Chapter 4
Regular Grammar and Regular Sets

(Solutions / Hints)

4.1 List all the strings of length up to five corresponding to the following regular expressions over $\{a, b\}$.
(a) $a(a + b)^*$
(b) $a(aa)^*$
(c) $(a + b)^*c$
(d) $(aa + bb)c(ab + ba)$
(e) $(aa + bb)^*c(ab + ba)$

Sol. **(a)**
a
aa, ab
aaa, aab, aba, abb
aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb
aaaaa, aaaab, aaaba, aaabb, aabaa, aabab, aabba, aabbb, abaaa, abaab, ababa, ababb, abbaa, abbab, abbba, abbbb

(b)
a, aaa, aaaaa

(c)
c
ac, bc
aac, abc, bac, bbc
aaac, aabc, abac, abbc, baac, babc, bbac, bbbc
aaaac,aaabc, aabac, aabbc, abaac, ababc, abbac, abbbc, baaac, baabc, babac, babbc, bbaac, bbabc, bbbac, bbbbc

(d) *aacab, bbcab, aacba, bbcba*

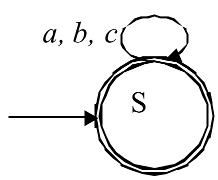
(e) *cab, cba, aacab, bbcba*

4.2 For the following sets, write the corresponding regular expression.
(a) $\{1, 12, 112, 1112, 11112, \ldots\}$
(b) $\{0, 1\}$
(c) $\{a^2, a^4, a^6, a^8, a^{10}, \ldots\}$
(d) The set of all strings over $\{a, b\}$ having exactly one a
(e) The set of all strings over $\{a, b\}$ ending with a
(f) The set of all strings over $\{a, b\}$ beginning with a
(g) The set of all strings over $\{a, b, c\}$ with exactly one c
(h) $\{a^x \mid x \text{ is divisible by 3 or 5}\}$
(i) The set of all strings over $\{a, b\}$ containing *bbbb* as a substring
(j) The set of all strings over $\{0, 1\}$ beginning with 0 and ending with 11
(k) The set of all strings over $\{0, 1\}$ containing no two consecutive 0s or 1s
(l) The set of all strings over $\{0, 1\}$ containing an odd number of characters
(m) The set of all strings over $\{0, 1\}$ containing an even number of characters

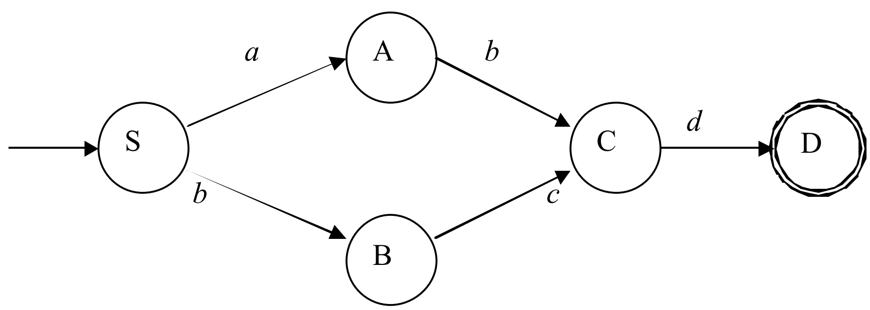
Sol. (a) 11*2
(b) 0+1
(c) *aa(aa)**
(d) b*ab*
(e) (a+b)*a
(f) a(a+b)*
(g) (a+b)*c(a+b)*
(h) *(aaa)*+(aaaaa)**
(i) *(a+b)*bbbb(a+b)**
(j) 0(0+1)*11
(k) 0(10)*1+0(10)*+1(01)*0+1(01)*
(l) (00+01+10+11)*(0+1)
(m) (00+01+10+11)*

4.3 Construct the finite automata for the following regular expressions.
(a) $(a + b + c)^*$
(b) $(ab + bc)d$
(c) $(ab + bc)^*k^*(d + e)$
(d) $a + bb + cc$

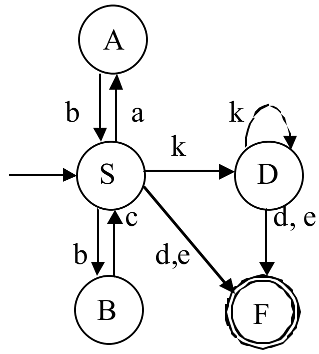
Sol. (a)



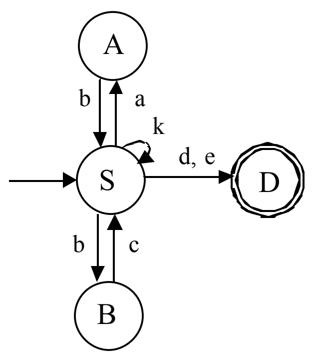
(b)



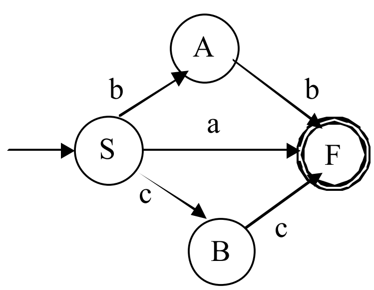
(c)



Caution: One may construct the FA as shown below. Such an automaton will accept all the valid strings in the language but it will also accept some strings that are not valid in the language. For example, repetition of k are allowed only after repetitions of ab or bc are over. Once k is reached repetitions of ab or bc are not allowed. But this automaton allows repetitions of ab, bc and k in any order which is not allowed. Hence, the above finite automaton.



(d)



4.4 Use pumping to prove that the language $L = \{0^n 1^n \mid n \geq 0\}$ is not regular.

Sol. Let L be a regular language with its corresponding finite automaton having m number of states.
Let $w=0^k 1^k$ be the string in the language L such that $|w|=2k > m$.
To use pumping lemma take $w = xyz$ with $|xy| \leq m$ and $|y| \neq 0$.

Now, the next step is to prove that the string xy^iz does not belong to L for all $i \geq 0$.

Let us take three possible cases.

Case I: y has only 0s. Let $y=0^t$ for some $t > 0$. Now, $xz=0^{k-t}1^k$.
Since $t > 0$ therefore $k-t < k$ and the string xz does not belong to L. Since at i=0, w=xyz=xz does not belong to L, hence L is not regular.

Case II: y has only 1s. Proof is similar to case I.

Case III : Let y contain both 0 and 1. Let $y=0^p1^s$. Now $x=0^{k-p}$ and $z=1^{k-s}$. At i=4, $xy^4z= 0^{k-p}\,0^p1^s\,0^p1^s\,0^p1^s\,0^p1^s1^{k-s}$. This string does not belong to L. Hence L is not regular.

4.5 Consider the following two regular expressions:

$R_1 = a^* + b^* \qquad R_2 = ab^* + ba^*$

Find a string w such that
(a) w belongs to R_1 but not to R_2
(b) w belongs to R_2 but not to R_1
(c) w belongs to both R_1 and R_2
(d) w belongs to neither R_1 nor R_2

Sol. (a) *aaa* belongs to R_1 but not to R_2.
bbb belongs to R_1 but not to R_2.
ϵ belongs to R_1 but not to R_2.

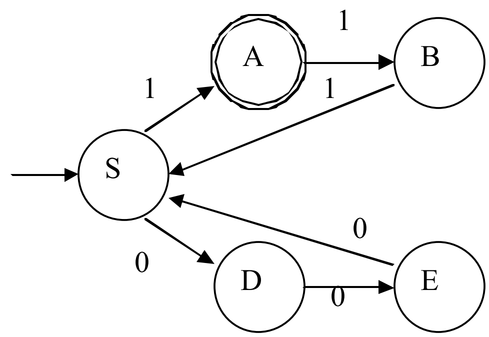
(b) *abbb* belongs to R_2 but not to R_1.
baaa belongs to R_2 but not to R_1.

(c) *a* belongs to bothR_1 and R_2.
b belongs to bothR_1 and R_2.

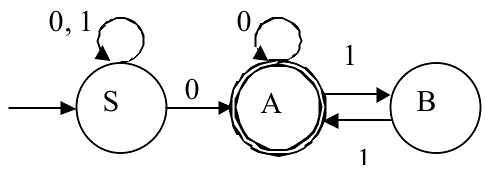
(d) *aba* belongs to neither R_1 nor R_2.
babb belongs to neither R_1 nor R_2.

4.6 For the following regular expressions, draw the corresponding finite automata:
(a) $(111 + 000)^*1$
(b) $(0 + 1)^*0(0 + 11)^*$
(c) $0 + 10^* + 001^*00$
(d) $(0 + 1)^*(01 + 1110)$

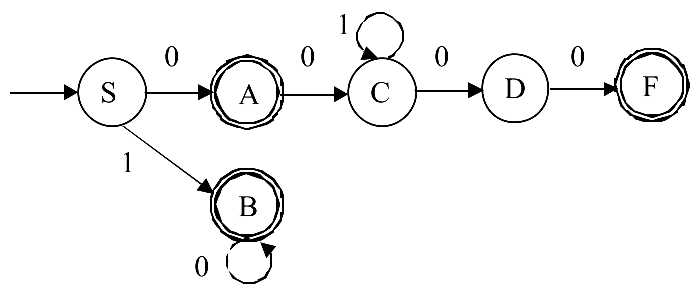
Sol. (a)



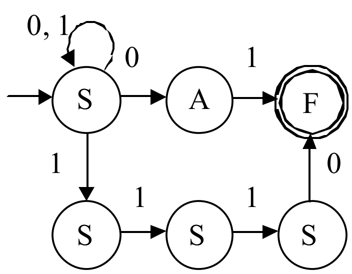
(b)



(c)

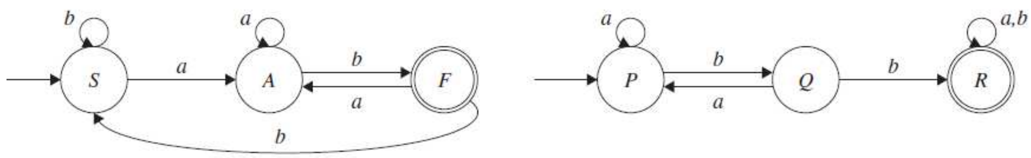


(d)



4.7 Let M_1 and M_2 be two finite automata accepting the languages L_1 and L_2 respectively as shown in the following figure. Construct the finite automata to accept the languages

(a) $L_1 \cup L_2$ (b) $L_1 \cap L_2$
(c) $L_1 - L_2$ (d) $L_2 - L_1$



Sol.
Q1={S, A, B}
Q2={P, Q, R}
Q1 X Q2= { (S, P), (S, Q), (S, R), (A, P), (A, Q), (A, R), (F, P), (F, Q), (F, R)}

Transition Table

Current State	Input Symbol	
	a	b
→ {S, P }	{A, P}	{S, Q}
{S,Q}	{A,P}	{S, R}
{S, R}	{A, R}	{S,R}
{A, P }	{A, P}	{F, Q}
{A,Q}	{A,P}	{F, R}
{A, R}	{A, R}	{F,R}
{F, P }	{A, P}	{S, Q}
{F,Q}	{A,P}	{S, R}
{F, R}	{A, R}	{S,R}

After removing the states that are not generated that is {A, Q} and {F, P}.

Current State	Input Symbol	
	a	b
→ {S, P }	{A, P}	{S, Q}
{A, P }	{A, P}	{F, Q}
{S,Q}	{A,P}	{S, R}

{F,Q}	{A,P}	{S, R}
{S, R}	{A, R}	{S,R}
{A, R}	{A, R}	{F,R}
{F, R}	{A, R}	{S,R}

Automata for the language L1 ∪ L2.

Current State	Input Symbol	
	a	b
→ {S, P }	{A, P}	{S, Q}
{A, P }	{A, P}	{F, Q}
{S,Q}	{A,P}	{S, R}
*{F,Q}	{A,P}	{S, R}
*{S, R} *{A, R}	{A, R} {A, R}	{S,R} {F,R}
*{F, R}	{A, R}	{S,R}

Automata for the language L1 ∩ L2

Current State	Input Symbol	
	a	b
→ {S, P }	{A, P}	{S, Q}
{A, P }	{A, P}	{F, Q}
{S,Q}	{A,P}	{S, R}
{F,Q}	{A,P}	{S, R}
{S, R}	{A, R}	{S,R}
{A, R}	{A, R}	{F,R}
*{F, R}	{A, R}	{S,R}

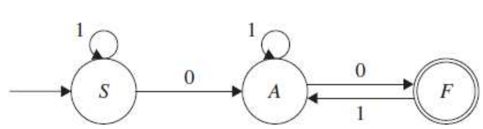
Automata for the language L1 - L2

Current State	Input Symbol	
	a	b
→ {S, P }	{A, P}	{S, Q}
{A, P }	{A, P}	{F, Q}
{S,Q}	{A,P}	{S, R}
* {F,Q}	{A,P}	{S, R}
{S, R}	{A, R}	{S,R}
{A, R}	{A, R}	{F,R}
{F, R}	{A, R}	{S,R}

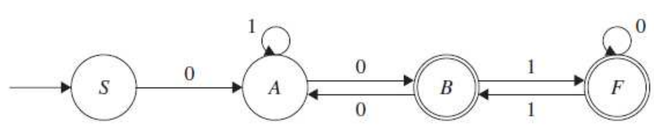
Automata for the language L2 – L1

Current State	Input Symbol	
	a	b
→ {S, P }	{A, P}	{S, Q}
{A, P }	{A, P}	{F, Q}
{S,Q}	{A,P}	{S, R}
{F,Q}	{A,P}	{S, R}
* {S, R}	{A, R}	{S,R}
* {A, R}	{A, R}	{F,R}
{F, R}	{A, R}	{S,R}

4.8 For the finite automata given in the following figures, find the corresponding regular expressions.



(a)



(b)

Sol. (a)

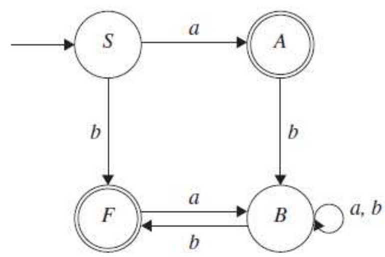
S= ϵ + S1= ϵ1* = 1*
F=A0
A=S0+ A1+ F1 =1*0 + A1+A01= 1*0+A(1+01)= 1*0(1+01)*
F=A0= 1*0(1+01)*0

Required regular expression: 1*0(1+01)*0

(b) S= ϵ
A=S0+ A1+ B0
F=B1+F0=B1(0*)*=B10*
B =A0 +F1=A0+B10*1=A0(10*1)*
A=S0+A1+B0= ϵ0+A1+ A0(10*1)*0=0+A1+ A0(10*1)*0= 0+ A(1+ 0(10*1)*0)
A=0(1+ 0(10*1)*0)*
B= A0(10*1)*= 0(1+ 0(10*1)*0)*0(10*1)*
F=B10*=0(1+ 0(10*1)*0)*0(10*1)*10*

Required regular expression:
0(1+ 0(10*1)*0)*0(10*1)*10* + 0(1+ 0(10*1)*0)*0(10*1)*

4.9 For the finite automaton given in the following figure, find the corresponding regular expression.

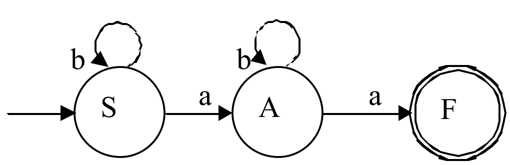


Sol. S= ϵ
A=Sa= ϵa=a
F= Sb+Bb= ϵb+Bb=b+Bb
B =Ab +B(a+b)+Fa=ab+B(a+b)+Fa=ab+Fa+B(a+b)=ab+(b+Bb)a+B(a+b)
=ab+ba+B(ba+a+b)=(ab+ba)(ba+a+b)*
F=b+(ab+ba)(ba+a+b)*b

Required regular expression: b+(ab+ba)(ba+a+b)*b +a

4.10 Draw a finite automaton M accepting the grammar $S \to bS \mid aA$, $A \to bA \mid a$. Find the regular expression corresponding to M.

Sol.



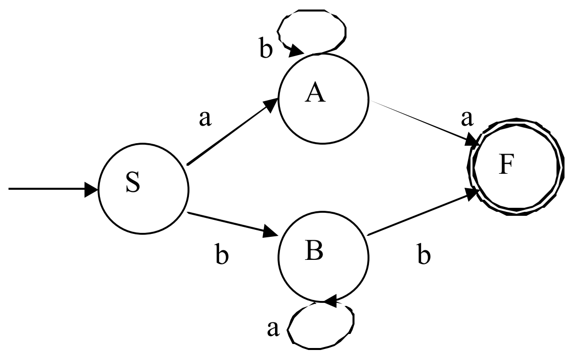
S= ϵ+Sb= ϵb*=b*
A=Sa+Ab=b*a+Ab=b*ab*

F=Aa=b*ab*a

Required regular expression: b*ab*a

4.11 Draw a finite automaton M accepting the grammar $S \to bB \mid aA$, $A \to bA \mid a$, $B \to aB \mid b$. Find the regular expression corresponding to M.

Sol.



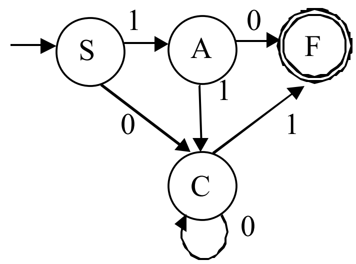
S= ϵ
A=Sa+Ab= ϵa+Ab=a+Ab=ab*
B= Sb+Ba= ϵb+Ba=b+Ba=ba*
F=Aa+Bb=ab*a+ba*b
Required regular expression: ab*a+ba*b

4.12 For the following languages over $\{a, b\}$, find the corresponding regular expression R.
(a) Every word in the language contains exactly three a's
(b) Every word in the language contains minimum three a's
(c) Every word contains alternate 00s and 11s
(d) $L = \{a^m b^n \mid m, n > 1\}$
(e) Every word begins and ends with 00

Sol. (a) b*ab*ab*ab*
(b) b*ab*ab*ab*(a+b)*
(c) 0011(0011)*(00)*+1100(1100)*(11)*
(d) aa*bb*
(e) 00(0+1)*00

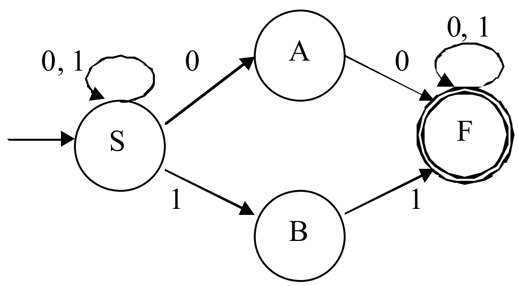
4.13 Design a finite automaton for the regular expression $10 + (0 + 11)0^*1$. If it is an NFA, then convert it into its equivalent DFA.

Sol. It is a DFA



4.14 Design a finite automaton for the regular expression (0 + 1)*(00 + 11)(0 + 1)*. If it is an NFA then convert it into its equivalent DFA.

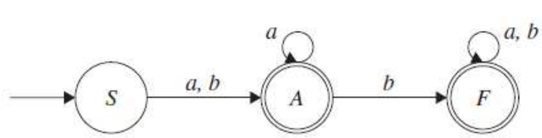
Sol. NFA corresponding to (0+1)*(00+11)(0+1)*



DFA corresponding to NFA

State	Input Symbol	
	0	1
→{S}	{S,A}	{S,B}
{S,A}	{S,A,F}	{S,B}
{S,B}	{S,A}	{S,B,F}
{S,A,F}	{S,A,F}	{S,B,F}
{S,B,F}	{S,A,F}	{S,B,F}

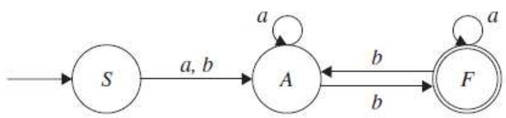
4.15 For the finite automaton in the following figure find the corresponding regular expression.



Sol.
S= ϵ
A=Sa+Sb+Aa= ϵa+ ϵb+Aa=a+b+Aa=(a+b)a*

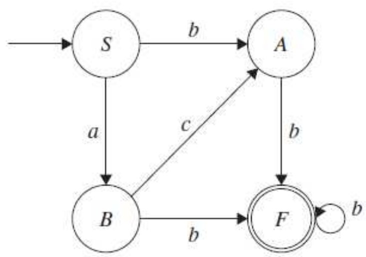
F=Ab+F(a+b)=(a+b)a*b+F(a+b)*=(a+b)a*b(a+b)*
Required regular expression: (a+b)a*b(a+b)*

4.16 For the finite automaton in the following figure find the corresponding regular expression.



Sol.
S= ϵ
A=Sa+Sb+Aa+Fb= ϵa+ ϵb+Aa+Fb=a+b+Aa+Fb
F=Ab+Fa=Aba*
A=a+b+Aa+Aba*b=a+b+A(a+ba*b)=(a+b)(a+ba*b)*
F=(a+b)(a+ba*b)*ba*
Required regular expression: (a+b)(a+ba*b)*ba*

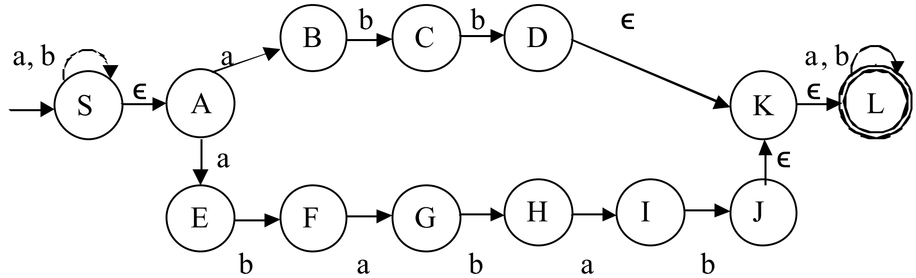
4.17 For the finite automaton in the following figure find the corresponding regular expression.

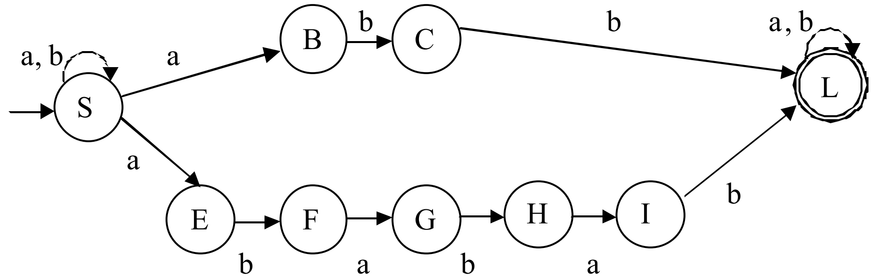


Sol.
S= ϵ
A=Sb+Bc= ϵb+Bc=b+Bc
B=Sa= ϵa=a
A=b+ac
F=Ab+Bb+Fb=(b+ac)b+ab+Fb=((b+ac)b+ab)b*
Required regular expression: ((b+ac)b+ab)b*

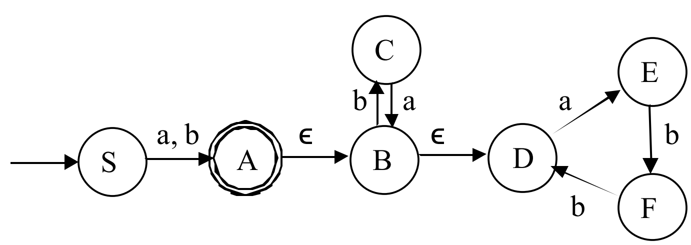
4.18 For the following regular expressions, draw an ϵ-NFA and convert them into their equivalent DFA.
(a) $(a + b)^*(abb + ababab)(a + b)^*$
(b) $(a + b)(ba)^*(abb)^*$

Sol.

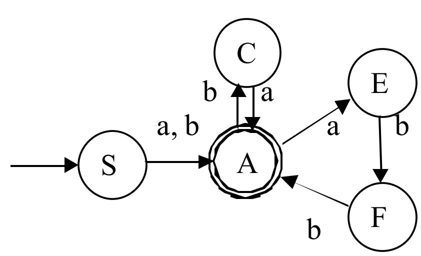




(b)



After removal of null moves, we have



Above finite automaton will accept all valid strings in the language, but it may also accept some other strings that are not in the language. For example, it will accept the repetitions of *ba* even after repetitions of *abb* which is not desirable. For example, *aabbbaba* will be a valid string which is not desirable. Hence in the actual automaton a path is so created that there is no way back once *abb* is reached. Hence, the above automaton is modified as given below: