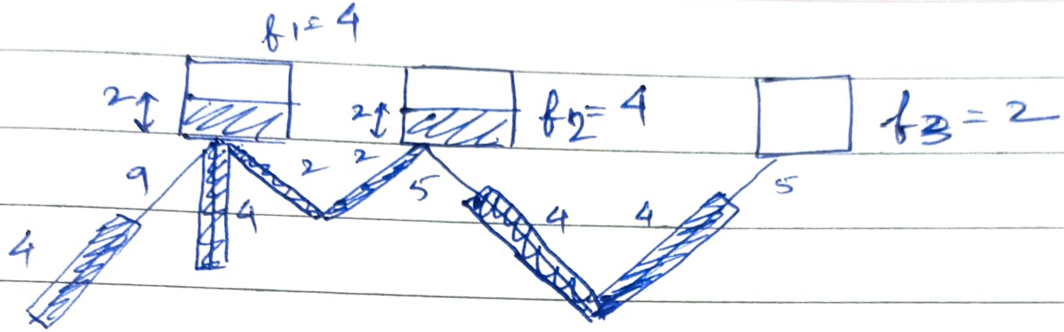
[$\lambda = 3$ we will see

Example

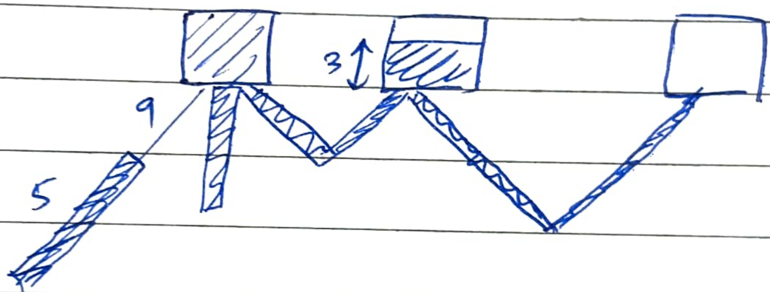




At $t = 2$



f1 = 4
f2 = 4
f3 = 2
2
2
9
2
2
5
5
4
4
4
4



3
9
5