

GOAL :- Linear SVM classifies two classes.

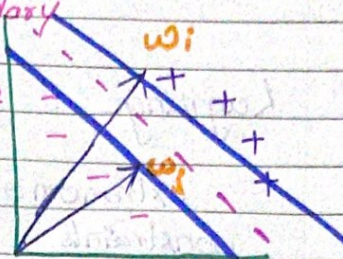
DATE

SVM

Hyperplane :- Decision boundary

Support Vector :- Data points nearest to Hyperplane

① Divide the negative and positive samples.



② Draw a vector w perpendicular to the median

+ve Sample

$$1 - (d + xw) \cdot \bar{w} \cdot \bar{u} \geq 0 \quad \begin{matrix} w = \text{weight} \\ u = \text{unknown sample} \\ c = \text{constant } (b) \end{matrix}$$

Decision

RULE

$$1. \bar{w} \cdot x_+ + b > 0$$

$$\bar{w} \cdot x + b > 1 \quad (\text{+ve Sample})$$

$$\bar{w} \cdot x_- + b \leq -1 \quad (\text{-ve Sample})$$

$$y_i (w \cdot x_i + b) > 1$$

$$y_i (w \cdot x_i + b) \geq 1$$

$y_i = \text{Variable}$

we multiplied -ve sample by -1 so the both the resultant equations are same.

$$2. y_i (x_i \cdot w + b) - 1 \geq 0$$

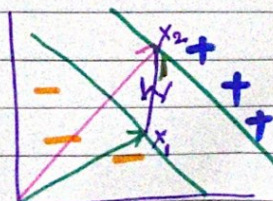
③ Maximum Margin

Width of the street

$$W = (x_+ - x_-) \times \frac{w}{\|w\|}$$

$$(1-b) \quad (1+b)$$

$$= \frac{2}{\|w\|} \quad \text{+ve sample}$$



$$\text{Max} = \frac{1}{2} \text{MIN} = \frac{1}{2} \|W\|^2$$

KKT

Lagrange :- if we are finding a extremum of functions with constraints, our new expression without thinking of constraints.

$$L = \frac{1}{2} \|W\|^2 - \sum d_i [y_i (w x_i + b) - 1]$$

= To find maximum, we find derivatives and set them to zero

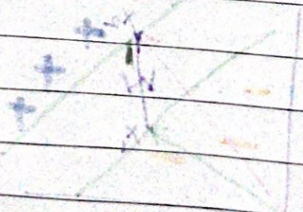
$$\frac{dL}{dW} = W - \sum d_i y_i x_i = 0$$

$$\Rightarrow W = \sum d_i y_i x_i$$

$$\frac{dL}{db} = - \sum d_i y_i = 0$$

$$\frac{W \cdot x}{\|W\|} = W$$

$$\frac{(d+1)}{(d-1)} = \frac{1}{\|W\|}$$



Substitute for w

$$L = \frac{1}{2} \left(\sum \alpha_i y_i x_i \right) \left(\sum \alpha_j y_j x_j \right)$$

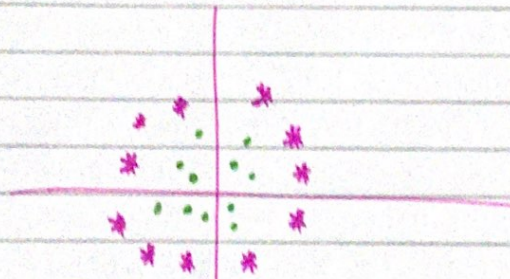
$$- \sum \alpha_i y_i x_i \left(\sum \alpha_j y_j x_j \right)$$

$$- \sum \alpha_i y_i b$$

→ constraint is zero.

$$L = \sum \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j x_i x_j$$

HARD MARGIN: Linear Separate, 100% classification
SOFT MARGIN: only Support Vector not outlier



To project data in higher dimension

$$Z = x^2 + y^2$$

mapping Z using x & $y \rightarrow$ Kernel Transformation