COO - Assignment

Q1) b) Function value for gradient descent algo in (a) was: 1.171875

whereas for (b), it was: 2.5536

- In (a) our step size reduced by factor of 0.5, while in (b) our step size t reduced by factor of beta = 0.1, in the backtracking algo.
  - We started with initial step size = 1 in both the cases but

  - In backtracking we iteratively reduce step size until we get a sufficiently reduced value of f.

  - In both the parts (a) & (b), when we backtrack in (a), we use $\beta = 0.1$ which results in a very crude & basic search while in (b) we use $\beta = 0.5$ keeping α value same in both (a) & (b), in (b), we get a much less crude search compared to (a)

Q1) c) The f values for Newton's descent is diff. than gradeint descent algo in (a).
Since Newton's algo applies much stronger constraints relative to gradeint descent, in terms of function differentiability criteria, it converges in far fewer steps and uses more info than gradeint which generally requires more no of iterations.

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Q2) minimize $f(x,y,z) = e^x + 2y^2 + 3z^2$

subj to $x - 5z = 1$

$y + z = 4$

we can form a KKT linear system as.

Assuming KKT is non singular,

$$\begin{bmatrix} \nabla^2 f(x) & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} \Delta x_{nt} \\ w \end{bmatrix} = \begin{bmatrix} -\nabla f(x) \\ 0 \end{bmatrix}$$

$w \rightarrow$ optimal dual variable

we get, $A \Delta x_{nt} = 0,$

$\nabla^2 f(x) \Delta x_{nt} + A^T w = -\nabla f(x)$

$$e^x \Delta x + w_1 = -e^x$$
$$4\Delta y + w_2 = -4y$$
$$6\Delta z - 5w_1 + w_2 = -6z$$
$$\Delta x - 5\Delta z = 0$$
$$\Delta y + \Delta z = 0$$

$$\begin{bmatrix} e^x & 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 & 1 \\ 0 & 0 & 6 & -5 & 1 \\ 1 & 0 & -5 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -e^x \\ -4y \\ -6z \\ 0 \\ 0 \end{bmatrix}$$

we have got final optimal values as

(1.14491625, 3.97101675, 0.02898325)

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$\hookrightarrow e^{(1.14491)} \Delta x + w_1 = -e^{(1.14491)}$

$\hookrightarrow (3.142)\Delta x + w_1 = -3.142$ ①

$4(\Delta y) + w_2 = -4(3.971)$

$4\Delta y + w_2 = -15.884$ ②

$6\Delta z - 5(w_1) + w_2 = -6(0.03)$

$6\Delta z - 5w_1 + w_2 = -0.18$ ③

$\Delta x = 5\Delta z$ ④

$\Delta y = -\Delta z$ ⑤

$\hookrightarrow$ on solving this values, & eq^n,

we get $w_1 = -3.14$, $w_2 = -15.88$. } optical dual var.

$\therefore$ Ans $\rightarrow$ $(-3.14, -15.88)$