

## ANOVA

- f<sub>2</sub>- distribution -

  - ↳ continuous prob. distribution

  - ↳ set has 2 parameters  $df_1$  &  $df_2$

  - ↳ Positively skewed

  - ↳ Used for testing equality of variance

Suppose -

$$\text{for } \chi^2_1 \rightarrow df_1$$

$$\chi^2_2 \rightarrow df_2$$

then

$$\frac{\chi^2_1 / df_1}{\chi^2_2 / df_2} \rightarrow \text{follows f-distribution}$$

- ⇒ One way ANOVA (Analysis of variance)

  - ↳ a statistical method used to compare means of 3 or more independent groups

  - ↳ It is extension of t-test

One-way ⇒ only 1 independent variable  
with multiple levels

- Steps - (with example)

  - i) Null hypothesis - all groups mean are equal

  - ii) Alternative " - at least one group mean is significantly diff

A	B	C
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3	1	8
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6	8	6
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3	9	10
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(i) calculate grand mean

$$\bar{x} = 3 + 6 + 3 + 1 + 8 + 9 + 8 + 6 + 10$$

$$= 6$$

iii) calculate mean of individual -

iv) calculate SST (Sum of Square total)

$$\frac{(6-3)^2 + (6-6)^2 + (6-3)^2 + (6-1)^2 + (6-8)^2 + (6-9)^2}{6} + (6-8)^2 + (6-6)^2 + (6-10)^2$$

= 76

$$\rightarrow df = n-1 = g-1 = 8$$

• Calculate SSW (Sum of square within)

$$(4-3)^1 + (4-6)^5 + (4-3)^2 + \\ (6-1)^2 + (6-8)^5 + (6-5)^2 + \\ (8-8)^5 + (8-6)^2 + (8-10)^2$$

$$SS\omega = 52$$

$$df = n-k = 9-3 = 6$$

## • Calendere SGB

$$\frac{3 \times (6 - 4)^2}{3} + 3 \times (6 - 6)^2 + 3 \times (6 - 8)^2$$

↓      ↓      ↓  
 # of elements in group A      mean of A      mean

= 24

$$df = k - 1 = 2$$

$$SST = SSW + SSB$$

v) Calculate test Statistic (for ratio)

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$$F\text{-ratio} = \frac{SSB}{df_{SSB}} / \frac{SSW}{df_{SSW}} = 1.4$$

$\rightarrow p\text{-value} \Rightarrow 0.31$

$0.31 > 0.05 \therefore \text{can't reject Null hypothesis}$

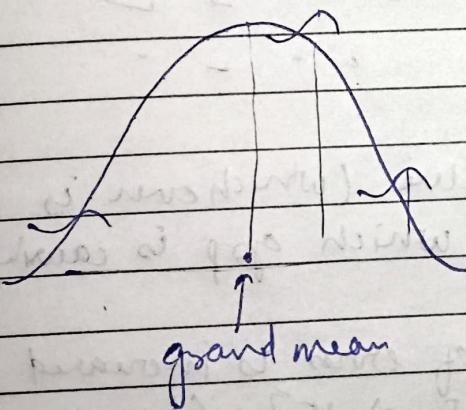
### Geometric intuition -

$$F = \frac{SSB}{df_{SSB}} \xrightarrow{\chi^2 \text{ dist}}$$

$$\frac{SSW}{df_{SSW}} \xrightarrow{\chi^2 \text{ dist}}$$

$\therefore F$  follows  $F$ -distribution

$\rightarrow H_0: \mu_A = \mu_B = \mu_C \Rightarrow$  all the three groups comes from same population



$SSB$  is dist. b/w grandmean & individual mean

$\rightarrow F \uparrow \text{then } SSB \uparrow \rightarrow \text{P-value is small}$

$\rightarrow SSW$  tells about variance of each individual  
 $SSW \uparrow \rightarrow F \downarrow$

(P-value is area to right in graph)

### Assumptions -

- observations should be independent
- The data within each group should be approx normally distributed
- Homogeneity of variance  
↳ var. of "popul" from each sample should be equal
- Post-hoc - Test  $\Rightarrow$   
↳ used in context of ANOVA  
↳ when we get significant diff in group means then we use it and tell because of which group problem is happening

#### i) Bonferroni's correction -

A - B  $\rightarrow$  do t-test  $\rightarrow$  p-val ✓

B - C  $\rightarrow$  " " - ✓

C - A  $\rightarrow$  " " - "

By seeing p-value (which ever is low)  
we can tell which grp is causing problem

Problem is P-value of error is increased to  
 $5+5+5 \Rightarrow 15\%$  (family wise Err rate FWER)

So we divide it by # groups i.e 3  
 $\Rightarrow 15/3 = 5$

#### 2) Tukey's HSD (Honestly Significant Difference) Test

• why t-test is not used for more than 3 categories?

- ↳ Increased Type-I error
- ↳ difficulty in interpreting result
- ↳ Inefficient