To Test the Difference between Mean of Two dependent Samples(Paired t Test)

- Let us now consider the case when (i) the sample sizes are equal (ii) the two samples are not independent but the same observations are pair together.
- This test is used in situations where there is a pairing of observations (x_i, y_i), like a stimulus is given to some patients and their blood pressure before and after giving the stimulus is noted, then with the help of this test it is analysed if the stimulus is effective, marks obtained by students of a class in two subjects, etc
- For dependent samples or related samples test, it is important that two samples taken in the study are of the same size.
- Let the null hypothesis be *Ho:* $\mu_1 = \mu_2$, i.e. the process is not effective. We define $d_i = y_i - x_i$, the difference in the observations for the i^{th} item

- Then we compute $\bar{d} = \frac{\sum d_i}{n}$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2$

$$= \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]$$

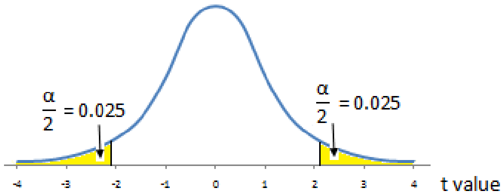
- The test statistic for paired observation is defined by the following formula

$$t = \frac{|\bar{d}|}{S/\sqrt{n}}$$

Where n is the number of pairs of difference

Student's t Distribution Table

For example, the t value for 18 degrees of freedom is 2.101 for 95% confidence interval (**2-Tail** $\alpha = 0.05$).



	90%	**95%**	**97.5%**	**99%**	**99.5%**	**99.95%**	**1-Tail Confidence Level**
	80%	**90%**	**95%**	**98%**	**99%**	**99.9%**	**2-Tail Confidence Level**
	0.100	**0.050**	**0.025**	**0.010**	**0.005**	**0.0005**	**1-Tail Alpha**
df	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.001**	**2-Tail Alpha**
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	

Example 1

IQ test was administered to 5 persons before and after they were trained. The results are given below:

IQ (Before training)	110	120	123	132	125
IQ (After training)	120	118	125	136	121

Test whether there is any change in IQ after training programme.

Example Solution:

Let us assume null hypothesis that There is no significant effect of traning

Now applying paired t- test

$$t = \frac{|\bar{d}|}{S/\sqrt{n}}$$

IQ before Training	IQ after Training	Difference (d_i)	d^2
110	120	10	100
120	118	-2	4
123	125	2	4
132	136	4	16
125	121	4	16
		10	140

$$t = \frac{|\bar{d}|}{S/\sqrt{n}}, \quad \bar{d} = \frac{\sum d_i}{n} = \frac{10}{5} = 2$$

Here $S^2 = \frac{1}{n-1}\left[\sum (d)^2 - \frac{(\sum d)^2}{n}\right]$

$$= \frac{1}{4}\left(140 - \frac{(10)^2}{5}\right) = \frac{1}{4}(120) = 30$$

$$S = \sqrt{30} = 5.477$$

$$t = \frac{2}{5.447/\sqrt{5}} = \frac{4.472}{5.477} = 0.816$$

As the calculated value of $|t| = 0.816 < t_{0.05}$ for 4 degree of freedom which is 2.78, Ho is Accepted

Decision: Accept Ho at α = 0.05(At 5%)

Conclusion:

There is no change after the training programme

Example 2

To test whether a course in statistics improved performance, a similar test was given to 12 participants, their scores both before and after the course are given below

Score (Before)	44	40	61	52	32	44	70	41	67	72	53	72
Score (After)	53	38	69	57	46	39	73	48	73	74	60	78

Test at 5%level of significance of the course was useful in terms of performance on the test

Example Solution:

Let us assume null hypothesis that There is no improvement due to course

Now applying paired t- test

$$t = \frac{|\bar{d}|}{S/\sqrt{n}}$$

Score before course	**Score after course**	**Difference (d_i)**	d^2
44	53	9	81
40	38	-2	4
61	69	8	64
52	57	5	25
32	46	14	196
44	39	-5	25
70	73	3	9
41	48	7	49
67	73	6	36
72	74	2	4
53	60	7	49
72	78	6	36
		60	578

$$t = \frac{|\bar{d}|}{S/\sqrt{n}}, \quad \bar{d} = \frac{\sum d_i}{n} = \frac{60}{12} = 5$$

Here $S^2 = \frac{1}{n-1}\left[\sum (d)^2 - \frac{(\sum d)^2}{n}\right]$

$$= \frac{1}{11}\left(578 - \frac{(60)^2}{12}\right) = \frac{1}{11}(278) = 25.27$$

$$S = \sqrt{25.27} = 5.026$$

$$t = \frac{5}{5.026/\sqrt{12}} = \frac{17.32}{5.026} = 3.446$$

As the calculated value of $|t| = 3.446 > t_{0.05}$ for 11 degree of freedom which is 1.79, Ho is rejected

Decision: Reject Ho at α = 0.05(At 5%)

Conclusion:

The course has improved performance