Digital Electronics- 2CS303

UNIT-1
Boolean algebra: Definitions, Theorems & Properties

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1. Boolean algebra: Definitions,
2. Theorems & Properties
3. Examples

Definition of a Boolean Algebra

- All the properties of Boolean functions and expressions that we have discovered also apply to **other mathematical structures** such as propositions and sets and the operations defined on them.
- If we can show that a particular structure is a Boolean algebra, then we know that all results established about Boolean algebras apply to this structure.
- For this purpose, we need an **abstract definition** of a Boolean algebra.

Basic Definitions

Binary Operators

- **AND**

  $z = x \bullet y = x\,y$ $\qquad z=1$ *if* $x=1$ *AND* $y=1$

- **OR**

  $z = x + y$ $\qquad z=1$ *if* $x=1$ *OR* $y=1$

- **NOT**

  $z = \overline{x} = x'$ $\qquad z=1$ *if* $x=0$

Boolean Algebra

- **Binary Variables: only ‘0’ and ‘1’ values**
- **Algebraic Manipulation**

POSTULATES OF BOOLEAN ALGEBRA:

- The Boolean algebra has its own set of fundamental laws, which differ from the traditional algebra. They are,

OR laws:

- $A+0=A$
- $A+1=1$
- $A+A=A$
- $A+\bar{A}=1$ (law of complementary)

AND laws:

- $A.0=0$
- $A.A=A$
- $A.1=A$
- $A.\bar{A}=0$ (law of complementary)

NOT laws:

- $\bar{0}=1$
- $\bar{1}=0$
- If $A=0$ then $\bar{A}=1$
- If $A=1$ then $\bar{A}=0$

Boolean Algebra Postulates

- **Commutative Law**

$$x \bullet y = y \bullet x \qquad\qquad x + y = y + x$$

- **Identity Element**

$$x \bullet 1 = x \qquad\qquad x + 0 = x$$

- **Complement**

$$x \bullet x' = 0 \qquad\qquad x + x' = 1$$

Boolean Algebra Theorems

★ Duality

- The *dual* of a Boolean algebraic expression is obtained by interchanging the AND and the OR operators and replacing the 1's by 0's and the 0's by 1's.

Example :

- $x \bullet (y + z) = (x \bullet y) + (x \bullet z)$
- $x + (y \bullet z) = (x + y) \bullet (x + z)$

Applied to a valid equation produces a valid equation

★ Theorem 1

- $x \bullet x = x$ $\qquad$ $x + x = x$

★ Theorem 2

- $x \bullet 0 = 0$ $\qquad$ $x + 1 = 1$

Boolean Algebra Theorems

Theorem 3: *Involution*

- $(x')' = x$; $(\overline{\overline{x}}) = x$

Theorem 4: *Associative*

- $(x \bullet y) \bullet z = x \bullet (y \bullet z)$; $(x + y) + z = x + (y + z)$

Theorem 5: *Distributive*

- $x \bullet (y + z) = (x \bullet y) + (x \bullet z)$;

  $x + (y \bullet z) = (x + y) \bullet (x + z)$

★ Theorem 6: *DeMorgan*

- $(x \bullet y)' = x' + y'$; $(x + y)' = x' \bullet y'$
- $\overline{(x \bullet y)} = \overline{x} + \overline{y}$; $\overline{(x + y)} = \overline{x} \bullet \overline{y}$

★ Theorem 7: *Absorption*

- $x \bullet (x + y) = x$; $x + (x \bullet y) = x$