Chapter 6
Pushdown Automata

(Solution/Hint)

6.1 Design a PDA accepting the following languages by null store.

$L_1 = \{a^m c b^m \mid m \geq 0\}$
$L_2 = \{a^m b^m \text{c} \mid m \geq 0\}$
$L_3 = \{c a^m b^m \mid m \geq 0\}$

Sol. PDA for L_1

$\delta(q_0, a, Z_0) \vdash (q_0, aZ_0)$
$\delta(q_0, a, a) \vdash (q_0, aa)$
$\delta(q_0, c, a) \vdash (q_1, a)$
$\delta(q_1, b, a) \vdash (q_2, \epsilon)$
$\delta(q_2, b, a) \vdash (q_2, \epsilon)$
$\delta(q_2, \epsilon, Z_0) \vdash (q_f, \epsilon)$
$\delta(q_0, c, Z_0) \vdash (q_f, \epsilon)$

PDA for L_2

$\delta(q_0, a, Z_0) \vdash (q_0, aZ_0)$
$\delta(q_0, a, a) \vdash (q_0, aa)$
$\delta(q_0, b, a) \vdash (q_1, \epsilon)$
$\delta(q_1, b, a) \vdash (q_1, \epsilon)$
$\delta(q_1, c, Z_0) \vdash (q_f, \epsilon)$

PDA for L_3

$\delta(q_0, c, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, a, Z_0) \vdash (q_1, aZ_0)$
$\delta(q_0, a, a) \vdash (q_1, aa)$
$\delta(q_1, b, a) \vdash (q_2, \epsilon)$
$\delta(q_2, b, a) \vdash (q_2, \epsilon)$
$\delta(q_2, \epsilon, Z_0) \vdash (q_f, \epsilon)$

6.2 Design a PDA accepting the following languages by null store.

$L_7 = \{a^n c b^m \mid n,m \geq 0\}$
$L_8 = \{a^n b^m \text{c} \mid n,m \geq 0\}$
$L_9 = \{c a^n b^m \mid n,m \geq 0\}$

Here, n and m are unrelated.

Sol.

PDA for L_7

$\delta(q_0, a, Z_0) \vdash (q_0, Z_0)$
$\delta(q_0, c, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, b, Z_0) \vdash (q_1, Z_0)$

$\delta(q_1, \epsilon, Z_0) \vdash (q_f, \epsilon)$

PDA for L_8
$\delta(q_0, a, Z_0) \vdash (q_0, Z_0)$
$\delta(q_0, b, Z_0) \vdash (q_0, Z_0)$
$\delta(q_0, c, Z_0) \vdash (q_f, \epsilon)$

PDA for L_9
$\delta(q_0, c, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, a, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, b, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, b, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, \epsilon, Z_0) \vdash (q_f, \epsilon)$

6.3 Design a PDA accepting the following languages by null store.
$L_{10} = \{a^n c b^m \mid n,m \geq 1\}$
$L_{11} = \{a^n b^m c \mid n,m \geq 1\}$
$L_{12} = \{c a^n b^m \mid n,m \geq 1\}$
Here, n and m are unrelated.

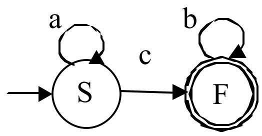
Sol. PDA for L_{10}
$\delta(q_0, a, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, a, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, c, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, b, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, \epsilon, Z_0) \vdash (q_f, \epsilon)$

PDA for L_{11}
$\delta(q_0, a, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, a, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, b, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, b, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, \square, Z_0) \vdash (q_f, \epsilon)$

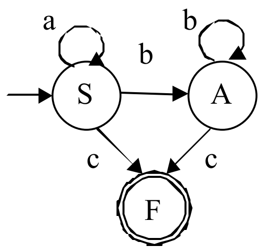
PDA for L_{12}
$\delta(q_0, c, Z_0) \vdash (q_1, Z_0)$
$\delta(q_1, a, Z_0) \vdash (q_2, Z_0)$
$\delta(q_2, a, Z_0) \vdash (q_3, Z_0)$
$\delta(q_3, b, Z_0) \vdash (q_4, Z_0)$
$\delta(q_4, b, Z_0) \vdash (q_4, Z_0)$
$\delta(q_4, \square, Z_0) \vdash (q_f, \epsilon)$

6.4 The languages L_7 to L_{12} given in 6.2 and 6.3 can also be implemented on finite automaton as n and m are unrelated. Design finite automata for each of these languages.

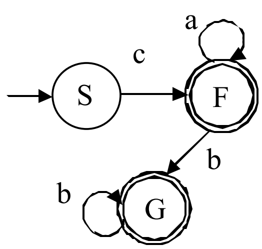
Sol.



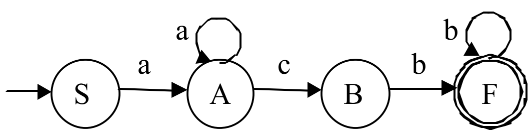
Finite Automaton for L_7



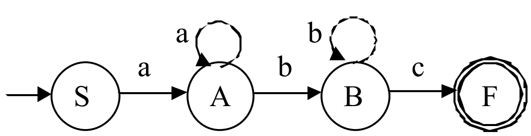
Finite Automaton for L_8



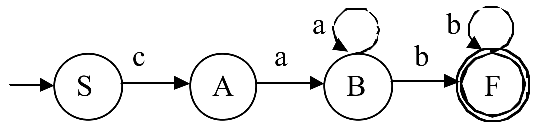
Finite Automaton for L_9



Finite Automaton for L_{10}



Finite Automaton for L_{11}



Finite Automaton for L_{12}

6.5 Design a PDA to accept the language L over $\Sigma = \{a, b\}$ consisting of all the strings with equal number of a's and b's.

Sol.
$\delta(q_0, a, Z_0) \vdash (q_1, aZ_0)$
$\delta(q_0, b, Z_0) \vdash (q_1, bZ_0)$
$\delta(q_1, a, a) \vdash (q_1, aa)$
$\delta(q_1, a, b) \vdash (q_1, \epsilon)$
$\delta(q_1, b, b) \vdash (q_1, bb)$
$\delta(q_1, b, a) \vdash (q_1, \epsilon)$
$\delta(q_1, \epsilon, Z_0) \vdash (q_f, \epsilon)$

6.6 Design a PDA corresponding to the following CFGs:
(a) $S \rightarrow 0S0 \mid 1S1 \mid A \quad A \rightarrow 2B3 \quad B \rightarrow 23 \mid 31$
(b) $S \rightarrow bX \mid aY \quad A \rightarrow bXX \mid aS \mid a \quad Y \rightarrow aYY \mid bS \mid b$
(c) $S \rightarrow 0Y \mid 1X \quad X \rightarrow 0S \mid 1XX \mid 0 \quad Y \rightarrow 1S \mid 0YY \mid 1$
Classify these PDA into deterministic and nondeterministic categories.

Sol. (a)
$\delta(q_0, \epsilon, S) \vdash (q_0, 0S0)$ $\quad$ $\delta(q_0, \epsilon, S) \vdash (q_0, 1S1)$
$\delta(q_0, \epsilon, S) \vdash (q_0, A)$ $\quad$ $\delta(q_0, \epsilon, A) \vdash (q_0, 2B3)$
$\delta(q_0, \epsilon, B) \vdash (q_0, 23)$ $\quad$ $\delta(q_0, \epsilon, B) \vdash (q_0, 31)$
$\delta(q_0, 0, 0) \vdash (q_0, \epsilon)$ $\quad$ $\delta(q_0, 1, 1) \vdash (q_0, \epsilon)$
$\delta(q_0, 2, 2) \vdash (q_0, \epsilon)$ $\quad$ $\delta(q_0, 3, 3) \vdash (q_0, \epsilon)$
Nondeterministic PDA

(b)
$\delta(q_0, \epsilon, S) \vdash (q_0, bX)$ $\quad$ $\delta(q_0, \epsilon, S) \vdash (q_0, aY)$
$\delta(q_0, \epsilon, A) \vdash (q_0, bXX)$ $\quad$ $\delta(q_0, \epsilon, A) \vdash (q_0, aS)$
$\delta(q_0, \epsilon, A) \vdash (q_0, a)$ $\quad$ $\delta(q_0, \epsilon, Y) \vdash (q_0, aYY)$
$\delta(q_0, \epsilon, Y) \vdash (q_0, bS)$ $\quad$ $\delta(q_0, \epsilon, Y) \vdash (q_0, b)$
$\delta(q_0, a, a) \vdash (q_0, \epsilon)$ $\quad$ $\delta(q_0, b, b) \vdash (q_0, \epsilon)$
Nondeterministic PDA

(c)
$\delta(q_0, \epsilon, S) \vdash (q_0, 0Y)$ $\quad$ $\delta(q_0, \epsilon, S) \vdash (q_0, 1X)$
$\delta(q_0, \epsilon, X) \vdash (q_0, 0S)$ $\quad$ $\delta(q_0, \epsilon, X) \vdash (q_0, 1XX)$
$\delta(q_0, \epsilon, X) \vdash (q_0, 0)$ $\quad$ $\delta(q_0, \epsilon, Y) \vdash (q_0, 1S)$
$\delta(q_0, \epsilon, Y) \vdash (q_0, 0YY)$ $\quad$ $\delta(q_0, \epsilon, Y) \vdash (q_0, 1)$

$\delta(q_0, 0, 0) \vdash (q_0, \epsilon)$ $\delta(q_0, 1, 1) \vdash (q_0, \epsilon)$

Nondeterministic PDA

6.7 Why cannot the following language be implemented on PDA?

$L = \{a^m b^m \mid m \geq 1\} \cup \{a^m b^{2m} \mid m \geq 1\}$

Sol. PDA can be designed to

- remove one *b* with one *a* or
- to remove *bb* with one *a*

but not both simultaneously.

6.8 Design a top-down parser to implement the following CFG and parse the string 0102313010.

$S \rightarrow 0S0 \mid 1S1 \mid A \quad A \rightarrow 2B3 \quad B \rightarrow 23 \mid 31$

Sol.

P1: $S \rightarrow 0S0$ P2: $S \rightarrow 1S1$ P3: $S \rightarrow A$

P4: $A \rightarrow 2B3$ P5: $B \rightarrow 23$ P6: $B \rightarrow 31$

It is a LL(1) grammar and can be straight forwarded implemented.

R1: $\delta(q, \epsilon, Z_0) \vdash (q, SZ_0)$ R2: $\delta(q, 0, \epsilon) \vdash (q_0, \epsilon)$

R3: $\delta(q, 1, \epsilon) \vdash (q_1, \epsilon)$ R4: $\delta(q, 2, \epsilon) \vdash (q_2, \epsilon)$

R5: $\delta(q, 3, \epsilon) \vdash (q_3, \epsilon)$ R6: $\delta(q_0, \epsilon, S) \vdash (q_0, 0S0)$

R7: $\delta(q_1, \epsilon, S) \vdash (q_1, 1S1)$ R8: $\delta(q_2, \epsilon, S) \vdash (q_2, A)$

R9: $\delta(q_2, \epsilon, A) \vdash (q_2, 2B3)$ R10: $\delta(q_2, \epsilon, B) \vdash (q_2, 23)$

R11: $\delta(q_3, \epsilon, B) \vdash (q_3, 31)$ R12: $\delta(q_0, \epsilon, 0) \vdash (q, \epsilon)$

R13: $\delta(q_1, \epsilon, 1) \vdash (q, \epsilon)$ R14: $\delta(q_2, \epsilon, 2) \vdash (q, \epsilon)$

R15: $\delta(q_3, \epsilon, 3) \vdash (q, \epsilon)$ R16: $\delta(q, \$, Z_0) \vdash (q, \epsilon)$

Current State	Unread input	Pushdown Store Contents	Rule used
Q	0102313010$	Z_0	------
Q q_0	0102313010$ 102313010$	SZ_0 SZ_0	R1 R2
q_0	102313010$	$0S0Z_0$	R6
Q	102313010$	$S0Z_0$	R12
q_1	02313010$	$S0Z_0$	R3
q_1	02313010$	$1S10Z_0$	R7
Q	02313010$	$S10Z_0$	R13
q_0	2313010$	$S10Z_0$	R2
q_0	2313010$	$0S010Z_0$	R6

Q	2313010$	S010Z_0	R12
q_2	313010$	S010Z_0	R4
q_2	313010$	A010Z_0	R8
q_2	313010$	2B3010Z_0	R9
Q	313010$	B3010Z_0	R14
q_3	13010$	B3010Z_0	R5
q_3	13010$	313010Z_0	R11
Q	13010$	13010Z_0	R15
q_1	3010$	13010Z_0	R3
Q	3010$	3010Z_0	R13
q_3	010$	3010Z_0	R5
Q	010$	010Z_0	R15
q_0	10$	010Z_0	R2
Q	10$	10Z_0	R12
q_1	0$	10Z_0	R3
Q	0$	0Z_0	R13
q_0	$	0Z_0	R2
Q	$	Z_0	R12
Q	□	□	R16

6.9 Convert the following grammar to LL(1) type:

$S \to S{+}A \quad S \to A \quad A \to A/B \quad A \to B \quad B \to a1 \mid a2 \mid a3$

where $\{a, 1, 2, 3, +, /\}$ is the set of terminals.

Sol. S→ S + A involves left factoring. This can be removed as follows.

S→AS'

S'→+AS' | ϵ

Since null productions are not allowed. Hence we modify the production

S'→+AS'| +A

A→ A/B involves left factoring. This can be removed as follows.

A→BA'

A'→/BA'| /B

B→a1| a2|a3 can be converted to:

$B \rightarrow aN$ $N \rightarrow 1$ $N \rightarrow 2$ $N \rightarrow 3$

6.10 Design a PDA to accept the language $L = \{a^n b a^n \mid n, m \geq 1\}$ by null store. Construct the corresponding CFG.

Sol. PDA corresponding to CFL $L=\{a^n b a^n \mid n \geq 1\}$

$\delta(q_0, a, Z_0) \vdash (q_1, aZ_0)$ $\delta(q_1, a, a) \vdash (q_1, aa)$
$\delta(q_1, b, a) \vdash (q_2, a)$ $\delta(q_2, a, a) \vdash (q_2, \epsilon)$
$\delta(q_2, \epsilon, Z_0) \vdash (q_f, \epsilon)$

Corresponding CFG:

S Productions
$S \rightarrow [q_0, Z_0, q_0]$
$S \rightarrow [q_0, Z_0, q_1]$
$S \rightarrow [q_0, Z_0, q_2]$
$S \rightarrow [q_0, Z_0, q_f]$

Productions corresponding to $\delta(q_0, a, Z_0) \vdash (q_1, aZ_0)$
$[q_0, a, q_0] \rightarrow a[q_0, a, q_0]\ [q_0, Z_0, q_0]$
$[q_0, a, q_0] \rightarrow a[q_0, a, q_1]\ [q_1, Z_0, q_0]$
$[q_0, a, q_0] \rightarrow a[q_0, a, q_2]\ [q_2, Z_0, q_0]$
$[q_0, a, q_0] \rightarrow a[q_0, a, q_f]\ [q_f, Z_0, q_0]$

$[q_0, a, q_1] \rightarrow a[q_0, a, q_0]\ [q_0, Z_0, q_1]$
$[q_0, a, q_1] \rightarrow a[q_0, a, q_1]\ [q_1, Z_0, q_1]$
$[q_0, a, q_1] \rightarrow a[q_0, a, q_2]\ [q_2, Z_0, q_1]$
$[q_0, a, q_1] \rightarrow a[q_0, a, q_f]\ [q_f, Z_0, q_1]$

$[q_0, a, q_2] \rightarrow a[q_0, a, q_0]\ [q_0, Z_0, q_2]$
$[q_0, a, q_2] \rightarrow a[q_0, a, q_1]\ [q_1, Z_0, q_2]$
$[q_0, a, q_2] \rightarrow a[q_0, a, q_2]\ [q_2, Z_0, q_2]$
$[q_0, a, q_2] \rightarrow a[q_0, a, q_f]\ [q_f, Z_0, q_2]$

$[q_0, a, q_f] \rightarrow a[q_0, a, q_0]\ [q_0, Z_0, q_f]$
$[q_0, a, q_f] \rightarrow a[q_0, a, q_1]\ [q_1, Z_0, q_f]$
$[q_0, a, q_f] \rightarrow a[q_0, a, q_2]\ [q_2, Z_0, q_f]$
$[q_0, a, q_f] \rightarrow a[q_0, a, q_f]\ [q_f, Z_0, q_f]$

Productions corresponding to $\delta(q_1, a, a) \vdash (q_1, aa)$

$[q_1, a, q_0] \rightarrow a[q_1, a, q_0]\ [q_0, Z_0, q_0]$
$[q_1, a, q_0] \rightarrow a[q_1, a, q_1]\ [q_1, Z_0, q_0]$
$[q_1, a, q_0] \rightarrow a[q_1, a, q_2]\ [q_2, Z_0, q_0]$
$[q_1, a, q_0] \rightarrow a[q_1, a, q_f]\ [q_f, Z_0, q_0]$

$[q_1, a, q_1] \rightarrow a[q_1, a, q_0]\ [q_0, Z_0, q_1]$

$[q_1, a, q_1] \rightarrow a[q_1, a, q_1]\ [q_1, Z_0, q_1]$
$[q_1, a, q_1] \rightarrow a[q_1, a, q_2]\ [q_2, Z_0, q_1]$
$[q_1, a, q_1] \rightarrow a[q_1, a, q_f]\ [q_f, Z_0, q_1]$

$[q_1, a, q_2] \rightarrow a[q_1, a, q_0]\ [q_0, Z_0, q_2]$
$[q_1, a, q_2] \rightarrow a[q_1, a, q_1]\ [q_1, Z_0, q_2]$
$[q_1, a, q_2] \rightarrow a[q_1, a, q_2]\ [q_2, Z_0, q_2]$
$[q_1, a, q_2] \rightarrow a[q_1, a, q_f]\ [q_f, Z_0, q_2]$

$[q_1, a, q_f] \rightarrow a[q_1, a, q_0]\ [q_0, Z_0, q_f]$
$[q_1, a, q_f] \rightarrow a[q_1, a, q_1]\ [q_1, Z_0, q_f]$
$[q_1, a, q_f] \rightarrow a[q_1, a, q_2]\ [q_2, Z_0, q_f]$
$[q_1, a, q_f] \rightarrow a[q_1, a, q_f]\ [q_f, Z_0, q_f]$

Productions corresponding to $\delta(q_1, b, a) \vdash (q_2, a)$
$[q_1, a, q_1] \rightarrow b[q_2, a, q_1]$
$[q_1, a, q_2] \rightarrow b[q_2, b, q_2]$

Productions corresponding to $\delta(q_2, a, a) \vdash (q_2, \epsilon)$
$[q_2, a, q_2] \rightarrow \epsilon$

Productions corresponding to $\delta(q_2, \epsilon, Z_0) \vdash (q_f, \epsilon)$
$[q_2, Z_0, q_f] \rightarrow \epsilon$