Balancing Communications and Computations in Gradient Tracking Algorithms for Distributed Optimization
Additional Numerical Experiments

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In this document, we present additional numerical experiments that accompany the manuscript "Balancing Communications and Computations in Gradient Tracking Algorithms for Distributed Optimization" [**?**]. These experiments illustrate the empirical performance of the methods defined in Table 1 using Python implementations[1]. We present results on two problems: (1) a synthetic strongly convex quadratic problem (Section 1); and, (2) binary classification logistic regression problems (Section 2) over multiple datasets summarized in Table 2. We investigated network structures (different mixing matrices $\mathbf{W}$) with $n = 16$ nodes summarized in Table 3.

The methods defined in Table 1 are denoted as `GTA`$-i(n_c, n_g)$, $i = 1, 2, 3$, where n_c and n_g are the number of communication and computation steps, respectively. The performance of the methods was measured in terms of the optimization error ($\|\bar{x}_k - x^*\|_2$) and the consensus error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$). We do not report the consensus error in auxiliary variable $\mathbf{y}_k$ ($\|\mathbf{y}_k - \bar{\mathbf{y}}_k\|_2$) as this measure does not provide any significant additional insights about the performance of the algorithms. We do not present the consensus error for Fully Connected Networks as in this case, it is always zero for `GTA-2` and `GTA-3`, and can be brought to zero for `GTA-1` in one communication step. The optimal solution x^* for quadratic problem was obtained analytically and for the logistic regression problems was obtained by running gradient descent in the centralized setting to high accuracy, i.e., $\|\nabla f(x^*)\|_2 \leq 10^{-12}$.

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[1]Our code will be made publicly available upon publication of the manuscript. Github repository: https://github.com/Shagun-G/Gradient-Tracking-Algorithmic-Framework.

Table 1: Special cases of Gradient Tracking Algorithms (`GTA`).

Method	Communication Matrices				Algorithms in literature
	$\mathbf{W}_1$	$\mathbf{W}_2$	$\mathbf{W}_3$	$\mathbf{W}_4$	$(n_c = n_g = 1)$
`GTA-1`	$\mathbf{W}$	I_n	$\mathbf{W}$	I_n	EXTRA[4] [8], DIGing [5]
`GTA-2`	$\mathbf{W}$	$\mathbf{W}$	$\mathbf{W}$	I_n	SONATA [9], NEXT [2], [7]
`GTA-3`	$\mathbf{W}$	$\mathbf{W}$	$\mathbf{W}$	$\mathbf{W}$	Aug-DGM [10], ATC-DIGing [6]

Table 2: Summary of Datasets

Dataset	d	$\sum_{i=1}^n n_i$
Mushroom [3]	117	8124
Australian [3]	41	690
Sonar [1]	60	208
Ionosphere [1]	34	351
Phishing [1]	68	11055

Table 3: Graphs Summary

Graph Name	$\|\mathbf{W} - \frac{11^T}{n}\|$
Full	0
Network 1	0.125
Network 2	0.247
Star	0.95
Cyclic	0.992
Line	0.998

1 Quadratic Problems

We first consider quadratic problems

$$f(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}x^T Q_i x + b_i^T x,$$

where $Q_i \in \mathbb{R}^{10\times 10}$, $Q_i \succ 0$ and $b_i \in \mathbb{R}^{10}$ is the local information at each node $i \in \{1, 2, .., n\}$, and $n = 16$. Each local problem is strongly convex and was generated using the procedure described in [4], with global condition number, $\kappa \approx 10^4$.

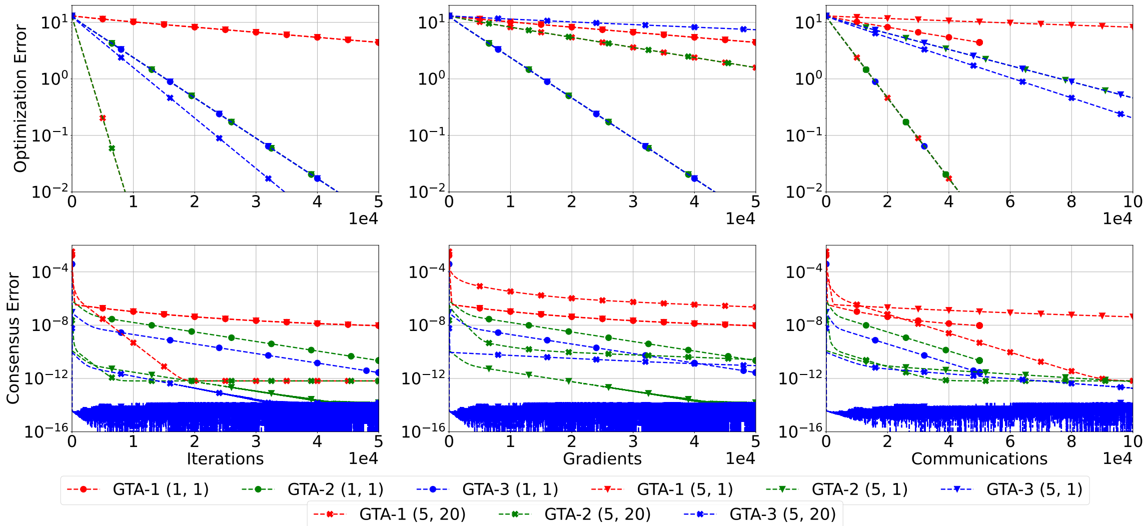

Figure 1: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for the synthetic quadratic problem over the Graph 1 network.

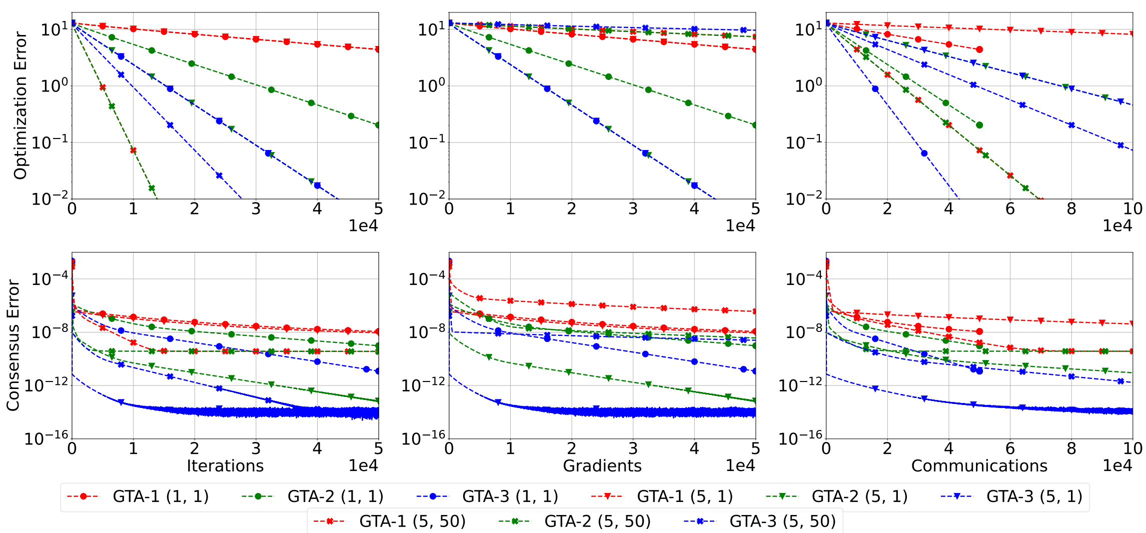

Figure 2: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for the synthetic quadratic problem over the Graph 2 network.

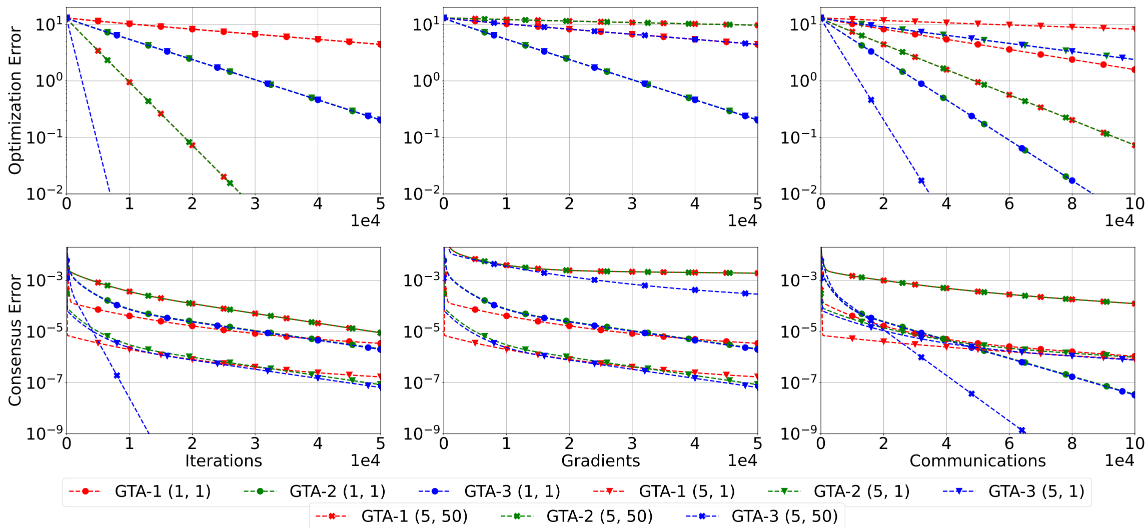

Figure 3: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for the synthetic quadratic problem over the star network.

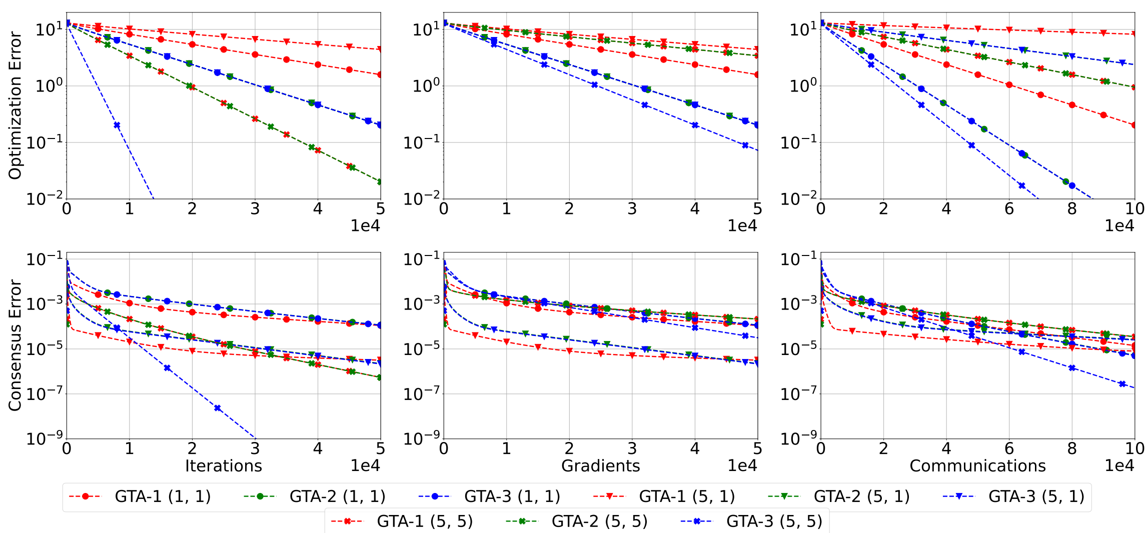

Figure 4: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for the synthetic quadratic problem over the cyclic network.

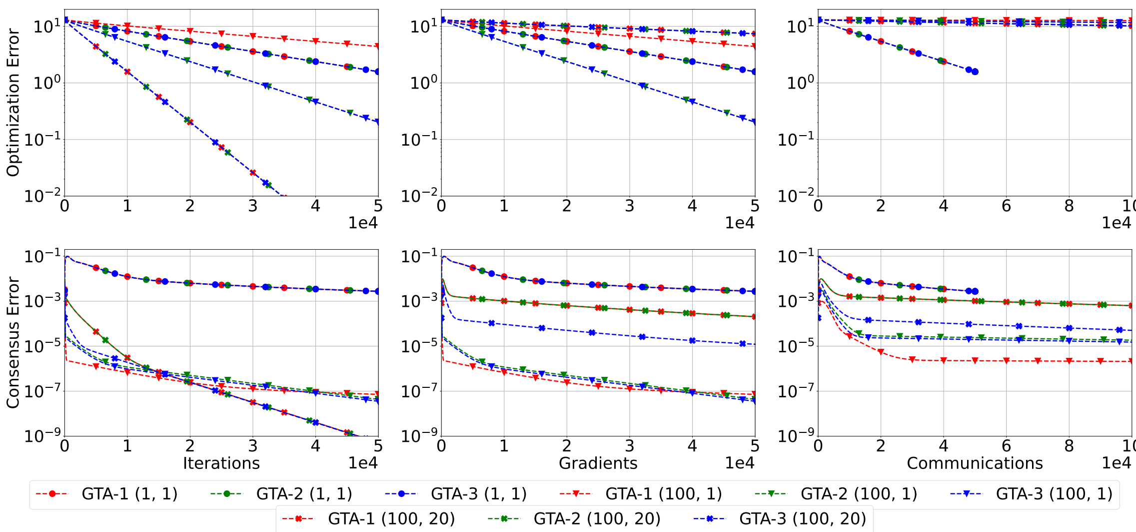

Figure 5: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for the synthetic quadratic problem over the line network.

2 Binary Classification Logistic Regression

Next, we consider ℓ_2-regularized binary classification logistic regression problems of the form

$$f(x) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{n_i} \log(1 + e^{-b_i^T A_i x}) + \frac{1}{n_i} \|x\|_2^2,$$

where each node $i \in \{1, 2, .., n\}$ has a portion of data samples $A_i \in \mathbb{R}^{n_i \times d}$ and corresponding labels $b_i \in \{0, 1\}^{n_i}$. Experiments were performed over the datasets summarised in Table 2.

2.1 Australian Dataset

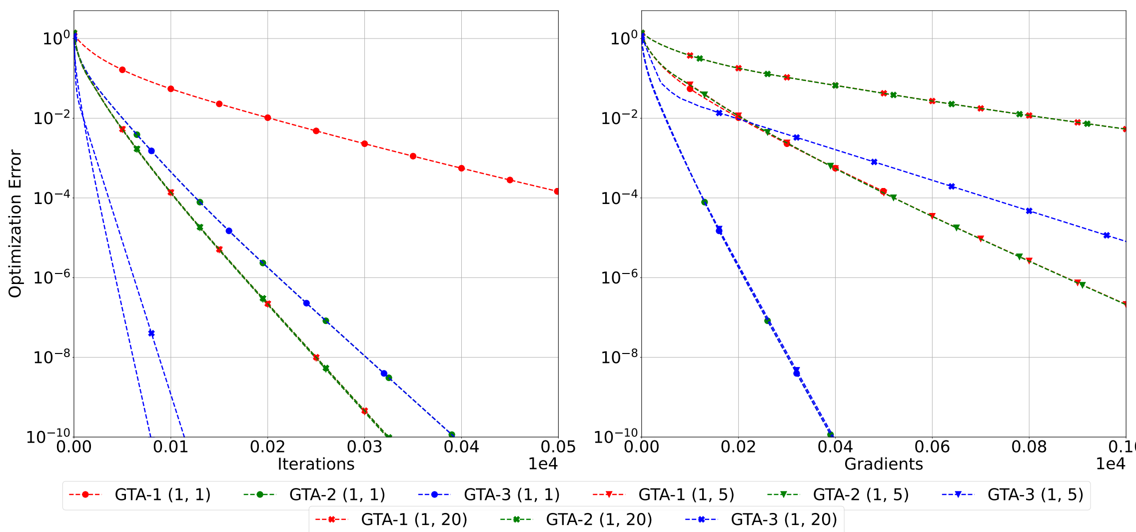

Figure 6: Optimization Error ($\|\bar{x}_k - x^*\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Australian dataset over the Full network.

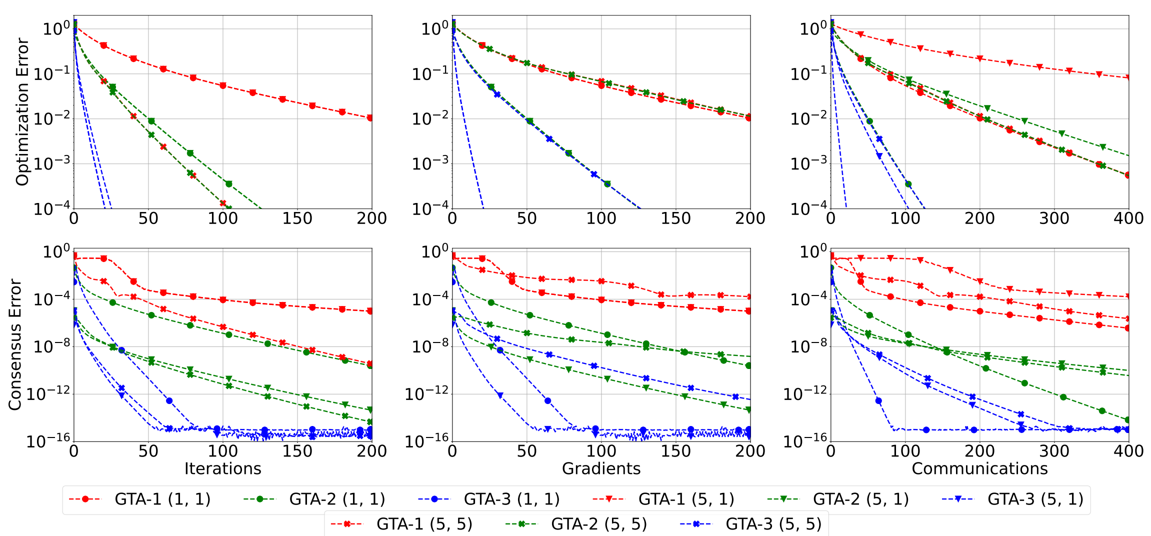

Figure 7: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Australian dataset over the Graph 1 network.

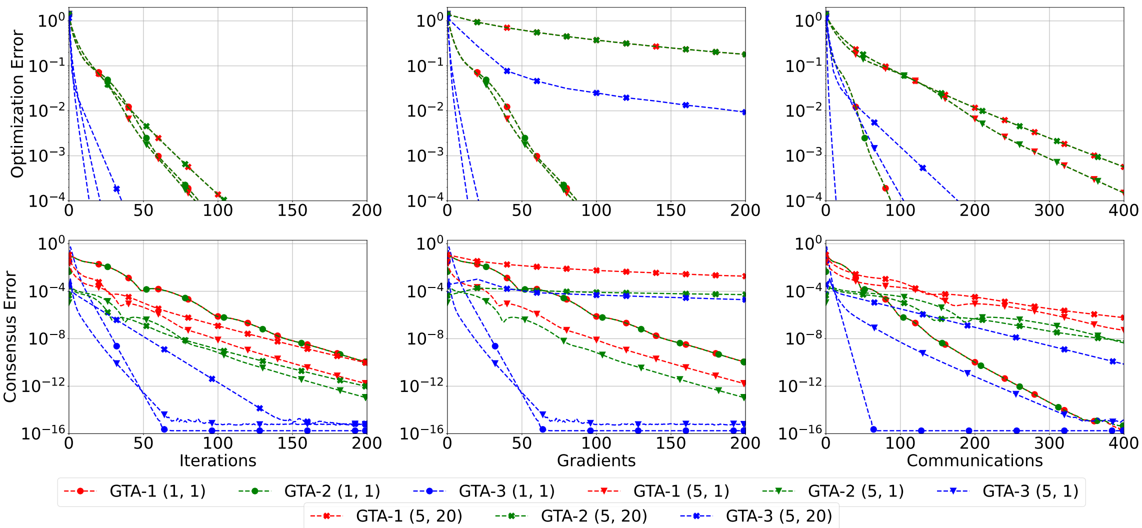

Figure 8: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Australian dataset over the Graph 2 network.

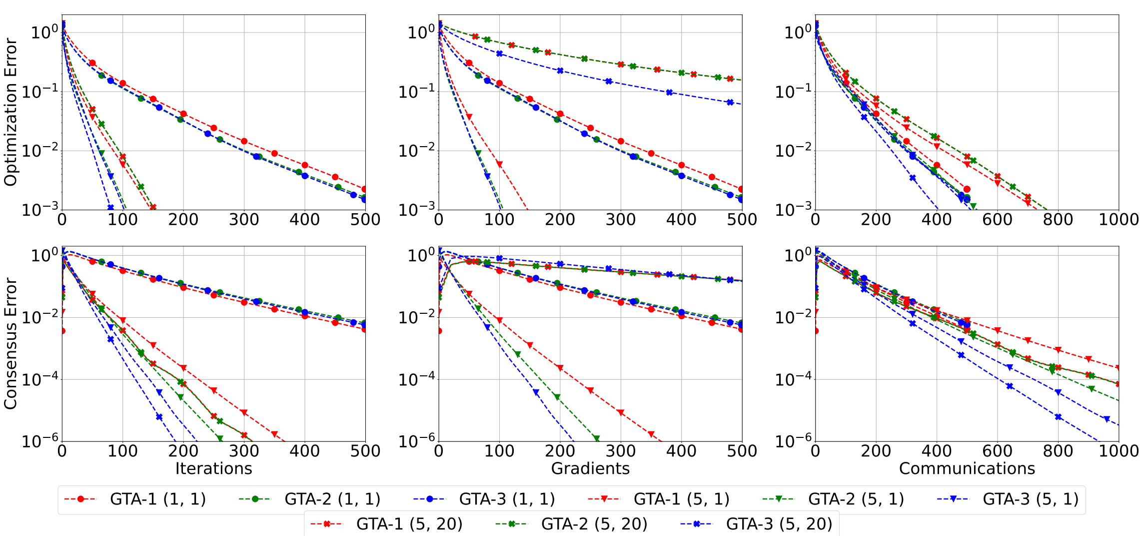

Figure 9: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations ffor binary logistic regression on Australian dataset over the star network.

2.2 Mushroom Dataset

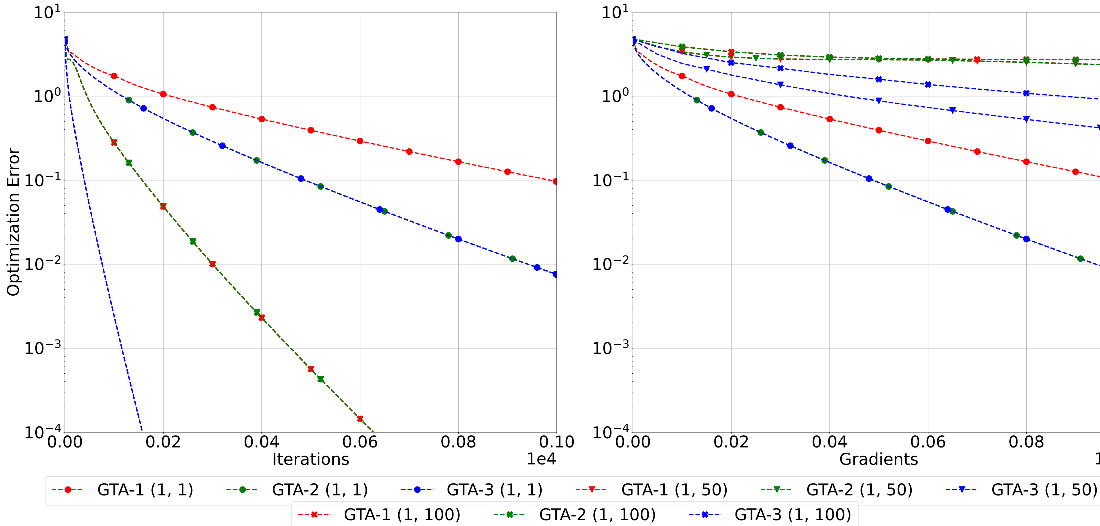

Figure 10: Optimization Error ($\|\bar{x}_k - x^*\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Mushroom dataset over the Full network.

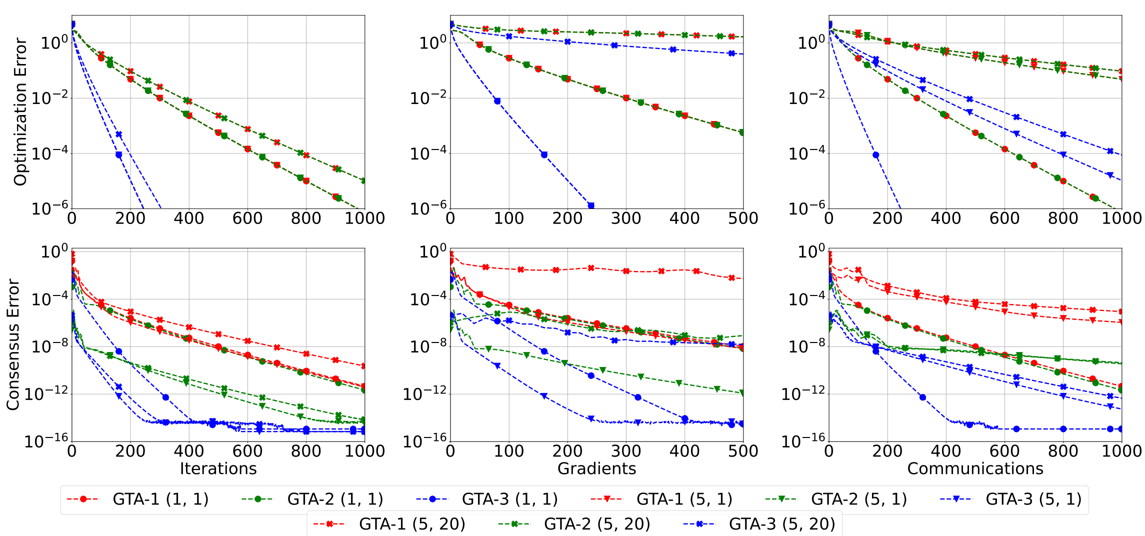

Figure 11: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Mushroom dataset over the Graph 1 network.

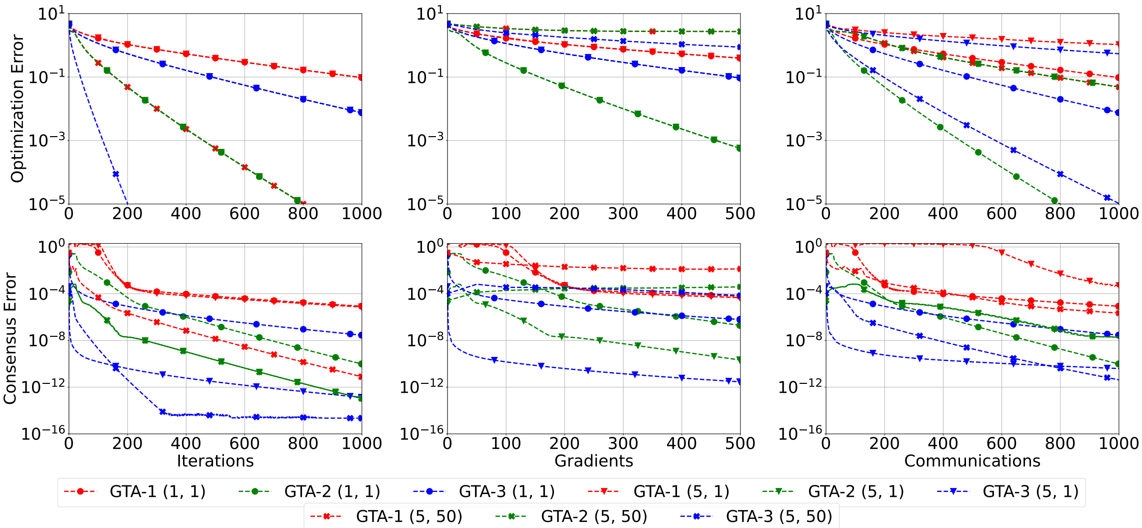

Figure 12: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Mushroom dataset over the Graph 2 network.

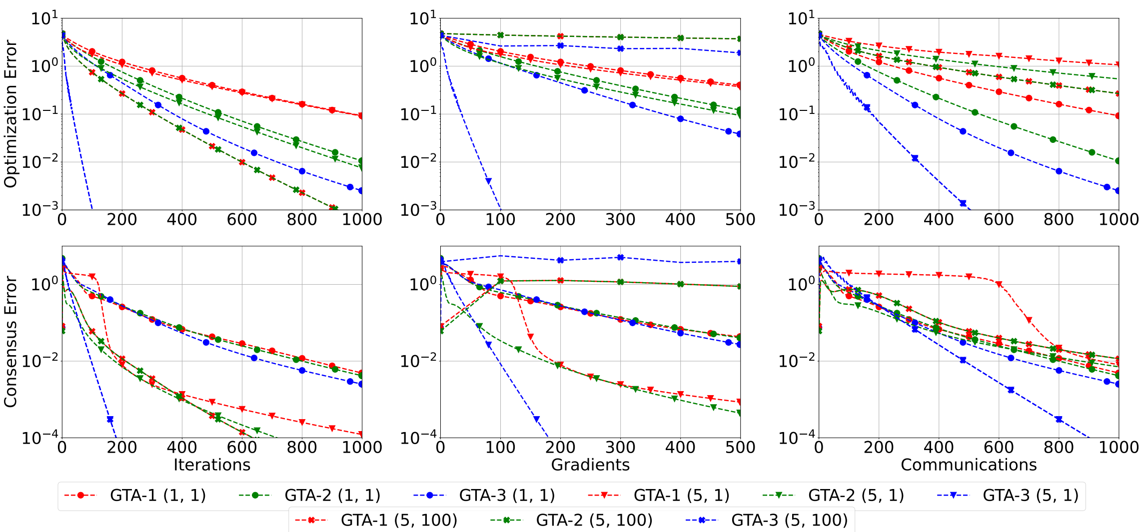

Figure 13: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations ffor binary logistic regression on Mushroom dataset over the star network.

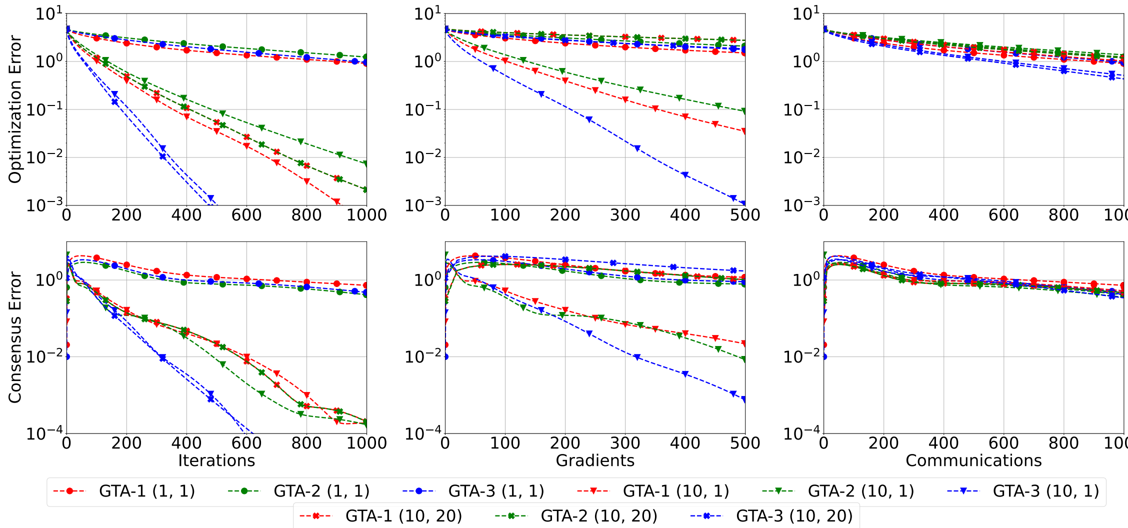

Figure 14: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations ffor binary logistic regression on Mushroom dataset over the cyclic network.

2.3 Phishing Dataset

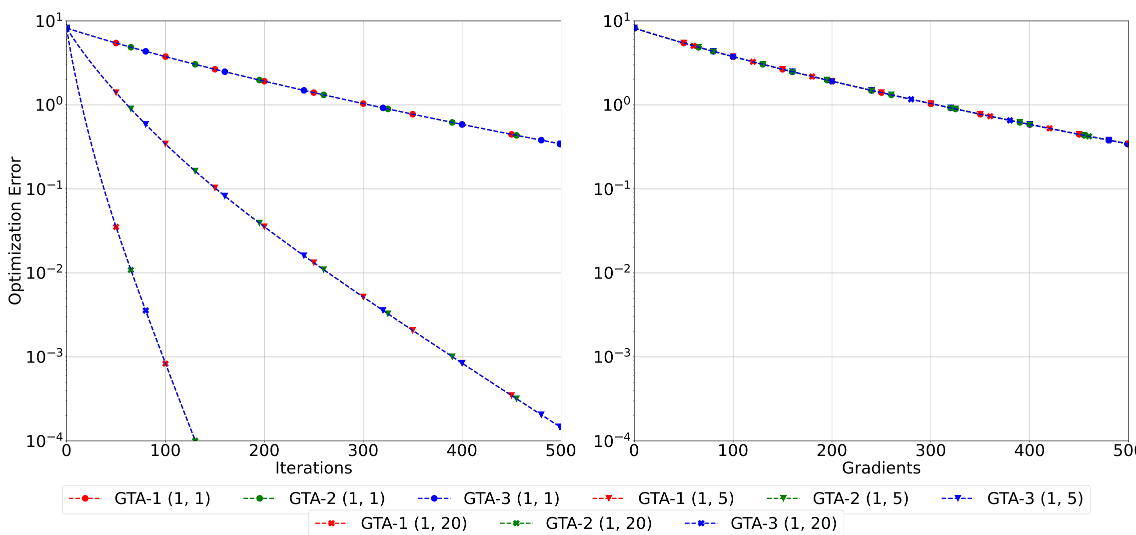

Figure 15: Optimization Error ($\|\bar{x}_k - x^*\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Phishing dataset over the Full network.

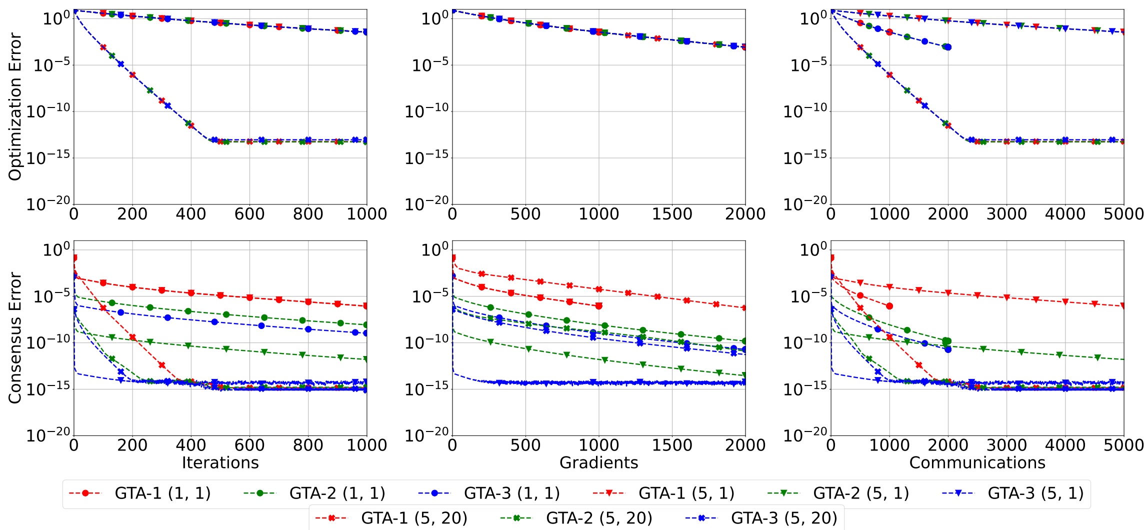

Figure 16: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Phishing dataset over the Graph 1 network.

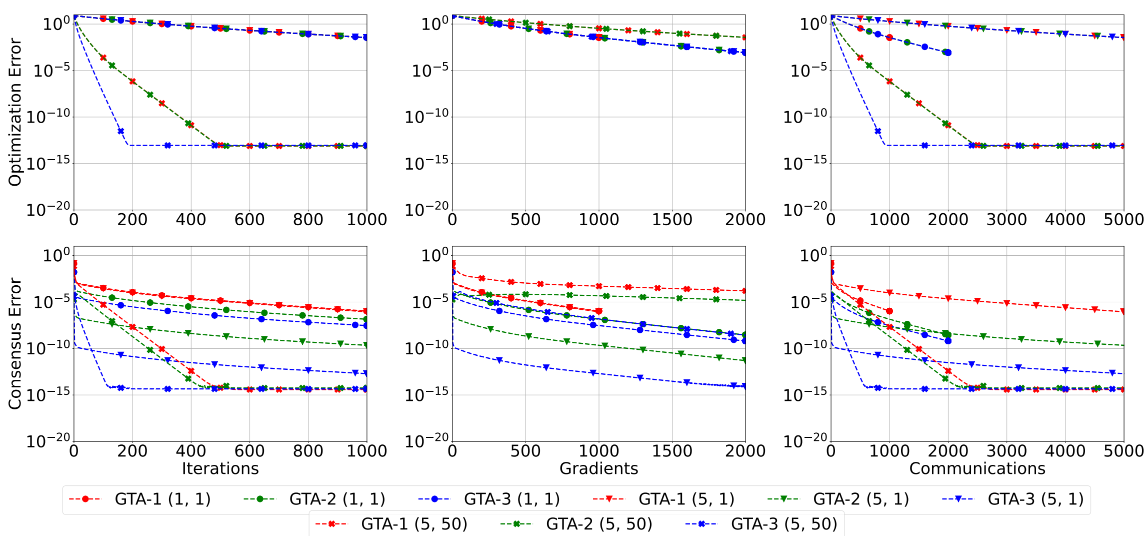

Figure 17: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Phishing dataset over the Graph 2 network.

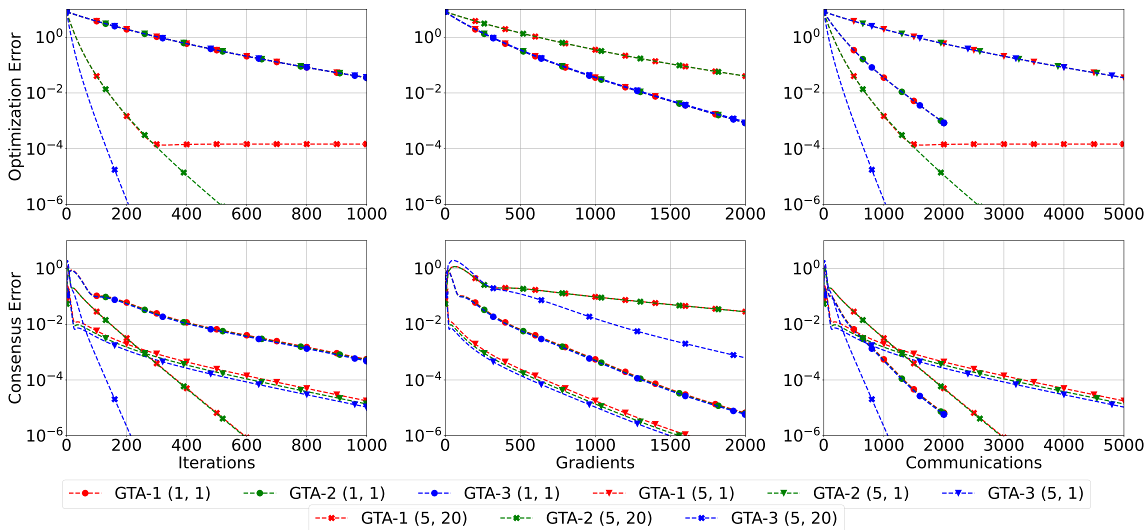

Figure 18: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Phishing dataset over the star network.

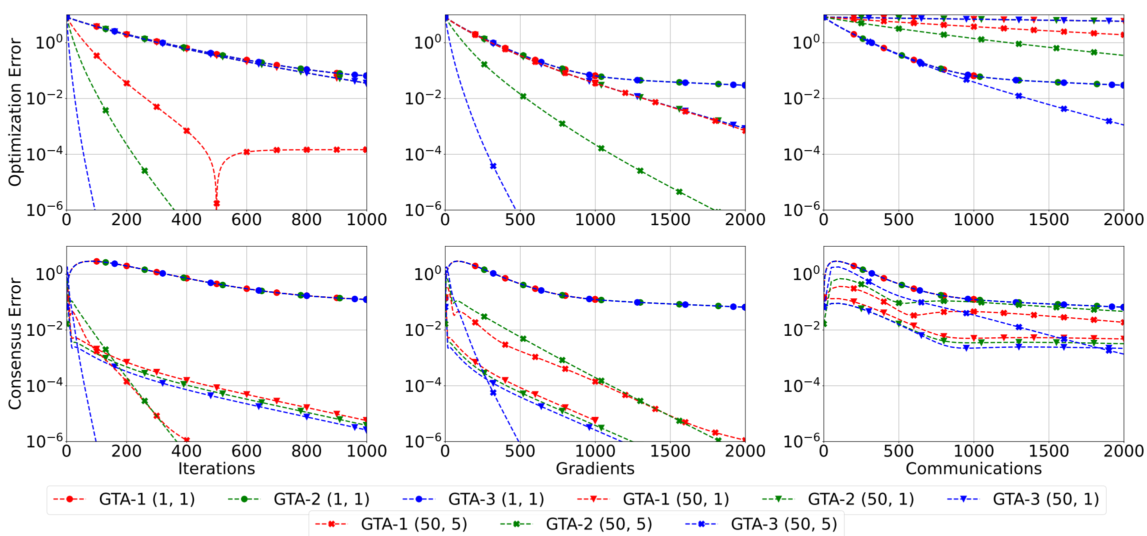

Figure 19: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Phishing dataset over the cyclic network.

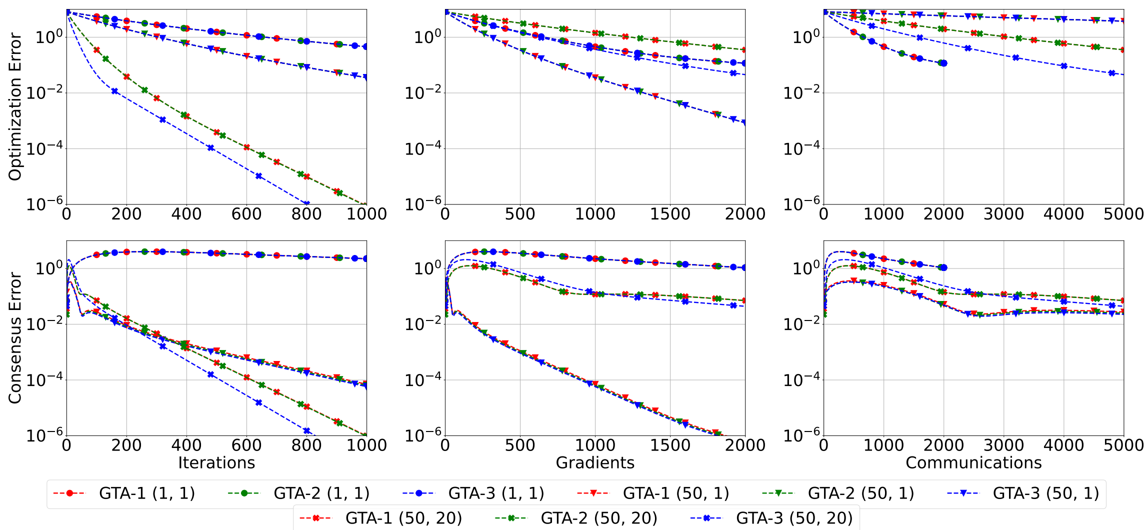

Figure 20: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Phishing dataset over the line network.

2.4 Sonar Dataset

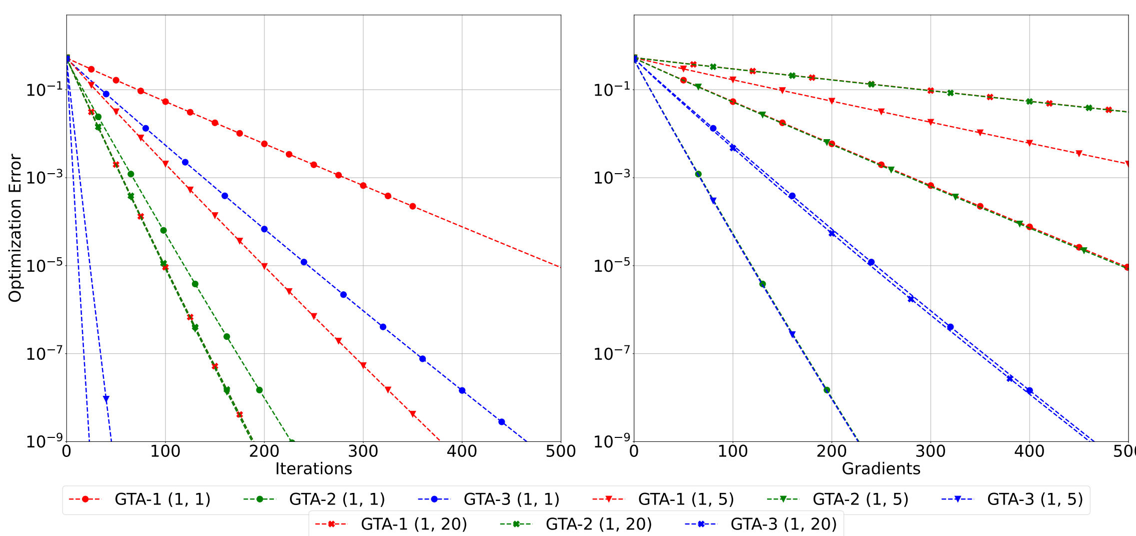

Figure 21: Optimization Error ($\|\bar{x}_k - x^*\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Sonar dataset over the Full network.

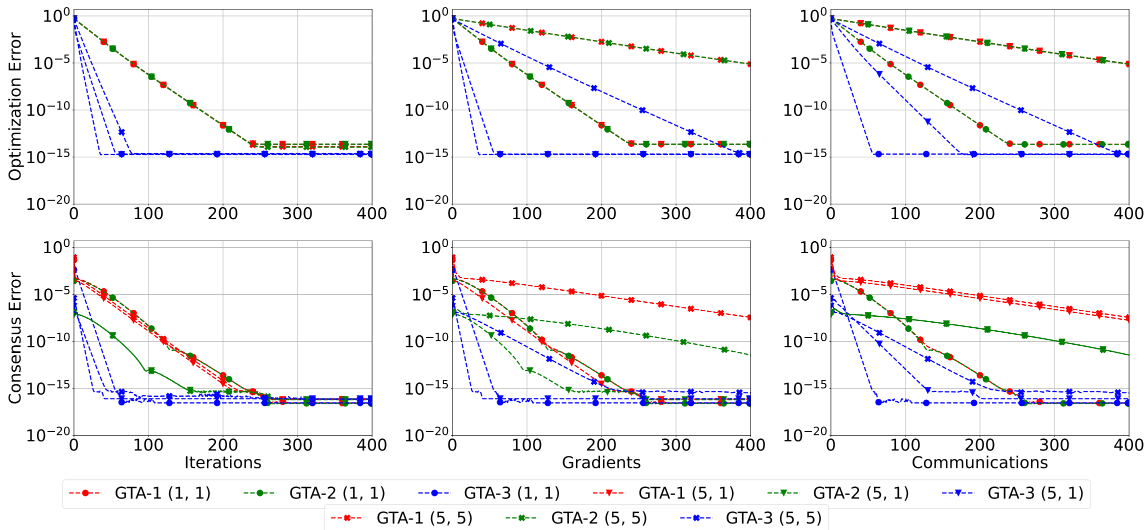

Figure 22: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Sonar dataset over the Graph 1 network.

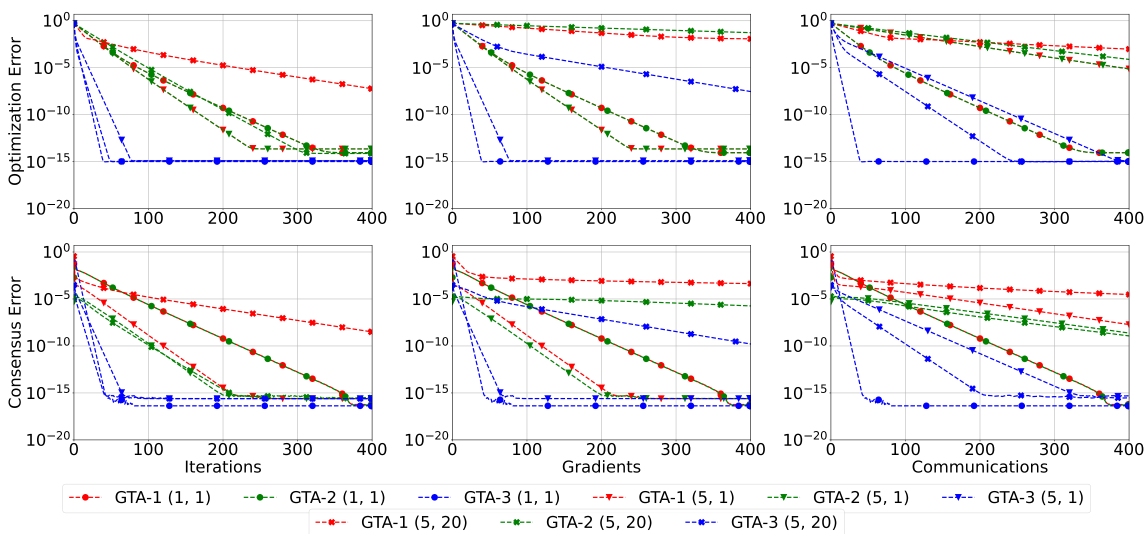

Figure 23: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Sonar dataset over the Graph 2 network.

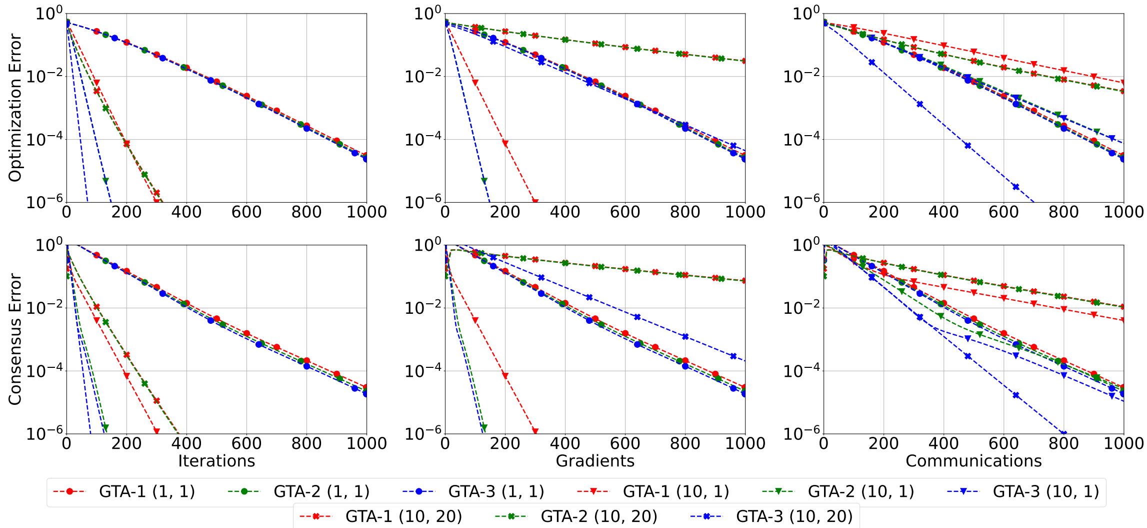

Figure 24: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Sonar dataset over the star network.

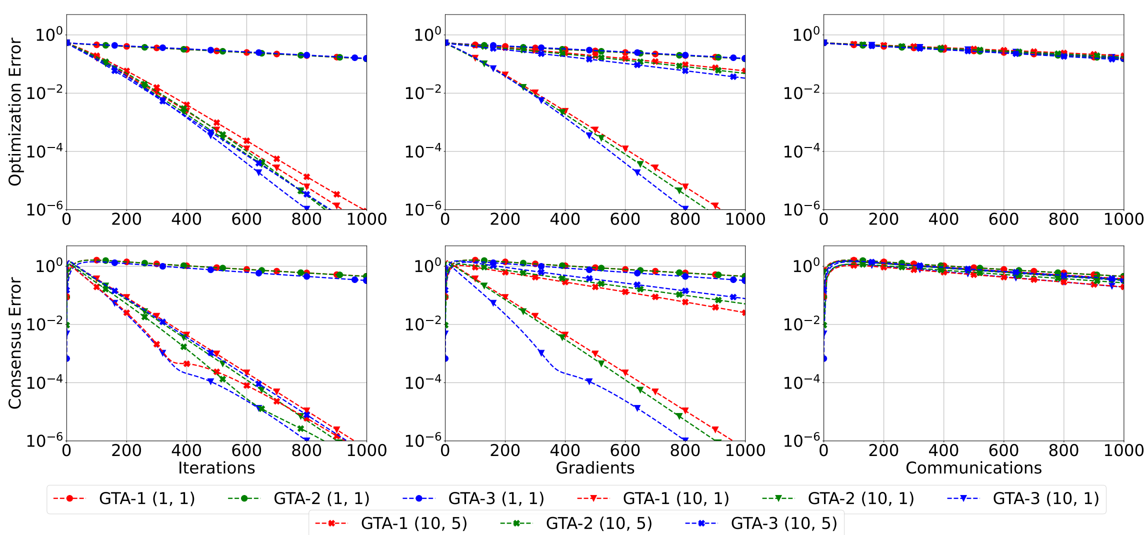

Figure 25: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Sonar dataset over the cyclic network.

2.5 Ionosphere Dataset

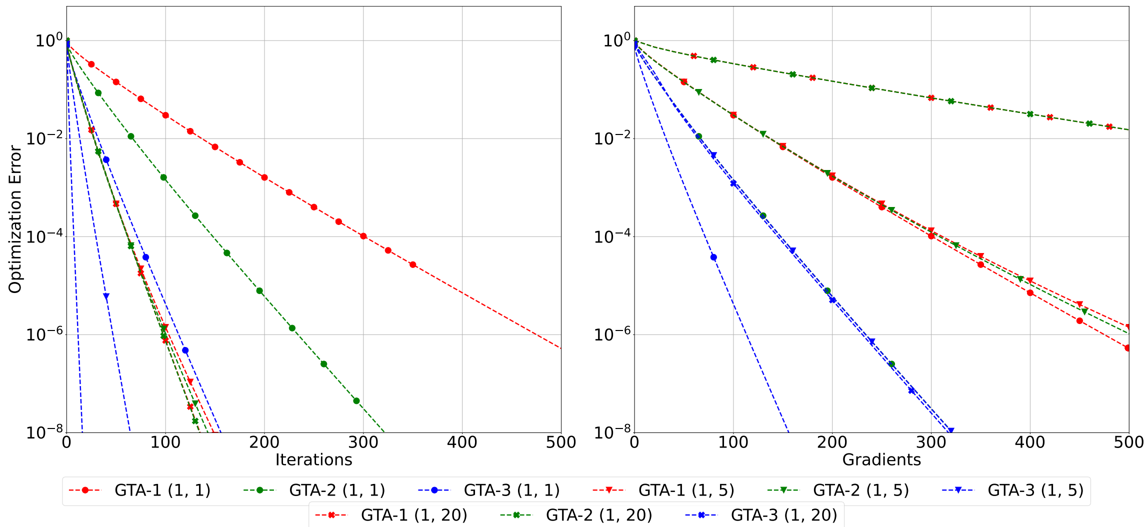

Figure 26: Optimization Error ($\|\bar{x}_k - x^*\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Ionosphere dataset over the Full network.

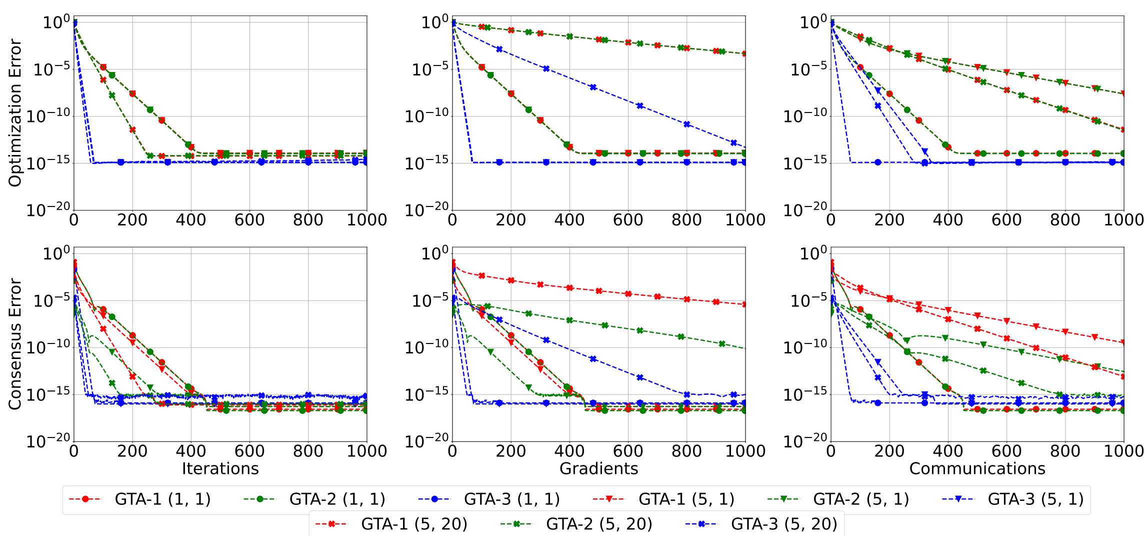

Figure 27: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Ionosphere dataset over the Graph 1 network.

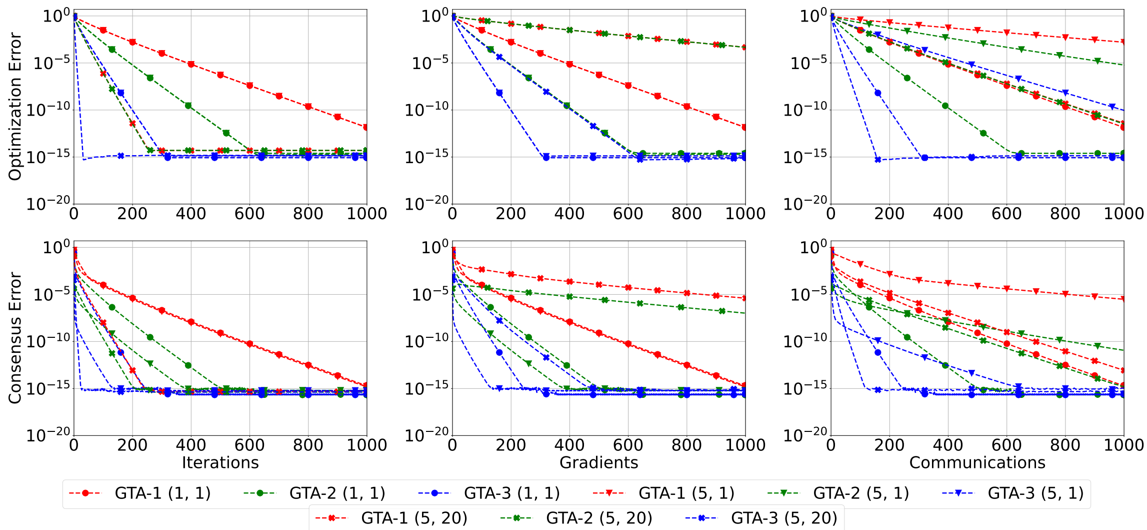

Figure 28: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Ionosphere dataset over the Graph 2 network.

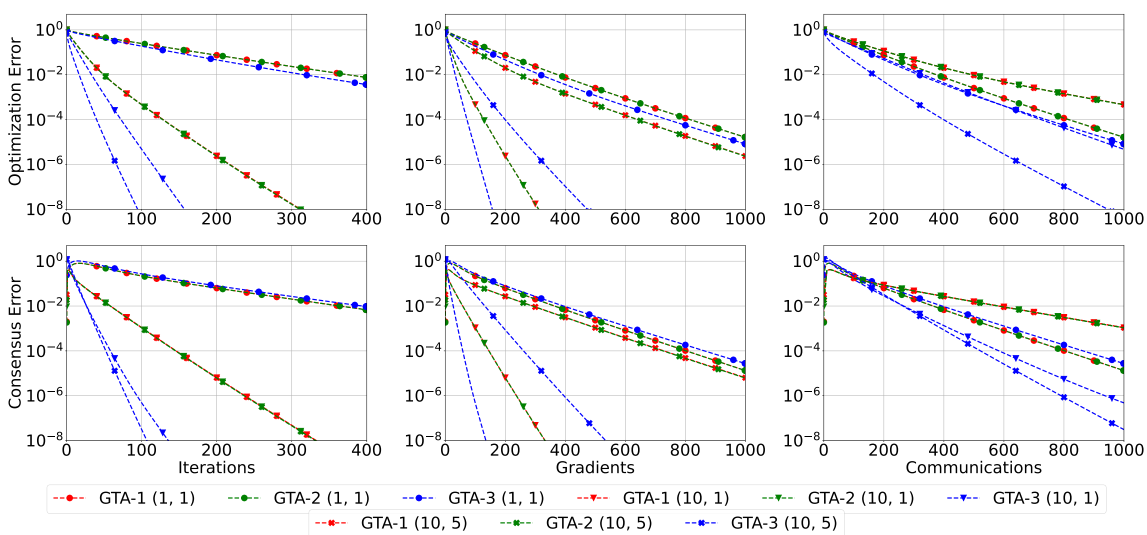

Figure 29: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Ionosphere dataset over the star network.

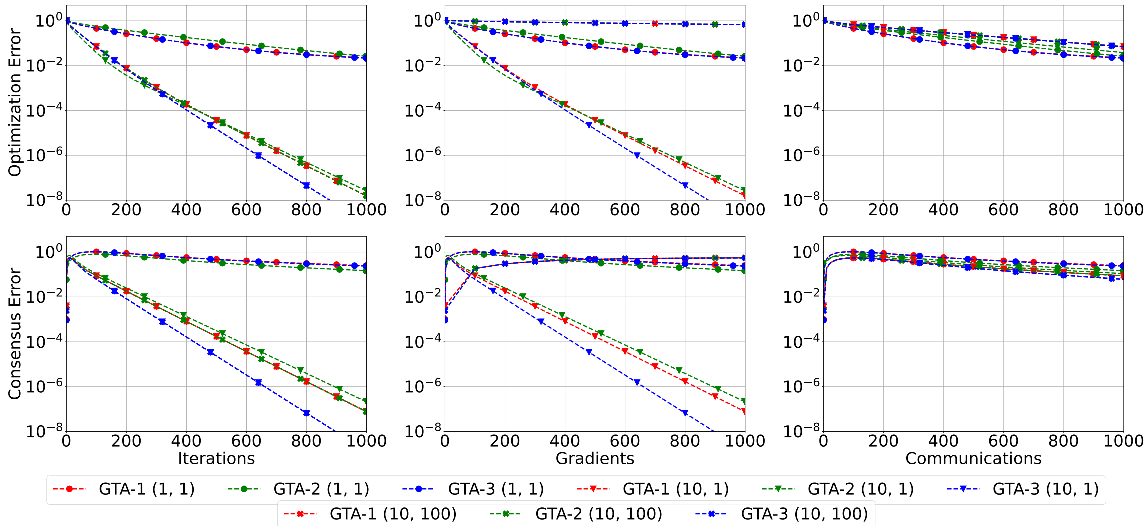

Figure 30: Optimization Error ($\|\bar{x}_k - x^*\|_2$) and Consensus Error ($\|\mathbf{x}_k - \bar{\mathbf{x}}_k\|_2$) of `GTA-1`, `GTA-2` and `GTA-3` with respect to number of iterations, communications and gradient evaluations for binary logistic regression on Ionosphere dataset over the cyclic network.

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