

ANOVA

• F-distribution -

- ↳ continuous prob. distribution
- ↳ It has 2 parameters df_1 & df_2
- ↳ Positively skewed
- ↳ Used for testing equality of variance

Suppose -

$$\begin{aligned} \text{for } & \chi_1^2 \rightarrow df_1 \\ & \chi_2^2 \rightarrow df_2 \end{aligned}$$

then

$$\frac{\chi_1^2 / df_1}{\chi_2^2 / df_2} \rightarrow \text{follows } F\text{-distribution}$$

⇒ one way ANOVA (Analysis of variance)

- ↳ a statistical method used to compare means of 3 or more independent groups
- ↳ It is extension of t -test

one-way ⇒ only 1 independent variable with multiple levels

• Steps - (with example)

- i) Null hypothesis - all groups mean are equal
 Alternate " - at least one group mean is significantly diff

A	B	C
3	1	8
6	8	6
3	9	10

(ii) Calculate grand mean

$$\bar{X} = \frac{3+6+3+1+8+9+8+6+10}{9}$$

$$= 6$$

iii) Calculate mean of individual -
 $\bar{x}_A = 4, \bar{x}_B = 6, \bar{x}_C = 8$

iv) Calculate SST (Sum of Square total)

$$(6-3)^2 + (6-6)^2 + (6-3)^2 + (6-1)^2 + (6-8)^2 + (6-9)^2 \\ + (6-8)^2 + (6-6)^2 + (6-10)^2$$

$$= 76$$

$$\rightarrow df = n-1 = 9-1 = 8$$

• Calculate SSW (Sum of Square within)

$$(4-3)^2 + (4-6)^2 + (4-3)^2 + \\ (6-7)^2 + (6-8)^2 + (6-9)^2 + \\ (8-8)^2 + (8-6)^2 + (8-10)^2$$

$$SSW = 52$$

$K = \# \text{ categories}$

$$df = n-k = 9-3 = 6$$

• Calculate SSB

$$3 \times (6-4)^2 + 3 \times (6-6)^2 + 3 \times (6-8)^2$$

of elements
in group A

grand
mean

$$= 24$$

$$df = k-1 = 2$$

$$SST = SSW + SSB$$

v) Calculate test Statistic (F-ratio)

$$f\text{-ratio} = \frac{SSB}{df_{SSB}} \bigg/ \frac{SSW}{df_{SSW}} = 1.4$$

$$\rightarrow p\text{-value} \Rightarrow 0.31$$

$0.31 > 0.05$ so can't reject Null hypothesis

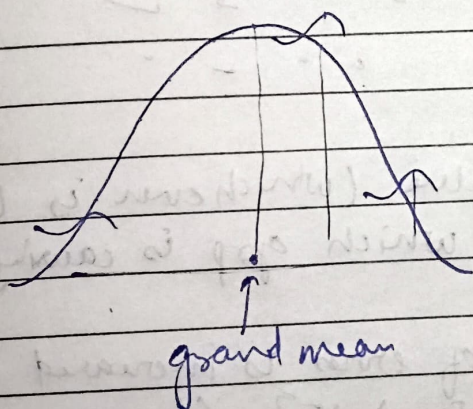
• Geometric intuition -

$$f = \frac{SSB}{df_{SSB}} \xrightarrow{\chi^2 \text{ dist}}$$

$$\frac{SSW}{df_{SSW}} \xrightarrow{\chi^2 \text{ dist}}$$

so F follows f -distribution

$\rightarrow H_0: \mu_A = \mu_B = \mu_C \Rightarrow$ all the three groups comes from same population



SSB is dist. b/w grandmean & individual mean

$\rightarrow F \uparrow$ then $SSB \uparrow \rightarrow p\text{-value is small}$

$\rightarrow SSW$ tells about variance of each individual
 $SSW \uparrow \rightarrow F \downarrow$

(p -value is area to right in graph)

Assumptions-

- observations should be independent
- The data within each group should be approx normally distributed
- Homogeneity of variance
 \hookrightarrow var of "popul" from each sample should be equal

→ Post-hoc - Test \Rightarrow

\hookrightarrow used in context of ANOVA

\hookrightarrow when we get significant diff in group means then we use it and tell because of which group problem is happening

i) Bonferroni's correctn-

A - B \rightarrow do t-test \rightarrow p-value ✓

B - C \rightarrow " " " ✓

C - A \rightarrow " " " "

By seeing p-value (whichever is low)

we can tell which grp is causing problem

Problem is Prob of error is increased to

$5 + 5 + 5 \Rightarrow 15\%$ (Family Wise Error rate FWER)

So we divide it by # groups i.e 3

$$\Rightarrow 15/3 = 5\%$$

2) Tukey's HSD (Honestly Significant Difference) Test

• why t-test is not used for more than 3 categories?

- ↳ Increased Type-I error
- ↳ Difficulty in interpreting result
- ↳ Inefficient