Decision Tree Examples

ID3

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

⊙ Values (outlook) = Sunny, overcast, Rain.

$$S \to 9+,\ 5- \qquad \text{Sunny} \to 2+,\ 3- \qquad \text{Overcast} \to 4+,\ 0- \qquad \text{Rain} \to 3+,\ 2-$$

$$\text{Entropy}(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = \underline{0.94}$$

$$\text{Entropy}(S=\text{sunny}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log\frac{3}{5} = 0.971$$

$\text{Entropy}(S=\text{overcast}) = 0$ (∵ all members belong to same class)

$$\text{Entropy}(S=\text{rain}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.971$$

$$\text{Gain}(S,\text{outlook}) = \text{Entropy}(S) - \sum_{v \in (\text{sunny, overcast, rain})} \frac{|S_v|}{|S|}\text{Entropy}(S_v)$$

$$= 0.94 - \frac{5}{14}(0.971) - \frac{4}{14}(0) - \frac{5}{14}(0.971)$$

$$= \underline{0.2464}$$

⊙ Values(Temp) = Hot, Mild, Cool

Hot: 2+ 2-   Mild: 4+ 2-   Cool: 3+ 1-

$$E(S=hot) = -\frac{2}{4}\log_2\frac{2}{4} - \frac{2}{4}\log_2\frac{2}{4} = 1$$

$$E(S=mild) = -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} = 0.9183$$

$$E(S=cool) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.8113$$

$$Gain(S, Temp) = Entropy(S) - \sum_{v \in \{Hot, mild, cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.94 - \frac{4}{14}(1) - \frac{6}{14}(0.9183) - \frac{4}{14}(0.8113)$$

$$= \underline{0.0289}$$

⊙ Values(Humidity) = High, Normal

High: 3+ 4-   Normal: 6+ 1-

$$E(S=high) = -\frac{3}{7}\log_2\frac{3}{7} - \frac{4}{7}\log_2\frac{4}{7} = 0.9852$$

$$E(S=normal) = -\frac{6}{7}\log_2\frac{6}{7} - \frac{1}{7}\log_2\frac{1}{7} = 0.5916$$

$$Gain(S, humidity) = Entropy(S) - \sum_{v \in (high, normal)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.94 - \frac{7}{14}(0.9852) - \frac{7}{14}(0.5916)$$

$$= \underline{0.1516}$$

⊙ Values (wind) = weak, strong

Weak → 6+, 2−

Strong → 3+, 3−

$E(S=\text{strong}) = 1$ (∵ all members are split equally)

$E(S=\text{weak}) = -\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8} = 0.8113$

$$\text{Gain}(S, \text{wind}) = \text{Entropy}(S) - \sum_{v \in (\text{weak, strong})} \frac{|S_v|}{|S|}\,\text{Entropy}(S_v)$$

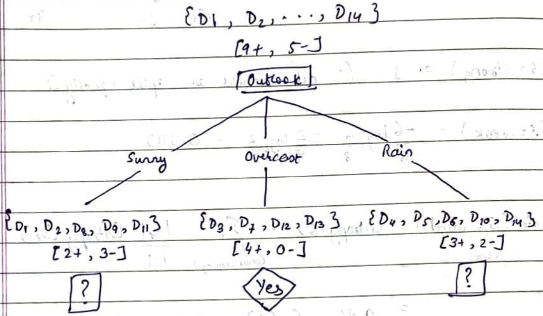
$$= 0.94 - \frac{6}{14}(1) - \frac{8}{14}(0.8113)$$

$$= \underline{0.0478}$$

ATTRIBUTE	GAIN
Outlook	0.2464
Temp	0.0289
Humidity	0.1516
Wind	0.0478

Now gain of outlook is highest. So we will partition on Outlook by making it as root node.

Tree after partition :-



For leftmost tree :-

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$$\text{Entropy (Sunny)} = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = \underline{\underline{0.97}}.$$

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⊙ Values(Temp) = Hot, Mild, Cool

Hot		Mild		Cool	
0+	2-	1+	1-	1+	0-

$E(S=hot) = 0$ (∵ ~~classes~~ members belongs to same class)
$E(S=mild) = 1$ (∵ all members are split equally)
$E(S=cool) = 0$ (∵ members belong to same class)

$$Gain(S_{sunny}, Temp) = Entropy(S) - \sum_{v \in (Hot, mild, cool)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.97 - \frac{2}{5}(0) - \frac{2}{5}(1) - \frac{1}{5}(0)$$

$$= \underline{0.57}$$

⊙ Values (Humidity) = High, normal

High		Normal	
0+	3-	2+	0-

$E(S=high) = 0$ (∵ members belong to same class)
$E(S=normal) = 0$ (" ")

$$Gain(S_{sunny}, Humidity) = Entropy(S) - \sum_{v \in (high, normal)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.97 - \frac{3}{5}(0) - \frac{2}{5}(0)$$

$$= \underline{0.97}$$

(6) Values (wind) = strong, weak

Strong: 1+ 1-

weak: 1+ 2-

$E(S = \text{strong}) = 1$ (∵ members are split equally)

$E(S = \text{weak}) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3} = 0.9183$

$$\text{Gain}(S_{sunny}, \text{wind}) = \text{Entropy}(S) - \sum_{v \in (\text{strong, weak})} \frac{|S_v|}{|S|}\text{Entropy}(S_v)$$

$$= 0.97 - \frac{2}{5}(1) - \frac{3}{5}(0.918)$$

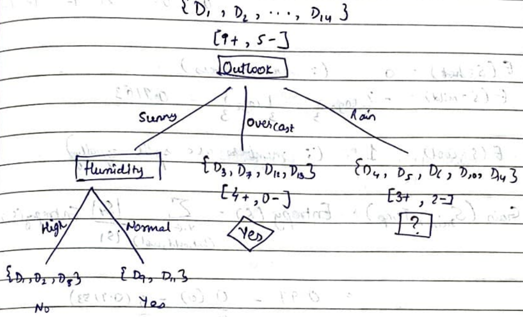
$$= \underline{0.0192}$$

For S_{sunny} :-

ATTRIBUTE	GAIN
Temp	0.57
Humidity	0.97
Wind	0.0192

Gain of humidity is highest. So split at humidity.

Resultant tree :-



For rightmost tree :-

Day	Temp	Humidity	Wind	Play Tennis
D_4	Mild	High	Weak	Yes
D_5	Cool	Normal	Weak	Yes
D_6	Cool	Normal	Strong	No
D_{10}	Mild	Normal	Weak	Yes
D_{14}	Mild	High	Strong	No

$$E(\text{Rain}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = \underline{\underline{0.97}}$$

⊙ Values(Temp) = Hot, Mild, Cool

Hot		Mild		Cool	
0+	0-	2+	1-	1+	1-

$E(S=hot) = 0$ (∵ zero members)

$E(S=mild) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.9183$

$E(S=cool) = 1$ (∵ members are split equally)

$$Gain(S_{rain}, Temp) = Entropy(S) - \sum_{v \in (hot, mild, cool)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.97 - \frac{0}{5}(0) - \frac{3}{5}(0.9183)$$

$$= \underline{0.0192}$$

⊙ Values(Humidity) = high, normal

High		Normal	
1+	1-	2+	1-

$E(S=high) = 1$ (∵ members are split equally)

$E(S=normal) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.9183$

$$Gain(S_{rain}, humidity) = Entropy(S) - \sum_{v \in (high, normal)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.97 - \frac{2}{5}(1) - \frac{3}{5}(0.918)$$

$$= \underline{0.0192}$$

⑥ Values(Wind) = strong, weak

Strong: 0+ 2-   Weak: 3+ 0-

E(S=strong) = 0   ($\because$ members belong to same class)
E(S = weak) = 0   ($\because$ members belong to same class)

$$Gain(S_{rain}, wind) = Entropy(S) - \sum_{v \in (strong, weak)} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$= 0.97 - \frac{2}{5}(0) - \frac{3}{5}(0)$$

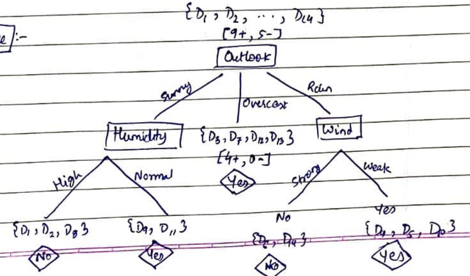
$$= \underline{0.97}$$

For S_{rain} :-

ATTRIBUTE	GAIN
Temp	0.0192
Humidity	0.0192
Wind	0.97

Gain of Wind is highest. Therefore partitioning at wind takes place.

Resultant tree :-
(FINAL)



$\boxed{C4.5}$

Same as ID3.

Just find gain ratio which is considered by :-

$$\text{Gain Ratio} = \frac{\text{Gain}(A)}{\text{SplitInfo}(A)}$$

$$= \frac{\text{Gain}(A)}{-\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)}$$

Consider an attribute as best attribute whose gain ratio is highest and make it root node of decision tree.

[CART]

Outlook	Temp	Humidity	Wind	Decision
Sunny	Hot	High	Weak	No
Sunny	Hot	High	Strong	No
Overcast	Hot	High	Weak	Yes
Rain	Mild	High	Weak	Yes
Rain	Cool	Normal	Weak	Yes
Rain	Cool	Normal	Strong	No
Overcast	Cool	Normal	Strong	Yes
Sunny	Mild	High	Weak	No
Sunny	Cool	Normal	Weak	Yes
Rain	Mild	Normal	Weak	Yes
Sunny	Mild	Normal	Strong	Yes
Overcast	Mild	High	Strong	Yes
Overcast	Hot	Normal	Weak	Yes
Rain	Mild	High	Strong	No

$$\text{Gini Index (Attribute = value)} = 1 - \sum_{i=1}^{N} (P_i)^2$$

$$\text{Gini Index (Attribute)} = \sum_{v=\text{values}} P_v \times GI(v)$$

(a) Values (outlook) = Sunny, overcast, rain

Sunny: 2+ 3−

Overcast: 4+ 0−

Rain: 3+ 2−

$$\text{Gini (outlook = sunny)} = 1-\left(\frac{2}{5}\right)^2-\left(\frac{3}{5}\right)^2 = 0.48$$

$$\text{Gini (outlook = overcast)} = 1-\left(\frac{4}{4}\right)^2-\left(\frac{0}{4}\right)^2 = 0$$

$$\text{Gini (outlook = rain)} = 1-(3/5)^2-(2/5)^2 = 0.48$$

Now calculate weighted sum of G.Is :-

$$\text{Gini (outlook)} = \frac{5}{14}(0.48) + \frac{4}{14}(0) + \frac{5}{14}(0.48)$$

$$= 0.171 + 0 + 0.171$$

$$= \underline{0.342}$$

Similarly, $$\text{Gini (Temperature)} = \frac{4}{14}(0.5) + \frac{4}{14}(0.375) + \frac{6}{14}(0.445)$$

$$= \underline{0.439}$$

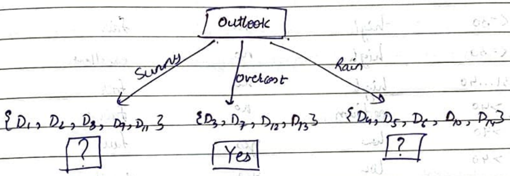
Similarly, $$\text{Gini (wind)} = \frac{8}{14}(0.375) + \frac{6}{14}(0.5) = \underline{0.428}$$

Similarly, $$\text{Gini (humidity)} = \frac{7}{14}(0.489) + \frac{7}{14}(0.244)$$

$$= \underline{0.367}$$

FEATURES	GINI INDEX
Outlook	0.342
Temperature	0.439
Humidity	0.367
Wind	0.428

Gini of outlook is lowest. Therefore we can branch at outlook.

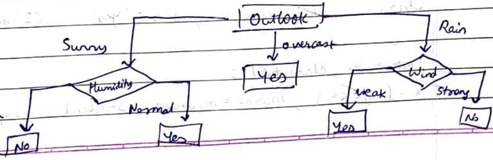


We will apply same principles to those subsets in following steps.
Focus on sub dataset for sunny outlook. We need to find G.I. scores for temperature, humidity and wind features respectively.

⋮

Tree is over for overcast outlook leaf as all answers result into Yes.

Final form =



NAIVE BAYES EXAMPLES

Gaussian Naive Bayes

⇒ CATEGORICAL

age	income	student	credit_rating	buys_comp
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

Class: C1: buys_computer = 'yes' C2: buys_computer = 'no'

Data to be classified:

X = (age <= 30, Income = medium, Student = yes, credit_rating = fair)

$P(C_i)$: $P(\text{buys\_computer} = \text{"yes"}) = 9/14 = 0.643$

$P(\text{buys\_computer} = \text{"no"}) = 5/14 = 0.357$

Compute $P(X|C_i)$ for each class

$P(\text{age} = \text{"<=30"} \mid \text{buys\_computer} = \text{"yes"}) = 2/9 = 0.222$

$P(\text{age} = \text{"<=30"} \mid \text{buys\_computer} = \text{"no"}) = 3/5 = 0.6$

$P(\text{income} = \text{"medium"} \mid \text{buys\_computer} = \text{"yes"}) = 4/9 = 0.444$

$P(\text{income} = \text{"medium"} \mid \text{buys\_computer} = \text{"no"}) = 2/5 = 0.4$

$P(\text{student} = \text{"yes"} \mid \text{buys\_computer} = \text{"yes"}) = 6/9 = 0.667$

$P(\text{student} = \text{"yes"} \mid \text{buys\_computer} = \text{"no"}) = 1/5 = 0.2$

$P(\text{credit\_rating} = \text{"fair"} \mid \text{buys\_computer} = \text{"yes"}) = 6/9 = 0.667$

$P(\text{credit\_rating} = \text{"fair"} \mid \text{buys\_computer} = \text{"no"}) = 2/5 = 0.4$

$X = (\text{age} <= 30, \text{income} = \text{medium}, \text{student} = \text{yes}, \text{credit\_rating} = \text{fair})$

$P(X/C_i)$: $P(X / \text{buys\_computer} = \text{"yes"}) = 0.222 \times 0.444 \times 0.667 \times 0.667$
$= 0.044$

$P(X \mid \text{buys\_computer} = \text{"no"}) = 0.6 \times 0.4 \times 0.2 \times 0.4$
$= 0.019$

$P(X/C_i) \times P(C_i)$: $P(X / \text{buys\_computer} = \text{"yes"}) \times P(\text{buys\_computer} = \text{"yes"})$
$= \boxed{0.028}$ ← greater

$P(X / \text{buys\_computer} = \text{"no"}) \times P(\text{buys\_computer} = \text{"no"}) = \boxed{0.007}$
↑
lesser

Therefore, X belongs to class ("buys_computer = yes")

⇒ CONTINUOUS

Gender	height (feet)	weight (lbs)	foot size (inches)
male	6	180	12
male	5.92	190	11
male	5.58	170	12
male	5.92	165	10
female	5	100	6
female	5.5	150	8
female	5.42	130	7
female	5.75	150	9

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{\Sigma x}{N} \qquad \sigma^2 = \frac{\Sigma (x-\mu)^2}{N}$$

Gender	mean (height)	variance (height)	mean (weight)	variance (weight)	mean (foot size)	variance (foot size)
male	5.855	3.5e-02	176.25	1.229e+02	11.25	9.1667e-01
female	5.4175	9.7e-02	132.5	5.583e+02	7.5	1.6667e+00

Instance to be classified :-

Gender	height (feet)	weight (lbs)	foot size (inches)
Sample	6	130	8

$P(\text{male}) = 0.5$

$$P(\text{height}/\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(6-\mu)^2}{2\sigma^2}\right) = 1.5789$$

$P(\text{weight} | \text{male}) = 5.9881 \times 10^{-6}$
$P(\text{foot size} | \text{male}) = 1.3112 \times 10^{-3}$

Posterior numerator (male) = their product $= 6.1984 \times 10^{-9}$

$P(\text{female}) = 0.5$
$P(\text{height} | \text{female}) = 2.2346 \times 10^{-1}$
$P(\text{weight} | \text{female}) = 1.6789 \times 10^{-2}$
$P(\text{foot size} | \text{female}) = 2.8669 \times 10^{-1}$

Posterior numerator (female) = their product $= \boxed{5.3778 \times 10^{-4}}$

greater

$\therefore$ Instance belongs to class = Female

Multinomial Naive Bayes

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors: $P(c) = \frac{3}{4}$ $P(j) = \frac{1}{4}$

Conditional Probabilities :

$P(\text{chinese} | c) = \frac{(5+1)}{(8+6)} = \frac{6}{14} = \frac{3}{7}$

$P(\text{Tokyo} | j) = \frac{(1+1)}{(3+6)} = \frac{2}{9}$

$P(\text{Tokyo} | c) = \frac{(0+1)}{(8+6)} = \frac{1}{14}$

$P(\text{Japan} | j) = \frac{(1+1)}{(3+6)} = \frac{2}{9}$

$P(\text{Japan} | c) = \frac{(0+1)}{(8+6)} = \frac{1}{14}$

$P(\text{chinese} | j) = \frac{(1+1)}{(3+6)} = \frac{2}{9}$

∴ Choosing a class :

$P(c | d_5) \propto \frac{3}{4} \times \left(\frac{3}{7}\right)^3 \times \frac{1}{14} \times \frac{1}{14} = \boxed{0.0003}$

↑ greater

$P(j | d_5) \propto \frac{1}{4} \times \left(\frac{2}{9}\right)^3 \times \frac{2}{9} \times \frac{2}{9} = 0.0001$

Therefore Test class belongs to $\boxed{\text{"c"}}$.

Multi-variate Bernoulli Naive Bayes

Consider set of documents, each of which is related either to Sports (S) or to Informatics (I). Given training set of 11 documents, we would like to estimate a Naive Bayes classifier, using the Bernoulli document model, to classify unlabelled documents as S or I.

We define vocabulary of eight words:

$$V = \begin{bmatrix} w_1 = \text{goal}, \\ w_2 = \text{tutor}, \\ w_3 = \text{variance}, \\ w_4 = \text{speed}, \\ w_5 = \text{drink}, \\ w_6 = \text{defence}, \\ w_7 = \text{performance}, \\ w_8 = \text{field} \end{bmatrix}$$

$$B^{sport} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$B^{Inf} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

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1. $b_1 = (1,0,0,1,1,1,0,1)^T$
2. $b_2 = (0,1,1,0,1,0,1,0)^T$

The total no. of documents in training set
$N = 11$; $N_S = 6$, $N_I = 5$

We estimate prior probabilities from training data as:

$$P(S) = \frac{6}{11} \; ; \; P(I) = \frac{5}{11}$$

Word counts in training data are:

$n_S(w_1) = 3$ $n_S(w_2) = 1$
$n_S(w_3) = 2$ $n_S(w_4) = 3$
$n_S(w_5) = 3$ $n_S(w_6) = 4$
$n_S(w_7) = 4$ $n_S(w_8) = 4$

$n_I(w_1) = 1$ $n_I(w_2) = 3$
$n_I(w_3) = 3$ $n_I(w_4) = 1$
$n_I(w_5) = 1$ $n_I(w_6) = 1$
$n_I(w_7) = 3$ $n_I(w_8) = 1$

We can estimate the word likelihoods by

$P(w_1|S) = 1/2$ 3/6 $P(w_2/S) = 1/6$
$P(w_3|S) = 1/3$ $P(w_4/S) = 1/2$
$P(w_5|S) = 1/2$ $P(w_6/S) = 2/3$
$P(w_7|S) = 2/3$ $P(w_8/S) = 2/3$

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And for class I:

$P(\omega_1/I) = 1/5$ $\qquad$ $P(\omega_2/I) = 3/5$

$P(\omega_3/I) = 3/5$ $\qquad$ $P(\omega_4/I) = 1/5$

$P(\omega_5/I) = 1/5$ $\qquad$ $P(\omega_6/I) = 1/5$

$P(\omega_7/I) = 3/5$ $\qquad$ $P(\omega_8/I) = 1/5$

1. $b_1 = (1, 0, 0, 1, 1, 1, 0, 1)^T$

$$P(S/b_1) \propto P(S) \prod_{l=1}^{8} [b_{1l}\, P(\omega_l/S) + (1-b_{1l})(1-P(\omega_l/S))]$$

$$\propto \frac{6}{11}\left(\frac{1}{2} \times \frac{5}{6} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3}\right)$$

$$= \frac{5}{891} = 5.6 \times 10^{-3}$$

$$P(I/b_1) \propto P(I) \prod_{l=1}^{8} [b_{1l}\, P(\omega_l/I) + (1-b_{1l})(1-P(\omega_l/I))]$$

$$\propto \frac{5}{11}\left(\frac{1}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{2}{5} \times \frac{1}{5}\right)$$

$$= \frac{8}{859375} = 9.3 \times 10^{-6}$$

Classify this document as S.

2. $b_2 = (0, 1, 1, 0, 1, 0, 1, 0)^T$

$$P(S|b_2) \propto P(S) \prod_{l=1}^{8} [b_{2l} P(\omega_l|S) + (1-b_{2l})(1-P(\omega_l|S))]$$

$$\propto \frac{6}{11}\left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}\right)$$

$$= \frac{12}{14256} = 8.4 \times 10^{-4}$$

$$P(I|b_2) \propto P(I) \prod_{l=1}^{8} [b_{2l} P(\omega_l|I) + (1-b_{2l})(1-P(\omega_l|I))]$$

$$\propto \frac{5}{11}\left(\frac{4}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{4}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{4}{5}\right)$$

$$= \frac{34560}{4296875}$$

$$= 8 \times 10^{-3}.$$

$\therefore$ Classify as I.