DAA Tut-2

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Q-1 Find $\Theta$ notation.

1. $f(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$

Let $0 \leq c_1\frac{n^3}{3} \leq \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \leq c_2 n^3$

dividing by $n^3$,

$$0 \leq c_1 \leq \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} \leq c_2$$

At $n=1$, we get max values

$\therefore c_2 \leq \frac{1}{3} + \frac{1}{2} + \frac{1}{6}$

$\therefore c_2 = 1$

At $n = \infty$, $c_1 = 1/3$

$n_0 \geq 1$

$\therefore f(n) = \Theta(n^3)$

2. $f(n) = 27n^2 + 16n$

$27n^2 \geq 27n^2 \quad \forall n \geq 1$

$16n^2 \geq 16n \quad \forall n \geq 1$

$\therefore f(n) \leq 43n^2$

Also $27n^2 \leq 27n^2$

$\therefore 27n^2 \leq 27n^2 + 16n \quad \forall n \geq 1$

$\therefore n^2 \leq 27n^2 + 16n \quad \forall n \geq 1$

$\therefore n^2 \leq f(n) \leq 43n^2 \quad \forall n \geq 1$

$\therefore c_1 = 1$ ; $c_2 = 43$ & $n_0 = 1$

$\therefore f(n) = \Theta(n^2)$

3. $f(n) = 3.2^n + 4^n + 5n + 3$

Q-2.

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here, $3 \cdot 2^n \leq 3 \times 2^n$

$\therefore 2^n \leq 3 \times 2^n$

$\therefore 2^n \leq 3 \cdot 2^n + 4n^2 + 5n + 3 \; ; \forall n \geq 1$

$\therefore 2^n \geq f(n)$ ... (1)

Also, $3 \cdot 2^n \geq 3 \cdot 2^n$

$8 \cdot 2^n \geq 4n^2$

$5 \cdot 2^n \geq 5n$

$3 \cdot 2^n \geq 3$

$\therefore (3+5+8+3) \cdot 2^n \geq f(n)$

$\therefore f(n) \leq 19 \cdot 2^n$ ... (2) $n \geq 1$

By (1) & (2) we get,

$2^n \leq f(n) \leq 19 \cdot 2^n$ $\forall n \geq 1$

$C_1 = 1$, $C_2 = 19$ $A_0 = 1$

$\therefore f(n) = \theta(2^n)$

Q-2. find O notation

1. $f(n) = 5n^3 + n^2 + 3n + 2$.

Now, $5n^3 \geq 5n^3$

$n^3 \geq n^2$

$3n^3 \geq 3n$

$2n^3 \geq 2$ $\forall n \geq 1$

$\therefore f(n) < 11n^3$ $\forall n \geq 1$

$\therefore C_0 = 11$ ; $n_0 = 1$

$\therefore f(n) = O(n^3)$

2. $f(n) = 3n^3 + 4n$

Now, $3n^3 \geq 3n^3$

$4n^3 \geq 4n$

$\therefore f(n) \leq 7n^3$ $\forall n \geq 1$

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$\therefore C = 7 \quad ; \quad n_0 = 1$

$\therefore f(n) = O(n^3)$

Q-3 Show that for any real number const. a and b where $b > 0$

$$(n+a)^b = \Theta(n^b)$$

Let $f(n) = (n+a)^b$

$g(n) = (n^b)$

Applying l'hopital's rule;

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} \frac{(n+a)^b}{n^b}$$

$$\lim_{n \to \infty} \frac{n^b (1 + a/n)^b}{n^b}$$

$$\lim_{n \to \infty} (1 + a/n)^b$$

$$= 1 \quad \in R$$

by limit rule we can say that

$f(n) = \Theta(g(n))$

$f(n) = \Theta(n^b)$

$\therefore (n+a)^b = \Theta(n^b)$

Hence, proved

Q-4 Is $2^{n+1} = O(2^n)$ ? Is $2^{2n} = O(2^n)$ ?

Here, $2^{n+1} = 2 \cdot 2^n$

$\therefore \frac{1}{2} \times 2^n \leq 2^n$

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$\therefore \quad f(n) \leq 2^n$

$\therefore \quad 2^{n+1} = O(2^n)$

Now, $\quad 2^{2n} = 2^n \cdot 2^n$

let $\quad 2^n \cdot 2^n \leq C \cdot 2^n$

$2^n \leq C$

but this case is not possible.

Hence, $\quad 2^{2n} \neq O(2^n)$.

Q5 Prove that $\quad n\log n - 2n + 13 = \Omega(n\log n)$

let $\quad cn\log n \leq n\log n - 2n + 13$

$cn\log n \leq n\log n - 2n$

dividing by $n\log n$

$c \leq 1 - \frac{2}{\log n} \quad \forall n \geq 1$

If $n \geq 8$, then $\quad 1 - \frac{2}{\log n} \leq \frac{2}{3}$

and if $c = 1/3$ then above condition holds true.

$\therefore \quad c = 1/3$ and $n_0 = 8$

$\therefore \quad 1/3 \, n\log n \leq n\log n - 2n$

$\therefore \quad 1/3 \, n\log n \leq n\log n - 2n + 13$

$\therefore \quad \frac{1}{3} n\log n \leq f(n)$

$\therefore \quad f(n) \ \Omega(n \log n)$

Hence, proved.

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Q-6 Prove that $\sum_{i=1}^{n} \log i$ is $O(n\log n)$

$\rightarrow f(n) = \sum_{i=1}^{n} \log(i)$

$= \log 1 + \log 2 + \log 3 + \dots + \log n$

$= \log(1 \cdot 2 \cdot 3 \cdots n)$

$\therefore f(n) = \log(n!)$

Now,

$\log(n!) = \log 1 + \log 2 + \log 3 + \dots + \log n$

$\therefore LHS \leq \log n + \log n + \log n + \dots + \log n$

$\therefore LHS \leq \log(n^n)$

$\therefore f(n) \leq n\log n \quad \dots (1)$

Now, $LHS \geq \log\left(\frac{n}{2}\right) + \log\left(\frac{n}{2}+1\right) + \dots + \log\left(\frac{n}{2}+n\right)$

$LHS \geq \log\left(\frac{n}{2}\right) + \log\left(\frac{n}{2}\right) + \dots + \log\left(\frac{n}{2}\right)$

$LHS \geq \log\left(\left(\frac{n}{2}\right)^{n/2}\right)$

$LHS \geq \frac{n}{2} \log \frac{n}{2}. \quad \dots (2)$

from (1) & (2) $f(n) = \theta(n\log n)$

Hence proved.