Digital Electronics- 2CS303

UNIT-1
Digital Systems and Binary Numbers

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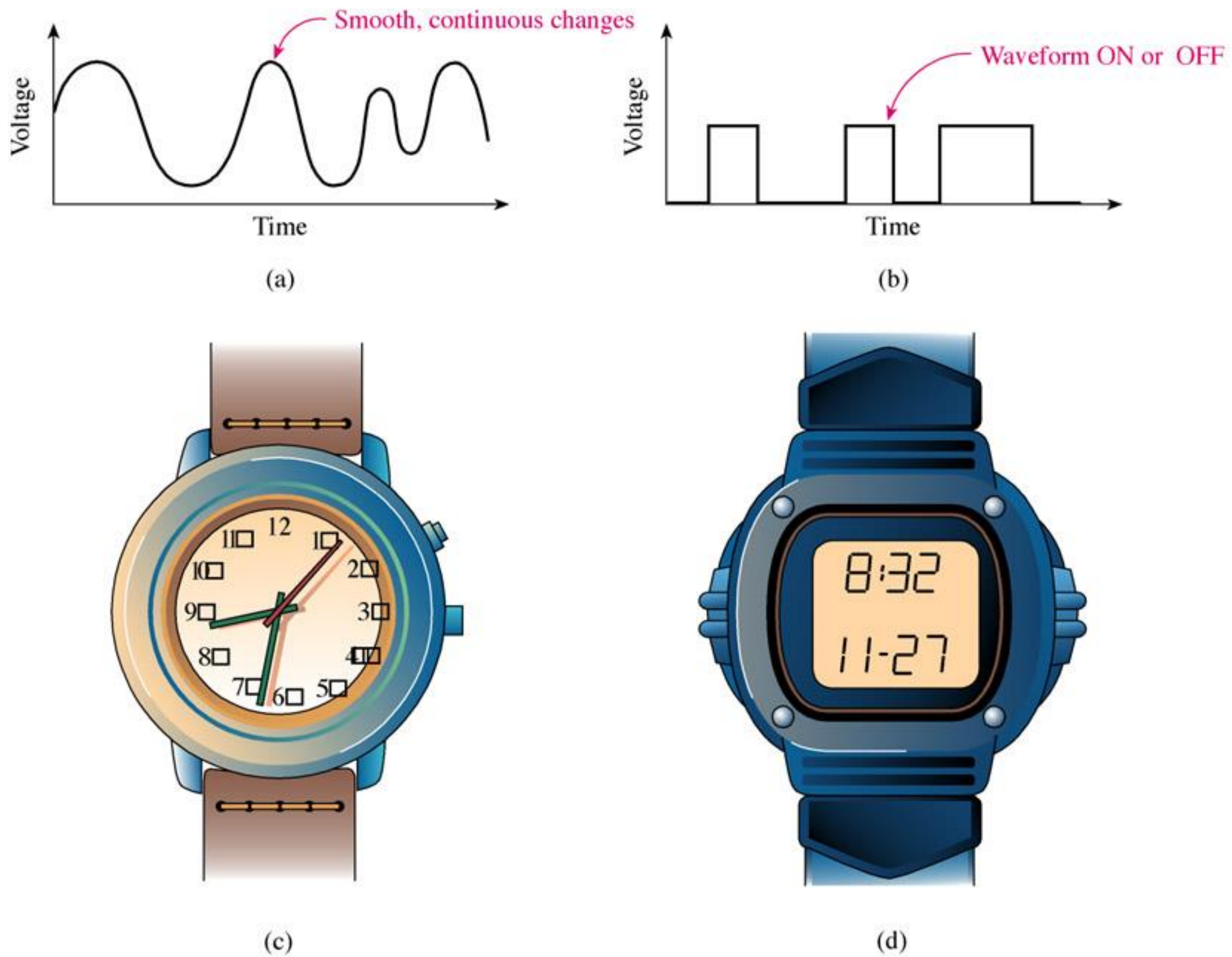
Outline

- 1.1 Digital Systems
- 1.2 Binary Numbers
- 1.3 Number-base Conversions
- 1.4 Octal and Hexadecimal Numbers

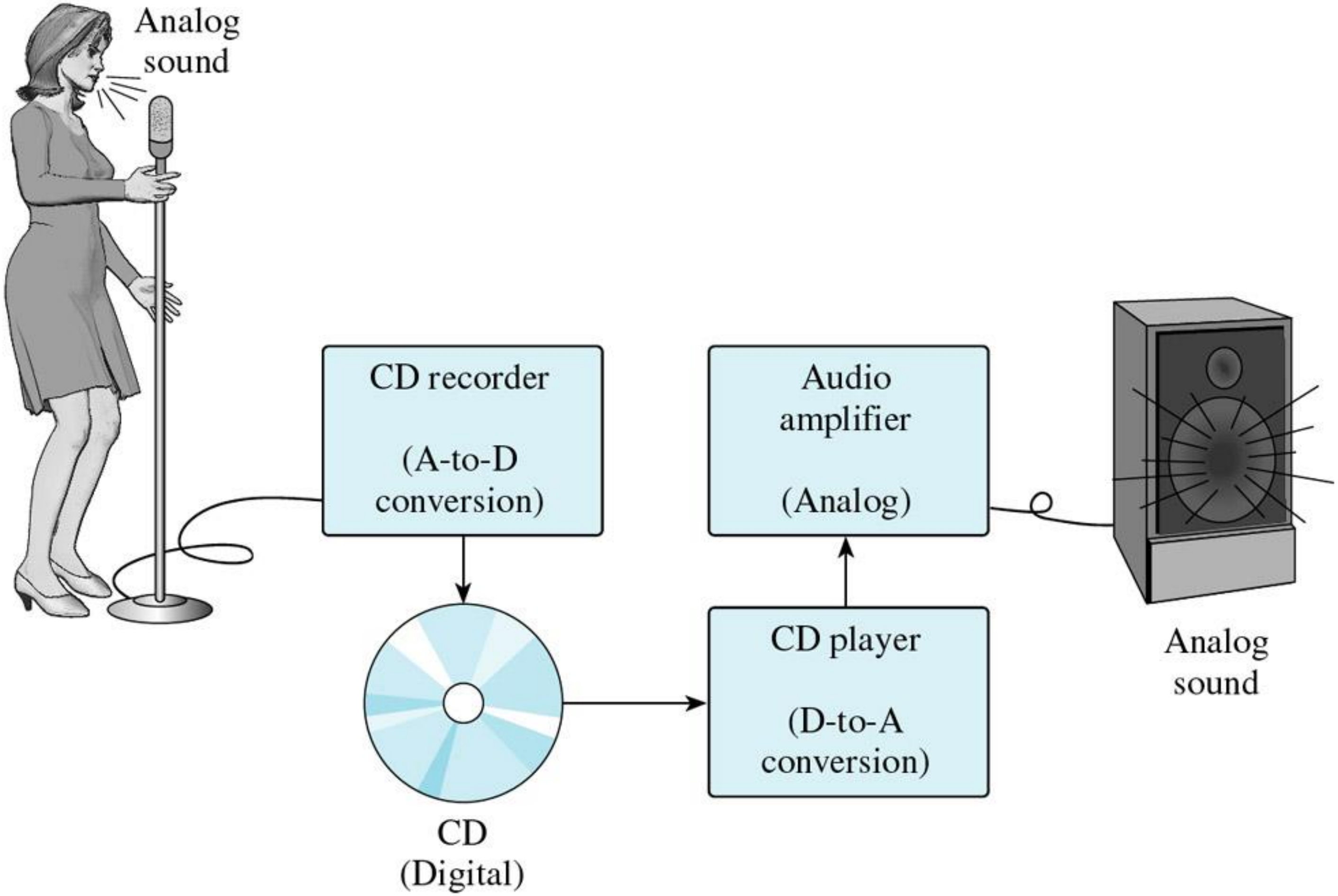
Digital versus Analog

- Digital
  - OFF and ON states that can be represented using 0s and 1s (respectively).
- Analog
  - Continuously varying
  - Examples: temperature, pressure, velocity

Digital vs. Analog

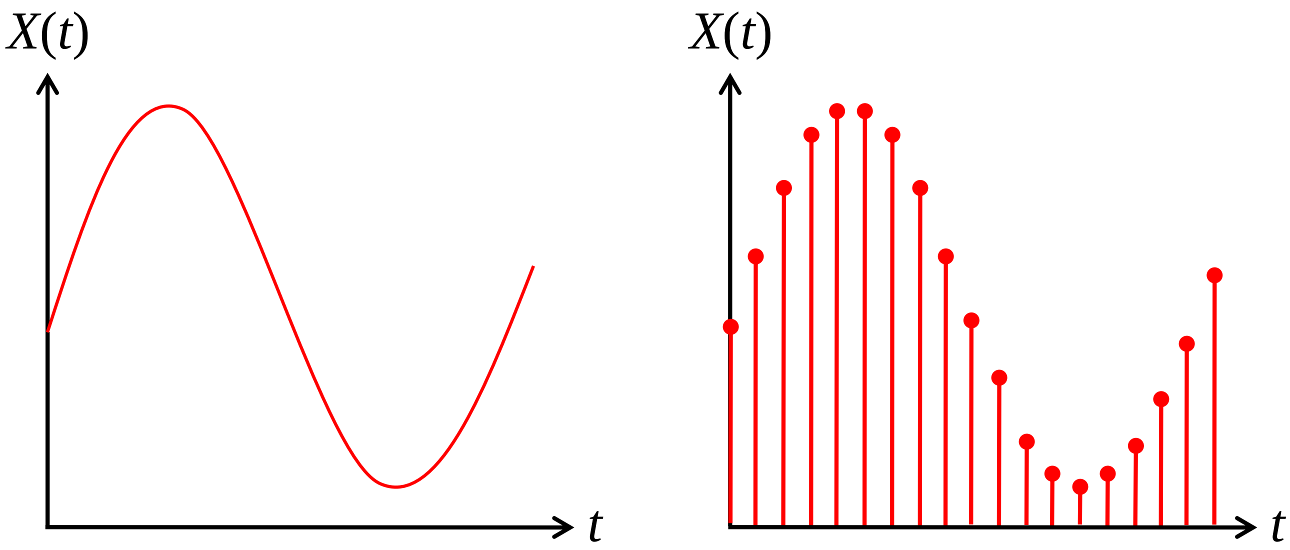


Digital-to-Analog and Back Again



Analog and Digital Signal

- Analog system
  - The physical quantities or signals may vary continuously over a specified range.
- Digital system
  - The physical quantities or signals can assume only discrete values.
  - Greater accuracy



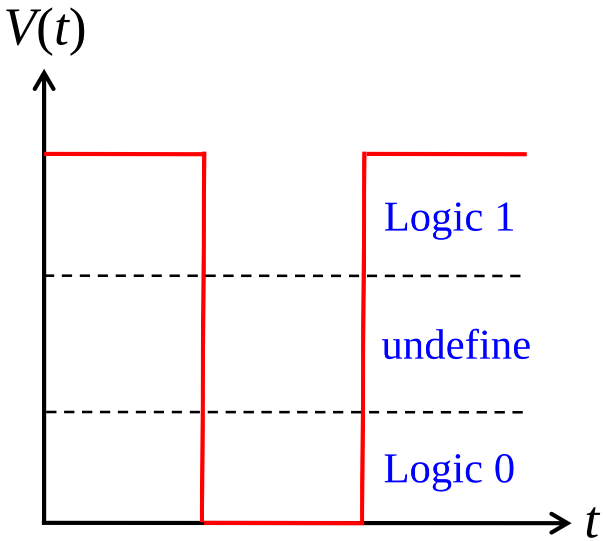
Analog signal Digital signal

Digital Systems and Binary Numbers

- Digital age and information age
- Digital computers
  - General purposes
  - Many scientific, industrial and commercial applications
- Digital systems
  - Telephone switching exchanges
  - Digital camera
  - Electronic calculators, PDA's
  - Digital TV
- Discrete information-processing systems
  - Manipulate discrete elements of information
  - For example, {1, 2, 3, …} and {A, B, C, …}…

Binary Digital Signal

- An information variable represented by physical quantity.
- For digital systems, the variable takes on discrete values.
  - Two level, or binary values are the most prevalent values.
- Binary values are represented abstractly by:
  - Digits 0 and 1
  - Words (symbols) False (F) and True (T)
  - Words (symbols) Low (L) and High (H)
  - And words On and Off
- Binary values are represented by values or ranges of values of physical quantities.



Binary digital signal

ANALOG-TO-DIGITAL CONVERSION

A digital signal is superior to an analog signal because it is more robust to noise and can easily be recovered, corrected and amplified. For this reason, the tendency today is to change an analog signal to digital data.

Number Systems

- Number Systems?
- What are the different forms to represent the number systems?
    - Decimal
    - Hexadecimal representation
    - Binary representation
    - Arithmetic Operations

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Decimal Number System

- Base (also called radix) = 10
  - 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- Digit Position
  - Integer & fraction
- Digit Weight
  - Weight = (*Base*) ^{Position}
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

2	1	0		-1	-2
5	1	2	.	7	4
100	10	1		0.1	0.01
			.		
500	10	2		0.7	0.04

$$d_2{*}B^2+d_1{*}B^1+d_0{*}B^0+d_{-1}{*}B^{-1}+d_{-2}{*}B^{-2}$$

$$(512.74)_{10}$$

Octal Number System

- Base = 8
  - 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }
- Weights
  - Weight = (*Base*) $^{\mathit{Position}}$
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

64	8	1		1/8	1/64
5	1	2	.	7	4
2	1	0		-1	-2

$$5 *8^{2}+1 *8^{1}+2 *8^{0}+7 *8^{-1}+4 *8^{-2}$$

$$=(330.9375)_{10}$$

$$(512.74)_{8}$$

Binary Number System

- Base = 2
  - 2 digits { 0, 1 }, called *b*inary dig*its* or "*bits*"
- Weights
  - Weight = $(Base)^{Position}$
- Magnitude
  - Sum of "*Bit* x *Weight*"
- Formal Notation
- Groups of bits

4 bits = *Nibble*

8 bits = *Byte*

4	2	1		1/2	1/4
1	0	1	.	0	1
2	1	0		-1	-2

$$1 * 2^{2} + 0 * 2^{1} + 1 * 2^{0} + 0 * 2^{-1} + 1 * 2^{-2}$$

$$= (5.25)_{10}$$

$$(101.01)_2$$

1 0 1 1

1 1 0 0 0 1 0 1

Hexadecimal Number System

- Base = 16
  - 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }
- Weights
  - Weight = $(Base)^{Position}$
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

256	16	1		1/16	1/256
1	E	5	.	7	A
2	1	0		-1	-2

$$1 * 16^2 + 14 * 16^1 + 5 * 16^0 + 7 * 16^{-1} + 10 * 16^{-2}$$

$$= (485.4765625)_{10}$$

$$(1E5.7A)_{16}$$

Common Powers (1 of 2)

- Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	µ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

Common Powers (2 of 2)

- Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- *What is the value of "k", "M", and "G"?*
- *In computing, particularly w.r.t. memory, the base-2 interpretation generally applies*

The Power of 2

n	2^n
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$

n	2^n	
8	$2^8=256$	
9	$2^9=512$	
10	$2^{10}=1024$	**Kilo**
11	$2^{11}=2048$	
12	$2^{12}=4096$	
20	$2^{20}=1M$	**Mega**
30	$2^{30}=1G$	**Giga**
40	$2^{40}=1T$	**Tera**

Binary Addition (1 of 4)

- Two 1-bit values

A	B	A + B	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

"two"

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Binary Addition (2 of 4)

- Two *n*-bit values
  - Add individual bits
  - Propagate carries
  - E.g.,

$\overset{1}{}\,1\,0\,1\,\overset{1}{0}\,1$	21
$+\ \ 1\,1\,0\,0\,1$	$+\ \ 25$
$1\,0\,1\,1\,1\,0$	46

Example-Addition (3 of 4)

- Decimal Addition

	1	**1**		← **Carry**
		5	5	
+		5	5	
	1	**1**	**0**	

= ***Ten*** $\geq$ ***Base***

➔ **Subtract a Base**

Example-Binary Addition (4 of 4)

- Column Addition

	1	1	1	1	1	1		
		1	1	1	1	0	1	= 61
+			1	0	1	1	1	= 23
	1	**0**	**1**	**0**	**1**	**0**	**0**	= 84

$\geq (2)_{10}$

Binary Subtraction (1 of 2)

- Two 1-bit values

A	B	A – B (Sub)	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Binary Subtraction (2 of 2)

- Borrow a “Base” when needed

$$
\begin{array}{cccccccc|l}
 & & 1 & & & 2 & & & \\
 & 0 & \cancel{2} & 2 & 0 & 0 & 2 & & = (10)_2 \\
 & \cancel{1} & 0 & 0 & \cancel{1} & \cancel{1} & 0 & 1 & = 77 \\
- & & & 1 & 0 & 1 & 1 & 1 & = 23 \\
\hline
 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & = 54
\end{array}
$$

Multiplication (1 of 3)

- Decimal (just for fun)

```
    35
 x 105
   175
  000
 35
  3675
```

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Multiplication (2 of 3)

- Binary, two 1-bit values

A	B	$A \times B$
0	0	0
0	1	0
1	0	0
1	1	1

Multiplication (3 of 3)-Example

- Binary, two *n*-bit values
  - As with decimal values
  - E.g.,

```
        1110
      x 1011
    --------
        1110
       1110
      0000
     1110
    --------
    10011010
```

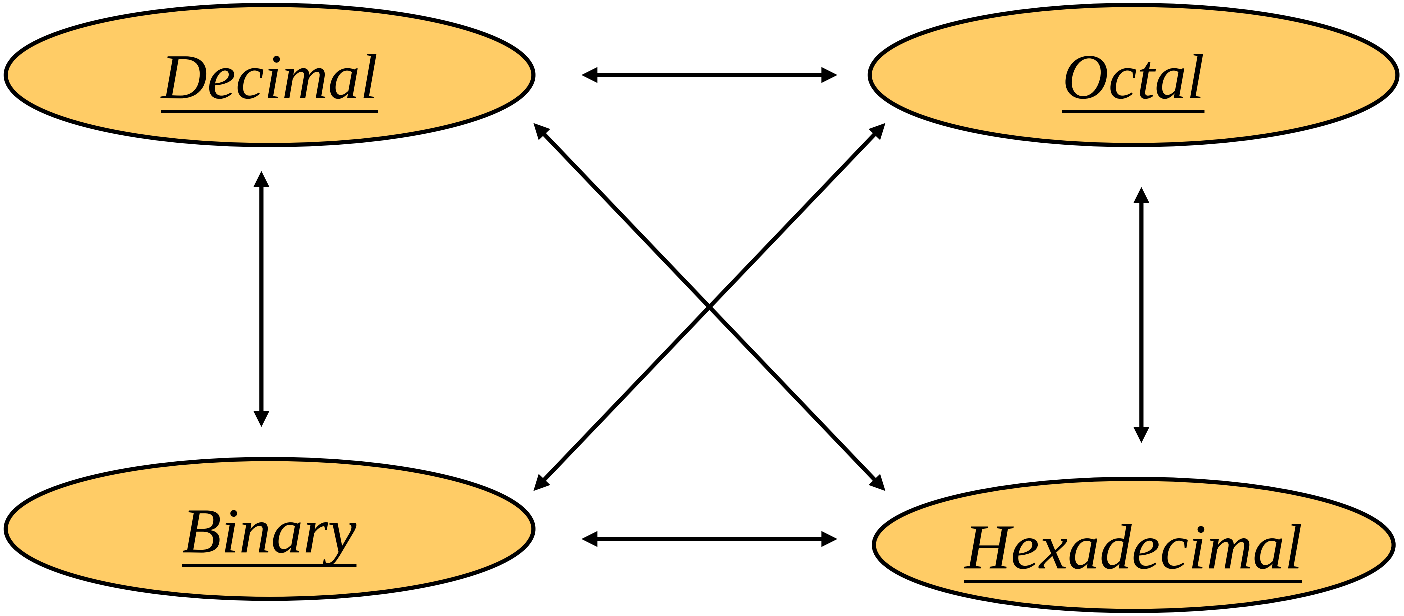
Binary Multiplication-Example

- Bit by bit

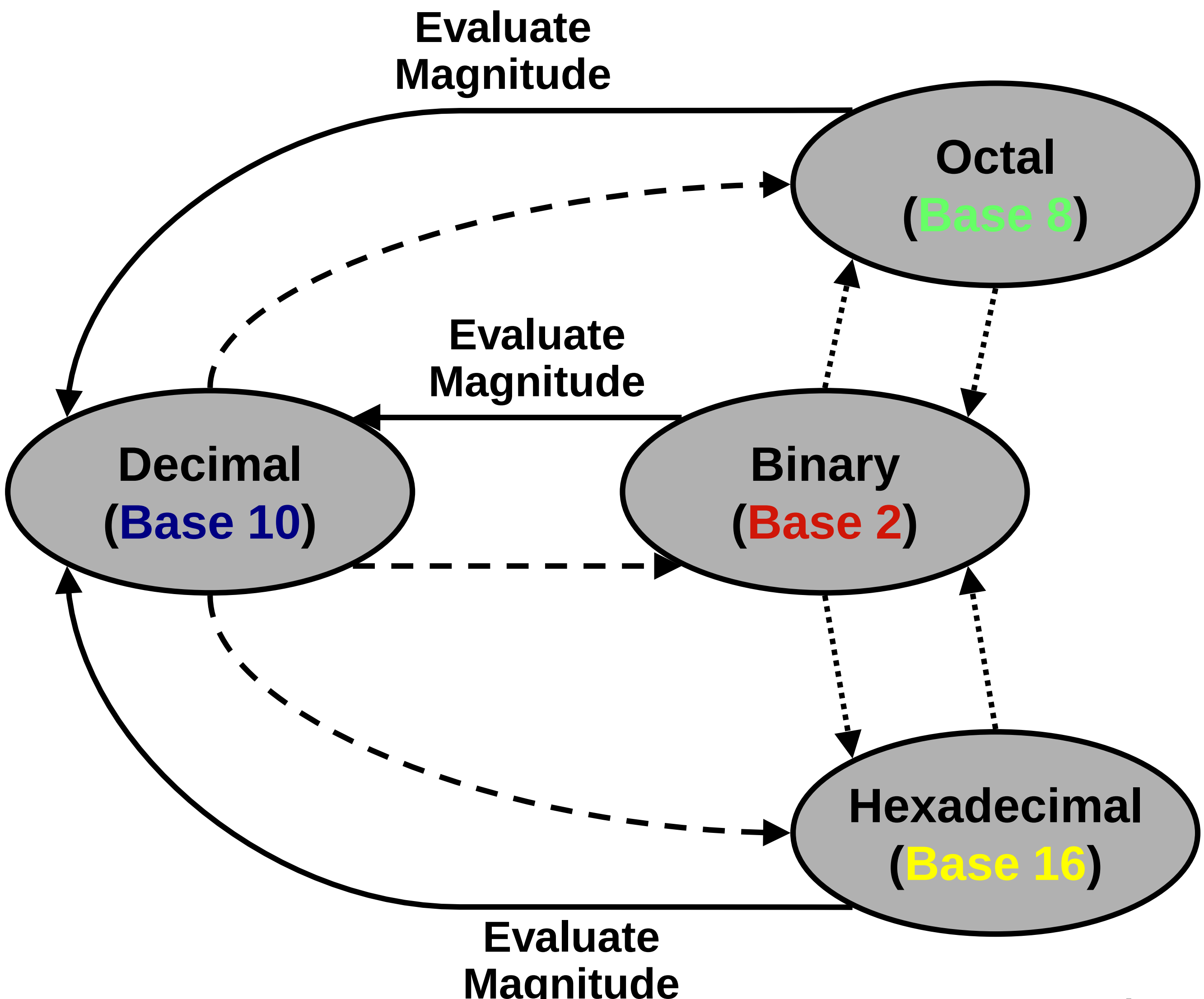
				1	0	1	1	1
x					1	0	1	0
				0	0	0	0	0
			1	0	1	1	1	
		0	0	0	0	0		
	1	0	1	1	1			
	1	1	1	0	0	1	1	0

Conversion Among Bases

- The possibilities:



Number Base Conversions



Decimal (*Integer*) to Binary Conversion

- Divide the number by the 'Base' (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

Example: $(13)_{10}$

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$a_1 = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$

Answer: $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

MSB LSB

Decimal (*Fraction*) to Binary Conversion

- Multiply the number by the ‘Base’ (=2)
- Take the integer (either 0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Example: $(0.625)_{10}$

		Integer		Fraction	Coefficient
0.625	* 2 =	1	.	25	$a_{-1} = 1$
0.25	* 2 =	0	.	5	$a_{-2} = 0$
0.5	* 2 =	1	.	0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3})_2 = (0.101)_2$

MSB LSB

Decimal to Octal Conversion

Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
$175 / 8 =$	21	7	$a_0 = 7$
$21 / 8 =$	2	5	$a_1 = 5$
$2 / 8 =$	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\, a_1\, a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

	Integer		Fraction	Coefficient
$0.3125 * 8 =$	2	.	5	$a_{-1} = 2$
$0.5 * 8 =$	4	.	0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_8 = (0.24)_8$

Octal to Decimal

- Technique
  - Multiply each bit by 8^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

724_8 =>	4×8^0 =	4
	2×8^1 =	16
	7×8^2 =	448
	Total=	468_{10}

Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

Example

$1234_{10} = ?_{16}$

$$
\begin{array}{r|l}
16 & 1234 \\ \hline
16 & 77 \quad 2 \\ \hline
16 & 4 \quad 13 = D \\ \hline
 & 0 \quad 4
\end{array}
$$

$1234_{10} = 4D2_{16}$

Hexadecimal to Decimal

- Technique
  - Multiply each bit by 16^n, where n is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$$
\begin{array}{llllllll}
ABC_{16} \Rightarrow & C \times 16^0 & = & 12 & \times & 1 & = & 12 \\
 & B \times 16^1 & = & 11 & \times & 16 & = & 176 \\
 & A \times 16^2 & = & 10 & \times & 256 & = & 2560 \\
 & \text{Total} = & & & & & & 2748_{10}
\end{array}
$$

Binary − Octal Conversion

- $8 = 2^3$
- Each group of 3 bits represents an octal digit

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Example:

Assume Zeros

$$(\ 1\ 0\ 1\ 1\ 0\ .\ 0\ 1\ )_2$$

$$(\ 2\ \ 6\ .\ 2\ )_8$$

Works **both** **ways (*Binary* to *Octal* & *Octal* to *Binary*)**

Binary − Hexadecimal Conversion

- $16 = 2^4$
- Each group of 4 bits represents a hexadecimal digit

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Example:

Assume Zeros

$$(\underbrace{1}_{1}\ \underbrace{0\,1\,1\,0}_{6}\,.\,\underbrace{0\,1}_{4})_2$$

$$(1\quad 6\ .\ 4)_{16}$$

Works both ways (*Binary* to *Hex* & *Hex* to *Binary*)

Octal – Hexadecimal Conversion

- Convert to Binary as an intermediate step

Example:

$$(\ 2\quad 6\ .\ 2\ )_8$$

Assume Zeros

Assume Zeros

$$(\ 0\,1\,0\,1\,1\,0\ .\ 0\,1\,0\ )_2$$

$$(\ 1\quad 6\ .\ 4\ )_{16}$$

Works both ways (*Octal* to *Hex* & *Hex* to *Octal*)

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Don't use a calculator!

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF