Chapter 1
Automata, Formal Languages, and Computability

(Solutions / Hints)

1.1 Given the grammar G with the following productions

$S \rightarrow S + S \mid S \times S \mid S - S \mid S/S \mid (S) \mid a \mid b \mid c \mid d$

derive the following strings

(a) $a + (b*c)/d$
(b) $a*(b + d)$
(c) $(a)*(b) + d$

Sol. Please read S x S as S*S

a) S → S+S →*a*+S→ *a* +S/S→ *a* +(S)/S→ *a*+(S*S)/S→ *a*+(*b***c*)/*d*
b) S → S*S →*a* *(S)→ *a**(S+S)→ *a**(*b*+*d*)
c) S → S+S →S*S+*d*→(S)*(S)+*d*→ (*a*)*(*b*)+*d*

1.2 Design a grammar for the language $L = \{a^n b^n \mid n \geq 0\}$.

Sol. S→*a*S*b*|∈

1.3 Find the language generated by the following grammars:

(a) $S \rightarrow aSb \mid aXb$ $\quad X \rightarrow bX \mid b$
(b) $S \rightarrow aA \mid bS \mid a \mid b$ $\quad A \rightarrow bA \mid bS \mid b$

Hint.

(a) The language set will contain the strings in which a substring of *b*'s follows the substring of *a*'s. The number of b's would be greater than the number of *a*'s. For example, *aabbb, aabbbb, abbbb,* etc.
(b) No sentence in the language will contain two consecutive *a*'s.

1.4 Design a grammar for the language $L = \{a^n b^{2n} \mid n \geq 1\}$.

Sol. S→*a*S*bb*|*abb*

1.5 Show that language generated by the following grammar is empty (does not contain any string).

$S \rightarrow 0AB \quad A \rightarrow 0A1 \mid 1S \quad B \rightarrow 00 \mid 11$

Sol. The non terminal A does not converge to any terminal, no sentence can be created and the language is empty.