

\* Risk & Return.

\* Return & Risk based on  
Past-Data.

$$R_t = \left[ \frac{P_t - P_{t-1} + \text{Div}_t}{P_{t-1}} \right] \times 100$$

\* Expected Return

$$\bar{R} = \frac{\sum R_i}{n}$$

\* Expected Risk

$$\sigma^2 = \frac{\sum (R_i - \bar{R})^2}{n-1} \quad \sigma = \sqrt{\sigma^2}$$

\* Risk & Return using  
Probabilities

$$\text{Expected Return} = \sum P_i R_i$$

$$\text{Expected Risk} = \sum P_i (R_i - \bar{R})^2$$

$$\sigma = \sqrt{\sigma^2}$$

# I. Annual Return

Year	Annual Return $R_t = \left[ \frac{P_t - P_{t-1} + \text{Div}}{P_{t-1}} \right] \times 100$	Return %. $R_i$	$R_i - \bar{R}$	$(R_i - \bar{R})^2$	Expected Risk
1988	$= \left( \frac{20.75 - 31.25 + 1.53}{31.25} \right) \times 100$	-28.7%	-80.74	6,518.95	
1989	$= \left( \frac{30.88 - 20.75 + 1.53}{20.75} \right) \times 100$	56.2%	4.16	17.31	$\sigma^2 = \frac{\sum (R_i - \bar{R})^2}{n-1}$
1990		123.45%	71.41	5,099.39	
1991		52.24%	0.2	0.04	
1992		57	4.96	24.60	
$E R_i = \frac{260.19}{5}$				11,660.29	$\sigma^2 = \frac{11,660.29}{4}$

Expected Return  $\bar{R} = \frac{E R_i}{n} = \frac{260.19}{5} = 52.04\%$

Range of Returns =  $-1.96$  to  $106.04$

$\sigma = \sqrt{2,915.07}$

$\sigma = 54\%$

4.

ACC

Prob. (P <sub>i</sub> )	Return (R <sub>i</sub> )	P <sub>i</sub> R <sub>i</sub>	R <sub>i</sub> - $\bar{R}$	(R <sub>i</sub> - $\bar{R}$ ) <sup>2</sup>	P <sub>i</sub> (R <sub>i</sub> - $\bar{R}$ ) <sup>2</sup>
0.5	1.94	0.97	-0.463	0.214	0.107
0.4	2.74	1.096	0.337	0.114	0.046
0.1	3.37	0.337	0.967	0.934	0.094

Expected Return =  $\bar{R} = \sum P_i R_i = \frac{2.403}{5} \%$ .

$$\sigma^2 = \sum P_i (R_i - \bar{R})^2 = 0.247 \quad \sigma = \sqrt{0.247} = 0.50 \%$$

Here

P <sub>i</sub>	R <sub>i</sub>	P <sub>i</sub> R <sub>i</sub>	R <sub>i</sub> - $\bar{R}$	(R <sub>i</sub> - $\bar{R}$ ) <sup>2</sup>	P <sub>i</sub> (R <sub>i</sub> - $\bar{R}$ ) <sup>2</sup>
0.5	5.1	2.55	-32.68	1067.98	534
0.4	74.92	29.97	37.14	1379.38	551.75
0.1	52.59	5.26	14.81	219.34	21.93

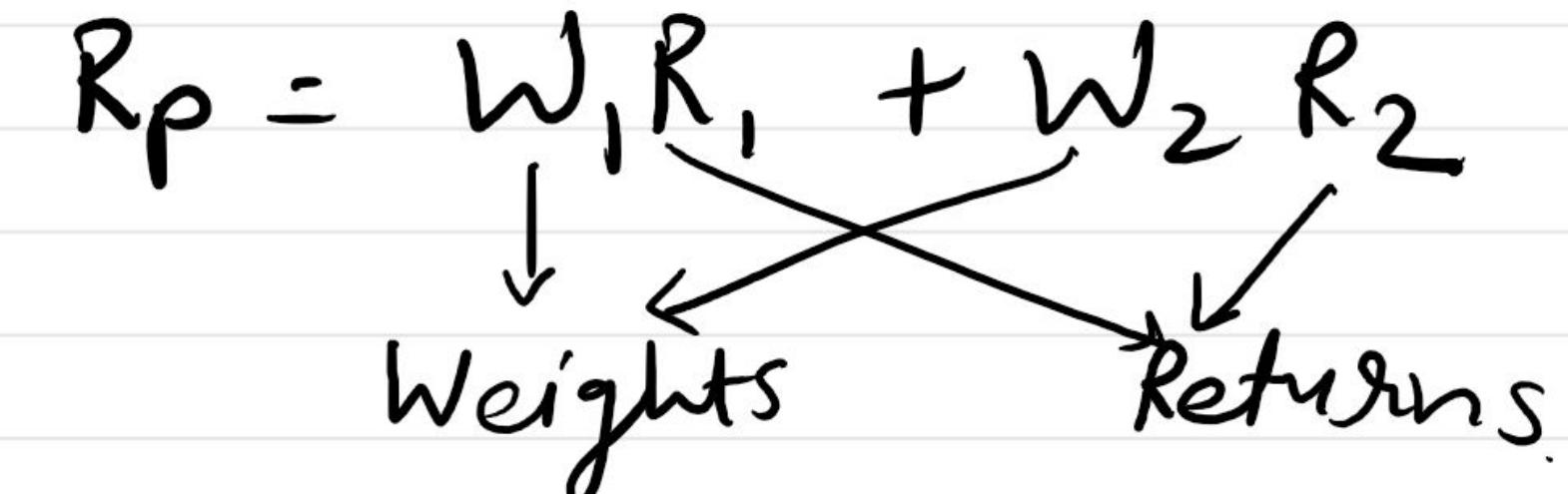
$$\bar{R} = \frac{37.78}{5}$$

$$\sigma^2 = \sum P_i (R_i - \bar{R})^2 = 1107.68$$

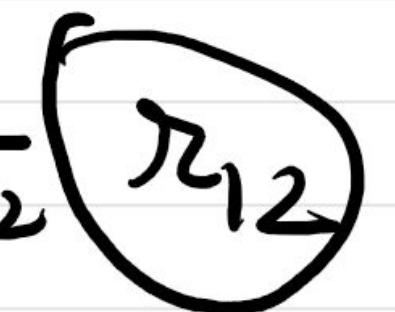
$$\sigma = \sqrt{1107.68} = 33.28$$

\* Expected Return and Risk of a Portfolio (2- Assets)

\* Expected Return  $R_p = w_1 R_1 + w_2 R_2$



\* Expected Risk  $\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2 r_{12}$



OR

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \text{COV}_{12}$$

$$\text{COV}_{12} = \sigma_1 \times \sigma_2 \times r_{12}$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

	X	Y
$R_i$	12%	15%
$\sigma_i$ (S.D.)	15%	20%
$w_1 = 0.4$		$w_2 = 0.6$

$$R_p = w_1 R_1 + w_2 R_2 = (0.4 \times 12) + (0.6 \times 15) = 4.8 + 9 = 13.8\%$$

\* Calculating Portfolio risk at different correlation co-efficient

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

$$\rho_{12} = +1$$

$$\begin{aligned}\sigma_p^2 &= (0.4)^2 (15)^2 + \\ &(0.6)^2 (20)^2 + (2 \times 0.4 \\ &\times 0.6 \times 15 \times 20 \times 1) \\ &= 36 + 144 + 144\end{aligned}$$

$$\sigma_p^2 = 324$$

$$\sigma_p = \sqrt{324} = 18\%$$

$$\begin{aligned}\sigma_p^2 &= (0.4)^2 (15)^2 \\ &+ (0.6)^2 (20)^2 + \\ &(2 \times 0.4 \times 0.6 \times 15 \times 20 \\ &\times 0.5) \\ &= 36 + 144 + 72\end{aligned}$$

$$\sigma_p^2 = 252$$

$$\sigma_p = \sqrt{252} = 15.87\%$$

$$\begin{aligned}\sigma_p^2 &= 36 + 144 \\ &+ (2 \times 0.4 \times 0.6 \times 15 \\ &\times 20 \times 0)\end{aligned}$$

$$\begin{aligned}&= 36 + 144 \\ &- 180\end{aligned}$$

$$\sigma_p^2 = 180$$

$$\sigma_p = \sqrt{180} = 13.42\%$$

$$\rho_{12} = -0.5$$

$$\begin{aligned}\sigma_p^2 &= 36 + 144 \\ &+ (2 \times 0.4 \times 0.6 \times \\ &15 \times 20 \times -0.5)\end{aligned}$$

$$\sigma_p^2 = 108$$

$$\begin{aligned}\sigma_p &= \sqrt{108} \\ &= 10.39\%\end{aligned}$$

$$\rho_{12} = -1$$

$$\begin{aligned}\sigma_p^2 &= \\ &36 + 144 - 144 \\ &= 36\end{aligned}$$

$$\sigma_p = \sqrt{36}$$

$$1 = 6\%$$

$$S.D. \quad X \quad Y$$

$$\sigma_{12} \quad 15\% \quad -0.7 \quad 20\%$$

$$w_i = 0.7 \quad 0.3$$

$$\sigma_p^2 = (0.7)^2 (15)^2 + (0.3)^2 (20)^2 + (2 \times 0.7 \times 0.3 \times 15 \times 20 \times -1)$$

$$= 110.25 + 36 - 126$$

$$\sigma_p^2 = 20.25$$

$$\sigma_p = 4.5\%$$

$$< 6\%$$

$$0.5 = \frac{50,000}{100000} = \frac{m}{n} \rightarrow \text{Invest in Asset}$$

$$\rightarrow \text{Total}$$

Invest 100% in X  
Risk = 15% 12%

$$\text{Max} = 15 + 12 = 27\%$$

$$\text{Min} = 15 - 12 = 3\%$$

$$\text{Range} = 3 \rightarrow 27\%$$

Invest 40% in X & 60% in Y

$$\text{Return} = 13.8\%$$

$$\text{Risk} = 6\%$$

$$\text{Range} = 7.8\% \rightarrow 19.8\%$$

$P_i$	$R_x$	$R_y$	$P_i R_x$	$P_i R_y$	$R_x - \bar{R}_x$	$P_i (R_x - \bar{R}_x)^2$	$R_y - \bar{R}_y$	$P_i (R_y - \bar{R}_y)^2$	$P_i (R_x - \bar{R}_x)(R_y - \bar{R}_y)$
0.1	20	14	2	1.4	-19.1	36.48	13.8	19.04	26.36
0.4	-16	-20	-6.4	-8	-16.9	114.24	-20.2	163.22	136.55
0.2	14	18	2.8	3.6	13.1	34.32	17.8	63.37	46.64
0.1	9	12	0.9	1.2	8.1	6.56	11.8	13.92	9.56
0.2	8	10	1.6	2	7.1	10.08	9.8	19.21	13.92
			0.9	0.2		201.68		278.76	233.03

$$\bar{R}_x = 0.9$$

$$\bar{R}_y = 0.2$$

$$\sigma_x^2 = \sum P_i (R_x - \bar{R}_x)^2 = 201.68 = \sigma_x \sqrt{201.68} = 14.20$$

$$\sigma_y^2 = 278.76 \quad \sigma_y = \sqrt{278.76} = 16.70$$

$$COV_{12} = \sum P_i (R_1 - \bar{R}_1)(R_2 - \bar{R}_2)$$

$$= 233.03$$

b)  $R_p = (0.5 \times 0.9) + (0.5 \times 0.2) + 0.55 \times \rho_{12}$

$$\rho_{12} = \frac{COV_{12}}{\sigma_1 \times \sigma_2} = \frac{233.03}{14.20 \times 16.70} = 0.98$$

## \* Capital Asset Pricing Model (CAPM)

Return of a Portfolio

$$R_p = R_f + \beta (R_m - R_f)$$

↓      ↑      ↑  
Risk-free Rate       $\beta$       Market Return

→ Market → Index (Sensex (BSE-30), Nifty (50), Bank Nifty etc.)  
↓  
 $\beta = 1$

Stock X has  $\beta = 1.5$

If the Return as per CAPM < Required Return, Stock is Undervalued.

If the Return as per CAPM > Required Return, Stock is Overvalued.

13.

$P_i$	$R_i$	$R_m$	$P_i R_i$	$P_i R_m$	$R_i - \bar{R}$	$P_i (R_i - \bar{R})^2$	$R_m - \bar{R}_m$	$P_i (R_m - \bar{R}_m)^2$	$P_i (R_i - \bar{R}_i)$ $(R_m - \bar{R}_m)$
0.4	20	16	8	6.4	7.8	24.34	4.2	7.06	13.104
0.4	13	12	5.2	4.8	0.8	0.26	0.2	0.02	0.064
0.2	-5	3	-1	0.6	-17.2	<u>59.17</u>	-8.8	<u>15.49</u>	<u>30.272</u>
			<u>12.2</u>	<u>11.8</u>		<u>83.77</u>		<u>22.57</u>	<u>43.440</u>

$$\bar{R} = \sum P_i R_i = 12.2\%$$

$$R_F = 7\% \quad (\text{Given in Ques})$$

$$\bar{R}_m = \sum P_i R_m = 11.8\% \quad (\text{Required Return})$$

$$\sigma_i^2 = \sum P_i (R_i - \bar{R}_i)^2 = 83.77 \quad \sigma_i = \sqrt{83.77} = 9.15\%$$

$$\sigma_m^2 = \sum P_i (R_m - \bar{R}_m)^2 = 22.57 \quad \sigma_m = \sqrt{22.57} = 4.75\%$$

$$(COV_{(i,m)}) = \sum P_i (R_i - \bar{R}_i) (R_m - \bar{R}_m) = 43.44$$

As Expected Return as per CAPM ( $16.26\%$ )  $> 12.2\%$  (Required Return, the XYZ stock is overvalued, therefore investment is not wise.)

$$\beta = \frac{COV_{(i,m)}}{\sigma_m^2} = \frac{43.44}{22.57} = 1.93$$

Expected Return as  
Per CAPM

$$R_i = R_F + \beta (R_m - R_F) = 7 + 1.93 (11.8 - 7) = 16.26\%$$

14.

$R_i$	$R_m$	$R_i - \bar{R}_i$	$(R_i - \bar{R}_i)^2$	$(R_m - \bar{R}_m)$	$(R_m - \bar{R}_m)^2$	$(R_i - \bar{R}_i)(R_m - \bar{R}_m)$
18	15	9	81	9	81	81
9	7	0	0	1	1	0
20	16	-11	121	-10	100	110
-10	-13	-19	361	-19	361	361
5	4	-4	16	-2	4	8
12	7	3	9	1	1	3
<u>54</u>	<u>36</u>		<u>588</u>		<u>548</u>	<u>563</u>

$$\bar{R}_i = \frac{\sum R_i}{n} = \frac{54}{6} = 9 \text{ yr.} \quad \sigma_i^2 = \frac{(\sum R_i - \bar{R}_i)^2}{n-1}$$

$$\sigma_m^2 = \frac{(\sum R_m - \bar{R}_m)^2}{n-1}$$

$$\bar{R} = \frac{\sum R_m}{n} = \frac{36}{6} = 6 \text{ yr.} \quad = \frac{588}{5} = 117.6 \quad = \frac{548}{5} = 109.6$$

$$\sigma_i = \sqrt{117.6} \quad \boxed{= 10.84 \text{ yr.}}$$

$$\sigma_m = \sqrt{109.6} \quad \boxed{= 10.47 \text{ yr.}}$$

14---(contd.)

$$(i) \text{COV}_{(i,m)} = \frac{\mathbb{E}(R_i - \bar{R}_i)(R_m - \bar{R}_m)}{n-1}$$
$$= \frac{863}{5} \quad \boxed{1 = 112.6}$$

$$\rho_{(i,m)} = \frac{\text{COV}_{(i,m)}}{\sigma_i \times \sigma_m}$$
$$= \frac{112.6}{10.84 \times 10.47} \quad \boxed{= 0.99}$$

$$(ii) \beta_i = \frac{\text{COV}_{(i,m)}}{\sigma_m^2} = \frac{112.6}{109.6} \quad \boxed{\text{OR}} \quad \beta = \frac{\rho_{(i,m)} \times \sigma_i}{\sigma_m}$$
$$\boxed{T = 1.03}$$

$$(iii) \text{Total Risk of Stock } s = 10.84 \times (\sigma_i)$$
$$\therefore \text{Variance} = \sigma_i^2 = 117.6$$

$$\sigma_i^2 = S.R. + U.S.R$$

$$= \frac{0.99 \times 10.84}{10.47} = \boxed{1.03}$$
$$\text{Systematic Risk} = \beta_i^2 \sigma_m^2$$
$$= (1.03)^2 (10.47)^2 = \boxed{116.27}$$

IF

$R_i$	$R_m$	$R_i - \bar{R}_i$	$(R_i - \bar{R}_i)^2$	$(R_m - \bar{R}_m)$	$(R_m - \bar{R}_m)^2$	$(R_i - \bar{R}_i)(R_m - \bar{R}_m)$
29	-10	15.2	231.04	-14	256	-212.8
31	24	17.2	295.84	20	400	344
10	11	-3.8	14.44	7	49	-26.6
6	-8	-7.8	60.84	-12	144	93.6
(-7)	3	-20.8	432.64	-1	1	20.8
	69	20	1034.8		790	219

$$\bar{R}_i = \frac{\sum R_i}{n} = \frac{69}{5} = 13.8 \quad \sigma_i^2 = \frac{\sum (R_i - \bar{R}_i)^2}{n-1} \quad \sigma_m^2 = \frac{\sum (R_m - \bar{R}_m)^2}{n-1}$$

$$\bar{R}_m = \frac{\sum R_m}{n} = \frac{20}{5} = 4 \quad = \frac{1034.8}{4} = \frac{790}{4} = 197.5$$

$$\sigma_i^2 = 258.7$$

$$\sigma_i = \boxed{16.08\%}$$

$$\sigma_m = \boxed{14.05\%}$$

$$\sigma_i^2 = 258.7 \quad \sigma_m^2 = 197.5 \quad \text{COV}_{i,m} = \frac{219}{4} = 54.75$$

$$\sigma_i = 16.08 \quad \sigma_m = 14.05 \quad \beta_i = \frac{\text{COV}_{i,m}}{\sigma_m^2} = \frac{54.75}{197.5} = 0.28$$

(d) Regression Line of Dependent Variable on Independent Variable.

Regression Line of  $y$  on  $x$

$$Y = \underset{\text{Alpha}}{\alpha} + \underset{\text{Beta}}{\beta} x + \hat{e} \leftarrow \text{Residual Error}$$

$$y = 12.68 + 0.28x \leftarrow (\text{Return of Xerox})$$

(Return of S&P 500)

$$\alpha = \bar{R}_i - \beta (\bar{R}_m)$$

$$= 13.8 - (0.28 \times 4)$$

$= 12.68$

Regression Equation  $y = 12.68 + 0.28x$

Return of S&P 500  
(Independent)

(a) Return of Xerox =  $17.7\%$   
Market Return =  $14\%$

Putting Values in  
Regression Eq.  $y = 12.68 + 0.28(14)$

$$= 12.68 + 3.92$$

Return of Xerox in  
Yr. 6  $\boxed{= 16.6\%}$

b) Total Variance of Xerox  $\sigma_i^2 = \underline{258.7}$

b)(ii) The proportion explained & not explained by Market.

→ Proportion of Total Risk which can be explained is calculated by  $r^2$  (Coefficient of Determination)

→ Proportion which can not be explained is  $1 - r^2$

$$r = \frac{\text{COV}_{(i,m)}}{\sigma_m \times \sigma_i} = \frac{54.75}{16.08 \times 14.05} = \boxed{0.25}$$

$r^2 = (0.25)^2 = 0.0625 \times 100 = 6.25\%$ . Proportion that is explained by S&P 500

$1 - r^2 = 0.9375 \times 100 = 93.75\%$ . Proportion not explained.