

1.) Probability that man speaks truth =  $3/4$ .

The probability that man lies is =  $1 - 3/4 = 1/4$

Probability of getting 6 on die =  $1/6$

Probability of not getting 6 =  $1 - 1/6 = 5/6$

Applying bayes theorem, we get the required probability as:

$$= \frac{1/6 \times 3/4}{(1/6 \times 3/4) + (5/6 \times 1/4)}$$

$$= \frac{3}{8}$$

here, applying formula:  $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

where  $P(B) = \left(\frac{1}{6} \times \frac{3}{4}\right) + \left(\frac{5}{6} \times \frac{1}{4}\right)$

and  $P(B|A) = 1/6$  and  $P(A) = 3/4$

where  $A \rightarrow$  suggest that man speaks truth.

$B \rightarrow$  suggest that we got 6 on die.

Q-1 B) 2

here,

Let  $E_1$  = plant A manufactures ~~see~~ bike. $E_2$  = plant B manufactures bike. $A$  = Bike is of standard quality.

$$P(E_1) = 70\% = \frac{70}{100} = 0.7 \quad (\because \text{bike manufactured by Plant A})$$

$$P(E_2) = 30\% = \frac{30}{100} = 0.3 \quad (\because \text{bike manufactured by Plant B})$$

$$\text{By } \downarrow \text{ } P(A/E_1) = 80\% = \frac{80}{100} = 0.8$$

$\downarrow$   
( $\because$  standard quality bikes manufactured by Plant A)

$$P(A/E_2) = 90\% = \frac{90}{100} = 0.9$$

$\downarrow$   
( $\because$  standard quality bikes manufactured by Plant B)

By applying Baye's theorem;

$$P(E_2/A) = \frac{P(E_2) P(A/E_2)}{P(E_2) P(A/E_2) + P(E_1) \cdot P(A/E_1)}$$

$$= \frac{0.3 \times 0.9}{(0.3 \times 0.9) + ((0.7)(0.8))}$$

$$= \frac{27}{83} \approx 0.325$$



Q-2

Probability of a defect per blade  $p = \frac{1}{500} = \text{or } 0.002$

$\therefore$  The mean number of defects  $= L = 10p = 0.02$   
 where  $p$  is the probability of the number of the defects,  $n=10$  which is packet of 10.  
 $L = np = 10p = 10 \times 0.002 = 0.02$

$$\therefore P(n) = \frac{L^n}{n!} e^{-L}$$

$$= \frac{(0.02)^n}{n!} e^{-0.02} = \frac{(0.02)^n}{n!} \times (0.98)$$

(i) The approximate number of packets containing zero ~~two~~ defective blades.

$$P(0) = \frac{(0.02)^0}{0!} e^{-0.02} \approx 0.98$$

$$\rightarrow \text{number of packets} = 10000 \times P(0) = 9800$$

(ii) The approx number of packets containing one defective blades:

$$P(1) = \frac{(0.02)^1}{1!} \times 0.98 = 0.0196$$

$$\rightarrow \text{number of packets} = 10000 \times P(1) = 196$$

(iii) The approx number of packets containing two defective blades =

$$P(2) = \frac{(0.02)^2}{2!} \times 0.98 = 1.96 \times 10^{-4}$$

$$\rightarrow \text{number of packets} = 10000 \times 1.96 \times 10^{-4} \approx 2$$

∴ answers:-

(1) two defective blades:

The approximate number of packets containing blades with two defective blades is 2

(2) at least two defective blades

$$= 1 - ({}^P C_{0 \text{ def. blades}} + {}^P C_{1 \text{ def. blades}})$$

$$= 1 - (0.98 + 0.0196)$$

$$= 4 \times 10^{-4}$$

∴ number of packets containing blades with at least two defective blades is

$$4 \times 10^{-4} \times 10000 = \underline{4}$$

Q-3 Total brass plugs = 1000

$$\mu = 0.7515 \text{ cm}$$

$$\sigma = 0.002 \text{ cm}$$

$$\text{approved diameter} = 0.752 \pm 0.004 \text{ cm}$$

$$Z = \frac{(x - \mu)}{\sigma}$$

• here for  $x$  we have to take LSL & USL



$$\text{where } LSL = 0.752 - 0.004 = 0.748$$
$$USL = 0.752 + 0.004 = 0.756$$

$$Z \text{ (for } x=LSL) = \frac{0.748 - 0.7515}{0.0020}$$
$$= \cancel{3.75} - 1.75$$

$$Z \text{ (for } x=USL) = \frac{0.756 - 0.7515}{0.002}$$
$$= 2.25$$

from table,

Area at  $z = 2.25$  is 0.4878

Area at  $z = 1.75$  is 0.4599

$$\therefore \text{Total area} = 0.4878 + 0.4599$$
$$= 0.9477.$$

$$\rightarrow \text{Brass plugs likely to be approved} =$$
$$\frac{0.9477 \times 1000}{1}$$
$$= 947.7 \approx 948 \text{ plugs.}$$

$$\therefore \text{The number of plugs likely to be rejected is}$$
$$1000 - 948$$
$$= \boxed{52} \text{ plugs.}$$

4.) Probability of man hitting target =  $\frac{1}{3}$ .

here ;  $p = \frac{1}{3}$

So suppose man fires  $n$  times, and let  $X$  denote the number of times he hits the target.

$$P(X=r) = {}^nC_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{n-r}, \text{ where } r=0, 1, 2, \dots, n$$

It is given that

$$P(X \geq 1) > \frac{3}{4}$$



( $\because$  man hits target atleast once) is gr

↗ ( $\because$  hits zero time)

$$1 - P(X=0) > \frac{3}{4}$$

$$1 - {}^nC_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{n-0} > \frac{3}{4}$$

$$1 - \left(\frac{2}{3}\right)^n > \frac{3}{4}$$

$$\frac{1}{4} > \left(\frac{2}{3}\right)^n$$

$\therefore$  so we can see that  $n$  should be 4

$$\text{so that } \frac{1}{4} \approx 0.25 > \left(\frac{2}{3}\right)^4 \approx 0.197$$

$\therefore$  The man must fire at least 4 times

$$\therefore \boxed{n=4}$$



4 B) Total questions : 8  
 smallest value of  $n = ?$   
 probability of guessing at least  $n$  correct answer =  $\frac{1}{2}$

here, let  $X =$  no of probab he guesses right

If he has to guess <sup>at least</sup>  $n$  correct answer, then  
 we can show that ~~be~~ ~~she~~

$$= 1 - (\text{guessing at most } n-1 \text{ correct answers})$$

$$= 1 - ({}^8C_0 \left(\frac{1}{2}\right)^8 + {}^8C_1 \left(\frac{1}{2}\right)^8 + \dots + {}^8C_{n-1} \left(\frac{1}{2}\right)^8)$$

[ $\therefore$  here by binomial probability distribution formula we applied.

$$P(n) = {}^8C_n \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{8-n} = {}^8C_n \left(\frac{1}{2}\right)^8]$$

in question it is given that this probability should be  $<$  less than  $\frac{1}{2}$ .

$$\therefore 1 - ({}^8C_0 \left(\frac{1}{2}\right)^8 + {}^8C_1 \left(\frac{1}{2}\right)^8 + \dots + {}^8C_{n-1} \left(\frac{1}{2}\right)^8) < \frac{1}{2}$$

$$\therefore 1 - \frac{1}{2} < \left(\frac{1}{2}\right)^8 ({}^8C_0 + {}^8C_1 + \dots + {}^8C_{n-1})$$

$$\therefore \frac{1}{2} < \left(\frac{1}{2}\right)^8 ({}^8C_0 + {}^8C_1 + \dots + {}^8C_{n-1})$$

$$\therefore {}^8C_0 + {}^8C_1 + \dots + {}^8C_{n-1} > (2)^7$$

for ~~by~~ trial and error we get  $n = 5$  at least.

5.)	2	0	1	2	3	4	5	6	7
f	0	k	2k	<del>2k</del>	<sup>2k</sup>	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> +k

here we know that  $\sum p(x) = 1$

$$\therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\therefore 10k^2 + 9k = 1$$

$$\therefore 10k^2 + 9k - 1 = 0 \quad \therefore 10k(k+1) - k(k+1) = 0$$

$$\therefore k = 0.100925 \quad \therefore k = \frac{1}{10}$$

$$\text{Mean} = 4k + k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$$

$$= 30k + 66k^2$$

$$\text{Mean} = 3.66$$

$$\begin{aligned} \sum (P(x^2)) &= k + 8k + 18k + 48k + 125k^2 + 72k^2 \\ &\quad + 343k^2 + 49k \end{aligned}$$

$$= 124 + 540k^2$$

$$= 124 + 4.4$$

$$16.8$$

$$\sigma^2 = 16.8 (\text{mean})^2$$

$$= 3.4$$