DIPLOMA SEM : 2
(4320001 / 4320002)

* If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$ then find $2A - 4B + C$.

$\Rightarrow \quad 2A - 4B + C$

$$= 2\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 & 2 \\ 0 & 2 & 0 \\ 14 & 16 & 18 \end{bmatrix} - \begin{bmatrix} -4 & -8 & 0 \\ 4 & 4 & -4 \\ 8 & 8 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+4+3 & 4+8+0 & 2-0+5 \\ 0-4+6 & 2-4+9 & 0+4-1 \\ 14-8+7 & 16-8+8 & 18-8-2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 12 & 7 \\ 2 & 7 & 3 \\ 13 & 16 & 8 \end{bmatrix}$$

* If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ then find 'X' from $X + A + B = 0$.

$\Rightarrow \quad X + A + B = 0$

$\Rightarrow \quad (-X) = A + B$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$$

$$\Rightarrow (-X) = \begin{bmatrix} 4 & 0 & 5 \\ 4 & 9 & 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -4 & 0 & -5 \\ -4 & -9 & -2 \end{bmatrix}.$$

* If $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & -4 \\ 5 & 1 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 1 \end{bmatrix}$ and $4A + 3C = B$ then find matrix 'C'.

$\Rightarrow \quad 4A + 3C = B$

$\Rightarrow \quad 3C = B - 4A$

$$\Rightarrow 3C = \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 1 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 & 0 \\ 3 & +2 & -4 \\ 5 & 1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 8 & -4 & 0 \\ 12 & 8 & -16 \\ 20 & 4 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 17-8 & -1+4 & 3-0 \\ -24-12 & -1-8 & -16+16 \\ -7-20 & 1-4 & 1-36 \end{bmatrix}$$

$$\Rightarrow 3C = \begin{bmatrix} 9 & 3 & 3 \\ -36 & -9 & 0 \\ -27 & -3 & -35 \end{bmatrix}$$

$$\Rightarrow C = \frac{1}{3}\begin{bmatrix} 9 & 3 & 3 \\ -36 & -9 & 0 \\ -27 & -3 & -35 \end{bmatrix}.$$

* If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ then prove that $(A+B)^T = A^T + B^T$.

$$\Rightarrow A + B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & 4-2 \\ 1+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$$

$$\Rightarrow (A+B)^T = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \quad \text{— } (1)$$

Now, $A^T = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$, $B^T = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$

$$A^T + B^T = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow A^T + B^T = \begin{bmatrix} 3-1 & 1+2 \\ 4-2 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \quad \text{— } (2)$$

from eq^n. (1) & (2) $(A+B)^T = A^T + B^T$.

* If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find AB and BA.

$$\Rightarrow AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+3 & 2+2+6 \\ 4+10+6 & 8+5+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 10 \\ 20 & 25 \end{bmatrix}.$$

$$\Rightarrow BA = \begin{bmatrix}1&2\\2&1\\1&2\end{bmatrix}\cdot\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$$

$$= \begin{bmatrix}1+8&2+10&3+12\\2+4&4+5&6+6\\1+8&2+10&3+12\end{bmatrix}$$

$$= \begin{bmatrix}9&12&15\\6&9&12\\9&12&15\end{bmatrix}.$$

* If $A = \begin{bmatrix}2&-2\\3&1\end{bmatrix}$, $B = \begin{bmatrix}-1&5\\4&-3\end{bmatrix}$ then prove that $(AB)^T = B^T\cdot A^T$.

$$\Rightarrow AB = \begin{bmatrix}2&-2\\3&1\end{bmatrix}\cdot\begin{bmatrix}-1&5\\4&-3\end{bmatrix}$$

$$= \begin{bmatrix}-2-8&10+6\\-3+4&15-3\end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix}-10&16\\1&12\end{bmatrix}$$

$$\Rightarrow (AB)^T = \begin{bmatrix}-10&1\\16&12\end{bmatrix} \text{ —— } 1$$

Now. $B^T = \begin{bmatrix}-1&4\\5&-3\end{bmatrix}$, $A^T = \begin{bmatrix}2&3\\-2&1\end{bmatrix}$

$$\Rightarrow B^T\cdot A^T = \begin{bmatrix}-1&4\\5&-3\end{bmatrix}\cdot\begin{bmatrix}2&3\\-2&1\end{bmatrix}$$

$$= \begin{bmatrix}-2-8&-3+4\\10+6&15-3\end{bmatrix}$$

$$\Rightarrow B^T\cdot A^T = \begin{bmatrix}-10&1\\16&12\end{bmatrix} \text{ —— } 2$$

from eqⁿ 1 & 2 $(AB)^T = B^T\cdot A^T$.

* If $A = \begin{bmatrix}1&2\\3&4\end{bmatrix}$ then prove that $A^2-5A-2I=0$.

$$\Rightarrow A^2 = A\cdot A$$

$$= \begin{bmatrix}1&2\\3&4\end{bmatrix}\cdot\begin{bmatrix}1&2\\3&4\end{bmatrix}$$

$$= \begin{bmatrix}1+6&2+8\\3+12&6+16\end{bmatrix} = \begin{bmatrix}7&10\\15&22\end{bmatrix}$$

$$L.H.S = A^2-5A-2I$$

$$= \begin{bmatrix}7&10\\15&22\end{bmatrix} - 5\begin{bmatrix}1&2\\3&4\end{bmatrix} - 2\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$= \begin{bmatrix}7&10\\15&22\end{bmatrix} - \begin{bmatrix}5&10\\15&20\end{bmatrix} - \begin{bmatrix}2&0\\0&2\end{bmatrix}$$

$$= \begin{bmatrix}7-5-2&10-10-0\\15-15-0&22-20-2\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix} = R.H.S.$$

* If $A = \begin{bmatrix}2&3\\-1&2\end{bmatrix}$ then prove that $A^2-4A+7I=0$.

$$\Rightarrow A^2 = A\cdot A$$

$$= \begin{bmatrix}2&3\\-1&2\end{bmatrix}\cdot\begin{bmatrix}2&3\\-1&2\end{bmatrix}$$

$$= \begin{bmatrix}4-3&6+6\\-2-2&-3+4\end{bmatrix} = \begin{bmatrix}1&12\\-4&1\end{bmatrix}$$

$$L.H.S = A^2-4A+7I$$

$$= \begin{bmatrix}1&12\\-4&1\end{bmatrix} - 4\begin{bmatrix}2&3\\-1&2\end{bmatrix} + 7\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$= \begin{bmatrix}1&12\\-4&1\end{bmatrix} - \begin{bmatrix}8&12\\-4&8\end{bmatrix} + \begin{bmatrix}7&0\\0&7\end{bmatrix}$$

$$= \begin{bmatrix}1-8+7&12-12+0\\-4+4+0&1-8+7\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix} = R.H.S.$$

* If $A = \begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$ then prove that $A^2-4A-5I=0$.

$$\Rightarrow A^2 = A\cdot A$$

$$= \begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}\cdot\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$$

$$= \begin{bmatrix}1+4+4&2+2+4&2+4+2\\2+2+4&4+1+4&4+2+2\\2+4+2&4+2+2&4+4+1\end{bmatrix} = \begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}.$$

$$L.H.S = A^2-4A-5I$$

$$= \begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix} - 4\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix} - 5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix} - \begin{bmatrix}4&8&8\\8&4&8\\8&8&4\end{bmatrix} - \begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$$

$$= \begin{bmatrix}9-4-5&8-8-0&8-8-0\\8-8-0&9-4-5&8-8-0\\8-8-0&8-8-0&9-4-5\end{bmatrix} = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$= R.H.S.$

* If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ then Prove that $A^2 = A$.

$\Rightarrow A^2 = A \cdot A$

$= \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1+3-5 & -3-9+15 & -5-15+25 \\ -1-3+5 & 3+9-15 & 5+15-25 \\ 1+3-5 & -3-9+15 & -5-15+25 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = A.$

* If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then Prove that

$Adj(A) = A$.

$\Rightarrow$ For Adj (A)

$A_{11} = \begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = 0-4 = -4 \quad A_{12} = \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 3-4 = -1$

$A_{13} = \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4-0 = 4 \quad A_{21} = \begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = -9+12 = 3$

$A_{22} = \begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix} = -12+12 = 0 \quad A_{23} = \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix} = -16+12 = -4$

$A_{31} = \begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix} = -3+0 = -3 \quad A_{32} = \begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} = -4+3 = -1$

$A_{33} = \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} = 0+3 = 3.$

$\Rightarrow$ matrix $= \begin{bmatrix} -4 & -1 & 4 \\ 3 & 0 & -4 \\ -3 & -1 & 3 \end{bmatrix} \Rightarrow$ matrix of co-factor $= \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}$

$\Rightarrow Adj(A) = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = A.$

* Find the inverse of $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 5 & 0 \end{vmatrix}$

$= 3(1+0) + 1(4+5) + 2(0-5)$

$= 3+9-10 = 2 \neq 0$

$\Rightarrow A^{-1}$ is exists.

$\Rightarrow$ For Adj (A)

$A_{11} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1+0 = 1 \quad A_{12} = \begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} = 4+5 = 9$

$A_{13} = \begin{vmatrix} 4 & 1 \\ 5 & 0 \end{vmatrix} = 0-5 = -5 \quad A_{21} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1-0 = -1$

$A_{22} = \begin{vmatrix} 3 & 2 \\ 5 & 1 \end{vmatrix} = 3-10 = -7 \quad A_{23} = \begin{vmatrix} 3 & -1 \\ 5 & 0 \end{vmatrix} = 0+5 = 5$

$A_{31} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1-2 = -1 \quad A_{32} = \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = -3-8 = -11$

$A_{33} = \begin{vmatrix} 3 & -1 \\ 4 & 1 \end{vmatrix} = 3+4 = 7.$

$\Rightarrow$ matrix $= \begin{bmatrix} 1 & 9 & -5 \\ -1 & -7 & 5 \\ -1 & -11 & 7 \end{bmatrix} \Rightarrow$ matrix of co-factor $= \begin{bmatrix} 1 & -9 & -5 \\ 1 & -7 & -5 \\ -1 & 11 & 7 \end{bmatrix}$

$\Rightarrow Adj(A) = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{Adj(A)}{|A|} = \dfrac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}.$

* Find the inverse of $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$\Rightarrow$ Let $A = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$

$= -1\begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix}$

$= -1(5-0) - 2(10-0) - 3(-4-4)$

$= -5-20+24 = -1 \neq 0.$

$\Rightarrow A^{-1}$ is exists.

$\Rightarrow$ For Adj (A).

$A_{11} = \begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} = 5+0 = 5 \quad A_{12} = \begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix} = 10-0 = 10.$

$A_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} = -4-4 = -8 \quad A_{21} = \begin{vmatrix} 2 & -3 \\ -2 & 5 \end{vmatrix} = 10-6 = 4$

$A_{22} = \begin{vmatrix} -1 & -3 \\ 4 & 5 \end{vmatrix} = -5+12 = 7 \quad A_{23} = \begin{vmatrix} -1 & 2 \\ 4 & -2 \end{vmatrix} = 2-8 = -6$

$A_{31} = \begin{vmatrix} 2 & -3 \\ 1 & 0 \end{vmatrix} = 0+3 = 3 \quad A_{32} = \begin{vmatrix} -1 & -3 \\ 2 & 0 \end{vmatrix} = 0+6 = 6$

$A_{33} = \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -1-4 = -5.$

$\Rightarrow$ matrix $= \begin{bmatrix} 5 & 10 & -8 \\ 4 & 7 & -6 \\ 3 & 6 & -5 \end{bmatrix} \Rightarrow$ matrix of co-factor $= \begin{bmatrix} 5 & -10 & -8 \\ -4 & 7 & 6 \\ 3 & -6 & -5 \end{bmatrix}.$

$\Rightarrow Adj(A) = \begin{bmatrix} 5 & -4 & 3 \\ -10 & 7 & -6 \\ -8 & 6 & -5 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{Adj(A)}{|A|} = -\dfrac{1}{1}\begin{bmatrix} 5 & -4 & 3 \\ -10 & 7 & -6 \\ -8 & 6 & -5 \end{bmatrix}.$

* If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

$\Rightarrow A \cdot B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$AB = \begin{bmatrix} 2+1 & 4+3 \\ 0+1 & 0+3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow |AB| = \begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix} = 9-7 = 2 \neq 0.$

$\Rightarrow (AB)^{-1}$ is exists.

$\Rightarrow Adj(AB) = \begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow (AB)^{-1} = \dfrac{Adj(AB)}{|AB|} = \dfrac{1}{2}\begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$ — (1)

$\Rightarrow$ For B^{-1}

$|B| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$

$= 6-4 = 2 \neq 0$

B^{-1} is exists.

$\Rightarrow Adj(B) = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow B^{-1} = \dfrac{Adj(B)}{|B|}$

$= \dfrac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow$ For A^{-1}

$|A| = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1-0 = 1 \neq 0$

A^{-1} is exists.

$\Rightarrow Adj(A) = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{Adj(A)}{|A|}$

$= \dfrac{1}{1}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$

Now, $B^{-1} \cdot A^{-1}$

$= \dfrac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} \cdot \dfrac{1}{1}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$= \dfrac{1}{2}\begin{bmatrix} 3+0 & -3-4 \\ -1+0 & 1+2 \end{bmatrix}$

$B^{-1} \cdot A^{-1} = \dfrac{1}{2}\begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$ — (2)

from eqn (1) & (2) $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

* Solve the equation $5x+3y=11$ and $3x-2y=-1$ using matrix method.

$\Rightarrow 5x+3y=11$

$3x-2y=-1$

$\Rightarrow \begin{bmatrix} 5 & 3 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -1 \end{bmatrix}$

$A \cdot X = B$

$\Rightarrow X = A^{-1} \cdot B$ — (1)

$\Rightarrow$ For A^{-1}

$|A| = \begin{vmatrix} 5 & 3 \\ 3 & -2 \end{vmatrix} = -10-9 = -19 \neq 0$

A^{-1} is exists.

$\Rightarrow Adj(A) = \begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{Adj(A)}{|A|} = -\dfrac{1}{19}\begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix}$

from (1) $X = A^{-1} \cdot B$

$= -\dfrac{1}{19}\begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 11 \\ -1 \end{bmatrix}$

$= -\dfrac{1}{19}\begin{bmatrix} -22+3 \\ -33-5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{19}\begin{bmatrix} -19 \\ -38 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -19/_{-19} \\ -38/_{-19} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\Rightarrow x = 1,\ y = 2.$

* Solve the equation $2x+3y=1$ and $y-4x=2$ using matrix method.

$\Rightarrow 2x+3y=1$

$-4x+y=2$

$\Rightarrow \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A \cdot X = B$

$\Rightarrow X = A^{-1} \cdot B$ — (1)

$\Rightarrow$ For A^{-1}

$|A| = \begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix} = 2+12 = 14 \neq 0$

A^{-1} is exists.

$\Rightarrow Adj(A) = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \dfrac{Adj(A)}{|A|} = \dfrac{1}{14}\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}.$

$\Rightarrow$ from (1)

$$X = A^{-1}.B$$

$$= \frac{1}{14}\begin{bmatrix}1 & -3\\ 4 & 2\end{bmatrix}.\begin{bmatrix}1\\2\end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix}1-6\\4+4\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{14}\begin{bmatrix}-5\\8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-5/14\\8/14\end{bmatrix}$$

$\Rightarrow x = -\frac{5}{14}$, $y = \frac{8}{14}$.

* Solve the equation $3x+2y=5$ and $2x-y=1$ using matrix method.

$$\Rightarrow \quad 3x+2y=5$$
$$2x-y=1$$

$$\Rightarrow \begin{bmatrix}3 & 2\\ 2 & -1\end{bmatrix}.\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}$$

$$A.X = B$$

$$\Rightarrow X = A^{-1}.B \text{ --- } (1)$$

$\Rightarrow$ For A^{-1}

$$|A| = \begin{vmatrix}3 & 2\\ 2 & -1\end{vmatrix} = -3-4 = -7 \neq 0$$

A^{-1} is exists.

$$\Rightarrow Adj(A) = \begin{bmatrix}-1 & -2\\ -2 & 3\end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{Adj(A)}{|A|} = -\frac{1}{7}\begin{bmatrix}-1 & -2\\ -2 & 3\end{bmatrix}$$

$\Rightarrow$ from (1)

$$X = A^{-1}.B$$

$$= -\frac{1}{7}\begin{bmatrix}-1 & -2\\ -2 & 3\end{bmatrix}.\begin{bmatrix}5\\1\end{bmatrix}$$

$$= -\frac{1}{7}\begin{bmatrix}-5-2\\-10+3\end{bmatrix}$$

$$= -\frac{1}{7}\begin{bmatrix}-7\\-7\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-7/-7\\-7/-7\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$$

$\Rightarrow x = 1$, $y = 1$.

* Solve the equation $2x+3y=6xy$ and $x-y=xy$ using matrix method.

$\Rightarrow 2x+3y=6xy$, $x-y=xy$

$\Rightarrow \frac{2x}{xy}+\frac{3y}{xy}=\frac{6xy}{xy}$, $\frac{x}{xy}-\frac{y}{xy}=\frac{xy}{xy}$

$\Rightarrow \frac{2}{y}+\frac{3}{x}=6$, $\frac{1}{y}-\frac{1}{x}=1$

Let, $\frac{1}{y}=a$, $\frac{1}{x}=b$

$\Rightarrow 2a+3b=6$, $a-b=1$

$$\Rightarrow \quad 2a+3b=6$$
$$a-b=1$$

$$\Rightarrow \begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}.\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}6\\1\end{bmatrix}$$

$$A.X = B$$

$$\Rightarrow X = A^{-1}.B \text{ --- } (1)$$

For A^{-1}

$$|A| = \begin{vmatrix}2 & 3\\ 1 & -1\end{vmatrix} = -2-3 = -5 \neq 0$$

A^{-1} is exists.

$$\Rightarrow Adj(A) = \begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{Adj(A)}{|A|} = -\frac{1}{5}\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}$$

$\Rightarrow$ From (1)

$$X = A^{-1}.B$$

$$= -\frac{1}{5}\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}.\begin{bmatrix}6\\1\end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix}-6-3\\-6+2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a\\b\end{bmatrix} = -\frac{1}{5}\begin{bmatrix}-9\\-4\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}-9/-5\\-4/-5\end{bmatrix} = \begin{bmatrix}9/5\\4/5\end{bmatrix}$$

$\Rightarrow a = 9/5$, $b = 4/5$

Now $\frac{1}{y} = a = 9/5$, $\frac{1}{x} = b = 4/5$

$\Rightarrow y = 5/9$, $x = 5/4$.

* If $y = e^x \cdot \sin x$ then find $\frac{dy}{dx}$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^x \cdot \sin x)$

$= e^x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(e^x)$

$= e^x \cdot \cos x + \sin x \cdot e^x$.

$\frac{dy}{dx} = e^x(\cos x + \sin x)$.

* If $y = x^3 \cdot \log x$ then find $\frac{dy}{dx}$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^3 \cdot \log x)$

$= x^3 \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x^3)$

$= x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2$

$\Rightarrow \frac{dy}{dx} = x^2 + \log x \cdot 3x^2$.

* If $y = \log(\sin 2x)$ then find $\frac{dy}{dx}$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log(\sin 2x)\right]$

$= \frac{1}{\sin 2x} \cdot \frac{d}{dx}(\sin 2x)$

$= \frac{1}{\sin 2x} \cdot \cos 2x \cdot \frac{d}{dx}(2x)$

$\frac{dy}{dx} = \cot 2x \cdot 2$.

* If $y = e^{2x} \cdot \sin 3x$ then find $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{d}{dx}(e^{2x} \cdot \sin 3x)$

$= e^{2x} \cdot \frac{d}{dx}(\sin 3x) + \sin 3x \cdot \frac{d}{dx}(e^{2x})$

$= e^{2x} \cdot \cos 3x \cdot \frac{d}{dx}(3x) + \sin 3x \cdot e^{2x} \cdot \frac{d}{dx}(2x)$

$= e^{2x} \cdot \cos 3x \cdot 3 + \sin 3x \cdot e^{2x} \cdot 2$

$\Rightarrow \frac{dy}{dx} = e^{2x}[3\cos 3x + 2\sin 3x]$.

* If $y = \frac{1-x^2}{1+x^2}$ then find $\frac{dy}{dx}$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$

$= \frac{(1+x^2) \cdot \frac{d}{dx}(1-x^2) - (1-x^2) \cdot \frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

$= \frac{(1+x^2)(0-2x) - (1-x^2)(0+2x)}{(1+x^2)^2}$

$= \frac{2x\left[(1+x^2)(-1) - (1-x^2)\right]}{(1+x^2)^2}$

$= \frac{2x\left[-1-x^2-1+x^2\right]}{(1+x^2)^2}$

$= \frac{2x(-2)}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}$.

* If $y = \frac{1+\tan x}{1-\tan x}$ then find $\frac{dy}{dx}$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\tan x}{1-\tan x}\right)$

$= \frac{(1-\tan x) \cdot \frac{d}{dx}(1+\tan x) - (1+\tan x) \frac{d}{dx}(1-\tan x)}{(1-\tan x)^2}$

$= \frac{(1-\tan x)(\sec^2 x) - (1+\tan x)(-\sec^2 x)}{(1-\tan x)^2}$

$= \frac{\sec^2 x\left[(1-\tan x) - (1+\tan x)(-1)\right]}{(1-\tan x)^2}$

$= \frac{\sec^2 x\left[1-\tan x + 1 + \tan x\right]}{(1-\tan x)^2}$

$= \frac{\sec^2 x(2)}{(1-\tan x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{2\sec^2 x}{(1-\tan x)^2}$.

* If $y = \frac{a+b\sin x}{b+a\sin x}$ then find $\frac{dy}{dx}$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{a+b\sin x}{b+a\sin x}\right)$

$= \frac{(b+a\sin x) \cdot \frac{d}{dx}(a+b\sin x) - (a+b\sin x) \cdot \frac{d}{dx}(b+a\sin x)}{(b+a\sin x)^2}$

$= \frac{(b+a\sin x)(b\cos x) - (a+b\sin x)(a\cos x)}{(b+a\sin x)^2}$

$= \frac{\cos x\left[(b+a\sin x)(b) - (a+b\sin x)(a)\right]}{(b+a\sin x)^2}$

$$= \frac{\cos x\left[b^2 + ab\sin x - a^2 - ab\sin x\right]}{(b+a\sin x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x\,(b^2-a^2)}{(b+a\sin x)^2}.$$

* If $y = \log(\sec x + \tan x)$ then find $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log(\sec x + \tan x)\right]$$

$$= \frac{1}{\sec x + \tan x}\cdot\frac{d}{dx}(\sec x + \tan x)$$

$$= \frac{1}{\sec x + \tan x}\cdot(\sec x\cdot\tan x + \sec^2 x)$$

$$= \frac{1}{\sec x + \tan x}\cdot\sec x\,(\tan x + \sec x)$$

$$\Rightarrow \frac{dy}{dx} = \sec x.$$

* If $y = x^3\cdot\sin(\log x)$ then find $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(x^3\cdot\sin(\log x)\right).$$

$$= x^3\cdot\frac{d}{dx}\left[\sin(\log x)\right] + \sin(\log x)\cdot\frac{d}{dx}(x^3)$$

$$= x^3\cdot\cos(\log x)\cdot\frac{d}{dx}(\log x) + \sin(\log x)\cdot 3x^2$$

$$= x^3\cdot\cos(\log x)\cdot\frac{1}{x} + \sin(\log x)\cdot 3x^2$$

$$\Rightarrow \frac{dy}{dx} = x^2\cdot\cos(\log x) + \sin(\log x)\cdot 3x^2.$$

* If $y = \frac{\sin(\log x)}{x}$ then find $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\frac{\sin(\log x)}{x}\right]$$

$$= \frac{x\cdot\frac{d}{dx}\left[\sin(\log x)\right] - \sin(\log x)\cdot\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\cdot\cos(\log x)\cdot\frac{d}{dx}(\log x) - \sin(\log x)(1)}{x^2}$$

$$= \frac{x\cdot\cos(\log x)\cdot\frac{1}{x} - \sin(\log x)}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(\log x) - \sin(\log x)}{x^2}.$$

* If $y = \log(\tan x) + \cos x + x$ then find $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log(\tan x) + \cos x + x\right]$$

$$= \frac{d}{dx}(\log\tan x) + \frac{d}{dx}(\cos x) + \frac{d}{dx}(x)$$

$$= \frac{1}{\tan x}\cdot\frac{d}{dx}(\tan x) - \sin x + 1$$

$$= \frac{1}{\tan x}\cdot\sec^2 x - \sin x + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{\tan x} - \sin x + 1.$$

* If $y = \log\left(\frac{\sin x}{1+\cos x}\right)$ then find $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{\sin x}{1+\cos x}\right)\right]$$

$$= \frac{1}{\frac{\sin x}{1+\cos x}}\cdot\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \frac{1+\cos x}{\sin x}\cdot\left[\frac{(1+\cos x)\cdot\frac{d}{dx}(\sin x) - \sin x\cdot\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{\sin x}\cdot\left[\frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{1+\cos x}\right]$$

$$= \frac{1}{\sin x}\cdot\left[\frac{\cos x + \cos^2 x + \sin^2 x}{1+\cos x}\right]$$

$$= \frac{1}{\sin x}\cdot\left[\frac{\cos x + 1}{1+\cos x}\right]$$

$$= \frac{1}{\sin x}.$$

$$\Rightarrow \frac{dy}{dx} = \operatorname{cosec} x.$$

* If $y = \log\sqrt{\frac{1-\sin x}{1+\sin x}}$ then find $\frac{dy}{dx}$.

$$\Rightarrow \quad y = \log\left(\frac{1-\sin x}{1+\sin x}\right)^{1/2}$$

$$= \frac{1}{2}\log\left(\frac{1-\sin x}{1+\sin x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left[\log\left(\frac{1-\sin x}{1+\sin x}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{\frac{1+\sin x}{1+\sin x}}\cdot\frac{d}{dx}\left(\frac{1-\sin x}{1+\sin x}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1+\sin x}{1-\sin x} \cdot \frac{(1+\sin x)\cdot\frac{d}{dx}(1-\sin x) - (1-\sin x)\cdot\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\sin x} \cdot \left(\frac{(1+\sin x)(-\cos x) - (1-\sin x)(\cos x)}{1+\sin x}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\sin x} \cdot \frac{\cos x\left[(1+\sin x)(-1) - (1-\sin x)\right]}{1+\sin x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\sin x} \cdot \frac{\cos x\left[-1-\sin x-1+\sin x\right]}{1+\sin x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\sin x} \cdot \frac{\cos x(-2)}{1+\sin x}\right]$$

$$= \frac{1}{2}\left[\frac{-2\cos x}{1-\sin^2 x}\right]$$

$$= \frac{-\cos x}{\cos^2 x}$$

$$= -\frac{1}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = -\sec x.$$

* If $y = \log\left[x + \sqrt{x^2+a^2}\right]$ then find $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log\left(x+\sqrt{x^2+a^2}\right)\right]$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left[1 + \frac{1}{2\sqrt{x^2+a^2}} \cdot \frac{d}{dx}(x^2+a^2)\right]$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left[1 + \frac{1}{2\sqrt{x^2+a^2}} \cdot 2x\right]$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left[1 + \frac{x}{\sqrt{x^2+a^2}}\right]$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left[\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}.$$

* If $y = x^x$ then find $\frac{dy}{dx}$.

$\Rightarrow y = x^x$

Taking log on both side.

$\Rightarrow \log y = \log x^x$

$\Rightarrow \log y = x.\log x$

Diff.. w.r.to 'x' both side.

$\Rightarrow \frac{d}{dx}(\log y) = \frac{d}{dx}(x.\log x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + \log x.(1)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$

$\Rightarrow \frac{dy}{dx} = y\left[1+\log x\right]$

$\Rightarrow \frac{dy}{dx} = x^x\left[1+\log x\right].$

* If $y = x^{\sin x}$ then find $\frac{dy}{dx}$.

$\Rightarrow y = x^{\sin x}$

Taking log on both side.

$\Rightarrow \log y = (\log x^{\sin x})$

$\Rightarrow \log y = \sin x.\log x.$

Diff. w.r.to. 'x' both side.

$\Rightarrow \frac{d}{dx}(\log y) = \frac{d}{dx}(\sin x.\log x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(\sin x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x.\frac{1}{x} + \log x.\cos x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \log x.\cos x$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{\sin x}{x} + \log x.\cos x\right]$

$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x.\cos x\right].$

* If $y = \sin x^{\tan x}$ then find $\frac{dy}{dx}$.

$\Rightarrow y = \sin x^{\tan x}$

Taking log on both side.

$\Rightarrow \log y = \log \sin x^{\tan x}$

$\Rightarrow \log y = \tan x.\log \sin x.$

Diff. w.r.to 'x' both side.

$\Rightarrow \frac{d}{dx}(\log y) = \frac{d}{dx}(\tan x.\log \sin x)$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan x \cdot \frac{d}{dx}(\log \sin x) + \log \sin x \cdot \frac{d}{dx}(\tan x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan x \cdot \frac{1}{\sin x}\cdot \frac{d}{dx}(\sin x) + \log \sin x \cdot \sec^2 x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \cdot \sec^2 x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan x \cdot \cot x + \log \sin x \cdot \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = y\left[1 + \log \sin x \cdot \sec^2 x\right]$$

$$\Rightarrow \frac{dy}{dx} = \sin x^{\tan x}\left[1 + \log \sin x \cdot \sec^2 x\right].$$

* If $x+y = \sin xy$ then find $\frac{dy}{dx}$.

$$\Rightarrow x+y = \sin xy$$

Diff. w.r. to 'x' both side.

$$\Rightarrow \frac{d}{dx}(x+y) = \frac{d}{dx}[\sin xy]$$

$$\Rightarrow 1+\frac{dy}{dx} = \cos xy \cdot \frac{d}{dx}(xy)$$

$$\Rightarrow 1+\frac{dy}{dx} = \cos xy \cdot \left[x\frac{d}{dx}(y) + y\cdot\frac{d}{dx}(x)\right]$$

$$\Rightarrow 1+\frac{dy}{dx} = \cos xy \cdot \left[x\frac{dy}{dx} + y\right]$$

$$\Rightarrow 1+\frac{dy}{dx} = \cos xy \cdot x\frac{dy}{dx} + \cos xy \cdot y$$

$$\Rightarrow \frac{dy}{dx} - x\cdot\cos xy\frac{dy}{dx} = y\cdot\cos xy - 1$$

$$\Rightarrow \frac{dy}{dx}\left[1 - x\cdot\cos xy\right] = y\cdot\cos xy - 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\cdot\cos xy - 1}{1 - x\cdot\cos xy}.$$

* If $x^3+y^3 = 3axy$ then find $\frac{dy}{dx}$.

$$\Rightarrow x^3+y^3 = 3axy$$

Diff. w.r. to 'x' both side.

$$\Rightarrow \frac{d}{dx}(x^3+y^3) = \frac{d}{dx}(3axy)$$

$$\Rightarrow 3x^2+3y^2\frac{dy}{dx} = 3a\frac{d}{dx}(xy)$$

$$\Rightarrow 3x^2+3y^2\frac{dy}{dx} = 3a\left[x\cdot\frac{d}{dx}(y) + y\cdot\frac{d}{dx}(x)\right]$$

$$\Rightarrow 3x^2+3y^2\frac{dy}{dx} = 3a\left[x\cdot\frac{dy}{dx} + y\right]$$

$$\Rightarrow 3x^2+3y^2\frac{dy}{dx} = 3ax\frac{dy}{dx} + 3ay$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 3ax\frac{dy}{dx} = 3ay - 3x^2$$

$$\Rightarrow \frac{dy}{dx}\left[3y^2 - 3ax\right] = 3ay - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3ay-3x^2}{3y^2-3ax} = \frac{3(ay-x^2)}{3(y^2-ax)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{ay-x^2}{y^2-ax}.$$

* If $y = \sin(x+y)$ then find $\frac{dy}{dx}$.

$$\Rightarrow y = \sin(x+y)$$

Diff. w.r. to 'x' both side.

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}\left[\sin(x+y)\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos(x+y)\cdot\frac{d}{dx}(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \cos(x+y)\cdot\left[1+\frac{dy}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos(x+y) + \cos(x+y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} - \cos(x+y)\frac{dy}{dx} = \cos(x+y)$$

$$\Rightarrow \frac{dy}{dx}\left[1-\cos(x+y)\right] = \cos(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(x+y)}{1-\cos(x+y)}.$$

* If $x\cdot\sin y + y\cdot\sin x = 5$ then find $\frac{dy}{dx}$.

$$\Rightarrow x\cdot\sin y + y\cdot\sin x = 5$$

Diff. w.r. to 'x' both side.

$$\Rightarrow \frac{d}{dx}(x\cdot\sin y + y\cdot\sin x) = \frac{d}{dx}(5)$$

$$\Rightarrow x\cdot\frac{d}{dx}(\sin y) + \sin y\cdot\frac{d}{dx}(x) + y\cdot\frac{d}{dx}(\sin x) + \sin x\cdot\frac{d}{dx}(y) = 0$$

$$\Rightarrow x\cdot\cos y\frac{dy}{dx} + \sin y + y\cdot\cos x + \sin x\cdot\frac{dy}{dx} = 0.$$

$$\Rightarrow x\cdot\cos y\frac{dy}{dx} + \sin x\frac{dy}{dx} = -\sin y - y\cdot\cos x$$

$$\Rightarrow \frac{dy}{dx}\left[x\cdot\cos y + \sin x\right] = -(\sin y + y\cos x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(\sin y + y\cos x)}{x\cdot\cos y + \sin x}$$

* If $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ then find $\frac{dy}{dx}$.

$$\Rightarrow x = a(\theta+\sin\theta)$$

Diff. w.r. to θ.

$$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(\theta+\sin\theta)\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a(1+\cos\theta)$$

$$y = a(1-\cos\theta)$$

Diff. w.r. to θ.

$$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(1-\cos\theta)\right]$$

$$\Rightarrow \frac{dy}{d\theta} = a\sin\theta.$$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$$= \frac{a\sin\theta}{a(1+\cos\theta)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sin\theta/2\cdot\cos\theta/2}{2\cos^2\theta/2}$$

$$\Rightarrow \frac{dy}{dx} = \tan\theta/2.$$

* If $x = at^2$, $y = 2at$ then find $\frac{d^2y}{dx^2}$.

$\Rightarrow x = at^2$

Diff. w.r.to 't'

$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}(at^2)$

$= a(2t)$

$\Rightarrow \frac{dx}{dt} = 2at$

$y = 2at$

Diff. w.r.to 't'.

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(2at)$

$\Rightarrow \frac{dy}{dt} = 2a1 = 2a.$

Now. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Again Diff. w.r.to 'x'

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right)$

$= \frac{d}{dt}\left(\frac{1}{t}\right) \cdot \frac{dt}{dx}$

$= -\frac{1}{t^2} \cdot \frac{1}{2at} \quad \left(\because \frac{dx}{dt} = 2at\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2at^3}.$

* If $x = \sec\theta + \tan\theta$, $y = \sec\theta - \tan\theta$ then find $\frac{dy}{dx}$.

$\Rightarrow x = \sec\theta + \tan\theta.$

Diff. w.r.to 'θ'

$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}(\sec\theta + \tan\theta)$

$= \sec\theta \cdot \tan\theta + \sec^2\theta$

$\Rightarrow \frac{dx}{d\theta} = \sec\theta(\tan\theta + \sec\theta)$

$\Rightarrow y = \sec\theta - \tan\theta.$

Diff. w.r.to 'θ'

$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}(\sec\theta - \tan\theta)$

$= \sec\theta \cdot \tan\theta - \sec^2\theta$

$\Rightarrow \frac{dy}{d\theta} = \sec\theta(\tan\theta - \sec\theta)$

Now. $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{\sec\theta(\tan\theta - \sec\theta)}{\sec\theta(\tan\theta + \sec\theta)}$

$= \frac{-(\sec\theta - \tan\theta)}{\sec\theta + \tan\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}.$

* If $y = A\cos pt + B\sin pt$ then Prove that $\frac{d^2y}{dt^2} + p^2y = 0.$

$\Rightarrow y = A\cos pt + B\sin pt$

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(A\cos pt + B\sin pt)$

$= \frac{d}{dt}(A\cos pt) + \frac{d}{dt}(B\sin pt)$

$\Rightarrow \frac{dy}{dt} = A(-\sin pt) \cdot \frac{d}{dt}(pt) + B\cos pt \cdot \frac{d}{dt}(pt)$

$= -A\sin pt \cdot p + B\cos pt \cdot p$

$\Rightarrow \frac{dy}{dt} = p[-A\sin pt + B\cos pt]$

Now $\frac{d^2y}{dt^2} = \frac{d}{dt}[p(-A\sin pt + B\cos pt)]$

$= p\left[\frac{d}{dt}(-A\sin pt) + \frac{d}{dt}(B\cos pt)\right]$

$= p\left[-A\cos pt \cdot \frac{d}{dt}(pt) + B(-\sin pt) \cdot \frac{d}{dt}(pt)\right]$

$= p[-A\cos pt \cdot p - B\sin pt \cdot p]$

$= -p^2[A\cos pt + B\sin pt]$

$\Rightarrow \frac{d^2y}{dt^2} = -p^2y$

$\Rightarrow \frac{d^2y}{dt^2} + p^2y = 0.$

* If $y = e^{2x}$ then Prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$\Rightarrow y = e^{2x}$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^{2x})$

$= e^{2x} \cdot \frac{d}{dx}(2x)$

$\Rightarrow \frac{dy}{dx} = e^{2x} \cdot 2$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(e^{2x} \cdot 2)$

$= 2e^{2x} \cdot \frac{d}{dx}(2x)$

$= 2 \cdot e^{2x} \cdot 2$

$\Rightarrow \frac{d^2y}{dx^2} = 4e^{2x}$

Now, L.H.S $= \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$= 4e^{2x} - 2e^{2x} - 2e^{2x}$

$= 4e^{2x} - 4e^{2x}$

$= 0 =$ R.H.S.

* If $f(x) = x^3 - 3x + 11$ then find maximum and minimum values.

$f(x) = x^3 - 3x + 11$

$\Rightarrow f'(x) = 3x^2 - 3$, $\quad f''(x) = 6x$

Take $f'(x) = 0$

$\Rightarrow 3x^2 - 3 = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$\Rightarrow x = 1 \quad , \quad x = -1$

Put the values of x in $f''(x)$.

$\Rightarrow$ For $x = 1$

$\Rightarrow f''(1) = 6(1)$

$= 6 > 0$

$\therefore f^n$ is minimum.

$\Rightarrow f(1) = (1)^3 - 3(1) + 11$

$= 1 - 3 + 11$

$= 9.$

$\Rightarrow$ For $x = -1$

$\Rightarrow f''(-1) = 6(-1)$

$= -6 < 0$

$\therefore f^n$ is maximum

$\Rightarrow f(-1) = (-1)^3 - 3(-1) + 11$

$= -1 + 3 + 11$

$= 13.$

* If $f(x) = 2x^3 - 3x^2 - 12x + 5$ then find Maximum and minimum values.

$f(x) = 2x^3 - 3x^2 - 12x + 5$

$\Rightarrow f'(x) = 6x^2 - 6x - 12 \quad , \quad f''(x) = 12x - 6$

Take $f'(x) = 0$.

$\Rightarrow 6x^2 - 6x - 12 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x - 2 = 0$ or $x + 1 = 0$.

$\Rightarrow x = 2$ or $x = -1$

Put the values of x in $f''(x)$.

$\Rightarrow$ For $x = 2$

$\Rightarrow f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f^n is minimum.

$\Rightarrow f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 5$

$= 2(8) - 3(4) - 24 + 5$

$= 16 - 12 - 24 + 5$

$f(2) = -15$

$\Rightarrow$ For $x = -1$

$\Rightarrow f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

f^n is maximum.

$\Rightarrow f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5$

$= 2(-1) - 3(1) + 12 + 5$

$= -2 - 3 + 12 + 5$

$f(-1) = 12.$

* If $f(x) = 2x^3 - 15x^2 + 36x + 10$ then find Maximum and minimum values.

$f(x) = 2x^3 - 15x^2 + 36x + 10.$

$\Rightarrow f'(x) = 6x^2 - 30x + 36 \quad , \quad f''(x) = 12x - 30.$

Take $f'(x) = 0$

$\Rightarrow 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x - 2 = 0$ or $x - 3 = 0$

$\Rightarrow x = 2$ or $x = 3.$

$\Rightarrow$ For $x = 2$

$\Rightarrow f''(2) = 12(2) - 30$

$= 24 - 30$

$= -6 < 0$

f^n is maximum.

$\Rightarrow f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 10$

$= 2(8) - 15(4) + 72 + 10$

$= 16 - 60 + 72 + 10$

$= 38.$

For $x = 3$

$\Rightarrow f''(3) = 12(3) - 30$

$= 36 - 30$

$= 6 > 0$

f^n is minimum.

$\Rightarrow f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 10$

$= 2(27) - 15(9) + 108 + 10$

$= 54 - 135 + 108 + 10$

$= 37.$

* If $f(x) = x + \frac{1}{x}$ then find maximum and minimum values.

$f(x) = x + \frac{1}{x}$

$\Rightarrow f'(x) = 1 - \frac{1}{x^2} \quad , \quad f''(x) = -\left(\frac{-2}{x^3}\right)$

$= \frac{2}{x^3}.$

$\Rightarrow$ Take $f'(x) = 0$

$\Rightarrow 1 - \frac{1}{x^2} = 0$

$\Rightarrow 1 = \frac{1}{x^2}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$\Rightarrow x = 1 \quad , \quad x = -1.$

Put the values of x in $f''(x)$

$\Rightarrow$ For $x = 1$

$\Rightarrow f''(1) = \frac{2}{(1)^3}$

$= 2 > 0$

f^n is minimum.

$\Rightarrow f(1) = 1 + \frac{1}{1}$

$= 1 + 1$

$= 2.$

For $x = -1$

$\Rightarrow f''(-1) = \frac{2}{(-1)^3}$

$= -2 < 0$

f^n is maximum

$\Rightarrow f(-1) = -1 + \frac{1}{-1}$

$= -1 - 1$

$= -2.$

* The equation of motion of Particle is $s = t^3 - 3t^2 + 4t + 3$ then find velocity and acceleration at $t = 2$.

$\Rightarrow s = t^3 - 3t^2 + 4t + 3$

$v = \frac{ds}{dt} = 3t^2 - 6t + 4$

$a = \frac{dv}{dt} = 6t - 6$

at $t=2$.

$\Rightarrow v = 3(2)^2 - 6(2) + 4$

$= 12 - 12 + 4$

$\boxed{v = 4 \text{ units.}}$

$\Rightarrow a = 6(2) - 6$

$= 12 - 6$

$\boxed{a = 6 \text{ units}}$

* The equation of motion of a Particle is $s = t^3 - 6t^2 + 9t + 6$. Find v when $a=0$.

$s = t^3 - 6t^2 + 9t + 6$

$v = \frac{ds}{dt} = 3t^2 - 12t + 9$

$a = \frac{dv}{dt} = 6t - 12.$

$\Rightarrow$ Now $a = 0$

$\Rightarrow 6t - 12 = 0$

$\Rightarrow 6t = 12$

$\Rightarrow t = \frac{12}{6} = 2$ sec.

Put $t=2$ in velocity.

$\Rightarrow v = 3(2)^2 - 12(2) + 9$

$= 12 - 24 + 9$

$\boxed{v = -3 \text{ units}}$

* The equation of motion of Particle is $s = t^3 + 3t$ then,

(i) Find velocity and acceleration at $t=3$.

(ii) when velocity and acceleration will become equal?

$s = t^3 + 3t$

$\Rightarrow v = \frac{ds}{dt} = 3t^2 + 3$

$a = \frac{dv}{dt} = 6t.$

(i) v and a at $t=3$.

$\Rightarrow v = 3(3)^2 + 3$

$= 27 + 3$

$\boxed{v = 30 \text{ units}}$

$\Rightarrow a = 6(3)$

$\boxed{a = 18 \text{ units}}$

(ii) velocity and acceleration are equal then $v = a$

$\Rightarrow 3t^2 + 3 = 6t$

$\Rightarrow 3t^2 - 6t + 3 = 0$

$\Rightarrow t^2 - 2t + 1 = 0$

$\Rightarrow (t-1)^2 = 0$

$\Rightarrow t - 1 = 0$

$\Rightarrow \boxed{t = 1 \text{ sec}}$

* The equation of motion of a Particle is $s = t^3 - 6t^2 + 9t + 6$ then

(i) Find v and a at $t=2$.

(ii) Find s and a when Particle change its direction.

$s = t^3 - 6t^2 + 9t + 6.$

$v = \frac{ds}{dt} = 3t^2 - 12t + 9$

$a = \frac{dv}{dt} = 6t - 12.$

(i) v and a at $t=2$.

$\Rightarrow v = 3(2)^2 - 12(2) + 9$

$= 12 - 24 + 9$

$\boxed{v = -3 \text{ units}}$

$\Rightarrow a = 6(2) - 12$

$= 12 - 12$

$\boxed{a = 0 \text{ unit}}$

(ii) when Particle change its direction

$v = 0$

$\Rightarrow 3t^2 - 12t + 9 = 0$

$\Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow (t-3)(t-1) = 0$

$\Rightarrow t - 3 = 0$ or $t - 1 = 0$

$\Rightarrow t = 3$ or $t = 1$.

$\Rightarrow$ at $t=1$

$\Rightarrow s = (1)^3 - 6(1)^2 + 9(1) + 6$

$= 1 - 6 + 9 + 6$

$\boxed{s = 10 \text{ units}}$

$\Rightarrow a = 6(1) - 12$

$= 6 - 12$

$\boxed{a = -6 \text{ units}}$

at $t=3$

$\Rightarrow s = (3)^3 - 6(3)^2 + 9(3) + 6$

$= 27 - 54 + 27 + 6$

$\boxed{s = 6 \text{ units}}$

$\Rightarrow a = 6(3) - 12$

$= 18 - 12$

$\boxed{a = 6 \text{ units}}$

* Evaluate: $\int \left(x-\frac{1}{x}\right)^2 dx$

$= \int \left(x^2 - 2x\cdot\frac{1}{x} + \frac{1}{x^2}\right) dx$

$= \int \left(x^2 - 2 + \frac{1}{x^2}\right) dx$

$= \int x^2 dx - 2\int 1 dx + \int \frac{1}{x^2} dx$

$= \frac{x^3}{3} - 2x - \frac{1}{x} + C.$

* Evaluate: $\int \frac{\operatorname{cosec}^2 x}{\sec x} dx$

$= \int \operatorname{cosec}^2 x \cdot \frac{1}{\sec x} dx$

$= \int \frac{1}{\sin^2 x} \cdot \cos x \, dx$

$= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$

$= \int \cot x \cdot \operatorname{cosec} x \, dx$

$= -\operatorname{cosec} x + C.$

* Evaluate: $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx.$

$= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x} dx \quad \left(\because \cos 2\theta = \cos^2\theta - \sin^2\theta\right)$

$= \int \left(\frac{\cos^2 x}{\cos^2 x \cdot \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \cdot \sin^2 x}\right) dx$

$= \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\right) dx$

$= \int \operatorname{cosec}^2 x \, dx - \int \sec^2 x \, dx$

$= -\cot x - \tan x + C.$

* Evaluate: $\int \frac{2 + 3\sin x}{\cos^2 x} dx.$

$= \int \left(\frac{2}{\cos^2 x} + \frac{3\sin x}{\cos^2 x}\right) dx$

$= 2\int \frac{1}{\cos^2 x} dx + 3\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$

$= 2\int \sec^2 x \, dx + 3\int \tan x \cdot \sec x \, dx$

$= 2\tan x + 3\sec x + C.$

* Evaluate: $\int \frac{4 + 3\cos x}{\sin^2 x} dx.$

$= \int \left(\frac{4}{\sin^2 x} + \frac{3\cos x}{\sin^2 x}\right) dx$

$= 4\int \frac{1}{\sin^2 x} dx + 3\int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$

$= 4\int \operatorname{cosec}^2 x \, dx + 3\int \cot x \cdot \operatorname{cosec} x \, dx$

$= 4(-\cot x) + 3(-\operatorname{cosec} x) + C$

$= -4\cot x - 3\operatorname{cosec} x + C.$

* Evaluate: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cdot \cos^2 x} dx.$

$= \int \left(\frac{\sin^3 x}{\sin^2 x \cdot \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cdot \cos^2 x}\right) dx$

$= \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}\right) dx$

$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx + \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$

$= \int \tan x \cdot \sec x \, dx + \int \cot x \cdot \operatorname{cosec} x \, dx$

$= \sec x - \operatorname{cosec} x + C.$

* Evaluate: $\int \frac{\cos x}{1 + \cos x} dx.$

$= \int \frac{\cos x}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x} dx$

$= \int \frac{\cos x (1 - \cos x)}{1 - \cos^2 x} dx$

$= \int \frac{\cos x - \cos^2 x}{\sin^2 x} dx$

$= \int \left(\frac{\cos x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}\right) dx$

$= \int \left(\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} - \cot^2 x\right) dx.$

$= \int \left(\operatorname{cosec} x \cdot \cot x - (\operatorname{cosec}^2 x - 1) \right) dx$

$= \int (\operatorname{cosec} x \cdot \cot x - \operatorname{cosec}^2 x + 1)\, dx$

$= \int \operatorname{cosec} x \cdot \cot x \, dx - \int \operatorname{cosec}^2 x \, dx + \int 1 \, dx$

$= -\operatorname{cosec} x + \cot x + x + C.$

* Evaluate :- $\int x \cdot e^x \, dx$

$\Rightarrow u = x$

$\frac{du}{dx} = \frac{d}{dx}(x) = 1$

$v = e^x$

$\int v\,dx = \int e^x dx = e^x$

$\int u \cdot v \, dx = u \cdot \int v\,dx - \int \left(\frac{du}{dx} \cdot \int v\,dx \right) dx$

$= x \cdot e^x - \int (1 \cdot e^x)\, dx$

$= x \cdot e^x - \int e^x dx$

$= x \cdot e^x - e^x + C$

$= e^x (x-1) + C.$

* Evaluate :- $\int \sin x \cdot x \, dx$

$\Rightarrow u = x$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x) = 1$

$v = \sin x$

$\int v\,dx = \int \sin x \, dx$

$= -\cos x$

$\int u \cdot v \, dx = u \cdot \int v\,dx - \int \left(\frac{du}{dx} \cdot \int v\,dx \right) dx$

$= x \cdot (-\cos x) - \int (1 \cdot (-\cos x))\, dx$

$= -x \cdot \cos x - \int (-\cos x)\, dx$

$= -x \cdot \cos x + \int \cos x \, dx$

$= -x \cdot \cos x + \sin x + C.$

* Evaluate :- $\int x^2 \cdot \log x \, dx$

$\Rightarrow u = \log x$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\log x)$

$= \frac{1}{x}$

$v = x^2$

$\int v\,dx = \int x^2 dx$

$= \frac{x^3}{3}$

$\int u \cdot v \, dx = u \cdot \int v\,dx - \int \left(\frac{du}{dx} \cdot \int v\,dx \right) dx$

$= \log x \cdot \frac{x^3}{3} - \int \left(\frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{3}\,2}}{3} \right) dx$

$= \log x \cdot \frac{x^3}{3} - \int \frac{x^2}{3} \, dx$

$= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx$

$= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + C$

$= \frac{x^3}{3} \left(\log x - \frac{1}{3} \right) + C.$

Evaluate:-

* $\int x^2 \cdot e^x \, dx$

$\Rightarrow u = x^2$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x^2)$

$= 2x$

$v = e^x$

$\int v\,dx = \int e^x dx$

$= e^x$

$\int u \cdot v \, dx = u \cdot \int v\,dx - \int \left(\frac{du}{dx} \cdot \int v\,dx \right) dx$

$= x^2 \cdot e^x - \int (2x \cdot e^x)\, dx$

$= x^2 \cdot e^x - 2 \int x \cdot e^x \, dx$

$= x^2 \cdot e^x - 2 \left[x \cdot \int e^x - \int \left(\frac{d}{dx}(x) \cdot \int e^x dx \right) dx \right]$

$= x^2 \cdot e^x - 2 \left[x \cdot e^x - \int (1 \cdot e^x)\, dx \right]$

$= x^2 \cdot e^x - 2 \left[x \cdot e^x - \int e^x dx \right]$

$= x^2 \cdot e^x - 2 \left[x \cdot e^x - e^x \right] + C$

$= x^2 \cdot e^x - 2 \left[e^x (x-1) \right] + C$

$= x^2 \cdot e^x - 2e^x (x-1) + C.$

* Evaluate:- $\int x \cdot e^{3x}\,dx$

$\Rightarrow u = x$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x)$

$= 1$

$v = e^{3x}$

$\int v\,dx = \int e^{3x}\,dx$

$= \frac{e^{3x}}{3}$

$$\int u \cdot v\,dx = u \cdot \int v\,dx - \int\left(\frac{du}{dx} \cdot \int v\,dx\right)dx$$

$$= x \cdot \frac{e^{3x}}{3} - \int\left(1 \cdot \frac{e^{3x}}{3}\right)dx$$

$$= x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx$$

$$= x \cdot \frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx$$

$$= x \cdot \frac{e^{3x}}{3} - \frac{1}{3}\,\frac{e^{3x}}{3} + C$$

$$= \frac{e^{3x}}{3}\left(x - \frac{1}{3}\right) + C.$$

* Evaluate:- $\int e^{\tan x} \cdot \sec^2 x\,dx$

Suppose $\boxed{u = \tan x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\tan x)$

$\Rightarrow \frac{du}{dx} = \sec^2 x$

$\Rightarrow \boxed{du = \sec^2 x\,dx}$

Now $\int e^{\tan x} \cdot \sec^2 x\,dx$

$= \int e^{u} \cdot du$

$= e^{u} + C$

$= e^{\tan x} + C.$

Evaluate:-

* $\int \cos x \cdot \sqrt{\sin x}\,dx$

Suppose $\boxed{u = \sin x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\sin x)$

$\Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow \boxed{du = \cos x\,dx}$

Now, $\int \cos x \cdot \sqrt{\sin x}\,dx$

$= \int \sqrt{\sin x} \cdot \cos x\,dx$

$= \int \sqrt{u} \cdot du$

$= \int (u)^{1/2}\,du$

$= \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{u^{3/2}}{3/2} + C$

$= \frac{(\sin x)^{3/2}}{3/2} + C.$

* Evaluate :- $\int \frac{\sin(\log x)}{x}\,dx$

Suppose $\boxed{u = \log x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\log x)$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\Rightarrow \boxed{du = \frac{1}{x}\,dx}$

Now, $\int \frac{\sin(\log x)}{x}\,dx$

$= \int \sin u \cdot du$

$= -\cos(u) + C$

$= -\cos(\log x) + C.$

* Evaluate: $\int \frac{(\log x)^5}{x} dx$

Suppose $\boxed{u = \log x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\log x)$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\Rightarrow \boxed{du = \frac{1}{x} dx}$

Now, $\int \frac{(\log x)^5}{x} dx$

$= \int (u)^5 . du$

$= \frac{u^6}{6} + C$

$= \frac{(\log x)^6}{6} + C.$

* Evaluate: $\int \frac{e^x(x+1)}{\sin^2(x \cdot e^x)} dx$

Suppose $\boxed{u = x \cdot e^x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x \cdot e^x)$

$= x \cdot \frac{d}{dx}(e^x) + e^x . \frac{d}{dx}(x)$

$= x \cdot e^x + e^x$

$\Rightarrow \frac{du}{dx} = e^x(x+1)$

$\Rightarrow \boxed{du = e^x(x+1) dx.}$

Now, $\int \frac{e^x(x+1)}{\sin^2(x \cdot e^x)} dx$

$= \int \frac{du}{\sin^2 u}$

$= \int \frac{1}{\sin^2 u} . du$

$= \int \operatorname{cosec}^2 u \, du$

$= -\cot(u) + C$

$= -\cot(x \cdot e^x) + C.$

* Evaluate: $\int \frac{e^x(x+1)}{\cos^2(x \cdot e^x)} dx$

Suppose $\boxed{u = x \cdot e^x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x \cdot e^x)$

$= x \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x)$

$= x \cdot e^x + e^x$

$\Rightarrow \frac{du}{dx} = e^x(x+1)$

$\Rightarrow \boxed{du = e^x(x+1) dx}$

Now, $\int \frac{e^x(x+1)}{\cos^2(x \cdot e^x)} dx$

$= \int \frac{du}{\cos^2 u}$

$= \int \frac{1}{\cos^2 u} . du$

$= \int \sec^2 u \, du$

$= \tan u + C$

$= \tan(x \cdot e^x) + C.$

* Evaluate: $\int \frac{2 \tan^{-1} x}{1+x^2} dx$

Suppose $\boxed{u = \tan^{-1} x}$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\tan^{-1} x)$

$= \frac{1}{1+x^2}$

$\Rightarrow \boxed{du = \frac{1}{1+x^2} dx}$

Now, $\int \frac{2 \tan^{-1} x}{1+x^2} dx$

$= \int 2 . u \, du$

$= 2 \int u \, du$

$= \cancel{2} . \frac{u^2}{\cancel{2}} + C$

$= u^2 + C$

$= (\tan^{-1} x)^2 + C.$

* $\int 2x\cdot(x^2+8)^8\,dx$

Suppose $u = x^2+8$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x^2+8)$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\cdot dx$

Now, $\int 2x\cdot(x^2+8)^8\,dx$

$= \int (x^2+8)^8\cdot 2x\,dx$

$= \int u^8\cdot du$

$= \frac{u^9}{9} + c$

$= \frac{(x^2+8)^9}{9} + c.$

* Evaluate:- $\int_0^1 \frac{x^3-8}{x-2}\,dx$

$= \int_0^1 \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}\,dx$

$= \int_0^1 (x^2+2x+4)\,dx$

$= \left[\frac{x^3}{3} + \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 4x\right]_0^1$

$= \left[\frac{x^3}{3} + x^2 + 4x\right]_0^1$

$= \left[\frac{(1)^3}{3} + (1)^2 + 4(1)\right] - \left[\frac{(0)^3}{3} + (0)^2 + 4(0)\right]$

$= \frac{1}{3} + 1 + 4 - 0$

$= \frac{1+3+12}{3}$

$= \frac{16}{3}.$

* $\int_1^e \frac{(\log x)^2}{x}\,dx$

Suppose $u = \log x$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\Rightarrow du = \frac{1}{x}dx$

Now. $\int_1^e \frac{(\log x)^2}{x}dx$

$= \int_1^e (u)^2 du$

$= \left[\frac{u^3}{3}\right]_1^e$

$= \left[\frac{(\log x)^3}{3}\right]_1^e$

$= \frac{(\log e)^3}{3} - \frac{(\log 1)^3}{3}$

$= \frac{(1)^3}{3} - \frac{0}{3}$

$= \frac{1}{3}.$

Evaluate:-

* $\int_1^3 (x^2+x+1)\,dx$

$= \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]_1^3$

$= \left[\frac{(3)^3}{3} + \frac{(3)^2}{2} + 3\right] - \left[\frac{(1)^3}{3} + \frac{(1)^2}{2} + 1\right]$

$= \left[\frac{27}{3} + \frac{9}{2} + 3\right] - \left[\frac{1}{3} + \frac{1}{2} + 1\right]$

$= \left[\frac{54+27+18}{6}\right] - \left[\frac{2+3+6}{6}\right]$

$= \left[\frac{99}{6}\right] - \left[\frac{11}{6}\right]$

$= \frac{99-11}{6} = \frac{88}{6}.$

* Evaluate:- $\int_0^1 \frac{x}{x+1}\,dx$

$= \int_0^1 \frac{x+1-1}{x+1}\,dx$

$= \int_0^1 \left(\frac{\cancel{x+1}}{\cancel{x+1}} - \frac{1}{x+1}\right)dx$

$= \int_0^1 \left(1 - \frac{1}{x+1}\right)dx$

$= \left[x - \log(x+1)\right]_0^1$

$= \left[1-\log(1+1)\right] - \left[0 - \log(0+1)\right]$

$= 1 - \log 2 + \log 1$

$= 1 - \log 2.$

* Evaluate:- $\int_{-4}^{-3} \frac{x}{7+x}\,dx$

$= \int_{-4}^{-3} \frac{7+x-7}{7+x}\,dx$

$= \int_{-4}^{-3} \left(\frac{\cancel{7+x}}{\cancel{7+x}} - \frac{7}{7+x}\right)dx$

$= \int_{-4}^{-3} \left(1 - \frac{7}{7+x}\right)dx$

$= \left[x - 7\log(7+x)\right]_{-4}^{-3}$

$= \left[-3 - 7\log(7-3)\right] - \left[-4 - 7\log(7-4)\right]$

$= -3 - 7\log 4 + 4 + 7\log 3$

$= 1 + 7\log 3 - 7\log 4$

$= 1 + 7(\log 3 - \log 4)$

$= 1 + 7\log\left(\frac{3}{4}\right)$

* $\int_0^1 \frac{2}{1+x^2}\,dx$

$= 2\int_0^1 \frac{1}{1+x^2}\,dx$

$= 2\left[\tan^{-1}x\right]_0^1$

$= 2\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= 2\left[\frac{\pi}{4} - 0\right]$

$= 2 \times \frac{\pi}{4}$

$= \frac{\pi}{2}.$

* Evaluate:- $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx \quad \text{—} (1)$

$= \int_0^{\pi/2} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)} + \sqrt{\cos(\frac{\pi}{2}-x)}}\,dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx \quad \text{—} (2)$

Adding (1) & (2)

$\Rightarrow 2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\right)dx$

$= \int_0^{\pi/2} \left(\frac{\cancel{\sqrt{\sin x} + \sqrt{\cos x}}}{\cancel{\sqrt{\sin x} + \sqrt{\cos x}}}\right)dx$

$= \int_0^{\pi/2} 1\,dx$

$= \left[x\right]_0^{\pi/2}$

$\Rightarrow 2I = \frac{\pi}{2} - 0$

$\Rightarrow I = \frac{\pi}{2\times 2} = \frac{\pi}{4}.$

* Evaluate:- $\displaystyle\int_0^{\pi/2} \frac{\tan x}{\tan x+\cot x}\,dx$

$$I=\int_0^{\pi/2} \frac{\tan x}{\tan x+\cot x}\,dx \quad -(1)$$

$$=\int_0^{\pi/2} \frac{\tan(\frac{\pi}{2}-x)}{\tan(\frac{\pi}{2}-x)+\cot(\frac{\pi}{2}-x)}\,dx$$

$$I=\int_0^{\pi/2} \frac{\cot x}{\cot x+\tan x}\,dx \quad -(2)$$

Adding (1) & (2)

$$\Rightarrow 2I=\int_0^{\pi/2} \left(\frac{\tan x}{\tan x+\cot x}+\frac{\cot x}{\tan x+\cot x}\right)dx$$

$$=\int_0^{\pi/2} \left(\frac{\cancel{\tan x+\cot x}}{\cancel{\tan x+\cot x}}\right)dx$$

$$=\int_0^{\pi/2} 1\,dx$$

$$=[x]_0^{\pi/2}$$

$$\Rightarrow 2I=\frac{\pi}{2}-0$$

$$\Rightarrow I=\frac{\pi}{4}.$$

* Evaluate:- $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}\,dx$

$$I=\int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}\,dx \quad -(1)$$

$$=\int_0^{\pi/2} \frac{\sqrt{\sec(\frac{\pi}{2}-x)}}{\sqrt{\sec(\frac{\pi}{2}-x)}+\sqrt{\operatorname{cosec}(\frac{\pi}{2}-x)}}\,dx$$

$$=\int_0^{\pi/2} \frac{\sqrt{\operatorname{cosec} x}}{\sqrt{\operatorname{cosec} x}+\sqrt{\sec x}}\,dx \quad -(2)$$

Adding (1) & (2)

$$\Rightarrow 2I=\int_0^{\pi/2} \left(\frac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}+\frac{\sqrt{\operatorname{cosec} x}}{\sqrt{\operatorname{cosec} x}+\sqrt{\sec x}}\right)dx$$

$$=\int_0^{\pi/2} \left(\frac{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}\right)dx$$

$$=\int_0^{\pi/2} 1\,dx$$

$$=[x]_0^{\pi/2}$$

$$\Rightarrow 2I=\frac{\pi}{2}-0$$

$$\Rightarrow I=\frac{\pi}{4}.$$

* $\displaystyle\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx$

$$=\int_0^{\pi/2} \frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}\,dx$$

$$=\int_0^{\pi/2} \frac{1}{\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}}}\,dx$$

$$I=\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad -(1)$$

$$=\int_0^{\pi/2} \frac{\sqrt{\cos(\frac{\pi}{2}-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)}+\sqrt{\sin(\frac{\pi}{2}-x)}}\,dx$$

$$I=\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad -(2)$$

Adding (1) & (2)

$$\Rightarrow 2I=\int_0^{\pi/2} \left(\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}+\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right)dx$$

$$=\int_0^{\pi/2} \left(\frac{\cancel{\sqrt{\cos x}+\sqrt{\sin x}}}{\cancel{\sqrt{\cos x}+\sqrt{\sin x}}}\right)dx$$

$$=\int_0^{\pi/2} 1\,dx$$

$$=[x]_0^{\pi/2}$$

$$\Rightarrow 2I=\frac{\pi}{2}-0$$

$$\Rightarrow I=\pi/4.$$

Evaluate:

$$* \int_0^{\pi/2} \log(\tan x)\, dx$$

$$I = \int_0^{\pi/2} \log(\tan x)\, dx \quad — (1)$$

$$= \int_0^{\pi/2} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx$$

$$I = \int_0^{\pi/2} \log(\cot x)\, dx \quad — (2)$$

Adding (1) & (2)

$$\Rightarrow 2I = \int_0^{\pi/2} \left[\log(\tan x) + \log(\cot x)\right] dx$$

$$= \int_0^{\pi/2} \log(\tan x \cdot \cot x)\, dx$$

$$= \int_0^{\pi/2} \log 1\, dx$$

$$= \int_0^{\pi/2} 0\, dx$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow 2I = 0.$$

$$* \text{ Evaluate: } \int_0^7 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}}\, dx$$

$$I = \int_0^7 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}}\, dx \quad — (1)$$

$$= \int_0^7 \frac{\sqrt{7-(7-x)}}{\sqrt{7-x} + \sqrt{7-(7-x)}}\, dx$$

$$= \int_0^7 \frac{\sqrt{7-7+x}}{\sqrt{7-x} + \sqrt{7-7+x}}\, dx$$

$$I = \int_0^7 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}}\, dx \quad — (2)$$

Adding (1) & (2)

$$\Rightarrow 2I = \int_0^7 \left(\frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}} + \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\right) dx$$

$$= \int_0^7 \left(\frac{\sqrt{7-x}+\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\right) dx$$

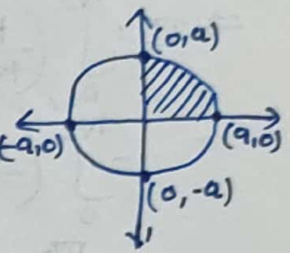
$$= \int_0^7 1\, dx$$

$$= [x]_0^7$$

$$\Rightarrow 2I = 7 - 0$$

$$\Rightarrow I = \frac{7}{2}.$$

* Find the area bounded by the curve $x^2+y^2 = a^2$.



$\Rightarrow$ Here, $x^2+y^2=a^2$

$$\Rightarrow y^2 = a^2 - x^2$$

$$\Rightarrow y = \sqrt{a^2-x^2}$$

$$A = \int_a^b y\, dx$$

$$= \int_0^a \sqrt{a^2-x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \left[\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right] - \left[\frac{0}{2}\sqrt{a^2-0} + \frac{a^2}{2}\sin^{-1}\left(\frac{0}{a}\right)\right]$$

$$= \left[0 + \frac{a^2}{2}\sin^{-1}(1)\right] - \left[0 + \frac{a^2}{2}\sin^{-1}(0)\right]$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} - \frac{a^2}{2} \cdot (0)$$

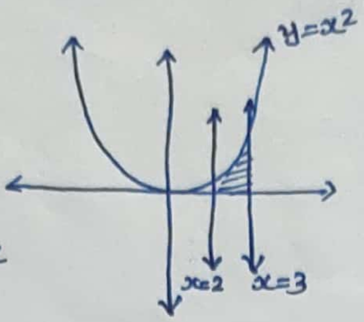
$$A = \frac{\pi a^2}{4}$$

Total Area of circle $= 4 \times A$

$$= 4 \cdot \frac{\pi a^2}{4}$$

$$= \pi a^2.$$

* Find the area bounded by the curve $y=x^2$ and lines $x=2, x=3$ with x-axis.



$\Rightarrow$ Here, $y=x^2$

$x=2$, $x=3$

Area $A=\int_a^b y\,dx$

$=\int_2^3 x^2\,dx$

$=\left[\frac{x^3}{3}\right]_2^3$

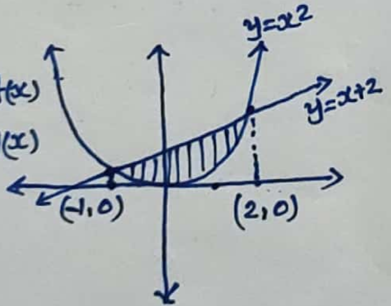
$=\frac{(3)^3}{3}-\frac{(2)^3}{3}$

$=\frac{27}{3}-\frac{8}{3}$

$=\frac{27-8}{3}$

$=\frac{19}{3}$ units.

* Find the area of the region bounded by the curves $y=x^2$ and the line $y=x+2$



$\Rightarrow$ Here $y=x^2=f(x)$

$y=x+2=g(x)$

$\Rightarrow x^2=x+2$

$\Rightarrow x^2-x-2=0$

$\Rightarrow (x-2)(x+1)=0$

$\Rightarrow x-2=0$ or $x+1=0$

$\Rightarrow x=2$ or $x=-1$.

$A=\int_a^b (f(x)-g(x))\,dx$

$=\int_{-1}^{2} (x^2-(x+2))\,dx$

$=\int_{-1}^{2} (x^2-x-2)\,dx$

$=\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_{-1}^{2}$

$=\left[\frac{(2)^3}{3}-\frac{(2)^2}{2}-2(2)\right]-\left[\frac{(-1)^3}{3}-\frac{(-1)^2}{2}-2(-1)\right]$

$=\left[\frac{8}{3}-\frac{4}{2}-4\right]-\left[\frac{-1}{3}-\frac{1}{2}+2\right]$

$=\left[\frac{16-12-24}{6}\right]-\left[\frac{-2-3+12}{6}\right]$

$=\left[\frac{16-36}{6}\right]-\left[\frac{-5+12}{6}\right]$

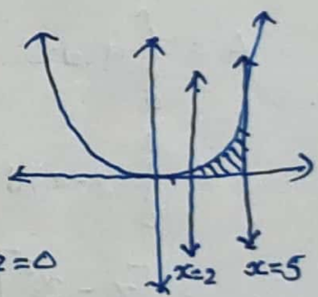
$=-\frac{20}{6}-\frac{7}{6}$

$=\frac{-20-7}{6}$

$=-\frac{27}{6}$ units.

$\therefore A=\frac{27}{6}$ units.

* Find the area of the region bounded by the curve $x^2-7x+10$ and x-axis.



Here $y=x^2-7x+10$

$\Rightarrow x^2-7x+10=0$

$\Rightarrow (x-5)(x-2)=0$

$\Rightarrow x-5=0$ or $x-2=0$

$\Rightarrow x=5$ or $x=2$.

$A=\int_a^b y\,dx$

$=\int_2^5 (x^2-7x+10)\,dx$

$=\left[\frac{x^3}{3}-7\frac{x^2}{2}+10x\right]_2^5$

$$=\left[\frac{(5)^3}{3}-\frac{7(5)^2}{2}+10(5)\right]-\left[\frac{(2)^3}{3}-\frac{7(2)^2}{2}+10(2)\right]$$

$$=\left[\frac{125}{3}-\frac{175}{2}+50\right]-\left[\frac{8}{3}-\frac{28}{2}+20\right]$$

$$=\left[\frac{250-525+300}{6}\right]-\left[\frac{16-84+120}{6}\right]$$

$$=\frac{25}{6}-\frac{52}{6}$$

$$=\frac{25-52}{6}$$

$$=-\frac{27}{6}$$

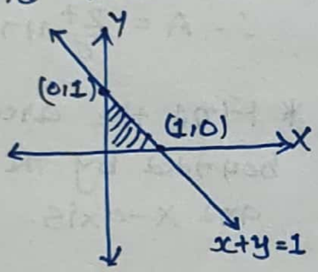
$A = \frac{27}{6}$ units.

* Find the area bounded by the line $x+y=1$ and axes.

$\Rightarrow x+y=1$

$\Rightarrow y=1-x$

$x=0,\ x=1.$



$$A=\int_a^b y\,dx$$

$$=\int_0^1 (1-x)\,dx$$

$$=\left[x-\frac{x^2}{2}\right]_0^1$$

$$=\left[1-\frac{(1)^2}{2}\right]-\left[0-0\right]$$

$$=1-\frac{1}{2}$$

$A=\frac{1}{2}$ units.