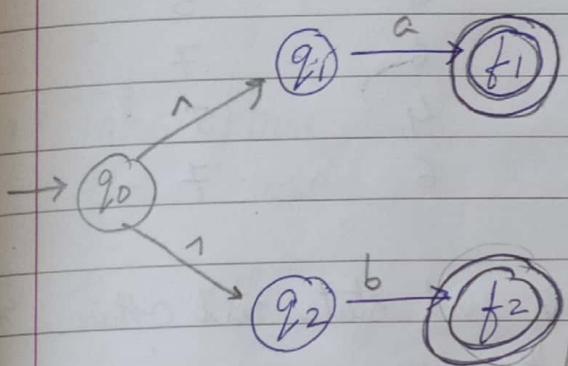


R.E. to NFA - 1

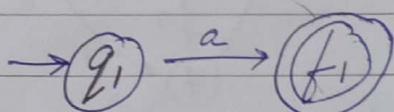
$$(a) ((ab)^* b + ab^*)^*$$

$\Rightarrow$  Kleen's Theorem.

Union :  $a+b$

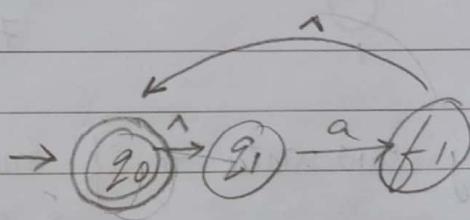


Kleen:  $a^*$



Step 3: Starting state  
q0 becomes

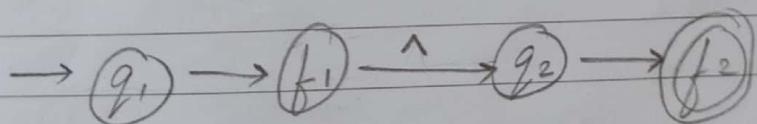
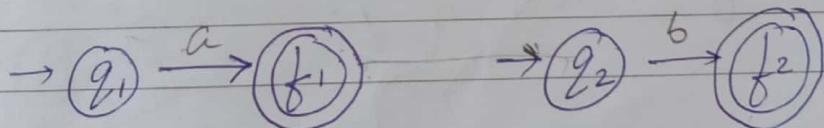
your accepting state.



Step 1: Add new starting state  $q_0$  and add  $\wedge$  transition  
from  $q_0$  to  $q_1$ .

Concatenation :  $a \cdot b$

Step 2: Add a transition from all accepting states to starting  
state by removing acceptance of existing acceptance state.



Step 1:

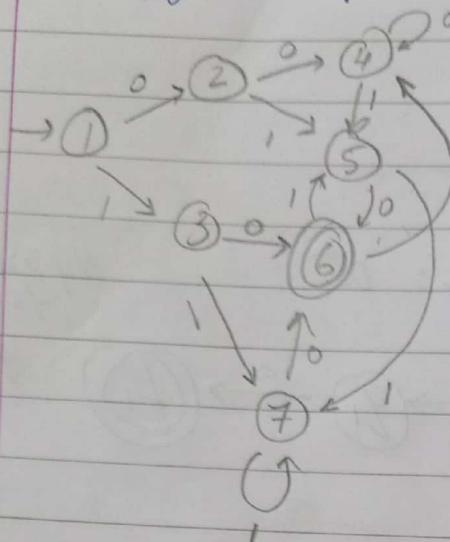
Remove accepting from  $f_1$

Step 2:

Add a transition from  $f_1$  to  $q_2$

⇒ Minimization of an FA.

Q.

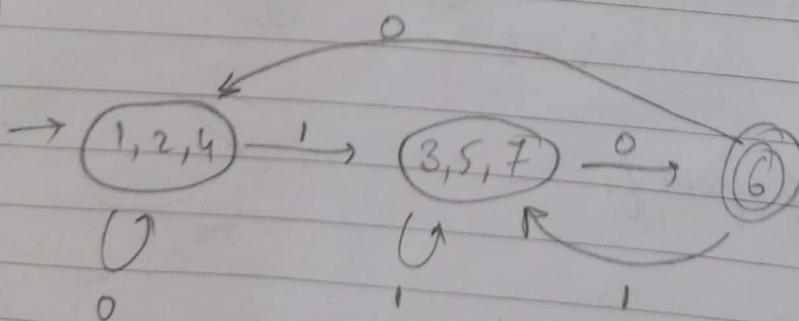
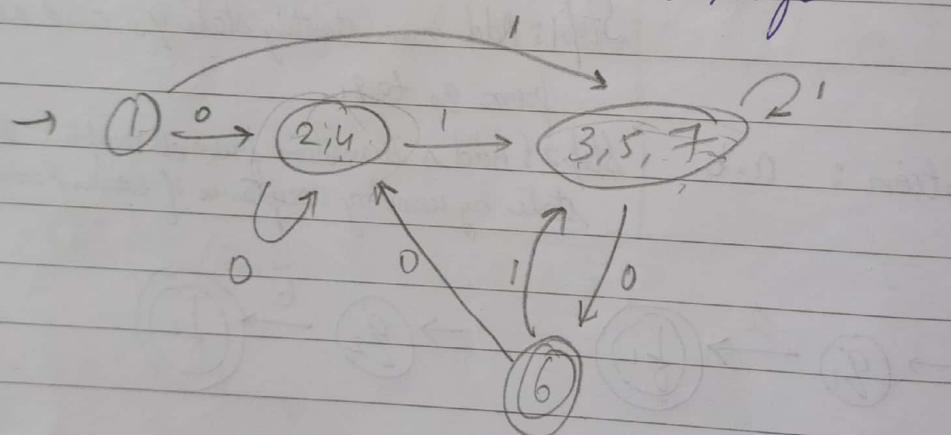


Status

	$s(q_0)$	$s(q_1)$
1	2	3
2	4	5
3	6	7
4	4	5
5	6	7
6	4	5
7	6	7

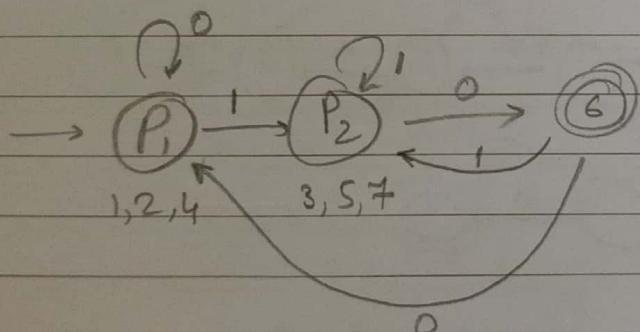
X = wrong.

Merging two states → if one is end state and other is not  
we can't move

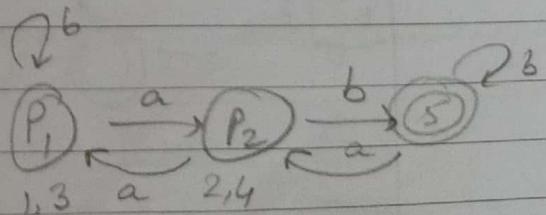
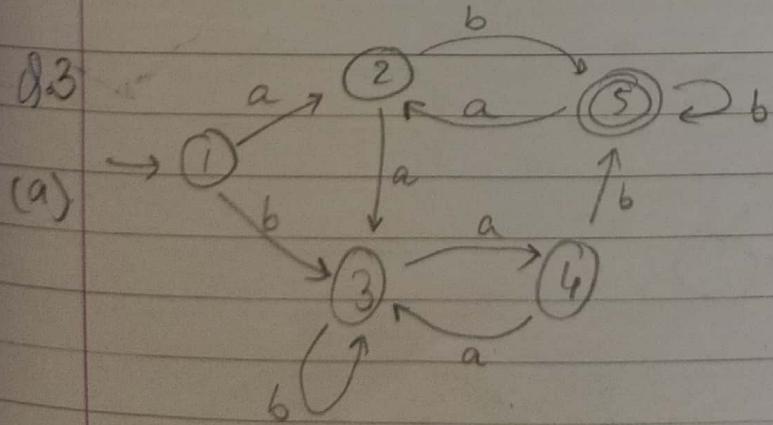


2	P <sub>1</sub>					
3		2	2			
4	P <sub>1</sub>	P <sub>1</sub>	2			
5	2	2	P <sub>2</sub>	2		
6	1	1	1	1	1	
7	2	2	P <sub>2</sub>	2	P <sub>2</sub>	1
	1	2	3	4	5	6

- Take pairs and if only one state is end state then mark 1
- For other pairs check (q, 0) and (q, 1) generated pair has only one end state then mark 2.
- For making pairs, check all the columns columnwise.

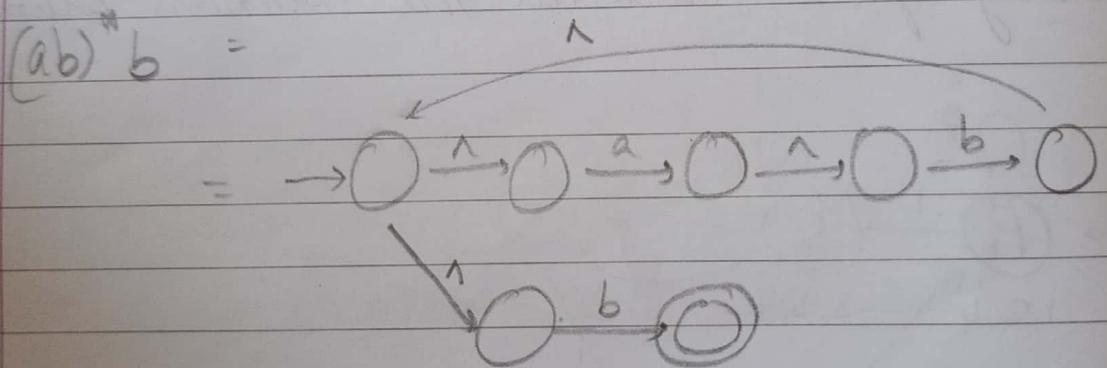
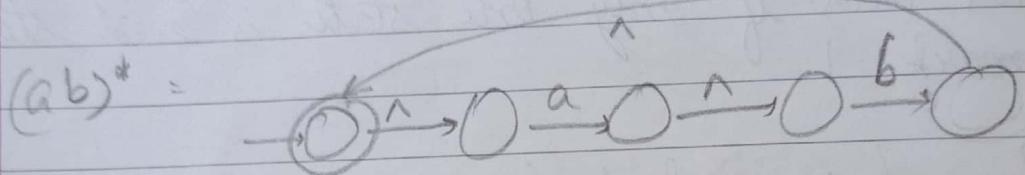
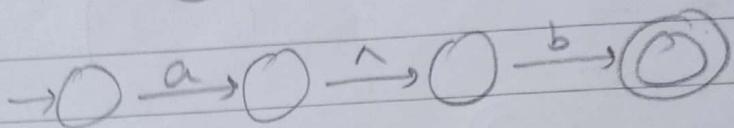
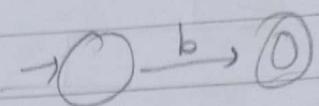
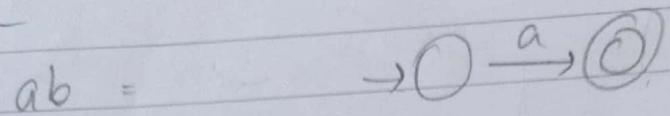


→ pairs such as (3, 3), (6, 6), (1, 1) won't be considered

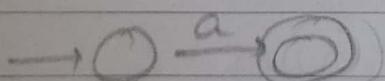


2	2				
3	P <sub>1</sub>	2			
4	2	P <sub>2</sub>	2		
5	1	1	1	1	
	1	2	3	4	

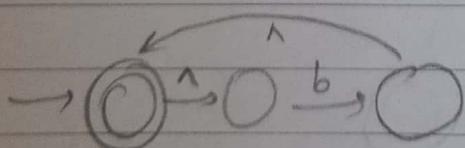
$$a. [(ab)^* b + ab^*]^*$$



$$ab^* =$$



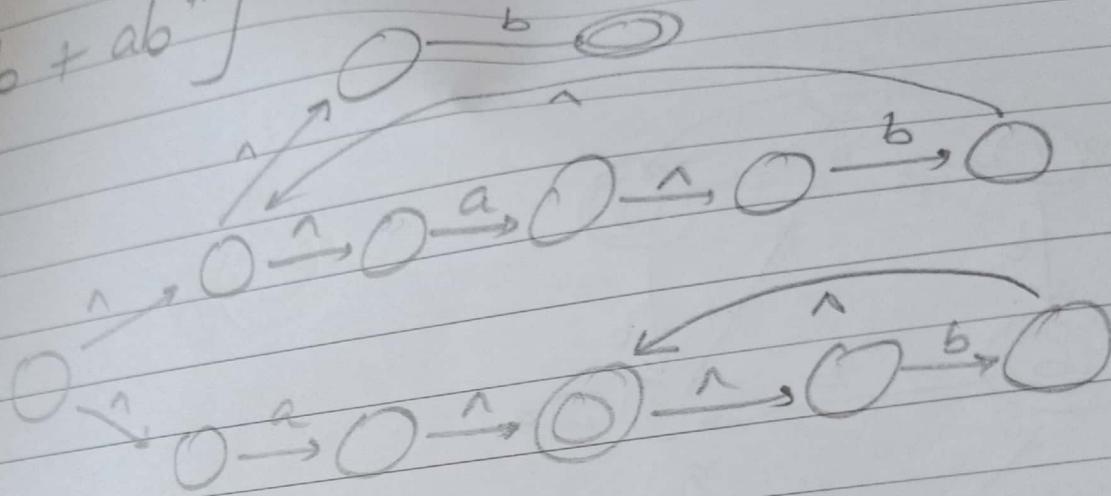
$$\boxed{b^*} =$$



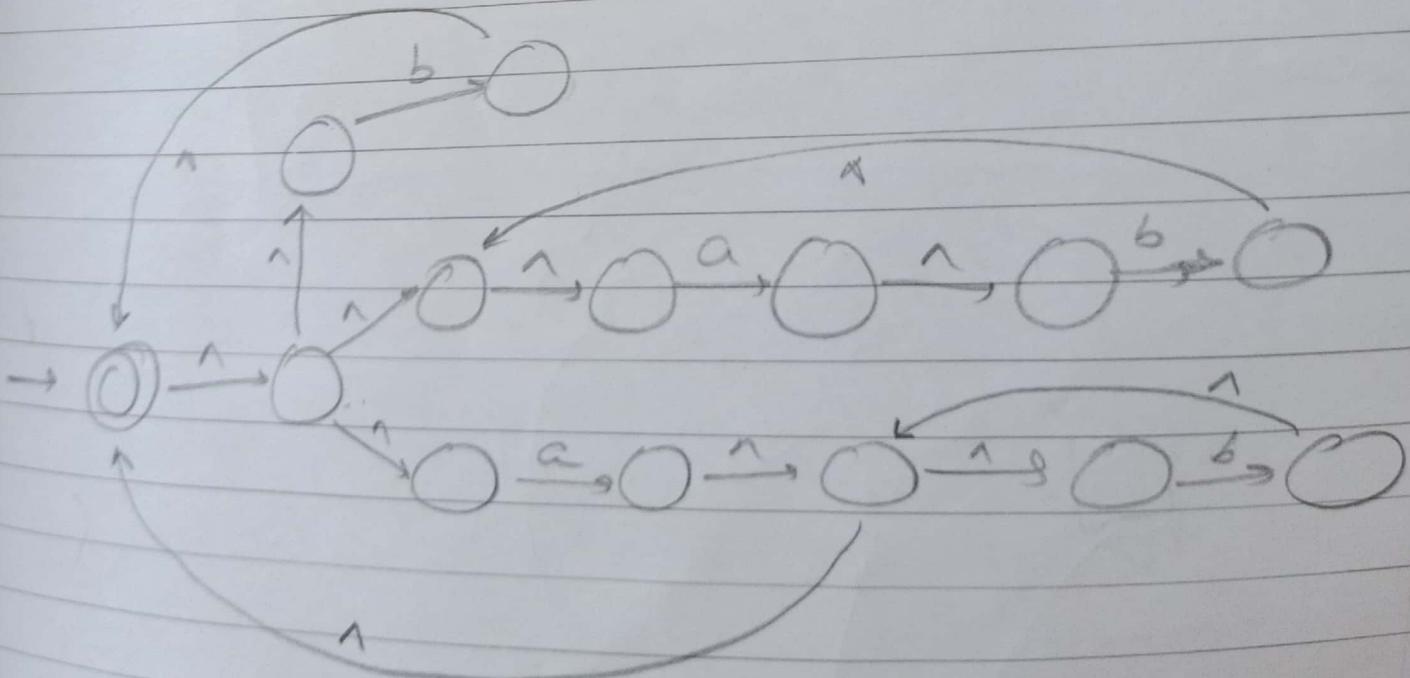
$\therefore ab^* = \rightarrow \textcircled{O} \xrightarrow{\wedge} \textcircled{O} \xrightarrow{\wedge} \textcircled{O} \xrightarrow{\wedge} \textcircled{O} \xrightarrow{b} \textcircled{O}$

Hence finally :

$$[(ab)^* b + ab^*]$$

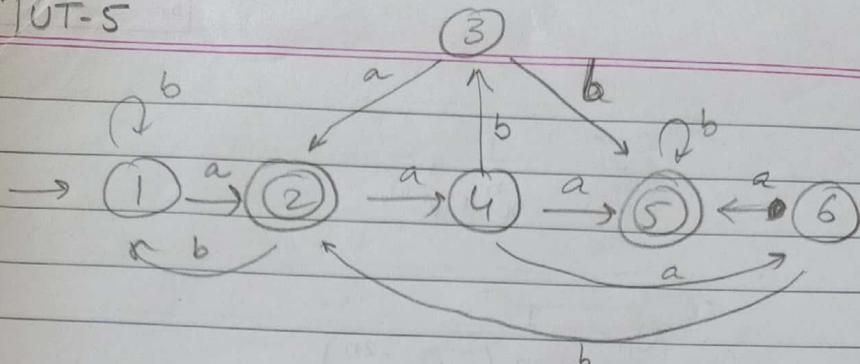


$$[(ab)^* b + ab^*]^* = \text{Final Answer}$$



83

(b)

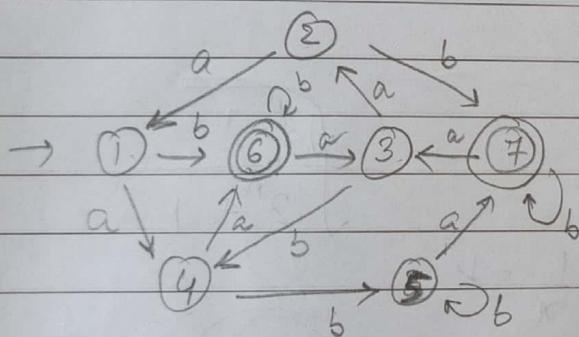


Hence  
this is  
already minimum.

2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	
6	X	X	X	X	X

1    2    3    4    5

(d)

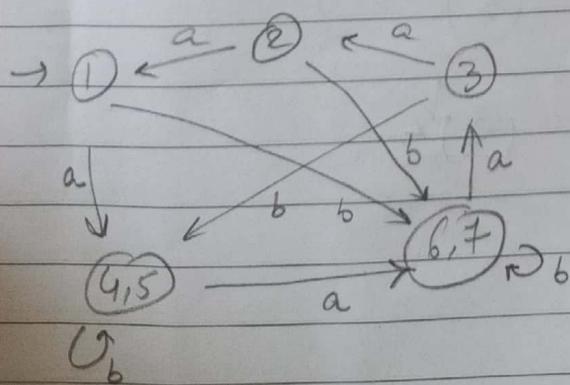


" . "

Hence (1, 2) is marked because after the first iteration point (1, 4) is marked and hence 1 & 2 on "a" we get (1, 4).

2	(1)					
3	X	X				
4	X	X	X			
5	X	X	X			
6	X	X	X	X	X	
7	X	X	X	X	X	X

1    2    3    4    5    6



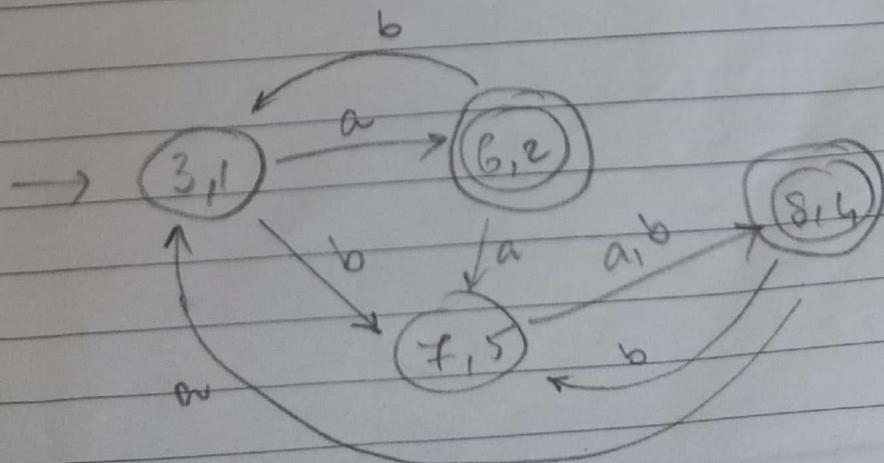
Hence 1, 2 & 3 are not included in any merging set hence they exist independently. (pairs having 1, 2 & 3 all are marked).

b.

	g.a	g.b
1	2	7
2	5	3
3	6	5
4	1	5
5	8	8
6	7	1
7	8	4
8	3	7

2	x						
3	(P <sub>1</sub> )	x					
4	x	(x)	x				
5	x	x	x	x			
6	x	(P <sub>2</sub> )	x	(x)	x		
7	x	x	x	x	(P <sub>4</sub> )	x	
8	x	(x)	x	(P <sub>2</sub> )	x	(x)	x
1	2	3	4	5	6	7	

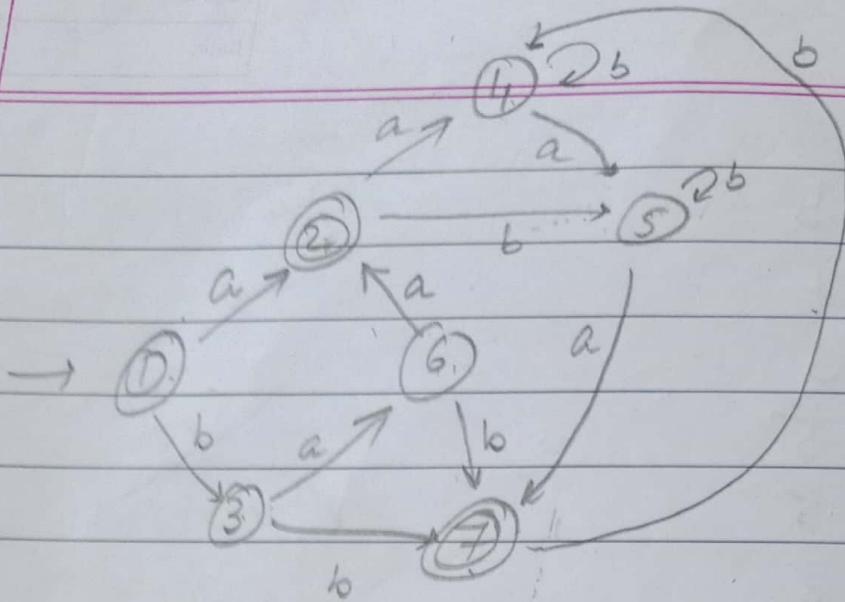
3,1      (6,2)      7,5      (8,4)  
 P<sub>1</sub>      P<sub>2</sub>      P<sub>4</sub>      P<sub>3</sub>



Q3.

Page No.:  
Date:

SL.



Already Minimized.

2	0					
3	X	X				
4	X	X	X			
5	X	X	X	X		
6	X	X	X	X	X	
7	0	0	X	X	X X	
	1	2	3	4	5	6

q<sub>0,a</sub>

q<sub>0,b</sub>

A

B

D

B

Φ

C

C

E

D

D

E

D

E

E

D

Φ

Φ

Φ

Tut - 5

M	T	W	T	F	S	S
Page No.:	YOUVA					
Date:						

Q1.

1'

a

$\delta(q, A)$

$\delta(q, B)$

A

B

D

B

$\emptyset$

C

C

E

D

D

E

D

R

E

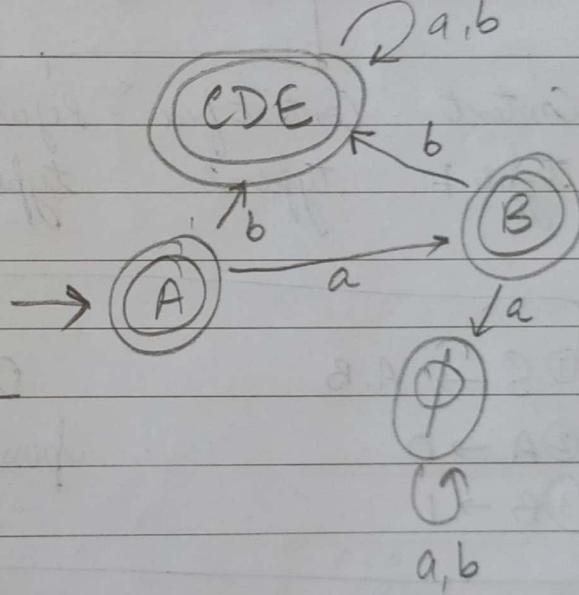
D

$\emptyset$

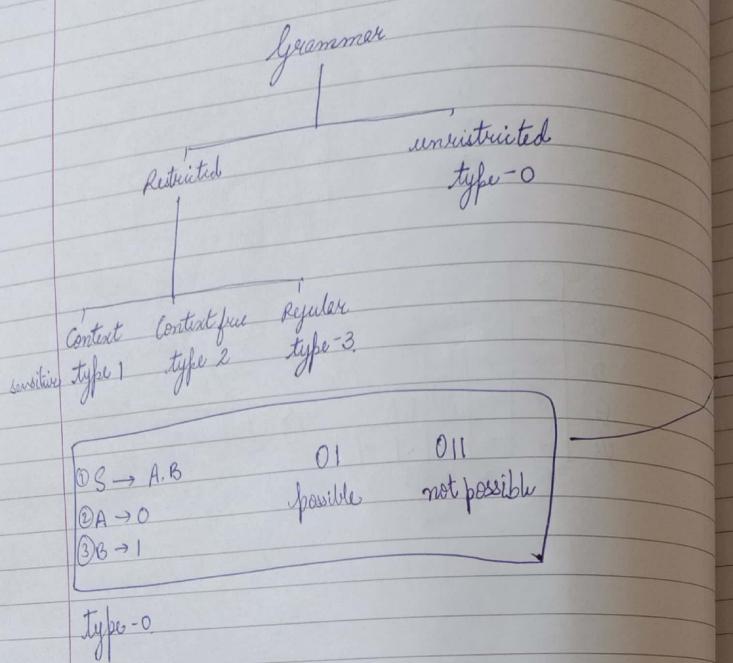
$\emptyset$

$\emptyset$

B	x				
C	x	x			
D	x	x	P <sub>1</sub>		
E	x	x	P <sub>1</sub>	P <sub>1</sub>	
$\emptyset$	x	x	x	x	x
	A	B	C	D	E



## Context free grammar (CFG)



type - 2  
 $A \rightarrow \alpha$   
 $A = \text{non-terminal}$   
 $\alpha = \text{non-terminal/terminal}$

type - 3  
 $A \rightarrow \alpha$   
 $A \rightarrow \alpha x$

CFG having 4 tuple  $G = (V, \Sigma, S, P)$

$$\begin{aligned} V &= S, A, B \\ \Sigma &= \{0, 1\} \\ S &= s \in V \\ P &= \text{production rules.} \end{aligned}$$

- Ex. ① CFG for either  $a$  or  $b$ . =  $a+b$
- $$S \rightarrow a/b$$
- ② for  $a^*$        $S \rightarrow \lambda/a^*$
- ③  $(ab)^+$        $S \rightarrow ab/abs$
- ④  $(ab)^*$        $S \rightarrow abS/\lambda$
- ⑤  $(a+b)^*$        $S \rightarrow \lambda/as/bs$
- ⑥  $a^*b^*$        $S \rightarrow \lambda/as/bs$
- ⑦  $a(a+b)^*$        $S \rightarrow a/as/(b)abs$   $\xrightarrow{S \rightarrow ax}$   
 $\xrightarrow{x \rightarrow ax/bx/\lambda}$
- ⑧  $a^*/b^*$        $S = A/B$        $A \rightarrow aA/\lambda$        $B \rightarrow bB/\lambda$
- ⑨  $(011+1)^*$        $(011+1)^* = S \rightarrow A \cdot B$

10. at least 3 times a.

$$S \rightarrow xaxaxax$$

$$X \rightarrow ax/bx/n$$

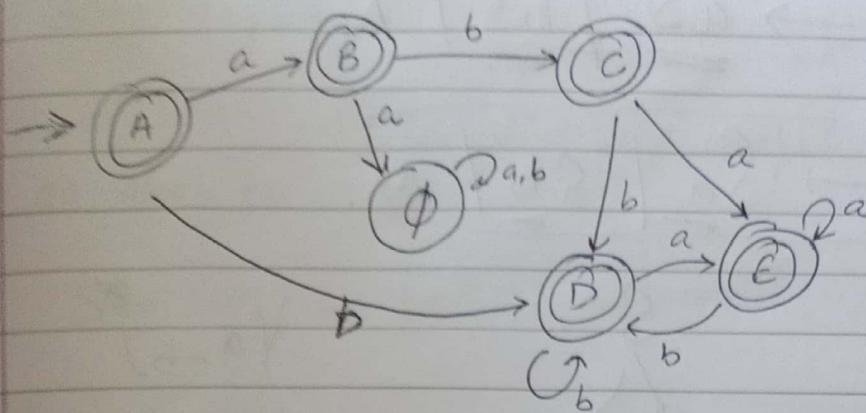
$$X = (a+b)^*$$

11. Starts and ends with same symbol.  
~~so it is so~~

$S \rightarrow OAO / IA / \text{I} / IO$   
 $A \rightarrow OA / IA / A$

12. Palindrome odd length / even length.

13. Equal no. of zeros and ones.  
 $S \rightarrow \text{FOSTER} / \wedge / \text{KISS} / 1SOS / OSIS.$



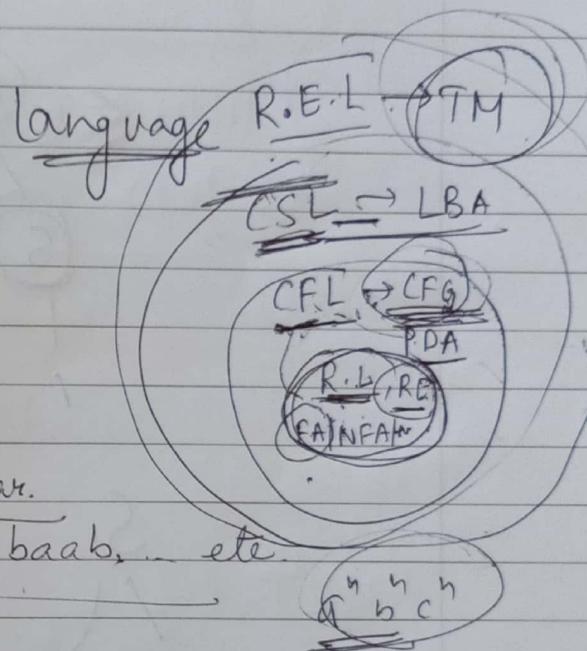
## Tutorial - 6

Q2.

$$1. S \rightarrow aSa \mid bSb \mid \lambda$$

Even length palindromes Non-regular.

abba, baaaab, aa, bb,  $\lambda$ , baab, etc.



$$2. S \rightarrow aSa \mid bSb \mid a \mid b$$

Odd length palindromes

aba, bab, aaa, ababa, a, b, etc.

$$3. S \rightarrow aSb \mid bSa \mid \lambda$$

Alternate a, b constituents a, b.

ab, ba, abab, baba,  
 $a^n b^n$

$$\overbrace{a^*}^{n} \overbrace{b}^{n} \quad \checkmark$$

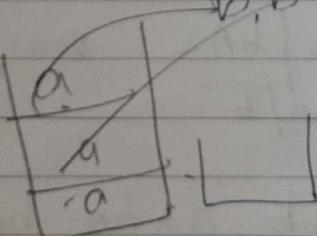
$$\checkmark \quad \overbrace{a^n b^n}^{0 \leq n \leq 3}$$

$\lambda + ab + aabb$

aaabb

Non Regular

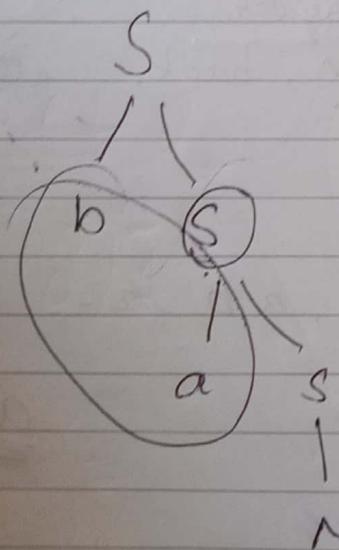
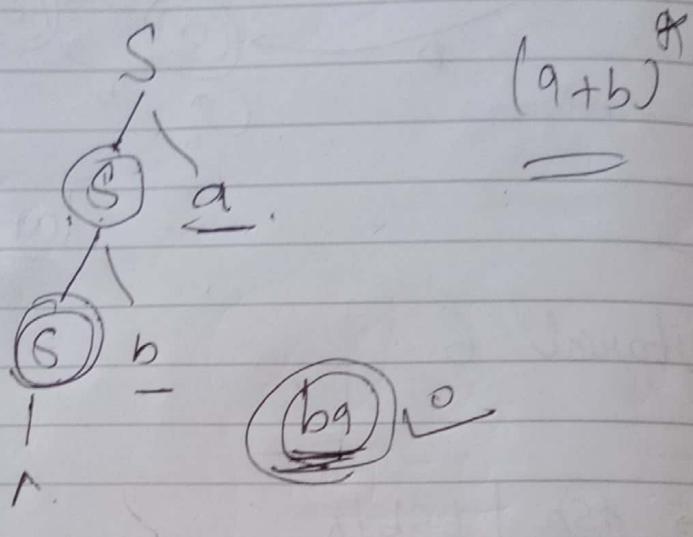
Context free



$a^n b^n$

Recursive Defi —

$$S \rightarrow \underline{a} S \mid \underline{b} S \mid \Lambda$$

$$S \rightarrow \underline{S_a} \mid \underline{S_b} \mid \Lambda$$


4. ~~S~~  $S \rightarrow a S_a \mid b S_b \mid a A b \mid b A a$  Non-palindrome.  
 $A \rightarrow a A a \mid b A b \mid \Lambda \mid a \mid b$

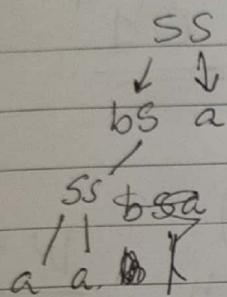
$S = ab, ba, aab, bba, abb, aaba, baaaabb, \dots$   
~~aaaa, bbbb~~.

$\rightarrow aba \& bab$  not possible

5.  $S \rightarrow aS / bS / a$

last digit is a compulsory.  
aa, aba, ba

6.  $S \rightarrow SS / bS / a$ .



baaa, ba, aa, aba, abba, --

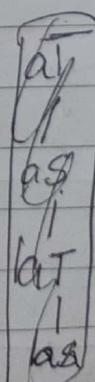
7.  $S \rightarrow SaS / b$ .

$\rightarrow S a S . a S a S$   
 $\rightarrow b a b a b a b , b a b , b a b a b$ .

8.  $S \rightarrow aT / bT / \lambda$

$T \rightarrow aS / bS$  even length

aa, bb, abba,  $\lambda$ , aaba, aaaa, bbbb, aabb



8.  $S \rightarrow OB | IA | \lambda$

Equal numbers of 0's and 1's.

$A \rightarrow OS | IAA$

$B \rightarrow IS | DBB$ .

OA	OB	OB
/	\	
OS	DBB	
\	/ /	
OB	IS	IS
\		
IS		

~~00000~~, 0011,

000111

2.  $S \rightarrow OS1 | ISO | SS | \lambda$

6.4.

(i)  $\rightarrow$  Set of odd length strings in  $a^*b^*$  with middle symbol a.

(ii)  $\rightarrow$  Set of even length strings over  $a^*b^*$  with two middle symbols equal.

(iii)  $\rightarrow$  The set of odd length strings in  $\{a,b\}^*$  whose first, middle and last symbols are same.

(i) Ans.  $S \rightarrow asa | asb | bSb | bSa | a$

(ii) Ans.  $S \rightarrow asa | asb | bSb | bSa | aa | bb$

(iii) Ans.  ~~$S \rightarrow asasa | bSbSb | aabb$~~

(iii)  $S \rightarrow aTa \mid bUb$

$$T \rightarrow aTB \mid bTa \mid bTb \mid aTBa$$

$$v \rightarrow bvb/bva|ava/b/ava/b.$$

6.5.

$$a) \quad S \rightarrow S01S \mid S10S \mid \Lambda$$

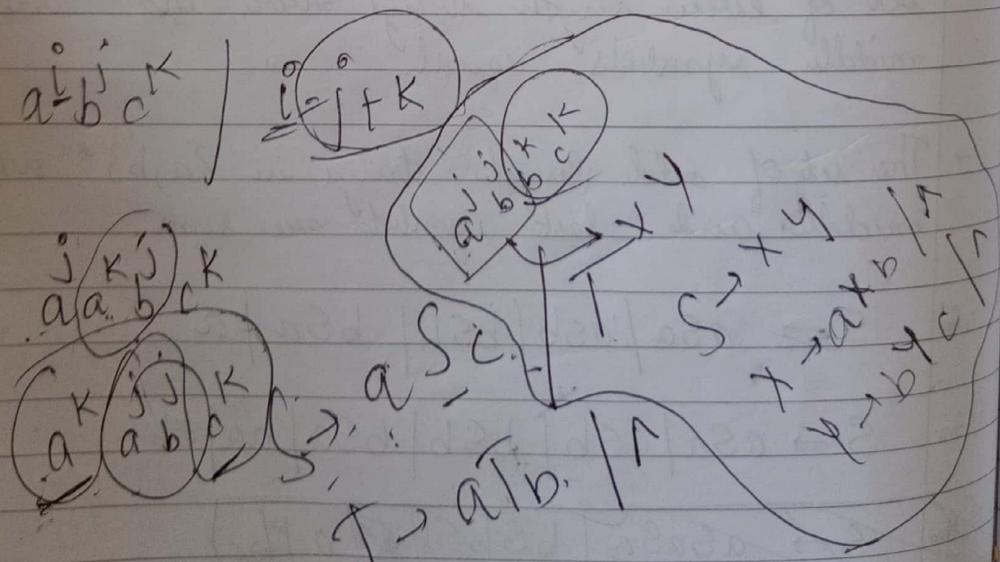
b)  $S \rightarrow OS1 \mid 1S0 \mid 01S \mid 10S \mid S01 \mid S10 \mid \lambda$

number of 0 = number of 1.

$$a \rightarrow 1100$$

b → 00111100

CFG for english language  
declarative sentence where you have subject, verb, object



## Recursive def<sup>n</sup>

①  $(a, b)^*$

Regular

1.  $\lambda \in L$
2. for any  $S \in L$ ,  $aS \in L$   
"  $S \in L$      $bS \in L$
3. No other strings are in  $L$

$$S \rightarrow \lambda / aS / bS$$

② Language of Palindrome

$$S \rightarrow \lambda / a/b / aSa / bSb$$

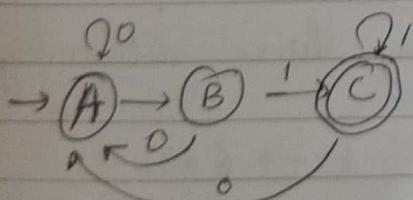
1.  $\lambda, a, b \in L$
2. for any  $S \in L$ ,  $aSa \in L$  and  $bSb \in L$
3. No other ...

③  $a^n b^n \quad n \geq 0$

Non- Regular

$$S \rightarrow \lambda / asb$$

1.  $\lambda \in L$
2. for any  $S \in L$ ,  $aSb \in L$
3. No other ...



$$\begin{array}{lll}
 A \rightarrow 0A & B \rightarrow 0A & C \rightarrow 1C \\
 A \rightarrow 1B & B \rightarrow 1C & C \rightarrow 0A
 \end{array}$$

011011

$$A \rightarrow 0A$$

$$01B$$

$$011C$$

$$0110A$$

$$01101B$$

- 011011(C) <sup>end state</sup> hence 011011 possible.

# Derivation tree

Eg

$$E \rightarrow E+E \mid E-E \mid a$$

Ambiguous grammar.

$$E \rightarrow E+E$$

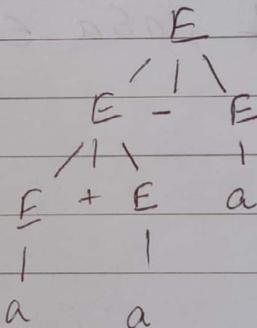
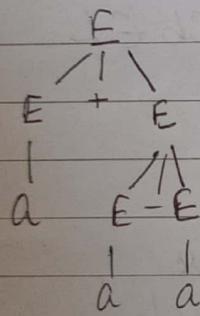
$$\begin{array}{c} \rightarrow E+E-E \\ \quad \quad \quad \quad | \\ \quad \quad \quad a \quad a \end{array}$$

$$E \rightarrow E-E$$

$$\begin{array}{c} \rightarrow E+E-E \\ \quad \quad \quad \quad | \\ \quad \quad \quad a \quad a \end{array}$$

$$\therefore a+a-a$$

$$\therefore a+a-a$$



Eg

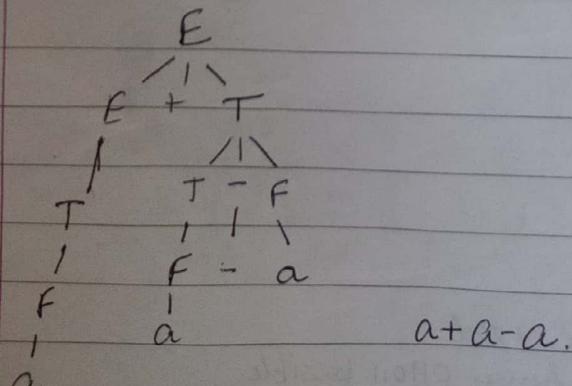
$$E \rightarrow E+T \mid T$$

$$a+a-a$$

$$T \rightarrow T-F \mid F$$

$$F \rightarrow a$$

only one way hence not ambiguous



$$a+a-a$$

$$\begin{array}{l} E \rightarrow E+T \\ \rightarrow T+T-F \\ F+F-a \\ a+a-a \end{array}$$

Write CFG

1.  $a^i b^j c^k \mid i = j+k$
2.  $a^i b^j c^k \mid j = i+k$
3.  $a^i b^j c^k \mid j=1 \text{ or } j=k$
4.  $a^i b^j c^k \mid i=j \text{ or } j=k$
5.  $a^i b^j c^k \mid i \leq j \leq 2i$

$\underline{a^i b^j b^k c^k} \quad f \rightarrow a s b / \cancel{f T}$   
 $f \rightarrow b T c / \cancel{f n}$

Q2.  
Ans.  $A \rightarrow a A b / \lambda$   
 $B \rightarrow b B c / \lambda$

Ans.  $a^{j+k} b^j c^k$   
 $a^k a^j b^j c^k$

$S \rightarrow a S c / T$   
 $T \rightarrow a T b / \lambda$

Q3.  $a^i b^j c^k$        $a^i b^k c^k$   
= Q4.  $A \rightarrow a A b / \lambda$        $\cancel{P} \rightarrow P a / \lambda$   
 $B \rightarrow B C / \lambda$        $\cancel{Q} \rightarrow b Q c / \lambda$

Ans Q.  $a^i b^j c^k$        $i < j$  or  $j > k$

$B \rightarrow b B / b$        $T \rightarrow a T b / \lambda$   
 $C \rightarrow c C / c$        $U \rightarrow a U c / V$   
 $A \rightarrow a A / a$        $V \rightarrow b V / \lambda$

$S \rightarrow T B C / A U$

$S_1 \rightarrow AB \cancel{PQ}$  or  $S_2 \rightarrow PQ$

$S \rightarrow AB | PQ$   
 $S \rightarrow \cancel{AB} \cancel{PQ}, S_1 | S_2$

$\circlearrowleft i < j$   
 $\underline{a^i b^j b^m c^k}, m > 0$

$A \rightarrow a A b / \lambda$

$B \rightarrow b B / b$

$C \rightarrow c C / \lambda$

$S_1 \rightarrow ABC$

$\circlearrowleft i > k$        $a^k a^m b^j c^k$   
 $P \rightarrow a P c / \cancel{b} / \lambda \quad m > 0$   
 $Q \rightarrow a Q / a$   
 $R \rightarrow b R / \lambda$   
 $S_2 \rightarrow PQR$        $S \rightarrow S_1 | S_2$

→ left most / right most derivation

$S \rightarrow A / B$       Striuy 00101  
 $A \rightarrow 0 A / \lambda$   
 $B \rightarrow 0 B | 1 B / \lambda$

$S \Rightarrow B$   
 $0B$   
 $00B$   
 $001B$   
 $0010B$   
 $00101B$       → 00101

Q.  $S \rightarrow AS / Sa / \lambda$   
String: aaaa.

$S \rightarrow AS$  ← right most  
a~~a~~as  
aaa's  
aaagas  
aaaa

$S \rightarrow Sa$  ← left most  
Saa  
Saaa  
Saaaa  
aaaa

Q.  $S \rightarrow SS / a/b$  Ambiguous  
String: abba.

$S \rightarrow SS$   
ss' ss  
a' b' b' a.

Q.  $S = A/B$  Ambiguous  
 $A \rightarrow aAb / ab$   
 $B \rightarrow abB / \lambda$

$S \rightarrow A$   
~~A~~  
ab

$S \rightarrow B$   
abB  
ab

Q.  $S \rightarrow AB / C$  Ambiguous.  
 $A \rightarrow aAb / ab$   
 $B \rightarrow cBd / cd$   
 $C \rightarrow aCd / ad \cancel{d}$   
 $D \rightarrow bDc / bc$

$S \rightarrow AB$   
a' Ab → cb~~d~~  
aabb ecdd.

$S \rightarrow C$   
acd = aabbcc  
~~acd~~  
~~aabbcc~~

8.  $S \rightarrow AS / aaB$  Ambiguous.  
 $A \rightarrow a / Aa$   
 $B \rightarrow b$   
aab.

$S \rightarrow AB$        $S \rightarrow aAB$   
 $/$   
 $AaB$   
aab.

8.  $S \rightarrow a / abSb / aAb$  Ambiguous.  
 $A \rightarrow bS / aAnb$

abababb

$S \rightarrow aAb$   
~~baAAbb~~  
abSb.  
ababSbb.  
abababb.

$S \rightarrow absb$   
ab aAbb  
ababSbb  
abababb

9.  $S \rightarrow asbs / bsas / \lambda$  Ambiguous.  
~~skij~~ abab

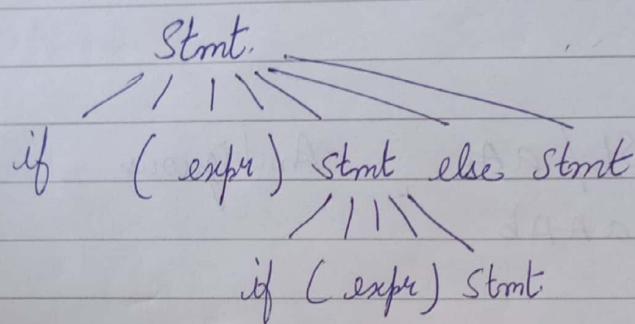
$\text{Q}$   
 $S \rightarrow asbs$   
absaSb  
abab.

$S \rightarrow asbs$   
 $\begin{matrix} \lambda \\ \wedge \end{matrix}$   
ab asbs  
absb.

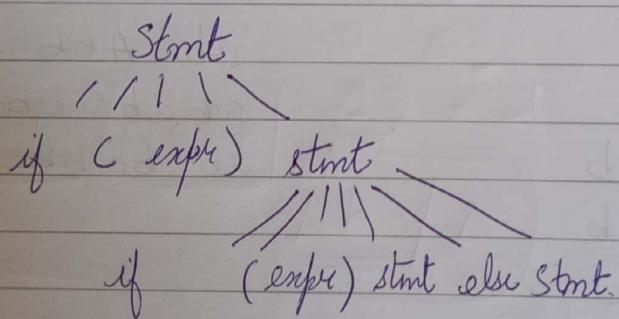
Dangling else.

Q. Statement → if (expr) Stmt /  
if (expr) Stmt else Stmt /  
other Stmt.  
Ambiguous.

stmt : if (expr) if (expr) stmt ;  
else stmt



OK.



Q Stmt  $\rightarrow$  s<sub>1</sub> | s<sub>2</sub>

$S_1 \rightarrow \text{if } (\text{expr}) \text{ stmt\_else } \text{stmt\_1} \text{ other\_stmt}$

$S2 \rightarrow \text{if expr) stmt} \mid \text{if expr) stmt else stmt.}$

$E \rightarrow E+E \mid E * E \mid id \longrightarrow$  Ambiguous  
 $E \rightarrow E + T \mid T$   
 $T \rightarrow + F \mid F$  } unambiguous

$$F \rightarrow id.$$

$\text{id} + \text{id} * \text{id}$ .

## Normal forms



Chomsky Normal form (CNF)

How to remove  $\lambda$

Q.

$$S \rightarrow S+a | S-a | \lambda$$

Nullable product as  $S$  can be having  $\lambda$ .

Ans.

$$S \rightarrow S+a | S-a | +a | -a$$

Q.

$$E \rightarrow E+E | E * E | id | \lambda$$

Ans.

$$E \rightarrow E+E | E * E | id | + | * | E+ | +E | E* | *E$$

### CNF

① Remove Null Productions

② Remove Unit production

③ Right side with single terminal  
or two or more non-terminals

$$E \rightarrow T | E+T$$

Here  $E \rightarrow T$

is the unit production

④ Right side can have 2 non-terminals.

Q.  $E \rightarrow E+T | T$  ~~T~~

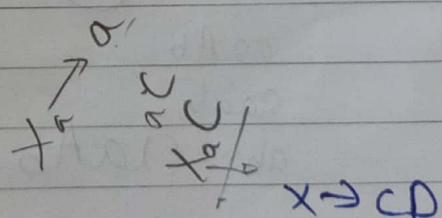
$$T \rightarrow T * F | \cancel{TF}$$

$$F \rightarrow id.$$

Ans.  $E \rightarrow E+T | T * F | id.$

$$T \rightarrow T * F | id.$$

$$F \rightarrow id.$$



Q.

$$S \rightarrow AACD$$

$$A \rightarrow aAb | ab$$

$$C \rightarrow ac | a$$

$$D \rightarrow aDa | bDb | aa | bb$$

①  $S \rightarrow AACD | A\cancel{CD} | AAC | c | CD | AC$

$$A \rightarrow aAb | ab$$

$$C \rightarrow ac | a$$

$$D \rightarrow aDa | bDb | aa | bb$$

$S \rightarrow AACD | A\cancel{CD} | AAC | \cancel{ac} | CD | AC | a$

$\cancel{ac}$

$x a \rightarrow a$

## Tutorial - 6.

Q1.

$$7. \quad a^i b^j \quad i < j \quad j = i+m \quad a^i b^j \quad 1 \leq i \leq 2j$$

$$a^i b^{i+m}$$

$$Q \rightarrow a^i b / \lambda$$

$$P \rightarrow P^i b / b$$

$$S \rightarrow aSb | aasb | aaasbb | Sb | b | ab$$

$$S \rightarrow QP.$$

$$6. \quad a^i b^j \quad 1 \leq i \leq 2j$$

$$l=2^i \text{ or } i=2^j + m$$

$$A \rightarrow aaAb / \lambda$$

$$B \rightarrow aB$$

$$l_a \leq 2^j$$

$$0 \leq 2^0$$

$$l \leq -1$$

$$\begin{array}{lll} l=0 & l=1 & l=2 \\ j=0 & j=1 & j=2 \\ 1 \leq 2^l & & 2 \leq 4 \end{array}$$

$$aaAb$$

$$aaab$$

$$ab$$

$$aaAb$$

(Dex)

8.  $a^i b^j \mid i \leq j \leq 2i$   $\wedge, ab, abb$

$0 \rightarrow 0 \quad 2 \rightarrow 2, 3, 4, 5, 6$   
 $1 \rightarrow 1, 2 \quad 3 \rightarrow 3, 4, 5, 6$

$3 \leq \leq 6$

$3, 4, 5, 6$

$A \rightarrow aAb \mid aAbb \mid \cancel{abb} \mid \wedge$

$i \leq j$

$aaabbbaa$

$aaabbb$

$aaa\ bbbb$

$aaa'$

Q6.  $\{a^i b^j \mid i \leq j\}$

$\begin{cases} i=0, & j=0, 1, 2, \dots \\ i=1, & j=1, 2, 3, \dots \\ i=2, & j=1, \dots \\ i=3, & j=2, \dots \end{cases}$

$A \rightarrow aAb \mid aaAb \mid Ab \mid \wedge$

$L = \{a^i b^i \mid i \geq 0\}$

$x = uvw$        $x = 0^n 1^n$   
 $u = 0^n \quad v = 1^k \quad w = 1^{n-k}$

1)  $UVW = 0^n 1^k 1^{n-k} = 0^n 1^n \in L$   
 2)  $UV^2W = 0^n 1^{2k} 1^{n-k} = 0^n 1^k 1^n \notin L$

So  $L$  is not regular.

Q.  $S \rightarrow AaA / CA / BaB$   
 $A \rightarrow aaBa / CDA / aa / DC$   
 $B \rightarrow bB / bAB / bb / aS.$   
 $C \rightarrow Ca / bc / D$   
 $D \rightarrow bD / \lambda$

Q. Remove Null.  
 $S \rightarrow ABA$   
 $A \rightarrow aA / \lambda$   
 $B \rightarrow bB / \lambda$

$S \rightarrow ABA / AB / BA / B / A / \cancel{AA}$   
 $A \rightarrow aA / a$   
 $B \rightarrow bB / b$

Q.  $S \rightarrow asa / bsb / \lambda$   
 $A \rightarrow aBb / bBa$   
 $B \rightarrow aB / bB / \lambda$

$S \rightarrow asa / bsb / aa / bb$   
 $A \rightarrow aBb / bBa / ab / ba$   
 $B \rightarrow aB / bB / a / b$ .

Q.  $S \rightarrow A / B / C$   
 $A \rightarrow AAa / B$   
 $B \rightarrow bB / bb / \lambda$   
 $C \rightarrow aCa / D$   
 $D \rightarrow baD / abD / aa$

$S \rightarrow A / B / C$   
 $A \rightarrow AAa / aa / B$   
 $B \rightarrow bB / bb / b$   
 $C \rightarrow aCa / D$   
 $D \rightarrow baD / aad / aa.$

# Push Down Automata (PDA)

$q_0 \rightarrow \text{Push}$

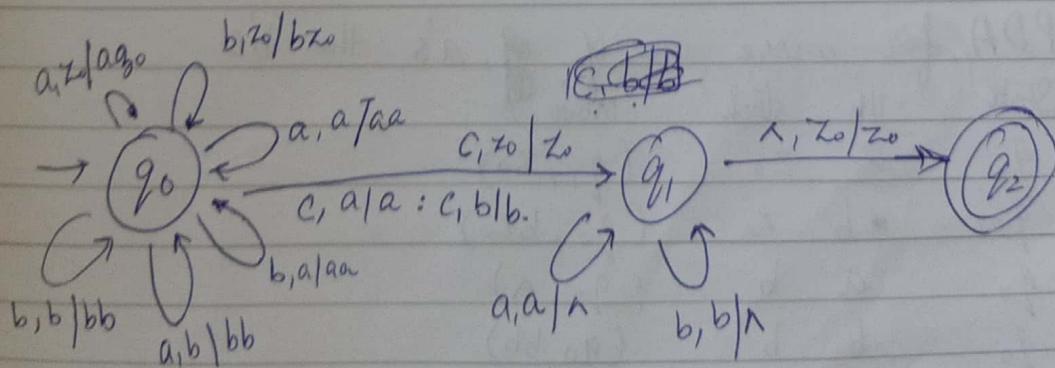
$q_1 \rightarrow \text{Pop}$

$q_2 \rightarrow \text{Accept.}$

For Palindrome, odd length

State    iff    stack    move

$q_0$	a	$z_0$	$(q_0, aq_0)$	<del>(<math>q_1, z_0</math>)</del>
$q_0$	a	a	$(q_0, aa)$	
$q_0$	b	$z_0$	$(q_0, bz_0)$	
$q_0$	b	b	$(q_0, bb)$	
$q_0$	a	b	$(q_0, a_1 b)$	
$q_0$	b	a	$(q_0, ba)$	
$q_0$	c	$z_0$	$(q_1, z_0)$	
$q_0$	c	a	$(q_1, a)$	
$q_0$	c	b	$(q_1, b)$	
$q_0$	$\lambda$	$z_0$		
$q_1$	a	a	$(q_1, \lambda)$	
$q_1$	b	b	$(q_1, \lambda)$	
$q_1$	$\lambda$	$z_0$	$(q_2, z_0) \rightarrow \text{accept.}$	



$t(q_0, abcbab, z_0)$   
 $r(q_0, bcba, az_0)$   
 $t(q_0, cba, ba z_0)$   
 $t(q_1, ba, ba z_0)$   
 $t(q_1, a, az_0)$   
 $t(q_1, \lambda, z_0) \rightarrow q_2$

Q. State iff stack move.

$q_0$	a	$z_0$	$(q_0, az_0)$	$(q_1, z_0)$
$q_0$	a	a	$(q_0, aa)$	$(q_1, a)$
$q_0$	b	$z_0$	$(q_0, bz_0)$	$(q_1, z_0)$
$q_0$	b	b	$(q_0, bb)$	$(q_1, b)$
$q_0$	a	b	$(q_0, ab)$	$(q_1, b)$
$q_0$	b	<del>b</del> a	$(q_0, ba)$	$(q_1, a)$
$q_0$	$\lambda$	$\lambda z_0$	$(q_1, z_0)$	
$q_0$	a	$z_0$	$(q_1, a)$	
$q_0$	b	$z_0$	$(q_1, b)$	
$q_1$	a	<del>b</del> a	$(q_1, \lambda)$	
$q_1$	b	b	$(q_1, \lambda)$	
$q_1$	$\lambda$	$z_0$	$(q_2, z_0)$	→ Accept.

Q. PDA for more no. of ~~a~~s as the ~~b~~s.

State	iff	stack	Move ↓
$q_0$	a	$z_0$	$(q_1, z_0)$
$q_0$	<del>ab</del>	$z_0$	$(q_0, bz_0)$
$q_0$	a	<del>b</del>	$(q_0, \lambda)$
$q_0$	b	b	$(q_0, bb)$
$q_1$	a	$z_0$	$(q_1, az_0)$
$q_1$	a	a	$(q_1, aa)$
$q_1$	b	a	$(q_1, \lambda)$

$(q_0, ababa, z_0)$

$(q_1, ababa, z_0)$

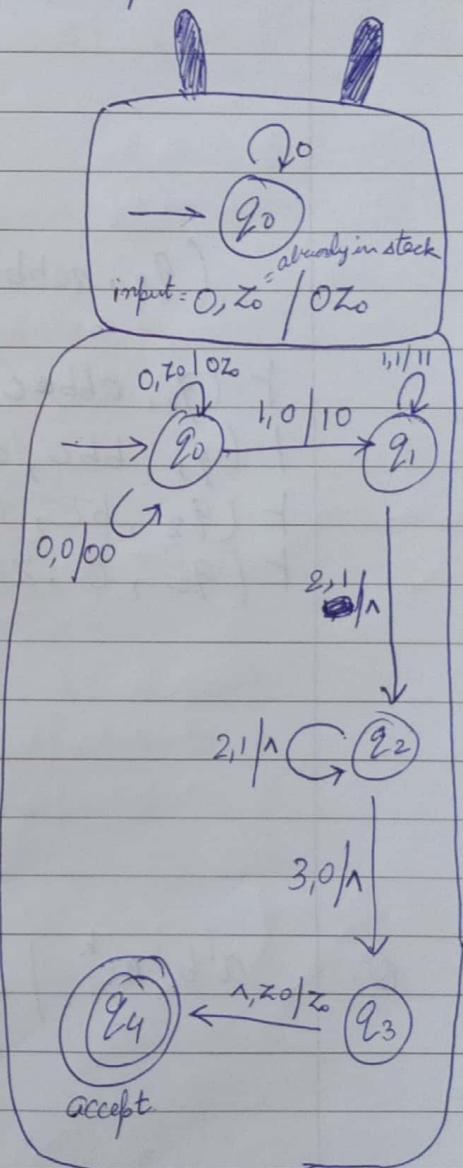
$(q_1, baba, az_0)$

$(q_1, aba, z_0)$

$(q_1, ba, a z_0)$

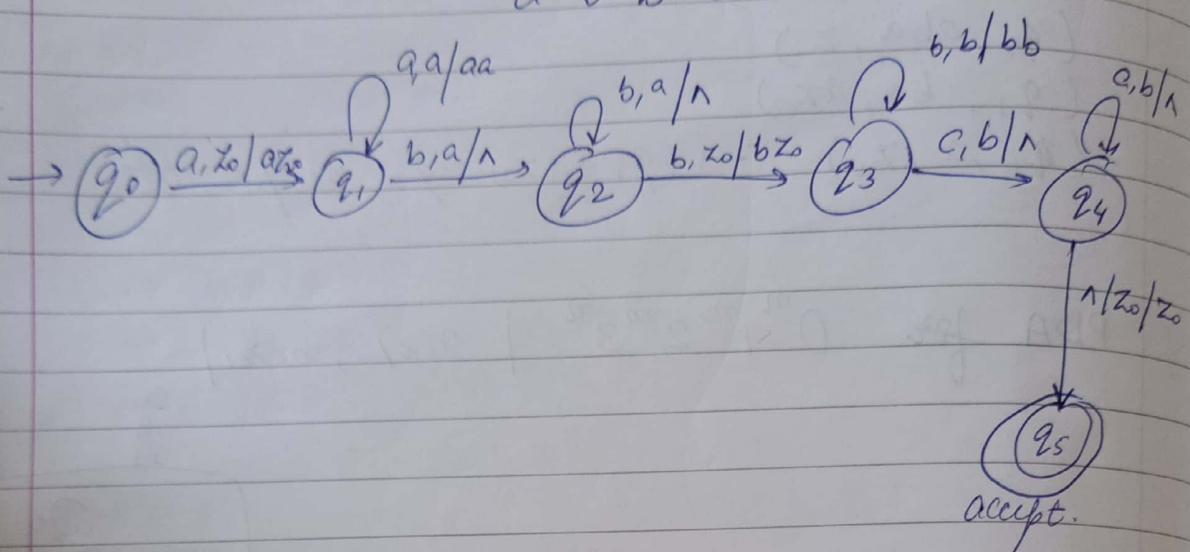
$(q_1, a, z_0) \rightarrow (q_1, az_0) = q_2$  accept state.

Q. PDA for  $0^n 1^m 2^m 3^n \mid n \geq 1, m \geq 1$



Q.  $a^n b^{n+m} c^m$        $n, m \geq 1$

$a^n b^m b^m c^m$



$(q_0, aabbcc, z_0)$

$(q_0, aabbcc, z_0)$

$\vdash (q_1, abbc, az_0)$

$\vdash (q_1, abbbcc, az_0)$

$\vdash (q_1, bcc, az_0)$

$\vdash (q_1, bbbbcc, az_0)$

$\vdash (q_2, bc, az_0)$

$\vdash (q_2, bbbcc, az_0)$

$\vdash (q_2, c, z_0) \times$

$\vdash (q_2, bbcc, z_0)$

$\vdash (q_3, bcc, bz_0)$

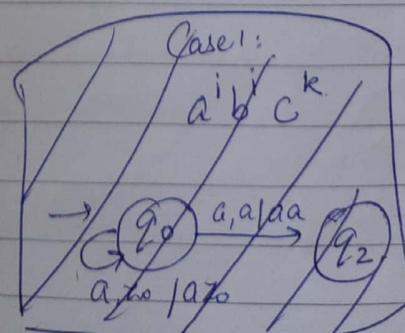
$\vdash (q_3, cc, bbz_0)$

$\vdash (q_4, c, bz_0)$

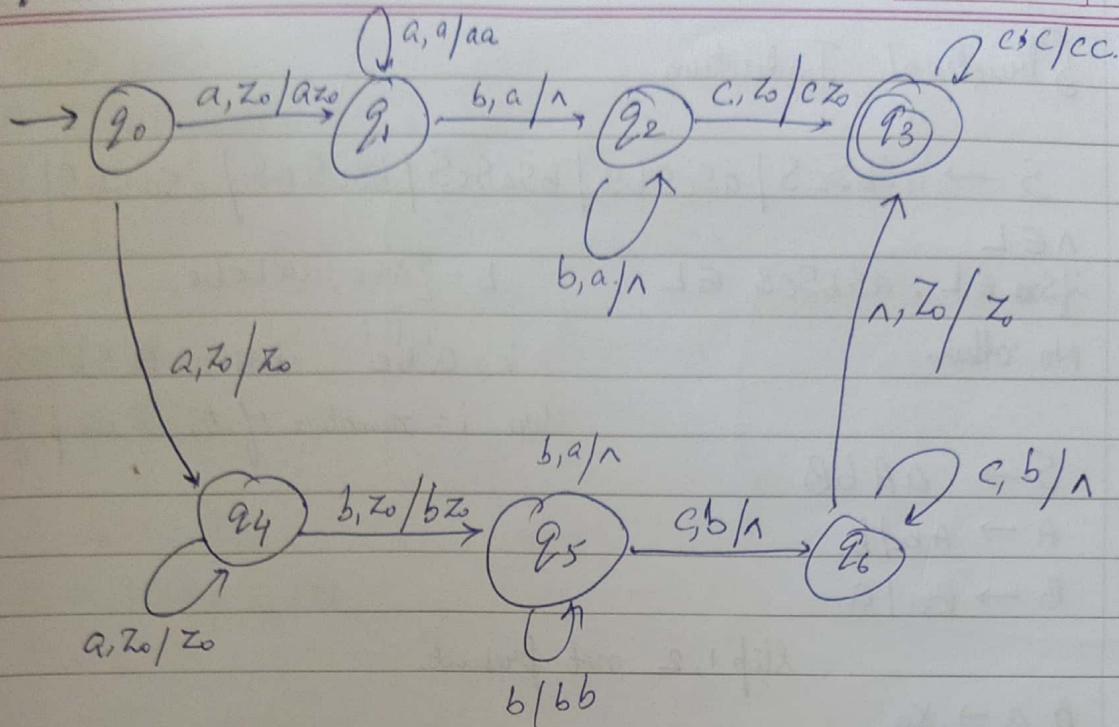
$\vdash (q_4, n, z_0) - \text{Accept}$

$q_5 = \text{accepted.}$

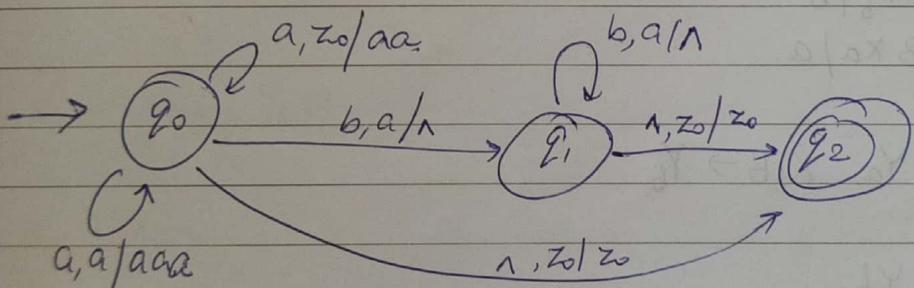
Q.  $a^i b^j c^k \mid i, j, k \geq 1 \quad \& \quad j=i \text{ or } j=k.$



NPDA



Q.  $a^n b^{2n} / n \geq 0$



$(q_0, ab^b, z_0)$

$\vdash (q_0, bb, aa z_0)$

$\vdash (q_1, b, a z_0)$

$\vdash (q_1, \lambda, z_0)$

$q_2$  - Accepted

## Q3. Structural Induction

$$S \rightarrow aSbScS \mid asScbs \mid bSaScS \mid bScSas \mid cSasbs \mid csbSas$$

1.  $\wedge L$
2. if  $S \in L$ ,  $aSbScS \in L$
3. No other.

$$L = \{abc, aabcbc, \dots\}$$

$$k = a^i b^i c^i \quad k+1 = a^i b^i c^{i+1}$$

Here  $i = \text{number of time we perform step 3.}$

## Q4.

$$1. S \rightarrow aAbB$$

$$A \rightarrow Ab \mid b$$

$$B \rightarrow Ba \mid a$$

Step 1, 2 not present.

$$a \rightarrow X_a$$

$$b \rightarrow X_b$$

Step 3.

$$\therefore S \rightarrow X_a A X_b B$$

$$A \rightarrow A X_b \mid b$$

$$B \rightarrow B X_a \mid a$$

$$X_a A \rightarrow Y_a \quad X_b B \rightarrow Y_b$$

$$S \rightarrow Y_a Y_b$$

$$A \rightarrow A X_b \mid b$$

$$B \rightarrow B X_a \mid a$$

$$2. S \rightarrow aA \mid bB$$

$$A \rightarrow bAA \mid a$$

$$B \rightarrow BBa \mid b$$

$$X_a = a$$

$$X_b = b$$

$$S \rightarrow X_a A \mid X_b B$$

$$A \rightarrow X_b AA \mid a$$

$$B \rightarrow BBX_b \mid b$$

M	T	W	T	F	S	S
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$$3. \quad S \rightarrow aAC$$

$$A \rightarrow aB \mid bAB$$

$$B \rightarrow b$$

$$C \rightarrow c$$

$$X_a \rightarrow a$$

$$X_b \rightarrow b$$

$$S \rightarrow X_a A C$$

$$A \rightarrow X_a B \mid X_b AB$$

$$B \rightarrow b$$

$$C \rightarrow c$$

$$X_a A \rightarrow Y_a$$

$$X_b A \rightarrow Y_b$$

$$S \rightarrow Y_a C$$

$$A \rightarrow X_a B \mid Y_b B$$

$$B \rightarrow b$$

$$C \rightarrow c.$$

TUT-7.

## Ambiguous and Non-Ambiguous

Q.

$$S \rightarrow SS/a/b$$

Ambiguous

$$S \rightarrow AS/bS/a/b$$

Unambiguous

Eg: String: aba

Q.

$$S \rightarrow S+S/S*S/a \quad \text{Ambiguous}$$

$$\left. \begin{array}{l} S \rightarrow S+T/T \\ T \rightarrow T*F/F \\ F \rightarrow S/a \end{array} \right\}$$

Unambiguous

Q6.

$$1. \quad S \rightarrow ABC/0$$

$$A \rightarrow 1$$

$$B \rightarrow C/0$$

$$C \rightarrow D$$

$$D \rightarrow E$$

$$E \rightarrow 2$$

$$S \rightarrow ABC/0$$

$$A \rightarrow 1$$

$$B \rightarrow 2/0$$

$$C \rightarrow 2$$

$$D \rightarrow 2$$

$$E \rightarrow 2$$

2.

$$S \rightarrow ABCD/0$$

$$A \rightarrow BC/1$$

$$B \rightarrow C$$

$$C \rightarrow D$$

$$D \rightarrow d$$

$$S \rightarrow ABCD/0$$

Q.

$$S \rightarrow ABA \quad \text{[Crossed out]}$$

$$A \rightarrow aA/\lambda$$

$$B \rightarrow abB/\lambda$$

Ambiguous.

$$S \rightarrow ABA / X$$

$$A \rightarrow aA/\lambda$$

$$B \rightarrow bB/b$$

$$X \rightarrow ax/\lambda$$

Unambiguous

Q.  $S \rightarrow A/B$   
 $A \rightarrow aAb/ab$   
 $B \rightarrow abB/\lambda$   
 Ambiguous

$S \rightarrow A/B$   
 $A \rightarrow aAb/ab$   
 $B \rightarrow abB/ab\lambda ab$   
 Unambiguous

Q.  $S \rightarrow aSb/aasb/\lambda$   
 Ambiguous

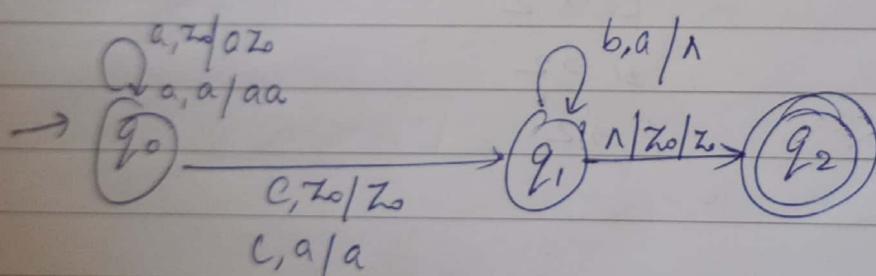
$\boxed{S \rightarrow axb feg}$   $S \rightarrow x/y/\lambda$

$X \rightarrow a\lambda b/\lambda$   
 $Y \rightarrow aaYb/\lambda$   
 $Z \rightarrow aabb/\lambda$

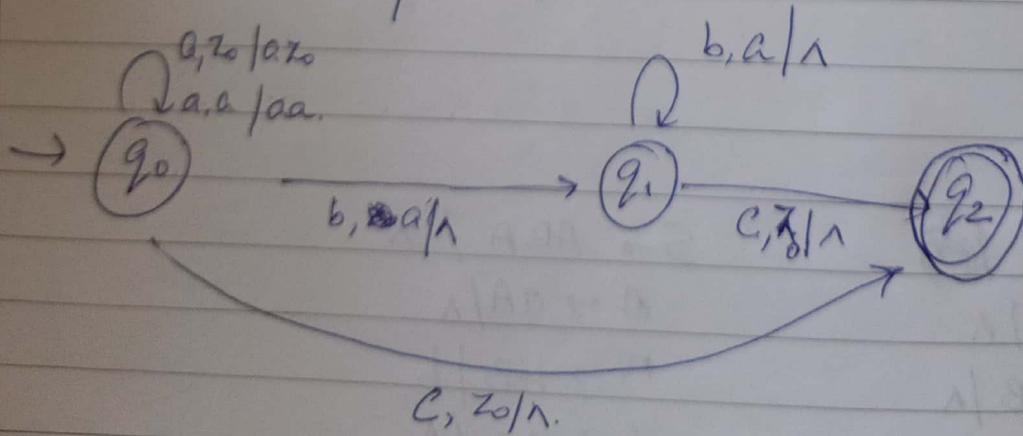
WT-7

Design PDA.

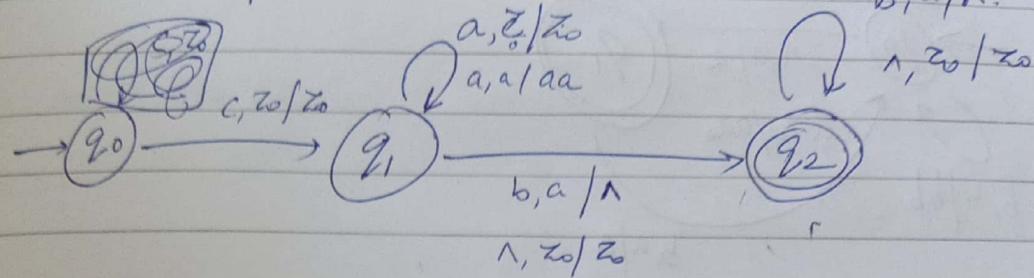
1).  $L_1 = \{a^m c b^m \mid m \geq 0\}$



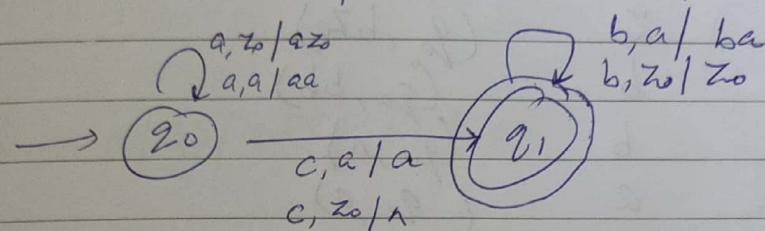
2).  $L_2 = \{a^m b^m c \mid m \geq 0\}$



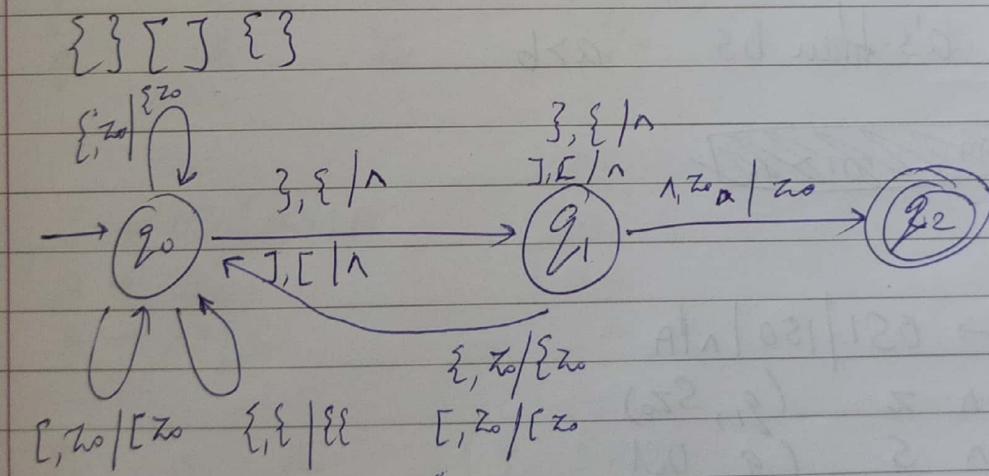
3).  $L_3 = \{ca^m b^m / m \geq 0\}$



4).  $L_4 = \{a^n cb^m / n, m \geq 0\}$



5).  $\{\}, [ ] \quad S \rightarrow \{S\} | [S] | SS | \lambda$



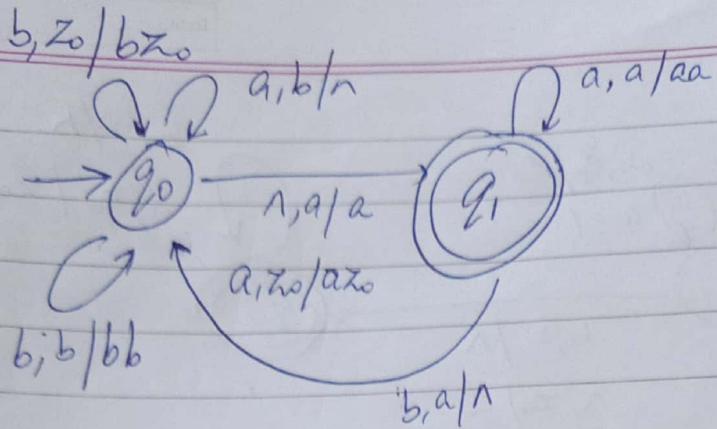
~~{} {} [ ] [ ]~~    {{ } } {{ } }    { }

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[ ] [ ]

{ } { } { } { }

Q.



$q_0$	a	$z_0$	$(q_1, a z_0)$
$q_0$	b	$z_0$	$(q_0, b z_0)$
$q_0$	b	b	$(q_0, b b)$
$q_0$	a	b	$(q_0, \lambda)$
$q_0$	$\lambda$	a	$(q_1, a)$
$q_1$	a	a	$(q_1, a a)$
$q_1$	b	a	$(q_0, \lambda)$

more a's than b's.  $a > b$

~~more a's than b's~~

$S \rightarrow OS1 | IS0 | \lambda | A$

- $q_0 \lambda z_0 (q_1, S z_0)$
- $q_1 \lambda S (q_1, O S I)$
- $(q_1, I S O)$
- $(q_1, \lambda)$

Q1. CFG to PDA

1. for each Non-terminal A, include transition  $\delta(z, \lambda, A) \rightarrow \{q_1\}$

$/A \rightarrow \infty$  is in ( $G_1$ )

2. for each terminal a, include a transition  $\delta(q_1, a, a) \rightarrow (q_1, \lambda)$ .

$S \rightarrow a / qS / bSS / SSb / SbS$

$q_0$	$\lambda$	$z_0$	$(q_1, Sz_0)$
$q_1$	$\lambda$	$S$	$(q_1, a)(q_2, qS)(q_1, bSS)(q_1, SSb)(q_1, SbS)$
$q_1$	$a$	$a$	$(q_1, \lambda)$
$q_1$	$b$	$b$	$(q_1, \lambda)$
$q_1$	$\lambda$	$z_0$	$(q_2, z_0)$ Accept.

Q.  $S \rightarrow 0S0 / 1S1 / A$

$A \rightarrow 2BB$

$B \rightarrow 23 / 31$

$q_0$	$\lambda$	$z_0$	$(q_1, Sz_0)$
$q_1$	$\lambda$	$S$	$(q_1, 0S0)(q_1, 1S1)(q_1, A)$
$q_1$	$\lambda$	$A$	$(q_1, 2BB)$
$q_1$	$\lambda$	$B$	$(q_1, 23)(q_1, 31)$
$q_1$	$0/1$	$0/1$	$(q_1, \lambda)$
	$3/2$	$3/2$	$(q_1, \lambda)$
$q_1$	$\lambda$	$z_0$	$(q_2, z_0)$ Accept state.

Deriv - ~~22~~ 22331

$S \rightarrow A$

$\Rightarrow 2BB$

$\Rightarrow 22331$

Q.

Design PDA for

$$\begin{aligned} S &\rightarrow bX/aY \\ X &\rightarrow bXX/aS/a \\ Y &\rightarrow aYY/bS/b \end{aligned}$$

$q_0$	$\lambda$	$z_0$	$(q_1, S z_0)$
$q_1$	$\lambda$	$X$	$(q_1, bXX)(q_1, aS), (q_1, a)$
$q_1$	$\lambda$	$Y$	$(q_1, aYY)(q_1, bS)(q_1, b)$
$q_1$	$\lambda$	$S$	$(q_1, bX)(q_1, aY)$
$q_1$	$a$	$a$	$(q_1, \lambda)$
$q_1$	$b$	$b$	$(q_1, \lambda)$
$q_1$	$\lambda$	$z_0$	$(q_2, z_0) \text{ Accept.}$

Tut-7

Q2.  $S \rightarrow ABCD$

$A \rightarrow a$

$B \rightarrow C/b$

$C \rightarrow D$

$D \rightarrow c$

$S \rightarrow ABCD$

$A \rightarrow a$

$B \rightarrow C/b$

$C \rightarrow c$

$D \rightarrow c$

$S \rightarrow ABCD$

$A \nrightarrow a$

$B \rightarrow c/b$

$C \rightarrow c$

$D \rightarrow c$

Q3.

1.  $S \rightarrow ABC/AoA$

$A \rightarrow oA/B_oC/ooo/B$

$C \rightarrow CA/AC$

$D \rightarrow \lambda$

2.  $S \rightarrow AAA/B$

$A \rightarrow oA/B$

$B \rightarrow \lambda$

