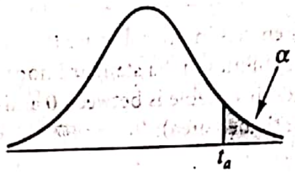
TABLE A.6 Critical Values from the t Distribution



df	Values of α for one-tailed test and $\alpha/2$ for two-tailed test					
	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2.000	2.390	2.660	3.232
70	1.294	1.667	1.994	2.381	2.648	3.211
80	1.292	1.664	1.990	2.374	2.639	3.195
90	1.291	1.662	1.987	2.368	2.632	3.183
100	1.290	1.660	1.984	2.364	2.626	3.174
150	1.287	1.655	1.976	2.351	2.609	3.145
200	1.286	1.653	1.972	2.345	2.601	3.131
∞	1.282	1.645	1.960	2.326	2.576	3.090

(*) t - test for testing the significance of a single mean :

Suppose we wish to test the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$; where μ is the mean of a normal population. we use the statistic

$$t = \frac{\sqrt{n}\,(\bar{x} - \mu)}{s} \quad \text{OR} \quad t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{--- (1)}$$

where $\bar{x}$ = sample mean

n = sample size

s - standard deviation of the sample.

$$\boxed{\bar{X} = \frac{\Sigma x_i}{n}} \quad \& \quad \boxed{s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}}}$$

$$\text{OR} \quad \boxed{s = \sqrt{\frac{1}{n-1} \left\{ \Sigma x^2 - \frac{(\Sigma x)^2}{n} \right\}}}$$

Note : 1) Here we use defined the standard deviation of a sample by dividing $\Sigma (x - \bar{x})^2$ by $(n-1)$ instead of n, because the resulting value gives a better estimate of σ, the population standard deviation.

2) The sampling distribution of the statistic t given in eqn (1) is a student's t distribution with $(n-1)$ degrees of freedom. Using the **table of** t - distribution, we find the value $t_{n-1,\,0.05}$. If $|t| \geq t_{n-1}$, we reject H_0 at 5% level of significance, otherwise we accept it. (not reject)

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Ex-1 A random sample of 20 tablets from a batch gives a mean active ingredient content 42 mg and standard deviation of 6 mg. Test the hypothesis that the population mean is 44 mg.

Solⁿ: Let $H_0 : \mu = 44$ mg, $H_1 : \mu \neq 44$ (two tailed test)

Given that $\bar{x} = 42$ & $s = 6$.

$$|t| = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| = \left|\frac{42-44}{6/\sqrt{20}}\right| = 1.49.$$

Degrees of freedom $= n-1 = 20-1 = 19$.

Now we find from table of t-distribution

$t_{19, 0.05} = 2.09$.

$\therefore |t| < t_{19, 0.05}$

$\therefore$ We accept H_0 at 5% level of significance and conclude that the population mean is 44 mg.

Ex-2 Eight items of a sample have the following values: 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of the 8 observations differ significantly from the assumed population mean of 48; use 5% level of significance.

Sol^n: We wish to test $H_0 : \mu = 48$ against $H_1 : \mu \neq 48$. (two tailed test)

To reduce the calculations, we subtract $A = 50$ from each observation. (other method to find $\bar{x}$ & s.)

$\boxed{A = 50}$

x	$d = x - A$	d^2
47	-3	9
50	0	0
52	2	4
48	-2	4
47	-3	9
49	-1	1
53	3	9
51	1	1
	$\Sigma d = -3$	$\Sigma d^2 = 37$

$$\bar{x} = A + \frac{\Sigma d}{n} = 50 + \frac{(-3)}{8} = 50 - 0.375 = 49.625$$

$$s = \sqrt{\frac{1}{n-1}\left\{\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right\}} = \sqrt{\frac{1}{8-1}\left\{37 - \frac{(-3)^2}{8}\right\}} = 2.2638$$

$$|t| = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| = \left|\frac{49.625 - 48}{2.2638/\sqrt{8}}\right| = 2.03$$

$d.f = n-1 = 8-1 = 7$.

from the table of t-distribution, we find

$t_{7,\, 0.05} = 2.36$.

$\therefore |t| < t_{7, 0.05}$

$\therefore$ We accept H_0 at 5% level of significance and conclude that the sample mean does not differ significantly from the population mean $\mu = 48$.

Ex-3 Measurements of body mass index (BMI) for a sample of 10 healthy adult males are shown in the following table.

Subject	1	2	3	4	5	6	7	8	9	10
BMI	21	23	32	24	47	22	45	37	24	35

On the basis of these data can we conclude that the BMI of the population from which the sample was drawn is 35?

Solⁿ: Let X denote the BMI of a subject.

$H_0 : \mu = 35$ against $H_1 : \mu \neq 35$. (two tailed test)

To calculate sample mean and S.D take $\boxed{A = 30}$

X	$d = X - A$	d^2
21	-9	81
23	-7	49
32	2	4
24	-6	36
47	17	289
22	-8	64
45	15	225
37	7	49
24	-6	36
35	5	25
	$\Sigma d = 10$	$\Sigma d^2 = 858$

Now $\bar{x} = A + \frac{\Sigma d}{n}$

$= 30 + \frac{10}{10}$

$\boxed{\bar{x} = 31}$

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$$s = \sqrt{\frac{1}{n-1}\left\{\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right\}} = \sqrt{\frac{1}{10-1}\left\{858 - \frac{(10)^2}{10}\right\}}$$

$$= 9.7068.$$

$$|t| = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| = \left|\frac{31-35}{9.71/\sqrt{10}}\right| = 1.30.$$

d.f $= n-1 = 10-1 = 9.$

From the table of t-distribution we find

$t_{9,\,0.05} = 2.262$

$\therefore |t| < t_{9,\,0.05}$

$\therefore$ we accept H_0 at 5% level of significance and conclude that mean BMI of the population is 35.

Ex-4 A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement campaign the mean sales per week per soap was 140 dozens. After the campaign, a sample of 26 shops was taken and the mean sales was found to be 147 dozens with standard deviation 16. Can you consider the advertisement campaign effective? (take the tabulated value of t = 1.708)

Solⁿ: Here $n = 26$, $s = 16$, $\bar{x} = 147$, $\mu = 140$.

to test the hypothesis

$H_0 : \mu = 140$ against $H_1 : \mu > 140$ (one tailed test)

$$|t| = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| = \left|\frac{147 - 140}{16/\sqrt{26}}\right| = 2.73.$$

d.f $= n-1 = 26-1 = 25$

we are given that $t_{25, 0.1} = 1.708$

$\therefore |t| > t_{25, 0.1}$

$\therefore$ we reject H_0 and conclude that the advertisement campaign was effective.

Ex-5 A drug manufacturer has installed a machine which automatically fills 5 gm of drug in each phial. A random sample of fills was taken and it was found to contain 5.02 gm. on an average in a phial. The standard deviation of the sample was 0.002 gms. Test at 5% level of significance if the adjustment in the machine is in order.

Solⁿ:

Here $n = 10$, $\bar{x} = 5.02$, $\mu = 5$, $s = 0.002$.

H_0: The adjustment in the machine is in order

i.e $H_0 : \mu = 5$.

$H_1 : \mu \neq 5$ (two tailed test).

$$|t| = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| = \left|\frac{5.02 - 5}{0.002/\sqrt{10}}\right| = 33.33.$$

d.f. = n-1 = 10-1 = 9.

From the table of t-distribution we find

$t_{9,0.05} = 2.262$

$|t| > t_{9,0.05}$

$\therefore H_0$ is rejected

i.e the adjustment in the machine is not in order.

Ex-6 The average breaking strength of steel rods is specified to be 18.5 thousand kg. For this a sample of 14 rods was tested. The mean and standard deviation obtained were 17.85 and 1.955, respectively. Test the significance of the deviation.

Soln Here $n = 14$, $\bar{x} = 17.85$, $\mu = 18.5$, $s = 1.955$

H_0: There is no significant deviation in the breaking strength.

i.e $H_0 : \mu = 18.5$

$H_1 : \mu \neq 18.5$ (two tailed test)

$$|t| = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| = \left|\frac{17.85-18.5}{1.955/\sqrt{14}}\right| = 1.24.$$

d.f = n-1 = 14-1 = 13.

From the table of t-distribution we find

$t_{13,0.05} = 2.16.$

As the calculated value of $|t| = 1.24 < t_{13, 0.05}$
$\therefore$ H_0 is accepted.

i.e
there is no significant deviation in the breaking strength.

Ex-7 Figures released by the U.S. department of Agriculture show that the average size of farms has increased since 1940. In 1940, the mean size of a farm was 174 acres; by 1997, the average size was 471 acres. Between those years the number of farms decreased but the amount of tillable land remained relatively constant, so now farms are bigger. This trend might be explained, in part, by the inability of small farms to compete with the prices and costs of large-scale operations and to produce a level of income necessary to support the farmer's desired standard of living. Suppose an agribusiness researcher believes the average size of farms increased from the 1997 mean figure of 471 acres. To test this notion, she randomly sampled 23 farms across the U.S and ascertained the size of each farm from country records. The data she gathered follow. Use a 5% level of significance to test her hypothesis.

445, 489, 474, 505, 553, 477, 454, 463, 466, 557, 502, 449, 438, 500, 466, 477, 557, 433, 545, 511, 590, 561, 560.

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Solⁿ: Given that $n = 23$, $\bar{x} = 498.78$, $s = 46.94$.
$\mu = 471$. (↑ from given data)

The resercher's hypothesis is that the average size of a U.S. farm is more than 471 acres. Because this theory is unproven, it is alternative hypothesis. The null hypothesis is that the mean is still 471 acres.

$H_0 : \mu = 471$
$H_1 : \mu > 471$ (one tailed test).

$$|t| = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| = \left|\frac{498.78 - 471}{46.94/\sqrt{23}}\right| = 2.84.$$

$\left(\bar{x} = \frac{\sum x_i}{n} = 498.78\right.$ calculate from given 23 observation$\left.\right)$

$d.f = n-1 = 23-1 = 22$.
From the table of t-distribution we find
$t_{22, 0.05} = 1.717$
As the calculated value of $|t| = 2.84$ is greater than 1.717.

i.e $|t| > t_{22, 0.05}$

$\therefore$ Ho is rejected
i.e The business researcher rejects the null hypothesis. she accepts the alternative hypothesis and concludes that the average size of a U.S. farm is now more than 471 acres.

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