Ex-1 suppose the waist measurements W of 800 girls are normally distributed with mean 66 cms and standard deviation 5 cms. Find the number N of girls with waists.

<i> between 65 and 70 cms;

<ii> greater than or equal to 72 cms.

Sol:→ Here W : waists measurements (in cms) of girls

W (in cms.)	65	70	72
$z = \frac{W-\mu}{\sigma} = \frac{W-66}{5}$ (standard normal variate)	$\frac{65-66}{5} = -0.2$	$\frac{70-66}{5} = 0.8$	$\frac{72-66}{5} = 1.2$

For table → Book : N. P. Bali (pg : 1651)

<i> The prob. that a girl has waist betn 65 cms. and 70 cms is given by :

$$P(65 \le W \le 70) = P(-0.2 \le z \le 0.8)$$

$$= P(-0.2 \le z \le 0) + P(0 \le z \le 0.8)$$

$$= P(0 \le z \le 0.2) + P(0 \le z \le 0.8)$$

($\because$ by symmetry)

$$= 0.0793 + 0.2881$$

$$= 0.3674$$

see table - VI pg.: 1347

Hence in a group of 800 girls; the expected number of girls with waists betn 65 cms and 70 cms. is given by

$= 800 \times 0.3674$

$= 293.92 \quad \simeq 294$

<ii> The probability that a girl has waist greater than or equal to 72 cms is given by

$P(W \geq 72) = P(Z \geq 1.2)$ ($\because$ by above table)

$= 0.5 - P(0 \leq Z \leq 1.2)$

$= 0.5 - 0.3849$

$= 0.1151$

$\therefore$ In a group of 800 girls; the expected number of girls with waists greater than or equal to 72 cms. is given by

$= 800 \times 0.1151$

$= 92.08$

$\simeq 92$

Ex-2 A sample of 100 dry battery cells tested to find the length of life produced the following results.

$\bar{x} = \mu = 12$ hrs. ; $\sigma = 3$ hrs.

Assuming the data to be normally distributed what percentage of battery cells are expected to have a life

<i> more than 15 hours.

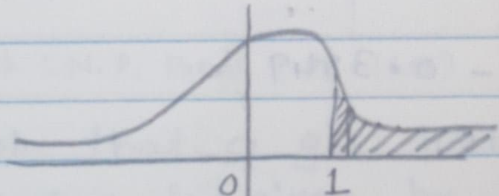
<ii> less than 6 hours.

<iii> between 10 and 14 hours.

Sol^n :→ Let x denotes the length of life of dry battery cells.

Also : $z = \frac{x-\mu}{\sigma} = \frac{x-12}{3}$

<i> When $x = 15$; $z = 1$



$\therefore P(x > 15) = P(z > 1)$

$= P(0 < z < \infty) - P(0 < z < 1)$

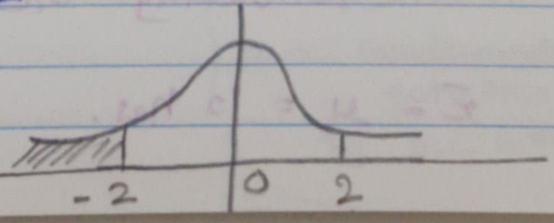
$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$= 15.87\ \%$

<ii> when $x = 6$; $z = -2$



$\therefore P(x < 6) = P(z < -2)$

$= P(z > 2)$ $(\because$ by symmetry$)$

$= P(0 < z < \infty) - P(0 < z < 2)$

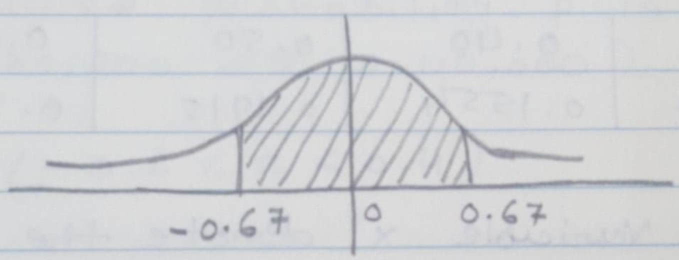
$= 0.5 - 0.4772$

$= 0.0228$

$= 2.28\%$

(iii) when $x = 10$; $z = -\frac{2}{3} = -0.67$

when $x = 14$; $z = \frac{2}{3} = 0.67$



$P(10 < x < 14) = P(-0.67 < z < 0.67)$

$= 2P(0 < z < 0.67)$ $(\because$ symmetry$)$

$= 2 \times 0.2487$

$= 0.4974$

$= 49.74\%$

Ex-3 : A sales Tax officer has reported that the average sales of the 500 business that he has to deal with during a year amount to Rs. 36,000 with a standard deviation of Rs. 10,000. Assuming that the sales in these business are normally distributed; find

<i> The number of businesses the sales of which are over Rs. 40,000

<ii> The percentage of businessess; the sales of which are likely to range between Rs. 30,000 and Rs. 40,000

<iii> The probability that the sales of business selected at random will be over Rs. 30,000

Proportions of the area under the normal curve:

z	0.25	0.40	0.50	0.60
area	0.0987	0.1554	0.1915	0.2257

Soln: Let the variable x denote the sales (in Rs.) of the business during a year

given:

$$\mu = 36{,}000 \quad \text{and} \quad \sigma = 10{,}000$$

<i> The probability that the sales of a business is over Rs. 40,000 is; given by $P(x > 40{,}000)$

when $x = 40{,}000$; $z = \dfrac{x-\mu}{\sigma} = \dfrac{40{,}000 - 36000}{10{,}000}$

$= 0.4$

$\therefore P(x > 40{,}000) = P(z > 0.4)$

$= 0.5 - P(0 \leq z \leq 0.4)$

$= 0.5 - 0.1554$
$= 0.3446$

$\therefore$ in a group of 500 businesses, the expected number of businesses with annual sales over Rs. 40,000 is

$= 500 \times 0.03446$
$= 172.3$
$\simeq 172$

<ii> Required probability p is given by
$P(30,000 < X < 40,000)$

$= P(-0.6 < z < 0.4)$

$\therefore$ when $X = 30,000$; $z = \frac{X - \mu}{\sigma} = \frac{30,000 - 36,000}{10,000}$
$= -0.6$

when $X = 40,000$; $z = 0.4$

$= P(-0.6 < z < 0.4)$

$= P(-0.6 < z \leq 0) + P(0 \leq z < 0.4)$

$= P(0 \leq z < 0.6) + P(0 \leq z < 0.4)$
($\because$ symmetry)

$= 0.2257 + 0.1554$

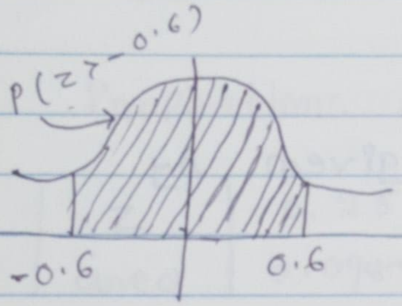
$= 0.3811$

$= 38.11\ \%$

<iii> The probability that the annual sales of a business selected at random will be over Rs. 30,000 is given by :

$P(X > 30{,}000) = P(z > -0.8)$

when $X = 30{,}000$; $z = \frac{x-\mu}{\sigma} = -0.6$



$= P(-0.6 < z < 0) + 0.5$

$= P(0 < z < 0.6) + 0.5$

$= 0.2257 + 0.5$

$= 0.7257$.

EX-4 Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be
<i> over six feet tall and
<ii> below 66 inches.
Assume heights to be normally distributed.

sol^n:→ Let the variable x denote the height (in inches) of the soldiers
Then we are given

mean $\mu = 68.22$
Variance $\sigma^2 = 10.8$

<i> A soldier will be over 6 feet tall if x is greater than 72.

($\because$ x is height in inches and 6 feet = 72 inches)

$\therefore$ when $x = 72$

$$z = \frac{x-\mu}{\sigma} = \frac{72 - 68.22}{\sqrt{10.8}} = \frac{3.78}{3.286} = 1.15$$

$\therefore$ The probability that a soldier is over 6 feet tall is given by

$$P(x > 72) = P(z > 1.15)$$
$$= 0.5 - P(0 \le z \le 1.15)$$
$$= 0.5 - 0.3749 \quad (\because \text{ by table}).$$
$$= 0.1251$$

$\therefore$ In a regiment of 1,000 soldiers; the number of soldiers over 6 feet tall is

$$= 1000 \times 0.1251$$
$$= 125.1$$
$$\simeq 125$$

<ii> The probability that a soldier is below 66 inches is given by:

$$P(x < 66) = P(z < -0.6756)$$

when $x = 66$; $z = \frac{x-\mu}{\sigma} = -0.6756$

$$= P(z > 0.6756) \quad (\because \text{symmetry})$$
$$= 0.5 - P(0 < z < 0.6756)$$

$= 0.5 - 0.2501$ ($\because$ by table)

$= 0.2499$

$\therefore$ The number of soldiers over 66 inches in a regiment of 1,000 soldiers is :

$= 1000 \times 0.2499$

$= 249.9$

$\simeq 250$

Ex-5 The average test marks in a particular class is 79. The standard deviation is 5. If the marks are distributed normally; how many students in a class of 200 did not receive marks between 75 and 82?

given :

$$\left.\begin{aligned} Pr.\{0 \le z \le 0.6\} &= 0.2257 \\ Pr.\{0 \le z \le 0.7\} &= 0.2580 \\ Pr.\{0 \le z \le 0.8\} &= 0.2881 \end{aligned}\right\}$$

where z is a standard normal variable.

Sol^n :- Let the variable x denotes the marks obtained by the students in the given test bet^n 75 and 82; then we are given :

$\mu = 79$; $\sigma = 5$

$$\left.\begin{aligned} \text{when } x = 75 \; ; \; z &= \frac{x-\mu}{\sigma} = -0.8 \\ x = 82 \; ; \; z &= \frac{x-\mu}{\sigma} = 0.6 \end{aligned}\right\}$$

$\therefore$ The probability that a one student gets marks bet^n 75 and 82 is given by:

$$P(75 < X < 82) = P(-0.8 < z < 0.6)$$

$$= P(-0.8 < z < 0) + P(0 < z < 0.6)$$

$$= P(0 < z < 0.8) + P(0 < z < 0.6)$$

($\because$ symmetry)

$$= 0.2881 + 0.2257$$

$$= 0.5138$$

$\therefore$ The prob. P that a student does not get marks between 75 and 82 is given by:

$$P = 1 - P(\text{student gets marks bet}^n \text{ 75 and 82})$$

$$= 1 - P(75 < X < 82)$$

$$= 1 - 0.5138$$

$$= 0.4862$$

Hence in a class of 200 students; the number of students who did not receive marks bet^n 75 and 82 is given by:

$$= 200 \times P$$

$$= 200 \times 0.4862$$

$$= 97.24$$

$$\simeq 97$$