Boolean Algebra

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Outline

- Error Detection Codes (Chapter 1)
- Boolean Algebra (Chapter 2)
  - Basic Boolean Equations
  - Multiple Level Logic Representation
  - Basic Identities
  - Algebraic Manipulation
  - Complements and Duals
- Practice Problems

Error Detection using Parity Bits

- Additional bits used to identify transmission errors, are called parity bits
- Each message is sent with an extra bit (parity bit) to make the total number of 1's transmitted either odd or even
- Used to detect bit reversal errors
- Limitations??

Error Detection using Parity Bits

Odd parity		Even parity	
Message	P	Message	P
0000	1	0000	0
0001	0	0001	1
0010	0	0010	1
0011	1	0011	0
0100	0	0100	1
0101	1	0101	0
0110	1	0110	0
0111	0	0111	1
1000	0	1000	1
1001	1	1001	0
1010	1	1010	0
1011	0	1011	1
1100	1	1100	0
1101	0	1101	1
1110	0	1110	1
1111	1	1111	0

count the sum of each digit
-> if sum is 1 then even parity's p is 1 and odd parity's p is 0
-> if sum is 0 then even parity's p is 0 and odd parity's p is 1

***ignore the carry if any**

History

- Invented by George Boole
- “spring of 1847 that he put his ideas into the pamphlet called *Mathematical Analysis of Logic*.” from wikipedia.com

Basic Boolean Equations

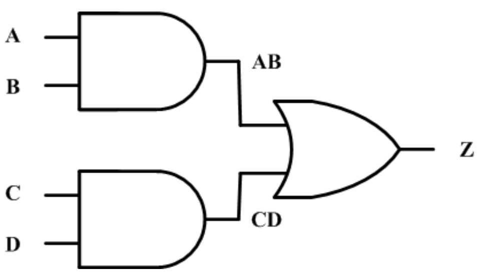
- For the basic gates/functions
- AND
  - $Z = A\ B$
  - $X = C\ D\ E$ 3 input gate
  - $Y = F\ G\ H\ K$ 4 input gate
- OR
  - $Z = A + B$
  - $Y = F + G + H + K$ 4 input gate
- NOT
  - $Z = \overline{A}$
  - $Y = \overline{(F\ G\ H\ K)}$ actually 2-level logic

2-Level Logic

- Consider the following logic equation
  - Z(A,B,C,D) = A B + C D
  - The Z(A,B,C,D) means that the output is a function of the four variables within the ().
  - The AB and CD are terms of the expression.
  - This form of representing the function is an algebraic expression.
  - For this function to be True, either both A AND B are True OR both C AND D are True.

Truth table expression

- Just like we had the truth tables for the basic functions, we can also construct truth tables for any function.



A	B	C	D	Z	AB	CD
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	1	0	1
0	1	0	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	1	1	1	0	1
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	1	0	1
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	1	1	0
1	1	1	1	1	1	1

Examples of Boolean Equations

- Some examples
  - F = AB + CD + BD'
  - Y = CD + A'B'
  - SUM = AB + A Cin + B Cin
  - $P = A_0A_1A_2A_3A_4B_0B_1B_2B_3B_4 + \ldots$

  - Equations can be very complex
  - Usually desire a minimal expression

Basic Identities of Boolean Algebra

- 1) $X + 0 = X$

OR +

A	0	RESULT
0	0	0
1	0	1

- 2) $X \cdot 1 = X$

AND •

A	1	RESULT
0	1	0
1	1	1

- 3) $X + 1 = 1$
- 4) $X \cdot 0 = 0$
- 5) $X + X = X$
- 6) $X \cdot X = X$

Basic Identities (2)

- 7) $X + X' = 1$

X	X'	RES
0	1	1
1	0	1

- 8) $X \cdot X' = 0$

X	X'	RES
0	1	0
1	0	0

- 9) $(X')' = X$

Basic Properties (Laws)

- Commutative
  - 10) $X + Y = Y + X$
- Associative
  - 12) $X+(Y+Z)=(X+Y)+Z$
- Distributive
  - 14) $X(Y+Z) = XY+XZ$
  - AND distributes over OR

- Commutative
  - 11) $X \cdot Y = Y \cdot X$
- Associative
  - 13) $X(YZ) = (XY)Z$
- Distributive
  - 15) $X+YZ=(X+Y)(X+Z)$
  - OR distributes over AND

Basic Properties (2)

- DeMorgan's Theorem
- Very important in simplifying equations
  - 16) $(X + Y)' = X' \cdot Y'$
  - 17) $(XY)' = X' + Y'$

X	Y	X+Y	$\overline{X+Y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X}\bullet\overline{Y}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Where to use?

- These properties (Laws and Theorems) can be used to simplify equations to their simplest form.
  - Simplify F=X'YZ+X'YZ'+XZ

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

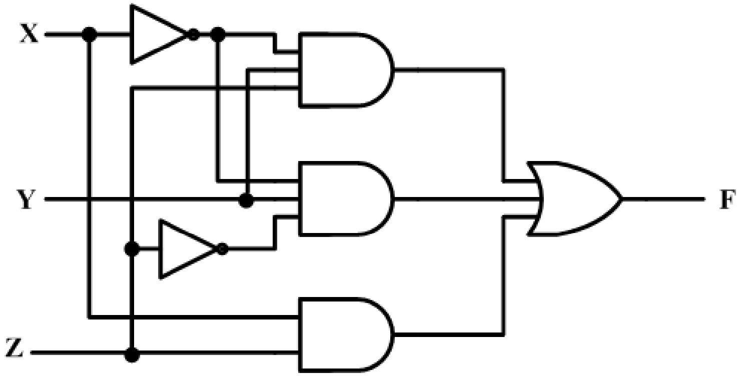
$$= \overline{X}Y(Z + \overline{Z}) + XZ \qquad \text{by identity 14}$$

$$= \overline{X}Y\bullet 1 + XZ \qquad \text{by identity 7}$$

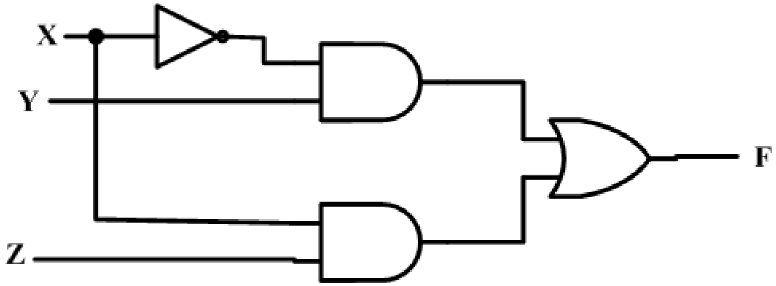
$$= \overline{X}Y + XZ \qquad \text{by identity 2}$$

Effect on implementation

- F = X'YZ + X'YZ' + XZ



- Reduces to F = X'Y + XZ



Other examples

- Examples from the text
  - 1) **X + XY**

    = X·1 + XY = X(1+Y) = X·1 = X

    Use 2 14 3 2
  - 2) **XY+XY'**

    = X(Y + Y') = X·1 = X

    Use 14 7 2
  - 3) **X+X'Y**

    = (X+X')(X+Y) = 1· (X+Y) = X+Y

    Use 15 7 2

Further Examples

- Examples from the text
  - 4) **$X\cdot (X+Y)$**
  - $=X\cdot X+X\cdot Y= X+XY=X(1+Y)=X\cdot 1=X$
    - Use 14 6 14 3 2
  - 5) **$(X+Y)\cdot(X+Y')$**
  - $=XX+XY'+XY+YY'=X+XY'+XY+0=X(1+Y'+Y)=X\cdot 1=X$
  - 6) **$X(X'+Y)$**
  - $= XX'+XY = 0 + XY = XY$

Consensus Theorem

- The Theorem gives us the relationship
  - $XY + X'Z + YZ = XY + X'Z$
  - How??

Application of Consensus Theorem

- Consider
  - $(A+B)(A'+C) = AA' + AC + A'B + BC$
  - $= AC + A'B + BC$
  - $= AC + A'B$

Complement of a function

- In real implementations, sometimes the complement of a function is needed.
  - Have F=X’YZ’+X’Y’Z

$$F = \overline{X}Y\overline{Z} + \overline{X}\,\overline{Y}Z$$

$$\overline{F} = \overline{\overline{X}Y\overline{Z} + \overline{X}\,\overline{Y}Z}$$

$$= (\overline{\overline{X}Y\overline{Z}})\bullet(\overline{\overline{X}\,\overline{Y}Z})$$

$$= (X+\overline{Y}+Z)\bullet(X+Y+\overline{Z})$$

Duals

- What is meant by the dual of a function?
  - The *dual* of a function is obtained by interchanging OR and AND operations and replacing 1s and 0s with 0s and 1s.
- Shortcut to getting function complement
  - Starting with the equation on the previous slide
  - Generate the dual $F=(X'+Y+Z')(X'+Y'+Z)$
  - Complement each literal to get:
  - $F'=(X+Y'+Z)(X+Y+Z')$

Exercise Prolems

- $X'Y' + Y'Z + XZ + XY + YZ'$
- Answer: $X'Y' + XZ + YZ'$
- $(AB + A'B')(CD+C'D') + (AC)'$
- Answer: $A' + C' + BD$