1. MATRICES

* Matrix :- A rectangular array of m rows and n columns enclosed by [ ] is called a matrix of order m × n.

$$A = \underbrace{[a_{ij}]}_{\text{element}}_{\boxed{m \times n}} \rightarrow \text{order of matrix}$$

i → row, j → column, m → row, n → column

* Difference between Determinant & Matrix

* Types of Matrices :-

* Square Matrix :- A matrix in which number of column equals number of rows is called a square matrix.

→ $[5]_{1\times1}$ $\quad \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}_{2\times2}$ $\quad \begin{bmatrix} -1 & 3 & 5 \\ 2 & 0 & 6 \\ 4 & -7 & 9 \end{bmatrix}_{3\times3}$

* Rectangle Matrix :- A matrix in which number of column does not equals number of rows is called Rectangle Matrix.

→ $\begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 3 & 5 \end{bmatrix}_{3\times2}$ $\quad \begin{bmatrix} -4 & -3 & 1 \\ -1 & 0 & 1 \end{bmatrix}_{2\times3}$

* Row Matrix :- A matrix having one row and any number of column is called row matrix.

→ $A = \begin{bmatrix} 6 & 8 \end{bmatrix}_{1\times2}$ $\quad B = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}_{1\times3}$ $\quad C = \begin{bmatrix} a & b & c & d & e \end{bmatrix}_{1\times5}$

* Column Matrix :- A matrix having one column and any number of row is called column Matrix.

→ $A = \begin{bmatrix} 3 \\ -4 \end{bmatrix}_{2\times1}$, $\quad B = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}_{3\times1}$ $\quad C = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}_{5\times1}$

* **Null Matrix :-** Any Matrix in which all the elements are zero is called a zero Matrix or Null Matrix.

→ $\begin{bmatrix} 0 & 0 \end{bmatrix}_{1\times 2}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

* **Diagonal Matrix :-** A square Matrix is called a digonal matrix if all its non-diagonal elements are zero.

→ $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

* **Scalar Matrix :-** A digonal matrix in which all the elements of its principal diagonal are equal is called Scalar Matrix.

→ $A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

* **Identity Matrix :-** A Scalar matrix in which all the elements of its principal diagonal are unity is called Identity Matrix.

→ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

$A = [a_{ij}]_{3\times 3}$
$a_{ij} = 1$ if $i = j$
$a_{ij} = 0$ if $i \neq j$

* **Transpose of a Matrix** (A^T or A') **:-** If the rows and columns of a matrix are interchanged then the new matrix is known as the transpose of the original Matrix. $(A^T)^T = A$

→ $A = \begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix}_{2\times 2}$ $\Rightarrow A^T = \begin{bmatrix} 3 & 4 \\ -1 & 0 \end{bmatrix}_{2\times 2}$

→ $B = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 7 & 9 \end{bmatrix}_{3\times 2}$ $\Rightarrow B^T = \begin{bmatrix} -2 & 4 & 7 \\ 3 & 5 & 9 \end{bmatrix}_{2\times 3}$

* **Symmetric Matrix :-** For a square matrix A if $A = A^T$ (A') then it is said to be symmetric Matrix.

$A = \begin{bmatrix} -1 & 3 & 0 \\ 3 & 2 & -2 \\ 0 & -2 & -5 \end{bmatrix}_{3\times 3}$ $\Rightarrow A^T = \begin{bmatrix} -1 & 3 & 0 \\ 3 & 2 & -2 \\ 0 & -2 & -5 \end{bmatrix}_{3\times 3}$

→ $A + A^T$ is always symmetric matrix.

* **Skew symmetric matrix** :- For a square matrix A if $A = -A^T$, then it is said to be skew symmetric Matrix.

$$A = \begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & -4 & -7 \\ 4 & 0 & 3 \\ 7 & -3 & 0 \end{bmatrix} \Rightarrow -A^T = \begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$$

→ $A - A^T$ is always skew symmetric Matrix.

* **Singular Matrix** :- For a square matrix A if $|A| = 0$, then it is said to be singular matrix.

$$A = \begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 6 & 3 \\ 4 & 2 \end{vmatrix} = 12 - 12 = 0 \Rightarrow |A| = 0.$$

so, Matrix A is singular Matrix.

* **Non-singular Matrix** :- For a square matrix A if $|A| \neq 0$, then it is said to be non-singular Matrix.

$$A = \begin{bmatrix} 7 & -2 \\ 5 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 7 & -2 \\ 5 & 4 \end{vmatrix} = 28 - (-10) = 38 \neq 0 \Rightarrow |A| \neq 0$$

so, Matrix A is non-singular Matrix.

* **Addition and Substraction of Matrices** :-

→ In addition and substraction matrix must have same order.

1) If $A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 0 & 4 \end{bmatrix}$ then find $A + B$ and $A - B$.

2) If $A = \begin{bmatrix} 4 & -7 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 5 \\ 2 & -1 \end{bmatrix}$ then find $2A + 3B$ and $3A - B$.

3) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ then find (i) $A + B$ (ii) $A - B$ (iii) $2A + 3B$ (iv) $3A - 2B$.

4) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \\ 2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 & 2 \\ 1 & 1 & -1 \\ 3 & 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 4 & -2 \\ -5 & 2 & -2 \end{bmatrix}$ then find

(i) $A + B + C$ (ii) $A + B - C$ (iii) $3A - 2B + C$ (iv) $2A + 3B - C$

5) If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 2 \\ -5 & 1 \end{bmatrix}$ and $2A+3B-C=0$ then find matrix 'C'.

6) If $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 & 3 \\ 4 & 4 & -1 \\ 3 & -1 & -2 \end{bmatrix}$ and $2A-B+C=0$ then find matrix 'C'.

7) If $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 5 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 8 & 2 \\ 3 & 2 & 4 \end{bmatrix}$ and $X+A+B=0$ then find matrix 'X'.

8) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 2 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 & 3 \\ -2 & 3 & 2 \end{bmatrix}$ and $3(X+B)+5A=0$ then find matrix 'X'.

9) If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ then prove that $(A+B)^T = A^T + B^T$.

10) If $3A-2B = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}$ and $B-4A = \begin{bmatrix} -1 & 2 \\ -4 & 4 \end{bmatrix}$ then find A and B.

* Matrix Multiplication :-

For two matrices A and B, multiplication AB is possible if the number of columns of A equals to the number of rows of B.

→ $A_{2\times3}$ $B_{3\times2}$ ⇒ $AB_{2\times2}$ is possible
⇒ $BA_{3\times3}$ is possible

→ $A_{2\times3}$ $B_{2\times3}$
⇒ AB is not possible
⇒ BA is not possible.

→ $A_{3\times3}$ $B_{2\times3}$ ⇒ AB is not possible
$BA_{2\times3}$ is possible

1) If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ then find AB and BA.

2) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find AB and BA whichever exists

3) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$ then find AB and BA whichever exist.

4) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ then P.T. $AB = AC$.

5) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find A^2.

6) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then find A^2.

7) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then find A^2.

8) If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ then P.T. $A^2 = A$.

9) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then P.T. $A^2 - 5A - 2I = 0$

10) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then P.T. $A^2 - 4A + 7I = 0$

11) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then P.T. $A^2 - 4A - 5I = 0$.

12) If $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ then P.T. $A^4 = I$.

13) If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ then P.T. $(AB)^T = B^T \cdot A^T$.

* Adjoint of Matrix (Adj (A))

→ Minor Matrix

→ Co-factor Matrix → Transpose of Matrix.

→ Adjoint Matrix

1) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ then find Adj(A).

2) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ then find Adj(A).

3) If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then P.T. Adj(A) = A.

→ 2×2

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. $\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

$a_{11} = d$

$a_{12} = c$

$a_{21} = b$

$a_{22} = a$

$Adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

* 3×3

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

* Inverse of Matrix (A^{-1})

$$A^{-1} = \frac{1}{|A|} Adj(A).$$

→ If $|A| = 0$ then A^{-1} does not exist.

→ If $|A| \neq 0$ then A^{-1} exist.

* If $A = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$ then find A^{-1}.

* If $A = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$ then find A^{-1}.

* If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ then find A^{-1}.

* If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ then find A^{-1}.

* If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ then find A^{-1}.

* If $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$ then find A^{-1}.

* If $A+B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A-B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ then find $(AB)^{-1}$.

* If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ then P. T. $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

* Solution of Linear Simultaneous Equations.

* Solve the eq^n $3x-y=1$ and $x+2y=5$ Using matrix method.

* Solve the eq^n $2x+5y=7$ and $8x-3y=5$ Using matrix method.

* Solve the eq^n $2x+3y=1$ and $y-4x=2$ Using matrix method.

* Solve the eq^n $2x-3y=-5$ and $3x+y=9$ Using matrix method.

* Solve the eq^n $3x+2y=7$ and $11x-4y=3$ Using matrix method.

* Solve the eq^n $3x+2y=5$ and $2x-y=1$ Using matrix method.

* Solve the eq^n $2x+3y=6xy$ and $x-y=xy$ using matrix method.