Tabulation Method

Tabulation Method

- As no of variables increases beyond 6 K-map is not practically possible because it is impossible to select best output.
- Tabulation solve this difficulty. It is step by step procedure that is gauranteed to produce simplified standard form expression for a function
- Suitable for m/c computation
- first formulated by Quine and later improved by McCluskey.

- It consist of two parts
  (I) find all terms that are candidate ~~for~~ to be included in simplified function
  These terms are called "Prime Implicants."
  (II) Chose among the prime implicants, those terms that ~~[illegible]~~ give an expression with the least number of literals.

* How to determine Prime Implicants.

- It use repeatedly $A+\bar{A}=1$
  'A = 1', '$\bar{A}$ = 0', absence of variable by '—'

Step-1 :- find the min terms, if not given, from the expression.

2. Compare each min term (in binary) with every other min term (next). If two min term differ in only one variable then combine them. Put — (dash) against eliminated ~~[illegible]~~ variable.

eg) Listing two minterm

A	B	C	D
1	1	1	0
1	1	1	1
1	1	1	—

Can combine differ in only one digit position

Put _ dash sign

A	B	C	D
1	1	0	1
1	1	1	0

Can't be combined because differ in more than one digit position.

- Number of 1's in a term referred to as its index.

eg) f(A,B,C,D)

0000 —— index0

0010, 1000 —— Index1

1010, 0011, 1001 —— Index2

1110, 1101 —— Index3

1111 —— Index4

The ...

Index 4

- The necessary Condition for Combining two terms is that the Index of the two term must differ by one Logic Variable which must also be the Same.

- ✓ means one term has been Combined with other term
- — dash Indicates missing Variable (eliminated during matching)
- Any unticked term in any list must be included in the final expression (except from the last list)

Problem-1

Prob: Simplify the following boolean function by using tabulation method.

Soln: To make the things easier change the function in to binary notations with index and decimal value

$f = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$

$F(A, B, C, D) = \Sigma(0000, 0001, 0010, 1000, 1010, 1011, 1110, 1111)$

Index value – $\begin{array}{cccccccc} 0 & 1 & 2 & 8 & 10 & 11 & 14 & 15 \\ (0, & 1, & 1, & 1, & 2, & 2, & 3, & 4) \end{array}$

decimal value $\Sigma(0, 1, 2, 8, 10, 11, 14, 15)$

first list

... value Σ(0, 1, 2, 8, 10, 11, 14, 15)

first list

i	d	A	B	C	D
0	0	0	0	0	0 ✓
	1	0	0	0	1 ✓
1	2	0	0	1	0 ✓
	8	1	0	0	0 ✓
2	10	1	0	1	0 ✓
	11	1	0	1	1 ✓
3	14	1	1	1	0 ✓
4	15	1	1	1	1 ✓

Second list

	A	B	C	D
0,1	0	0	0	_
0,2	0	0	_	0 ✓
0,8	_	0	0	0 ✓
2,10	_	0	1	0 ✓
8,10	1	0	_	0 ✓
10,11	1	0	1	_ ✓
10,14	1	_	1	0 ✓
11,15	1	_	1	1 ✓
14,15	1	1	1	_ ✓

Third list

	A	B	C	D
0,2,8,10	_	0	_	0
0,8,2,10	_	0	_	0
10,11,14,15	1	_	1	_
10,14,11,15	1	_	1	_

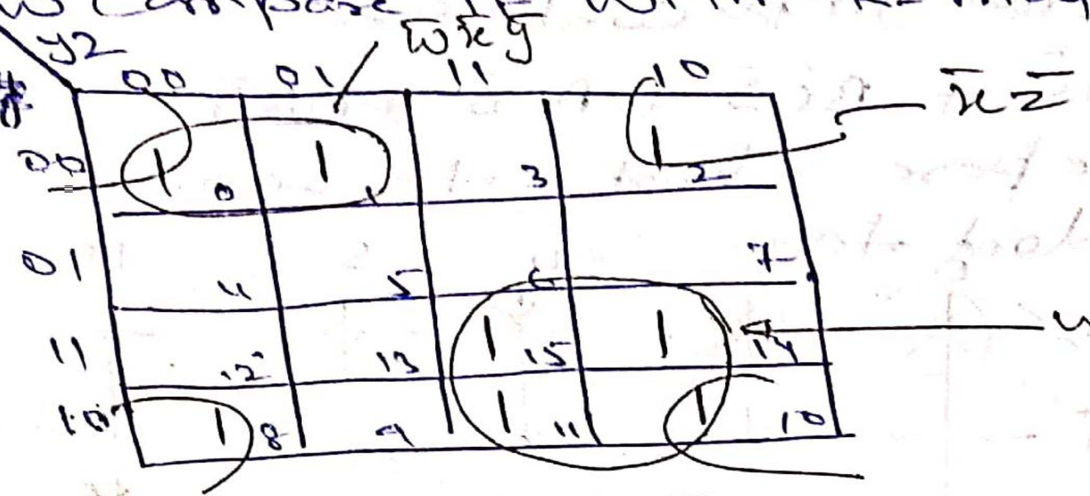
For the given example, the sum of prime implicants gives the minimized function in SOP for

$F = \bar{A}\bar{B}\bar{C} + \bar{B}\bar{D} + AC$

or

$F = \bar{w}\bar{x}\bar{y} + \bar{x}\bar{z} + wy$

Now compare it with K-map.



$F = \bar{x}\bar{z} + wy + \bar{w}\bar{x}\bar{y}$

In most of the cases the sum of prime implicants does not necessarily form the expression with minimum no of terms.

Example-2

Ex Determine the prime Implicants of the function

Soln $f(w,x,y,z) = \sum_m(1,4,6,7,8,9,10,11,15)$

Binary $= \sum(0001, 0100, 0110, 0111, 1000, 1001, 1010, 1011, 1111)$

Index value $= \sum(1, 1, 2, 3, 1, 2, 2, 3, 4)$

first list

	d	w	x	y	z	
1	1	0	0	0	1	✓
	4	0	1	0	0	✓
	8	1	0	0	0	✓
2	6	0	1	1	0	✓
	9	1	0	0	1	✓
	10	1	0	1	0	✓
3	7	0	1	1	1	✓
	11	1	0	1	1	✓
4	15	1	1	1	1	✓

Second list

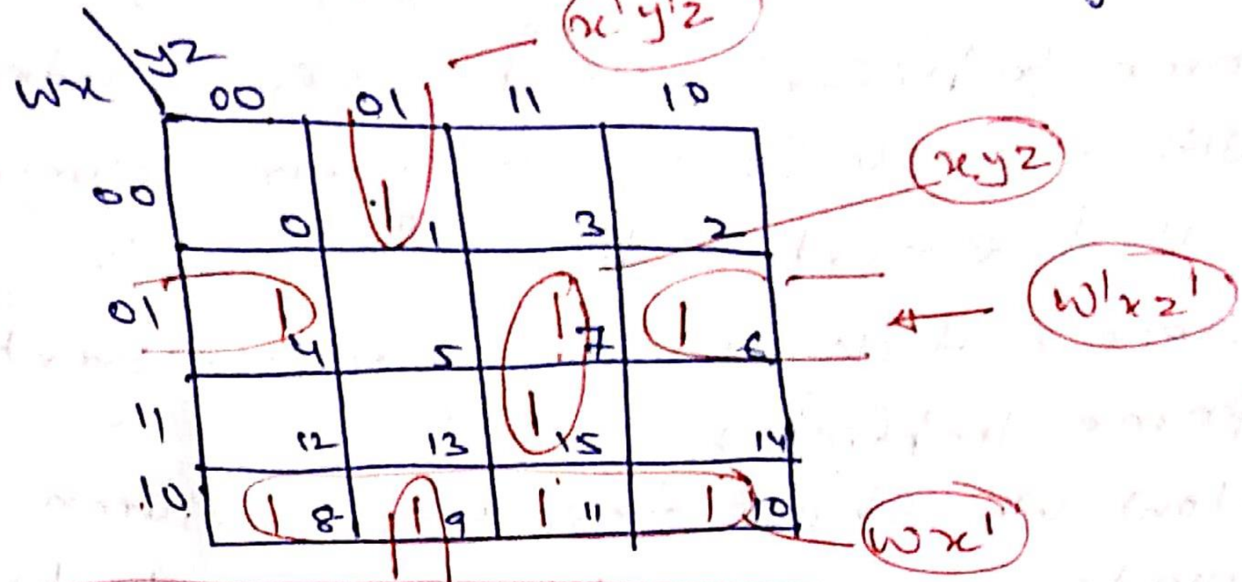
	w	x	y	z	
1,9	–	0	0	1	
4,6	0	1	–	0	
8,9	1	0	0	–	✓
8,10	1	0	–	0	✓
6,7	0	1	1	–	
9,11	1	0	–	1	✓
10,11	1	0	1	–	✓
7,15	–	1	1	1	
11,15	1	–	1	1	

Third list

	w	x	y	z
8,9,10,11	1	0	–	–
8,10,9,11	1	0	–	–

$F = x'y'z + wx'z' + w'xy' + xyz' + wy'z + wx'$

But if we try to form the K map and Reduce the same exp then we get



$$\boxed{f = x'y'z + xyz + w'xz' + wx'}$$

It consist of four prime implicants out of six derived by Tabular method

So we have to select the prime implicants those give minimized function.

w, x, y, z

Selection of prime Implicants :-

- Generate a prime Implicant table
- In this table each prime Implicant is represented in a row and each min term in a column.
- Crosses are placed in each row to show the composition of minterms that make the prime Implicant

	1	4	6	7	8	9	10	11	15
1,9 - $\bar{x}\bar{y}z$ ✓	X					X			
4,6 - $\bar{w}x\bar{z}$ ✓		X	X						
6,7 - $\bar{w}xy$			X	X					
7,15 - xyz				X					X
11,15 - wyz								X	X
8,9,10,11 - $w\bar{x}$ ✓					X	X	X	X	
	✓	✓	✓	1	✓	✓	✓	✓	1

- After that we select X only one [illegible]

Essential prime Implicants

- first scan the table to find the column containing only a single cross
- Prime implicants that cover minterms with a single cross in their column are called essential prime implicants.

Place a tick mark against essential prime implicants

→ Now we check the each column whose minterm is covered by the selected essential prime implicants.

eg) $\bar{x}\bar{y}z$ cover 1, 9 so place ✓

$\bar{w}x\bar{z}$ covers 4 & 6 so place ✓

$w\bar{x}$ covers 8, 9, 10, 11 so place ✓

Selected essential prime implicants covers all the minterms of the function except 7 and 15.

These two minterms must be ~~selected~~ included by selection of one or more prime implicants

In this example prime implicant xyz ✓ covers both minterm 7 & 15 so minimized function is

$$\boxed{F = \bar{x}\bar{y}z + \bar{w}x\bar{z} + w\bar{x} + xyz}$$

- Tabulation method can also be adapted to give a simplified exp in POS form.
- We have to start with the complement of the function by taking 0's as the initial list of minterms. This list contain those minterms not included in the original function which are numerically equal to maxterm of the function)
- The Tabulation method carried out with the 0's of the function and terminate with a simplified expression in SOP of the complement of function. By taking complement again, we obtain the simplified POS expression

We use don't care when prime implicants are determined but do not use don't care terms when prime implicants table is setup because don't care terms not have to be covered by the selected prime implicants.

→ Reason for not included don't care terms in prime implicants table

Example-3

① find the minimal expression for
(P, Q, R, S) $f = \prod M\,(2, 3, 8, 12, 13) \cdot d\,(10, 14)$
Using Tabulation method.

Soln

$\prod M\,(0010, 0011, 1000, 1100, 1101)$
$\cdot\, d\,(1010, 1110)$

Index value $\prod m\,(1, 2, 1, 2, 3) \cdot d\,(2, 3)$

Here use 0 — A, 1 — $\bar{A}$
write the term in sum form

Step 1

i	d	P	Q	R	S	
1	2	0	0	1	0	✓
	8	1	0	0	0	✓
2	3	0	0	1	1	✓
	10	1	0	1	0	✓
	12	1	1	0	0	✓
3	13	1	1	0	1	✓
	14	1	1	1	0	✓

Prime Impl...

Step 2

d	P	Q	R	S	
2, 3	0	0	1	—	
2, 10	—	0	1	0	
8, 10	1	0	—	0	✓
8, 12	1	—	0	0	✓
10, 14	1	—	1	0	✓
12, 13	1	1	0	—	
12, 14	1	1	—	0	✓

Step-3

d	P	Q	R	S
8, 12, 10, 14	1	—	—	0

except last

Note

- Do not use don't care when prime implicant table is formed

Prime Implicants Chart Table is formed

		2	3	8	12	13
2, 3	$\bar{P}\bar{Q}R$ ✓	×	×			
2, 10	$\bar{Q}R\bar{S}$	×				
12, 13	$PQ\bar{R}$ ✓				×	×
8, 10, 12, 14	$P\bar{S}$ ✓			×	×	
		✓	✓	✓	✓	✓

Col. contain only one single cross (3, 8, 13)

$$F = \bar{P}\bar{Q}R + PQ\bar{R} + P\bar{S}$$

$$= (A + B + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + D)$$ ✓

Example-4

(1) Minimize the following expression using Tabular Method / Quine McCluskey Method

$Y = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$

→ find binary equivalent of each minterms

Σ(0000, 0010, 0011, 0100, 0111, 1000, 1010, 1100, 1101)

→ find index value of each minterm in binary

Σ(0, 1, 2, 2, 3, 1, 2, 2, 3) (for 0, 2, 3, 6, 7, 8, 10, 12, 13)

Step 1

Group	Minterm	A	B	C	D	
0	0	0	0	0	0	✓
1	2	0	0	1	0	✓
	8	1	0	0	0	✓
2	3	0	0	1	1	✓
	6	0	1	1	0	✓
	10	1	0	1	0	✓
	12	1	1	0	0	✓
3	7	0	1	1	1	✓
	13	1	1	0	1	✓

Step-2

	A	B	C	D	
0,2	0	0	–	0	✓
0,8	–	0	0	0	✓
2,3	0	0	1	–	✓
2,6	0	–	1	0	✓
2,10	–	0	1	0	✓
8,10	1	0	–	0	✓
8,12	1	–	0	0	→ $A\bar{C}\bar{D}$,
3,7	0	–	1	1	✓
6,7	0	1	1	–	✓
12,13	1	1	0	–	→ $AB\bar{C}$

Step 3

	A	B	C	D
0,2,8,10	–	0	–	0
0,8,2,10	–	0	–	0
2,3,6,7	0	–	1	–
2,6,3,7	0	–	1	–

$\bar{B}\bar{D} + \bar{A}C$,

3. Check only one × column and put ✓ in front of minterm

1. So all unticked min terms will form the candidates for the minimized exp.
2. Then prepare the prime implicant table

... the prime implicant table

		0	2	3	6	7	8	10	12	13
0, 2, 8, 10	$\bar{B}\bar{D}$ ✓	×	×				×	×		
2, 3, 6, 7	$\bar{A}C$ ✓		×	×	×	×				
12, 13	$AB\bar{C}$ ✓								×	×
8, 12	$A\bar{C}\bar{D}$						×		×	
		✓	✓	✓	✓	✓	✓	✓	✓	✓

$f = \bar{B}\bar{D} + \bar{A}C + AB\bar{C}$

If any column remaining unticked then we must include that in minimized ~~expre ssion~~ / Select minimum no of primes that cover all the minterms

Example-5

Simplify the following function using QMC method in SOP form

$Y(A,B,C,D) = \Pi M (1,4,6,9,10,11,14,15)$

Bin...

$... \Pi M (1,4,6,9,10,11,14,15)$ form

Binary of all maxterms | minterms

$\Sigma m (0,2,3,5,7,8,12,13)$

Σm (0000, 0010, 0011, 0101, 0111, 1000, 1100, 11..

Index value

0 2 3 5 7 8 12 13

$\Sigma m (0,1,2,2,3,1,2,3)$

Step 1

i	d	A	B	C	D	
0	0	0	0	0	0	✓
1	2	0	0	1	0	✓
	8	1	0	0	0	✓
2	3	0	0	1	1	✓
	5	0	1	0	1	✓
	12	1	1	0	0	✓
3	7	0	1	1	1	✓
	13	1	1	0	1	✓

Step 2

	A	B	C	D
0,2	0	0	–	0
0,8	–	0	0	0
2,3	0	0	1	–
8,12	1	–	0	0
3,7	0	–	1	1
5,7	0	1	–	1
5,13	–	1	0	1
12,13	1	1	0	–

Step 3

NIL

		0	2	3	5	7	8	12	13
0,2	$\bar{A}\bar{B}\cdot\bar{D}$ ✓	×	×						
0,8	$\bar{B}\bar{C}\bar{D}$ ✓	×					×		
2,3	$\bar{A}\bar{B}\cdot C$		×	×					
8,12	$A\bar{C}\cdot\bar{D}$						×	×	
3,7	$\bar{A}C\cdot D$ ✓			×		×			
5,7	$\bar{A}BD$ ✓				×	×			
5,13	$B\bar{C}D$ ✓				×				×
12,13	$AB\bar{C}$ ✓							×	×
		✓	✓				✓		

In this Case Select the minimum number of primes that must Cover all the min terms.

SOP → $f_{min} = AB\bar{C} + B\bar{C}D + \bar{A}BD + \bar{A}CD + \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{D}$

POS → $(\bar{A}+\bar{B}+C)(\bar{B}+C+\bar{D})(A+\bar{B}+\bar{D})(A+\bar{C}+\bar{D})(B+C+D)(A+B+D)$

STEP 1 - EXAMPLE

$f^{on} = \{m_0, m_1, m_2, m_3, m_5, m_8, m_{10}, m_{11}, m_{13}, m_{15}\} = \sum (0, 1, 2, 3, 5, 8, 10, 11, 13, 15)$

Minterm	Cube			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0
3	0	0	1	1
5	0	1	0	1
10	1	0	1	0
11	1	0	1	1
13	1	1	0	1
15	1	1	1	1

STEP 1 - EXAMPLE

$f^{on} = \{m_0, m_1, m_2, m_3, m_5, m_8, m_{10}, m_{11}, m_{13}, m_{15}\} = \sum(0, 1, 2, 3, 5, 8, 10, 11, 13, 15)$

Minterm	Cube				
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
10	1	0	1	0	✓
11	1	0	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

Minterm	Cube			
0,1	0	0	0	-
0,2	0	0	-	0
0,8	-	0	0	0
1,3	0	0	-	1
1,5	0	-	0	1
2,3	0	0	1	-
2,10	-	0	1	0
8,10	1	0	-	0
3,11	-	0	1	1
5,13	-	1	0	1
10,11	1	0	1	-
11,15	1	-	1	1
13,15	1	1	-	1

STEP 1 - EXAMPLE

$f^{on} = \{m_0, m_1, m_2, m_3, m_5, m_8, m_{10}, m_{11}, m_{13}, m_{15}\} = \sum (0, 1, 2, 3, 5, 8, 10, 11, 13, 15)$

Minterm	Cube				
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
10	1	0	1	0	✓
11	1	0	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

Minterm	Cube				
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,3	0	0	-	1	✓
1,5	0	-	0	1	
2,3	0	0	1	-	✓
2,10	-	0	1	0	✓
8,10	1	0	-	0	✓
3,11	-	0	1	1	✓
5,13	-	1	0	1	
10,11	1	0	1	-	✓
11,15	1	-	1	1	
13,15	1	1	-	1	

Minterm	Cube			
0,1,2,3	0	0	-	-
0,8,2,10	-	0	-	0
2,3,10,11	-	0	1	-

STEP 1 - EXAMPLE

$f^{on} = \{m_0, m_1, m_2, m_3, m_5, m_8, m_{10}, m_{11}, m_{13}, m_{15}\} = \sum(0, 1, 2, 3, 5, 8, 10, 11, 13, 15)$

Minterm	Cube				
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
10	1	0	1	0	✓
11	1	0	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

Minterm	Cube				
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,3	0	0	-	1	✓
1,5	0	-	0	1	PI=D
2,3	0	0	1	-	✓
2,10	-	0	1	0	✓
8,10	1	0	-	0	✓
3,11	-	0	1	1	✓
5,13	-	1	0	1	PI=E
10,11	1	0	1	-	✓
11,15	1	-	1	1	PI=F
13,15	1	1	-	1	PI=G

Minterm	Cube				
0,1,2,3	0	0	-	-	PI=A
0,8,2,10	-	0	-	0	PI=C
2,3,10,11	-	0	1	-	PI=B

***Question:* Can this be done on a CCM? How modified?**

$f^{on} = \{A,B,C,D,E,F,G\}$ = {00--, -01-, -0-0, 0-01, -101, 1-11, 11-1}

STEP 2 – Construct Cover Table

- PIs Along Vertical Axis (in order of # of literals)
- Minterms Along Horizontal Axis

	0	1	2	3	5	8	10	11	13	15
A	x	x	x	x						
B			x	x			x	x		
C	x		x			x	x			
D		x			x					
E					x				x	
F								x		x
G									x	x

NOTE: Table 4.2 in book is incomplete

Questions

1. Explain Quine-McCluskey Method.
2. How to program QM Method in CCM?
3. Methods to solve Covering Problem.
4. Possible Applications of Covering.
5. Reduction of Covering to Petrick Function.
6. Merging Operator.