Examples

Simplify the Boolean function

$F = A'B'C' + B'CD' + A'BCD' + AB'C'$

IInd Method

Convert SOP to SSOP

$F = \bar{A}\bar{B}\bar{C}(D+\bar{D}) + \bar{B}C\bar{D}(A+\bar{A}) + \bar{A}BC\bar{D} + A\bar{B}\bar{C}(D+\bar{D})$

$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

$= \bar{A}\bar{B}$

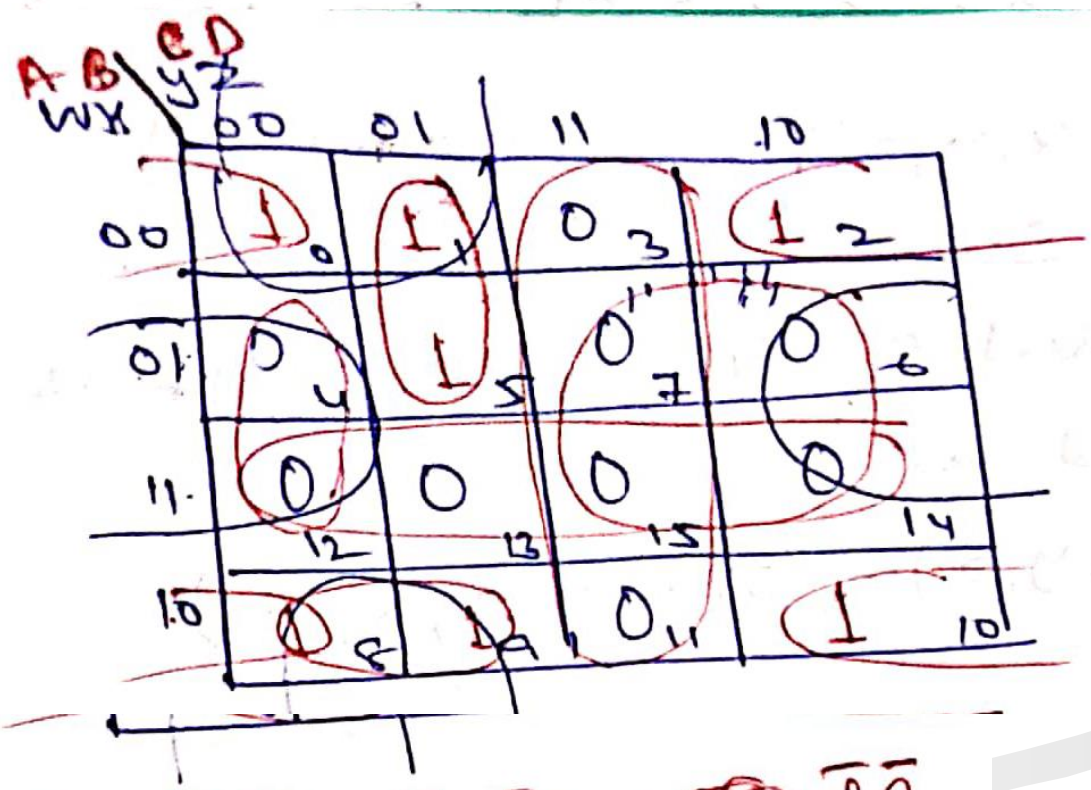
SSOP

Examples

Simplify the Boolean function

$F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$

a) SOP b) POS form



a) $f = \bar{B}\bar{D} + \bar{A}\bar{C}D + \bar{B}\bar{C}$

Examples

If Squares marked with 0's are combined there we obtained Simplified Complemented function

$$f' = AB + CD + B\bar{D}$$

~~$= AB(C+\bar{C})(D+\bar{D}) +$~~

$$F'' = \overline{AB + CD + B\bar{D}}$$ — Apply Demorgan

$$= \bar{A}\bar{B} \cdot \bar{C}\bar{D} \cdot \overline{B\bar{D}}$$ — again apply Demorgan

(b) $$F = (\bar{A}+\bar{B})(\bar{C}+\bar{D})(\bar{B}+D')$$

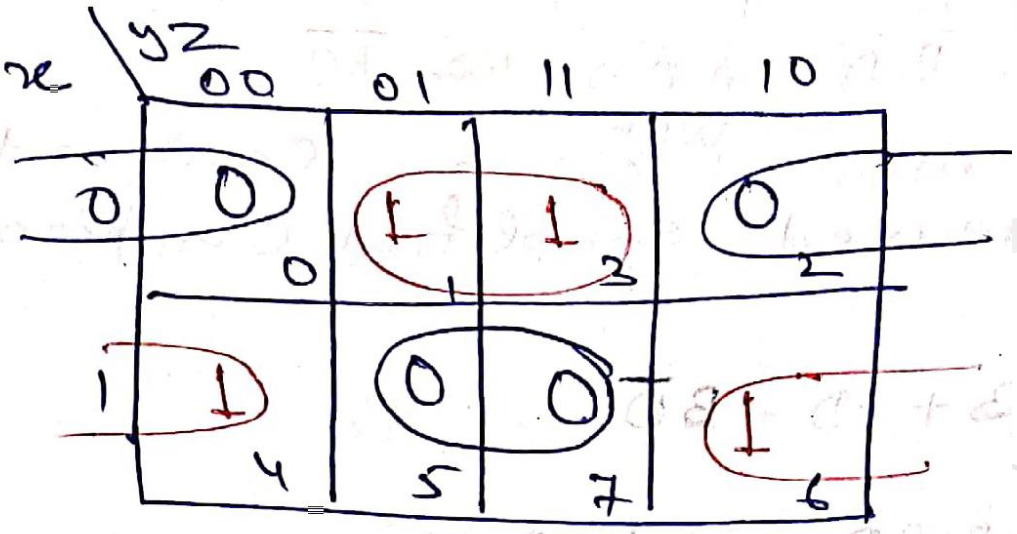
(POS form Simplified)

Examples

$$f(x,y,z) = \Sigma(1,3,4,6)$$

In product of maxterms, it is expressed as

$$f(x,y,z) = \Pi(0,2,5,7)$$



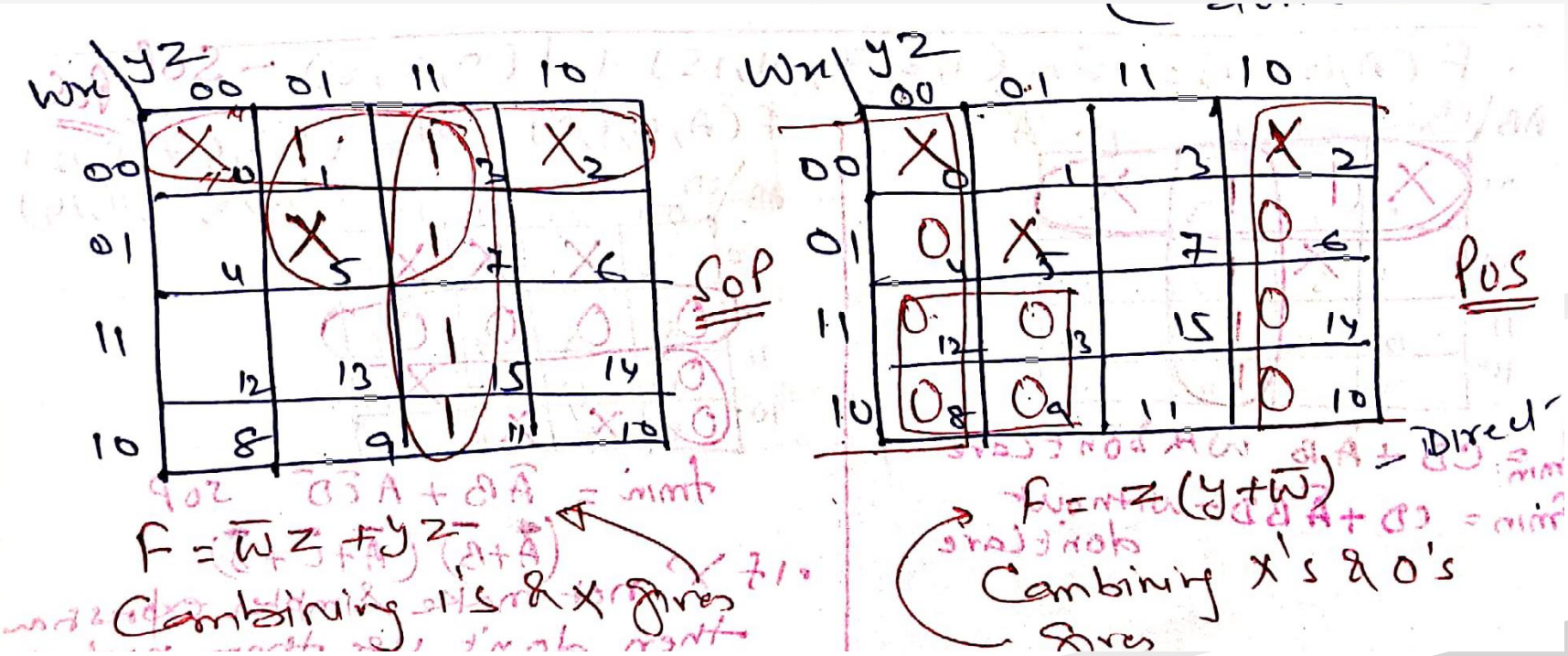
$$f = \bar{x}z + x\bar{z} \quad - \text{SOP}$$

$$f = (x+z)(\bar{x}+\bar{z})$$

Examples

Simplify the Boolean function

$f(w,x,y,z) = \Sigma(1,3,7,11,15) + d(0,2,5)$ don't care



Alternate method to find out POS expression from SOP

The Simplified SOP function

$$f = \bar{w}z + yz$$

In Pos form we obtained

$$f' = \bar{z} + w\bar{y}$$

Complementing again we get-

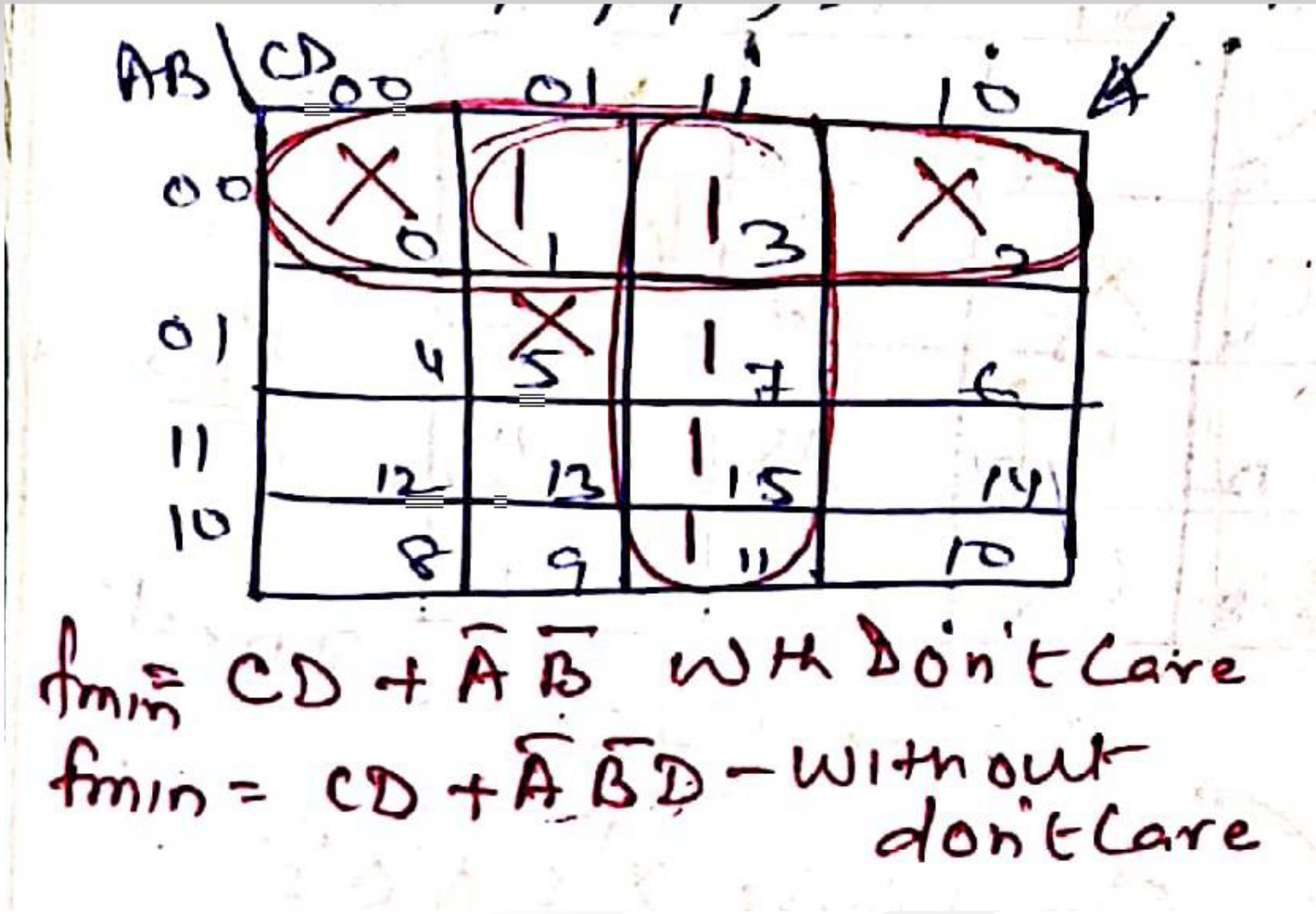
$$f'' = f = (\bar{\bar{z}}) \cdot (\overline{w\bar{y}})$$

$$\boxed{f = z(\bar{w} + y)}$$ - Equal to the Pos exp

But in even

Examples

$F(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$



Examples

$F(A,B,C,D) = \Pi_M (4,5,6,7,8,12) \cdot d(1,2,3,9,11,14)$ — POS

AB \ CD: 00, 01, 11, 10

$F(A,B,C,D) = \Pi_M (4,5,6,7,8,12) \cdot d(1,2,3,9,11,14)$ — POS

AB \ CD	00	01	11	10
00	0	X_1	X_3	X_2
01	0_4	0_5	0_7	0_6
11	0_{12}	13	15	X_{14}
10	0_8	X_9	X_{11}	10

$f_{min} = \bar{A}B + A\bar{C}\bar{D}$ SOP

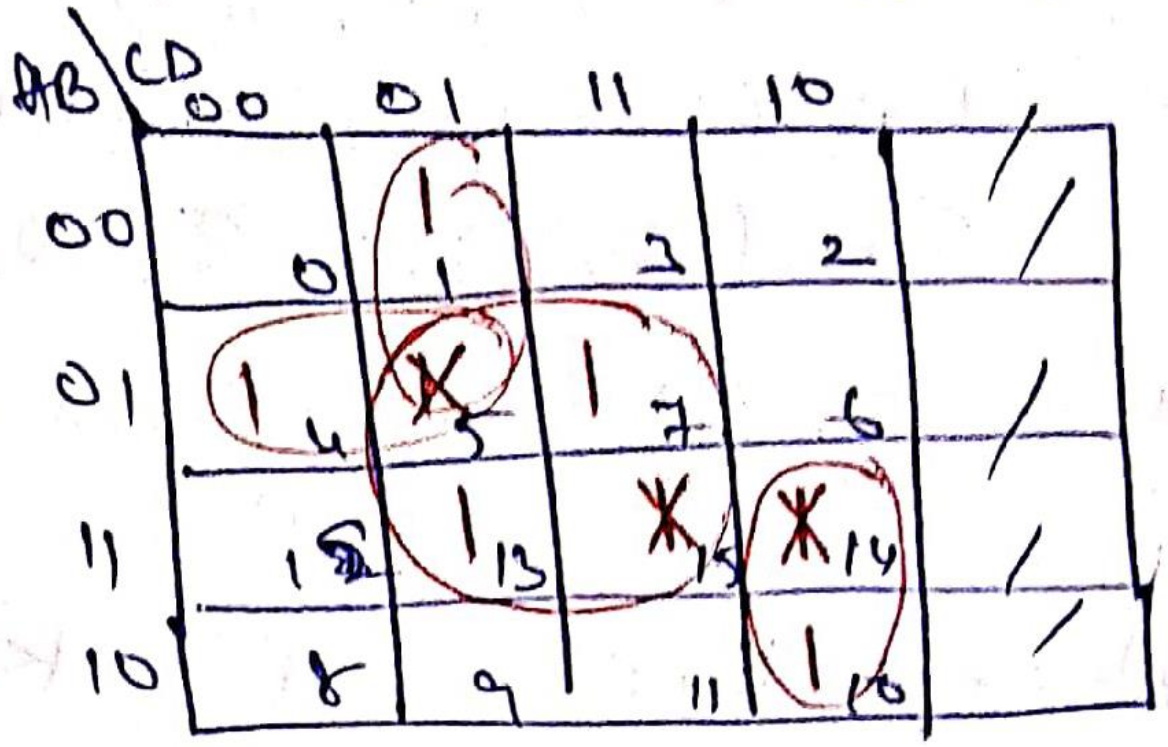
$(\bar{A}+B)(\bar{A}+\bar{C}+\bar{D})$

If 'X' not make simpler expression then don't use them neglect them. :)

Examples

Ex minimize the expression using k-maps

$F(A,B,C,D) = \Sigma_m(1,4,7,10,13) + \Sigma d(5,14,15)$



$F = BD + \bar{A}\bar{C}D + \bar{A}B\bar{C} + AC\bar{D}$

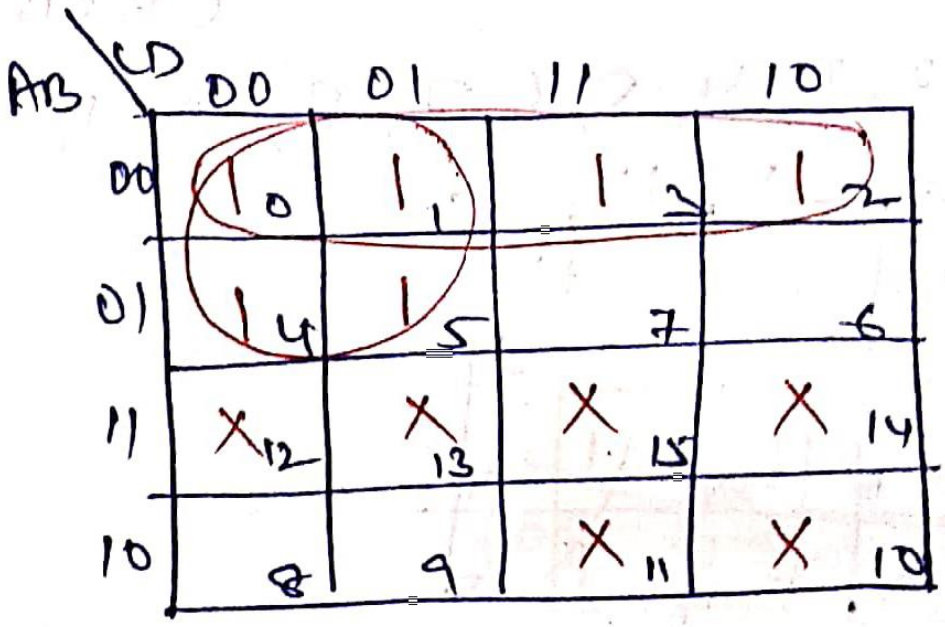
SOP K-maps

Examples

SOP K-map

Ex Obtained the Simplified expression in POS form.

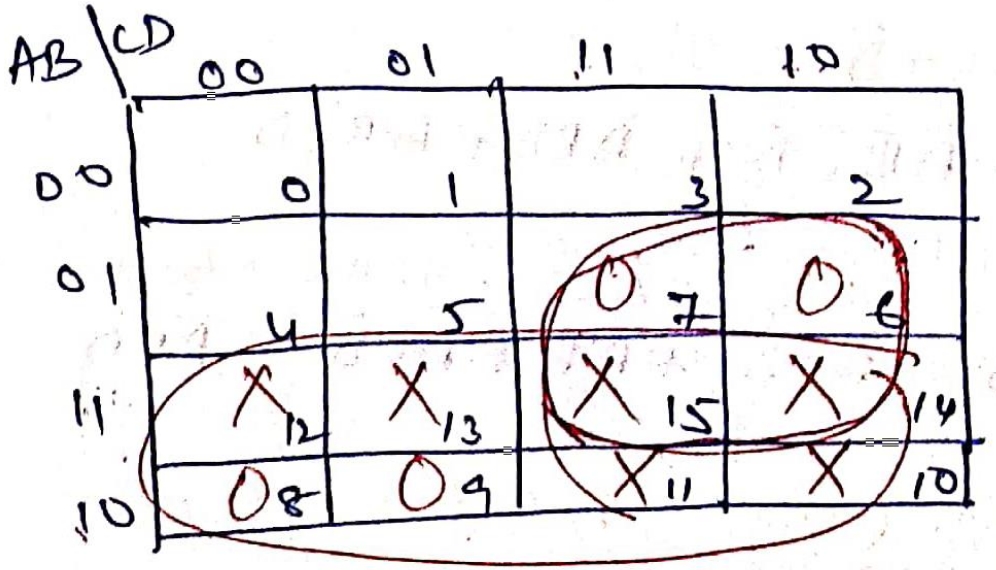
$$f(A,B,C,D) = \Sigma m(0,1,2,3,4,5) + d(10,11,12,13,14,15)$$



$F = \bar{A}\bar{B} + \bar{A}\bar{C}$

Ex $\Pi M(6,7,8,9)$ — SOP

Examples



Pos

$F = \overline{A} \cdot (\overline{B} + \overline{C})$

Same by indirect method

$F' = A + BC$

$F'' = \overline{A + BC}$

$F = \overline{A} \cdot (\overline{B} + \overline{C})$

Examples

(Ex) i) Simplify Boolean expression, using K-map

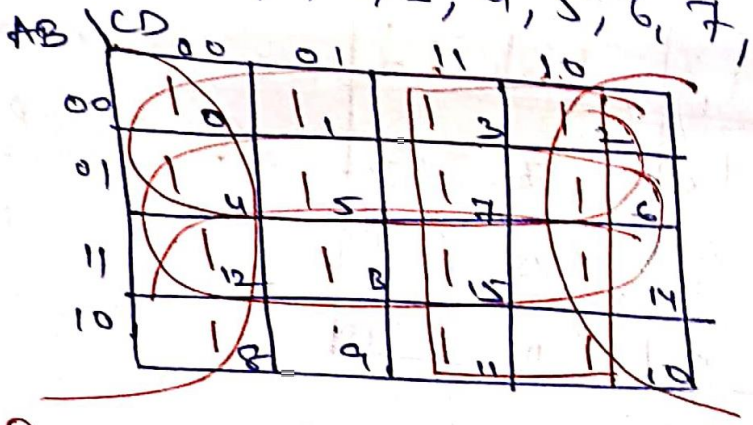
$(\bar{x}+y+w)$

Soln $f = \bar{A} + AB + AB\bar{D} + A\bar{B}\bar{D} + C$

Expand SOP in to SSOP from (SOP form)

$f = \bar{A}(B+\bar{B})(C+\bar{C})(D+\bar{D}) + AB(C+\bar{C})(D+\bar{D}) + AB\bar{D}(C+\bar{C}) + A\bar{B}\bar{D}(C+\bar{C}) + C(A+\bar{A})(B+\bar{B})(D+\bar{D})$

$f = \sum_m (0,1,2,3,4,5,6,7,8,10,11,12,13,14,15)$



DE MORGAN'S $f = C + \bar{D} + \bar{A} + B$

$= \bar{A} + B + C + \bar{D}$

Examples

ii) $f = \bar{A}\bar{B}D + \bar{A}B\bar{C}\bar{D} + \bar{A}BD + AB\bar{C}D$

Soln $\Sigma m(0, 2, 5, 7, 11)$

$f_{min} = \bar{A}\bar{B}\bar{D} + \bar{A}BD + AB\bar{C}D$

iii) $f = A\bar{B}D + \bar{A}\bar{C}\bar{D} + \bar{A}B + \bar{A}C\bar{D} + A\bar{B}D$

Soln $\Sigma m(0, 2, 4, 5, 6, 7, 9, 11, 13, 15)$

$f_{min} = \bar{A}\bar{D} + BD + AD$

Examples

Ex PROBLEMS

1) Obtained the Simplified exp in POS form

$f(A,B,C,D) = \Sigma m(5,6,7,8,9,12,13,14)$

Ans: $(B+\bar{C})(A+B)(A+C+D)(\bar{A}+B+\bar{C})$

2) $f(W,X,Y,Z) = \Sigma m(0,1,5,7,8,10,14,15)$

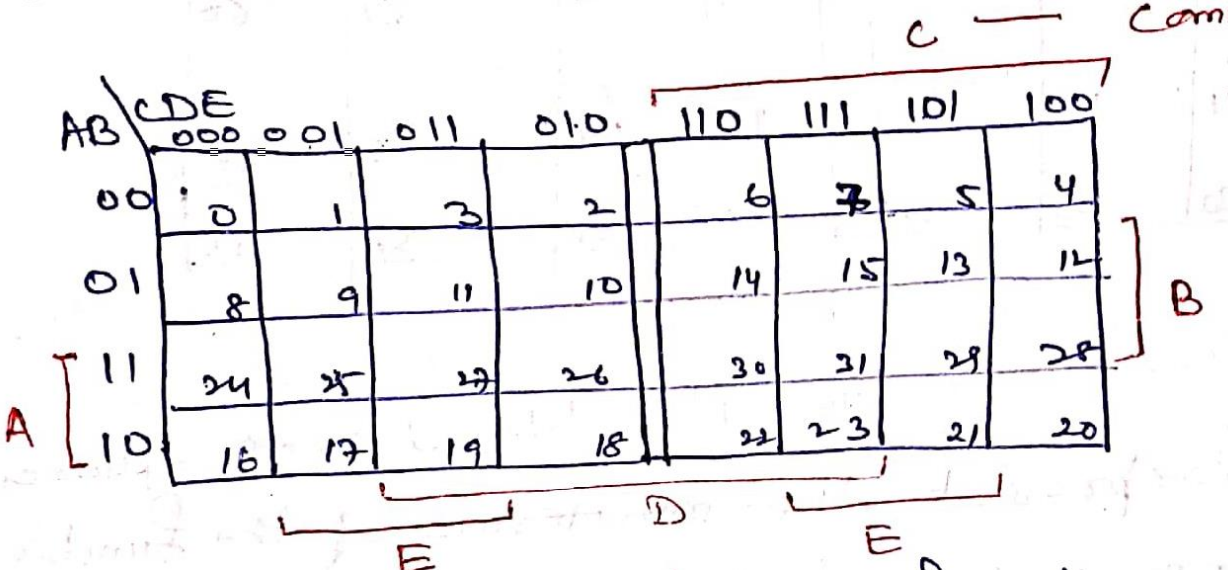
ANS: $(W+\bar{Y}+Z)(X+\bar{Y}+\bar{Z})(\bar{W}+Y+\bar{Z})$
$(\bar{X}+Y+W)$

Five variables K Map

- The center double line is like a mirror with each square being adjacent, not only to its four neighboring squares, but also to its mirror image.

  eg) min term -31 in five variable map is adjacent to min term 30, 15, 29, 23 and 27

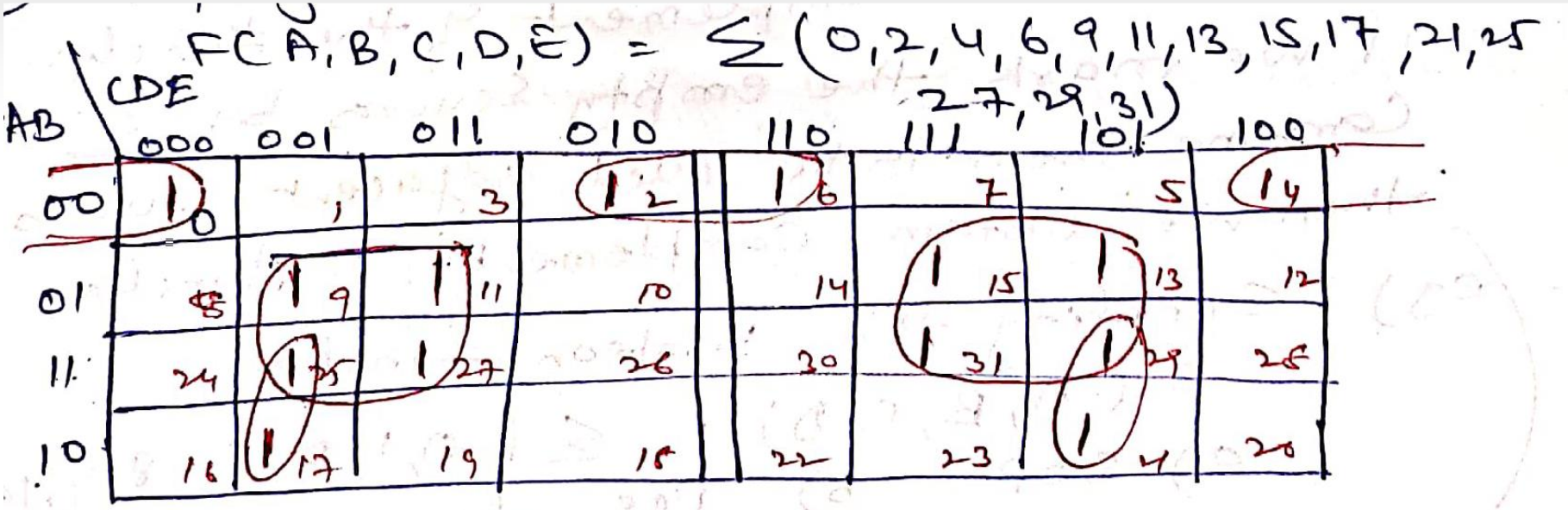
- The same is followed in 6x variable map



Examples

Ex Simplify the Boolean function

$F(A,B,C,D,E) = \sum(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$



$F(A,B,C,D,E) = BE + A\bar{D}E + \bar{A}\bar{B}\bar{E}$

Examples

Ex Reduce the following expression to the simplest possible POS and SOP.

$f = \Sigma m(6, 9, 13, 16, 19, 25, 27, 29, 31) + d(2, 3, 11, 15, 17, 24, 28)$

Soln

The given exp in POS form is

$f = \Pi M(0, 1, 4, 5, 7, 8, 10, 12, 14, 16, 20, 21, 22, 23, 26, 30) \cdot \Pi d(2, 3, 11, 15, 17, 24, 28)$

AB \ CDE	000	001	011	010	110	111	101	100
00	0 (0)	0 (1)	X (2)	X (3)	6	0 (7)	0 (5)	0 (4)
01	0 (8)	9	X (11)	0 (10)	0 (14)	X (15)	13	0 (12)
11	X (24)	25	27	0 (26)	0 (30)	31	29	X (28)
10	0 (16)	X (17)	19	18	0 (22)	0 (23)	0 (21)	0 (20)

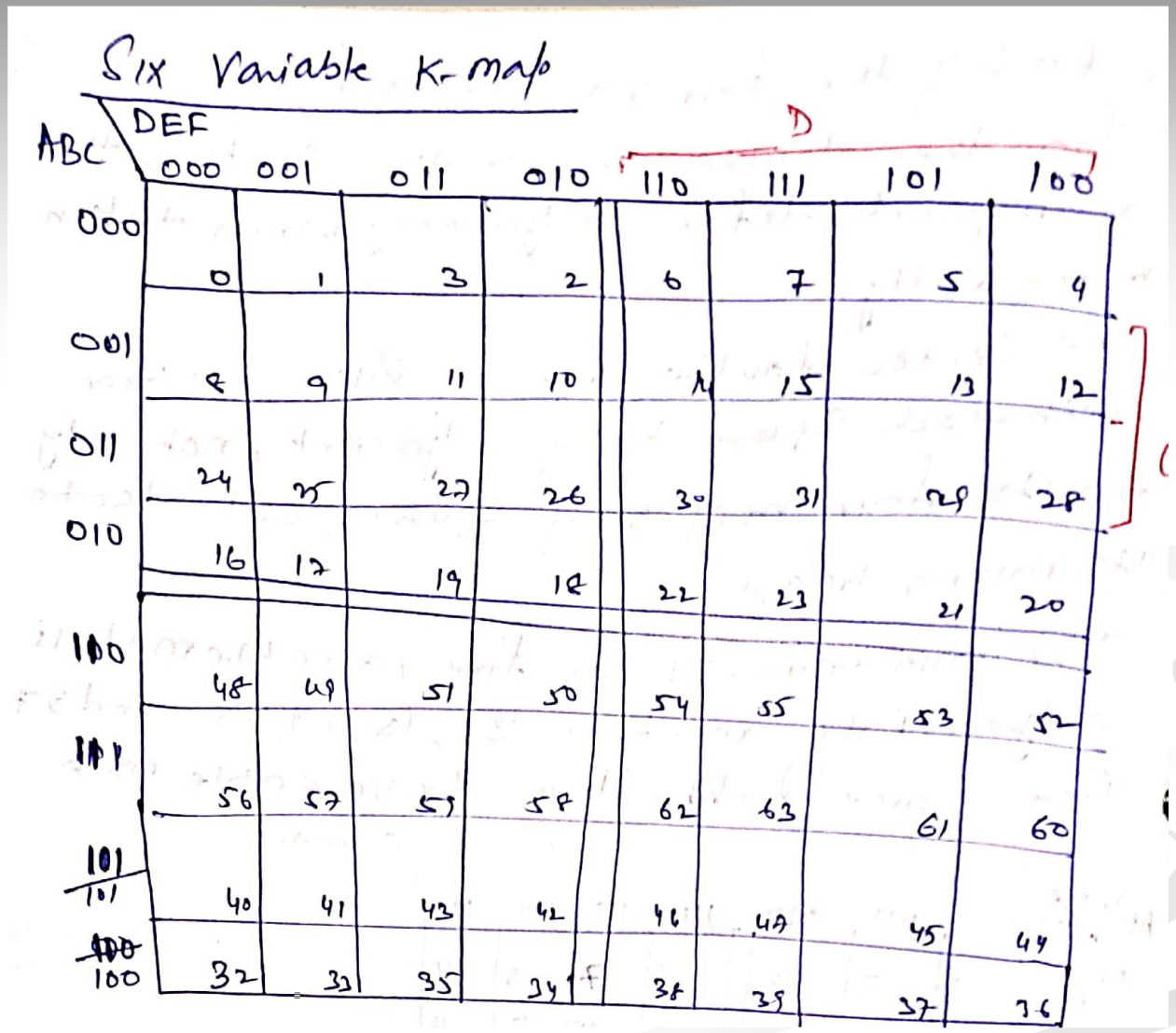
POS

$F = (D+E)(\bar{B}+E)(A+B+\bar{E})(B+\bar{C}+\bar{D})$

$(\bar{A}+B+\bar{C}$?

$f_{min} = BE + \bar{B}\bar{C}D + \bar{A}\bar{B}D\bar{E}$ — SOP

Six variables K map



Limitation of K-Map

(1) As the no of variables increases / exceeds 6 then it become difficult to make judgement about which combination form the minimal expression.

(2) It is impossible to work with problem of 7 or more variables using K-map

(3) K-map simplification is a manual technique and simplification process depends on human ability

- Alternate to this is Tabular method to simplify Boolean expression. (Quine-Mc Cluskey method)

Some extra examples

ii) $f = \bar{A}\bar{B}D + \bar{A}B\bar{C}\bar{D} + \bar{A}BD + AB\bar{C}D$

Soln $\Sigma m(0, 2, 5, 7, 11)$

$f_{min} = \bar{A}\bar{B}\bar{D} + \bar{A}BD + AB\bar{C}D$

iii) $f = A\bar{B}D + \bar{A}\bar{C}\bar{D} + \bar{A}B + \bar{A}C\bar{D} + A\bar{B}D$

Soln $\Sigma m(0, 2, 4, 5, 6, 7, 9, 11, 13, 15)$

$f_{min} = \bar{A}\bar{D} + BD + AD$