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IRS

* Performance evaluation.

→ Accuracy
→ Error rate
→ Precision (should be higher)
→ Recall (should be higher)
→ F_1 measure
→ ROC. (Receiver Operating character)

→ Relevant $\subseteq$ Retrieved

For IR system
1. Precision
2. Avg. precision
3. Recall
4. ~~Avg. recall~~
5. F- score
6. Rank precision
7. P-R curve.

– → non relevant doc.

+ → Relevant doc

Rank(i)	Status	Precision (i)	Recall (i).
1	+	1/1 = 100%.	1/5 = 20%.
2	+	2/2 = 100%.	2/5 = 40%.
3	–	2/3 = 66.67%.	2/5 = 40%.
4	+	3/4 = 75%.	3/5 = 60%.
5	–	3/5 = 60%.	3/5 = 60%.
6	–	3/6 = 50%	3/5 = 60%.
7	+	4/7 = 56%.	4/5 = 80%
8	–	4/8 = 50%.	4/5 = 80%.
9	–	4/9 = 45%	4/5 = 80%.
10	–	4/10 = 40%	4/5 = 80%.

Precision at rank position (i) ⇒ $\dfrac{\text{\# relevant doc. retrieved. so far}}{\text{\# docs. retrieved so far.}}$

Recall at rank position (i) ⇒ $\dfrac{\text{\# relevant docs retrieved so far.}}{\text{\# relevant docs as per ground truth.}}$ } will be given.

Average precision = average of relevant precision only

$= \dfrac{100 + 100 + 75 + 56.6}{4}$

=

F-score ⇒ not relevant.

Rank precision ⇒ Precision for arbitrary rank i.

P-R Breakeven point ⇒ Value of rank position where precision = recall

i	Relevant	P(i)	R(i)
1	+	1/1	1/5
2	-	1/2	1/5
3	+	2/3	2/5
4	+	3/4	3/5
5	-	3/5	3/5
6	+	4/6	4/5
7	+	5/7	5/5
8	-	5/8	5/5
9	-	5/9	5/5
10	-	5/10	5/5

ground truth = 5

Avg. precision ⇒ $\frac{100 + 66.6 + 75 + 66.6 + 70}{5}$

PR Breakeven point = 5.

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* Precision - Recall curve

→

→ for the P-R curve, we use 11 standard recall levels, 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

We cannot obtain exact recall levels, thus, intercolation is used to obtain precision value at such recall levels.

∴ For intercolation

Let r_i be a recall level, where $r_i \in \{0, 100\}$.

$P(r_i)$ be precision value at recall level i. Then $P(r_i)$ is computed as

$$P(r_i) = \max_{r_i \le r \le r_{10}} (p(r))$$

i	P(i)	r(i)	P(r_i)
0		0	
1		10	
2		20	
3		30	
4		40	
5		50	
6		60	
7		70	
8		80	
9		90	
10		100	

Rank	P(i)	R(i)
1	100	20
2	100	40
3	100	60
4	75	60
5	80	80
6	66	80
7	56	80
8	63	100
9	56	100
10	50	100

i	r(i)	P(r(i))
0	0	100
1	10	100
2	20	100
3	30	100
4	40	100
5	50	100
6	60	100
7	70	80
8	80	80
9	90	63
10	100	63.

ground truth = 8

i	1	2	3	4	5	6	7	8	9	10
Rel	+	+	+	–	+	–	+	–	+	+
P(i)	100	100	100	75	80	66.6	70	62	66	70
r(i)	12	25	37	37	50	50	62	62	75	87

i	11	12	13	14	15	16	17	18	19	20
Rel	–	–	+	–	–	–	–	–	–	–
P(i)	63	58	8/13							
r(i)	87	87	100	100	100	100	100	100	100	100

we take P-R curve for only 0–10 only.

5/7
5/7
4/5
4/4
4/6
2/3
5/7
5/8
6/9
7/11 6/10
7/11
6/11
7/13
70
66
50

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* Some special case.

(1) 100% precision, 100% recall
(2) 100% precision, 0% recall
(3) 0% precision, 100% recall
(4) 0% precision, 0% recall.

IDEAL 1.) precision: all retrived documents are relavant.
recall: all relavant documents as per ground truth are retrived.

Not possible 2.) all retrived docs are relavant.
none of the retrived docs is relavant.

Not possible 3.) all irrelavant docs are ~~relavant~~ retrived.
all ~~retrived~~ relavant docs are retrived.

No rel. docs retrived

4.) If 1000 docs are retrived none of them is relavant.
not able to fetch any relavant docs.

0% precision ⇒ 0% recall.
0% precision ⇐ 0% recall

Goal: higher precision > higher recall.

- If recall increases, precision also increases.
- If precision increases, recall also increases.

ground truth: 4.

		p(l)	r(l)
+	1	1/1 100%	1/4 25%
+	2	2/2 100%	2/4 50%
−	3	2/3 67%	2/4 50%
+	4	3/4 75%	3/4 75%
−	5	3/5 60%	3/4 75%

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▷ Does stemming increase recall or decreased it?
→ stemming reduces the size of vocabulary.
|V| = {jump, go, today, jumped, went, jumping}
without stemming
|V| = {jump, go, today, went}

query :
|Q| = <jumping>

D1 : today went jumping.
D2 : Go jump today
D3 : Jumped today.

|V| = {go, jump, today, went}

cosine (d1, q)
cosine (d2, q)
cosine (d3, q)

B1 = <0, 1, 1, 1>
B2 = <1, 1, 1, 0>
B3 = <0, 1, 1, 0>
Q = <0, 1, 0, 0>

$\cos(D_1, Q) = \frac{1}{\sqrt{3}}$.

$\cos(D_2, Q) = \frac{1}{\sqrt{3}}$

$\cos(D_3, Q) = \frac{1}{\sqrt{2}}$.

▷ Relavance Feedback

1.) Closed loop Sysytem
2.) Feedback based Control System.
3.) Backpropagation → closed loop
FF → open loop

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Input ⟶ Process ⟶ O/P

Feedback

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RELAVANCE FEEDBACK

- query ⟶ Search Engine ⟶ Responses

revised query

Examine the responses.

- Lablled learning
  - ↳ supervised learning.

1. Relavant
2. Non relavant

- query : set of Keywords.
- Revisions in original query.
  1) removing certain ~~query~~ keywords. → in non-relavant docs.
  2) adding certain keywords. → in relavant docs.

explicit from user asking feedback

1) Rochhio method. → supervised learning
2) Semi-Supervised learning
   - ① L-U learning
     - ↳ Labelled - Unlabellebd
   - ② P-U learning
     - ↳ positive (relavant) unlabelled
       - ↳ (doesn't mean non-relavant) (NOT REVIEWED.)
3) Implicit Feedback.
   - If user only clicks perticular links, we assume that it will be relavant only.

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- Pseudo- Relavance feedback.
  extract keywords from top-ranked documents.

- Meta Search.
  more than one search engines are involved.

Judges/ Search Engines:

		J_1	J_2	J_3	J_4
Participating Search Responces.	P_1	1	2	5	4
	P_2	2	1	4	3
	P_3	3	4	1	2
	P_4	4	3	2	1
	P_5	5	5	3	5

P - candidates
J - voters.

1.) Borda (1770) election by order of merit
2.) Reciprocal
3.) Condorcet.

(1.) Borda.

candidate 1 : 5 + 4 + 1 + 2 = 12
2 : 4 + 5 + 2 + 3 = 14
3 : 3 + 2 + 5 + 4 = 14
4 : 2 + 3 + 4 + 5 = 14
5 : 1 + 1 + 3 + 1 = 6

(2, 3, 4) Tie.

In case of tie, we randomly select the winner.
↳ If there is no weightage of ranks.

- If there are candidates unranked by a voter, then the remaining points are divided evenly among the unranked candidates.

not ranked.

1	2	-	4
2	1	-	3
3	4	1	2
4	3	2	1
5	5	3	5

$\Rightarrow$ 3 remaining points, 2 ~~ranked~~ unranked candidates $\Rightarrow \frac{3}{2} = 1.5$ $\downarrow$ evenly distributed

$\Rightarrow$ $5 + 4 + 1.5 + 2 = 12.5$

$4 + 5 + 1.5 + 3 = \cancel{14}\ 13.5$

$3 + 2 + 5 + 4 = 14$

$2 + 3 + 4 + 5 = 14$

$1 + 1 + 3 + 1 = 6$

(2.) Reciprocal

↳ 1 point to the highest ranked candidate.

$1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{4} = \frac{39}{20}$

$\frac{1}{2} + 1 + \frac{1}{4} + \frac{1}{3} = \frac{13}{12} + 1 = \frac{25}{12}$

$\frac{1}{3} + \frac{1}{4} + 1 + \frac{1}{2} = \frac{25}{12}$

$\frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1 = \frac{25}{12}$

$\frac{1}{5} + \frac{1}{5} + \frac{1}{3} + \frac{1}{5} = \frac{14}{15}$

→ If some candidates are not ranked than put score for that candidate as 'zero'.

28|3|22.

- Meta Search
  - Example on borda & Reciprocal.
  - Understanding condercet method.

BORDA

System 1	a b c d	
System 2	b a d c	
System 3	c b a d	
System 4	c b d	
System 5	c b	

a : 4 + 3 + 2 + 1 + 1.5 = 11.5 (3) ($1 \rightarrow 1/1$, $1.5 \rightarrow 3/2$)

b : 3 + 4 + 3 + 3 + 3 = 16 (1)

c : 2 + 1 + 4 + 4 + 4 = 15 (2)

d : 1 + 2 + 1 + 2 + 1.5 = 7.5 (4) ($1.5 \rightarrow 3/2$)

final ranking = B, C, A, D

- Reciprocal

System a : $1 + 1/2 + 1/3 + 0 + 0 = 11/6 = 1.83$

b : $1/2 + 1 + 1/2 + 1/2 + 1/2 = 3 = 3$

c : $1/3 + 1/4 + 1 + 1 + 1 = 43/12 = 3.5$

d : $1/4 + 1/3 + 1/4 + 1/3 + 0 = 7/6 = 1.16$

Final Ranking : ~~BCDA~~ C, B, A, D.

- Condorcet.

step 1 : yield an NxN matrix (no. of candidates / docs) for all pair wise comparisions.

step 2 : Calculate the number of wins, loses and tie from each non diagonal entry of the matrix and

step 3 : determine pairwise winners.

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STEP:1

Pairs:

Pairs	S_1	S_2	S_3	S_4	S_5
a, b	a	b	b	b	b
a, c	a	a	c	c	c
a, d	a	a	a	d	–
b, c	b	b	c	c	c
b, d	b	b	b	b	b
c, d	c	d	c	c	c

→ TIE

[win : lose : tie.]

STEP:2

	a	b	c	d.
a	-	1:4:0	2:3:0	3:1:1
b	4:1:0	–	2:3:0	5:0:0
c	3:2:0	3:2:0	-	4:1:0
d	1:3:1	0:5:0	1:4:0	-

STEP 3

	win	loses	tie.
a	1	2	0
b	2	1	0
c	3	0	0.
d	0	3	0

Final Ranking: C B A D.

→ if same no. of wins, then go for less no. of losses → higher ranke.

→ if both same, then do tie or random selection.

	J1	J2	J3	J4	~~J5~~
P1	1	2	5	4	
P2	2	1	4	3	
P3	3	4	1	2	
P4	4	3	2	1	
P5	5	5	3	5	

Pairs	J1	J2	J3	J4
$P_1 P_2$:	P_1	P_2	P_2	P_2
$P_1 P_3$:	P_1	P_1	P_3	P_3
$P_1 P_4$:	P_1	P_1	P_4	P_4
$P_1 P_5$:	P_1	P_1	P_5	P_1
$P_2 P_3$:	P_2	P_2	P_3	P_3
$P_2 P_4$:	P_2	P_2	P_4	P_4
$P_2 P_5$:	P_2	P_2	P_5	P_2
$P_3 P_4$:	P_3	P_4	P_3	P_4
$P_3 P_5$:	P_3	P_3	P_3	P_3
$P_4 P_5$:	P_4	P_4	P_4	P_4

J_1 : P1, P2, P3, P4, P5

J_2 : P2, P1, P4, P3, P5

J_3 : P3, P4, P5, P2, P1

J_4 : P4, P3, P2, P1, P5

	P_1	P_2	P_3	P_4	P_5
P_1	-	1:3:0	2:2:0	2:2:0	3:1:0
P_2	3:1:0	-	2:2:0	2:2:0	3:1:0
P_3	2:2:0	2:2:0	-	2:2:0	4:0:0
P_4	2:2:0	2:2:0	2:2:0	-	4:0:0
P_5	1:3:0	1:3:0	0:4:0	0:4:0	-

	win	lose	tie
P_1	1	1	2
P_2	2	0	2
P_3	1	0	3
P_4	1	0	3
P_5	0	4	0

final Ranking: P_2, P_3, P_4, P_1, P_5
(or)
P_2, P_4, P_3, P_1, P_5

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- Linear Algebra
- Vectors

$v_1 \langle 2,3,1 \rangle$ $\qquad ||\vec{v_1}|| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$

$v_2 \langle 1,0,4 \rangle$ $\qquad ||\vec{v_2}|| = d \qquad = \sqrt{17}$

$v_3 \langle 3,3,5 \rangle$ $\qquad ||\vec{v_3}|| = d$

$\vec{v_1} \cdot \vec{v_2} = dot(v_1, v_2) = 2\cdot1 + 3\cdot0 + 1\cdot4$
$= 2+0+4$
$= 6.$

$$\begin{array}{ccc} \vec{v_1} & \vec{v_2} & \vec{v_3} \end{array}$$
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = 1(45-48) - 2(36-42) + 3(32-35)$$
$= 0.$

orthonormal = orthogonal + normal.

- gram-schmidt orthonormalization

- plane : (orthonormal) ~~ortho~~

$v_1 \langle 1,0,0 \rangle$
$v_2 \langle 0,1,0 \rangle$
$v_3 \langle 0,0,1 \rangle$

- eigen vectors and eigen values.

Singular
Value
Decomposition
(SVD)

$A = USV^T$ $\qquad$ Input $\xrightarrow{A}$ $\boxed{SVD}$ $\longrightarrow$ U, S, V

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$Av = \lambda v$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \lambda v_1 \\ \lambda v_2 \end{bmatrix}$$

$$\begin{bmatrix} 2v_1 + v_2 \\ v_1 + 2v_2 \end{bmatrix} = \begin{bmatrix} \lambda v_1 \\ \lambda v_2 \end{bmatrix}$$

$2v_1 + v_2 = \lambda v_1$

$v_1 + 2v_2 = \lambda v_2$

$2v_1 + v_2 = \lambda v_1$

$2v_1 + 4v_2 = 2\lambda v_2$

$- \quad - \quad -$

$-3v_2 = \lambda(v_1 - 2v_2)$

$\lambda = \frac{-3}{v_1 - 2v_2}$

$(2-\lambda)v_1 + v_2 = 0$

$v_1 + (2-\lambda)v_2 = 0.$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$4 - 4\lambda + \lambda^2 - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0.$

$\lambda_1 = 3$

$\lambda_2 = 1.$

$\Downarrow$

$v_1 = -v_2$

$v_1 = v_2.$

Gram - Schmidt

Orthonormalization.

method of converting ~~seta~~ of vectors into set of orthonormal vectors. into a set of orthonormal vectors.

~~It~~ It basically begins by normalizing first vector under consideration and iteratively rewriting the remaining vectors in terms of themselves minus a multiplication of the already normalized

$$\vec{W_k} = \vec{V_k} - \sum_{i=1}^{k-1} \vec{u_i} \cdot \vec{V_k} * \vec{u_i}$$

$$= [2,2,3,1] - \left[\frac{1}{\sqrt{6}}, 0, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right] \cdot [2,2,3,1]$$

$$= [2,2,3,1] - \frac{9}{\sqrt{6}}\left[\frac{1}{\sqrt{6}}, 0, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right]$$

$$= [2,2,3,1] - \left[\frac{3}{2}, 0, 3, \frac{3}{2}\right]$$

$$\vec{w_2} = \left[\frac{1}{2}, 2, 0, -\frac{1}{2}\right]$$ $\vec{u_2}$ = normalize $\vec{w_2}$

$$\vec{w_3} = \vec{V_3} - (\vec{u_1} \cdot \vec{V_3} * \vec{u_1} - \vec{u_2} \cdot \vec{V_3} * \vec{u_2}]$$

=

$\vec{u_3}$ = Normalise $\vec{w_3}$

$$\begin{array}{ccc} \vec{u_1} & \vec{u_2} & \vec{u_3} \end{array}$$
$$\begin{bmatrix} 1/\sqrt{6} & \sqrt{2}/\sqrt{6} & \\ 0 & 2\sqrt{2}/3 & \\ 2/\sqrt{6} & 0 & \\ 1/\sqrt{6} & -\sqrt{2}/6 & \end{bmatrix}$$

$$\vec{w_3} = [4/9, -2/9, 0, -4/9]$$

- Singular Value Decomposition
  - SVD is a method for transforming corelated variables into a set of uncorrelated variables that better exposed the relationship about the original data. it's a method for identyfying

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and ordering the dimensions along which data points exhibits most variations. therefore, it is possible to find the best approximation of the original data points using fewer dimensions.

Using SVD, the matrix A can be broken down in to the product of 3 matrices. and orthogonal matrix U, a diagonal matrix S and transpose of orthogonal matrix V

diagonal

$$A = U S V^T$$

orthogonal.

where, $U^T U = I$ and
$V^T V = I$

The columns of U are orthonormal eigen vectors of $A \cdot A^T$.

The columns of V are orthonormal eigen vectors of $A \cdot A^T$ and S (the diagonal matrix) containing the square roots of eigen values from U or V in descending order.

ex $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$ $\quad \underset{2\times 3}{A} = \underset{2\times p}{U} \; \underset{p\times p}{S} \; \underset{p\times 3}{V^T}$

▸ The columns of U are orthonormal eigenvectors of $A \cdot A^T$

$\rightarrow A \cdot A^T = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 11 & 1 \\ 1 & 11 \end{bmatrix}$

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(1.)
$$\begin{bmatrix} 11 & 1 \\ 1 & 11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \lambda V_1 \\ \lambda V_2 \end{bmatrix}$$

$$\begin{bmatrix} 11V_1 + V_2 \\ V_1 + 11V_2 \end{bmatrix} = \begin{bmatrix} \lambda V_1 \\ \lambda V_2 \end{bmatrix}$$

$11V_1 + V_2 = \lambda V_1$

$V_1 + 11V_2 = \lambda V_2$.

$\Rightarrow$ $(11-\lambda)V_1 + V_2 = 0$

$V_1 + (11-\lambda)V_2 = 0$.

$$\begin{vmatrix} (11-\lambda) & 1 \\ 1 & (11-\lambda) \end{vmatrix} = 0$$

$(11-\lambda)^2 - 1 = 0$

$121 - 22\lambda + \lambda^2 - 1 = 0$

$\lambda^2 - 22\lambda + 120 = 0$

$\lambda^2 - 10\lambda - 12\lambda + 120 = 0$

$(\lambda - 12)(\lambda - 10) = 0$

$\lambda = 12, 10$

$\xrightarrow{\lambda=12}$ $-V_1 + V_2 = 0$

$V_1 - V_2 = 0$

$V_1 = V_2$. $[1, 1]$

$\xrightarrow{\lambda=10}$ $V_1 + V_2 = 0$

$V_1 + V_2 = 0$

$V_1 = -V_2$ $[1, -1]$

(2.) eigenvectors: $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

this is orthogonal but not orthonormal.

$\Rightarrow$ ~~ortho~~ normalising: $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

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$$A^T A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 & 4 \\ 0 & 10 & 4 \\ 2 & 4 & 2 \end{bmatrix}$$

$\lambda_1 = 12$

$\lambda_2 = 10$

$\lambda_3 = 0.$

$$\begin{bmatrix} 10 & 6 & 4 \\ 0 & 10 & 4 \\ 2 & 4 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \lambda V_1 \\ \lambda V_2 \\ \lambda V_3 \end{bmatrix}$$

$\cancel{(10-\lambda)V_1 + 6}$ $(10-\lambda)V_1 + 6V_2 + 4V_3 = 0$

$(10-\lambda)V_2 + 4V_3 = 0.$

$(2)V_1 + 4V_2 + (2-\lambda)V_3 = 0.$

$\lambda = 12$

$\longrightarrow$ $2V_1 + 6V_2 + 4V_3 = 0$

$-2V_2 + 4V_3 = 0.$

$2V_1 + 4V_2 - 10V_3 = 0$

$\rightarrow V_1 + 3V_2 + 2V_3 = 0$

$2V_3 = V_2$

$V_1 + 2V_2 - 5V_3 = 0$

$\rightarrow V_1 + 6V_3 + 2V_3 = 0$

$V_1 + 4V_3 - 5V_3 = 0$

$V_1 + 8V_3 = 0.$

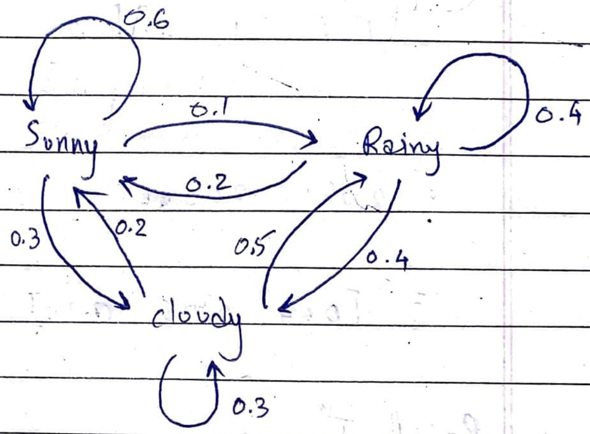
6/4/22

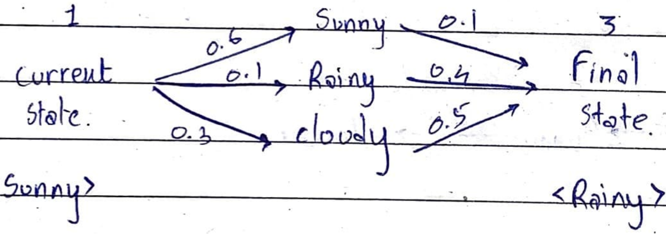
▷ Apple

11/4 ▷ Markov Models

- weather
- Prediction.
  - Sunny
  - Rainy
  - Cloudy

$\boxed{n=3}$





$$\begin{array}{r} 0.06 \\ +\ 0.04 \\ +\ 0.15 \\ \hline 0.25 \end{array}$$

$$T = P\% = \begin{array}{c} \\ S \\ R \\ C \end{array} \begin{array}{c} \begin{array}{ccc} S & R & C \end{array} \\ \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.2 & 0.4 & 0.4 \\ 0.2 & 0.5 & 0.3 \end{bmatrix} \end{array}$$

▷ State Transition Matrix

Summation of probability : 1 across rows ~~and columns~~

$P_0 = [P_1 \ P_2 \ P_3]$ → Initial state matrix

$P_0[0\ 1\ 0] \longrightarrow P_0 = [0.3 \ 0.4 \ 0.3]$

$P_0[1\ 0\ 0] \longrightarrow P_1 = [P_1 \ P_2 \ P_3]$ $[0.6 \ 0.1 \ 0.3]$

$P_1 = P_0 T$

$(1\times3) \ (3\times3)$

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$$P_1 = [0.6 \quad 0.1 \quad 0.3]$$

$$P_2 = P_1 T$$

$$= [0.6 \quad 0.1 \quad 0.3] \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.2 & 0.4 & 0.4 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}$$

$$= 0.36 + 0.02 + 0.06 \qquad 0.06 + 0.04 + 0.15 \qquad 0.18 + 0.04 + 0.09$$

$$= [0.44 \quad 0.25 \quad 0.31]$$

$$P_n = P_0 \cdot T^n.$$

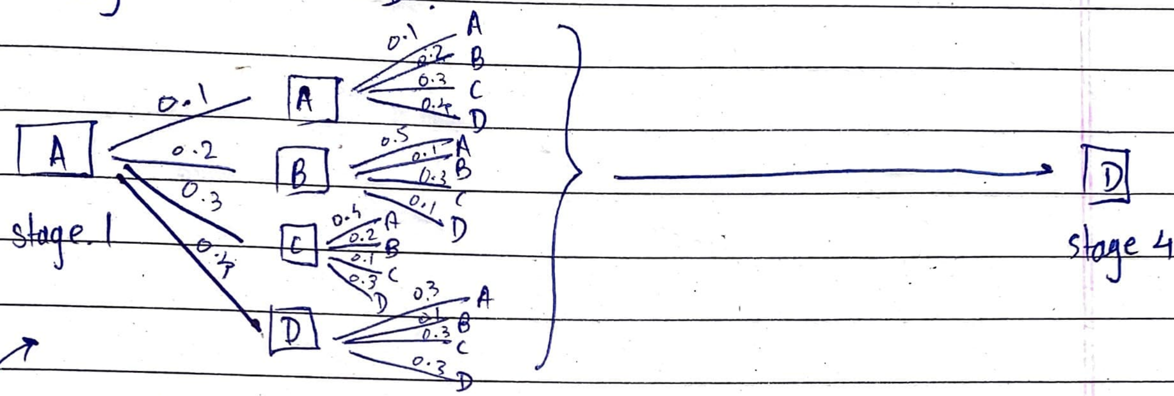
$$\text{Rainy} \xrightarrow{0.2} \text{Sunny} \xrightarrow{0.3} \text{Cloudy}.$$

yesterday - Rainy
today - Sunny
tomorrow - cloudy

$$T = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.5 & 0.1 & 0.3 & 0.1 \\ 0.4 & 0.2 & 0.1 & 0.3 \\ 0.3 & 0.1 & 0.3 & 0.3 \end{bmatrix} \end{array}$$

→ valid matrix → as row wise summation is one.
→ four states : 16 transitions.
→ $P(ABCD) = P(B|A) \times P(C|B) \times P(D|C)$
$= 0.2 \times 0.3 \times 0.3$
$= 0.018$

→ initial stage 1 is A. what is the probability of stage 4 is D.



This is the probability tree.

A	O	O	D.
	A	ABCD	
	B	ABCD	
	C	ABCD	
	D	ABCD.	

first possible path : AAAD

last possible path : ADDD