

L3> MODE :-

The mode refers to that value in a distribution which occurs most frequent. It is an actual value which has the highest concentration of items in and around it.

In the words of croxton and cowden "The mode of a distribution is the value at the point around which the items tend to be most heavily concentrated. It may be regarded as the most typical of a series of values."

* Merits of mode :-

- 1> Mode is readily comprehensible, commonly understood and easily calculated.
- 2> It is not affected by extreme values.
- 3> It can be conveniently obtained in the case of open end classes which do not pose any problem here.
- 4> It is based on all the values of the variable.
- 5> Mode can be conveniently located even if the frequency distribution has class-interval of unequal magnitudes provided the mode-class and the classes preceding and succeeding it are of the same magnitude.

* Demerits of mode:

- (1) Mode is not rigidly defined. It is ill-defined and indeterminate.
- (2) It is not based on all the observations of a series.
- (3) Mode is not suitable for further mathematical treatment.
- (4) As compared with mean; mode is affected to a great extent by the fluctuations of Sampling.
- (5) In many cases it may be impossible to get a definite value of mode. There may be 2, 3 or more modal values.

* Relation betⁿ. Mean, Median and mode.

In case of a symmetrical distribution
Mean, median and mode coincide

$$\text{ie } \text{Mean} = \text{Median} = \text{Mode}$$

However for a moderately assymetrical distribution; Mean and Mode usually lie on the two ends and Median lie in between them and they obey the following important empirical relationship given by prof. kurt pearson :

$$\text{Mode} = \text{Mean} - 3(\text{Mean} - \text{Median})$$

$$\Rightarrow \text{Mode} = \text{Mean} - 3\text{Mean} + 3\text{Median}$$

$$\Rightarrow \boxed{\text{Mode} = 3\text{Median} - 2\text{Mean}}$$

This formula is specially useful to determine the value of mode in case it is ill-defined.

Ques: Define and relate Mean, Median and Mode.

★ Calculation of Mode :-

1) In the case of discrete frequency distribution Mode is the value of the variable corresponding to the maximum frequency.

But in any one of the following cases:

- If the maximum frequency is repeated.
- If the maximum frequency occurs in the very beginning or at the end of the distri.
- If there are irregularities in the distri.

→ The value of mode is determined by the method of grouping.

2) In the case of continuous frequency distribution the class corresponding to the maximum

frequency is called the "modal class" and the value of mode is obtained by the formula.

$$\text{Mode} = l + h \left[\frac{f_1 - f_0}{(f_1 - f_0) - (f_2 - f_1)} \right]$$

$$= l + h \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right]$$

Mode formula

where;

l = lower limit of the modal class.

f_1 = frequency of the modal class.

f_0 = frequency of the class preceding modal class.

f_2 = frequency of the class succeeding modal class.

h = magnitude of the modal class.

{3} If $2f_1 - f_0 - f_2 = 0$; use the formula.

$$\text{Mode} = l + h \cdot \left[\frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \right]$$

Ex-5: Find the mode of the following frequency distribution.

size(x) : 4 5 6 7 8 9 10 11 12 13
freq(f) : 2 5 8 9 12 14 14 15 11 13

Sol:

size (x)	I	II	III	IV	V	VI
4	2					
5		5				
6		8				
7		9				
8		12				
9		14				
10						
11						
12		11				
13		13				

Frequency.

Arrows indicate the movement from the first column to the second, then to the third, and so on, with some loops and specific points circled in blue.

- From 10 to 14
- From 11 to 15
- From 15 to 29
- From 29 to 28
- From 28 to 40
- From 40 to 40
- From 40 to 43

Circled values: 29, 28, 40, 40, 43.

columns.	size of item having max. freq.
I	11
II	<u>10, 11</u>
III	<u>9, 10</u>
IV	<u>10, 11, 12</u>
V	8, 9, 10
VI	9, <u>10, 11</u> .

since the item 10 occurs maximum number of times.

$$\therefore \boxed{\text{Mode} = 10}$$

Ex-6 compute the mode of the following frequency distribution.

$$\begin{array}{llllllllllllll} x: & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ f: & 3 & 8 & 10 & 12 & 16 & 14 & 10 & 8 & 17 & 5 & 4 & 1 \end{array}$$

Ans Mode = 6.

Ex-7 Compute the mode of the following frequency distribution.

$$\begin{array}{llllllllllll} x: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ f: & 3 & 19 & 12 & 4 & 4 & 3 & 7 & 6 & 3 & 4 & 5 & 2 \end{array}$$

Ans Mode = 2.

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

(or calculate Mean, Median and Mode).

Ex-8. Calculate Mode from the following data.

Marks.	No. of students.	Marks.	No. of students.
Below 10	4	Below 60	86
" 20	6	" 70	96
" 30	24	" 80	99
" 40	46	" 90	100.
" 50	67		

Sol: since we are given the cumulative frequency distribution of marks; first we shall convert it into the frequency distribution as under-

Marks.	Frequency.	Marks.	Frequency.
0-10	4	50-60	86-67 = 19
10-20	6-4 = 2	60-70	96-86 = 10
20-30	24-6 = 18	70-80	99-96 = 3
30-40	46-24 = 22	80-90	100-99 = 1
40-50	67-46 = 21		

Further; since the frequencies first decrease; then increase and again decrease; the distribution is irregular hence the modal-class is located by the method of grouping as explained under.

Marks

Frequencies.

	I	II	III	IV	V	VI
0-10	4					
10-20	2	6		24		
20-30	18	40	20		42	
30-40	22		43			61
40-50	21	40		62		
50-60	19		29		50	
60-70	10	13				32
70-80	3		4	14		
80-90	1					

Column No.	Max. Frequency	Class corr. maximum freq.
I	22	<u>30-40</u>
II	40	20-30, <u>30-40</u> , 40-50, 50-60.
III	43	<u>30-40</u> , 40-50,
IV	62	<u>30-40</u> , 40-50, 50-60
V	50	40-50, 50-60, 60-70.
VI	61	20-30, <u>30-40</u> , 40-50.

Now; In the above table there are two classes viz : 30-40 and 40-50 which are repeated maximum number (5) of items and as such we cannot decide about the modal class.

Thus, even the method of grouping fails to give the modal class.

We say that in the above example Mode is ill-defined and we locate it by formula:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$$

(*)

First find mean and median.

(Let $A = 45$)

Marks	Mid-Value (X)	Frequency (F)	C.F.	$d = \frac{x-A}{10}$	F.d.
0-10	5	4	4	-4	-16
10-20	15	2	6	-3	-6
20-30	25	18	24	-2	-36
30-40	35	22	46	-1	-22
40-50	45	21	67	0	0
50-60	55	19	86	1	19
60-70	65	10	96	2	20
70-80	75	3	99	3	9
80-90	85	1	100	4	4
	$N = 100$			$\sum = -28$	

$$\textcircled{1} \text{ Mean} = A + h \cdot \frac{\sum f_d}{N}$$

$$= 45 + 10 \left[\frac{-28}{100} \right]$$

$$= 45 - 2.8$$

$$\text{OP. } \bar{x} = \frac{\sum f_i x_i}{N}$$

$$= 42.20$$

$$\boxed{\text{Mean} = 42.20}$$

$\textcircled{2}$ Now $N/2 = 50$;
since c.f. just greater than 50 is 67;
the corr. class 40-50 is Median class.

$$\therefore \text{Median} = l + \frac{h}{F} \left[\frac{N}{2} - c \right]$$

$$= 40 + \frac{10}{21} \left[50 - 46 \right]$$

$$= 40 + 1.9$$

$$= 41.90$$

$$\therefore \boxed{\text{Median} = 41.90}$$

Hence from eqn. $\textcircled{1}$

$$\begin{aligned} \text{Mode} &= 3(\text{Median}) - 2(\text{Mean}) \\ &= 3(41.90) - 2(42.20) \\ &= 125.7 - 84.4 \\ &= 41.30. \end{aligned}$$

OP. by formula.

$$= l + h \cdot \frac{f_1 - f_0}{2f_1 - f_{0+} - f_{0-}}$$

$$= 38.$$



Partition Values :-

Just as a median divides the distribution into two equal parts; there are other positional measures which partition a series into still smaller parts say : 4, 10 or 100. The values which break the series into a number of equal parts are called the partition values.

Q1) Quartiles :-

The values which divide the given data into four equal parts are known as quartiles. Obviously there will be three such points Q_1, Q_2, Q_3 such that $Q_1 \leq Q_2 \leq Q_3$. Q_1 known as the lower or first quartile has 25% of the item of the distribution below it and 75% of the item are greater than it. Incidentally Q_2 ; the second quartile coincides with the median and has an equal number of observations above it and below it. Q_3 known as the upper or third quartile; has 75% of the observations below it and 25% of the observations above it.

→ The formula for computation are:

$$Q_1 = l + \frac{h}{f} \left[\frac{N}{4} - c \right]$$

Q₂

$$\text{Median} = l + \frac{h}{f} \left[\frac{N}{2} - c \right]$$

where;

l = lower limit of the class containing Q₁ or Q₃

f = The freq. of the class cont. Q₁ / Q₃.

h = The magnitude of the class cont. Q₁ / Q₃.

c = Cumulative freq. of the class preceding the class containing Q₁ / Q₃.

N/2 or 3N/2

b) Deciles :-

Deciles are those values of the variate which divide the series in to ten equal parts.

obviously there are nine deciles; D₁, D₂, D₃, ..., D₉ such that : D₁ ≤ D₂ ≤ D₃ ≤ ... ≤ D₉.

The fifth decile D₅ is the median.

→ The formula for find deciles are :

$$D_i = l + \frac{h}{f} \left[\frac{i \times N}{10} - c \right]$$

(i = 1, 2, 3, ..., 9)

where;

l = lower limit of the class containing D_i

f = frequency " " " "

h = magnitude " " " "

c = The cumulative frequency of the class preceding the class containing D_i

Q.C. Percentiles :-

Percentiles are those values of the variate which divides the total frequency in to 100 equal parts. Obviously there are 99 percentiles. The 50th percentile P_{50} is the median.

→ The formula for find percentiles are:

$$P_i = l + \frac{h}{f} \left[\frac{ixN}{100} - c \right]$$

($i = 1, 2, 3, \dots, 99$)

where the letters l, h, f, N, c have been used in the same sense in which they have been used in the formula for median.

Ex-16

Find the (mean), Median, Quartiles, 7th decile and 82nd percentile from the following distribution

wages in Rs.	0-10	10-20	20-30	30-40	40-50
No. of workers.	22	38	46	35	20

Sol :-

Class	Frequency (F)	c.f.	Mid point(x)	f.x
0-10	22	22	5	
10-20	38	60	15	
20-30	46	106	25	
30-40	35	141	35	
40-50	20	161	45	
	N = 161			

$$\text{1) Mean } \bar{x} = \frac{\sum f_i x_i}{N} = 24.56$$

2) Median : Here $\frac{N}{2} = 80.5$

∴ Median class is 20-30.

$$l = 20, h = 10, F = 46, C = 60.$$

$$\therefore \text{Median} = l + \frac{h}{F} \left[\frac{N}{2} - c \right]$$

$$= 20 + \frac{10}{46} [80.5 - 60]$$

$$= 20 + 4.46$$

$$= 24.46 \text{ Rs.}$$

Q1.: Here $N_4 = 40.25$

\therefore class of Q_1 is : 10-20.

$$\therefore Q_1 = l + \frac{h}{f} \left[\frac{N}{4} - c \right]$$

$$= 10 + \frac{10}{38} [40.25 - 22]$$

$$= 10 + 4.8$$

$$= 14.8 \text{ Rs.}$$

Q3.: Here $\frac{3N}{4} = 120.75$

\therefore class of Q_3 is : 30-40.

$$\therefore Q_3 = l + \frac{h}{f} \left[\frac{3N}{4} - c \right]$$

$$= 30 + \frac{10}{35} [120.75 - 106]$$

$$= 30 + 4.21$$

$$= 34.21 \text{ Rs.}$$

(5) calculate D_7 : $(D_2 = 16.92)$

Here $\frac{7N}{10} = 112.7$.

\therefore class of D_7 is : 30-40.

$$\therefore D_7 = l + \frac{h}{F} \left[\frac{7N}{10} - c \right]$$

$$= 30 + \frac{10}{35} [112.7 - 106]$$

$$= 30 + 1.91$$

$$= 31.91 \text{ Rs.}$$

(6) calculate P_{82} : $(P_{20} = 12.60)$

Here $\frac{82N}{100} = 132.02$.

\therefore class of P_{82} is : 30-40.

$$\therefore P_{82} = l + \frac{h}{F} \left[\frac{82N}{100} - c \right]$$

$$= 30 + \frac{10}{35} [132.02 - 106]$$

$$= 30 + 7.43$$

$$= 37.43 \text{ Rs.}$$