UNIT-2: Boolean Function Simplification

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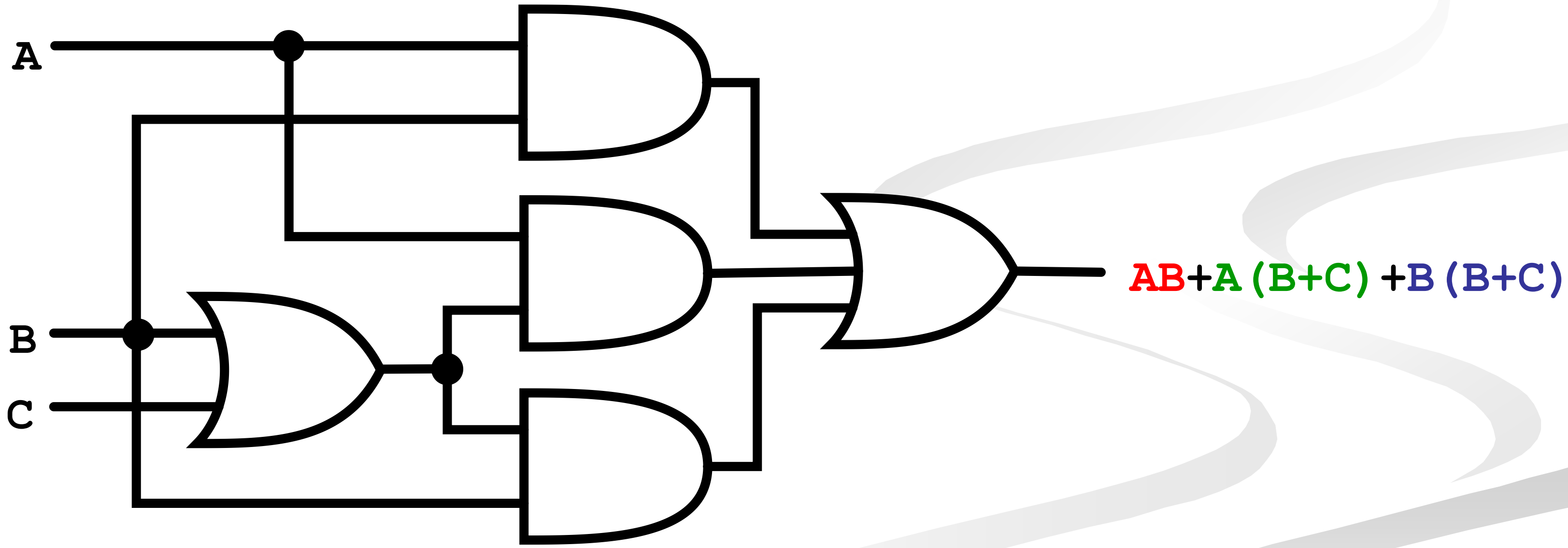
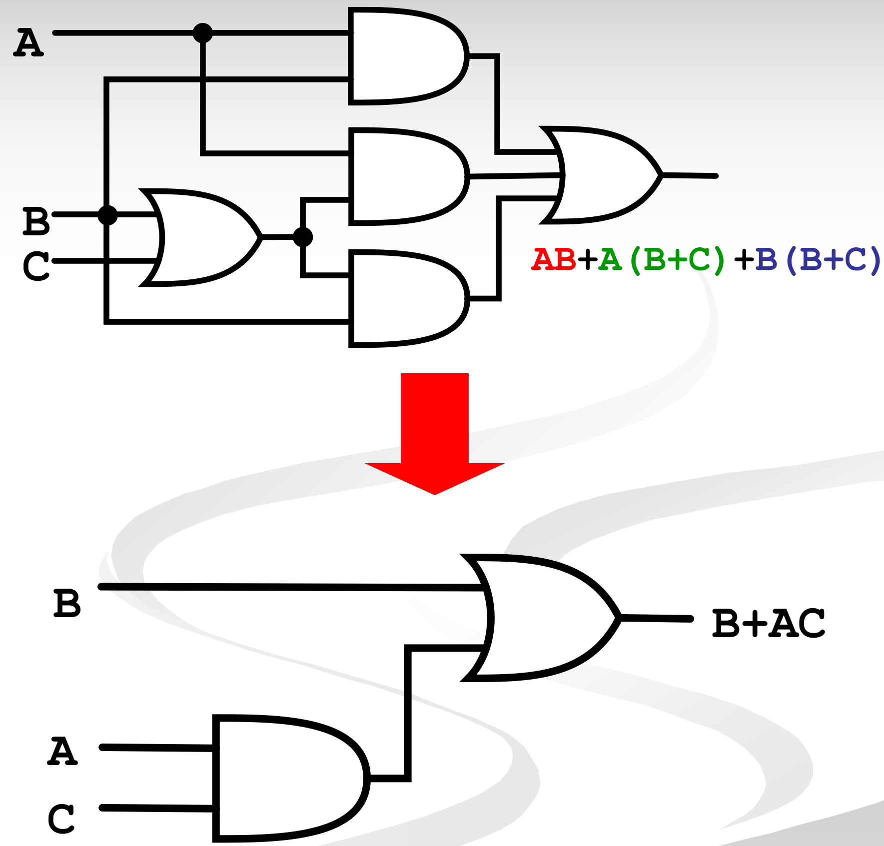
Simplification Using Boolean Algebra

- A simplified Boolean expression uses the fewest gates possible to implement a given expression.



Simplification Using Boolean Algebra

- AB+A(B+C)+B(B+C)
  - (distributive law)
    - AB+AB+AC+BB+BC
  - (rule 7; BB=B)
    - AB+AB+AC+**B**+BC
  - (rule 5; AB+AB=AB)
    - **AB**+AC+B+BC
  - (rule 10; B+BC=B)
    - AB+AC+**B**
  - (rule 10; AB+B=B)
    - **B**+AC



Simplification Using Boolean Algebra

- Try these:

$$[A\overline{B}(C+BD)+\overline{A}\,\overline{B}]C \longrightarrow \text{B}'\text{C}$$

$$\overline{A}BC+A\overline{B}\,\overline{C}+\overline{A}\,\overline{B}\,\overline{C}+A\overline{B}C+ABC \longrightarrow \text{BC + B}'\text{(A+C}'\text{)}$$

$$\overline{AB+AC}+\overline{A}\,\overline{B}C \longrightarrow \text{A}'\text{+B}'\text{C}'$$

Standard Forms of Boolean Expressions

- All Boolean expressions, regardless of their form, can be converted into either of two standard forms:
  - The sum-of-products (SOP) form
  - The product-of-sums (POS) form
- Standardization makes the evaluation, simplification, and implementation of Boolean expressions much more systematic and easier.

Sum-of-Products (SOP)

The Sum-of-Products (SOP) Form

- An SOP expression → when two or more product terms are summed by Boolean addition.
  - Examples:

    $$AB + ABC$$

    $$ABC + CDE + \overline{B}C\overline{D}$$

    $$\overline{A}B + \overline{A}B\overline{C} + AC$$

  - Also:

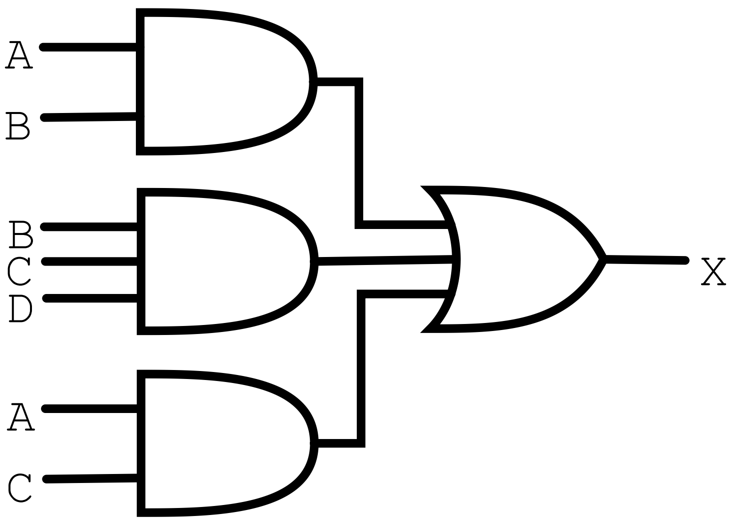
    $$A + \overline{A}\overline{B}C + BC\overline{D}$$

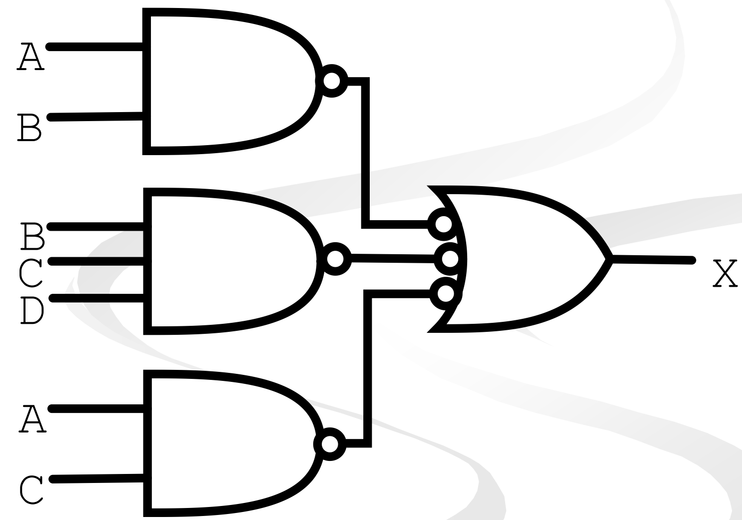
- In an SOP form, a single overbar cannot extend over more than one variable; however, more than one variable <u>in a term</u> can have an overbar:
  - example: $\overline{A}\overline{B}\overline{C}$ is OK!
  - **But not:** $\overline{ABC}$

Implementation of an SOP

$X = AB + BCD + AC$

- AND/OR implementation
- NAND/NAND implementation





General Expression → SOP

- Any logic expression can be changed into SOP form by applying Boolean algebra techniques.

  ex:

$$A(B+CD) = AB + ACD$$

$$AB + B(CD + EF) = AB + BCD + BEF$$

$$(A+B)(B+C+D) = AB + AC + AD + BB + BC + BD$$

$$\overline{(\overline{A+B})+C} = (\overline{\overline{A+B}})\overline{C} = (A+B)\overline{C} = A\overline{C} + B\overline{C}$$

The Standard SOP Form

- A standard SOP expression is one in which *all* the variables in the domain appear in each product term in the expression.
  - Example:

$$A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + AB\overline{C}\,\overline{D}$$

- Standard SOP expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

Converting Product Terms to Standard SOP

- **Step 1:** Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms.
  - As you know, you can multiply anything by 1 without changing its value.
- **Step 2:** Repeat step 1 until all resulting product term contains all variables in the domain in either complemented or uncomplemented form. In converting a product term to standard form, the number of product terms is doubled for each missing variable.

Converting Product Terms to Standard SOP (example)

- Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$A\overline{B}C = A\overline{B}C(D+\overline{D}) = \boxed{A\overline{B}CD + A\overline{B}C\overline{D}}$$

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C+\overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

$$\overline{A}\,\overline{B}C(D+\overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D+\overline{D}) = \boxed{\overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}}$$

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = \boxed{A\overline{B}CD + A\overline{B}C\overline{D}} + \boxed{\overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}} + AB\overline{C}D$$

Binary Representation of a Standard Product Term

- A standard product term is equal to 1 for only one combination of variable values.
  - Example: $A\overline{B}C\overline{D}$ is equal to 1 when A=1, B=0, C=1, and D=0 as shown below

$$A\overline{B}C\overline{D} = 1 \bullet \overline{0} \bullet 1 \bullet \overline{0} = 1 \bullet 1 \bullet 1 \bullet 1 = 1$$

  - And this term is 0 for all other combinations of values for the variables.

Converting Product Terms to Standard SOP (example)

eg) Convert SOP to SSOP and expand it to min term

variable B is missing

$\overline{A} + \overline{B}$

variable A is missing

$\overline{A}(B+\overline{B}) + \overline{B}(A+\overline{A})$

$\overline{A}B + \overline{A}\,\overline{B} + A\overline{B} + \overline{A}\,\overline{B}$

$\overline{A}B + A\overline{B} + \overline{A}\,\overline{B}$ — SSOP

minterm

$01 + 10 + 00$

$m_1 + m_2 + m_0$

$\Sigma m(0,1,2)$

Another DIRECT method in which minterms for any Expression can be written

— Given expression Can be directly written in term of its minterm by using following

(II) Method

Steps

(1) Write down all the terms

(2) ~~Hanand~~ ~~Stap~~ Put xs in term where ~~...~~ variable must be inserted to form a minterm

(3) Replace the non Complemented variable by 1's and the complemented variables by 0's and use all combinations of xs in term of 0's and 1's to generate minterms

(4) Drop out the redundant term

Converting Product Terms to Standard SOP (example)

Another way to find minterm is

$$\bar{A}+\bar{B} = \bar{A}\cdot X + X\cdot\bar{B}$$

$$= 0\cdot X + X\cdot 0$$

$$= 00 + 01 + 00 + 10$$

$$= 00 + 01 + 10$$

$$\Sigma m(0,1,2)$$

$0\cdot0 + 0\cdot0$
$0\cdot0 + 0\cdot1 + 10$
$+ 0\cdot1 + 10$
00, 01, 10

The minterm m_3 is missing in SOP form ∴ the maxterm M_3 will be present in the POS form.

Product-of-Sums (POS)

The Product-of-Sums (POS) Form

- When two or more sum terms are multiplied, the result expression is a product-of-sums (POS):
  - Examples:

$$(\overline{A}+B)(A+\overline{B}+C)$$

$$(\overline{A}+\overline{B}+\overline{C})(C+\overline{D}+E)(\overline{B}+C+D)$$

$$(A+B)(A+\overline{B}+C)(\overline{A}+C)$$

  - Also:

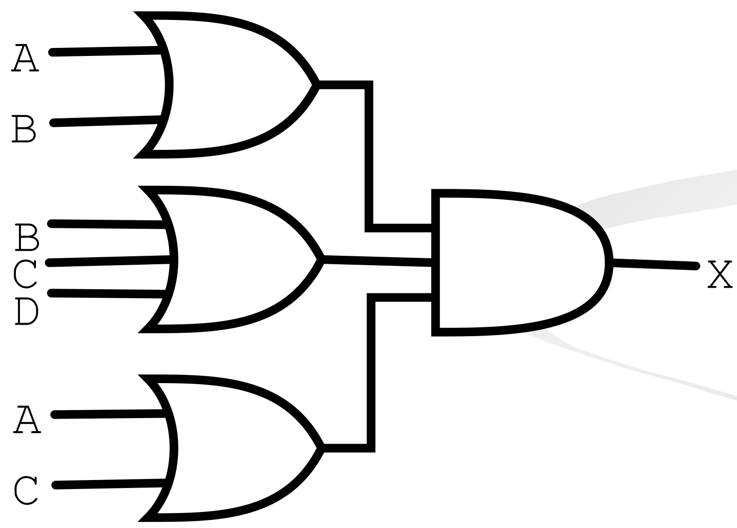
$$\overline{A}(\overline{A}+\overline{B}+C)(B+C+\overline{D})$$

- In a POS form, a single overbar cannot extend over more than one variable; however, more than one variable <u>in a term</u> can have an overbar:
  - example: $\overline{A}+\overline{B}+\overline{C}$ is OK!
  - **But not:** $\overline{A+B+C}$

Implementation of a POS

$X=(A+B)(B+C+D)(A+C)$

- OR/AND implementation



The Standard POS Form

- A standard POS expression is one in which *all* the variables in the domain appear <u>in each sum term</u> in the expression.
  - Example: $(\overline{A}+\overline{B}+\overline{C}+\overline{D})(A+\overline{B}+C+D)(A+B+\overline{C}+D)$
- Standard POS expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

Converting a Sum Term to Standard POS

- **Step 1:** Add to each nonstandard product term a term made up of the product of the missing variable and its complement. This results in two sum terms.
  - As you know, you can add 0 to anything without changing its value.
- **Step 2:** Apply rule 12 → $A+BC=(A+B)(A+C)$.
- **Step 3:** Repeat step 1 until all resulting sum terms contain all variable in the domain in either complemented or uncomplemented form.

Converting a Sum Term to Standard POS (example)

- Convert the following Boolean expression into standard POS form:

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

Tip → **Just simply split that term in 2 terms by adding missing variable in one and its complement in another one**

$$A+\overline{B}+C = A+\overline{B}+C+D\overline{D} = \boxed{(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})}$$

$$\overline{B}+C+\overline{D} = \overline{B}+C+\overline{D}+A\overline{A} = \boxed{(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})}$$

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})\boxed{(A+\overline{B}+\overline{C}+D)} =$$

$$\boxed{(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})}\boxed{(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})}\boxed{(A+\overline{B}+\overline{C}+D)}$$

Binary Representation of a Standard Sum Term

- A standard sum term is equal to 0 for only one combination of variable values.
  - Example: $A+\overline{B}+C+\overline{D}$ is equal to 0 when A=0, B=1, C=0, and D=1 as shown below

$$A+\overline{B}+C+\overline{D}=0+\overline{1}+0+\overline{1}=0+0+0+0=0$$

  - And this term is 1 for all other combinations of values for the variables.

Converting a Sum Term to Standard POS (example)

Numerical

Expand $A(\bar{B}+A)B$ to max term and min term

min term

Soln This exp is in 2 variable POS form

$(A+B\bar{B})(\bar{B}+A)(B+A\bar{A})$

Since B is missing in 1st term, A is missing in 2nd and ok 3rd

Distributed Law $\longrightarrow (A+B)(A+\bar{B})(A+\bar{B})(A+B)(\bar{A}+B)$ — Redundant term eliminated

$(A+B)(A+\bar{B})(\bar{A}+B)$

$(00)\ (01)\ (10)$

$(M_0)(M_1)(M_2) \quad \pi(0,1,2)$

The Maxterm M_3 is missing in POS so SOP form will contain only one minterm m_3

$\Sigma m(3)$

Converting a Sum Term to Standard POS (example)

Another way to found maxterm is

$A + (\bar{B} + A) B$

$A \times (A + \bar{B}) \times B$

missing terms

$0 \cdot \times (0 + 1) \times \cdot 0$

$(0\ 0)(0\ 1)\ (0\ 0)\ |\ (0\ 1)\ (1\ 0)$

$00 \quad 01 \quad 10$

$0,\ 1,\ 2$

$\Pi M(0, 1, 2)$

Use all combination of X in terms of 0's and 1's

Converting a Sum Term to Standard POS (example)

Expand

$A.(\bar{A}+B)(\bar{A}+B+\bar{C})$ to maxterm and minterm

$(A+B\bar{B}+C\bar{C})(\underline{\bar{A}+B}+C\bar{C})(\bar{A}+B+\bar{C})$

$((A+B)(A+\bar{B})+C\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

$(A+B+C\bar{C})(A+\bar{B}+C\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$
$(\bar{A}+B+\bar{C})$

$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})$
$(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

$(000)\ (001)\ (010)\ (011)\ (100)$

$0, 1, 2, 3, 4$

ΠM **(0,1,2,3,4,5)**

Σm **(6,7)**