Time Value of Money (TVM)

→ TVM is the value of a unit of money which is differend in <u>different time period</u>.

→ The value of money today is more than value of money in future.

$\boxed{\text{OR}}$

→ The value of money in future is less than the value of money today.

→ Value of money/rupee received today is more valuable, so, a rational investor/individual would prefer current receipts to future receipts.
This is called Time Preference of Money.

→ Money received today has investment opportunity higher then money received in future.

Time Value of Money Techniques

Future Value (Compounding Technique) | Present Value (Discounting Technique)

* Future Value of Single Amount (Lumpsum)

$F.V. = P.V. (1+i)^n$ ← yrs to maturity $\boxed{OR}$ $F.V. = P.V. \times FVIF_{(i\%, n\,yrs)}$

F.V. → Future Value; P.V. → Present Value; i → interest Rate

* Future Value of Annuity.

→ An Annuity is a stream of Cash Flows of Equal Amount at regular intervals.

→ When the Cash flows occur at the end of period, it is called Deferred Annuity and when C.F. occur at the begining of period, it is called Annuity Due.

* Future Value of Deferred Annuity (End of Period)

$$F.V. = A\left[\frac{(1+i)^n - 1}{i}\right] \quad \boxed{OR} \quad F.V. = A \times FVIFA_{(i\%, n\,yrs)}$$

* Future Value of Annuity Due (Begining of Period)

$$F.V. = A\left[\frac{(1+i)^n - 1}{i}\right] \times (1+i) \quad \boxed{OR} \quad F.V. = A \times FVIFA_{(i\%, n\,yrs)} \times (1+i)$$

* Doubling Period :- Shows how long will it take to double the investment at given rate of interest.

Rule of 72

$$n = \frac{72}{i}$$

Rule of 69

$$n = 0.35 + \frac{69}{i}$$

* The rate of interest is to be taken in decimal

6.1 $P.V. = 1,000 \quad n = 5$ yrs

a) $i = 8\% = 0.08$

$F.V. = P.V. \times (1+i)^n$

$= 1,000 \times (1+0.08)^5$

$= 1000 \times (1.08)^5$

$= 1,000 \times 1.469$

$F.V. \boxed{= 1,469}$

OR

$F.V. = P.V. \times FVIF_{(i\%, n\,yrs)}$

$= 1000 \times 1.469$

$\boxed{= 1,469}$

b) 1,611

c) 1,762

d) 2,011

6.2. $n = \frac{72}{i}$

$= \frac{72}{12}$

$n = \boxed{6 \text{ yrs}}$

Investment today = 5,000

F.V. = 1,60,000

↓

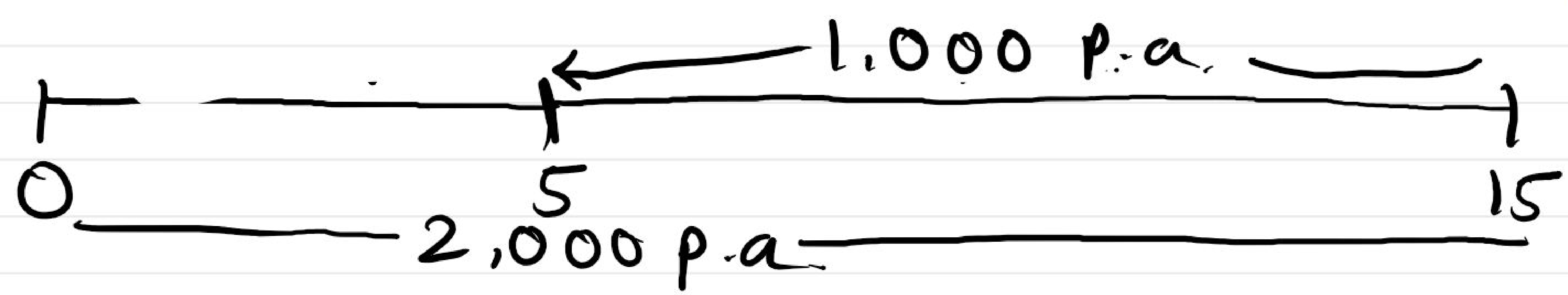
The amount will grow to 1,60,000 in 30 yrs.

6.4.

Savings ₹2,000 p.a. for 5 yrs & ₹3,000 p.a. for 10 yrs. thereafter.

Total $n = 15$ yrs.

$i = 10\%$



F.V. = 2,000 × $FVIFA_{(10\%, 15 yrs)}$ + 1000 × $FVIFA_{(10\%, 10 yrs)}$ =

$$F.V. = A\left[\frac{(1+i)^n - 1}{i}\right] + A\left[\frac{(1+i)^n - 1}{i}\right]$$

2000, 15 yrs   1,000, 10 yrs.

$$= 2,000\left[\frac{(1.10)^{15} - 1}{0.10}\right] + 1,000\left[\frac{(1.10)^{10} - 1}{0.10}\right]$$

$(2000 \times 31.772) + (1000 \times 15.937) = 79,481$

65

F.V. = 10,00,000

n = 10 yrs

i = 12%

A = ?

$$F.V. = A \times FVIFA_{(i\%, n\,yrs)}$$

= 56,983

6.6.

F.V. = 10,000

A = 1,000

n = 6 yrs

i = ?

F.V. = A × FVIFA$_{(i\%, n\,yrs)}$

∴ 10,000 = 1,000 × FVIFA$_{(i\%, 6\,yrs)}$

∴ FVIFA$_{(i\%, 6\,yrs)}$ = 10

From the Table

FVIFA$_{(20\%, 6\,yrs)}$ = 9.930 → i

FVIFA$_{(21\%, 6\,yrs)}$ = 10.183

* Using Liner Interpolation

$$i = \text{Lower Rate of Return} + \text{Diff. B/w Two Rates}\left[\frac{\text{Val. Reqd.} - \text{Val.@ Lower Rate}}{\text{Val.@ higher Rate} - \text{Val.@ Lower Rate}}\right]$$

$$= 20 + 1\left[\frac{10 - 9.930}{10.183 - 9.930}\right]$$

$$= 20 + 1\times\left(\frac{0.07}{0.253}\right)$$

$$\boxed{= 20.28\%}$$

* Present Value (Discounting)

P.V. of Single Amount (Lumpsum)

$$P.V. = \frac{F.V.}{(1+i)^n}$$

OR

$$P.V. = FV \times PVIF_{(i\%, n\,yrs)}$$

P.V. of Annuity (Deferred)

$$P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

OR

$$PV = A \times PVIFA_{(i\%, n\,yrs)}$$

P.V. of Annuity Due

$$PV = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \times (1+i)$$

OR

$$PV = (A \times PVIFA_{(i\%, n\,yrs)}) \times (1+i)$$

P.V. of Perpetuity (Infinite Period)

$$PV = \frac{A}{i}$$

P.V. of Growing Annuity

$$PV = A(1+g)\left[\frac{1 - \frac{(1+g)^n}{(1+i)^n}}{i-g}\right]$$

P.V. of Uneven Cash Flows

$$PV = \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \cdots \frac{A_n}{(1+i)^n}$$

6.8

$F.V. = 10,000$

$n = 8$ yrs

$i = 10\% = 0.10$

$$P.V. = \frac{FV}{(1+i)^n}$$

$$= \frac{10,000}{(1.10)^8}$$

$$= \boxed{4665.07}$$

(All decimal)

OR

$PV = FV \times PVIF_{(10\%, 8 yrs)}$

$= 10,000 \times 0.467$

$$\boxed{= 4,670}$$

6.9

$A = 2,000$

$i = 10\% = 0.1$

$n = 5$ yrs

$$P.V. = A \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 2,000 \left[\frac{(1.10)^5 - 1}{0.10\,(1.10)^5}\right]$$

$$= 2,000 \left[\frac{0.61051}{0.161051}\right]$$

$2,000 \times 3.791$

$$\boxed{= 7,582}$$

OR

$PV = A \times PVIFA_{(10\%, 5 yrs)}$ $= 2,000 \times 3.791$

$$\boxed{= 7,582}$$

6.10 Mr. Jingo has 2 Alternatives

Alt. 1 Pension of 10,000 p.a. for 15 yrs @ 15% &

Alt. 2 Lumpsum of 50,000 today → $P.V._0$

Cal. P.V. of Annuity for Alt. 1

$P.V. = A \times PVIFA_{(i\%, n\, yrs)}$

$= 10,000 \times PVIFA_{(15\%, 15\, yrs)}$

$= 10,000 \times 5.847$

$PV_0 = \boxed{58,470}$

Mr. Jingo should select Alt. -1 as P.V. is ~~higher~~ than Alt. 2

" " " " " F.V " " "

Alternative Solution

Alt. -1 F.V. of Annuity

$F.V. = A \times FVIFA_{(15\%, 15\, yrs)}$

$= 10,000 \times 47.580$

$\boxed{= 4,75,800}$ 1 0

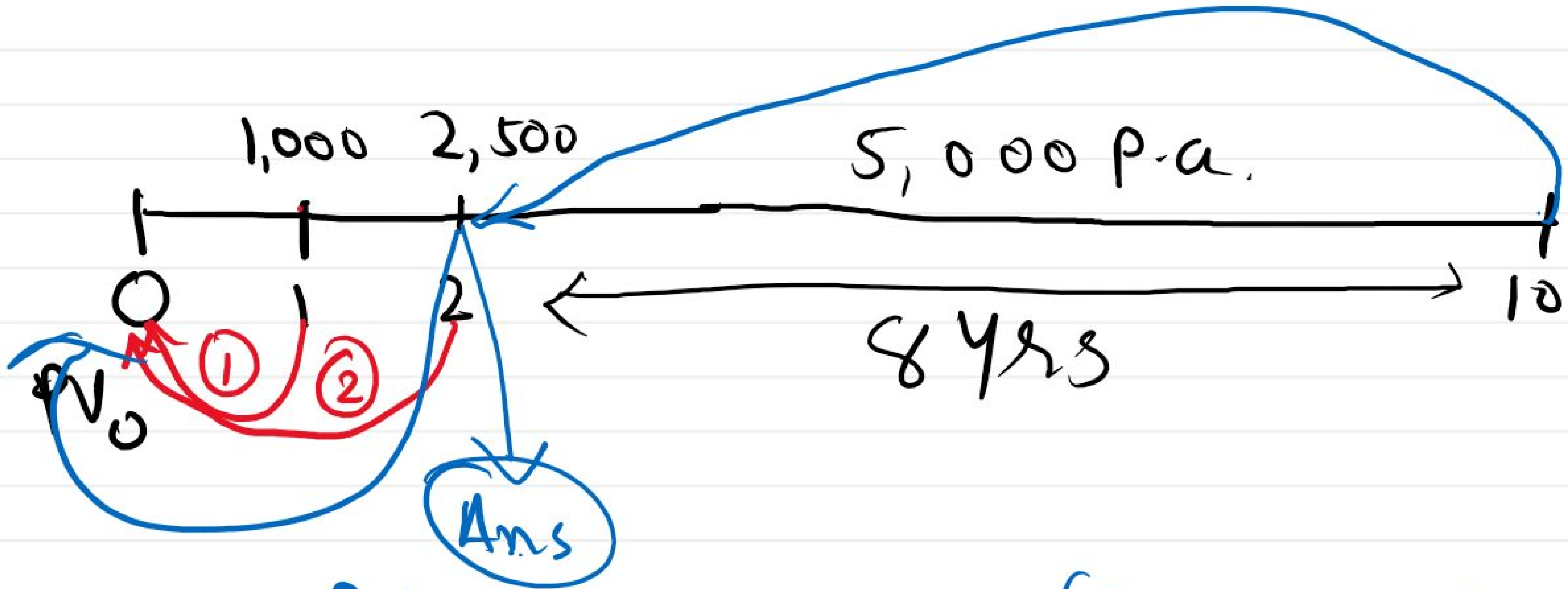
Alt. 2 F.V. of single Amt.

$F.V = P.V. \times FVIF_{(15\%, 15\, yrs)}$

$= 50,000 \times 8.137$

$\boxed{= 4,06,850}$

6.12

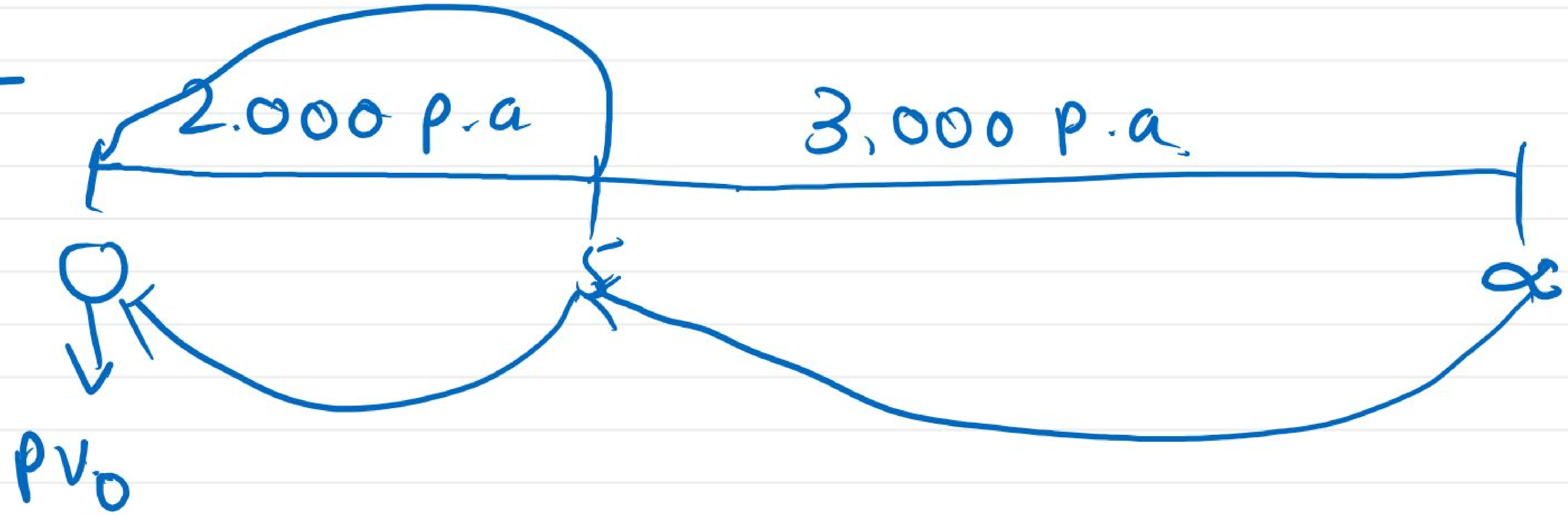


$$P.V._0 = \left(1{,}000 \times PVIF_{(12\%,\ 1yr)}\right) + \left(2{,}500 \times PVIF_{(12\%,\ 2yrs)}\right) + \left(5{,}000 \times PVIFA_{(12\%,\ 8yr)} \times PVIF_{(12\%,\ 2yr)}\right)$$

$$= 893 + 1{,}992.5 + 19{,}797.5$$

$$\boxed{= 22{,}683} \simeq 22{,}692.98$$

6.13

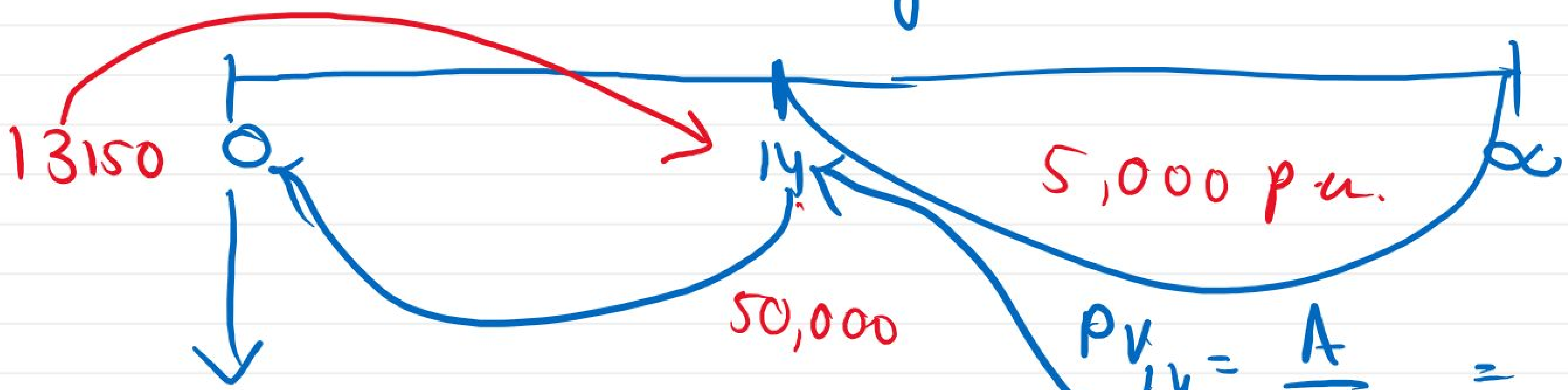


$$PV_0 = \left(2{,}000 \times PVIFA_{(10\%,\,5yrs)} + \left(\frac{A}{i} \times PVIF_{(10\%,\,5yrs)}\right)\right)$$

$$\boxed{= 26{,}212}$$

6.14 To earn an annual income of 5,000 starting from the end of 15 yrs @ 10%.

→ First we have cal. P.V. of Perpetuity which gives the P.V. @ the end of 14 years and then discount it to yr. 0.



$$PV_{14} = \frac{A}{i} = \frac{5,000}{0.10}$$

$$\boxed{= 50,000}$$

$$PV_0 = 50000 \times PVIF_{(10\%, 14 yrs)}$$

$$\boxed{= 13,150}$$

$$FV_{14} = 13150 \times (1.10)^{14}$$

$$= \text{Approx. } 50,000$$

* Loan Amortization

$$P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore 1,00,000 = A\left[\frac{(1.095)^5 - 1}{0.095 \times (1.095)^5}\right]$$

$$\therefore 1,00,000 = A \times \left(\frac{0.574}{0.14955268}\right)$$

$$\therefore 1,00,000 = 3.838\,A$$

$$\therefore A = \frac{1,00,000}{3.838}$$

$$= 26,054.48$$

$$\boxed{\simeq 26,055}$$

P.V. = 1,00,000

i = 9.5% = 0.095

n = 5 yrs

A = ?

* Loan Amortization Schedule

1 Yrs.	2 O/S Amount (Opening)	3 Installment	4 Interest @ 9.5%	5 Principal (3−4)	6 O/S Amt (Closing) (2−5)
1	1,00,000	26,055	9,500	16,555	83,445
2	83,445	26,055	7,927	18,128	65,317
3	65,317	26,055	6,205	19,850	45,467
4	45,467	26,055	4,319	21,736	23,731
5	23,731	26,055	2,324	23,731	0

6.34 $A = 12$ cr.

$i = 12\% = 0.12$

$g = -3\% = -0.03$

(i) Pipeline is used Forever (Perpetuity)

$$P.V. = \frac{A}{i-g} = \frac{1,20,00,000}{0.12-(-0.03)}$$

$$\boxed{= 80 \text{ cr.}}$$

(ii) If Pipeline is scrapped after 25 yrs

$$P.V. = A(1+g)\left[\frac{1-\frac{(1+g)^n}{(1+i)^n}}{i-g}\right]$$

$$= 12 \text{ cr } \cancel{(1-0.03)}\left[\frac{1-\frac{(1-0.03)^{25}}{(1.12)^{25}}}{0.12-(-0.03)}\right]$$

$$\boxed{= 75.468 \text{ cr}} \quad \boxed{= 77.8 \text{ cr}}$$

6.35 No. of barrels produced per year = 50,000

growth for barrels = −5% = −0.05

Oil Prices = $ 50

growth for oil prices = 3% = 0.03

n = 15 yrs

i = 10% = 0.10

P.V. of Well's Production

$$PV_0 = A(1+g)\left[\frac{1-\frac{(1+g)^n}{(1+i)^n}}{i-g}\right]$$

Net growth ↓ barrels −5% Oil Price +3%

$$g = (1-0.05)\times(1+0.03)-1$$
$$= (0.95\times 1.03)-1$$
$$\boxed{g = -0.0215}$$

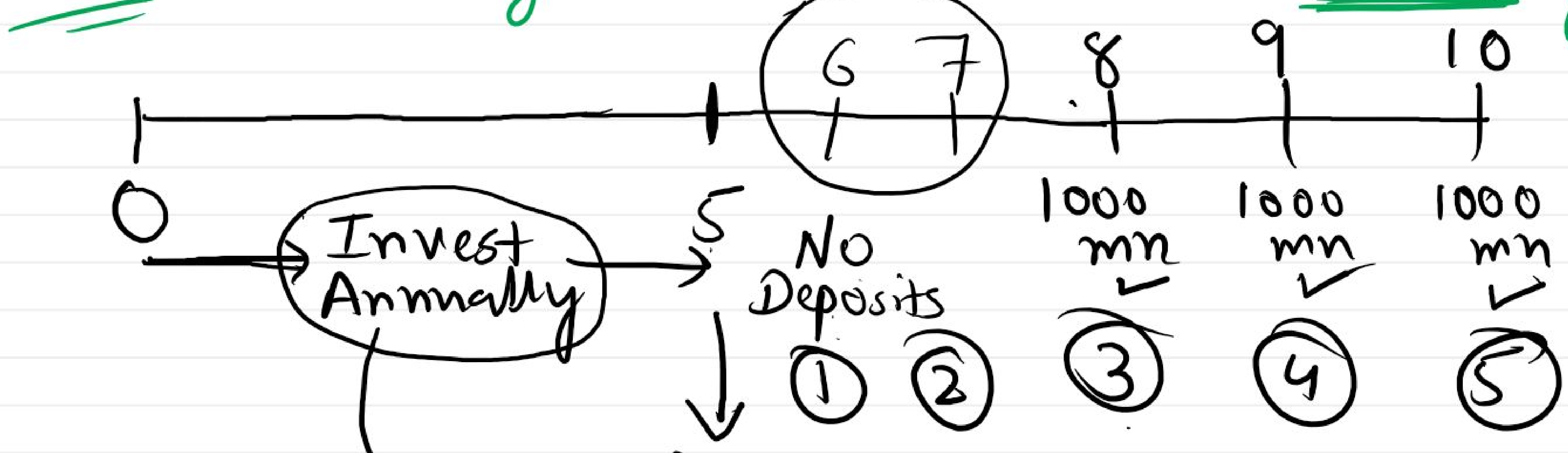
$$= (50{,}000\times 50)\times(1-0.0215)\left[\frac{1-\frac{(1-0.0215)^{15}}{(1.10)^{15}}}{0.10-(-0.0215)}\right]$$

$$= 24{,}46{,}250\times\left[\frac{0.827}{0.1215}\right]$$

$$= 24{,}46{,}250\times 6.807$$

$$PV = \boxed{1{,}66{,}51{,}623.75}$$

6.28 Sinking Fund (F.V. of Annuity)



$PV_5 = 2210$ mn

Part A Discounted Value of Debentures to be redeemed/repaid b/w 8 to 10 yrs. is calculated at the end of 5th yr.

$PV_5 = 1000\text{ mn} \times \left(PVIF_{(8\%, 3yrs)} + PVIF_{(8\%, 4yrs)} + PVIF_{(8\%, 5yrs)} \right)$

$= 1000\text{ mn} \times (0.794 + 0.735 + 0.681)$

$= 1000 \times 2.21$

$\boxed{PV_5 = 2{,}210\text{ mn}}$

Part B Annual Deposit to be made in sinking fund from yr. 0 to 5

$\therefore PV_5 = FV$

$\therefore FV = A \times FVIFA_{(i\%, n yrs)}$

$\therefore 2210\text{ mn} = A \times FVIFA_{(8\%, 5yrs)}$

$\therefore 2210\text{ mn} = 5.867\,A$

$\therefore A = \dfrac{2210}{5.867}$

$\therefore \boxed{A = 376.68\text{ mn}}$

6.28 Explanation A = 376.68 mn i = 8%

yr.			
1	376.68 × 1.08	= 406.8144	→ 2
2	(406.8144 + 376.68) × 1.08	= 846.1740	3
3	(846.1740 + 376.68) × 1.08	= 1,320.68	~~3~~ 4
4	(1320.68 + 376.68) × 1.08	= 1,833.15	5
5	(1833.15 + 376.68) ×	= 2209.83	= 2210
6	2210 × 1.08	= 2,386.8	
7	2,386.8 × 1.08	= 2,577.74	
8	(2,577.74 × 1.08	= 2,783.96 − 1,000mn	= 1,783.96
9	(1,783.96 × 1.08	= 1,926.68 − 1,000 mn	= 926.68
10	926.68 × 1.08	= 1,000.81 − 1000mn	= 0

* Muti-Period Compounding

Single Amount

$$F.V. = P.V. \times \left[1 + \frac{i}{m}\right]^{n \times m}$$

m = No. of Compoundings

Semi-Annually $m = 2$ Daily $m = 365$

Quarterly $m = 4$

monthly $m = 12$

weekly $m = 52$

Annuity $F.V. = A \left[\frac{\left(1 + \frac{i}{m}\right)^{m \times n} - 1}{\frac{i}{m}}\right]$

* Effective Interest Rate (EIR)

→ EIR shows how the compounding frequency impacts on the EIR.

→ The effect of increasing the freq. of compounding, additional gains is in the form of interest.

$$EIR = \left[1 + \frac{i}{m}\right]^{m} - 1$$

6.17 P.V. = 10,000

$i = 16\% = 0.16$

$n = 5$ yrs

Quarterly compounding $m = 4$

$$F.V. = PV\left(1+\frac{i}{m}\right)^{m \times n}$$

$$= 10,000\left(1+\frac{0.16}{4}\right)^{5 \times 4}$$

$$= 10,000\,(1.04)^{20}$$

$$\boxed{= 21,911.23}$$

Cal. EIR

$$EIR = \left(1+\frac{i}{m}\right)^{m} - 1$$

$$= \left(1+\frac{0.16}{4}\right)^{4} - 1$$

$$= 0.16985 \text{ or } \boxed{16.985\%}$$

$$F.V. = 10,000 \times (1.16985)^{5}$$

$$\boxed{= 21,911.23}$$

6.22

$P.V. = 10,000$

$i = 10\% = 0.10$

$n = 10$ yrs

$m = 2$

$$F.V = PV \left[1 + \frac{i}{m}\right]^{m \times n}$$

$$= 10,000 \left[1 + \frac{0.10}{2}\right]^{10 \times 2}$$

$$\boxed{FV_{10} = 26,532.98}$$

Value of FV_{10} in terms of current rupee if Inflation Rate is 8%.

$$P.V. = \frac{FV}{(1+i)^n}$$

$$= \frac{26,532.98}{(1.08)^{10}}$$

$$\boxed{= 12,289.90}$$