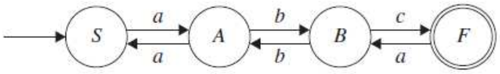
Chapter 3
Finite Automata

(Solution/Hints)

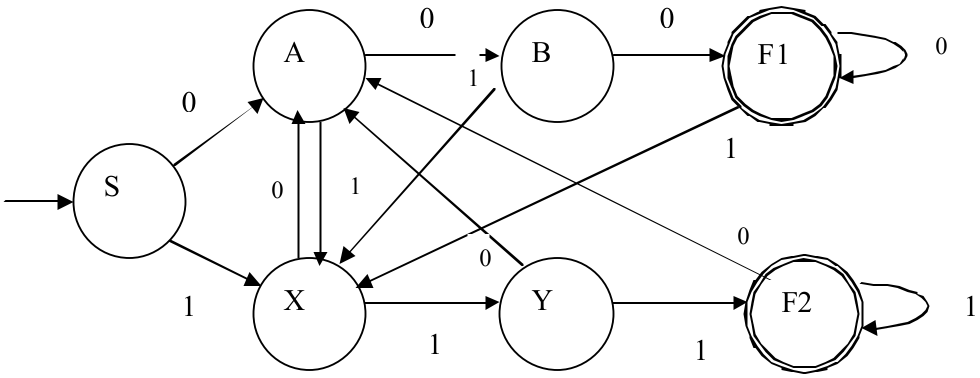
3.1 Find the path for the strings *abb, abca, aa, abb*, abbc in the finite automaton shown in the following figure:



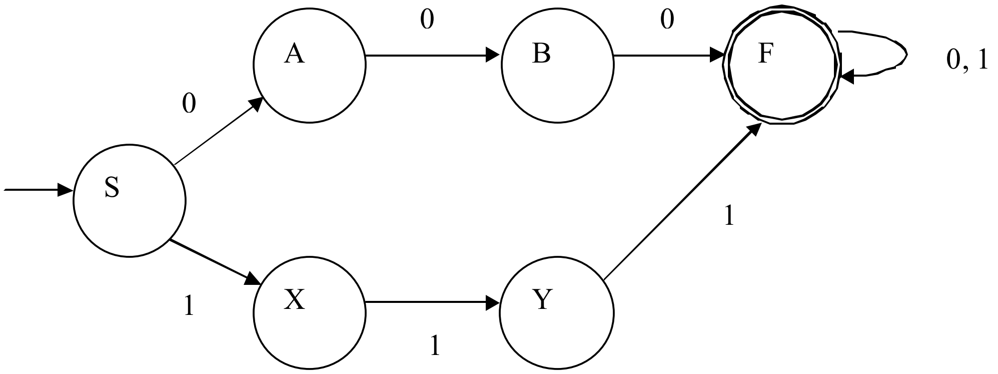
Sol. *abb*: S→A→B→A
abca: S→A→B→F→B
aa: S→A→S
abb: S→A→B→A
abbc: S→A→B→A (halts at A, no path for input symbol *c*)

3.2 Design a finite automaton *M* over {0, 1} to accept all strings satisfying the following conditions:
- (a) Ending with 111 or 000
- (b) Starting with 111 or 000
- (c) Containing the substring 000 or 111

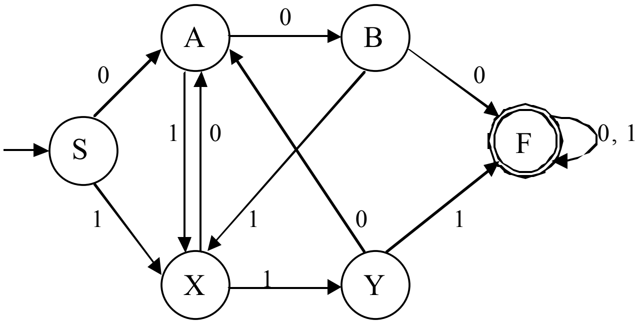
Sol. (a)



Sol. (b)

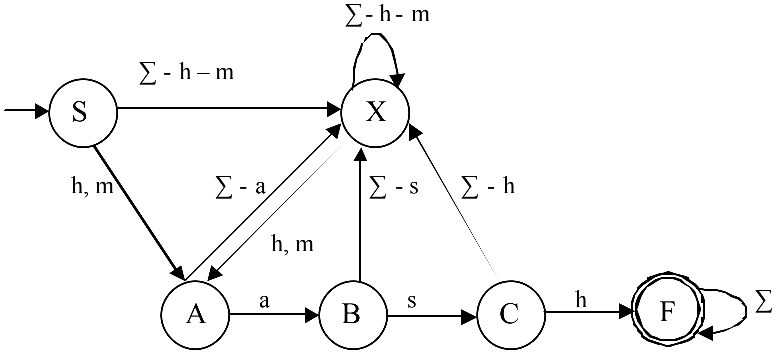


Sol. (c)



3.3 Design a finite automaton *M* over {*a*, *b*, ..., *z*} such that each string accepted by *M* contains a substring *hash* or *mash*.

Sol. Let ∑= {a, b, c, …………z}.



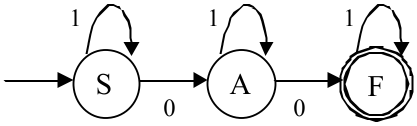
3.4 Design a finite automaton *M* over {0, 1} to accept all strings satisfying the following conditions:

(a) Containing exactly two 0s

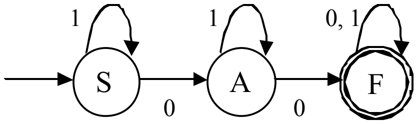
(b) Containing at least two 0s

(c) Containing at the most two 0s

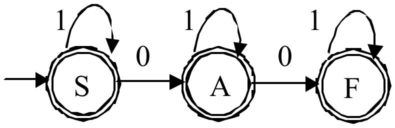
Sol. (a)



Sol.
(b)



Sol.
(c)



3.5 Design the DFA equivalent for the NFA given in the following table:

Current state	Input symbol	
	a	b
$\rightarrow q_0$	q_0, q_1	q_0, q_2
q_1	—	q_3
q_2	q_0, q_3	q_1
$\textcircled{q_3}$	q_2	—

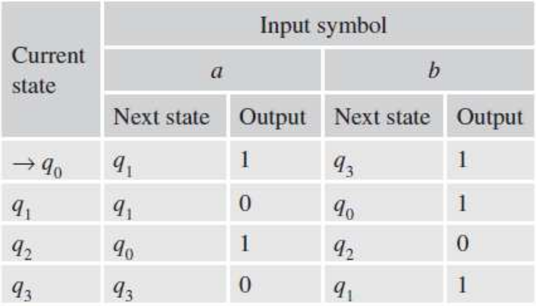
Sol.

Current State	Input Symbol	
	a	b
→{q0}	{q0 , q1}	{q0 , q2}
{q0 , q1}	{q0 , q1}	{q0 , q2 , q3}
{q0 , q2}	{q0 , q1 , q3}	{q0 , q1 , q2}
*{q0 , q2 , q3}	{q0 , q1, q2 , q3}	{q0 , q1 , q2}

*{q_0 , q_1 , q_3}	{q_0 , q_1 , q_2}	{q_0 , q_2 , q_3}
{q_0 , q_1 , q_2}	{q_0 , q_1 , q_3}	{q_0 , q_1, q_2 , q_3}
*{q_0 , q_1, q_2 , q_3}	{q_0 , q_1, q_2 , q_3}	{q_0 , q_1, q_2 , q_3}

* indicates final state.

3.6 For the Mealy machine given in the following table, find the equivalent Moore machine.

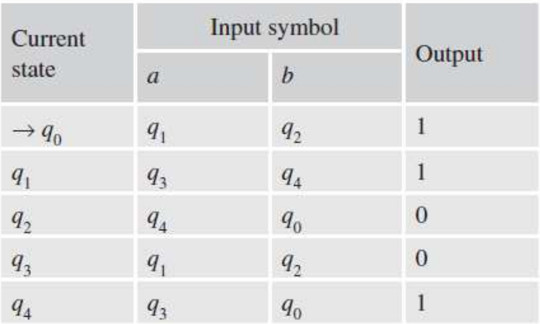


Current state	Input symbol			
	a		*b*	
	Next state	Output	Next state	Output
→ q_0	q_1	1	q_3	1
q_1	q_1	0	q_0	1
q_2	q_0	1	q_2	0
q_3	q_3	0	q_1	1

Sol.

Current State	Input Symbol		Output
	a	b	
→q_0	q_{11}	q_{31}	1
q_{10}	q_{10}	q_0	0
q_{11}	q_{10}	q_0	1
q_2	q_0	q_2	0
q_{30}	q_{30}	q_{11}	0
q_{31}	q_{30}	q_{11}	1

3.7 For the Moore machine given in the following table, find the equivalent Mealy machine.



Current state	Input symbol		Output
	a	*b*	
→ q_0	q_1	q_2	1
q_1	q_3	q_4	1
q_2	q_4	q_0	0
q_3	q_1	q_2	0
q_4	q_3	q_0	1

Sol

Current	Input Symbol

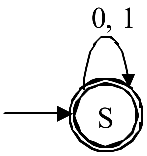
State	*a*		*b*	
	Next state	Output	Next state	Output
→q_0	q_1	1	q_2	0
q_1	q_3	0	q_4	1
q_2	q_4	1	q_0	1
q_3	q_1	1	q_2	0
q_4	q_3	0	q_0	1

3.8 In the finite automaton $M(Q, \Sigma, q_0, \delta, F)$, for every state $q_i \in Q$ there is a string w such that $\delta(q_i, w) = q_i \in F$. Describe the language accepted by the finite automaton.

Sol. Every string in the language ends with the substring *w*.

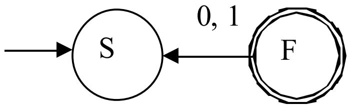
3.9 Design a finite automaton to accept to all possible strings over {0, 1}.

Sol.



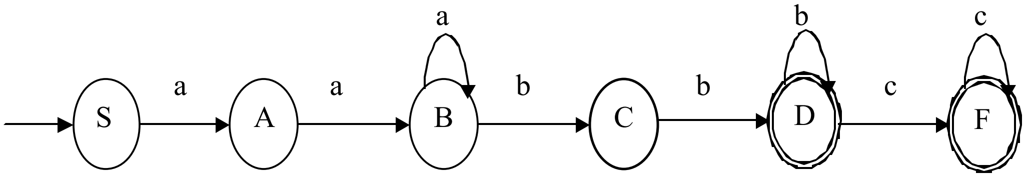
3.10 Design a finite automaton over {0, 1}, which does not accept any string.

Sol.

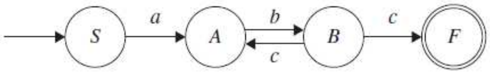


3.11 Design a finite automaton over $\{a, b, c\}$ to accept the language $L = \{a^i b^j c^k \mid i, j > 1 \text{ and } k \geq 0\}$.

Sol.

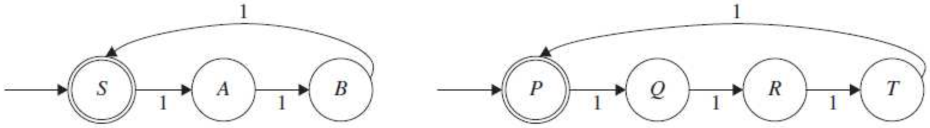


3.12 For the finite automaton given in the following figure, write the corresponding type-3 production system.



Sol. S→aA
A→bB
B→cA| c

3.13 Check whether the two finite automata given in the following figures are equivalent.

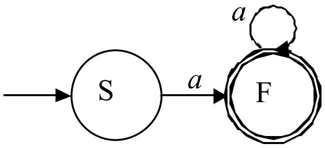


Sol.

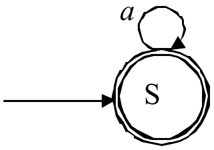
State	Input Symbol
	1
(S,P)	(A,Q)
(A,Q)	(B,R)
(B,R)	(S,T)

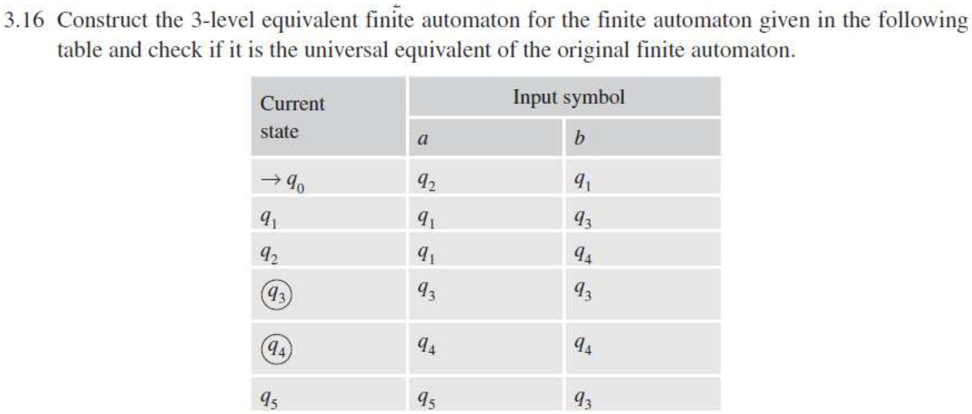
(S,T) is a non equivalent pair. Hence the Finite automata are not equivalent.

3.14 Construct a DFA for the language $L_1 = \{a, aa, aaa, aaa, aaaa, \ldots\}$.
Sol.



3.15 Construct a DFA for the language $L_2 = \{\epsilon, a, aa, aaa, aaa, aaaa, \ldots\}$.
Sol.





3.16 Construct the 3-level equivalent finite automaton for the finite automaton given in the following table and check if it is the universal equivalent of the original finite automaton.

Current state	Input symbol	
	a	*b*
$\rightarrow q_0$	q_2	q_1
q_1	q_1	q_3
q_2	q_1	q_4
$\textcircled{q_3}$	q_3	q_3
$\textcircled{q_4}$	q_4	q_4
q_5	q_5	q_3

Sol. **0 level equivalence**

$\Pi_0 = \{\{q_0, q_1, q_2, q_5\}, \{q_3, q_4\}\}$

1 level equivalence

$\Pi_1 = \{\{q_0\}, \{q_1, q_2, q_5\}, \{q_3, q_4\}\}$

2 level equivalence

$\Pi_2 = \{\{q_0\}, \{q_1, q_2, q_5\}, \{q_3, q_4\}\}$

3 level equivalence

$\Pi_3 = \{\{q_0\}, \{q_1, q_2, q_5\}, \{q_3, q_4\}\}$

Let $\{q_0\} = S_0$ $\{q_1, q_2, q_5\} = S_1$ $\{q_3, q_4\} = S_2$

Transition table for 3 level equivalence

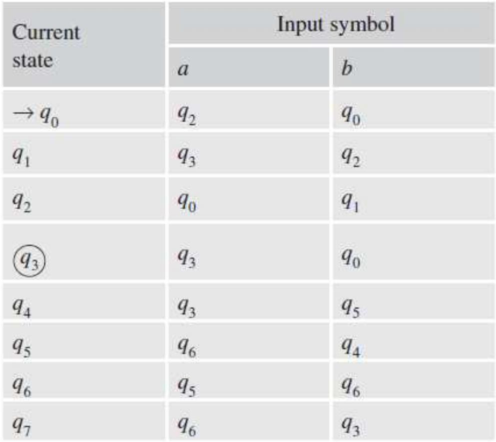
	a	*b*
$\rightarrow S_0$	S_1	S_1
S_1	S_1	S_2
$*S_2$	S_2	S_2

Comparison Table to check for universal equivalence

State Pair	*a*	*b*
$\rightarrow(q_0, S_0)$	(q_2, S_1)	(q_1, S_1)
(q_1, S_1)	(q_1, S_1)	(q_3, S_2)
(q_2, S_1)	(q_1, S_1)	(q_4, S_2)
(q_3, S_2)	(q_3, S_2)	(q_3, S_2)
(q_4, S_2)	(q_4, S_2)	(q_4, S_2)

All state pairs are equivalent, hence 3 level equivalence is universal equivalence.

3.17 Construct the 3-level equivalent finite automaton for the finite automaton given in the following table.



Current state	Input symbol	
	a	*b*
→ q_0	q_2	q_0
q_1	q_3	q_2
q_2	q_0	q_1
q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

Sol. **0 level equivalence**

$\Pi_0 = \{\{q_0, q_1, q_2, q_4, q_5, q_6, q_7\}, \{q_3\}\}$

1 level equivalence

$\Pi_1 = \{\{q_0, q_2, q_5, q_6\}, \{q_1, q_4\}, \{q_3\}, \{q_7\}\}$

2 level equivalence

$\Pi_2 = \{\{q_0, q_6\}, \{q_2, q_5\}, \{q_1, q_4\}, \{q_3\}, \{q_7\}\}$

3 level equivalence

$\Pi_3 = \{\{q_0, q_6\}, \{q_2, q_5\}, \{q_1, q_4\}, \{q_3\}, \{q_7\}\}$

Let $\{q_0, q_6\} = S_0$
$\{q_2, q_5\} = S_1$
$\{q_1, q_4\} = S_2$
$\{q_3\} = S_3$
$\{q_7\} = S_4$

Transition table for 3 level equivalence

	a	*b*
→S_0	S_1	S_0
S_1	S_0	S_2
S_2	S_3	S_1
*S_3	S_3	S_0
S_4	S_0	S_3

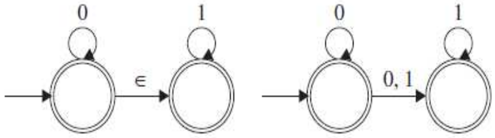
Comparison Table to check for universal equivalence

State Pair	*a*	*b*
→(q_0, S_0)	(q_2, S_1)	(q_0, S_0)

(q_2, S_1)	(q_0, S_0)	(q_1, S_2)
(q_1, S_2)	(q_3, S_3)	(q_2, S_1)
(q_3, S_3)	(q_3, S_3)	(q_0, S_0)

All state pairs are equivalent. 3 level equivalence is universal equivalence.

3.18 Check if the two finite automata given in the following figures are equivalent. Give reasons to support your answer.



Sol. Make DFA for both. First one accepts the string 0101second one does not. Hence they are not equivalent.

3.19 Design a finite automaton that does not accept any string.

Sol. There can be many versions for such an automaton. There can be one with no final state or one with path reaching to final state. See exercise 3.10.