## **Digital Electronics- 2CS303**

# UNIT-1 Boolean algebra: Definitions, Theorems & Properties

Dr. Sudeep Tanwar

M

- 1.Boolean algebra: Definitions,
- 2. Theorems & Properties
- 3.Examples

# Definition of a Boolean Algebra

- All the properties of Boolean functions and expressions that we have discovered also apply to other mathematical structures such as propositions and sets and the operations defined on them.
- If we can show that a particular structure is a Boolean algebra, then we know that all results established about Boolean algebras apply to this structure.
- ■For this purpose, we need an abstract definition of a Boolean algebra.



#### **Basic Definitions**

# **Binary Operators**

AND

$$z = x \cdot y = x y$$

$$z=1$$
 if  $x=1$  AND  $y=1$ 

• OR

$$z = x + y$$

$$z=1$$
 if  $x=1$  OR  $y=1$ 

NOT

$$z=\overline{x}=x'$$

$$z=1$$
 if  $x=0$ 

# **Boolean Algebra**

- Binary Variables: only '0' and '1' values
- Algebraic Manipulation

# w

# POSTULATES OF BOOLEAN ALGEBRA:

The Boolean algebra has its own set of fundamental laws, which differ from the traditional algebra. They are,

# OR laws:

- $\rightarrow$  A+0=A
- > A+1=1
- $\rightarrow$  A+A=A
- > A+Ā=1 (law of complementary)

# AND laws:

- > A.0=0
- A=A.A=A
- > A.1=A
- > A.Ā=0 (law of complementary)

# **NOT laws**:

- $> \overline{0}=1$
- > 1=0
- > If A=0 then Ā=1
- ➤ If A=1 then Ā=0



# **Boolean Algebra Postulates**

#### **★** Commutative Law

$$x \bullet y = y \bullet x$$

$$x + y = y + x$$

# **★ Identity Element**

$$x \cdot 1 = x$$

$$x + 0 = x$$

# **\*** Complement

$$x \cdot x' = 0$$

$$x + x' = 1$$



# **Boolean Algebra Theorems**

# **★ Duality**

• The *dual* of a Boolean algebraic expression is obtained by interchanging the AND and the OR operators and replacing the 1's by 0's and the 0's by 1's.

#### **Example:**

$$\bullet x \bullet (y+z) = (x \bullet y) + (x \bullet z)$$

•  $x + (y \cdot z) = (x + y) \cdot (x + z)$ 



Applied to a valid equation produces a valid equation

#### **★** Theorem 1

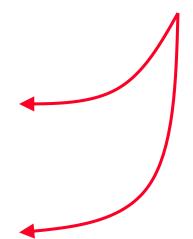
$$\bullet$$
  $x \bullet x = x$ 

$$x + x = x$$

#### **★ Theorem 2**

$$\bullet$$
  $x \bullet 0 = 0$ 

$$x + 1 = 1$$





## **Boolean Algebra Theorems**

**★** Theorem 3: *Involution* 

• 
$$(x')' = x$$
 ;  $(\overline{x}) = x$ 

**★** Theorem 4: Associative

• 
$$(x \cdot y) \cdot z = x \cdot (y \cdot z)$$
;  $(x + y) + z = x + (y + z)$ 

**★** Theorem 5: *Distributive* 

• 
$$x \cdot (y+z) = (x \cdot y) + (x \cdot z);$$
  
 $x + (y \cdot z) = (x+y) \cdot (x+z)$ 



# **★** Theorem 6: *DeMorgan*

•  $(x \cdot y)' = x' + y'$ ;

$$(x+y)'=x'\cdot y'$$

 $\bullet \quad (\overline{x \bullet y}) = \overline{x} + \overline{y} \qquad ;$ 

$$(\overline{x+y}) = \overline{x} \cdot \overline{y}$$

# **★** Theorem 7: Absorption

$$\bullet \quad x \bullet (x + y) = x$$

$$x + (x \cdot y) = x$$