Chapter 5
Context-free Grammars and Languages

(Solutions/Hints)

5.1 Identify the nonterminals and terminals in the following grammars:

(a) $S \to ABa \mid b$ $A \to BB \mid aa$ $B \to bB \mid c$ $C \to cC \mid d$

(b) $S \to XY1 \mid 0$ $X \to 00X \mid 1$ $Y \to 1X1$

(c) $S \to XY$ $X \to YSY$ $X \to YY \mid a$ $Y \to aXb \mid b$

(d) $S \to XY$ $X \to YSY$ $X \to YY \mid 1$ $Y \to 0X1 \mid 1$

Sol. (a) Terminals={*a, b, c, d*}Nonterminals={S, A, B, C}
(b) Terminals={1, 0} Nonterminals={S, X, Y}
(c) Terminals={*a, b*} Nonterminals={S, X, Y}
(d) Terminals={1, 0} Nonterminals={S, X, Y}

5.2 Remove unit productions from the following grammars and generate equivalent grammar:

(a) $S \to ABC \mid 0, A \to 1, B \to C \mid 0, C \to D, D \to E, E \to 2$

(b) $S \to ABCD \mid 0, A \to BC \mid 1, B \to C, C \to D, D \to d$

Sol. (a) S→ABC|0 A→1 B→C|0 C→2

(b) S→ABBB|0 A→BB|1 B→d

5.3 Convert the following CFGs to GNF:

(a) $S \to XY1 \mid 0$ $X \to 00X \mid 1$ $Y \to 1X1$

(b) $S \to XY$ $X \to YSY$ $X \to YY \mid 1$ $Y \to 0X1 \mid 1$

(c) $S \to Xa$ $X \to aY$ $Y \to Xa \mid b$

Sol. (a) Let us introduce two new variables P and Q such that P→1 and Q→0.
Now,
Y→1XP X→0QX|1 S→XYP|0
S→XYP|0 leads to S→0QXYP| 1YP | 0

Total production set in GNF:
S→0QXYP| 1YP | 0 X→0QX|1 Y→1XP

(b)X→1 and Y→1 are already in GNF.
Introduce a new variable P such that P→1, it is different from X and Y in the manner that they have other productions X→YY and Y→0X1 as well.
Now, we have Y→0XP and P→1 both of which are in GNF.
New production set
X→YY|YSY|1 leads to X→0XPY| 1Y| 0XPSY| 1SY|1
S→XY leads to S→0XPYY| 1YY| 0XPSYY| 1SYY|1Y
Total grammar in GNF is
S→0XPYY| 1YY| 0XPSYY| 1SYY| 1Y
X→0XPY| 1Y| 0XPSY| 1SY| 1
Y→0XP| 1
P→1

(c) X→*a*Y and Y→*b* are already in GNF.
To convert this grammar into GNF, we modify the grammar as follow:
S→XP X→*a*Y Y→XP|*b* P→*a*
Now the grammar can be written in GNF as follow:
S→*a*YP X→*a*Y Y→*a*YP|*b* P→*a*

5.4 Convert the following CFG to CNF:
(a) $S \rightarrow aAC$ $A \rightarrow aB \mid bAB$ $B \rightarrow b$ $C \rightarrow c$
(b) $S \rightarrow 0X1Y$ $X \rightarrow 0X \mid 0$ $Y \rightarrow 1Y \mid 1$
(c) $S \rightarrow abSab \mid a \mid aAAb$ $A \rightarrow bS \mid aAAb \mid c$

Sol. (a) B→b and C→c are already in CNF.
A→XB| ZB where X→a, Z→YA and Y→b.
S→WC where W→XA.
Production set in CNF:
S→WC W→XA A→XB| ZB X→a Z→YA
Y→b B→b C→c

(b) S→PQ P →RX Q →TY R→0 T→1
X →RX|0 Y→TY| 1

(c) S→WZ| a| PQ X→a Y→b W→ZS
P→XA Q→AY Z→XY A→YS|PQ|c

5.5 Identify and remove the nonreachable nonterminals from the following grammars:
(a) $S \rightarrow XY1 \mid 0$ $X \rightarrow 00X \mid 1$ $Y \rightarrow 1X1$ $Z \rightarrow 00$
(b) $S \rightarrow XZ \mid 0$ $X \rightarrow YA \mid 1$ $Y \rightarrow Z1 \mid A2$ $A \rightarrow 01$ $B \rightarrow X \mid 2$

Sol. (a) V={S, X, Y, Z}
W1= {S}
W2= {S} ∪ {X, Y}={S, X,Y}
W3=W2
Non reachable terminals= V-W3=Z
Grammar without non reachable terminals:
S→ XY1 | 0 X→00X | 1 Y→1X1

(b) V={S, X, Y, Z, A, B }
W1= {S}
W2= {S} ∪ {X, Z}={S, X, Z}
W3= {S, X, Z} ∪ {Y, A}={S, X, Z, Y, A}
W4= {S, X, Z, Y, A} ∪ {Z}= {S, X, Z, Y, A}
W5=W4
Non reachable terminals= V-W5=B
Grammar without non reachable terminals:
S→ XZ | 0 X→YA | 1 Y→Z1| A2 A→01

5.6 Identify the nonterminals from the following grammars, which fail to generate terminals:

(a) $S \rightarrow XY1 \mid 0 \qquad X \rightarrow 00X \qquad Y \rightarrow 1X1 \mid 2$

(b) $S \rightarrow XZ \mid 0 \qquad X \rightarrow YA \mid 1 \qquad Y \rightarrow Z1 \mid A2 \mid 3 \qquad Z \rightarrow 3Z$

Sol. (a) V={S, X, Y}
W1= {S, Y}
W2=W1
Terminals which fail to generate terminals:
V- W2={X}

(b) V={S, X, Y, Z, A}
W1= {S, X, Y}
W2=W1
Terminals which fail to generate terminals:
V- W2={Z, A}

5.7 Consider the following grammar:

$S \rightarrow ASA \mid BSB \mid ASB \mid BSA \mid 1 \qquad A \rightarrow 0 \qquad B \rightarrow 1$

Derive the strings 010, 111, 00101, and 11100 using both left and right derivation.

Sol. String 010
Left derivation: S ⇒ ASA ⇒ 0SA ⇒ 01A ⇒ 010
Right derivation : S ⇒ ASA ⇒ AS0 ⇒ A01 ⇒ 010

String 111
Left derivation: S ⇒ BSB ⇒ 1SB ⇒ 11B ⇒ 111
Right derivation : S ⇒ BSB ⇒ BS1 ⇒ B11 ⇒ 111

String 00101
Left derivation: S ⇒ ASB ⇒ 0SB ⇒ 0ASAB ⇒ 00SAB ⇒ 001AB ⇒ 0010B ⇒ 00101
Right derivation :
S ⇒ ASB ⇒ AS1 ⇒ AASA1 ⇒ AAS01 ⇒ AAS01 ⇒ AA101 ⇒ A0101 ⇒ 00101

String 11100
Left derivation:
S ⇒ BSA ⇒ BBSAA ⇒ 1BSAA ⇒ 11SAA ⇒ 111AA ⇒ 1110A ⇒ 11100
Right derivation : S ⇒ BSA ⇒ BS0 ⇒ BBSA0 ⇒ BBS00 ⇒ BB100 ⇒ B1100 ⇒ 11100