NIRMA UNIVERSITY
Institute Of Technology, Ahmedabad
B.Tech. 3rd CE/IT (ODD 2020-21)
2CS305: Discrete Mathematics
Tutorial
Topic-First Order Logic

Q.1 What is the first order predicate calculus statement equivalent to the following?

Every teacher is liked by some student

(A) $\forall$(x) [teacher (x) → $\exists$ (y) [student (y) → likes (y, x)]]
(B) $\forall$ (x) [teacher (x) → $\exists$ (y) [student (y) ^ likes (y, x)]]
(C) $\exists$ (y) $\forall$ (x) [teacher (x) → [student (y) ^ likes (y, x)]]
(D) $\forall$ (x) [teacher (x) ^ $\exists$ (y) [student (y) → likes (y, x)]]

Q.2 Some oranges in the box are more ripen than apples.

Write down the above statement by using Quantifiers.

Q.3 Consider the following:

I. $\neg\forall x (P(x))$ II. $\neg\exists x (P(x))$
III. $\neg\exists x (\neg P(x))$ IV. $\exists x (\neg P(x))$

Which of the above two are equivalent?

A. I and III

B. I and IV

C. II and III

D. II and IV

Q.4 Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x) be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement: “Not every graph is connected”?

(A) $\neg\forall x\,(Graph(x) \Rightarrow Connected(x))$

(B) $\exists x\,(Graph(x) \wedge \neg Connected(x))$

(C) $\neg\forall x\,(\neg Graph(x) \vee Connected(x))$

(D) $\forall x\,(Graph(x) \Rightarrow \neg Connected(x))$

Q.5 Which one of the first order predicate calculus statements given below correctly express the following English statement?

Tigers and lions attack if they are hungry or threatened

(A) $\forall x\left[(\text{tiger}(x) \wedge \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}\right]$

(B) $\forall x\left[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \wedge \text{attacks}(x)\}\right]$

(C) $\forall x\left[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{\text{attacks}(x) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x))\}\right]$

(D) $\forall x\left[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}\right]$

Q.6 The CORRECT formula for the sentence, “not all rainy days are cold”

(A) $\forall d\,(\text{Rainy}(d) \wedge \sim\text{Cold}(d))$

(B) $\forall d\,(\sim\text{Rainy}(d) \rightarrow \text{Cold}(d))$

(C) $\exists d\,(\sim\text{Rainy}(d) \rightarrow \text{Cold}(d))$

is (D) $\exists d\,(\text{Rainy}(d) \wedge \sim\text{Cold}(d))$

Q.7 Let apple(x) be the predicate that x is an apple. Let green(x) be the predicate that the color of x is green. Which of the following statements does not represent the given statement? "Not every apple is green"

(A) $\neg\forall x(apple(x) \rightarrow green(x))$

(B) $\exists x(apple(x) \wedge \neg green(x))$

(C) $\neg\forall x(\neg apple(x) \vee green(x))$

(D) $\forall x(apple(x) \rightarrow \neg green(x))$

Q.8 Which one of the following is the most appropriate logical formula to represent the statement? "Gold and silver ornaments are precious". The following notations are used: G(x): x is a gold ornament S(x): x is a silver ornament P(x): x is precious

A. $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$

B. $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$

C. $\exists x((G(x) \wedge S(x)) \rightarrow P(x)$

D. $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

Q.9 Suppose U is the power set of the set S = {1,2,3,4,5,6}. For any T ∈ U, let |T| denote the number of elements in T and T′ denote the complement of T. For any T, R ∈ U, let TR be the set of all elements in T which are not in R. Which one of the following is true?

(A) $\forall X \in U(|X| = |X'|)$

(B) $\exists X \in U \exists Y \in U(|X| = 5, |Y| = 5 \text{ and } X \cap Y = \phi)$

(C) $\forall X \in U \forall Y \in U(|X| = 2, |Y| = 3 \text{ and } X \setminus Y = \phi)$

(D) $\forall X \in U \forall Y \in U(X \setminus Y = Y' \setminus X')$