Hypothesis Testing for Single Population Proportion

The population proportion P is the ratio of the number of elements possessing characteristics to the total number of elements in the population

$$P = \frac{Number\ of\ elements\ possessing\ the\ characteristics}{Total\ number\ of\ elements\ in\ the\ population}$$

Whereas the sample proportion p is the ratio of the number of elements possessing a characteristics to the total number of elements in the sample

$$p = \frac{Number\ of\ elements\ possessing\ the\ characteristics}{Total\ number\ of\ elements\ in\ the\ sample}$$

Hypothesis Testing for Single Population Proportion

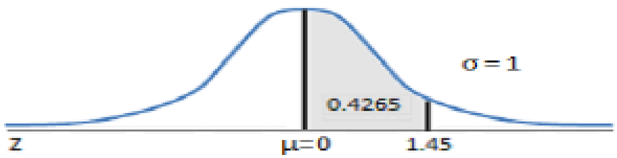
(continued)

This test is used to find the significance between proportion of the sample and the population. Test Statistics for sample of size n is

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Example 1: Two Tail Test

A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Check whether the die can be regarded as an unbiased one?(Test at 5% level of significance)

Example Solution: Two Tail

Here P = Population proportion

= Prob. of getting 3 or 4 on die = $\frac{2}{6} = \frac{1}{3}$

And Q = 1 – P = $\frac{2}{3}$

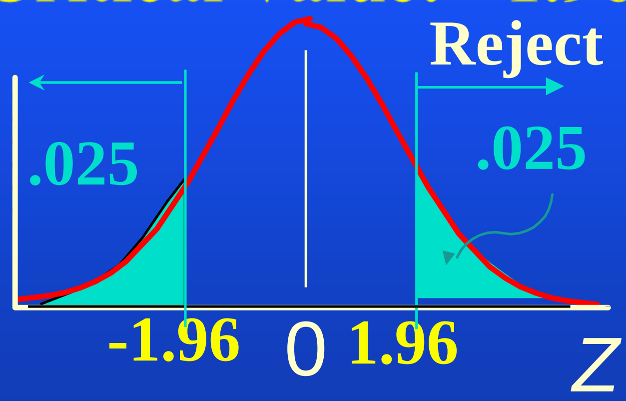
Here n = sample size=9000

p = Sample Proportion = $\frac{3240}{9000}$ = 0.36

$\alpha = 0.05$

$\alpha/2 = 0.025$

Critical Value: ±1.96



H_0: The Die is unbiased

i.e. $H_0 : P = \frac{1}{3}$

$H_1: P \neq \frac{1}{3}$ (Two tailed test)

Under H_0: $Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.36-0.33}{\sqrt{\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9000}}}$

$= 0.03496 < 1.96$

Decision:

Accept Ho at α = 0.05(At 5%)

Conclusion:

The Die is Unbiased

Example 2:

E.g.1 An insurance company states that 90% of its claims are settled within 30 days. A consumer group selected a simple random sample of 75 of the company's claims to test this statement. The consumer group found that 55 of the claims were settled within 30 days. *A*t the 0.05 significance level, test the company's claim that 90% of its claims are sett*l*ed within 30 days.

Example Solution:

Here P = Population proportion

= Proportion of Claims settled within 30 days = 0.9 (90 %)

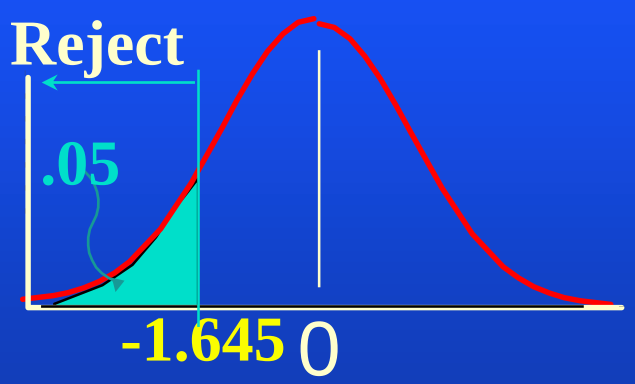
And Q = 1 − P = 0.1

Here n = sample size

p = Sample Proportion = $\frac{55}{75}$ = 0.73

$\alpha = 0.05$

Critical Value: 1.645



H_0: Claim that 90% of the company's claims are settle within 30 days is true

i.e. $H_0: P \geq 0.9$

H_1: $P < 0.9$ (Left tailed test)

Under H_0: $Z = \dfrac{p-P}{\sqrt{\frac{PQ}{n}}} = \dfrac{0.73-0.9}{\sqrt{0.9 \times 0.1 \times \frac{1}{75}}}$

$= -4.811 < -1.645$

Decision:

Reject Ho at α = 0.05(At 5%)

Conclusion:

Claim that 90% of the company's claims are settle within 30 days is not acceptable

Example 3

A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Test the claim of the wholesaler at 5%.

Example 4

In a sample of 1000 people, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice eater and wheat eater are equally popular at 1% level of significance?

Example 5

In a sample of 1000 people, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice eater and wheat eater are equally popular at 1% level of significance?