

Examples

Simplify the Boolean function
 $F = A'B'C' + B'CD' + A'BCD' + AB'C'$

Yund Method

Convert SOP to SSOP

$$F = \bar{A}\bar{B}\bar{C}(D+\bar{D}) + \bar{B}C\bar{D}(A+\bar{A}) + \bar{A}B\bar{C}\bar{D} + A\bar{D}\bar{C}(D+\bar{D})$$

$$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D}$$

$$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D}$$

SSOP

Examples

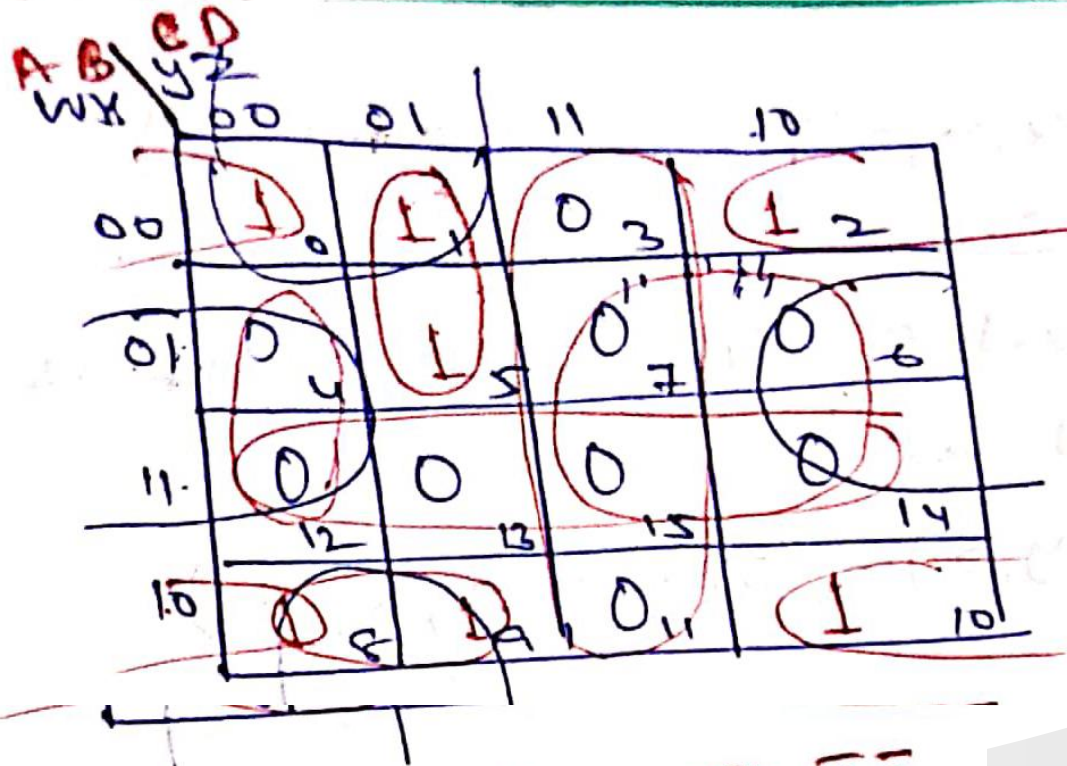
Simplify the Boolean function

$$F(A, B, C, D) = \sum (0, 1, 2, 5, 8, 9, 10)$$

a) SOP

b) Pos

form



a) $F = \overline{B}\overline{D} + \overline{A}C\overline{D} + \overline{A}B\overline{C}$

Examples

If Squares marked with 0's are combined then we obtained Simplified Complemented function

$$F' = AB + CD + B\bar{D}$$

~~$$= \overline{AB}(\bar{C} + \bar{C})(\bar{C}D + \bar{C}\bar{D}) +$$~~

$$F'' = \overline{AB + CD + B\bar{D}}$$

- Apply DeMorgan

$$= \bar{A}\bar{B} * \bar{C}\bar{D} * \overline{B\bar{D}}$$

- again apply DeMorgan

$$F = (\bar{A} + \bar{B})(\bar{C} + \bar{D})(\bar{B} + D)$$

(b) POS form Simplified

CSA
4.24

(b)

Has 0010

Examples

$$F(x, y, z) = \sum (1, 3, 4, 6)$$

In product of 5 maxterms, it is expressed as

$$F(x, y, z) = \prod (0, 2, 5, 7)$$

$x \backslash yz$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$$F = \bar{x}z + x\bar{z} \quad \text{--- SOP}$$

$$F = (x+z)(\bar{x}+\bar{z})$$



Examples

Simplify the Boolean Function

$$F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

+ d(0, 2, 5) don't care

wx \ yz	00	01	11	10
00	X ₀	1 ₁	1 ₃	X ₂
01		X ₅	1 ₇	X ₆
11			1 ₁₁	
10			1 ₁₅	X ₁₀

SOP

$$F = \bar{w}z + yz$$

Combining 1's & X gives

wx \ yz	00	01	11	10
00	X ₀			X ₂
01	0 ₄	X ₅		0 ₆
11	0 ₁₂	0 ₁₃		
10	0 ₈	0 ₉		0 ₁₀

POS

$$F = z(y + \bar{w})$$

Combining X's & 0's gives



Alternate method to find out POS expression from SOP

The Simplified SOP function

$$F = \bar{w}z + yz$$

In POS form we obtained

$$F' = \bar{x} + w\bar{y}$$

Complementing again we get-

$$F'' = F = (\bar{\bar{x}}) \cdot (\overline{w\bar{y}})$$

$$\boxed{F = z(\bar{w} + y)} \quad \text{Equal to the POS exp}$$

But in even rule

Examples

$$F(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + d(0, 2, 5)$$

AB \ CD	00	01	11	10
00	X ₀	1 ₁	1 ₃	X ₂
01	4	X ₅	1 ₇	6
11	12	13	1 ₁₅	14
10	8	9	1 ₁₁	10

$f_{min} = CD + \bar{A}\bar{B}$ WITH Don't Care

$f_{min} = CD + \bar{A}\bar{B}D$ - without don't care

Examples

$$F(A, B, C, D) = \prod_m (4, 5, 6, 7, 8, 12)$$

AB \ CD 00 01 11 10 d (1, 2, 3, 9, 11, 14)

$$F(A, B, C, D) = \prod_m (4, 5, 6, 7, 8, 12)$$

AB \ CD 00 01 11 10 d (1, 2, 3, 9, 11, 14)

00		X ₁	X ₃	X ₂
01	0 ₄	0 ₅	0 ₇	0 ₆
11	0 ₁₂	1 ₃	1 ₅	X ₁₄
10	0 ₈	X ₉	X ₁₁	1 ₀

$$f_{min} = \bar{A}B + A\bar{C}\bar{D} \text{ SOP}$$

if 'x' not make simpler expression
then don't use them neglect them.

Examples

Ex minimize the expression using k-maps

$$F(A, B, C, D) = \sum_m (1, 4, 7, 10, 13) + \sum_d (5, 14, 15)$$

AB \ CD	00	01	11	10	
00	0	1	2	2	/
01	1	X	1	6	/
11	1	1	X	X	/
10	8	9	11	1	/

$$F = BD + \bar{A}\bar{C}D + \bar{A}B\bar{C} + AC\bar{D}$$

✓ SOP k-map



Examples

Ex Obtained the Simplified expression in pos form.
 $F(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5) +$
 $d(10, 11, 12, 13, 14, 15)$

AB \ CD	00	01	11	10
00	1 ₀	1 ₁	1 ₂	1 ₃
01	1 ₄	1 ₅	7	6
11	X ₁₂	X ₁₃	X ₁₅	X ₁₄
10	8	9	X ₁₁	X ₁₀

$\Pi m(6, 7, 8, 9)$

SOP

$$F = \bar{A}\bar{B} + \bar{A}\bar{C}$$

Examples

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	X ₁₂	X ₁₃	X ₁₅	X ₁₄
10	8	9	X ₁₁	X ₁₀

Pos

$$F = \bar{A} \cdot (\bar{B} + \bar{C})$$

Same by indirect method

$$\begin{cases} F' = A + BC \\ F'' = \frac{A + BC}{\bar{A} \cdot (\bar{B} + \bar{C})} \\ F = \end{cases}$$

Examples

(Ex) Simplify Boolean expression using K-map $(\bar{x}+y+w)$

Soln $F = \bar{A} + AB + A\bar{B}\bar{D} + A\bar{B}\bar{D} + C$

Expand SOP in to SSOP form

$$F = \bar{A}(B+\bar{B})(C+\bar{C})(D+\bar{D}) + AB(C+\bar{C})(D+\bar{D}) + A\bar{B}\bar{D}(C+\bar{C}) + A\bar{B}\bar{D}(C+\bar{C}) + C(A+\bar{A})(B+\bar{B})(D+\bar{D})$$

$$F = \sum m(0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15)$$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

LEM's $F = C + \bar{D} + \bar{A} + B$
 $= \bar{A} + B + C + \bar{D}$



Examples

ii)

$$f = \bar{A}\bar{B}D + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}BD + A\bar{B}\bar{C}D$$

Soln

$$\sum m(0, 2, 5, 7, 11)$$

iii)

$$f = ABD + \bar{A}\bar{C}\bar{D} + \bar{A}B + \bar{A}\bar{C}\bar{D} + A\bar{B}D$$

Soln

$$\sum m(0, 2, 4, 5, 6, 7, 9, 11, 13, 15)$$

$$f_{min} = \bar{A}\bar{D} + BD + AD$$

Examples

Ex
PROBLEM'S

Obtained the simplified exp in Pos form

$$1) F(A, B, C, D) = \sum m(5, 6, 7, 8, 9, 12, 13, 14)$$

Ans:

$$(B+C)(A+B)(A+C+D) \cdot (\bar{A} + \bar{B} + \bar{C})$$

$$2) F(W, X, Y, Z) = \sum m(0, 1, 5, 7, 8, 10, 14, 15)$$

Ans:

$$(W + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})(\bar{W} + Y + \bar{Z}) \\ (\bar{X} + Y + W)$$

Five variables K Map

- The center double line is like a mirror with each square being adjacent, not only to its four neighboring squares, but also to its mirror image

eg) min term - 31 in five variable map is adjacent to min term 30, 15, 29, 23 and 27
 The same happened in six variable map

C — Common

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

A [11 10] B

E D E

Examples

Ex) Simplify the Boolean function

$$F(A, B, C, D, E) = \sum (0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)$$

$F(A, B, C, D, E) = \sum (0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)$

AB \ CDE	000	001	011	010	110	111	101	100
00	1 0	1 1	1 3	1 2	1 6	1 7	1 5	1 4
01	1 8	1 9	1 11	1 10	1 14	1 15	1 13	1 12
11	1 24	1 25	1 27	1 26	1 20	1 31	1 29	1 28
10	1 16	1 17	1 19	1 18	1 22	1 23	1 4	1 20

$$F(A, B, C, D, E) = BE + A\bar{D}E + \bar{A}\bar{B}E$$

Examples

(Ex) Reduce the following expression to the simplest possible pos and sop.

$$f = \sum m(6, 9, 13, 16, 19, 25, 27, 29, 31)$$

$$+ d(2, 3, 11, 15, 17, 24, 28)$$

Soln

The given exp in pos form is

$$f = \prod m(0, 1, \cancel{2}, \cancel{3}, 4, 5, 7, 8, 10, \cancel{11}, 12, 14, 16, 20, 21$$

$$22, 23, 26, 30). \quad \prod d(2, 3, 11, 15, 17, 24, 28)$$

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	X ₂	X ₃		4	5	6
01	7	8	X ₉	10	11	X ₁₂	13	14
11	X ₁₅	16	17	18	19	20	21	X ₂₂
10	23	X ₂₄	25	26	27	28	29	30

pos

$$F = (D+E)(\bar{B}+E)(A+B+\bar{E})(B+\bar{C}+\bar{D})$$

$$f_{min} = BE + \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}DE - \text{SOP} \quad (A+B+\bar{C}?)$$

Six variables K map

Six Variable K-map

ABC	DEF							
	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	9	11	10	14	15	13	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
100	48	49	51	50	54	55	53	52
101	56	57	59	58	62	63	61	60
111	40	41	43	42	46	47	45	44
110	32	33	35	34	38	39	37	36

Handwritten annotations: A red bracket labeled 'D' spans columns 110 and 111. A red bracket labeled 'C' spans rows 001, 011, and 010.

Limitation of K-map

- ① As the no of variables increases/exceeds 6 then it becomes difficult to make judgement about which combination form the minimal expression.
 - ② It is impossible to work with problem of 7 or more variables using K-map.
 - ③ K-map simplification is a manual technique and simplification process depends on human ability.
- Alternate to this is Tabular method to simplify Boolean expression. (Quine-McCluskey method)

Some extra examples

ii)

$$F = \bar{A}\bar{B}D + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}BD + A\bar{B}\bar{C}D$$

Soln

$$\sum m(0, 2, 5, 7, 11)$$

iii)

$$F = ABD + \bar{A}\bar{C}\bar{D} + \bar{A}B + \bar{A}\bar{C}\bar{D} + A\bar{B}D$$

Soln

$$\sum m(0, 2, 4, 5, 6, 7, 9, 11, 13, 15)$$

$$F_{min} = \bar{A}\bar{D} + BD + AD$$