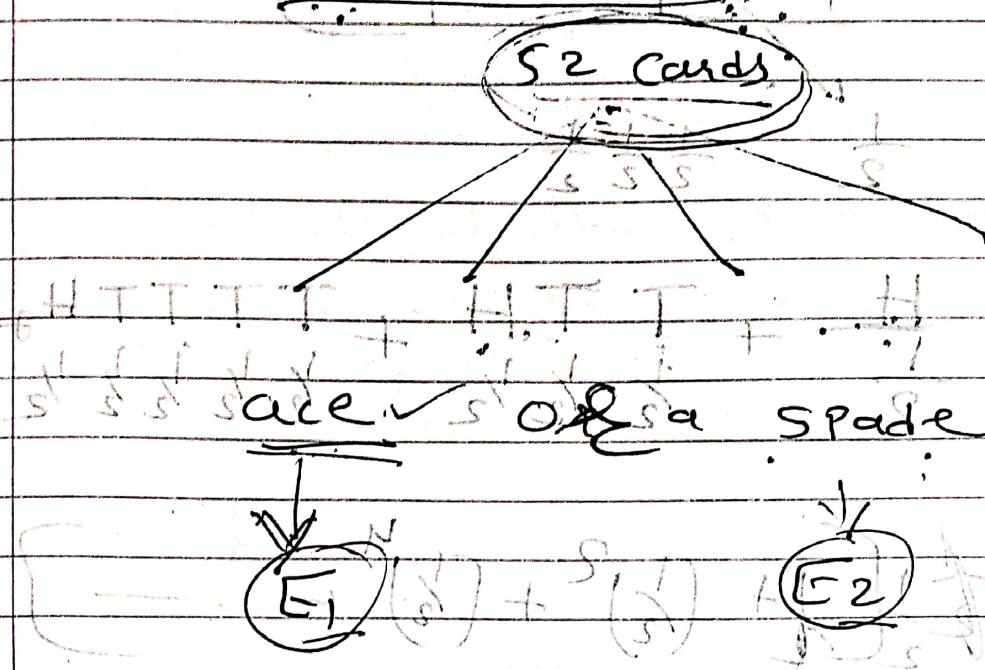


Probability



$$P(E_1) = \frac{4}{52} \quad P(E_2) = \frac{13}{52}$$

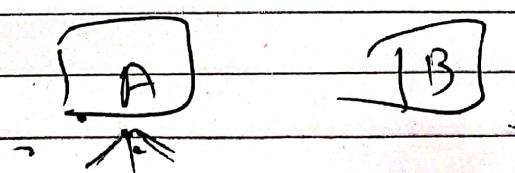
$$E = E_1 \cup E_2$$

$$\begin{aligned} P(E) &= P(E_1 \cup E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \end{aligned}$$

$$= \frac{16}{52}$$

e.g. A, B toss an unbiased coin alt. such that first who gets the head wins.

If A starts the game. What is the prob of A's winning?



A

B

$$\frac{1}{2} + \frac{1}{2} \frac{1}{2} \frac{1}{2}$$

$$\frac{H}{2} + \frac{T}{2} \frac{T}{2} \frac{H}{2} + \frac{T}{2} \frac{T}{2} \frac{T}{2} \frac{H}{2}$$

$$\frac{1}{2} [1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4] =$$

$$S_1 = (A)q \quad S_2 = (A)q$$

$$\frac{1}{2} \left[\frac{9}{1-2} \right] = \frac{1}{2} \left[\frac{1}{1-2} \right]$$

$$(k=1) \cdot 2^0 = (q)$$

$$(A)q = (A)q + (A)q =$$

$$\frac{1}{2} = \frac{1}{2} + \left[\frac{1}{2} \right] =$$

$$= \frac{2}{3} \frac{1}{2} A's$$

bisected by 120° A \checkmark

$$120^\circ + 60^\circ + 60^\circ = 180^\circ \checkmark$$

each angle is 60° only

thus each angle is 60° and it is

perimeter of A to 120° and P

81

81

$$\text{eg } A = \frac{1}{2} \text{ & } \text{Solving } \left\{ \begin{array}{l} B = \frac{1}{3} \\ C \rightarrow \frac{1}{4} \end{array} \right. \text{ Prob}$$

What is the prob that
Prob. will be solved.

$$\text{Ans. } \times \frac{3}{4}$$

~~(A) good knowledge~~ $\rightarrow P(A)$
~~1 M Ed. exam~~

* Conditional Prob:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Baye's Thm

A \Rightarrow E_1, E_2, \dots, E_n

$P(E_i)$ & $P(A|E_i)$ are given

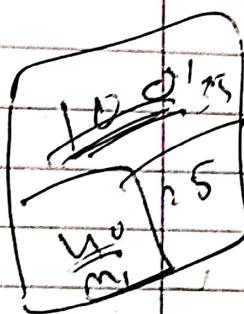
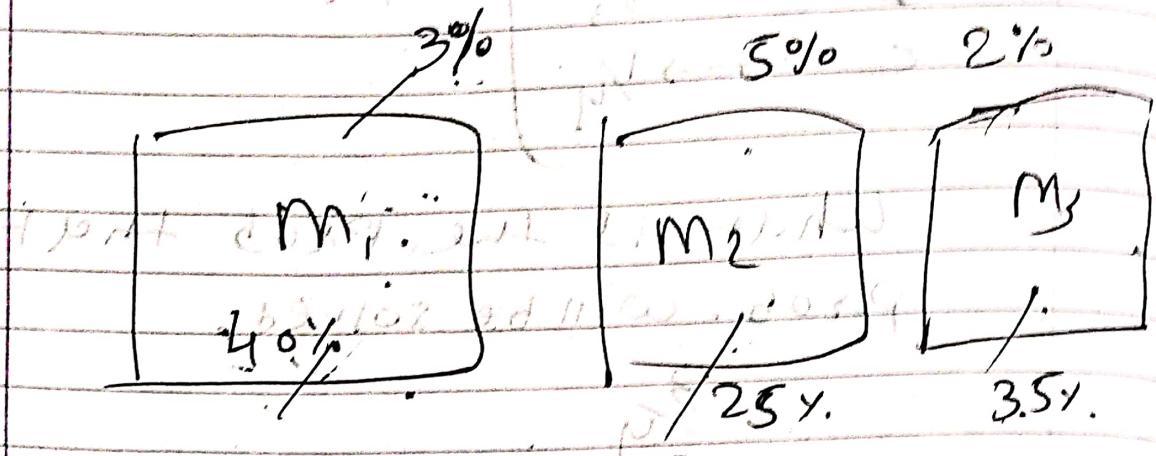
~~2nd method~~ $P(E_i|A) = P(E_i) \cdot P(A|E_i)$

$$\therefore P(A) = (\sum P(E_i) \cdot P(A|E_i))$$

$$20.0 = (2|A)$$

$$50.0 = (3|A)$$

Laptop Bugs - ~~(A)~~



E₁: Selected Bag is man. by M₁

E₂: " " M₂
 E₃: " " M₃

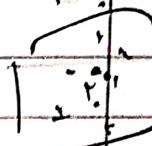
$$P(E_1) = \frac{40}{100} = 0.4$$

$$\checkmark P(E_2) = 0.25$$

$$\checkmark P(E_3) = 0.35$$

(A) Selected Bag is defective

$$(B) P(A|E_1) = \frac{3}{100} = 0.03$$



$$P(A|E_2) = 0.05$$

$$P(A|E_3) = 0.02$$

8th for exams and next
topic is Probability

$$P(E_1 | A) = \frac{P(A|E_1) \cdot P(E_1)}{\sum P(A|E_i) \cdot P(E_i)}$$

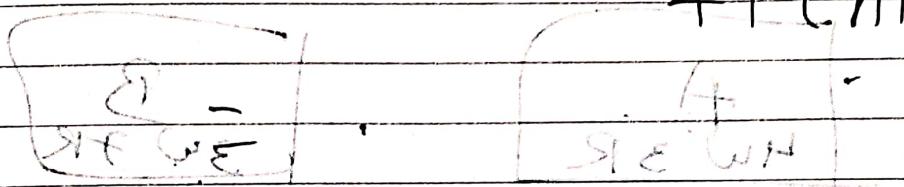
+ more's need \sum ~~$P(A|E_i) \cdot P(E_i)$~~

most number \rightarrow ~~$P(A|E_i) \cdot P(E_i)$~~

$$P(E_1 | A) = \frac{P(A|E_1) \cdot P(E_1)}{\sum P(A|E_i) \cdot P(E_i)}$$

$$= \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)}$$

$$+ P(A|E_3) \cdot P(E_3)$$



$$P_{SV}(E_3 | A)$$

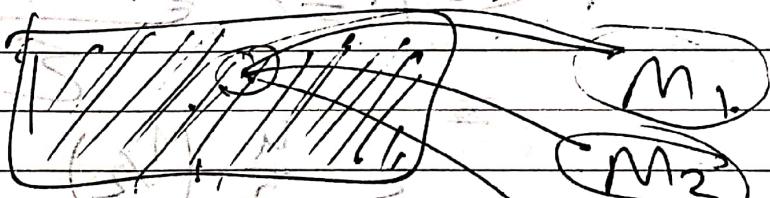
$$P(A \cap B) = P(A|B)$$

$$P(A \cap B) = P(A \cap B) / P(B)$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$(A)^q \cdot (B)^q = (A \cap B)^q$$



$$\delta = (\frac{1}{3})^3 + (\frac{1}{3})^3$$

Q9

There are 2 boxes A & B containing 4 white & 3 Red
of 3 white & 7 red balls.

resp-

A box is chosen random. &
a ball is drawn from it.

If the ball is white find

(the) Prob that it is from
box A.

A
4W 3R

B
3W 7R

$$P(E_1) = P(G_1) = \frac{1}{2}$$

$$P(E_2) = P(G_2) = \frac{1}{2}$$

$\checkmark A$ - Seven ball is white

$$\frac{1}{2} \left(\frac{4}{7}\right) + \frac{1}{2} \left(\frac{3}{10}\right) = \frac{40}{70}$$

$$\frac{1}{2} \left(\frac{4}{7}\right) + \left(\frac{1}{2}\right) \left(\frac{3}{10}\right) = \frac{61}{70}$$

$$P(A|G) = \frac{1}{7}, P(A|G_2) = \frac{3}{10}$$

$$P(E_1|A) = P(A|E_1) \cdot P(G_1)$$

$$= \frac{4}{7} \left(\frac{1}{2}\right)$$

$$\frac{4}{7} \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{3}{10}\right) = \frac{61}{70}$$

TA

- ex) A microchip company has
- ex) A letter is known to have come either from TATANAGAR or from CALCUTTA.

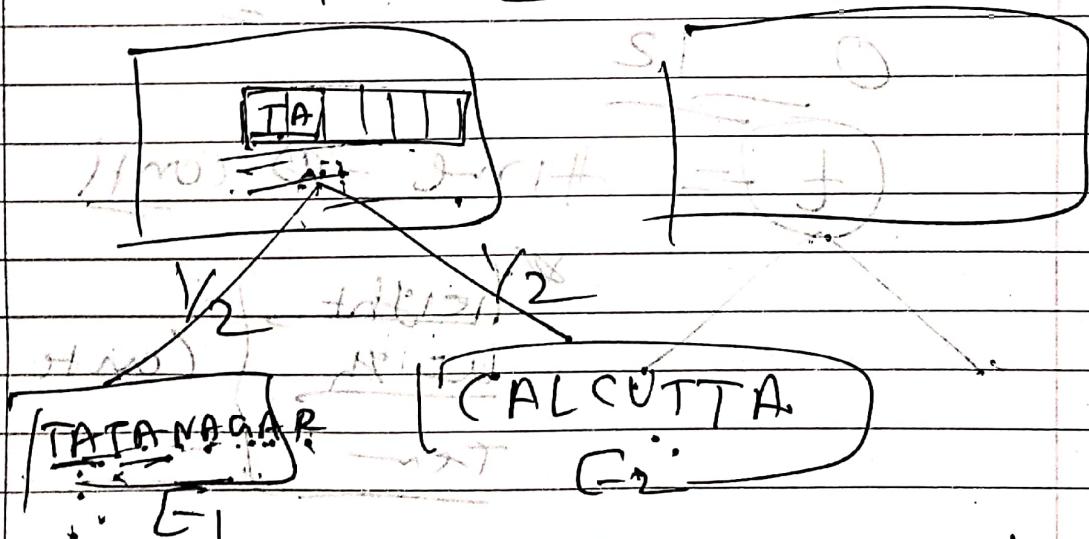
On the envelope just two consecutive letters TA are visible. What is the prob. that the letter came from CALCUTTA.

$$\text{Total cases} = 2 - \text{TA} + \text{TA} = 8$$

$$P(G_1) = \frac{2}{8} \quad P(G_2) = \frac{1}{4}$$

$$P(A|G_1) = \frac{2}{8} \quad P(A|G_2) = \frac{1}{4}$$

$$P(G_1) = \frac{1}{2} \quad P(G_2) = \frac{1}{2}$$



$$P(G_1) = \frac{1}{2} \quad P(G_2) = \frac{1}{2}$$

A : TA

$$P(A|G_1) = \frac{2}{8} \quad P(A|G_2) = \frac{1}{4}$$

Random Variables

Variable

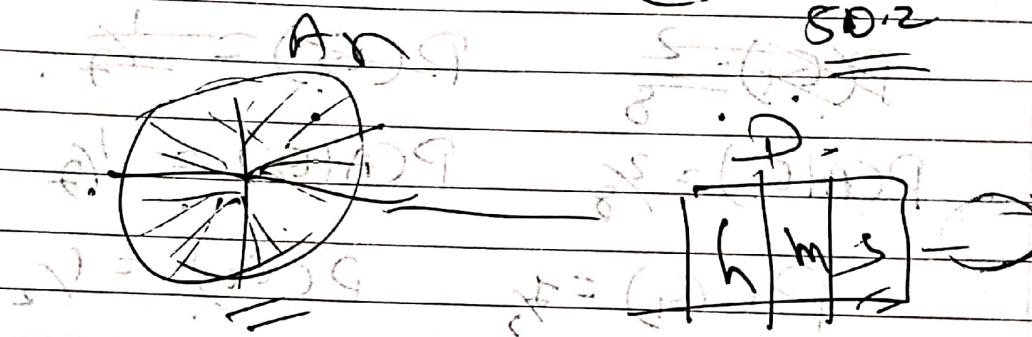
CHARACTERISTICS Model

Discrete Continuous

Mark Marks Attitude

Grades Marks

A B C D E Attitude (vs 50, 100)



0 1 2
 $t = \text{time}$ → conti

height
weight } (conti)
Attitude }
Time }

$X = (0, 1, 2)$ marks (2) disc

Grades Disc

$t = (0, 1, 2)$ → (no of sec)

X - Conti Discrete

finitely many values

[1, 2, 3, 4, 5, ...]

[0.1, 0.2, 0.3, ...] is conti

continuous

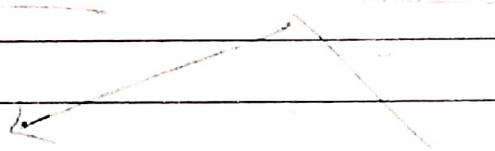
few many [-1, 0, 1, ...]

and so forth

Discrete Random Variable

Conti

random variable



partition

probability

random variable must sum to 1

ex. go to another point at

fixed value & probability is 1

probability

HT, TH, TT, HH

must be 0.25 each

based on probability

27/01/21

Date: / /
Page No.

*

X -

(x) marks on I.C.T. test

$$x = 0 \text{ to } 30$$

(31) Values

y: height of S. m

$$[120, 180]$$

*

Random Variable:

Discrete

Continuous

→ Rule that assigns a number to each outcome of an exp.

→ Eg Exp: Tossing 2 Unbrn coins simultaneously.

HH, TT, HT, TH

Interested in no of Heads.

X: no of heads.

| Outcomes | $\text{No. of heads} = X$ |
|----------|---------------------------|
| H.H | 2 |
| H.T | 1 |
| T.H | 1 |
| T.T | 0 |

$$X \in \{0, 1, 2\}$$

Ex. If X = no. of tail

(X) may take values 0, 1, 2

Expt. Selecting one student from CSE IV sem. [A, B, C, D].

X = marks obtained by student in CT.

Range of X 0, 1, ..., 30

Discrete R-X

Continuous R-X Height of student

[120 cm, 180 cm]

Continuous R-X

Height

Position

$$f(x) = x^2$$

* Distribution:

Probability

Outcome

x

$f(x)$

x

$f(x)$

0

0

0

0

$1/4$

1

1

1

1

$2/4$

2

2

2

2

$1/4$

$$1 = \sum f(x)$$

| X | $P(X_i)$ |
|-----|----------|
| H.H | |
| T.H | |
| H.T | |
| T.T | |

D.R.V. X has x_1, x_2, \dots, x_n as theProb. Distribution $P(X)$ probabilities $P(X_i)$ pto each x_i

[H.H, T.H, H.T, T.T] are the four math

(a) $0 \leq P(X_i) \leq 1$

(b) $\sum P(X_i) = 1$

Prob. Distribution(1) Discrete Prob. Distribution(2) Continuous~~most~~ X . ~~X is discontinuous~~DiscreteContBinomialNormalPoissonexp(x)UnifExp(x)UnifG.A.P(x)UnifGauss(x)UnifQ.E.D(x)Unif

Probability mass function

Discrete X

$$P(f(x)) = P(X=x_i) \quad P.m.f.$$

TTQ
(HTT, THT, THH)

$$P(X=0) = 1/4$$

HT, TH

$$P(X=1) = 2/4$$

HTH

$$P(X=2) = 1/4$$

HHH

$$P(X=3) = 1/4$$

X - Cont. R.V.

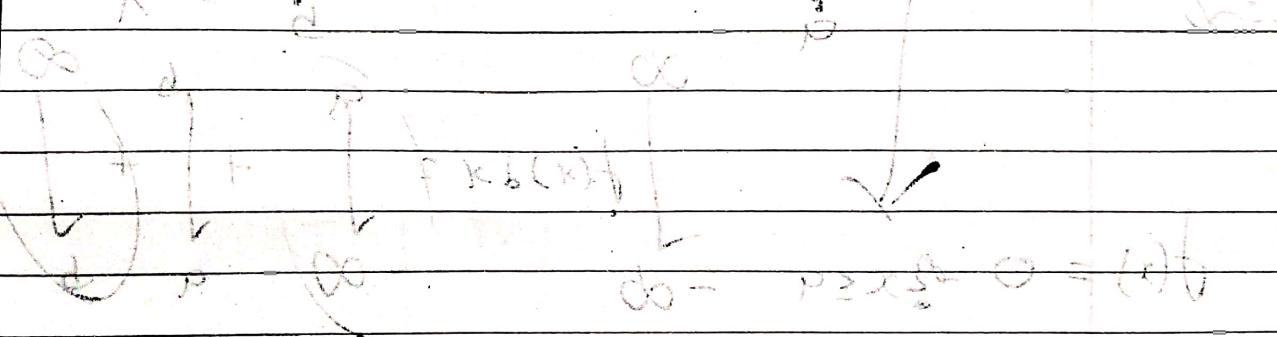
(169) cont. interval - drops

then $f(x) =$ Prob-density fn

$$P(X=25) = \frac{8}{\infty} = \frac{8}{n}$$

$$P(Y=155) = ?$$

$$P(150 \leq Y \leq 160) = ?$$



28/1/21.

Maths 2020-21

 X DISCRETE

$$\text{Prob. } P(x) = f(x)$$

$$f(x) = P(x)$$

 X CONTINUOUSProb. mass funⁿ. (pmf)

$$P(X = x) = f(x)$$

 X - conti

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

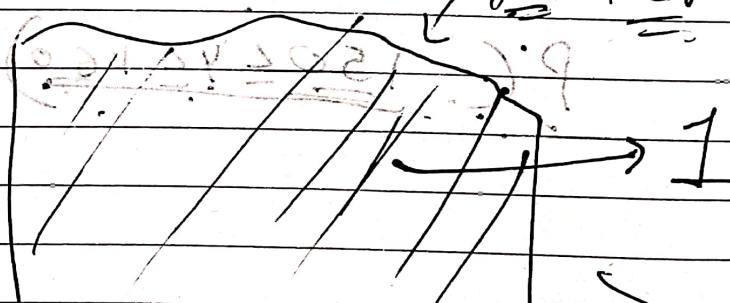
Prob. density funⁿ. (pdf).Prob. density funⁿ. (pdf).

$$f(x) \geq 0$$

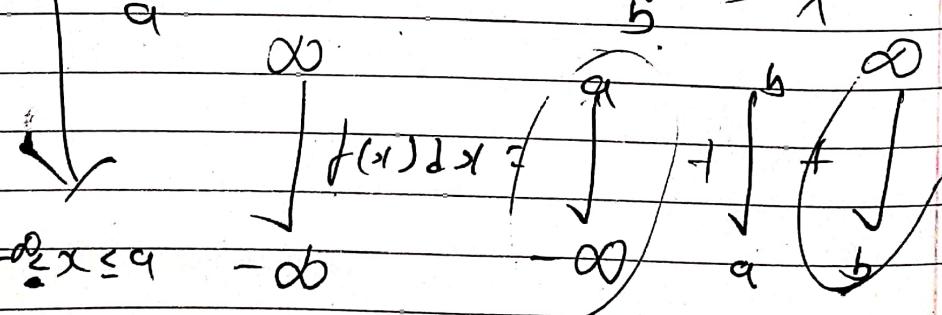
 $\int_{-\infty}^{\infty}$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$f(x) = (2x + 1)^{-1} \quad f(x) \text{ pdf}$$



$$f(x) = 0 \quad x \leq -0.5$$



Mathematical Expectation

Mean of ...

Expected Value of R.V.

$$E(X) := \sum x_i P(x_i)$$

Two coins are tossed.

X : No. of heads

| x | $P(x)$ | $x \cdot P(x)$ | Toss |
|-----|---------------|----------------|--|
| 0 | $\frac{1}{4}$ | 0 | $\begin{matrix} H & H \\ T & T \end{matrix}$ |
| 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\begin{matrix} H & T \\ T & H \end{matrix}$ |
| 2 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\begin{matrix} H & H \end{matrix}$ |

$$E(X) = \sum x P(x) = 1$$

$$\boxed{E(X) = 1}$$

Toss 3 coins. X : no. of heads

| x | $P(x)$ | $x \cdot P(x)$ | X : No of defec |
|-----|---------------|----------------|-------------------|
| 0 | $\frac{1}{8}$ | 0 | |
| 1 | $\frac{3}{8}$ | $\frac{3}{8}$ | |
| 2 | $\frac{3}{8}$ | $\frac{6}{8}$ | |
| 3 | $\frac{1}{8}$ | $\frac{3}{8}$ | |

$$\boxed{E(X) = 1}$$

$$\text{Mean: } E(X) = \sum x P(x) = \frac{1+2+3}{8} = \frac{3}{2} = 1.5$$

X Y_x

$$\bar{X} = E(X) = \sum x_i P(x_i) \text{ discrete}$$

$$\bar{X} = E(X) = \int x f(x) dx \quad (\text{cont})$$

$$(n) \sum x_i P(x_i) = E(X)$$

* Properties of Mathematical Expectation

(1)

$$E(c) = c$$

(2)

$$E(cx) = cE(x)$$

(3)

$$E(c_1x + c_2) = c_1 E(x) + c_2$$

(4)

$$E(x+y) = E(x) + E(y)$$

(5)

~~x & y are independent~~

(6)

$$E(xy) = E(x) \cdot E(y)$$

(7)

$$E(g(x)) = \sum g(x) \cdot P(x)$$

(8)

$$g(x) = (x^2 + 5)$$

$$E(g) = 21 - (0.25) \cdot 5 = 20.75$$

* Variance of Random Var

$$\text{Var}(X) := E(X^2) - (E(X))^2$$

$$\text{Var} = \frac{(S.D)^2}{n} = \frac{\sum (x_i - \bar{x})^2}{n}$$

Average
mean

$$\text{Var}(X) = E((X - \bar{X})^2)$$

$$= E(X^2 - 2\bar{X}X + \bar{X}^2)$$

$$= E(X^2) - 2\bar{X}E(X) + \bar{X}^2$$

$$= E(X^2) - 2\bar{X}^2 + \bar{X}^2$$

$$= E(X^2) - \bar{X}^2$$

$$\boxed{\text{Var}(X) = E(X^2) - (E(X))^2}$$

$$\sigma = \sqrt{\text{Var}(X)}$$