

# Part 9: Text Classification; The Naïve Bayes algorithm



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Most of these slides comes from the  
course:

Information Retrieval and Web Search,  
Christopher Manning and Prabhakar  
Raghavan

# Content

- Introduction to Text Classification
- Bayes rule
- Naïve Bayes text classification
- Feature independence assumption
- Multivariate and Multinomial approaches
- Smoothing (avoid overfitting)
- Feature selection
  - Chi square and Mutual Information
- Evaluating NB classification.

# Standing queries

- The path from information retrieval to text classification:
  - You have an information need, say:
    - *"Unrest in the Niger delta region"*
  - You want to rerun an appropriate query periodically to find new news items on this topic
  - You will be sent new documents that are found
    - I.e., it's classification not ranking
- Such queries are called ***standing queries***
  - Long used by “information professionals”
  - A modern mass instantiation is **Google Alerts.**

# Google alerts



[FAQ](#) | [Sign in](#)

## Welcome to Google Alerts

Google Alerts are email updates of the latest relevant Google results (web, news, etc.) based on your choice of query or topic.

Some handy uses of Google Alerts include:

- monitoring a developing news story
- keeping current on a competitor or industry
- getting the latest on a celebrity or event
- keeping tabs on your favorite sports teams

Create an alert with the form on the right.

You can also [sign in to manage your alerts](#)

### Create a Google Alert

Enter the topic you wish to monitor.

Search terms:

Type:

How often:

Email length:

Your email:

Google will not sell or share your email address.

# Spam filtering: Another text classification task

From: "" <takworld@hotmail.com>

Subject: real estate is the only way... gem oalvgkay

Anyone can buy real estate with no money down

Stop paying rent TODAY !

There is no need to spend hundreds or even thousands for similar courses

I am 22 years old and I have already purchased 6 properties using the methods outlined in this truly INCREDIBLE ebook.

Change your life NOW !

=====

Click Below to order:

<http://www.wholesaledaily.com/sales/nmd.htm>

=====

# Categorization/Classification

- Given:

- A **description of an instance**,  $x \in X$ , where  $X$  is the *instance language* or *instance space*

- Issue: how to represent text documents – the representation determines what information is used for solving the classification task

- A fixed **set of classes**:

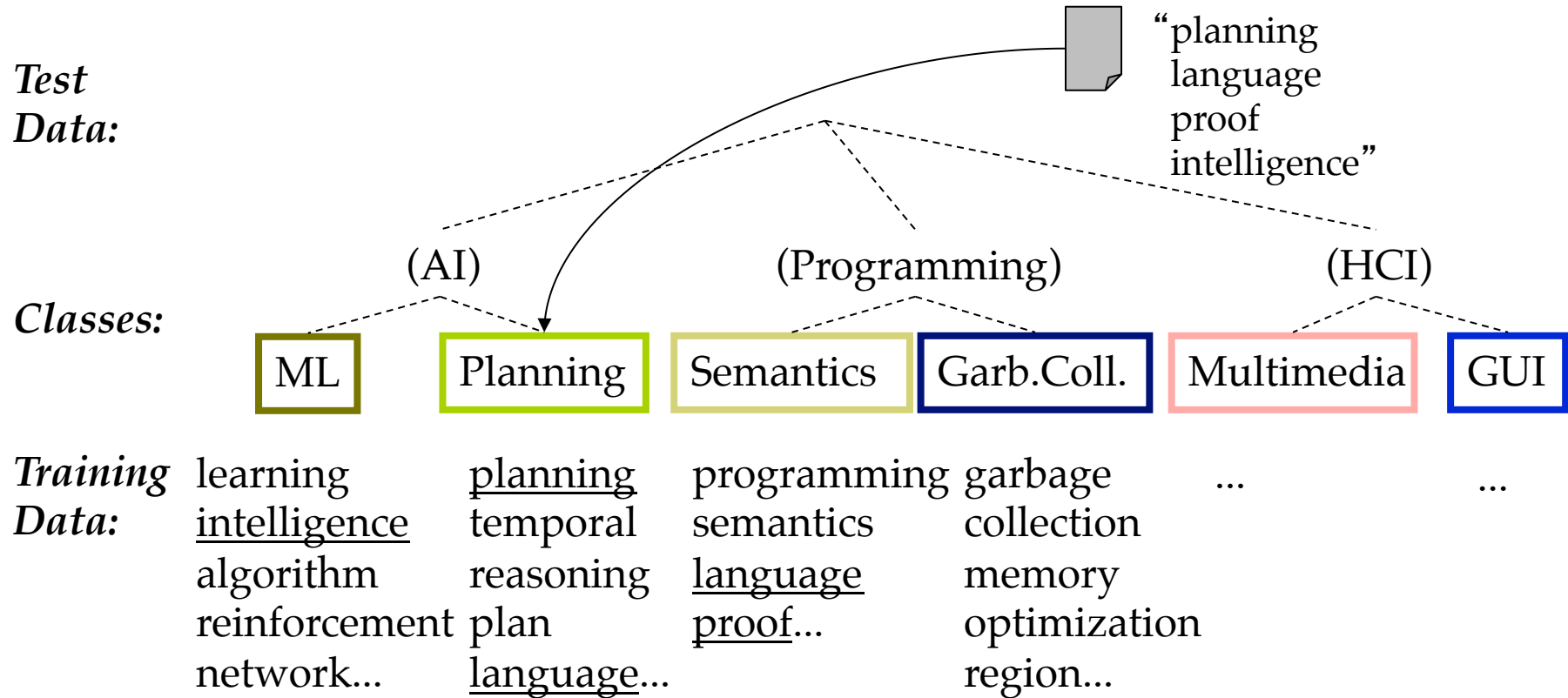
$$C = \{c_1, c_2, \dots, c_J\}$$

- Determine:

- The class of  $x$ :  $c(x) \in C$ , where  $c(x)$  is a **classification function** whose domain is  $X$  and whose range is  $C$

- We want to know how to build classification functions (“classifiers”).

# Document Classification



(Note: in real life there is often a hierarchy, not present in the above problem statement; and also, you get papers on "ML approaches to Garb. Coll.")

# More Text Classification Examples

□ Many **search engine** functionalities use **classification**

□ **Assign labels** to each **document** or web-page:

- Labels are most often **topics** such as Yahoo-categories  
*e.g., "finance," "sports," "news>world>asia>business"*
- Labels may be **genres**  
*e.g., "editorials" "movie-reviews" "news"*
- Labels may be **opinion** on a person/product  
*e.g., "like", "hate", "neutral"*
- Labels may be **domain-specific**  
*e.g., "interesting-to-me" : "not-interesting-to-me"*  
*e.g., "contains adult language" : "doesn't"*  
*e.g., language identification: English, French, Chinese, ...*  
*e.g., "link spam" : "not link spam"*  
*e.g., "key-phrase" : "not key-phrase"*

topics  
genres  
opinion  
domain-specific



# Classification Methods (1)

## □ Manual classification

- Used by Yahoo! (originally; now present but downplayed), Looksmart, about.com, ODP, PubMed
- **Very accurate** when job is done by **experts**
- Consistent when the **problem size** and **team** is **small**
- Difficult and **expensive to scale**
  - Means we need automatic classification methods for big problems.

## Classification Methods (2)

### ■ Hand-coded rule-based systems

- One technique used by CS dept's spam filter, Reuters, CIA, etc.
- Companies (Verity) provide "IDE" for writing such rules
- Example: *assign category if document contains a given Boolean combination of words*
- **Standing queries:** Commercial systems have complex query languages (everything in IR query languages + accumulators)
- Accuracy is often very high if a **rule has been carefully refined** over time by a subject expert
- Building and maintaining these rules is expensive!

# Verity topic (a classification rule)

```
comment line      # Beginning of art topic definition
top-level topic   art ACCRUE
                  /author = "fsmith"
topic definition modifiers {
                  /date   = "30-Dec-01"
                  /annotation = "Topic created
                           by fsmith"
subtopic topic    * 0.70 performing-arts ACCRUE
evidencetopic     ** 0.50 WORD
topic definition modifier /wordtext = ballet
evidencetopic     ** 0.50 STEM
topic definition modifier /wordtext = dance
evidencetopic     ** 0.50 WORD
topic definition modifier /wordtext = opera
evidencetopic     ** 0.30 WORD
topic definition modifier /wordtext = symphony
subtopic          * 0.70 visual-arts ACCRUE
                  ** 0.50 WORD
                  /wordtext = painting
                  ** 0.50 WORD
                  /wordtext = sculpture
subtopic          * 0.70 film ACCRUE
                  ** 0.50 STEM
                  /wordtext = film
subtopic          ** 0.50 motion-picture PHRASE
                  *** 1.00 WORD
                  /wordtext = motion
                  *** 1.00 WORD
                  /wordtext = picture
                  ** 0.50 STEM
                  /wordtext = movie
subtopic          * 0.50 video ACCRUE
                  ** 0.50 STEM
                  /wordtext = video
                  ** 0.50 STEM
                  /wordtext = vcr
                  # End of art topic
```

## □ Note:

- maintenance issues (author, etc.)
- Hand-weighting of terms
- But it is easy to explain the results.

## Classification Methods (3)

- **Supervised learning** of a document-label assignment function
- Many systems partly rely on machine learning (Autonomy, MSN, Verity, Enkata, Yahoo!, ...)
  - k-Nearest Neighbors (simple, powerful)
  - Naive Bayes (simple, common method)
  - Support-vector machines (new, more powerful)
  - ... plus many other methods
- No free lunch: **requires hand-classified training data**
- Note that many commercial systems use a mixture of methods.

## Recall a few probability basics

□ For events  $a$  and  $b$ :

□ **Bayes' Rule**

$$p(a, b) = p(a \cap b) = p(a | b)p(b) = p(b | a)p(a)$$

$$p(a | b) = \frac{p(b | a)p(a)}{p(b)} = \frac{p(b | a)\overset{\text{Prior}}{p(a)}}{\sum_{x=a, \bar{a}} p(b | x)p(x)}$$

↑  
Posterior

□ **Odds:**

$$O(a) = \frac{p(a)}{p(\bar{a})} = \frac{p(a)}{1 - p(a)}$$

## Bayes' Rule Example

$$P(C, E) = P(C | E)P(E) = P(E | C)P(C)$$

$$P(C | E) = \frac{P(E | C)P(C)}{P(E)}$$

$P(\text{pass exam} | \text{attend classes}) = ?$

$= P(\text{pass exam}) * P(\text{attend classes} | \text{pass exam}) / P(\text{attend classes})$

$= 0.7 * 0.9 / 0.78$

$= 0.7 * 1.15 = 0.81$

Initial estimation

Correction based on a ratio

# Example explained

Pass 70%	Not pass 30%
Attend 90% = $P(\text{attend} \text{pass})$	Attend 50%
Not attend 10%	Not attend 50%

$$P(\text{pass}) = 0.7$$

$$\begin{aligned} P(\text{attend}) &= P(\text{attend} | \text{pass})P(\text{pass}) + P(\text{attend} | \text{not pass})P(\text{not pass}) \\ &= 0.9 \cdot 0.7 + 0.5 \cdot 0.3 = 0.63 + 0.15 = 0.78 \end{aligned}$$

$$\begin{aligned} p(\text{pass} | \text{attend}) &= p(\text{pass}) \cdot p(\text{attend} | \text{pass}) / p(\text{attend}) \\ &= 0.7 \cdot 0.9 / 0.78 = 0.81 \end{aligned}$$

# Bayesian Methods

- Our focus this lecture
- Learning and classification methods based on **probability theory**
- **Bayes theorem** plays a critical role in probabilistic learning and classification
- Uses **prior** probability of each category given no information about an item
- Obtains a **posterior probability** distribution over the possible categories **given a description of an item.**



# Naive Bayes Classifiers

- Task: Classify a new instance  $D$  based on a tuple of attribute values  $D = \langle x_1, x_2, \dots, x_n \rangle$  into one of the classes  $c_j \in C$

$$c_{MAP} = \operatorname{argmax}_{c_j \in C} P(c_j | x_1, x_2, \dots, x_n)$$

Maximum A  
Posteriori  
class

$$= \operatorname{argmax}_{c_j \in C} \frac{P(x_1, x_2, \dots, x_n | c_j) P(c_j)}{P(x_1, x_2, \dots, x_n)}$$

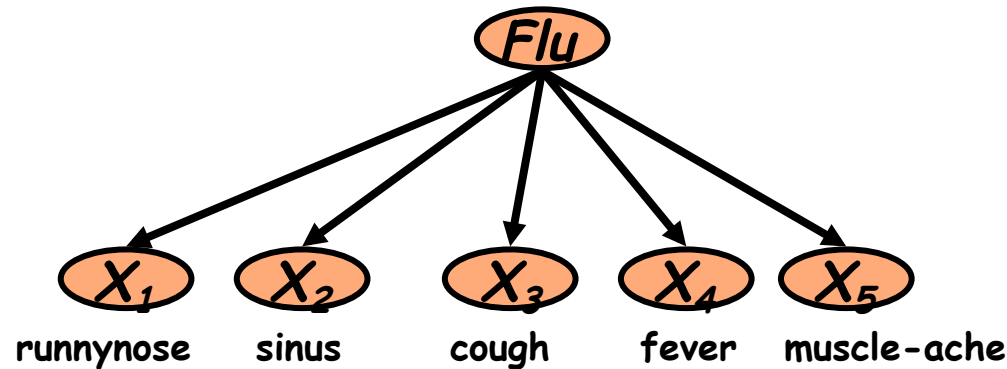
$$= \operatorname{argmax}_{c_j \in C} P(x_1, x_2, \dots, x_n | c_j) P(c_j)$$

# Naïve Bayes Assumption

- $P(c_j)$ 
  - Can be estimated from the frequency of classes in the training examples
- $P(x_1, x_2, \dots, x_n | c_j)$ 
  - $O(|X|^n |C|)$  parameters (*assuming  $X$  finite*)
  - Could only be estimated if a very, very large number of training examples was available – or?
- **Naïve Bayes Conditional Independence Assumption:**
  - Assume that the probability of observing the **conjunction** of attributes is equal to the **product** of the individual probabilities  $P(x_i | c_j)$ .

$$c_{MAP} = \operatorname{argmax}_{c_j \in C} P(c_j) \prod_{i=1}^n P(x_i | c_j)$$

# The Naïve Bayes Classifier



- **Conditional Independence Assumption:**  
features detect term presence and are independent of each other given the class:

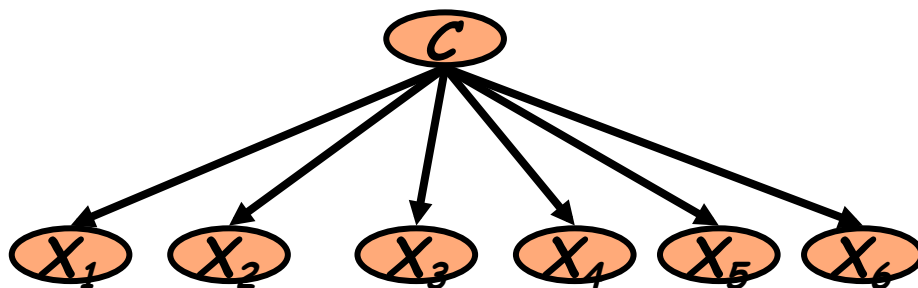
$$P(x_1, \dots, x_5 | C) = P(x_1 | C) \cdot P(x_2 | C) \cdot \dots \cdot P(x_5 | C)$$

- This model is appropriate for **binary** variables
  - Multivariate Bernoulli model

= many variables

= only 2 values – T or F

# Learning the Model



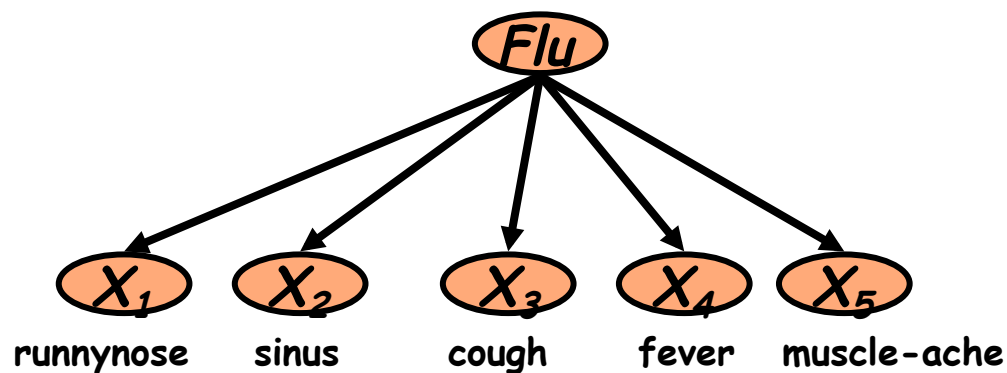
- First attempt: maximum likelihood estimates
  - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{N(C = c_j)}{N}$$

$$\hat{P}(x_i | c_j) = \frac{N(X_i = x_i, C = c_j)}{N(C = c_j)}$$

Estimated conditional probability that the attribute  $X_i$  (e.g. Fever) has the value  $x_i$  (True or False) – we will also write  $P(\text{Fever} | c_j)$  instead of  $P(\text{Fever} = T | c_j)$

# Problem with Max Likelihood



$$P(x_1, \dots, x_5 | C) = P(x_1 | C) \cdot P(x_2 | C) \cdot \dots \cdot P(x_5 | C)$$

- What if we have seen no training cases where patient had flu and muscle aches?

$$\hat{P}(X_5 = T | C = flu) = \frac{N(X_5 = T, C = flu)}{N(C = flu)} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$\ell = \arg \max_c \hat{P}(c) \prod_i \hat{P}(x_i | c)$$

# Smoothing to Avoid Overfitting

$$\hat{P}(x_i | c_j) = \frac{N(X_i = x_i, C = c_j) + 1}{N(C = c_j) + k}$$

# of values of  $X_i$

- Somewhat more subtle version

$$\hat{P}(x_i | c_j) = \frac{N(X_i = x_i, C = c_j) + mP(X_i = x_i)}{N(C = c_j) + m}$$

overall fraction in  
data where  $X_i = x_i$

extent of  
“smoothing”<sup>23</sup>

# Example

	docID	words in document	in $c = \textit{China}$ ?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

$$\begin{aligned}
 \hat{P}(\text{Chinese}|c) &= (3 + 1)/(3 + 2) = 4/5 \\
 \hat{P}(\text{Japan}|c) = \hat{P}(\text{Tokyo}|c) &= (0 + 1)/(3 + 2) = 1/5 \\
 \hat{P}(\text{Beijing}|c) = \hat{P}(\text{Macao}|c) = \hat{P}(\text{Shanghai}|c) &= (1 + 1)/(3 + 2) = 2/5 \\
 \hat{P}(\text{Chinese}|\bar{c}) &= (1 + 1)/(1 + 2) = 2/3 \\
 \hat{P}(\text{Japan}|\bar{c}) = \hat{P}(\text{Tokyo}|\bar{c}) &= (1 + 1)/(1 + 2) = 2/3 \\
 \hat{P}(\text{Beijing}|\bar{c}) = \hat{P}(\text{Macao}|\bar{c}) = \hat{P}(\text{Shanghai}|\bar{c}) &= (0 + 1)/(1 + 2) = 1/3
 \end{aligned}$$

$$\begin{aligned}
 \hat{P}(c|d_5) &\propto \hat{P}(c) \cdot \hat{P}(\text{Chinese}|c) \cdot \hat{P}(\text{Japan}|c) \cdot \hat{P}(\text{Tokyo}|c) \\
 &\quad \cdot (1 - \hat{P}(\text{Beijing}|c)) \cdot (1 - \hat{P}(\text{Shanghai}|c)) \cdot (1 - \hat{P}(\text{Macao}|c)) \\
 &= 3/4 \cdot 4/5 \cdot 1/5 \cdot 1/5 \cdot (1 - 2/5) \cdot (1 - 2/5) \cdot (1 - 2/5) \\
 &\approx 0.005
 \end{aligned}$$

# Exercise

	docID	words in document	in $c = \textit{China}$ ?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

$$\begin{aligned}
 \hat{P}(\text{Chinese}|c) &= (3 + 1)/(3 + 2) = 4/5 \\
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 \hat{P}(\text{Beijing}|\bar{c}) = \hat{P}(\text{Macao}|\bar{c}) = \hat{P}(\text{Shanghai}|\bar{c}) &= (0 + 1)/(1 + 2) = 1/3
 \end{aligned}$$

Estimate the probability that the test document does not belong to class  $c$



# Exercise

	docID	words in document	in $c = \textit{China}$ ?
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 \hat{P}(\text{Beijing}|\bar{c}) = \hat{P}(\text{Macao}|\bar{c}) = \hat{P}(\text{Shanghai}|\bar{c}) &= (0 + 1)/(1 + 2) = 1/3
 \end{aligned}$$

$$\begin{aligned}
 \hat{P}(\bar{c}|d_5) &\propto 1/4 \cdot 2/3 \cdot 2/3 \cdot 2/3 \cdot (1 - 1/3) \cdot (1 - 1/3) \cdot (1 - 1/3) \\
 &\approx 0.022
 \end{aligned}$$

## Multinomial Naive Bayes Classifiers: Basic method

- Attributes ( $X_i$ ) are text positions, values ( $x_i$ ) are words:

$$\begin{aligned}c_{NB} &= \operatorname{argmax}_{c_j \in C} P(c_j) \prod_{i=1}^n P(x_i | c_j) \\ &= \operatorname{argmax}_{c_j \in C} P(c_j) P(X_1 = \text{"Our"} | c_j) \cdots P(X_n = \text{"text."} | c_j)\end{aligned}$$

- Still too many possibilities
- Assume that classification is *independent* of the positions of the words

$$P(X_i = w | c) = P(X_j = w | c)$$

# Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*
- Calculate required  $P(c_j)$  and  $P(x_k | c_j)$  terms
  - For each class  $c_j$  in  $C$  do
    - $docs_j \leftarrow$  subset of documents for which the target class is  $c_j$

$$P(c_j) \leftarrow \frac{|docs_j|}{\text{total \# documents}}$$

- $Text_j \leftarrow$  single document containing all  $docs_j$ 
  - for each word  $x_k$  in *Vocabulary*
  - $n_{jk} \leftarrow$  number of occurrences of  $x_k$  in  $Text_j$
  - $n_j \leftarrow$  number of words in  $Text_j$

$$P(x_k | c_j) \leftarrow \frac{n_{jk} + \alpha}{n_j + \alpha |Vocabulary|}$$

Assume  $\alpha = 1$ ;  
this is for  
smoothing

# Multinomial Naïve Bayes: Classifying

- $\text{positions} \leftarrow$  all word positions in **current document** which contain tokens found in *Vocabulary*
- Return  $c_{NB}$  such that:

$$c_{NB} = \operatorname{argmax}_{c_j \in C} P(c_j) \prod_{i \in \text{positions}} P(x_i | c_j)$$

# Example

	docID	words in document	in $c = \textit{China}$ ?
training set	1	Chinese Beijing Chinese	yes
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	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

$$\hat{P}(\text{Chinese}|c) = (5 + 1)/(8 + 6) = 6/14 = 3/7$$

$$\hat{P}(\text{Tokyo}|c) = \hat{P}(\text{Japan}|c) = (0 + 1)/(8 + 6) = 1/14$$

$$\hat{P}(\text{Chinese}|\bar{c}) = (1 + 1)/(3 + 6) = 2/9$$

$$\hat{P}(\text{Tokyo}|\bar{c}) = \hat{P}(\text{Japan}|\bar{c}) = (1 + 1)/(3 + 6) = 2/9$$

$$\hat{P}(c|d_5) \propto 3/4 \cdot (3/7)^3 \cdot 1/14 \cdot 1/14 \approx 0.0003.$$

$$\hat{P}(\bar{c}|d_5) \propto 1/4 \cdot (2/9)^3 \cdot 2/9 \cdot 2/9 \approx 0.0001.$$

# Naive Bayes: Time Complexity

- **Training Time:** if  $L_d$  is the average length of a document in  $D$

Number of conditional probabilities to estimate

- $O(|D|L_d + |C||V|)$

Scan the documents to compute the vocabulary and the frequencies of words

- Assumes that  $V$  and all  $docs_j$ ,  $n_j$ , and  $n_{jk}$  are computed in  $O(|D|L_d)$  time during one pass through all of the data
- Generally just  $O(|D|L_d)$  since usually  $|C||V| < |D|L_d$
- **Test Time:**  $O(|C| L_t)$  - where  $L_t$  is the average length of a test document
- **Very efficient** overall, linearly proportional to the time needed to just read in all the data.

# Underflow Prevention: log space

- ❑ Multiplying lots of probabilities, which are between 0 and 1 by definition, can result in **floating-point underflow**
- ❑ Since  $\log(xy) = \log(x) + \log(y)$ , it is better to perform all computations by summing logs of probabilities rather than multiplying probabilities
- ❑ Class with highest final un-normalized log probability score is still the most probable

$$c_{NB} = \operatorname{argmax}_{c_j \in C} \left( \log P(c_j) + \sum_{i \in \text{positions}} \log P(x_i | c_j) \right)$$

- ❑ Note that model is now just max of sum of weights...

Sounds  
familiar?

## Summary - Two Models: Multivariate Bernoulli

- One feature  $X_w$  for each word in dictionary
- $X_w = \text{true}$  in document  $d$  if  $w$  appears in  $d$
- **Naive Bayes assumption:**
  - Given the document's topic (class), appearance of one word in the document tells us nothing about chances that another word appears (independence)



## Summary - Two Models: Multinomial

- One feature  $X_i$  for each word positions in document
  - feature's values are all words in dictionary
- Value of  $X_i$  is the word in position  $i$
- **Naïve Bayes assumption:**
  - Given the document's topic (class), word in one position in the document tells us nothing about words in other positions
- **Second assumption:**
  - Word appearance does not depend on position - for all positions  $i, j$ , word  $w$ , and class  $c$

$$P(X_i = w | c) = P(X_j = w | c)$$

- Just have one multinomial feature predicting all words.

# Parameter estimation

## ▣ Multivariate Bernoulli model:

$$\hat{P}(X_w = \textit{true} \mid c_j) =$$

fraction of documents of topic  $c_j$  in which word  $w$  appears

## ▣ Multinomial model:

$$\hat{P}(X_i = w \mid c_j) =$$

fraction of times in which word  $w$  appears across all documents of topic  $c_j$

- Can create a mega-document for topic  $j$  by concatenating all documents in this topic
- Use frequency of  $w$  in mega-document.

# Classification

- Multinomial vs Multivariate Bernoulli?
- **Multinomial** model is almost always **more effective** in text applications!
  - See results figures later
- See *IIR* sections 13.2 and 13.3 for worked examples with each model

# Feature Selection: Why?

- Text collections have a large number of features
  - 10,000 – 1,000,000 unique words ... and more
- May make using a particular classifier unfeasible
  - Some classifiers can't deal with 100,000 of features
- Reduces training time
  - Training time for some methods is quadratic or worse in the number of features
- Can improve generalization (performance)
  - Eliminates noise features
  - Avoids overfitting.

# Feature selection: how?

- Two ideas:

- **Hypothesis testing statistics:**

- Are we confident that the value of one categorical variable is associated with the value of another

- *Chi-square test* (  
<http://faculty.vassar.edu/lowry/webtext.html> chapter 8)

- **Information theory:**

- How much information does the value of one categorical variable give you about the value of another

- *Mutual information*

## $\chi^2$ statistic (CHI) – testing independence of class and term

	<i>Term = jaguar</i>	<i>Term <math>\neq</math> jaguar</i>
<i>Class = auto</i>	2	500
<i>Class <math>\neq</math> auto</i>	3	9500

- If the term "jaguar" is **independent** from the class "auto" we should have:
- $P(C(d)=auto, d \text{ contains jaguar}) = P(C(d)= auto) * P(d \text{ contains jaguar})$
- $2/10005 = ? \quad 502/10005 * 5/10005$
- $0.00019 = ? \quad 0.0501 * 0.00049 = 0.000025 \quad \text{NOT REALLY!}$
- To be independent we should have more documents that contains jaguar but are not in class auto (38).

# $\chi^2$ statistic (CHI)

- $\chi^2$  is interested in  $(f_o - f_e)^2 / f_e$  summed over all table entries: is the observed number what you'd expect given the marginals?

$$\chi^2(j,a) = \sum (O - E)^2 / E = (2 - .25)^2 / .25 + (3 - 4.75)^2 / 4.75 + (500 - 502)^2 / 502 + (9500 - 9498)^2 / 9498 = 12.9 \ (p < .001)$$

- The null hypothesis (the two variables are independent) is rejected with confidence .999,
- since  $12.9 > 10.83$  (the critical value for .999 confidence for a 1 degree of freedom  $\chi^2$  distribution ).

	<i>Term = jaguar</i>	<i>Term <math>\neq</math> jaguar</i>
<i>Class = auto</i>	2 (0.25)	500 (502)
<i>Class <math>\neq</math> auto</i>	3 (4.75)	9500 (9498)

Expected value, for instance in the up-left cell is:

$$P(c=auto) * P(T=jaguar) * \# \text{ of cases} = 502/10005 * 5/10005 * 10005 = 0.2508$$

## $\chi^2$ statistic (CHI)

There is a “simpler” formula for 2x2  $\chi^2$ :

$$\chi^2(t, c) = \frac{N \times (AD - CB)^2}{(A + C) \times (B + D) \times (A + B) \times (C + D)}$$

$A = \#(t, c)$	$C = \#(\neg t, c)$
$B = \#(t, \neg c)$	$D = \#(\neg t, \neg c)$

$$N = A + B + C + D$$



# Feature selection via Mutual Information

- In training set, choose  $k$  words which give most info on the knowledge of the categories
- The **Mutual Information** between a word, class is:

$$I(U, C) = \sum_{e_w \in \{0,1\}} \sum_{e_c \in \{0,1\}} p(U = e_w, C = e_c) \log \frac{p(U = e_w, C = e_c)}{p(U = e_w) p(C = e_c)}$$

- $U=1$  ( $U=0$ ) means the document (does not) contains  $w$
- $C=1$  ( $C=0$ ) the document is (not) in class  $c$
- For each word  $w$  and each category  $c$
- $I(X, Y) = H(X) - H(X|Y)$   
 $= -\sum_i p(x_i) \log p(x_i) + \sum_j p(y_j) H(X|Y=y_j)$
- $H$  is called the Entropy

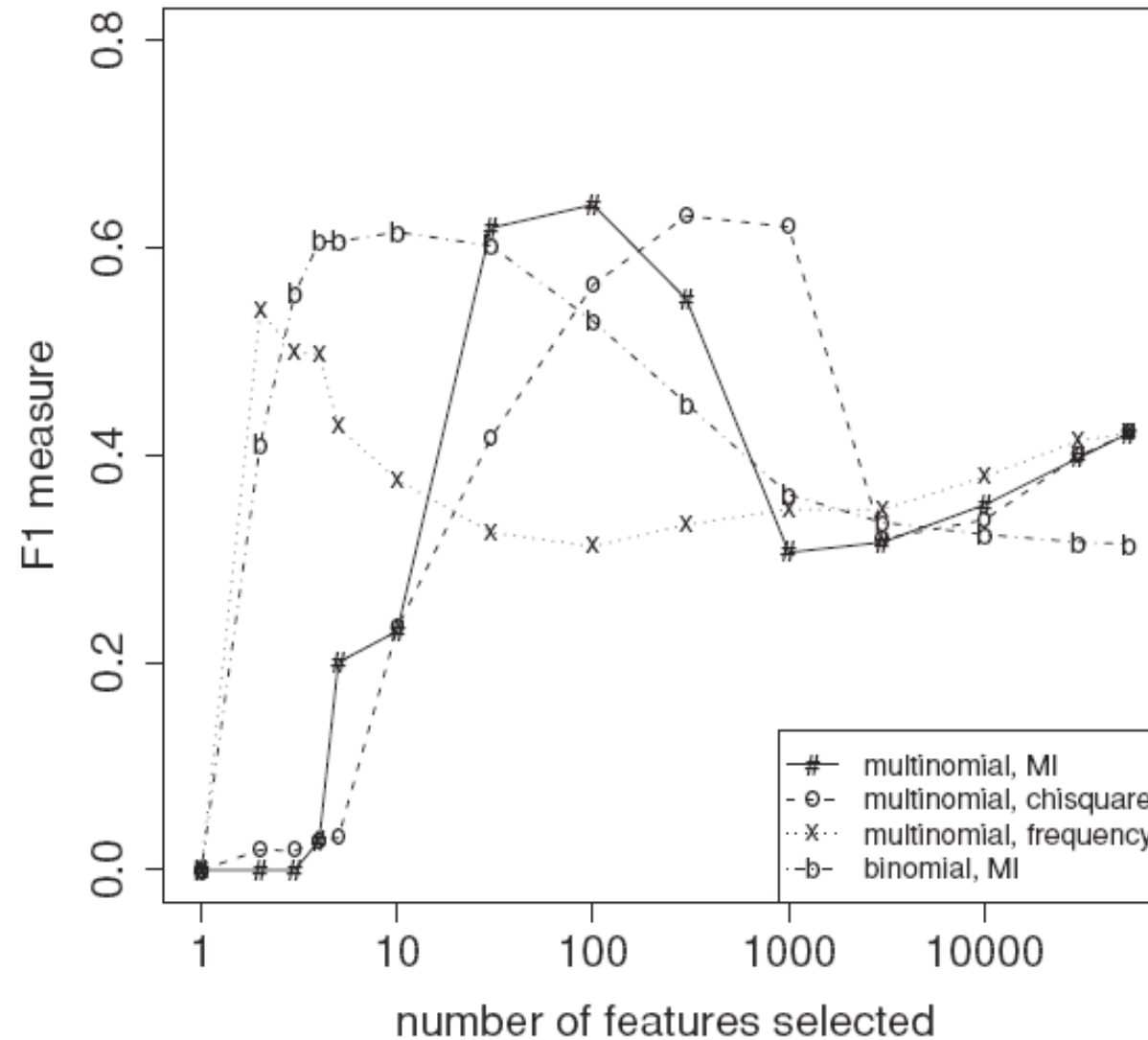
## Feature selection via MI (contd.)

- For each category we build a list of  $k$  most discriminating terms
- For example (on 20 Newsgroups):
  - ***sci.electronics:*** circuit, voltage, amp, ground, copy, battery, electronics, cooling, ...
  - ***rec.autos:*** car, cars, engine, ford, dealer, mustang, oil, collision, autos, tires, toyota, ...
- Greedy: does not account for correlations between terms
- Why?

# Feature Selection

- Mutual Information
  - Clear information-theoretic interpretation
  - May select very slightly informative **frequent** terms that are not very useful for classification
- Chi-square
  - Statistical foundation
  - May select **rare statistically correlated** but uninformative terms
- Just use the commonest terms?
  - No particular foundation
  - In practice, this is often 90% as good.

# Example



## Feature selection for NB

- In general feature selection is *necessary* for **multivariate Bernoulli** NB
- Otherwise you suffer from noise, multi-counting
- “Feature selection” really means something different for multinomial NB - it means **dictionary truncation**
  - The multinomial NB model only has 1 feature
- This “feature selection” normally isn’t needed for multinomial NB, but may help a fraction with quantities that are badly estimated.

# Evaluating Categorization

- ❑ Evaluation must be done on **test data** that are **independent** of the **training data** (usually a disjoint set of instances).
- ❑ *Classification accuracy*:  $c/n$  where  $n$  is the total number of test instances and  $c$  is the number of test instances correctly classified by the system
  - Adequate if one class per document (*and positive and negative examples have similar cardinalities*)
  - Otherwise **F measure** for each class
- ❑ Results can vary based on sampling error due to different training and test sets
- ❑ Average results over multiple training and test sets (splits of the overall data) for the best results.

# WebKB Experiment (1998)

- Classify webpages from CS departments into:
  - student, faculty, course, project
- Train on ~5,000 hand-labeled web pages
  - Cornell, Washington, U.Texas, Wisconsin
- Crawl and classify a new site (CMU)



## □ Results:

	Student	Faculty	Person	Project	Course	Department
Extracted	180	66	246	99	28	1
Correct	130	28	194	72	25	1
Accuracy:	72%	42%	79%	73%	89%	100%

Actually this is not accuracy but ...

# Most relevant features: MI

Faculty

associate	0.00417
chair	0.00303
member	0.00288
ph	0.00287
director	0.00282
fax	0.00279
journal	0.00271
recent	0.00260
received	0.00258
award	0.00250

Students

resume	0.00516
advisor	0.00456
student	0.00387
working	0.00361
stuff	0.00359
links	0.00355
homepage	0.00345
interests	0.00332
personal	0.00332
favorite	0.00310

Courses

homework	0.00413
syllabus	0.00399
assignments	0.00388
exam	0.00385
grading	0.00381
midterm	0.00374
pm	0.00371
instructor	0.00370
due	0.00364
final	0.00355

Departments

departmental	0.01246
colloquia	0.01076
epartment	0.01045
seminars	0.00997
schedules	0.00879
webmaster	0.00879
events	0.00826
facilities	0.00807
eople	0.00772
postgraduate	0.00764

Research Projects

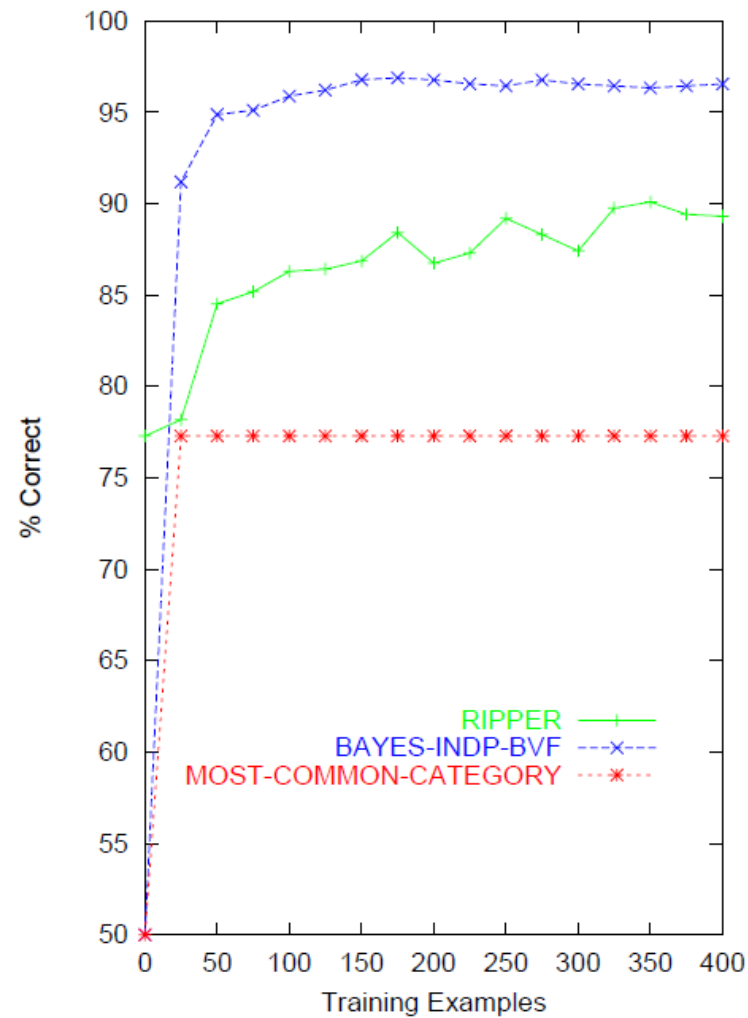
investigators	0.00256
group	0.00250
members	0.00242
researchers	0.00241
laboratory	0.00238
develop	0.00201
related	0.00200
arpa	0.00187
affiliated	0.00184
project	0.00183

Others

type	0.00164
jan	0.00148
enter	0.00145
random	0.00142
program	0.00136
net	0.00128
time	0.00128
format	0.00124
access	0.00117
begin	0.00116



# Naïve Bayes on spam email



# Naïve Bayes Posterior Probabilities

- **Classification results** of naïve Bayes (the class with maximum posterior probability) **are usually fairly accurate**
- However, due to the inadequacy of the conditional independence assumption, the **actual posterior-probability numerical estimates are not**
  - Output probabilities are commonly very close to 0 or 1
- Correct estimation  $\Rightarrow$  accurate prediction, but correct probability estimation is **NOT** necessary for accurate prediction (just need right ordering of probabilities).

# Naive Bayes is Not So Naive

- Naïve Bayes: First and Second place in KDD-CUP 97 competition, among 16 (then) state of the art algorithms

Goal: Financial services industry direct mail response prediction model:  
Predict if the recipient of mail will actually respond to the advertisement  
– 750,000 records.

- Robust to Irrelevant Features

Irrelevant Features cancel each other without affecting results  
Instead Decision Trees can heavily suffer from this.

- Very good in domains with many equally important features

Decision Trees suffer from *fragmentation* in such cases – especially if little data

- A good dependable baseline for text classification (but not the best)!

- Optimal if the Independence Assumptions hold: If assumed independence is correct, then it is the Bayes Optimal Classifier for problem

- Very Fast: Learning with one pass of counting over the data; testing linear in the number of attributes, and document collection size

- Low Storage requirements

# Resources

- IIR 13
- Tom Mitchell, Machine Learning. McGraw-Hill, 1997.
  - Clear simple explanation of Naïve Bayes