Branch and Bound

Introduction: Branch and Bound is an algorithm design paradigm which is extensively used for solving "Combinatorial Optimization problems".

→ **Combinatorial optimization** :- It is a subfield of mathematical optimization. It is related to :

1) Operations Research
2) Algorithm theory
3) Computational complexity theory.

- It is a method of finding an optimal object (solution) from a finite set of objects (solutions).
- In many combinatorial optimization problems, the exhaustive search (brute-force) is not tractable.
- Typical combinatorial optization problems are :- TSP, MST and the Knapsack problem.
- Applications of Combinatorial optimization :

1) Artificial Intelligence 2) Machine Learning
3) Software Engineering 4) Theoretical Computer Science.

- Formally, a combinatorial optimization problem A is a quadruple (I, f, m, g), where

  $I \rightarrow$ set of instances.

  $f \rightarrow$ given an instance $x \in I$, $f(x)$ is the finite set of feasible solutions.

  $m \rightarrow$ given an instance x and a feasible solution y of x, $m(x, y)$ denotes the measure of y. (usually a positive real number).

  $g \rightarrow$ the goal function (either min or max)

The goal is to find for some instance x an optimal solution, that is, a feasible solution y with

$$m(x, y) = g\{m(x, y') \mid y' \in f(x)\}.$$

For each combinatorial optimization problem, there is a corresponding decision problem that asks whether there is a feasible solution for some particular measure m_0.

For eg:- if there is a graph G which contains vertices u and v, an optimization problem might be "Find a path from u to v that uses the fewest edges." This problem might have an answer (say 4). The corresponding decision problem would be "Is there a path from u to v that uses 10 or fewer edges?" This problem can be answered with a simple "Yes" or "No".

- Combinatorial optimization problems are typically exponential in terms of time complexity and may require exploring all possible permutations in the worst case.

The Branch and Bound algorithm design paradigm (technique) solves these problems relatively quickly.

Principle: The Branch and Bound approach is based on the principle that the total set of feasible solutions can be partitioned into

smaller subsets of solutions. These smaller subsets can then be evaluated systematically until the best solution is found.

The goal of a branch and bound algorithm is to find a value x that maximizes or minimizes the value of a real-valued function $f(x)$, called an objective/goal function, among some set S of admissible/feasible/candidate solutions. The set S is known as the "search space" or "feasible region".