Digital Electronics- 2CS303

UNIT-1

Boolean Algebra Problems

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Boolean Algebric Theorems

Boolean Algebric Theorem.

1. $A + 0 = A$
2. $A \cdot 1 = A$
3. $A + 1 = 1$
4. $A \cdot 0 = 0$
5. $A + A = A$
6. $A \cdot A = A$
7. $A + \bar{A} = 1$
8. $A \cdot \bar{A} = 0$
9. $A(B+C) = AB + AC$
10. $A + BC = (A+B) \cdot (A+C)$
11. $A + AB = A$
12. $A \cdot (A+B) = A$
13. $\boxed{A + \bar{A}B = (A+B)}$
14. $A(\bar{A}+B) = AB$
15. $AB + A\bar{B} = A$
16. $(A+B) \cdot (A+\bar{B}) = A$
17. $(AB + \bar{A}C) = (A+C)(\bar{A}+B)$
18. $(A+B)(\bar{A}+C) = AC + \bar{A}B$
19. $\boxed{AB + \bar{A}C + BC = AB + \bar{A}C}$
20. $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$
21. $\overline{A \cdot B \cdot C \cdots} = \bar{A} + \bar{B} + \bar{C} + \cdots$
22. $\overline{A + B + C + \cdots} = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdots$

Problem-1

$$AB + ABC + A\bar{B} = A.$$

$$AB + ABC + A\bar{B}$$

$$AB(1+C) + A\bar{B} \qquad (1+C) = 1.$$

$$AB \cdot 1 + A\bar{B} = AB + A\bar{B}$$

$$A(B+\bar{B}) = A \qquad , (B+\bar{B}) = 1.$$

Problem-2

$$(B+A)(B+D)(C+A)(C+D) = BC + AD.$$

$$(B+A)(B+D)(C+A)(C+D)$$

$$(B+AD)(C+AD) \qquad \text{Because } (A+B)(A+C) = (A+BC)$$

$$(AD+BC).$$

Problem-3

$$(\bar{A}+B)\,\bar{A}\,\bar{B}\,\bar{C} = \overline{A+B+C}$$

$$(\bar{A}+B)\,\bar{A}\,\bar{B}\,\bar{C}$$

$$\bar{A}\cdot\bar{A}\,\bar{B}\,\bar{C} + B\cdot\bar{A}\,\bar{B}\,\bar{C}$$

$$\bar{A}\cdot\bar{B}\cdot\bar{C} + 0 \quad , \quad \bar{A}\cdot\bar{A} = \bar{A} \quad \& \quad B\cdot\bar{B} = 0$$

$\bar{A}\,\bar{B}\,\bar{C} = \overline{A+B+C}$ from De-Morgan's theorem.

Problem-4

$$\overline{(A+\bar{A}B)(C+\bar{D})} = \bar{A}\bar{B} + \bar{C}D.$$

$\overline{(A+\bar{A}B)(C+\bar{D})}$ use De-Morgan's theorem.

$$\overline{(A+\bar{A}B)} + \overline{(C+\bar{D})}$$

$$\bar{A}\cdot\overline{\bar{A}B} + \bar{C}\cdot\bar{\bar{D}}$$

Because $\bar{\bar{A}} = A$.

$$\bar{A}\cdot(\bar{\bar{A}}+\bar{B}) + \bar{C}\cdot D$$

$$\bar{A}\cdot(A+\bar{B}) + \bar{C}D. = \bar{A}\cdot A + \bar{A}\bar{B} + \bar{C}D$$

$\bar{A}\bar{B} + \bar{C}D$ Because $\bar{A}\cdot A = 0$.

Problem-5

$$\bar{A}(A+B)+\bar{C}+BC = B+\bar{C}$$

$$\bar{A}(A+B)+\bar{C}+BC$$

$$\bar{A}A+\bar{A}B+\bar{C}+BC$$

$$\bar{A}B+\bar{C}+BC \qquad , \bar{B}\cdot B = 0$$

$$\bar{A}B+\bar{C}(1+B)+BC$$

$$\bar{A}B+\bar{C}+B\bar{C}+BC = \bar{A}B+\bar{C}+B \qquad , C+\bar{C}=1$$

$$\bar{A}B+\bar{C}+B\cdot 1 = \bar{A}B+\bar{C}+B(1+\bar{A})$$

$$\bar{A}B+\bar{C}+B+\bar{A}B \Rightarrow B(A+\bar{A})+\bar{C}+B$$

$$B+\bar{C}+B \Rightarrow B+\bar{C}$$

Problem-6

Simplify it:
F=X'YZ+X'YZ'+XZ

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$$= \overline{X}Y(Z + \overline{Z}) + XZ \quad \text{by identity 14}$$

$$= \overline{X}Y \bullet 1 + XZ \quad \text{by identity 7}$$

$$= \overline{X}Y + XZ \quad \text{by identity 2}$$

Problem-7

Using Boolean algebra techniques, simplify the following expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

$$\overline{AB}.\overline{AC} + \overline{A}\,\overline{B}C$$

$$(\overline{A} + \overline{B}).(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$

$$\overline{A}.\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

$$\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

$$\overline{A} + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

$$\overline{A}(1 + \overline{B}C) + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

$$\overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

$$\overline{A}(1 + \overline{B}) + \overline{B}\,\overline{C}$$

$$\overline{A} + \overline{B}\,\overline{C}$$

Problem-8

$$(A + C)(AD + A\overline{D}) + AC + C$$

$$- (A + C)(AD + A\overline{D}) + AC + C$$
$$= (A + C)\,A\,(D + \overline{D}) + AC + C$$
$$= (A + C)\,A + C$$
$$= AA + AC + C$$
$$= A + C$$

Problem-9

Simplifying:

$$F = \overline{A}\,\overline{B}\,C + A\,(\overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} + B\,C)$$

$$F = \overline{A}\,\overline{B}\,C + A\,(\overline{B}\,(\overline{C} + C) + B\,(\overline{C} + C))$$

$$F = \overline{A}\,\overline{B}\,C + A\,(\overline{B} + B)$$
$$F = \overline{A}\,\overline{B}\,C + A$$
$$F = \overline{B}\,C + A$$

Problem-10

$$(\overline{A} + B).(\overline{A}.(B + A))$$

$(\overline{A} + B).(\overline{A}.B + \overline{A}.A)$ [expansion of brackets]

$(\overline{A} + B).(\overline{A}.B)$ [use of identities $X.\overline{X} = 0$ and $X+0 = X$]

$\overline{A}.\overline{A}.B + B.\overline{A}.B$ [expansion of brackets]

$\overline{A}.B + \overline{A}.B$ [use of identity $X.X = X$ twice]

$\overline{A}.B$ [use of identity $X + X = X$]

Problem-11

$$\overline{(A\overline{B} + \overline{A}B)}(A + B)$$

$$\overline{A\overline{B}}\,\overline{\overline{A}B}(A+B)$$

$$= (\overline{A}+B)(A+\overline{B})(A+B)$$

$$= (\overline{A}+B)(AA+AB + \overline{B}A + \overline{B}B)$$

$$= (\overline{A}+B)(A + AB + A\overline{B} + \overline{B}B)$$

$$= (\overline{A}+B)(A(1 + B + \overline{B}) + \overline{B}B)$$

$$= (\overline{A}+B)(A(1) + \overline{B}B)$$

$$= (\overline{A}+B)A$$

$$= A\overline{A} + AB$$

$$= AB$$

Problem-12

$$AB + \overline{A}C + BC = AB + \overline{A}C \text{ (Consensus Theorem)}$$

$AB + \overline{A}C + BC$		$AB + ABC + \overline{A}C + \overline{A}CB$	Commutative
$= AB + \overline{A}C + 1 \cdot BC$	Identity element	$AB \cdot 1 + ABC + \overline{A}C \cdot 1 + \overline{A}CB$	Identity element
$AB + \overline{A}C + (A + \overline{A}) \cdot BC$	Complement	$AB\,(1+C) + \overline{A}C\,(1 + B)$	Distributive
$AB + \overline{A}C + ABC + \overline{A}BC$	Distributive	$AB \,.\, 1 + \overline{A}C \,.\, 1$	$1+X = 1$
		$AB + \overline{A}C$	Identity element

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Problem-13

$$AB + A(B + C) + B(B + C)$$

- Apply distributive law,

$$AB + AB + AC + BB + BC$$

- Apply rule 7 (BB = B), and rule 5 (AB + AB = AB)

$$AB + AC + B + BC$$

- Apply rule 10 (B + BC = B)

$$AB + AC + B$$

Apply (AB + B = B) to the first and third terms.

$$B + AC$$

Problem-14

$$Z = (A + \overline{B} + \overline{C})(A + \overline{B}C)$$

$$Z = AA + A\overline{B}C + A\overline{B} + \overline{B}\overline{B}C + A\overline{C} + \overline{B}C\overline{C}$$

$$Z = A(1 + \overline{B}C + \overline{B} + \overline{C}) + \overline{B}C + \overline{B}C\overline{C}$$

$$Z = A + \overline{B}C$$

Problem-15

$$Z = A'BC + AB'C' + AB'C + ABC' + ABC$$

$= AB'C + AB'C' + A'BC + ABC' + ABC$	rearrange
$= AB'(C + C') + A'BC + AB(C' + C)$	distributive
$= AB' + A'BC + AB$	comp.
$= AB' + AB + A'BC$	rearrange
$= A(B' + B) + A'BC$	distributive
$= A + A'BC$	comp.

Problem-16

$$F_4 = P S + P \overline{Q} \overline{S} + P Q S$$

$F_4 = P(S + \overline{Q}\,\overline{S}) + P Q S$; Theorem #12A

$F_4 = P(S + \overline{Q}) + P Q S$; Theorem #13C

$F_4 = P S + P\overline{Q} + P Q S$; Theorem #12A

$F_4 = PS(1 + Q) + P\overline{Q}$; Theorem #12A

$F_4 = PS(1) + P\overline{Q}$; Theorem #6

$\boxed{F_4 = PS + P\overline{Q}}$; Theorem #2

Problem-17

Simplify $A.(\overline{A} + B) + A.\overline{B}$.

$$= A.\overline{A} + A.B + A.\overline{B}$$

$$= A.B + A.\overline{B}$$

$$= A(B + \overline{B})$$

$$= A(1)$$

$$= A$$

Problem-18

Simplify - AB + A(B + C) + B(B + C)

AB + AB + AC + BB + BC

AB + AB + AC + B + BC

AB + AC + B + BC

AB + AC + B

B+AC

Problem-19

$$F_1 = A(\overline{A} + AB)$$

$$F_1 = A(\overline{A} + B)$$

$$F_1 = A\overline{A} + AB$$

$$F_1 = 0 + AB$$

$$F_1 = AB$$

Problem-20

$$F_2 = X\overline{Y}Z + \overline{X}\,\overline{Y}Z + \overline{X}YZ$$

$$F_2 = \overline{Y}Z(X + \overline{X}) + \overline{X}YZ$$

$$F_2 = \overline{Y}Z(1) + \overline{X}YZ$$

$$F_2 = \overline{Y}Z + \overline{X}YZ$$

$$F_2 = Z(\overline{Y} + Y\overline{X})$$

$$F_2 = Z(\overline{Y} + \overline{X})$$

$$F_2 = \overline{Y}Z + \overline{X}Z$$

Problem-21

$$F_4 = (B + \overline{B})(A\overline{B} + A\overline{B}\overline{C})$$
$$F_4 = (1)(A\overline{B} + A\overline{B}\overline{C})$$
$$F_4 = A\overline{B}(1 + \overline{C})$$
$$F_4 = A\overline{B}(1)$$
$$F_4 = A\overline{B}$$

$$F_6 = JK + (\overline{J} + \overline{K})L + JK$$
$$F_6 = JK + JK + (\overline{JK})L$$
$$F_6 = JK + (\overline{JK})L$$
$$F_6 = JK + L$$

Problem-22

$$F_6 = JK + (\overline{J} + \overline{K})L + JK$$
$$F_6 = JK + \overline{J}L + \overline{K}L + JK$$
$$F_6 = JK + \overline{J}L + \overline{K}L$$
$$F_6 = JK + \overline{J}L(K + \overline{K}) + \overline{K}L$$
$$F_6 = JK + \overline{J}KL + \overline{J}\overline{K}L + \overline{K}L$$
$$F_6 = JK + \overline{J}KL + \overline{K}L(\overline{J} + 1)$$
$$F_6 = JK + \overline{J}KL + \overline{K}L(1)$$
$$F_6 = JK + \overline{J}KL + \overline{K}L$$
$$F_6 = K(J + \overline{J}L) + \overline{K}L$$
$$F_6 = K(J + L) + \overline{K}L$$
$$F_6 = JK + KL + \overline{K}L$$
$$F_6 = JK + L(K + \overline{K})$$
$$F_6 = JK + L(1)$$
$$F_6 = JK + L$$

Problem-23

$$F_8 = (N + \overline{N}M)(\overline{N} + NM)(N + M)$$

$$F_8 = (N + M)(\overline{N} + M)(N + M)$$

$$F_8 = (N\overline{N} + NM + \overline{N}M + MM)(N + M)$$

$$F_8 = (0 + MN + M\overline{N} + M)(N + M)$$

$$F_8 = (M + MN + M\overline{N})(N + M)$$

$$F_8 = (M + M\overline{N})(N + M)$$

$$F_8 = (M)(N + M)$$

$$F_8 = MN + MM$$

$$F_8 = MN + M$$

$$F_8 = M$$

Problem-24

$$F_4 = (B + \overline{B})(A\overline{B} + A\overline{B}\overline{C})$$

$$F_4 = (1)(A\overline{B} + A\overline{B}\overline{C})$$

$$F_4 = A\overline{B}(1 + \overline{C})$$

$$F_4 = A\overline{B}(1)$$

$$F_4 = A\overline{B}$$

Problem-25

$$X = ABC + \bar{A}C$$
$$= C(AB + \bar{A})$$
$$= C(\bar{A} + B)$$
$$= C\bar{A} + C$$

Problem-26

$$Y = (Q + R)(\bar{Q} + \bar{R})$$
$$= 0 + Q\bar{R} + R\bar{Q} + 0$$

Problem-27

$$Q = \overline{RST}\,(\overline{R + S + T})$$
$$= \bar{R} + \bar{S} + \bar{T}\,(\bar{R}\bar{S}\bar{T})$$
$$= \bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T}$$
$$= \bar{R}\bar{S}\bar{T}$$

Problem-28 Try these three and simplify them

$$[A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

$$\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$\overline{AB + AC} + \bar{A}\bar{B}C$$