⭐ Never blindly trust anything.

If you find any mistake, kindly correct it and if possible inform in our grp too.

Thank you
18BCE120.

Plz, don't download, use directly from drive, so that If you get most recent and updated answers.

Update / Correction will be mentioned here.

Date

28-03-21 → Q. 2.1

30-03-21 → Q 3

31-03-21 → Q. 2.3.

2-04-21 → Q. 2.3, 2.5.

4-04-21 → Q. 2.3, 2.6, 2.f, 2.g.

05-04-2021 Q2.4

Tutorial-6

Page No.:
Date:

1

Design CFG for given languages :-

(a) $\{a^i b^j c^k \mid i = j+k\}$

$S \rightarrow aS_1c \mid \varepsilon$

$S_1 \rightarrow aS_1b \mid \varepsilon$

(b) $\{a^i b^j c^k \mid j = i+k\}$

$S_0 \rightarrow aS_1bS_2 \mid S_1bS_2c \mid \varepsilon$

$S_1 \rightarrow aS_1b \mid \varepsilon$

$S_2 \rightarrow bS_2c \mid \varepsilon$

(c) $\{a^i b^j c^k \mid j = i \text{ or } j = k\}$

$S_0 \rightarrow aS_1bS_2 \mid S_3bS_4c \mid S_2 \mid S_3$

$S_1 \rightarrow aS_1b \mid \varepsilon$

$S_2 \rightarrow cS_2 \mid \varepsilon$

$S_3 \rightarrow aS_3 \mid \varepsilon$

$S_4 \rightarrow bS_4c \mid \varepsilon$

(d) $\{ a^i b^j c^k \mid i = j \text{ or } i = k \}$

$S_0 \to aS_1b \mid aS_3c \mid S_2 \mid S_4$

$S_1 \to aS_1b \mid \varepsilon$

$S_2 \to cS_2 \mid \varepsilon$

$S_3 \to aS_3c \mid S_4$

$S_4 \to bS_4 \mid \varepsilon.$

(e) $\{ a^i b^j c^k \mid i < j \text{ or } i > k \}$

$S_0 \to S_1b \mid aS_3$

$S_1 \to S_1b \mid aS_1b \mid S_2$

$S_2 \to cS_2 \mid \varepsilon$

$S_3 \to aS_3c \mid S_3c \mid S_4$

$S_4 \to bS_4 \mid \varepsilon$

(f) $\{ a^i b^j \mid i \leq 2j \}$

$S_0 \to aaS_0b \mid aS_0b \mid S_0b \mid \wedge.$

(6) $\{a^i b^j \mid i < 2j\}$

(7)

$S_0 \rightarrow S_1 b$

$S_1 \rightarrow aaS_1 b \mid aS_1 b \mid S_1 b \mid \wedge$

~~$S \rightarrow bS \mid \varepsilon$~~

(8) $\{a^i b^j \mid i \leq j \leq 2i\}$

$S_0 \rightarrow aS_0 S_1 \mid \varepsilon$

$S_1 \rightarrow bb \mid b$.

(1) $S \rightarrow aSa \mid bSb \mid \wedge$ (2)

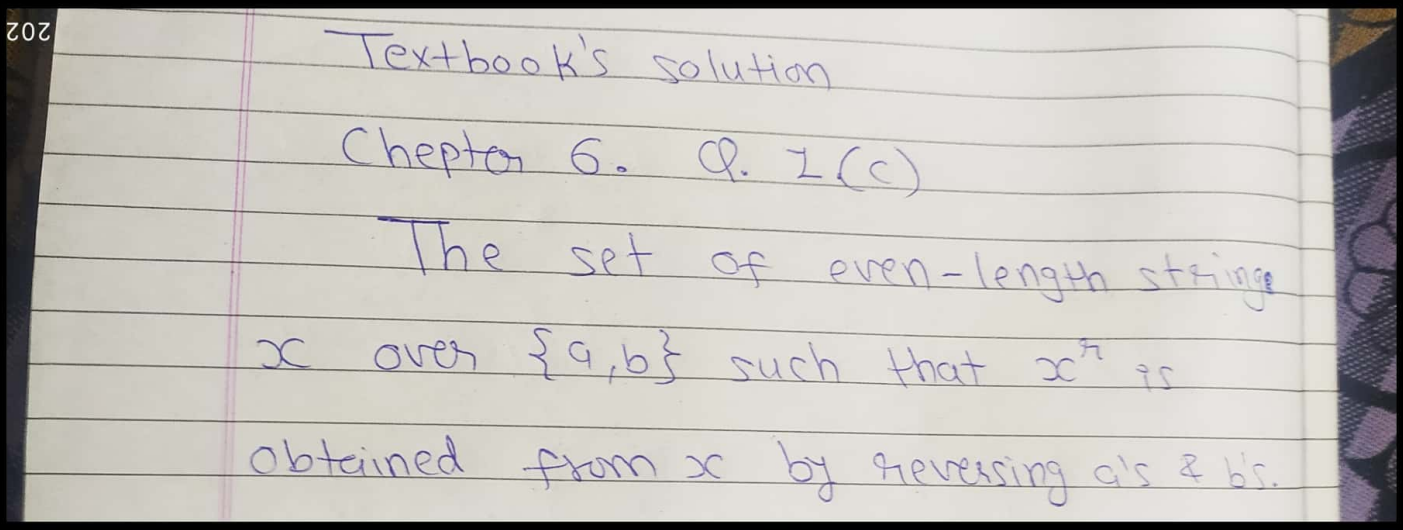
$\{S \mid S \in (a,b)^*$ and S is palindrom of even length$\}$

(2) $S \rightarrow aSa \mid bSb \mid a \mid b$

$\{S \mid S \in (a,b)^*$ and S is palindrome of odd length$\}$

(3) $S \rightarrow$

TextBook's Solution which I have



Textbook's solution

Chepter 6. Q. 1(c)

The set of even-length strings x over {a,b} such that x^R is obtained from x by reversing a's & b's.

3) My Solution

$S \rightarrow aSb \mid bSa \mid \wedge$

$L = \{ XY \mid Y$ is compliment of reverse of $X \}$. $\Sigma(a,b)$

To derive Y. 1) Reverse X
2) then replace a with b & b with a.

aababb, ab, ba
aaabbb,

terminal.

Textbook Chapter 6 Q. 1. d

(4) $S \rightarrow aSa \mid bSb \mid aAb \mid bAa$

$A \rightarrow aAa \mid bAb \mid a \mid b \mid \wedge$

$L = \{ s \mid s \in (a,b)^* \text{ and } s \text{ is not palindrome} \}$

but could be made into palindrome by changing one symbol.

a to b or b to a

5th & 6th.

sol^{n} $L = \{ s \mid s \in (a,b)^*$, s ends with 'a' and $n_a \geq 1$, $n_b \geq 0 \}$

(7) $S \to aSa \mid b$

$\{ a^n b^n a^n \mid n \geq 1 \}$

(8)

$S \to aT \mid bT \mid \wedge$

$T \to aS \mid bS$

~~[illegible]~~

$\Longrightarrow$ ~~even length string~~

$L = \{ S \mid S \in (a,b)^* \text{ and } |S| \text{ is even} \}$

[If this kind of Question asked in written exam : Define Lang. for given grammar.

2 Mark : write only lang

2 Mark : show the steps & write lang.

$S \to aSbS \mid \wedge$

3 . Yes

→ Proof not yet done.

No

→ as it can not generate string aabbcc.

CNF $A \to BC$ or $A \to b$

4

1. $S \to aAbB$
   $A \to Ab \mid b$
   $B \to Bala$

Step 1: Remove null production ✓

Step 2: Remove unit production ✓

Step 3:

Reduce $S \to aAbB$

$\therefore S \to aH_0$ $\quad H_0 \to AbB$

Reduce $H_0 \to AbB$

$\therefore H_0 \to AH_1$ $\quad H_1 \to bB$

Reduce

Hence, after step 3.

$S \to aH_0$
$H_0 \to AH_1$
$H_1 \to bB$
$A \to Ab \mid b$
$B \to Bala$

Step 4: convert to $A \to BC$ form

convert $S \to aH_0$

$\therefore S \to H_2H_0$ $\quad H_2 \to a$

convert $H_2 \rightarrow bB$

$\therefore H_1 \rightarrow H_3B \quad H_3 \rightarrow b$

convert $A \rightarrow Ab$

$\therefore A \rightarrow AH_3$

convert $B \rightarrow Bq$

$\therefore B \rightarrow BH_2$ ~~$\rightarrow q$~~

CNF

$S \rightarrow H_2H_0$

$A \rightarrow AH_3 \mid b$

$B \rightarrow BH_4 \mid q$

$H_0 \rightarrow AH_1$

$H_1 \rightarrow H_3B$

$H_2 \rightarrow q$

$H_3 \rightarrow b$

~~$H_4 \rightarrow q$~~

(2)

$S \rightarrow aA \mid bB$
$A \rightarrow bAA \mid a$
$B \rightarrow BBa \mid b$

→ Step 1: Null production Remove ✓

→ Step 2: Unit production Remove ✓

→ Step 3:
~~Reduce to A → BC form~~

Reduce $A \rightarrow bAA$

$\therefore A \rightarrow bH_0 \quad H_0 \rightarrow AA$

$\therefore A \rightarrow bH_0 \mid a \quad H_0 \rightarrow AA$

Reduce $B \rightarrow BBa$

$\therefore B \rightarrow BH_1 \quad H_1 \rightarrow Ba$

$\therefore B \rightarrow BH_1 \mid b \quad H_1 \rightarrow Ba$

→ after Step 3.

$S \rightarrow aA \mid bB$
$A \rightarrow bH_0 \mid a$
$B \rightarrow BH_1 \mid b$
$H_0 \rightarrow AA$
$H_1 \rightarrow Ba$

→ step 4:
convert to $A \to BC$ or $A \to b$ form

convert
$S \to aA$

$\therefore S \to H_2A \quad H_2 \to a$

convert $S \to bB$

$\therefore S \to H_3B \quad H_3 \to b$

$\therefore S \to H_2A \mid H_3B$

$H_2 \to a, \quad H_3 \to b$

convert $A \to bH_0$

$\therefore A \to H_3H_0 \mid a$.

convert $H_1 \to Ba$

$\therefore H_1 \to BH_2$

⟹ CNF

$S \to H_2A \mid H_3B$
$A \to H_3H_0 \mid a$
$B \to BH_2 \mid b$
$H_0 \to AA$
$H_1 \to BH_2$
$H_2 \to a$
$H_3 \to b$

(3)

$S \rightarrow aAC$
$A \rightarrow aB \mid bAB$
$B \rightarrow b$
$C \rightarrow c$

Step 1: Null production Remove ✓
Step 2: Unit production Remove ✓
Step 3:

Reduce $S \rightarrow aAe$
$\therefore S \rightarrow aH_0 \quad H_0 \rightarrow Ae$

Reduce $A \rightarrow bAB$
$\therefore A \rightarrow bH_2 \quad H_1 \rightarrow AB$

$\therefore A \rightarrow aB \mid bH_2, \; H_1 \rightarrow AB$

after Step 3

$S \rightarrow aH_0$
$A \rightarrow aB \mid bH_1$
$B \rightarrow b$
$C \rightarrow c$
$H_0 \rightarrow Ae$
$H_1 \rightarrow AB$

$\Longrightarrow$ Step 4:
Convert $S \rightarrow aH_0$
$\therefore S \rightarrow H_2H_0 \quad H_2 \rightarrow a$

Convert $A \rightarrow aB \mid bH_1$
$\therefore A \rightarrow H_2B \mid BH_1$.

⟹ CNF

$S \to H_2 H_0$
$A \to H_2 B \mid BH_1$
$B \to b$
$C \to c$
$H_0 \to AC$
$H_1 \to AB$
$H_2 \to a$

(4) $S \to OXIY$
$X \to OX \mid O$
$Y \to IY \mid I$

→ Step 1 & 2 ✓

→ Step 3:

Reduce $S \to OXIY$

$\therefore S \to OH_0 \quad H_0 \to XIY$

Reduce $H_0 \to XIY$

$\therefore H_0 \to XH_2 \quad H_2 \to IY$

~~Reduce~~

→ after step 3

S → OHO
X → OX | O
Y → ZYYZ
Ho → XH1
H1 → ZY

⇒ Step 4 :

convert S → OHO

∴ S → H2Ho H2 → O.

convert X → OX

∴ X → H2X

∴ X → H2X | O.

convert Y → ZY

∴ Y → H3Y H3 → 1

∴ Y → H3X | Z H3 → Z

convert H1 → ZY

∴ H1 → H3Y.

CNF

$S \to H_2 H_0$

$X \to H_2 X \mid 0$

$Y \to H_3 Y \mid z$

$H_0 \to X H_1$

$H_1 \to H_3 Y$

$H_2 \to 0$

$H_3 \to 1$

(5) $S \to ab\,S\,ab \mid a \mid a\,A\,A\,b$

$A \to b\,S \mid a\,A\,A\,b \mid c$

⟹ Step 1 & 2 : ✓

⟹ Step 3:

Reduce $S \to ab\,S\,ab$

$\therefore S \to a H_0 \quad H_0 \to b\,S\,ab$

Reduce $H_0 \to b\,S\,ab$

$\therefore H_0 \to b H_1 \quad H_1 \to S\,ab$

Reduce $H_1 \to S\,ab$

$\therefore H_1 \to S H_2 \quad H_2 \to ab$

Reduce $S \to aAb$

$\therefore S \to aH_3 \quad H_3 \to AAb$

Reduce $H_3 \to AAb$

$\therefore H_3 \to AH_4 \quad H_4 \to Ab$

Reduce $A \to aAAb$

$\therefore A \to aH_3$

$\Rightarrow$ after step 3

$S \to aH_0 \mid a \mid aH_3$
$A \to bS \mid aH_3 \mid c$
$H_0 \to bH_1$
$H_2 \to SH_2$
$H_2 \to ab$
$H_3 \to AH_4$
$H_4 \to Ab$

~~$\Rightarrow$~~ Step 4

Convert $S \to aH_0$

$\therefore S \to H_5 H_0 \quad H_5 \to a$

Convert $S \to aH_3$

$S \to H_5 H_3$

Convert $A \to bS$

$\therefore A \to H_6 S \quad H_6 \to b$

Convert $A \rightarrow aH_3$

$\therefore A \rightarrow H_5H_3$

Convert $H_0 \rightarrow bH_1$

$\therefore H_0 \rightarrow H_6H_1$

Convert $H_2 \rightarrow ab$

$\therefore H_2 \rightarrow H_1H_6$

Convert $H_4 \rightarrow Ab$

$\therefore H_4 \rightarrow AH_6$

CNF

$S \rightarrow H_5H_0 \mid a \mid H_5H_3$

$A \rightarrow H_6S \mid H_5H_3 \mid c$

$H_0 \rightarrow H_6H_1$

$H_1 \rightarrow SH_2$

$H_2 \rightarrow H_5H_6$

$H_3 \rightarrow AH_4$

$H_4 \rightarrow AH_6$

$H_5 \rightarrow a$

$H_6 \rightarrow b$

(5) Ambiguity

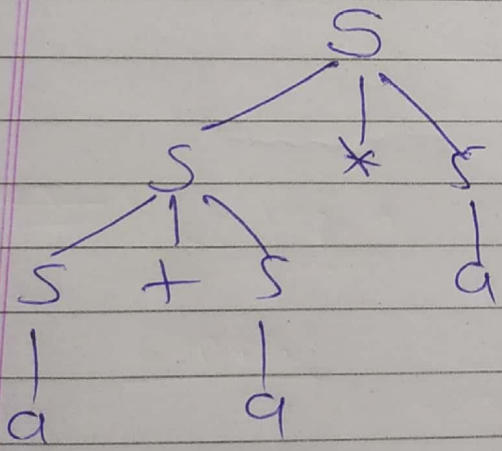
⟹ If a context free grammer (CFG) has more than one derivation tree for some string $\omega \in L(G)$, it is called an ambigous grammer.

⟹ There exist multiple rightmost or left most derivations for some string generated from that grammer

⟹ $S \rightarrow S+S \mid S-S \mid S*S \mid S-S \mid a$

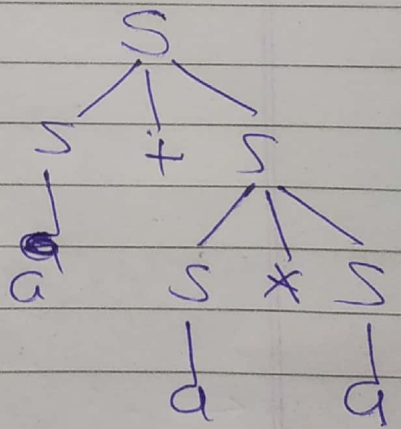
$\omega = a+a*a \quad \omega \in L(G)$.

Left-most derivation



a + a * a

Right-most derivation



a + a * a

As, there are two parse tree for single string, the given grammer is ambigay.

Remove unit production

6 (I)

$S \to ABC \mid 0$
$A \to 1$
$B \to C \mid 0$
$C \to D$
$D \to E$
$E \to 2$

unit production $D \to E$,

Remove $D \to E$

So, $S \to ABC \mid 0$
$A \to 1$
$B \to C \mid 0$
$C \to E$
$E \to 2$

Remove $C \to E$

$S \to ABE \mid 0$
$A \to 1$
$B \to E \mid 0$
$E \to 2$

unit production $D \to E$, $C \to D$, $B \to C$

remove $D \to E$, So, $S \to ABC \mid 0$
$A \to 1$
$B \to C \mid 0$
$C \to D$
$D \to 2$

remove $C \to D$

So, $S \to ABC \mid 0$
$A \to 1$
$B \to C \mid D$
$C \to 2$

remove $B \to C$

$S \to ABC \mid 0$
$A \to 1$
$B \to 2 \mid 0$
$C \to 2$
$D \to 2$
$E \to 2$

$\Rightarrow$ final answer

$S \to ABC \mid 0$
$A \to 1$
$B \to 2 \mid 0$
$C \to 2$

(2)

$S \to ABCD \mid 0$
$A \to BC \mid 1$
$B \to C$
$C \to D$
$D \to d$

Unit productions $C \to D$, $B \to C$

remove $C \to D$, so, $S \to ABCD \mid 0$
$A \to BC \mid 1$
$B \to C$
$C \to d$ $D \to d$

remove $B \to C$

$S \to ABCD \mid b$
$A \to BC \mid Z$
$B \to d$
$C \to d$
$D \to d$

↑ final answer