Probability

52 cards

ace or a spade

E_1 E_2

$P(E_1) = \frac{4}{52}$ $P(E_2) = \frac{13}{52}$

$$E = E_1 \cup E_2$$

$$P(E) = P(E_1 \cup E_2)$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

eg2 A, B toss an unbiased coin alt. such that first who gets the head wins.
If A starts the game. What is the prob of A's winning?

$\boxed{A}$ $\boxed{B}$

A B

$\frac{1}{2}$ $\quad \frac{1}{2}\frac{1}{2}\frac{1}{2}$

$$\frac{H}{\frac{1}{2}} + \frac{T\ T\ H}{\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}} + \frac{T\ T\ T\ T\ H}{\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}}$$

$$\frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \text{---}\right]$$

$$\frac{1}{2}\left[\frac{a}{1-r}\right] = \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$(r = \frac{1}{4}) < 1$ (cgt)

$$= \frac{1}{2}\left[\frac{4}{3}\right]$$

$$= \frac{2}{3} \quad A's$$

$1 - \frac{2}{3} = \frac{1}{3}$ B

eg A — $\frac{1}{2}$ } Saving
B — $\frac{1}{3}$ } prob
$C \longrightarrow \frac{1}{4}$ }

What is the prob that prob. will be solved.

$\frac{3}{4}$

* Conditional Prob:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

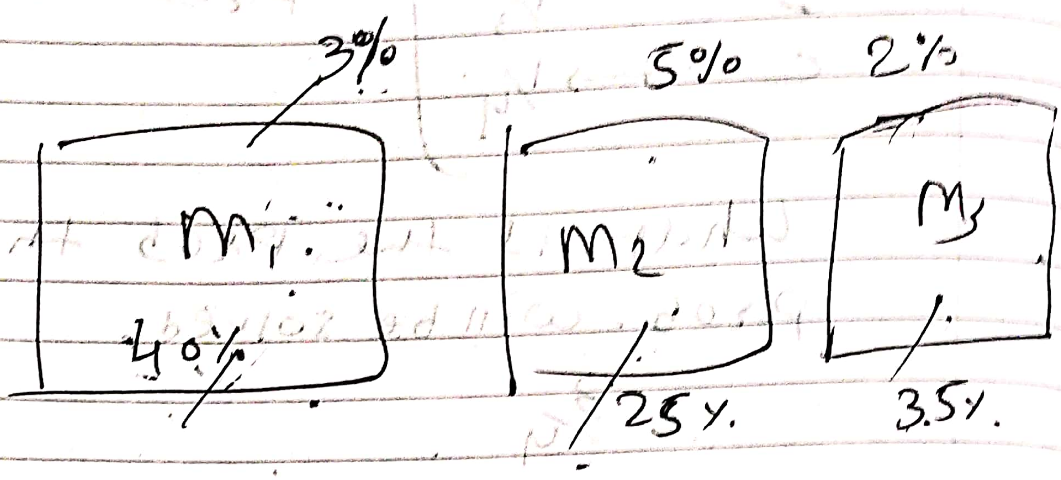
Baye's Thm:

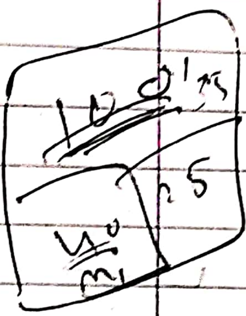
A $E_1, E_2 \cdots E_n$

$P(E_i)$ & $P(A|E_i)$ are given

$$\boxed{P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum P(E_i) \cdot P(A|E_i)}}$$

Laptop Bags





E_1 : Selected Bag is man. by M_1

E_2 : " M_2

E_3 : " " M_3

$P(E_1) = \frac{40}{100} = 0.4$

$P(E_2) = 0.25$

$P(E_3) = 0.35$

$\underline{A}$: Selected Bag is defective

m_1 $P(A|E_1) = \frac{3}{100} = 0.03$

$P(A|E_2) = 0.05$

$P(A|E_3) = 0.02$

$P \quad A$

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{\sum_i P(A|E_i) \cdot P(E_i)}$$

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2) + P(A|E_3) \cdot P(E_3)}$$

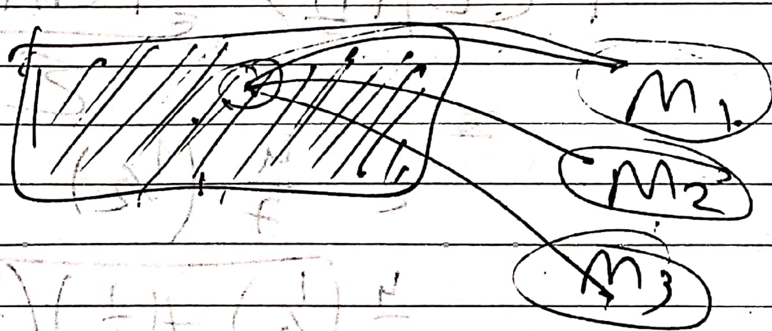
$P(E_3|A)$

$P(A \cap B) \; \_ \; P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = P(A|B) \cdot P(B)$

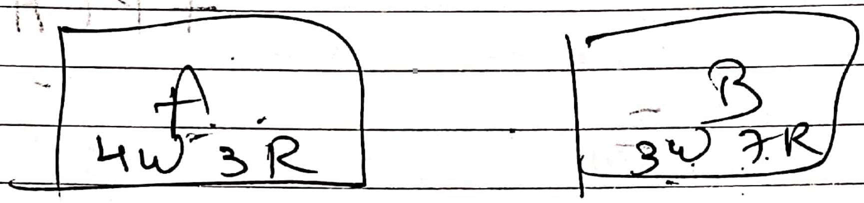
$P(A \cap B) = P(A) \cdot P(B)$



eg There are 2 boxes A & B containing 4 white & 3 Red & 3 white & 7 red balls.

resp:

A box is chosen random. & a ball is drawn from it. If the ball is white find the prob that it is from box A.



E_1 — $P(E_1) = 1/2$

E_2 — $P(E_2) = 1/2$

A — seen ball is white

$$\frac{\frac{1}{2}\left(\frac{4}{7}\right)}{\frac{1}{2}\left(\frac{4}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right)} = \frac{40}{61}$$

$P(A|E_1) = \frac{4}{7}$ $\quad P(A|E_2) = \frac{3}{10}$

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{\Sigma}$$

$$= \frac{\frac{4}{7}\left(1/2\right)}{\frac{4}{7}\left(1/2\right) + \left(\frac{1}{2}\right)\left(3/10\right)} = \frac{40}{61}$$

TA

eg ~~A microchip company has~~

eg A letter is known to have come either from TATANAGAR or from CALCUTTA.

On the envelope just two consecutive letters TA are visible. What is the prob. that the letter came from CALCUTTA.

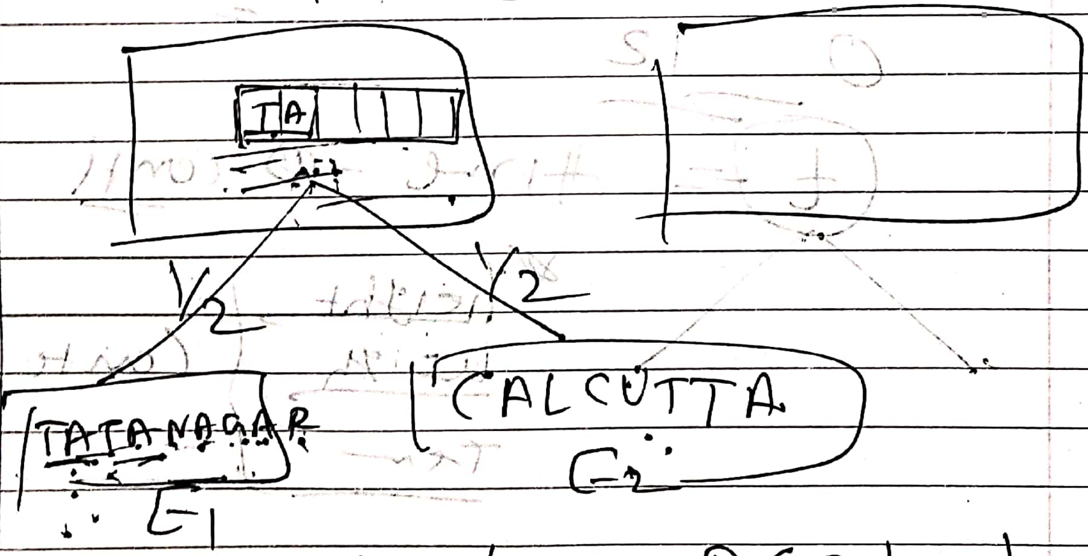
2 - TA T : 8

1 TA 7

$\cancel{P(E_1) = \frac{2}{8}}$ $\cancel{P(E_2) = \frac{1}{7}}$.

$P(A|E_1) = 2/8$ $P(A|E_2) = 1/7$

$P(E_1) = 1/2$ $P(E_2) = 1/2$



$P(E_1) = 1/2$ $P(E_2) = 1/2$

A : TA

$P(A|E_1) = \frac{2}{8}$ $P(A|E_2) = 1/7$

* # Random Variable

Variable

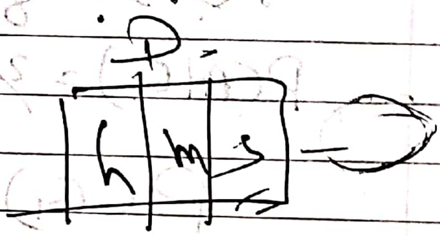
Discrete — Continuous

Grades — Marks

A B C D F — 0 vs 50. 100

80.2

An



0 12

t = time — conti

height
weigh
Tem
} conti

Marks — Discr

Grades — Discrete

No of Stu

X — ~~Conti~~ Discr

finitely many values [. '''''']

Y is conti

Inf many [. . .]

Discrete Random Var

Conti "

27/01/21

* X —

X : marks CT

X = 0 to 30 [0, 30]

31 Values

y : height of stu. cm

[120, 180]

Inf

* Random Variable :

Discrete

Continuous

→ Rule that assigns a number to each outcome of an exp.

→ eg Exp: Tossing 2 Unbi coins simultaneously.

HH, TT, HT, TH

Interested in No of Heads.

X : No of heads.

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outcomes	X = No of heads
H H	2
H T	1
T H	1
T T	0

X : 0, 1, 2

Exp: selecting one student from CSE IV sem [A, B, C, D].

X : marks obtaind ~~by~~ in CT.

Range of X 0, 1, -- 30 (31).

Discrete R.V.

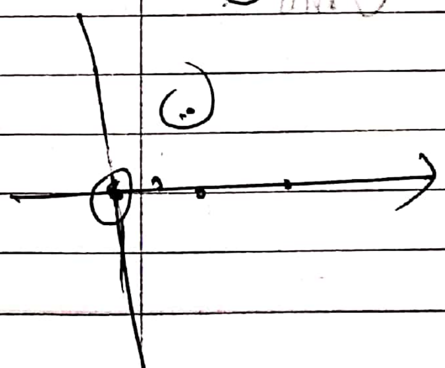
Y : Height of student

[120 cm — 180 cm]

Continuous R.X.

$\boxed{f(x) = x^2}$

* **Distribution.**



X	$f(x)$
~~0~~	~~0~~
~~1~~	~~1~~
~~2~~	~~4~~

X	$g(x)$ ~~$f(x)$~~
0	1/4
1	2/4
2	1/4

$1 = \sum f(x)$

X	$P(x_i)$

D.R.V.

X has $x_1, x_2, \ldots x_n$ the

Prob. Distribution $P(X)$

$P(x_i)$ to each x_i

(a) $0 \le P(x_i) \le 1$

(b) $\sum P(x_i) = 1$

Prob. Distribution:

(1) Discrete Prob. Distribution

(2) Continuous " "

X → Discrete, Conti

Discrete

Binomial

Poisson

Conti

Normal

exp

Uniform

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Probability mass function

Discrete X	$f(x) = P(X = x_i)$ P.m.f.
TT 0	$P(X=0) = 1/4$
HT, TH 1	$P(X=1) = 2/4$
HH 2	$P(X=2) = 1/4$

X → Conti. R.V.

then $f(x)$ - Prob. density fnⁿ.

$P(X=25) = \frac{\boxed{1/2}}{n} = \frac{2}{n}$

$P(Y = 155) = 0$

$P(150 \leq Y \leq 160) =$

28/1/21.

X Discrete

$f(x) = P(x)$.

Prob. mass funⁿ. (pmf)

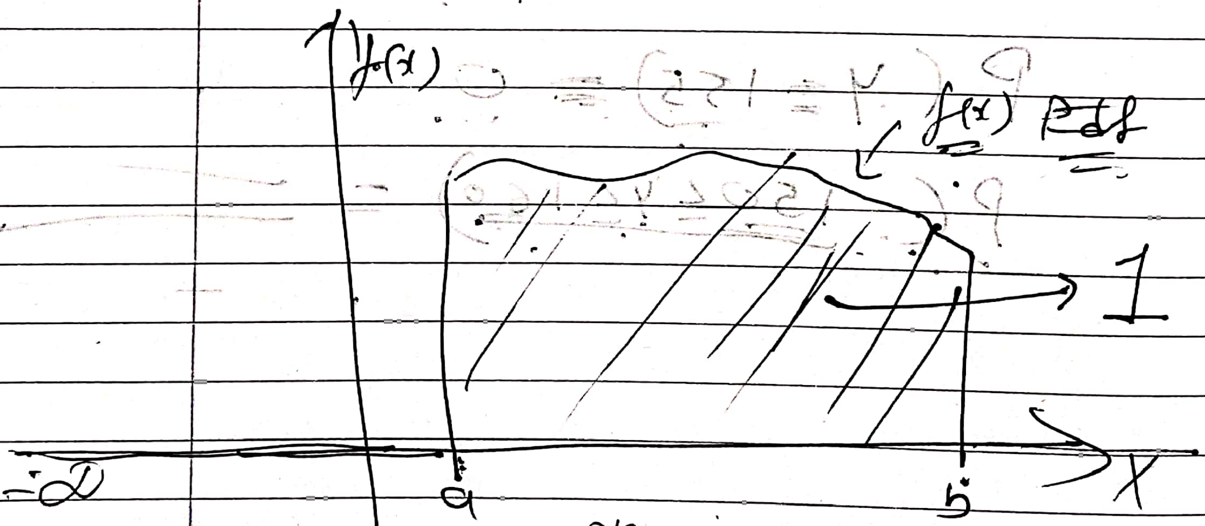
X - conti

$$P(a < X \le b) = \int_a^b f(x)\,dx$$

Prob. density funⁿ (pdf).

$$f(x) \geq 0.$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



$$f(x) = 0 \quad -\infty \le x \le a$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} + \int_a^b + \int_b^{\infty}$$

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Mathematical Expectation

Mean of

Expected value of r.v

$$E(X) = \sum x_i P(x_i)$$

Toss two coins.

X: No of heads

x	P(x)	x·P(x)
0	1/4	0
1	1/2	1/2
2	1/4	1/2

Toss
H 1/2
T 1/2
2 2

6 → 1/6
$\boxed{X}$ → 5/6

$E(X) = \sum x P(x) = 1$

$E(X) = 1$

Toss 3 coins. X: no of heads
n

X	P(x)	xP(x)
0	1/8	0
1	3/8	3/8
2	3/8	6/8
3	1/8	3/8

X: No of defe

$E(X) = \boxed{\ }$

Mean $= E(X) = \sum x P(x) = 1.5 = \boxed{3/2} =$

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$\bar{X}$ μ_x

$\bar{x} = E(X) = \sum x \cdot p(x)$ Discrete

$\bar{x} = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$ conti

Properties of mathematical Expectation

1. $E(c) = c$

2. $E(cX) = c \cdot E(x)$.

3. $E(c_1 X + c_2) = c_1 E(x) + c_2$

4. $E(x+y) = E(x) + E(y)$.

5. X & Y are independent
   $E(X \cdot Y) = E(X) \cdot E(Y)$.

6. $\boxed{E(g(x)) = \sum g(x) \cdot p(x)}$.

$g(x) = (x^2 + x)$

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Variance of Random Var

$$Var(X) := E(X^2) - [E(X)]^2$$

$$Var = \boxed{(S.D)^2 = \sum \frac{(x_i - \bar{x})^2}{n}}$$ ~~Average~~ mean

$$Var(X) = E(\cancel{X} - \cancel{\bar{X}})^2$$

$$= E(X^2 - 2X\bar{X} + \bar{X}^2)$$

$$= E(X^2) - 2\bar{X}E(X) + E(\bar{X}^2)$$

$$= E(X^2) - 2\bar{X}^2 + \bar{X}^2$$

$$= E(X^2) - \bar{X}^2$$

$$\boxed{Var(X) = E(X^2) - (E(X))^2}$$

$$\sigma = \sqrt{Var(X)}$$