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- In some applications, it doesn't matter which member is used as the representative; we care only that if we ask for the representative of a dynamic set twice without modifying the set between the requests, we get the same answer both times.

- Other applications may require a prespecified rule for choosing the representative, such as choosing the smallest member in the set (assuming, of course, that the elements can be ordered).

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    - We assume that the two sets are disjoint prior to the operation.
    - The representative of the resulting set is any member of Sx U Sy, although many implementations of UNION specifically choose the representative of either Sx or Sy as the new representative.

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    - In practice, we often absorb the elements of one of the sets into the other set.

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- Letting x denote an object, we want to support the following operations:
  - FIND-SET(x)
    - returns a pointer to the representative of the (unique) set containing x.

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  - m >= n (since the MAKE-SET operations are included in the total number of operations m)
  - It is assumed that n MAKE-SET operations are the first n operations performed.

Data Structures for Disjoint Set

- An Application of Disjoint-Set Data Structures & its Operations
  - Determining the connected components (a connected component of an undirected graph is a subgraph in which each pair of nodes is connected with each other via a path.) of undirected graph

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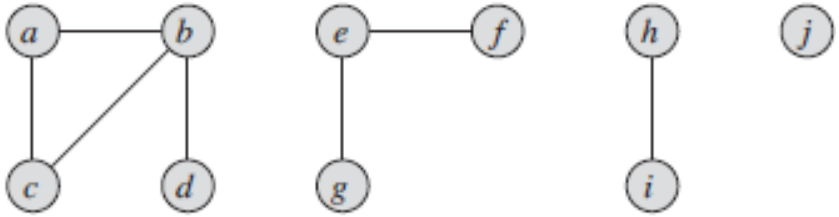
    - Following figure shows a graph with four connected components.



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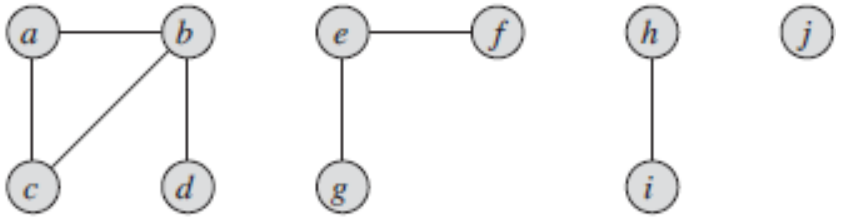

    - The procedure CONNECTED-COMPONENTS uses the disjoint-set operations to compute the connected components of a graph. (See Table)

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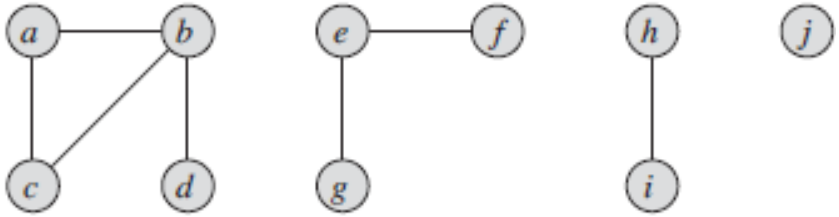


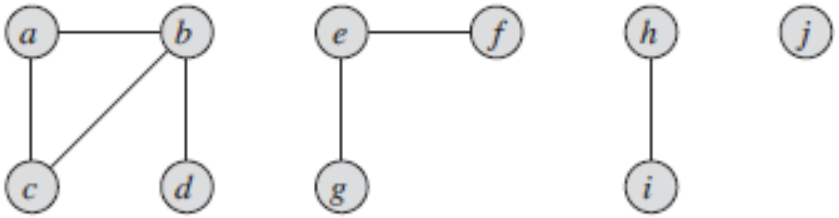
    - The procedure CONNECTED-COMPONENTS uses the disjoint-set operations to compute the connected components of a graph. (See Table)
    - Once CONNECTED-COMPONENTS has pre-processed the graph, the procedure SAME-COMPONENT answers queries about whether two vertices are in the same connected component.

Data Structures for Disjoint Set

- An Application of Disjoint-Set Data Structures & its Operations
  - Determining the connected components of undirected graph

```
CONNECTED-COMPONENTS(G)
1  for each vertex v ∈ G.V
2      MAKE-SET(v)
3  for each edge (u, v) ∈ G.E
4      if FIND-SET(u) ≠ FIND-SET(v)
5          UNION(u, v)
```

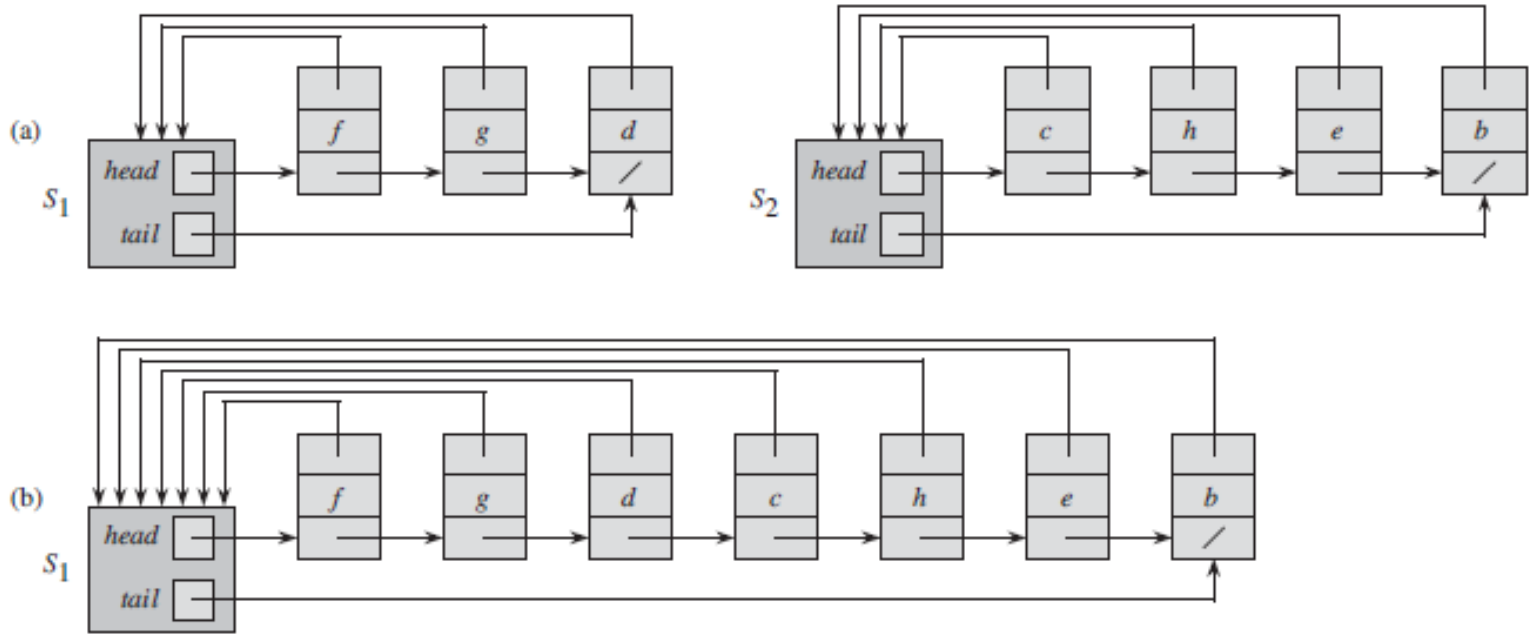
```
SAME-COMPONENT(u, v)
1  if FIND-SET(u) == FIND-SET(v)
2      return TRUE
3  else return FALSE
```



Edge processed	Collection of disjoint sets									
initial sets	{a}	{b}	{c}	{d}	{e}	{f}	{g}	{h}	{i}	{j}
(b,d)	{a}	{b,d}	{c}		{e}	{f}	{g}	{h}	{i}	{j}
(e,g)	{a}	{b,d}	{c}		{e,g}	{f}		{h}	{i}	{j}
(a,c)	{a,c}	{b,d}			{e,g}	{f}		{h}	{i}	{j}
(h,i)	{a,c}	{b,d}			{e,g}	{f}		{h,i}		{j}
(a,b)	{a,b,c,d}				{e,g}	{f}		{h,i}		{j}
(e,f)	{a,b,c,d}				{e,f,g}			{h,i}		{j}
(b,c)	{a,b,c,d}				{e,f,g}			{h,i}		{j}

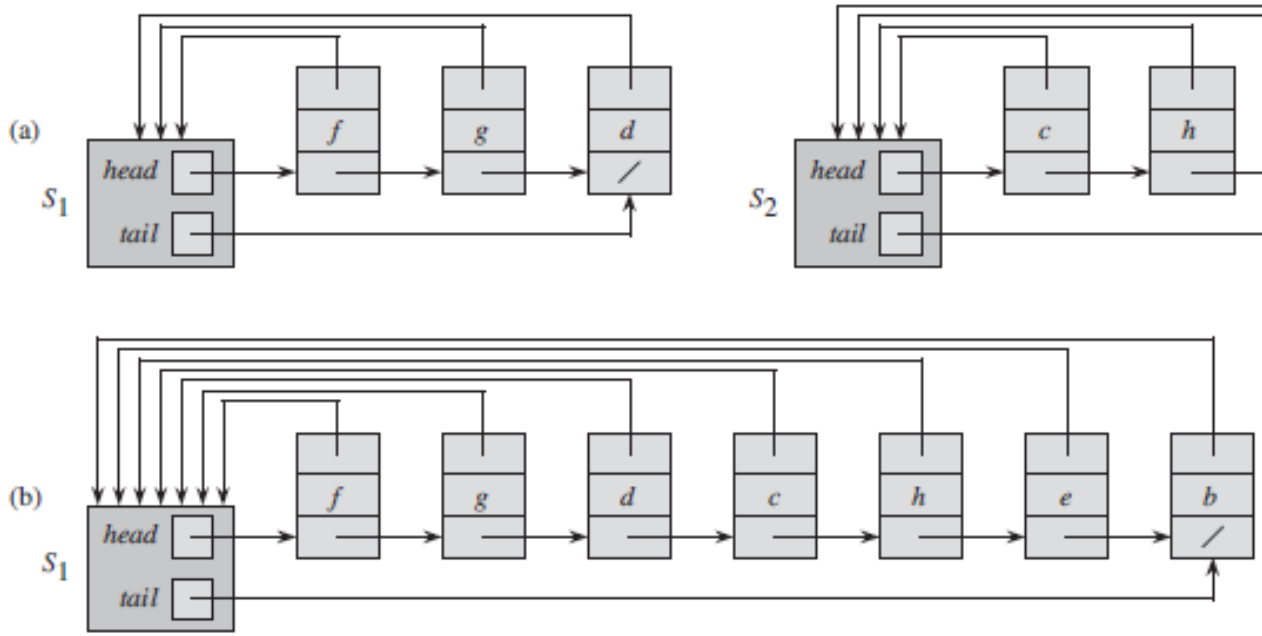
Data Structures for Disjoint Set

- Linked List Representation of Disjoint Sets
  - Figure shows a simple way to implement a disjoint-set data structure: **each set is represented by its own linked list**.
  - The object for each set has attributes head, pointing to the first object in the list, and tail, pointing to the last object.



Data Structures for Disjoint Set

- Linked List Representation of Disjoint Sets
  - Each object in the list contains a set member, a pointer to the next object in the list, and a pointer back to the set object.
  - Within each linked list, the objects may appear in any order.
  - The representative is the set member in the first object in the list.



Data Structures for Disjoint Set

- Linked List Representation of Disjoint Sets (Look at Union Later)

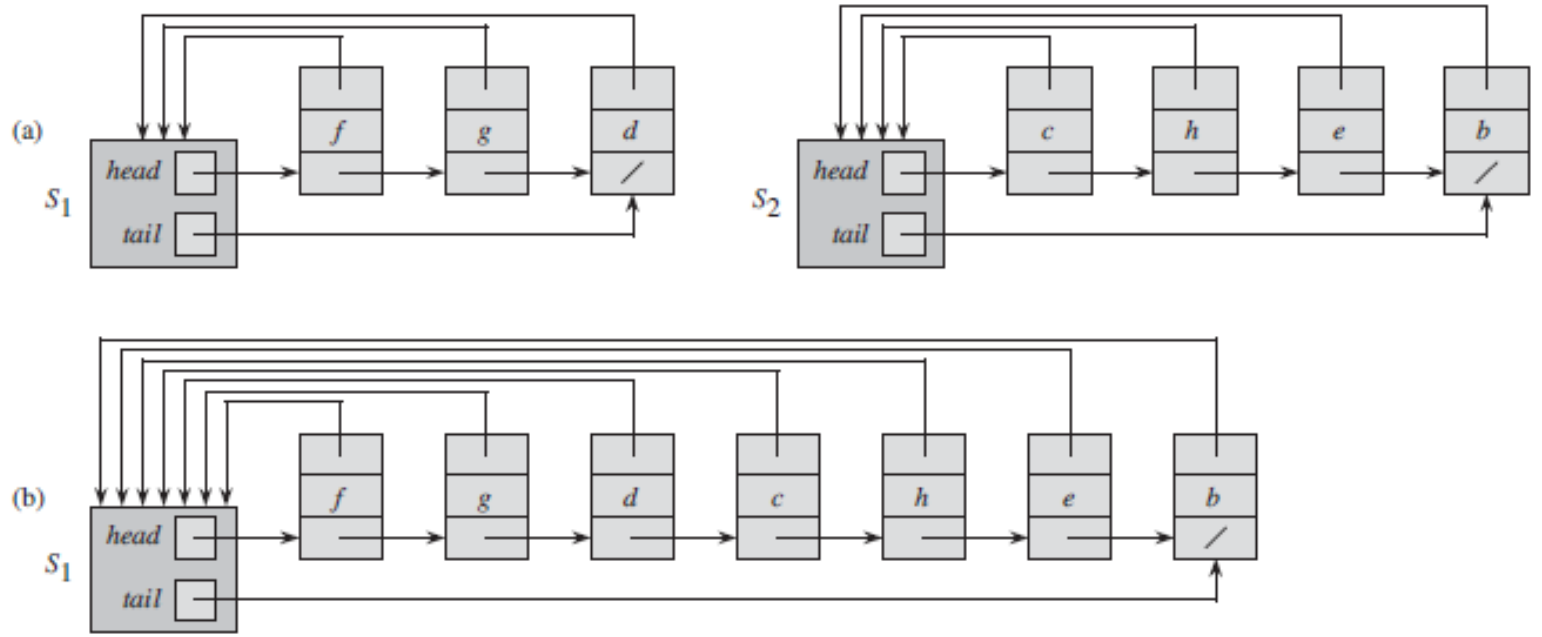


Figure 21.2 (a) Linked-list representations of two sets. Set S_1 contains members d, f, and g, with representative f, and set S_2 contains members b, c, e, and h, with representative c. Each object in the list contains a set member, a pointer to the next object in the list, and a pointer back to the set object. Each set object has pointers *head* and *tail* to the first and last objects, respectively. (b) The result of UNION(g, e), which appends the linked list containing e to the linked list containing g. The representative of the resulting set is f. The set object for e's list, S_2, is destroyed.

Data Structures for Disjoint Set

- Linked List Representation of Disjoint Sets
  - With this linked-list representation, both MAKE-SET and FIND-SET are easy, requiring O(1) time.

  - To carry out MAKE-SET(x), we create a new linked list whose only object is x. (Here x is a value of the object for which a node is to be created. This procedure creates a set object and a node with x as a value and appropriate pointers. It also takes care of head & tail pointer)

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  - For FIND-SET(x), we just follow the pointer from x back to its set object and then return the member in the object that head points to. (Here, x is a pointer referring to the node containing x)
  - For example, in Figure on last slide, the call FIND-SET(g) would return f.

Data Structures for Disjoint Set

- Linked List Representation of Disjoint Sets
  - A simple implementation of union
    - The simplest implementation of the UNION operation using the linked-list set representation takes significantly more time than MAKE-SET or FIND-SET.

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    - Because all members of y's list join x's list, we can destroy the set object for y's list. **(Note: Just set object is destroyed, other nodes will remain in their initial memory location)**
    - Unfortunately, we must update the pointer to the set object for each object originally on y's list, which takes time linear in the length of y's list.

Data Structures for Disjoint Set

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  - A simple implementation of union
    - In Figure 21.2, for example, the operation UNION(g, e) causes pointers to be updated in the objects for b, c, e, and h.

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    - Suppose that we have objects $x_1, x_2, \ldots, x_n$.
    - Assume that we have executed the sequence of n MAKE-SET operations to create these objects.
    - Now let us assume that these MAKE-SET operations are followed by n - 1 UNION operations as shown in following figure, so that m = 2n - 1.

Operation	Number of objects updated
MAKE-SET(x_1)	1
MAKE-SET(x_2)	1
⋮	⋮
MAKE-SET(x_n)	1
UNION(x_2, x_1)	1
UNION(x_3, x_2)	2
UNION(x_4, x_3)	3
⋮	⋮
UNION(x_n, x_{n-1})	$n-1$

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$$\sum_{i=1}^{n-1} i = \Theta(n^2)$$

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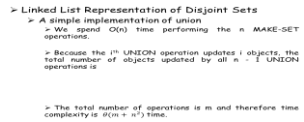

    - The total number of operations is m and therefore time complexity is $\theta(m + n^2)$ time.

Data Structures for Disjoint Set

- Linked List Representation of Disjoint Sets
  - A weighted-union heuristic
    - Through this heuristic, we will try to achieve a tighter upper bound on a sequence of m MAKE-SET, UNION and FIND-SET operations.

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    - Consider a particular object x
    - We know that each time x's pointer was updated, x must have started in the smaller set.
    - The first time x's pointer was updated, therefore, the resulting set must have had at least 2 members.

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    - Similarly, the next time x's pointer was updated, the resulting set must have had at least 4 members.

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    - Continuing on, we observe that for any $k \leq n$, after x's pointer has been updated $\lceil \lg k \rceil$ times, the resulting set must have at least k members.

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    - Since the largest resulting set can have at most n members, **each** object's pointer is updated at most $\lceil \lg n \rceil$ times over all the UNION operations.

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    - Since the largest resulting set can have at most n members, **each** object's pointer is updated at most $\lceil \lg n \rceil$ times over all the UNION operations.
    - Thus the total time spent updating object pointers over all UNION operations is O(n lg n) (as there are n objects).

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    - We must also account for updating the tail pointers and the list lengths, which take only $\theta(1)$ time per UNION operation.

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    - The time for the entire sequence of m operations follows easily. Each MAKESET and FIND-SET operation takes O(1) time, and there are O(m) of them.

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    - The total time spent in all UNION operations is thus $O(n \lg n)$.
    - The time for the entire sequence of m operations follows easily. Each MAKESET and FIND-SET operation takes $O(1)$ time, and there are $O(m)$ of them.
    - The total time for the entire sequence is thus $O(m + n \lg n)$

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - In a faster implementation of disjoint sets, we represent sets by rooted trees, with each node containing one member and each tree representing one set.

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - In a faster implementation of disjoint sets, we represent sets by rooted trees, with each node containing one member and each tree representing one set.
  - In a disjoint-set forest, illustrated in Figure 21.4(a), each member points only to its parent.

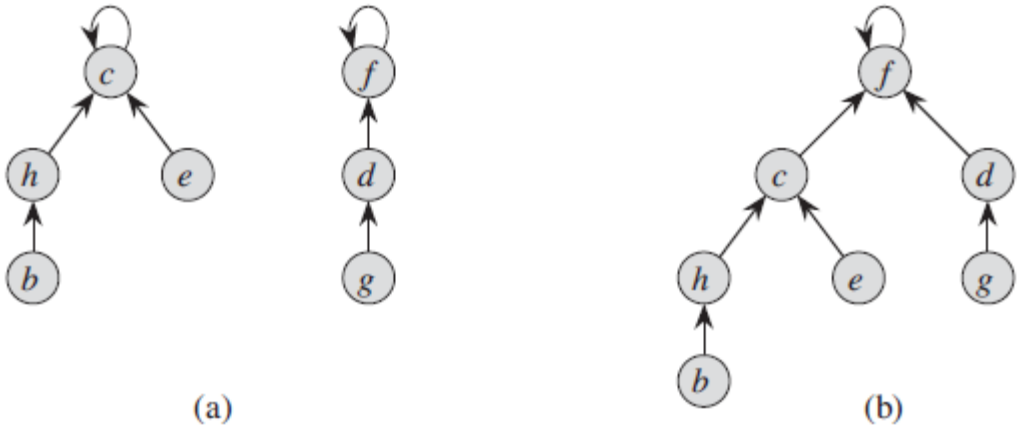

Figure 21.4 A disjoint-set forest. **(a)** Two trees representing the two sets of Figure 21.2. The tree on the left represents the set $\{b, c, e, h\}$, with c as the representative, and the tree on the right represents the set $\{d, f, g\}$, with f as the representative. **(b)** The result of UNION(e, g).

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - The root of each tree contains the representative and is its own parent.
  - As we shall see, although the straightforward algorithms that use this representation are no faster than ones that use the linked-list representation, by introducing two heuristics—"union by rank" and "path compression"—we can achieve an asymptotically optimal disjoint-set data structure.

Data Structures for Disjoint Set

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  - We perform the three disjoint-set operations as follows.

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  - The nodes visited on this simple path toward the root constitute the find path.
  - A UNION operation, shown in Figure 21.4(b), causes the root of one tree to point to the root of the other. (depends on find set)

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    - So far, we have not improved on the linked-list implementation.

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      - Idea is to make the root of the tree with less height point to the root of the tree with more height.
      - For each node, we maintain a rank, which is an upper bound on the height of the node.
      - In union by rank, we make the root with smaller rank point to the root with larger rank during a UNION operation.

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - Heuristics to improve the running time
    - Path Compression
      - We use it during FIND-SET operations to make each node on the find path point directly to the root. We do not change any ranks.

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - Path Compression

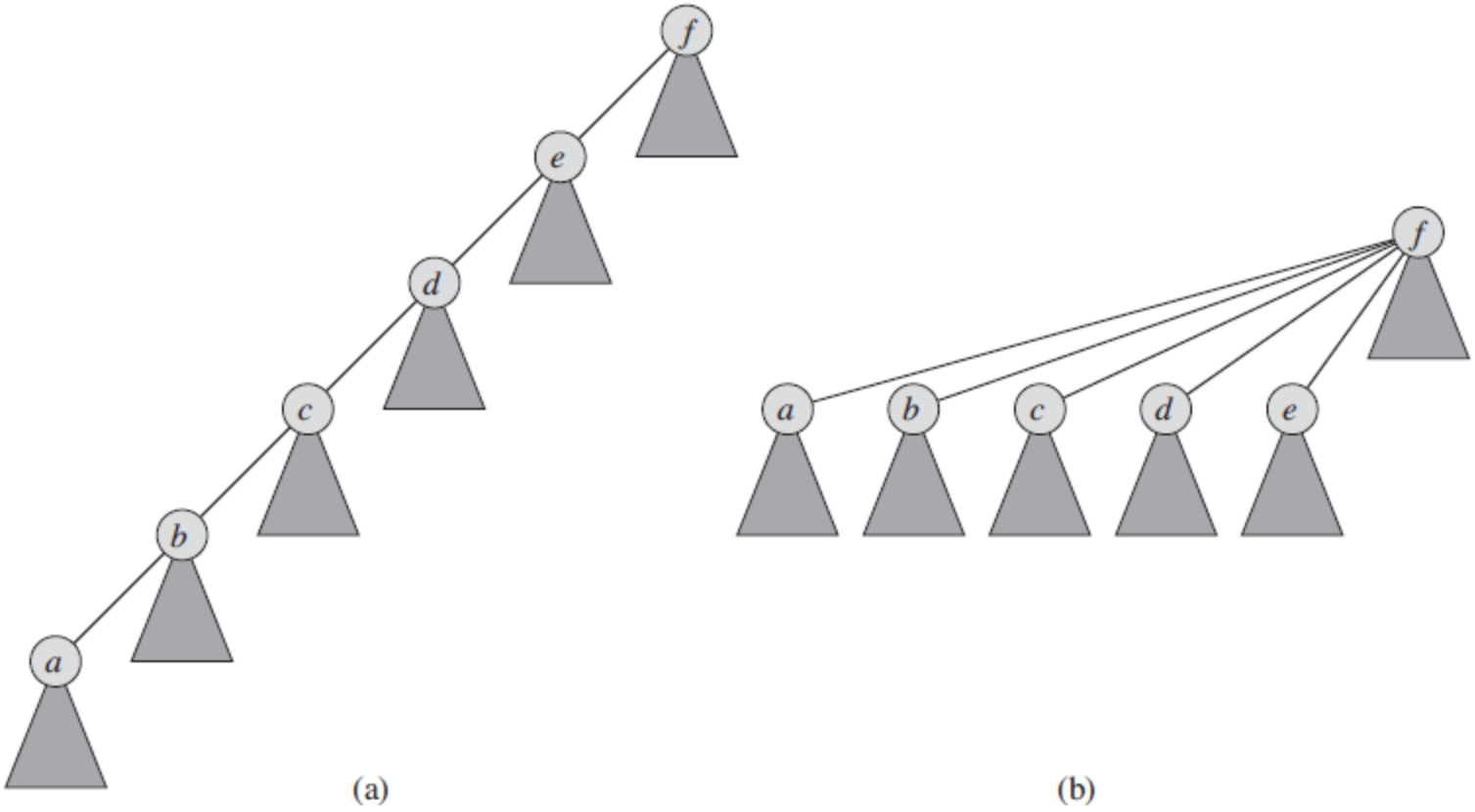

Figure 21.5 Path compression during the operation FIND-SET. Arrows and self-loops at roots are omitted. **(a)** A tree representing a set prior to executing FIND-SET(a). Triangles represent subtrees whose roots are the nodes shown. Each node has a pointer to its parent. **(b)** The same set after executing FIND-SET(a). Each node on the find path now points directly to the root.

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - Pseudocode

```
MAKE-SET(x)
1  x.p = x
2  x.rank = 0

UNION(x, y)
1  LINK(FIND-SET(x), FIND-SET(y))

LINK(x, y)
1  if x.rank > y.rank
2       y.p = x
3  else x.p = y
4       if x.rank == y.rank
5            y.rank = y.rank + 1
```

The FIND-SET procedure with path compression is quite simple:

```
FIND-SET(x)
1  if x ≠ x.p
2       x.p = FIND-SET(x.p)
3  return x.p
```

Data Structures for Disjoint Set

- Rooted Tree Representation of Disjoint Sets
  - Running Time Analysis
    - When we use both union by rank and path compression, the worst-case running time is $O(m\,\alpha(n))$, where $\alpha(n)$ is a *very* slowly growing function.

      In any conceivable application of a disjoint-set data structure, a(n) <= 4; thus, we can view the running time as linear in m in all practical situations.

Disclaimer

- Contents of this presentation are not original and they have been prepared from various sources just for the teaching purpose.