The Karnaugh Map

The Karnaugh Map

- Feel a little difficult using Boolean algebra laws, rules, and theorems to simplify logic?
- A K-map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the <u>minimum expression</u>.

What is K-Map

- It's similar to truth table; instead of being organized (i/p and o/p) into columns and rows, the K-map is an array of cells in which each cell represents a binary value of the input variables.
- The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells.
- K-maps can be used for expressions with 2, 3, 4, and 5 variables.
  - 3 and 4 variables will be discussed to illustrate the principles.

The 2 Variable K-Map

Two and three variable:

$2^2 = 4$ so four cells
$2^3 = 8$ so eight cells

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	0	1
A 1	2	3

m_0	m_1
m_2	m_3

x \ y	0($\bar{y}$)	1(y)
($\bar{x}$) 0	0	1
(x) 1	2	3

– Note

- x appeared primed in Row 0 and unprimed in Row 1
- llly y appeared primed in Col 0 and unprimed in column 1.

The 3 Variable K-Map

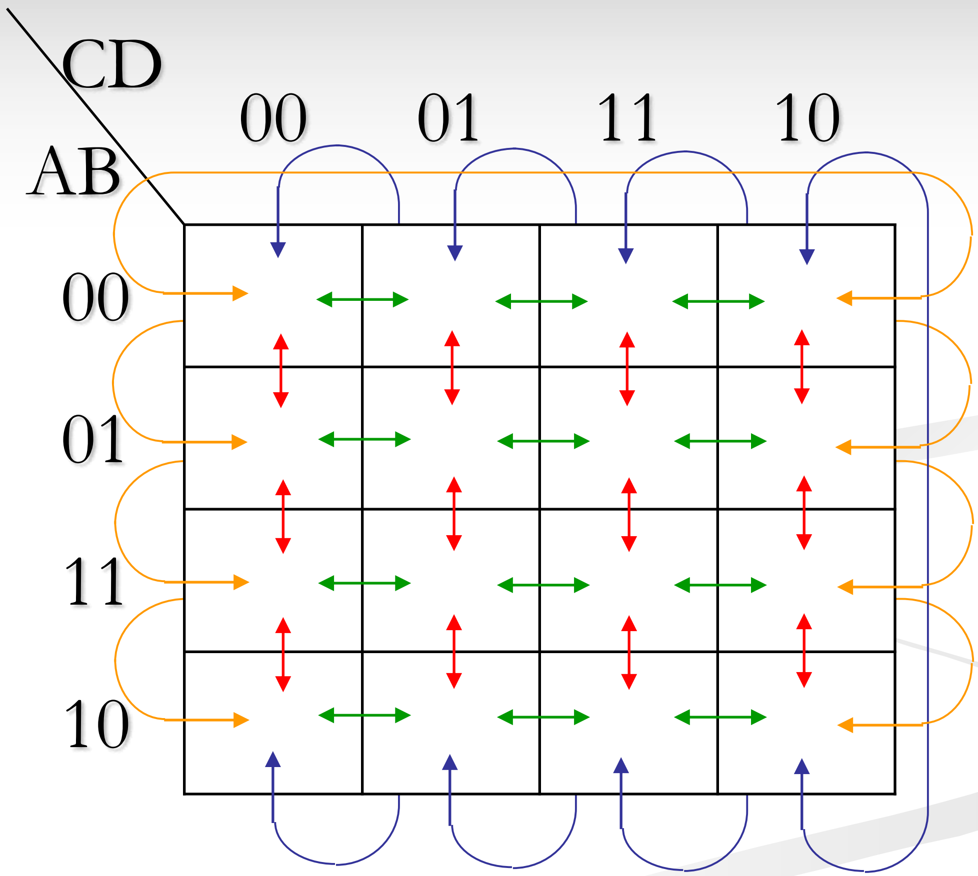
- There are 8 cells as shown:

AB \ C	0	1
00	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$
01	$\overline{A}B\overline{C}$	$\overline{A}BC$
11	$AB\overline{C}$	ABC
10	$A\overline{B}\,\overline{C}$	$A\overline{B}C$

The 4-Variable K-Map

AB \ CD	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}\,\overline{B}CD$	$\overline{A}\,\overline{B}C\overline{D}$
01	$\overline{A}B\overline{C}\,\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
11	$AB\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

Cell Adjacency



K-Map SOP Minimization

- The K-Map is used for simplifying Boolean expressions to their minimal form.
- A minimized SOP expression contains the fewest possible terms with fewest possible variables per term.
- Generally, a minimum SOP expression can be implemented with fewer logic gates than a standard expression.

Mapping a Standard SOP Expression

- For an SOP expression in standard form:
  - A 1 is placed on the K-map for each product term in the expression.
  - Each 1 is placed in a cell corresponding to the value of a product term.
  - Example: for the product term $A\overline{B}C$, a 1 goes in the 101 cell on a 3-variable map.

AB \ C	0	1
00	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$
01	$\overline{A}B\overline{C}$	$\overline{A}BC$
11	$AB\overline{C}$	ABC
10	$A\overline{B}\,\overline{C}$	$A\overline{B}C$ 1

Mapping a Standard SOP Expression (full example)

The expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + AB\overline{C} + A\overline{B}\,\overline{C}$$

000 001 110 100

AB \ C	0	1
00	1	1
01		
11	1	
10	1	

Practice:

$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$

$\overline{A}BC + A\overline{B}C + A\overline{B}\,\overline{C}$

$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$

Mapping a Nonstandard SOP Expression

- A Boolean expression must be in standard form before you use a K-map.
  - If one is not in standard form, it must be converted.
- You may use the procedure mentioned earlier or use numerical expansion.

Mapping a Nonstandard SOP Expression

- Numerical Expansion of a Nonstandard product term
  - Assume that one of the product terms in a certain 3-variable SOP expression is $A\overline{B}$.
  - It can be expanded numerically to standard form as follows:
    - **Step 1:** Write the binary value of the two variables and attach a 0 for the missing variable $\overline{C}$: 100.
    - **Step 2:** Write the binary value of the two variables and attach a 1 for the missing variable C: 100.
  - The two resulting binary numbers are the values of the standard SOP terms → $A\overline{B}\overline{C}$ and $A\overline{B}C$.
- If the assumption that one of the product term in a 3-variable expression is B. How can we do this?

Mapping a Nonstandard SOP Expression

- Map the following SOP expressions on K-maps:

$$\overline{A} + A\overline{B} + AB\overline{C}$$

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

K-Map Simplification of SOP Expressions

- After an SOP expression has been mapped, we can do the process of *minimization:*
  - Grouping the 1s
  - Determining the minimum SOP expression from the map

Grouping the 1s

- You can group 1s on the K-map according to the following rules by enclosing those adjacent cells containing 1s.
- **The goal** is to maximize the size of the groups and to minimize the number of groups.

Grouping the 1s (rules)

1. A group must contain either 1,2,4,8,or 16 cells (depending on number of variables in the expression)
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

Grouping the 1s (example)

AB \ C	0	1
00	1	
01		1
11	1	1
10		

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

Grouping the 1s (example)

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

Determining the Minimum SOP Expression from the Map

- The following rules are applied to find the minimum product terms and the minimum SOP expression:
  1. Group the cells that have 1s. Each group of cell containing 1s creates one product term composed of all variables that occur in only one form (either complemented or complemented) within the group. Variables that occur both complemented and uncomplemented within the group are eliminated → called *contradictory variables*.

Determining the Minimum SOP Expression from the Map

2. Determine the minimum product term for each group.
   - For a 3-variable map:
     1. A 1-cell group yields a 3-variable product term
     2. A 2-cell group yields a 2-variable product term
     3. A 4-cell group yields a 1-variable product term
     4. An 8-cell group yields a value of 1 for the expression.
   - For a 4-variable map:
     1. A 1-cell group yields a 4-variable product term
     2. A 2-cell group yields a 3-variable product term
     3. A 4-cell group yields a 2-variable product term
     4. An 8-cell group yields a a 1-variable product term
     5. A 16-cell group yields a value of 1 for the expression.

Determining the Minimum SOP Expression from the Map

3. When all the minimum product terms are derived from the K-map, they are summed to form the minimum SOP expression.

Determining the Minimum SOP Expression from the Map (example)

$$B + \overline{A}C + A\overline{C}D$$

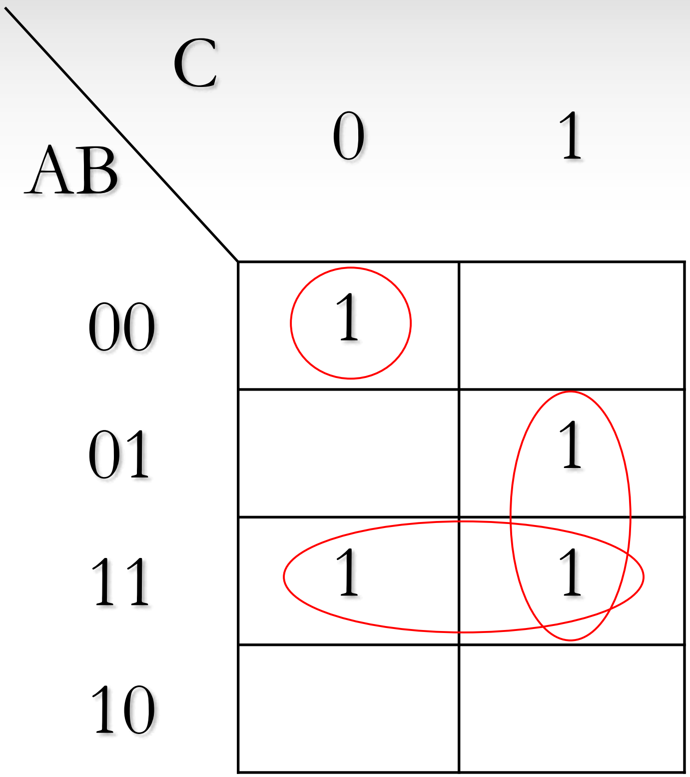
AB \ CD	00	01	11	10
00			1	1
01	1	1	1	1
11	1	1	1	1
10		1		

$\overline{A}C$

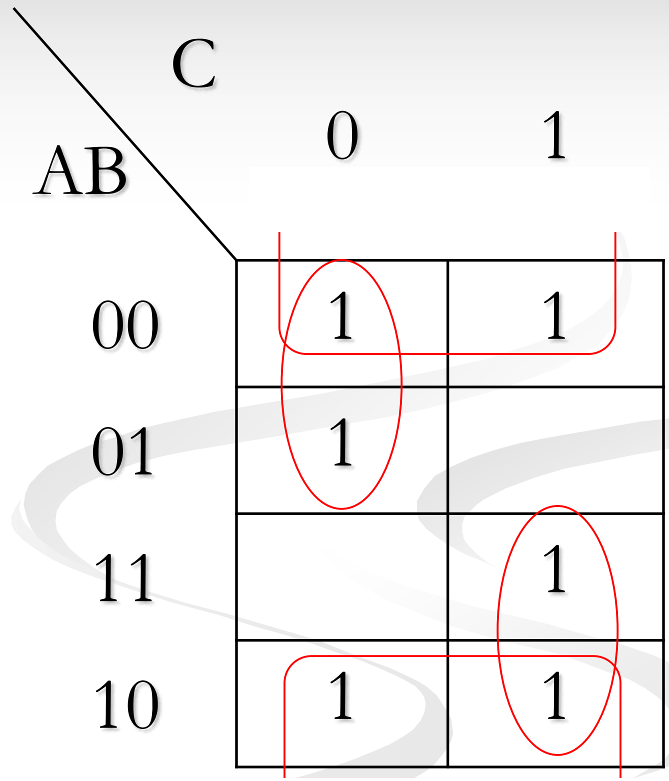
B

$A\overline{C}D$

Determining the Minimum SOP Expression from the Map (exercises)



$AB + BC + \overline{A}\,\overline{B}\,\overline{C}$



$\overline{B} + \overline{A}\,\overline{C} + AC$

Determining the Minimum SOP Expression from the Map (exercises)

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

$$\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$$

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

$$\overline{D} + A\overline{B}C + B\overline{C}$$

Practicing K-Map (SOP)

$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

$$\overline{B} + \overline{A}C$$

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD +$$
$$\overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

$$\overline{D} + \overline{B}C$$

Mapping Directly from a Truth Table

I/P			O/P
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

AB \ C	0	1
00	1	
01		
11	1	1
10	1	

“Don’t Care” Conditions

- Sometimes a situation arises in which some input variable combinations are not allowed, i.e. BCD code:
  - There are six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111.
- Since these unallowed states will never occur in an application involving the BCD code → they can be treated as “don’t care” terms with respect to their effect on the output.
- The “don’t care” terms can be used to advantage on the K-map (how? see the next slide).

“Don’t Care” Conditions

INPUTS				O/P
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

AB \ CD	00	01	11	10
00				
01			1	
11	X	X	X	X
10	1	1	X	X

Without “don’t care”

$$Y = A\bar{B}\bar{C} + \bar{A}BCD$$

With “don’t care”

$$Y = A + BCD$$

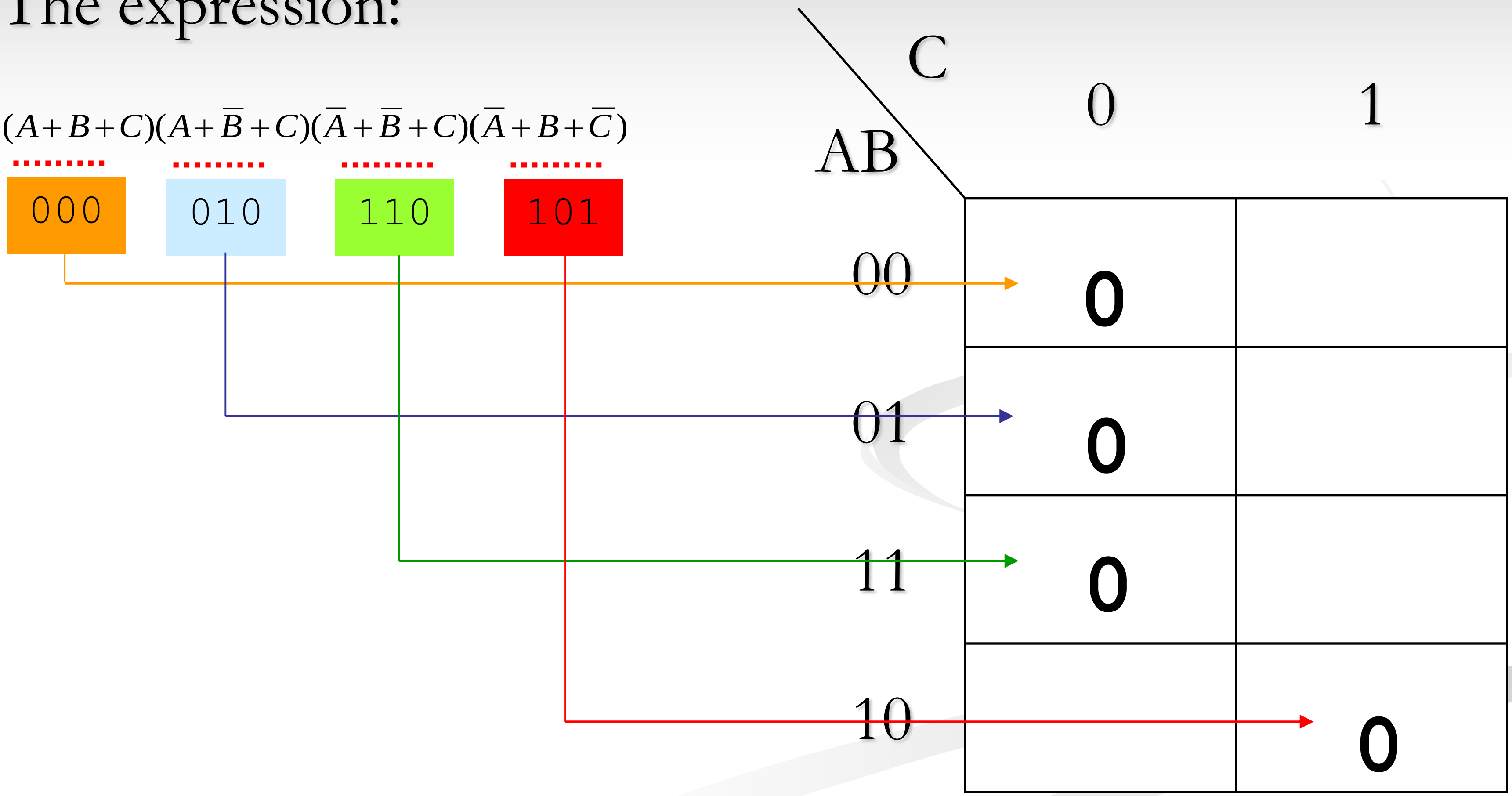
K-Map POS Minimization

- The approaches are much the same (as SOP) except that with POS expression, 0s representing the standard sum terms are placed on the K-map instead of 1s.

Mapping a Standard POS Expression (full example)

The expression:

$$(A+B+C)(A+\overline{B}+C)(\overline{A}+\overline{B}+C)(\overline{A}+B+\overline{C})$$



K-map Simplification of POS Expression

$$(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+C)$$