Canonical Forms & Logic Operations

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1) Express the boolean Fn $F = A + B'C$ in a sum of minterms

- Add missing variables

$A = A(B+B') = AB + AB'$

$AB(C+C') = ABC + ABC'$

$AB'(C+C') = AB'C + AB'C'$

$B'C = (A+A') \cdot B'C$
$= AB'C + A'B'C$

SO now $F = ABC + ABC' + AB'C + AB'C' \cancel{+ AB'C} + A'B'C$

$\{x + x = x\}$

$= m_7 + m_6 + m_5 + m_4 + m_1$

$\therefore F(A,B,C) = \Sigma(1,4,5,6,7)$

Alternate way: Derive F using Truth Table

Q:- Express $(xy+z)(y+xz)$ in sum of Minterms.

- Repeat for $(A'+B)(B'+C)$

Express $F = xy + x'z$ in a product of maxterms form.

$$F = xy + x'z = (xy + x')(xy + z)$$
$$= \cancel{(x + x')}(x' + y)$$
$$(x + z)(y + z)$$

Add Missing variables

$$x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$$
$$x + z = x + z + yy' = (x + y + z)(x + y' + z)$$
$$y + z = xx' + y + z = \cancel{(x + y + z)}\,\cancel{(x' + y + z)}$$

$$F = (x' + y + z)(x' + y + z')(x + y' + z)$$
$$(x + y + z)$$
$$= M_0 M_2 M_4 M_5 = \Pi(0, 2, 4, 5)$$

Alternate way:

Find F in sum of minterms

$$F = xy(z + z') + x'z(y + y')$$
$$= \underset{m_7}{xyz} + \underset{m_6}{xyz'} + \underset{m_3}{x'yz} + \underset{m_1}{x'y'z}$$
$$(F') = x'y'z' + x'yz' + xy'z' + xy'z$$
$$= \overset{M_0}{(x + y + z)}\,\overset{M_2}{(x + y' + z)}\,\overset{M_4}{(x' + y + z)}$$
$$(F')' =$$
$$(x' + y + z')$$
$$= M_0 M_2 M_4 M_5$$

Q: Represent $y'z + wxy' + wxz' + w'x'z$ in a prod. of maxterms.

Exercise

1) Find the complement of the following in sum of minterms.

a. $F(A, B, C, D) = \Sigma(0, 2, 6, 11, 13, 14)$

b. $F(x, y, z) = \Pi(0, 3, 6, 7)$

2) convert the following to the other canonical form:

a. $F(x, y, z) = \Sigma(1, 3, 7)$

b. $F(A, B, C, D) = \Pi(0, 1, 2, 3, 4, 6, 12)$

Logic operations

⇒ Special properties of Logic gates

For n variables, 2^{2^n} logic F^ns can be defined

Boolean F^n	Symbol	Name	Meaning
$F_0 = 0$		Null	Binary constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x but not y
$F_3 = x$		Transfer	x
$F_4 = x' \cdot y$	y/x	Inhibition	y but not x
$F_5 = y$		Transfer	y
$F_6 = x'y + xy'$	$x \oplus y$	EX-OR	x or y but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x+y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$x \odot y$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary const 1

Extension to Multiple Inputs

AND } Commutative, } For any no. of
OR } Associative } i/ps

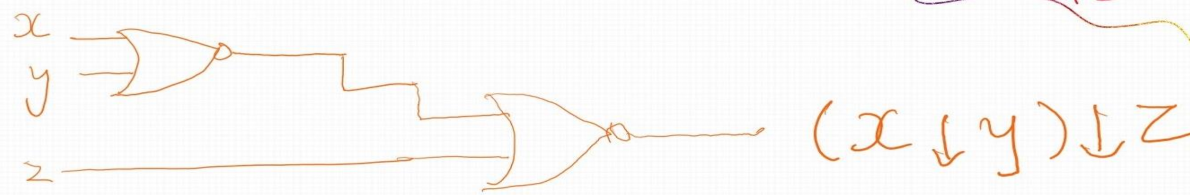
NAND ↑ } Commutative } For Multiple
NOR ↓ } } i/ps

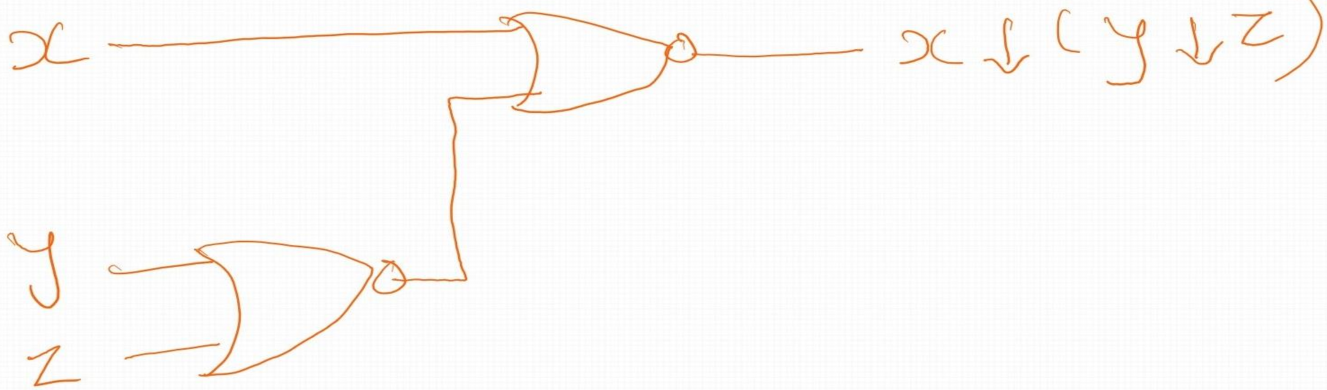
Associative ???

$$(x \downarrow y) \downarrow z = [(x+y)' + z]'$$
$$= (x+y) . z'$$
$$= xz' + yz'$$
$$x \downarrow (y \downarrow z) = [x + (y+z)']'$$
$$= x'(y+z)$$
$$= x'y + x'z$$

Not equal

$\therefore$ NOT Associative



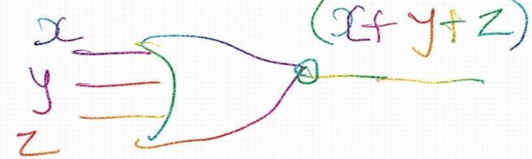


To overcome this difficulty,

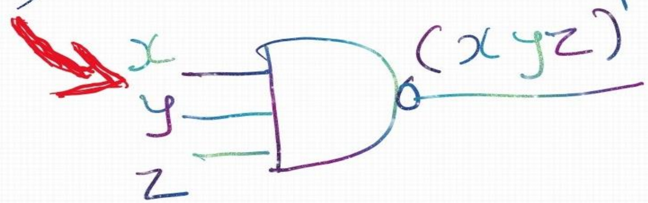
Multiple NOR

" NAND

$x \downarrow y \downarrow z = (x+y+z)'$



$x \uparrow y \uparrow z = (xyz)'$



EX-OR , EX-NOR
(Equivalence)

Commutative } ?? → CHECK ??
Associative }

EX-OR is an ODD Fn.

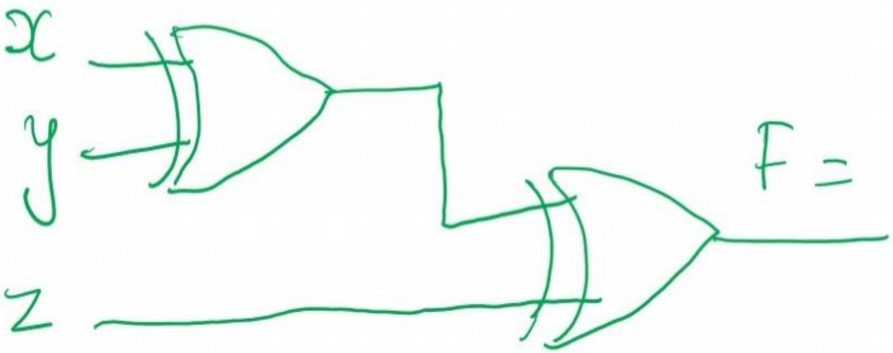
It is equal to 1 if the i/p variables have an odd no. of 1's

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

3-i/p EX-OR operation

$F = x \oplus y \oplus z$

equal ?



$F = (x \oplus y) \oplus z$

What about EX_NOR ??

Odd/even ?

Exercise:

1> The dual of EX-OR is equal to its complement.

State True / False.
Justify the answer

2> Equivalence is an even F^n.

3> Inhibition operation is neither commutative nor associative.