SOP/POS

Converting Standard SOP to Standard POS

- The Facts:
  - The binary values of the product terms in a given standard SOP expression are not present in the equivalent standard POS expression.
  - The binary values that <u>are not</u> represented in the SOP expression are present in the equivalent POS expression.

Converting Standard SOP to Standard POS

- What can you use the facts?
  - Convert from standard SOP to standard POS.
- How?
  - **Step 1:** Evaluate each product term in the SOP expression. That is, determine the binary numbers that represent the product terms.
  - **Step 2:** Determine all of the binary numbers not included in the evaluation in Step 1.
  - **Step 3:** Write the equivalent sum term for each binary number from Step 2 and express in POS form.

Converting Standard SOP to Standard POS (example)

- Convert the SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

  - The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

  - There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

Boolean Expressions & Truth Tables

- All standard Boolean expression can be easily converted into truth table format using binary values for each term in the expression.
- Also, standard SOP or POS expression can be determined from the truth table.

Converting SOP Expressions to Truth Table Format

- Recall the fact:
  - An SOP expression is equal to 1 only if at least one of the product term is equal to 1.
- Constructing a truth table:
  - **Step 1:** List all possible combinations of binary values of the variables in the expression.
  - **Step 2:** Convert the SOP expression to standard form if it is not already.
  - **Step 3:** Place a 1 in the output column (X) for each binary value that makes the standard SOP expression a 1 and place 0 for all the remaining binary values.

Converting SOP Expressions to Truth Table Format (example)

- Develop a truth table for the standard SOP expression

$$\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	
0	0	1	1	$\overline{A}\,\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

Converting POS Expressions to Truth Table Format

- Recall the fact:
  - A POS expression is equal to 0 only if at least one of the product term is equal to 0.
- Constructing a truth table:
  - **Step 1:** List all possible combinations of binary values of the variables in the expression.
  - **Step 2:** Convert the POS expression to standard form if it is not already.
  - **Step 3:** Place a 0 in the output column (X) for each binary value that makes the standard POS expression a 0 and place 1 for all the remaining binary values.

Converting POS Expressions to Truth Table Format (example)

- Develop a truth table for the standard POS expression

$$(A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+C)$$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	$(A+B+C)$
0	0	1	1	
0	1	0	0	$(A+\overline{B}+C)$
0	1	1	0	$(A+\overline{B}+\overline{C})$
1	0	0	1	
1	0	1	0	$(\overline{A}+B+\overline{C})$
1	1	0	0	$(\overline{A}+\overline{B}+C)$
1	1	1	1	

Determining Standard Expression from a Truth Table

- To determine the standard **SOP expression** represented by a truth table.
- Instructions:
  - **Step 1:** List the binary values of the input variables for which the output is 1.
  - **Step 2:** Convert each binary value to the corresponding product term by replacing:
    - each 1 with the corresponding variable, and
    - each 0 with the corresponding variable complement.
- Example: 1010 → $A\overline{B}C\overline{D}$

Determining Standard Expression from a Truth Table

- To determine the standard **POS expression** represented by a truth table.
- Instructions:
  - **Step 1:** List the binary values of the input variables for which the output is 0.
  - **Step 2:** Convert each binary value to the corresponding product term by replacing:
    - each 1 with the corresponding variable complement, and
    - each 0 with the corresponding variable.
- Example: 1001 → $\overline{A}+B+C+\overline{D}$

Determining Standard Expression from a Truth Table (example)

I/P			O/P
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

SOP

- There are four 1s in the output and the corresponding binary value are 011, 100, 110, and 111.

$$011 \rightarrow \overline{A}BC$$
$$100 \rightarrow A\overline{B}\,\overline{C}$$
$$110 \rightarrow AB\overline{C}$$
$$111 \rightarrow ABC$$

$$X = \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

POS

- There are four 0s in the output and the corresponding binary value are 000, 001, 010, and 101.

$$000 \rightarrow A + B + C$$
$$001 \rightarrow A + B + \overline{C}$$
$$010 \rightarrow A + \overline{B} + C$$
$$101 \rightarrow \overline{A} + B + \overline{C}$$

$$X = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

Examples

Convert SOP to SSOP & then to SPOS

1) $AB + \bar{A}BD + \bar{A}C\bar{D}$

$AB(C+\bar{C})(D+\bar{D}) + \bar{A}BD(C+\bar{C}) + \bar{A}C\bar{D}(B+\bar{B})$

$ABC + AB\bar{C}(D+\bar{D}) + \bar{A}BCD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}B\bar{C}\bar{D}$

$ABCD + ABC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + \bar{A}BCD + \bar{A}B\bar{C}D$

$+\bar{A}BC\bar{D} + \bar{A}B\bar{C}\bar{D}$

SSOP to SPOS

$1111 + 1110 + 1100 + 1101 + 0111 + 0101$

$+ 0110 + 0010$

Because 4 variable so total combinations are 16

0000, 0001, 0011, 0100, 1000, 1001, 1010, 1101,

POS. expression is

$(A+B+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+\bar{D})(A+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})$

$(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+C+\bar{D})$

Examples

Express the Boolean function

$F = A + B'C$ in Sum of minterm (SOP)

$F = A(B+\bar{B})(C+\bar{C}) + B\bar{C}(A+\bar{A})$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C}$

~~000~~, 110, 101, 100, 110, 010

$\sum m = 7, 6, 5, 4, 6, 2$

Summation stands = 2, 4, 5, 6, 7 (Common / Redundant)

$m_2 + m_4 + m_5 + m_6 + m_7$

Examples

Express the Boolean function

$F = xy + \bar{x}z$ in a product of Max term form (POS)

1. first SOP $\xrightarrow{\text{SSOP}}$ POS —

$$xy(z+\bar{z}) + \bar{x}z(y+\bar{y})$$

$$xyz + xy\bar{z} + \bar{x}yz + \bar{x}\bar{y}z$$

111, 110, 011, 001

2- 8 Total Combinations

POS must contain 04 no which are

000, 010, 100, 101

$$(x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$$

$$\Pi M = M_0 \cdot M_2 \cdot M_4 \cdot M_5$$

So $F(x,y,z) = \Pi(0,2,4,5)$

Denotes the Anding of Maxterms and Numbers are the Maxterm of the function

Examples

Express the function in to Sum of minterm and product of maxterm

$(xy+z)(y+xz)$

$xyy + xyz + yz + xzz$

$xy + xyz + yz + xz$ — SOP

$xy(z+\bar{z}) + xyz + yz(x+\bar{x}) + xz(y+\bar{y})$

$xyz + xy\bar{z} + xyz + x\bar{y}z + \bar{x}yz + xyz + x\bar{y}z$

$xyz + xy\bar{z} + \bar{x}yz + x\bar{y}z$ — SSOP

111, 110, 011, 101

So minterms are

$\Sigma m = m_3 + m_5 + m_6 + m_7$

and

$\Pi M = M_0 \cdot M_1 \cdot M_2 \cdot M_4$

$\Pi M(0,1,2,4)$

Examples

Express the following function in Sum of minterm (SOP)

$F(A,B,C,D) = \Sigma(3,5,9,11,15)$

0011, 0101, 1001, 1011, 1111

$\bar{A}\bar{B}CD + \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + A\bar{B}CD + ABCD$

→ 0, 1, 2, 4, 6, 7, 8, 10, 12, 13, 14

Examples

Convert this exp in to SOP and POS.

$(A+B+C)(B+\bar{C}D)$

$ABB + AB\bar{C}D + BC + C\bar{C}D$

$AB + AB\bar{C}D + BC$

$AB(1+\bar{C}D) + BC$

$AB + BC$ (SOP) from

$B(A+C)$ — POS from

Examples

Convert the following exp into Sum of product and Product of Sum

$\bar{x} + x(x+\bar{y})(y+\bar{z})$ Distributed law

$\bar{x} + x(x+\bar{y})(y+\bar{z})$ Distributed law

$(x+\bar{x})[\bar{x} + (x+\bar{y})(y+\bar{z})]$ $A+BC = (A+B)(A+C)$

~~$\bar{x} + x\bar{x} + x\bar{z} + x\bar{y} + \bar{y}\bar{z}$~~

~~$\bar{x} + x + x\bar{z} + x\bar{y} + \bar{y}\bar{z}$~~ again distributed

$(\bar{x} + x + \bar{y})(\bar{x} + y + \bar{z})$

$(1+\bar{y})(\bar{x} + y + \bar{z})$

$\bar{x} + y + \bar{z}$ — SOP form

— again in POS form