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Aayush Shah Tut-3.

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1.) $T(n) - 4T(n-1) + 4T(n-2) = 0$

Acc to the form $a_0 t_n + a_1 t_{n-1} + \ldots + a_k t_{n-k} = 0$.

$a_0 = 1,\ a_1 = -4,\ a_2 = a_k = 4,\ k = 2.$

$\therefore x^2 - 4x + 4 = (x-2)^2 = 0 \qquad x = 2, 2$

roots : $r = 2,\ m = 2.$

$t_n = c_1 2^n + c_2 n 2^n$

when $n = 0,\ c_1 = t_0$

$\therefore t_n = t_0 2^n + c_2 n 2^n$

2. $T(n) - 5T(n-1) + 6T(n-2) = 0$

$a_0 = 1,\ a_1 = -5,\ a_2 = 6 = a_k,\ a = 2.$

$\therefore x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

Roots $r_1 = 2,\ m = 2\ \&\ r_2 = 3,\ m = 2$

$\therefore t_n = c_1 2^n + c_2 3^n$

3. $T(n) + 5T(n-1) + 6T(n-2) = 3n^2 - 2n + 1$

$a_0 = 1 \quad a_1 = 5 \quad a_2 = a_k = 6 \quad k = 2.$

$b = 1 \quad d = 2 \quad P(n) = 3n^2 - 2n + 1$

$\therefore (x^2 + 5x + 6)(x-1)^2 = 0$

$(x+2)(x+3)(x-1)^3 = 0$

roots, $r_1 = -2,\ m = 1,\ r_2 = 3,\ m = 1,\ r_3 = 1,\ m = 3$

$\therefore t_n = c_1(-2)^n + c_2(-3)^n + c_3 + c_4 n + c_5 n^2$

③ 1.) $T(n) = 8T(n/2) + 1000n^2$

$a = 8 \quad b = 2 \quad f(n) = 1000n^2$

$n^{\log_b a} = n^{\log_2 8} = n^3$

Here, $f(n) = O(n^{\log_b a - \varepsilon})$, where $\varepsilon = 1$

$\therefore T(n) = \Theta(n^3)$

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$\therefore T(n) = \Theta(n^3)$

2) $T(n) = 2T(n/2) + 10n$

$a = 2 \quad b = 2 \quad f(n) = 10n$

$n^{\log_b a} = n^{\log_2 2} = n$

Here, $f(n) = \Theta(n)$

$\therefore T(n) = \Theta(n \log n)$

3.) $T(n) = 2T(n/2) + n^2$

$a = 2 \quad b = 2 \quad f(n) = n^2$

$n^{\log_b a} = n$

Here, $f(n) = \Omega(n^{\log_b a + \varepsilon})$ where $\varepsilon = 1$

$\therefore T(n) = \Theta(n^2)$

4.) $T(n) = 3T(n/3) + n\log n$

$a = 3, \ b = 3 \ ; \quad f(n) = n\log n$

$n^{\log_a b} = n$

Here. $f(n)$ is asymptotically larger than n but not polynomially larger.

$\therefore$ Master method doesn't apply to this recurrence.

5.) $T(n) = 9T(n/3) + n$

$a = 9 \quad b = 3 \quad f(n) = n$

$n^{\log_a b} = n^2$

Here $f(n) = O(n^{\log_a b - \varepsilon})$ where $\varepsilon = 1$

$\therefore T(n) = \Theta(n^2)$

6) $T(n) = 27T(n/3) + n^3$

$a = 27 \quad b = 3 \quad f(n) = n^3$

Here $f(n) = \Theta(n^3)$

$\therefore T(n) = \Theta(n^3 \log n)$

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7.) $T(n) = 8T(n/2) + n^3 \log n$.

$a = 8 \quad b = 2 \quad f(n) = n^3 \log n$.

$n^{\log_b a} = n^3$.

Here $\frac{f(n)}{n^3} = \frac{1}{\log n}$.

$\therefore$ Master method doesn't apply.

8.) $T(n) = 2T(n/2) + n \log n$

$a = 2 \quad b = 2 \quad f(n) = n \log n$.

$n^{\log_a b} = n$

Here $\frac{f(n)}{n} = \frac{1}{\log n}$

Master method doesn't apply.

9.) $T(n) = 0.5\, T(n/2) + 1/n$

$a = 1/2 \quad b = 2 \quad f(n) = 1/n$

$n^{\log_a b} = n^{\log_2 1/2} = -1$

Here $f(n) = \theta(1/n)$

$T(n) = \theta(\log n / n)$.

10.) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1 \quad b = 2 \quad f(n) = n(1 - \cos n)$

$n^{\log_a b} = n^0 = 1$

Here, $f(n) = \Omega(n^{\log_a b + \varepsilon})$, $\varepsilon = 1$

$\therefore f(n/b) \leq c f(n)$

$\frac{n}{2}(2 - \cos n/2) \leq c(\cos n)$

let $n = 360$

$\frac{1}{2}(2+1) \leq (2-1)$

$3/2 \leq c$; not possible.

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6.) $T(n) = nT^2(n/2)$

let $T(2^i) = 2^i T^2(n/2 \; 2^{i-1})$

$t_i = 2^i t_{i-1}^2$

let $u_i = \log t_i = \log(2^i t_{i-1}^2)$

$u_i = i + 2\log t_{i-1}$

$u_i = i + 2 u_{i-1}$

$\therefore u_i - 2u_{i-1} = i$

$\therefore a_0 = 1$; $a_2 = a_x = 2$; $k=1$ $b=1$ $d=1$

$\therefore (x-2)(x-1)^2 = 0$

Roots, $r_1 = 2$ $m_1 = 1$ $r_2 = 1$ $m_2 = 2$

Here. $u_i = C_1 2^i + C_2 + iC_3$

Here, $u_i - 2u_{i-1} = i$

$(C_1 2^i + C_2 + iC_3) - 2(C_1 2^{i-1} + C_2 + (i-1)C_3) = i$

$\therefore C_2 - 2C_2 + iC_3 - 2iC_3 - 2C_3 = 1$

$\therefore C_3 = -1$, $C_2 = -2$

$u_i = C_1 2^i - 2 - i$

Now, $t_i = 2^{u_i}$

$T(n) = t_{\log t_i} = 2^{C_1 i - \log i - 2}$

$T(n) = \dfrac{2^{C_1 n}}{4n}$

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Now, $T(1) = 1/3 = \frac{2^{c_1}}{4}$

$2^{c_1} = \frac{4}{3}$

$c_1 = \log 4/3$

$c_1 = 2 - \log 3$

$c_1 = 2 - \log 3$

$\therefore T(n) = \frac{2^{2n - n\log 3}}{4n}$

$\therefore T(n) = \frac{2^{2n}}{4n \cdot 3^n}$