To Test the Difference between Mean of Two independent Samples

- When sample size is small and samples are independent (not related) and population standard deviation is unknown, the t-statistic can be used to test the hypothesis for the difference between two population means.
- This technique is based on the assumption that the characteristic being studied is normally distributed for both the population.
- Suppose two independent samples of sizes n_1 and n_2 with means $\overline{x_1}$ and $\overline{x_2}$ and standard deviations s_1 and s_2 we may be interested in testing:

  (i) whether the two independent samples have been drawn from the population with the same means

  (ii) the sample mean $\overline{x_1}$ and $\overline{x_2}$ do not differ significantly

- **To carry out the test, we calculate t as follows**

$$t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- Where $\overline{x_1}$ is the mean of first sample and $\overline{x_2}$ is the mean of second sample and

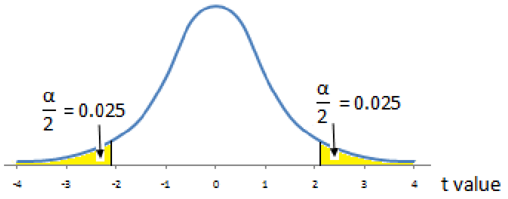
$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2\right]$$

$$= \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

- Degree of freedom = $n_1 + n_2 - 2$

Student's t Distribution Table

For example, the t value for 18 degrees of freedom is 2.101 for 95% confidence interval (**2-Tail** α = 0.05).



	90%	**95%**	**97.5%**	**99%**	**99.5%**	**99.95%**	**1-Tail Confidence Level**
	80%	**90%**	**95%**	**98%**	**99%**	**99.9%**	**2-Tail Confidence Level**
	0.100	**0.050**	**0.025**	**0.010**	**0.005**	**0.0005**	**1-Tail Alpha**
df	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.001**	**2-Tail Alpha**
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	

Example 1

The mean life of a random sample of 10 light bulbs was found to be 1456 hours with a S.D. of 423 hours. A second sample of 17 bulbs chosen at random from a different batch showed a mean life of 1,280 hours with a *S.D*. of 398 hours. Is there significant difference between the mean life of the two batches?

Example Solution:

Here $n_1 = 10, n_2 = 17, \overline{x_1} = 1456, \overline{x_2} = 1280, s_1$=423, $s_2 = 398$

Let us assume null hypothesis that There is no significant difference in the mean life of bulbs of the two batches

Now

$$t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Here $S^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2\right]$

$$= \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2} = \frac{1}{10+17-2}(10 \times 423^2 + 17 \times 398^2)$$

$$= \frac{4482158}{25} = 179286.32$$

$$S = \sqrt{179286.32} = 423.42$$

As $t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{1456 - 1280}{423.42\sqrt{\frac{1}{10} + \frac{1}{17}}} = \frac{176}{423.42 \times 0.3985} = \frac{176}{168.69} = 1.04$

As tabulated value of t at 5% level for 25 degree of freedom is $t_{0.05} = 2.06$. And the calculated value of $|t| = 1.04 < t_{0.05}$ for 25 degree of freedom Ho is Accepted

Decision: Accept Ho at α = 0.05(At 5%)

Conclusion:

There is no significant difference between the mean life of bulbs of the two batches

Example 2

Below are given the gain of weights (in lbs.) of lions on two diet X and Y:

Diet X	**25**	**32**	**30**	**32**	**24**	**14**	**32**			
Diet Y	24	34	22	30	42	31	40	30	32	35

Test at 5% level of significance whether the two diets differ significantly in increasing weight.

Example Solution:

Let us assume null hypothesis that two means do not differ significantly

Now we have $\overline{x_1} = \frac{\sum x_i}{n} = \frac{189}{7} = 27$ and $\overline{x_2} = \frac{\sum x_i}{n} = \frac{320}{10} = 32$

	Diet X			Diet Y	
x_1	$x_1 - \overline{x_1}$	$(x_1 - \overline{x_1})^2$	x_2	$x_2 - \overline{x_2}$	$(x_2 - \overline{x_2})^2$
25	-2	4	24	-8	64
32	-5	25	34	2	4
30	3	9	22	-10	100
32	5	25	30	-2	4
24	-3	9	42	10	100
14	-13	169	31	-1	1
32	5	25	40	8	64
			30	-2	4
			35	3	9
189		266	320		350

$$t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Here $S^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2\right]$

$$= \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2} = \frac{1}{7+10+2}(266 + 350)$$

$$= \frac{616}{15} = 41.066$$

$$S = \sqrt{41.066} = 6.408$$

As $t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{27 - 32}{6.408\sqrt{\frac{1}{7} + \frac{1}{10}}} = \frac{5}{6.408 \times 0.4928} = -\frac{3}{3.157} = -1.583$

As tabulated value of t at 5% level for 15 degree of freedom is $t_{0.05} = 2.13$. And the calculated value of $|t| = 1.583 < t_{0.05}$ for 15 degree of freedom Ho is Accepted

Decision: Accept Ho at α = 0.05(At 5%)

Conclusion:

Two diets do not differ significantly

Example 3

A group of seven week-old chickens reared on a high protein diet weigh 12, 15, 11, 16, 14

and 16 ounces, a second group of five chickens similarly treated except that they receive a low protein diet weighted 8, 10, 14, 10 and 13 ounces. Test whether there is sufficient evidence that additional protein has increased the weight of the chickens.

Example 4

The heights of six randomly chosen sailors are in inches: 63, 65, 68, 69, 71 and 72. Those of

10 randomly chosen soldiers are 61, 62, 65, 66, 69, 70, 71, 72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers.