[C] NORMAL DISTRIBUTION

The above Distributions; viz. Binomial distribution and poisson distribution are discrete probability distributions; since the variables under study were discrete random variables.

Now we confine the discussion to continuous probability distributions which arise when the underlying variable is a continuous one.

Normal distribution is one of the most important continuous theoretical distributions in statistics.

Def^n :→ If x is a continuous random variable following by normal Probability distribution with mean μ and standard deviation σ; then its probability density f^{n} is given by:

$$\boxed{P(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

where $-\infty < x < \infty$

Here π and e are absolute constants with values 3.14159 and 2.71828 respectively

→ The mean μ and standard deviation σ

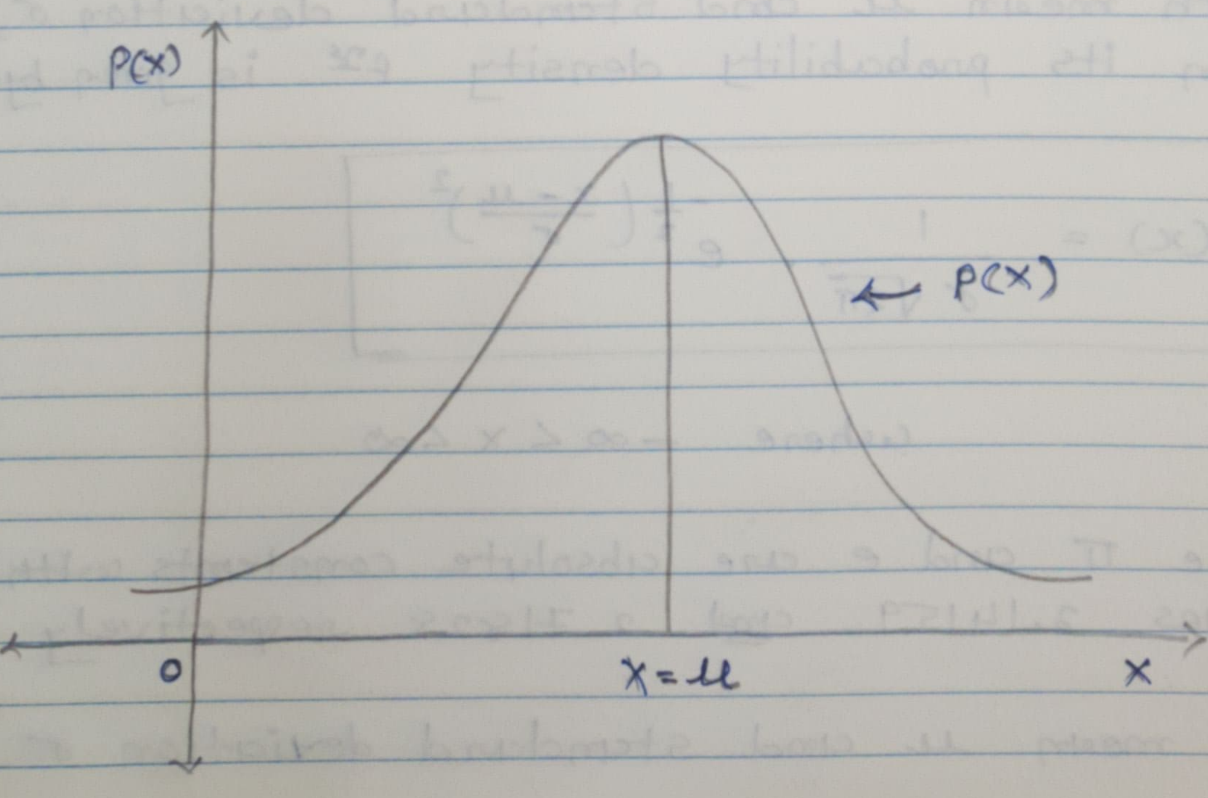
are called the parameters of the Normal distribution.

→ If x is a random variable followed by normal distribution with mean μ and standard deviation σ; then the random variable z defined as :

$$z = \frac{x - \mu}{\sigma}$$

is called the standard normal variate.

→ For given values of the parameters μ and σ; the shape of the curve corresponding to normal probability density f^n^ p(x) is the famous bell shaped curve as shown in the diagram as under.



⊛ Properties of Normal distribution :

<1> It is perfectly symmetrical about the mean μ and is bell - shaped.

<2> since the distribution is symmetrical; mean; median and mode coincide.

i.e

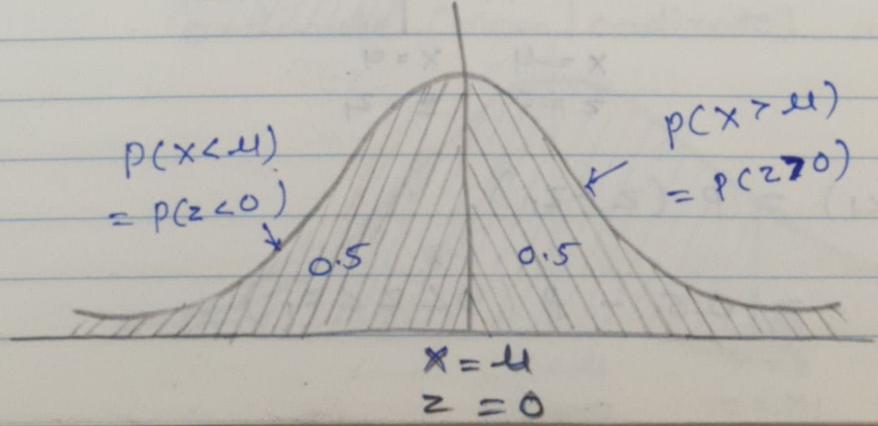
$$\text{Mean} = \text{Median} = \text{Mode}$$

<3> Distribution is unimodal ; since the only mode occuring at $x = \mu$.

<4> Since Mean = Median = μ ; the ordinate at $x = \mu$ $(z = 0)$ divides the whole (area) region in to two equal parts.

Also ; since total area under normal Probability curve is 1 ; the area to the right of the ordinate as well as to the left of the ordinate at $x = \mu$ (or $z = 0$) is 0.5

<5> since total probability is always 1 ; we have the total area under the normal probability curve is 1



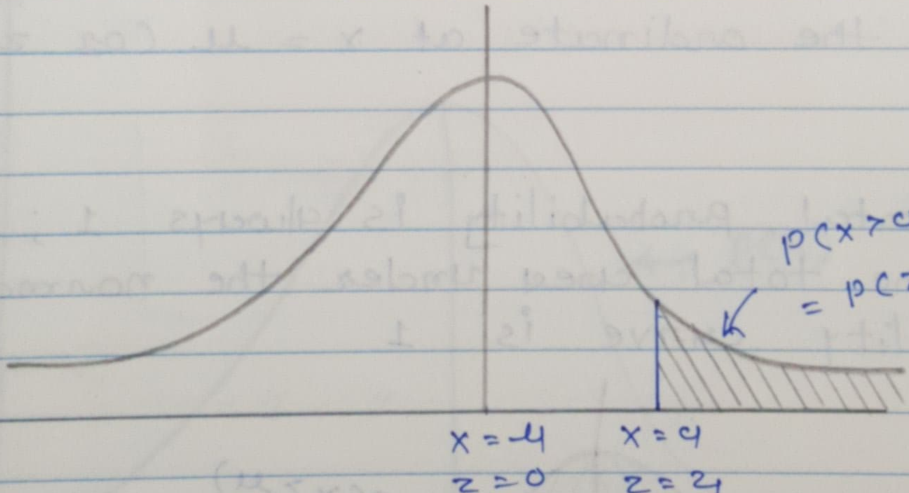
Note :→ For practical problems we dont deal with the variable x but first convert it in to standard normal variate z. Next we try to convert the required area in the form $P(0 < z < z_1)$ by using the following results.

$$\left.\begin{aligned} P(x > \mu) &= P(z > 0) = 0.5 \\ P(x < \mu) &= P(z < 0) = 0.5 \end{aligned}\right\}$$ ($\because$ by above figure)

and making use of the symmetry property of the distribution.

(A) computation of area to the right of the ordinate at $x = a$ i.e to find $\{P(x > a)$

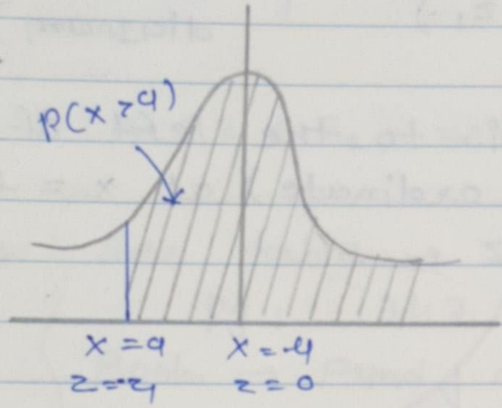
(*) case → (i) : $a > \mu$; i.e a is to the right of the mean ordinate.

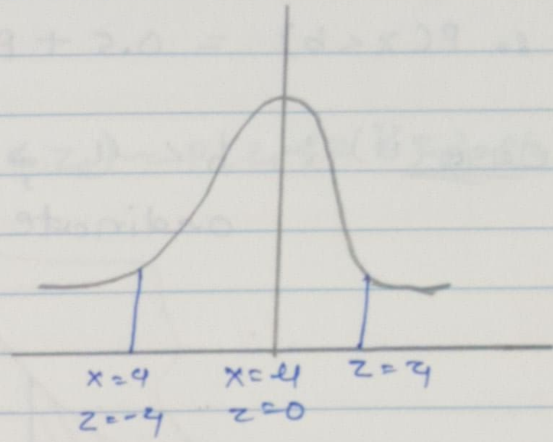


$\therefore P(x > a) = P(z > z_1)$

$= 0.5 - P(0 < z < z_1)$

⊛ <u>case (ii)</u> : $a < \mu$; i.e a is to the left of the mean ordinate.





since $a < \mu$; the value of z corresponding to $x = a$ will be negative

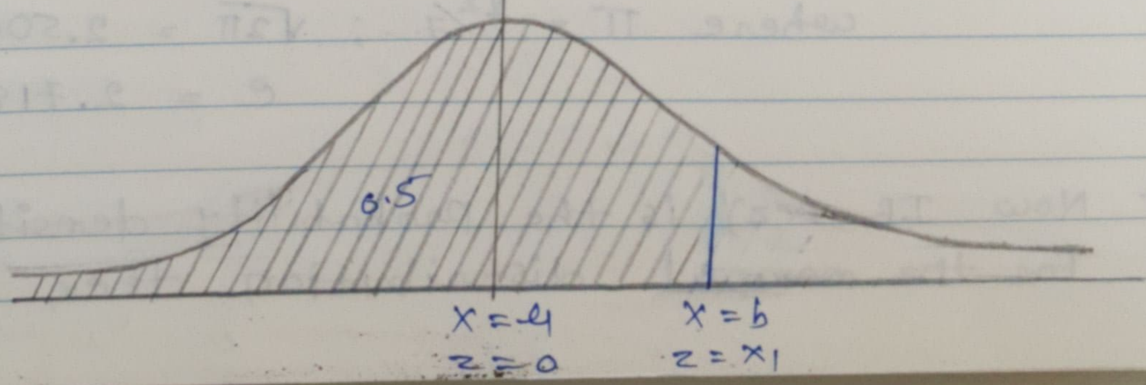
when $x = a$; $z = \dfrac{a - \mu}{\sigma} = -z_1$ (say)

$\therefore P(x > a) = 0.5 + P(-z_1 < z < 0)$ ($\because$ from diagram)

$= 0.5 + P(0 < z < z_1)$ ($\because$ By symmetry)

(B) Computation of the area to the left of the ordinate $x = b$ i.e to find $P(x < b)$

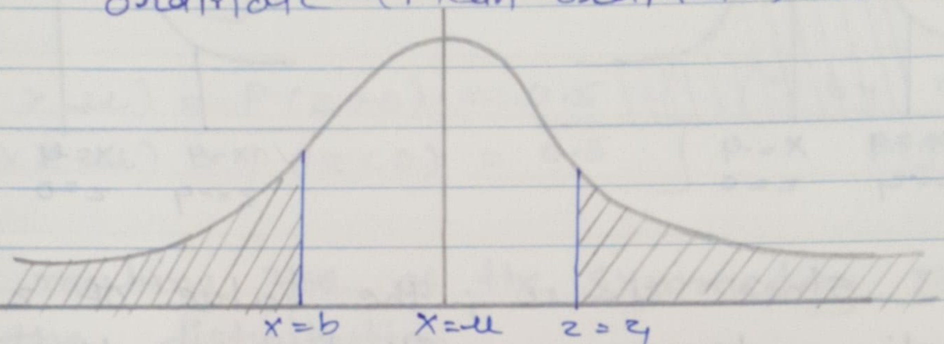
⊛ <u>case (i)</u> : $b > \mu$; i.e b is to the right of the ordinate (mean ordinate) at $x = \mu$.



when $x = b$; $z = \frac{b - \mu}{\sigma} = z_1$ (say)

$\therefore P(x < b) = 0.5 + P(0 < z < z_1)$ ($\because$ from diagram)

* Case (ii) :- $b < \mu$; ie b is to the left of the ordinate (Mean ordinate) at $x = \mu$



$\therefore P(x < b) = P(z < -z_1)$
$= P(z > z_1)$
$= 0.5 - P(0 < z < z_1)$ ($\because$ by Symmetry)

Note :→ If $z = \frac{x - \mu}{\sigma}$ is the standard normal variate then ;

The probability density f^{ns} for the normal distribution in standard form is given by

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}}$$

where $\pi = 22/7$; $\sqrt{2\pi} = 2.5066$
$e = 2.71828$

→ Now If $f(z)$ is the probability density f^{ns} for the normal distribution then

$$P(z_1 \le z \le z_2) = \int_{z_1}^{z_2} f(z)\,dz$$

$\rightarrow P(z_1 \le z \le z_2) = P(z_1 \le z < z_2) = P(z_1 < z \le z_2) = P(z_1 < z < z_2)$