

For  $B_i$  at  $k^{th}$  level we have  $C_k$  number of nodes.

→ Binomial heaps is a collection of binary tree.

\* ⇒ Disjoint Set Data Structure.

3 operations performed by ~~that~~ Disjoint Set DS

- i) Make-Set
- ii) Find-Set
- iii) Union.

i) Make set - It will convert set elements into singleton set (For e.g.  $\{1, 2, 3\} \rightarrow \{1\}, \{2\}, \{3\}$   
For function call write.  
Make-Set(1) or Make-Set(2) or Make-Set(3)

ii) Find-Set :- It will return the representative element of the particular ~~set~~ element.

iii) Union :- It will be applicable only when Find-Set of 2 elements are different.

For e.g.  $X = \{a, b, c\}$  For  $X$ , let  $\{a\}$  is the representative  
 $Y = \{x, y, z\}$  For  $Y$ , let  $\{x\}$  " " " " " "

~~As X~~ For same set elements we cannot do union as ~~the~~ <sup>any</sup> particular set has only 1 representative.

Not possible → Union of (Find-Set(a), Find-Set(b))  
Possible → Union of (Find-Set(a), Find-Set(c))

~~##~~

⇒ Applications of Disjoint-Set DS

I) Finding connected components in a graph (undirected)

→ Connected components means between 2 vertices at least 1 path should be there.

→ For Algorithm we will go edge by edge in line number 3, 4, 5



→ ~~e~~ e & g are different so they can be Unioned

→ e, g are now joined consider any one as representative & they are joined with f.

II) See ppt

⇒ List of Paradigms

- 1) Backtracking
- 2) Branch & bound
- 3) Brute-force search
- 4) Divide & conquer
- 5) Dynamic programming
- 6) Greedy Algorithm
- 7) Prune & search.



## Non-Conflicting Activities

→ 2 activities are said to be non-conflicting if  $s_i \geq f_j$  or  $s_j \geq f_i$  where  $s_i$  &  $s_j$  denote the starting time of activities  $i$  &  $j$  respectively &  $f_i$  &  $f_j$  refers to the finishing time of the activities  $i$  &  $j$  respectively.

→ Time complexity will be  $N \log n$

eg.

	0	1	2	3
S	0	3	5	11
F	4	7	8	13

Activity 0, 2, 3 will be selected  
 For activity 1,  $F_0 > S_1$  ∴ Activity 1 will not be selected  
 For activity 2,  $F_0 < S_2$  ∴ 2 will be selected  
 For activity 3,  $F_2 < S_3$  ∴ 3 will be selected.

⇒ Job sequencing with deadline

We will arrange all the profits in non-increasing order.

→ Refer gatevidyalay website.

eg.

Jobs	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$	$J_6$
Deadlines	5	3	3	2	4	2
Profits	200	180	190	300	120	100

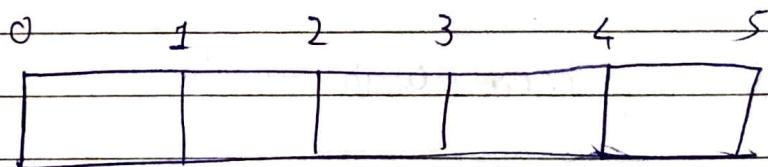
Sol<sup>n</sup>

Descending order of profit:

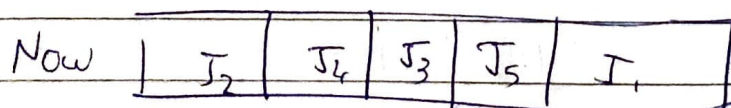
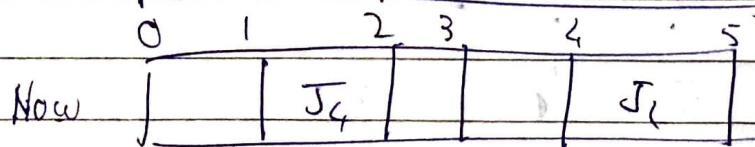
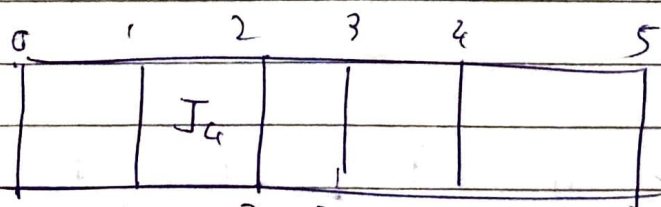
$J_4, J_1, J_3, J_2, J_5, J_6$

Jobs	Deadline	Profit
$J_4$	2	300
$J_1$	5	200
$J_3$	3	190
$J_2$	3	180
$J_5$	4	120
$J_6$	2	100

→ Now  $J_1$  has got more time i.e. 5 days or 5 slots comparatively to other jobs.



→ You have to keep your job initially as far as possible in deadline





# ⇒ Matrix chain Multiplication

ex:  $\langle P_0, P_1, P_2, P_3, P_4, P_5 \rangle$   
 $\langle 4, 10, 3, 12, 20, 7 \rangle$

$$n = 5$$

$$m[1, 5] = m[1, 1] + m[2, 5] + P_0 P_1 P_5 \quad - \quad (k=1) = 1350 + 280 = 1680$$

$$\begin{aligned} m[1, 5] &= m[1, 2] + m[3, 5] + P_0 P_2 P_5 \quad - \quad (k=2) = 120 + 1160 = 1280 \\ &= m[1, 3] + m[4, 5] + P_0 P_3 P_5 \quad - \quad k=3 = 264 + 1680 = 1944 \\ &= m[1, 4] + m[5, 5] + P_0 P_4 P_5 \quad - \quad k=4 = 1224 + 150 = 1374 \end{aligned}$$

$$\begin{aligned} m[2, 5] &= m[2, 2] + m[3, 5] + P_1 P_2 P_5 \quad (k=2) = 1140 + 210 = 1350 \\ &= m[2, 3] + m[4, 5] + P_1 P_3 P_5 \quad k=3 = 360 + 1680 = 2040 \\ &= m[2, 4] + m[5, 5] + P_1 P_4 P_5 \quad k=4 = 1320 + 1400 = 2720 \end{aligned}$$

$$\begin{aligned} m[3, 5] &= m[3, 3] + m[4, 5] + P_2 P_3 P_5 = 1680 + 252 = 1932 \\ &= m[3, 4] + m[5, 5] + P_2 P_4 P_5 \quad (k=4) = 720 + 420 = 1140 \end{aligned}$$

$$m[4, 5] = P_3 P_4 P_5 = 1680$$

$$m[3, 4] = P_2 P_3 P_4 = 720$$

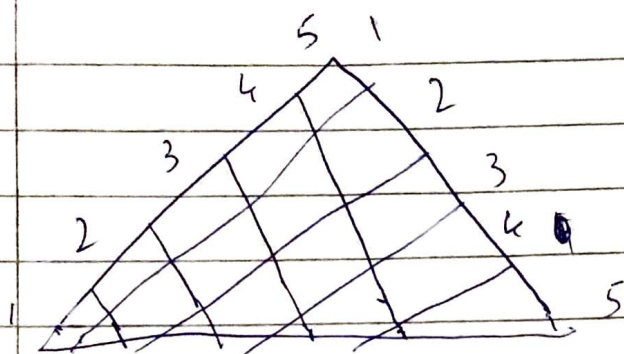
$$m[2, 3] = P_1 P_2 P_3 = 360$$

$$m[1, 2] = P_0 P_1 P_2 = 120$$

$$\begin{aligned} m[1, 3] &= m[1, 1] + m[2, 3] + P_0 P_1 P_3 \quad \text{save the value} \quad 480 + 360 = 840 \\ &= m[1, 2] + m[3, 3] + P_0 P_2 P_3 \quad (k=2) = 120 + 144 = 264 \end{aligned}$$

$$\begin{aligned} m[1, 4] &= m[1, 1] + m[2, 4] + P_0 P_1 P_4 = 1320 + 800 = 2120 \\ &= m[1, 2] + m[3, 4] + P_0 P_2 P_4 \quad (k=3) = 120 + 720 + 240 = 1080 \\ &= m[1, 3] + m[4, 4] + P_0 P_3 P_4 = 264 + 960 = 1224 \end{aligned}$$

$$\begin{aligned} m[2, 4] &= m[2, 2] + m[3, 4] + P_1 P_2 P_4 = 1140 + 600 = 1740 \\ &= m[2, 3] + m[4, 4] + P_1 P_3 P_4 = 360 + 2400 = 2760 \end{aligned}$$



$$k=3 \quad s[1,6] \rightarrow s[1,3] \rightarrow s[1,7]$$

$$A_1 A_2 A_3 A_4 A_5 A_6$$

$m[1,1]$	$m[2,2]$	$m[3,3]$	$m[4,4]$	$m[5,5]$
$m[1,2]$	$m[2,3]$	$m[3,4]$	$m[4,5]$	
$m[1,3]$	$m[2,4]$	$m[3,5]$		
$m[1,4]$	$m[2,5]$			
$m[1,5]$				

For right side  $k=5$   
 Now  $s[4,6]$   
 $\rightarrow A_1 A_2 A_3 A_4 A_5 A_6$

Now  $k=5$   
 Now  $s[5,6]$   
 $\rightarrow (A_1)(A_2 A_3)(A_4 A_5) A_6$

So for left side we have to decrement  $J$   
 & for right side we have to increment  $I$



# ⇒ Longest Common Subsequence

A subseq of a given sequence is just a sequence with one or more char left out.

eg:- X:- BDCABA  
Y :- ABCBDAB

BCBA  
BCAB  
BCBA

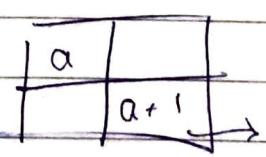
As here string is matched we have put 1.

Initially add extra rows of 0.

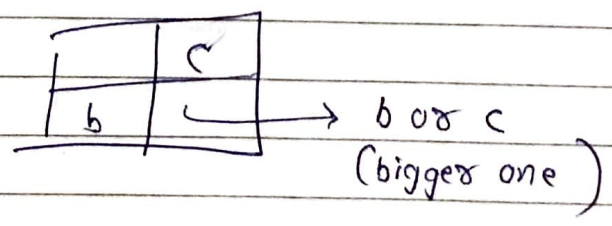
	B	D	C	A	B	A
A	0	0	0	0	1	1
B	0	1	1	1	2	2
C	0	1	1	2	2	2
B	0	1	1	2	2	3
D	0	1	2	2	2	3
A	0	1	2	2	3	4
B	0	1	2	2	3	4

BCBA

For match



For no match



For moving

either left or top. and write answer from right to left

dx

 $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$  $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$ 

0	1	0	0	1	0	1	0	0
0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1
1	0	1	1	2	2	2	2	2
0	0	1	2	2	3	3	3	3
1	0	1	2	3	3	4	4	4
1	0	1	2	3	3	4	4	4
0	0	1	2	3	4	4	5	5
1	0	1	<del>1</del>	3	4	5	5	5
1	0	1	<del>2</del>	3	4	5	5	5
0	0	1	2	3	4	5	6	6