Ex-1 Suppose the waist measurements W of 800 girls are normally distributed with mean 66 cms and standard deviation 5 cms. Find the number N of girls with waists.

(i) between 65 and 70 cms;

(ii) greater than or equal to 72 cms.

Sol:→ Here W : waists measurements (in cms) of girls

W (in cms.)	65	70	72
$z = \frac{W-\mu}{\sigma} = \frac{W-66}{5}$ (standard normal variate)	$\frac{65-66}{5} = -0.2$	$\frac{70-66}{5} = 0.8$	$\frac{72-66}{5} = 1.2$

For table → Book : N. P. Bali (pg : 1651)

(i) The prob. that a girl has waist bet^n 65 cms. and 70 cms is given by :

$$P(65 \le W \le 70) = P(-0.2 \le z \le 0.8)$$

$$= P(-0.2 \le z \le 0) + P(0 \le z \le 0.8)$$

$$= P(0 \le z \le 0.2) + P(0 \le z \le 0.8)$$

(∵ by symmetry)

$$= 0.0793 + 0.2881$$

$$= 0.3674$$

see table - VI
pg. : 1347

Hence in a group of 800 girls; the expected number of girls with waists bet^{n} 65 cms and 70 cms. is given by

$= 800 \times 0.3674$

$= 293.92 \simeq 294$

(ii) The probability that a girl has waist greater than or equal to 72 cms is given by

$P(W \geq 72) = P(z \geq 1.2)$ ($\because$ by above table)

$= 0.5 - P(0 \leq z \leq 1.2)$

$= 0.5 - 0.3849$

$= 0.1151$

$\therefore$ In a group of 800 girls; the expected number of girls with waists greater than or equal to 72 cms. is given by

$= 800 \times 0.1151$

$= 92.08$

$\simeq 92$

Ex-2 A sample of 100 dry battery cells tested to find the length of life produced the following results.

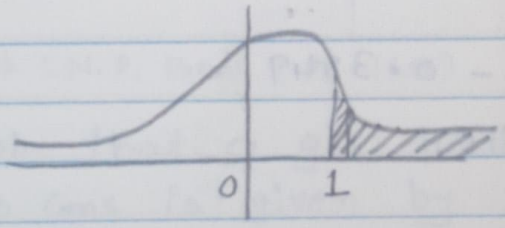
$\bar{x} = \mu = 12$ hrs. ; $\sigma = 3$ hrs.

Assuming the data to be normally distributed what percentage of battery cells are expected to have a life
(i) more than 15 hours,
(ii) less than 6 hours,
(iii) between 10 and 14 hours.

Sol :→ Let x denotes the length of life of dry battery cells.

Also : $z = \frac{x - \mu}{\sigma} = \frac{x - 12}{3}$

(i) When $x = 15$; $z = 1$



$\therefore P(x > 15) = P(z > 1)$

$= P(0 < z < \infty) - P(0 < z < 1)$

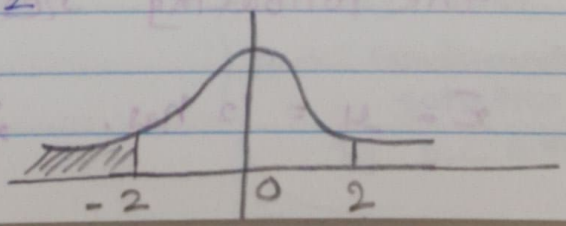
$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$= 15.87\ \%$

(ii) when $x = 6$; $z = -2$



$\therefore P(x < 6) = P(z < -2)$

$= P(z > 2)$ (∵ by symmetry)

$= P(0 < z < \infty) - P(0 < z < 2)$

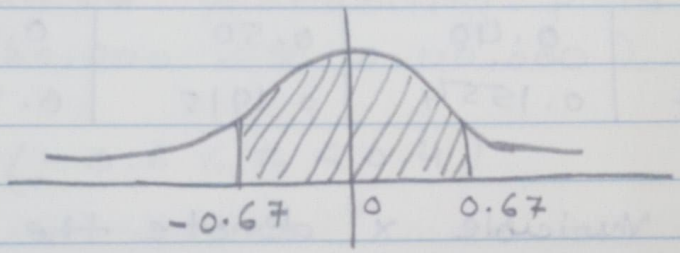
$= 0.5 - 0.4772$

$= 0.0228$

$= 2.28\ \%$

(iii) when $x = 10$; $z = -\frac{2}{3} \approx -0.67$

when $x = 14$; $z = \frac{2}{3} = 0.67$



$P(10 < x < 14) = P(-0.67 < z < 0.67)$

$= 2P(0 < z < 0.67)$ (∵ symmetry)

$= 2 \times 0.2487$

$= 0.4974$

$= 49.74\ \%$

Ex-3 : A sales Tax officer has reported that the average sales of the 500 business that he has to deal with during a year amount to Rs. 36,000 with a standard deviation of Rs. 10,000. Assuming that the sales in these

business are normally distributed ; find
(i) The number of businesses the sales of which are over Rs. 40,000
(ii) The percentage of businessess ; the sales of which are likely to range between Rs. 30,000 and Rs. 40,000
(iii) The probability that the sales of business selected at random will be over Rs. 30000

Proportions of the area under the normal curve:

z	0.25	0.40	0.50	0.60
area	0.0987	0.1554	0.1915	0.2257

Sol:- Let the variable x denote the sales (in Rs.) of the business during a year

given :

$\mu = 36,000$ and $\sigma = 10,000$

(i) The probability that the sales of a business is over Rs. 40,000 is ; given by $P(x > 40,000)$

when $x = 40,000$; $z = \frac{x - \mu}{\sigma} = \frac{40000 - 36000}{10,000}$

$= 0.4$

$\therefore P(x > 40,000) = P(z > 0.4)$

$= 0.5 - P(0 \le z \le 0.4)$

$= 0.5 - 0.1554$

$= 0.3446$

∴ In a group of 500 businesses, the expected number of businesses with annual sales over Rs. 40,000 is

$= 500 \times 0.03446$

$= 172.3$

$\simeq 172$

<ii> Required probability p is given by

$P(30,000 < X < 40,000)$

$= P(-0.6 < z < 0.4)$

∴ when $X = 30,000$; $z = \frac{x - \mu}{\sigma} = \frac{30,000 - 36,000}{10,000}$

$= -0.6$

when $X = 40,000$; $z = 0.4$

$= P(-0.6 < z < 0.4)$

$= P(-0.6 < z \leq 0) + P(0 \leq z < 0.4)$

$= P(0 \leq z < 0.6) + P(0 \leq z < 0.4)$

(∵ Symmetry)

$= 0.2257 + 0.1554$

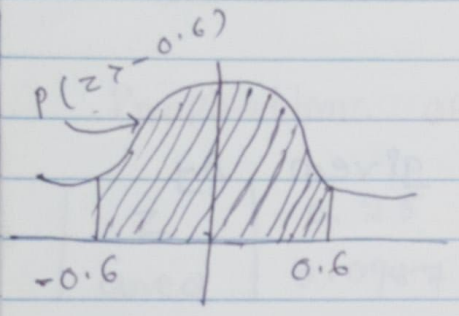
$= 0.3811$

$= 38.11\ \%$

(iii) The probability that the annual sales of a business selected at random will be over Rs. 30,000 is given by:

$$P(x > 30,000) = P(z > -0.8)$$

when $x = 30,000$; $z = \frac{x-\mu}{\sigma} = -0.6$



$$= P(-0.6 < z < 0) + 0.5$$
$$= P(0 < z < 0.6) + 0.5$$
$$= 0.2257 + 0.5$$
$$= 0.7257.$$

Ex-4 Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be
(i) over six feet tall and
(ii) below 66 inches.
Assume heights to be normally distributed.

Sol:→ Let the variable x denote the height (in inches) of the soldiers
Then we are given

$$\left.\begin{aligned} \text{mean } \mu &= 68.22 \\ \text{Variance } \sigma^2 &= 10.8 \end{aligned}\right\}$$

(i) A soldier will be over 6 feet tall if x is greater than 72.

(∵ x is height in inches and 6 feet = 72 inches)

∴ when $x = 72$

$$z = \frac{x - \mu}{\sigma} = \frac{72 - 68.22}{\sqrt{10.8}} = \frac{3.78}{3.286} = 1.15$$

∴ The probability that a soldier is over 6 feet tall is given by

$$P(x > 72) = P(z > 1.15)$$
$$= 0.5 - P(0 \leq z \leq 1.15)$$
$$= 0.5 - 0.3749 \quad (\because \text{by table})$$
$$= 0.1251$$

∴ In a regiment of 1,000 soldiers ; the number of soldiers over 6 feet tall is

$$= 1000 \times 0.1251$$
$$= 125.1$$
$$\simeq 125$$

(iii) The probability that a soldier is below 66 inches is given by :

$$P(x < 66) = P(z < -0.6756)$$

when $x = 66$; $z = \frac{x - \mu}{\sigma} = -0.6756$

$$= P(z > 0.6756) \quad (\because \text{symmetry})$$
$$= 0.5 - P(0 < z < 0.6756)$$

$= 0.5 - 0.2501$ (∵ by table)

$= 0.2499$

∴ The number of soldiers over 66 inches in a regiment of 1,000 soldiers is :

$= 1000 \times 0.2499$

$= 249.9$

≈ 250

Ex-5 The average test marks in a Particular class is 79. The standard deviation is 5. If the marks are distributed normally ; how many students in a class of 200 did not receive marks between 75 and 82 ?

given :

$Pr.\{0 \leq z \leq 0.6\} = 0.2257$

$Pr.\{0 \leq z \leq 0.7\} = 0.2580$

$Pr.\{0 \leq z \leq 0.8\} = 0.2881$

where z is a standard normal variable.

Soln :→ Let the variable x denotes the marks obtained by the students in the given test bet^n 75 and 82 ; then we are given :

$\mu = 79 \quad ; \quad \sigma = 5$

when $x = 75 \quad ; \quad z = \frac{x-\mu}{\sigma} = -0.8$

$x = 82 \quad ; \quad z = \frac{x-\mu}{\sigma} = 0.6$

∴ The probability that a one student gets marks betⁿ 75 and 82 is given by:

$$P(75 < X < 82) = P(-0.8 < z < 0.6)$$

$$= P(-0.8 < z < 0) + P(0 < z < 0.6)$$

$$= P(0 < z < 0.8) + P(0 < z < 0.6) \quad (\because \text{symmetry})$$

$$= 0.2881 + 0.2257$$

$$= 0.5138$$

∴ The prob. P that a student does not get marks between 75 and 82 is given by:

$$P = 1 - P(\text{student gets marks betⁿ 75 and 82})$$

$$= 1 - P(75 < X < 82)$$

$$= 1 - 0.5138$$

$$= 0.4862$$

Hence in a class of 200 students; the number of students who did not receive marks betⁿ 75 and 82 is given by:

$$= 200 \times P$$

$$= 200 \times 0.4862$$

$$= 97.24$$

$$\simeq 97$$