Applied Physics

Assignment: 02

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Problem 1

$m = 1.75\ kg$

$x = 4\cos(1.33t + \pi/5)$

(a) amplitude $= x_0 = ?$, frequency $= f = ?$
angular frequency $= \omega = ?$, time period $= T = ?$

(b) equation of velocity

(c) equation of acceleration

(d) spring constant $= k = ?$

Sol:- $x = 4\cos(1.33t + \pi/5)$

$\because x = x_0\cos(\omega t + \phi)$

Therefore;

$\boxed{x_0 = 4m}$, $\boxed{\omega = 1.33\ rad/s}$, $\phi = \pi/5$ radians

$\because \omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{1.33}{2\pi}$

$\boxed{f = 0.211\ Hz}$

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$\therefore$ Time period $= T = \frac{1}{f}$

$T = \frac{1}{0.211}$

$\boxed{T = 4.727 \text{ seconds}}$

(b) $x = x_0 \cos(\omega t + \phi)$

$v = -x_0 \omega \cos(\omega t + \phi)$

$v = -4(1.33) \sin(1.33t + \pi/5)$

(c) $a = -x_0 \omega^2 \sin(\omega t + \phi)$

$= -4(1.33)^2 \cos(1.33t + \pi/5)$

(d) $\omega = \sqrt{\frac{k}{m}}$

$\omega^2 = \frac{k}{m}$

$k = \omega^2 m$

$k = (1.33)^2 (1.75)$

$\boxed{k = 3.045 \text{ N/m}}$

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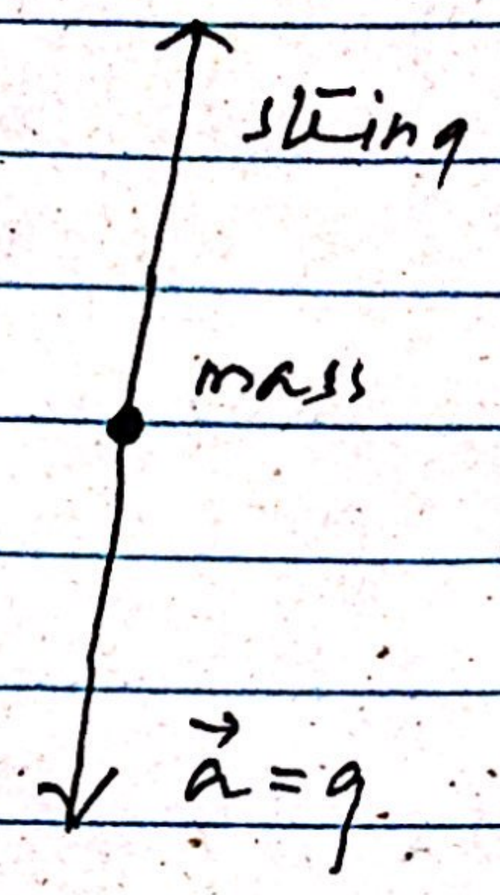
Problem 2

$k = 900\ N/m$

$m = 6.00\ kg$

$x_0 = ?$



Sol:

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m} \quad -(i)$$

and $a = \omega^2 x$

$$\omega^2 = \frac{a}{x} \quad -(ii)$$

Comparing (i) and (ii)

$$\frac{a}{x} = \frac{k}{m} \quad -(A)$$

Acceleration due to SHM should be equal to acceleration due to gravity, Therefore; $a = g$

$$(A) \Rightarrow \frac{g}{x} = \frac{k}{m}$$

$$x = \frac{(g)(m)}{(k)}$$

$$x = \frac{(9.8)(6.00)}{(900)}$$

$$\boxed{x = 0.147\ m}$$

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Question 3:-

At equilibrium point all P.E is converted into K.E therefore;

$$P.E = K.E$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$$

Since there are two springs

$$\frac{1}{\cancel{2}}(\cancel{2}k)x_0^2 = \frac{1}{2}mv^2$$

$$kx_0^2 = \frac{1}{2}mv^2$$

$$\sqrt{v^2} = \sqrt{\frac{2kx_0^2}{m}}$$

$$v = \sqrt{\frac{2k}{m}} \cdot x_0 \quad m.s^{-1}$$

(b) We know that

$$v = \omega x_0$$

$$\sqrt{\frac{2k}{m}}\cancel{x_0} = \omega \cancel{x_0}$$

$$\omega = \sqrt{\frac{2k}{m}} \quad rad.s^{-1}$$

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Problem 4:-

$m = 0.50\,kg$

$F_s = 75.0\,N$

(a) amplitude $= x_0 = ?$

(b) period $= T = ?$

(c) $\vec{a}_{max} = ?$

(d) $K.E_{max} = ?$

Sol: (a) from graph

$$\boxed{x_m = 0.30\,m}$$

(b) $F = kx$

$$k = \frac{F}{x}$$

$$k = \frac{75.0}{0.30}$$

$$k = 250\,N/m$$

Now $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{0.50}{250}}$$

$$\boxed{T = 0.280 \text{ seconds}}$$

(c) $\vec{a}_{max} = \omega^2 x$

$$\vec{a}_{max} = \left(\frac{2\pi}{T}\right)^2 x$$

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$$\vec{a}_{max} = \left(\frac{2\pi}{0.280}\right)^2 (0.30)$$

$$\boxed{\vec{a}_{max} = 151.055 \ m/s^2}$$

(d) $K.E = \frac{1}{2} k x_0^2$

$$= \frac{1}{2}(250)(0.30)^2$$

$$\boxed{K.E_{max} = 11.25 \text{ Joules}}$$

<u>Problem 5:-</u>

$m = 15.0 kg$

$x_0 = 12.0 cm = 0.12 m$

$f = 2 Hz$

a) $k = ?$

b) $w = ?$

Sol:- $\because w = 2\pi f$ → We have a range can not use

$w = 2\pi(2)$

$= 12.56$ rad/seconds

$w = \sqrt{\frac{k}{m}}$

$w^2 = \frac{k}{m} \Rightarrow k = w^2 m$

$k = (12.56)^2 (15.0)$

$k =$

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Sol:- (a) $a = \frac{k}{m} x$

$\because a = g$

$g = \frac{k}{m} \cdot x$

$k = \frac{g \cdot m}{x}$

$k = \frac{(9.8)(15)}{(0.12)}$

$k = 1225 \ N/m$

(b) $\omega = \sqrt{\frac{k}{m}}$ $\quad \because \omega = 2\pi f$

$= 2\pi (2) = 4\pi$

$4\pi = \sqrt{\frac{1225}{m}}$

$(4\pi)^2 = \frac{1225}{m}$

$m = \frac{1225}{(4\pi)^2}$

$m = 7.75 \ kg$

Then $\because W = mg$

$W = (7.75)(9.8)$

$\boxed{W = 76.02 \ N}$

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Problem 6:-

a) transverse speed = ?] → at $t = 0.200\,s$
acceleration = ?] located at $x = 1.60\,m$

(b) $\lambda = ?$
$T = ?$
speed of propagation $= v = ?$

Sol:

$$\because\ y = (0.120\,m)\sin\left[\left(\frac{\pi x}{8}\right) + 4\pi t\right] \quad -(i)$$

for speed derivate (i)

$$(i) \Rightarrow v = 0.120\cos\left(\frac{\pi x}{8} + 4\pi t\right)\left(\frac{\pi}{8} + 4\pi\right) \quad -(A)$$

Put $t = 0.200$ and $x = 1.60$ in (A)

$$(A) \Rightarrow v = (0.120)\left(\frac{33\pi}{8}\right)\cos\left[\left(\frac{\pi}{8}\right)(1.60) + 4\pi(0.2)\right]$$

$$v = 1.552\ m/s$$

for acceleration derivate (A)

$$(A) \Rightarrow \vec{a} = -(0.120)\left(\frac{33\pi}{8}\right)^2 \sin\left[\left(\frac{\pi x}{8}\right)(1.60) + 4\pi(0.2)\right]$$

$$\vec{a} = \cancel{10.44\ m/s^2}\ 0\,m/s^2$$

$$\because\ y = (0.120\,m)\sin\left[\left(\frac{\pi x}{8}\right) + 4\pi t\right]$$

$$y = x_0 \sin(kx - \omega t)$$

$k = \frac{\pi}{8}$ and $\omega = -4\pi$

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$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\cancel{\pi}}{4\cancel{\pi}}$$

$$\boxed{T = 0.5 \text{ seconds}}$$

For wavelength :-

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{2\pi}{\pi/8}$$

$$\boxed{\lambda = 16 \text{ m}}$$

For total speed :-

$$v = \frac{\omega}{k}$$

$$v = \frac{4\pi}{\pi/8}$$

$$\boxed{v = 32 \text{ ms}^{-1}}$$

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Problem 7:-

Two identical waves moving in same direction results in combined wave having amplitude 1.50 times that that of common amplitude.

(a) ϕ in degree = ?

(b) ϕ in radians = ?

(c) ϕ in terms of wave length = ?

Sol:- $y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$

$y = y_m [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$ —(i)

let $kx - \omega t = \alpha$

$\phi = \beta$

(i) $\Rightarrow y = y_m [\sin(\alpha - 0) + \sin(\alpha - \beta)]$

$y = y_m \, 2$

$y = y_m [\sin(kx - \omega t) + [\sin(kx - \omega t)\cos\phi + \cos(kx - \omega t)\sin\phi]]$

$y = 2y_m \sin(kx - \omega t) \left[\frac{1 + \cos\phi}{2} + \cos(kx - \omega t)\sin\phi\right]$

$y = 2y_m \sin(kx - \omega t) \left(\cos^2\frac{\phi}{2} + \cos(kx - \omega t)\, 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)$

$y = 2y_m \left[\sin(kx - \omega t)\cos\frac{\phi}{2} + \cos(kx - \omega t)\sin\frac{\phi}{2}\right]\cos\frac{\phi}{2}$

$y = 2y_m \cos\frac{\phi}{2} \cdot \sin\left(kx - \omega t + \frac{\phi}{2}\right)$

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(a)

$y = 2y_m \cos\frac{\phi}{2}$ ∵ for amplitude

$1.5\,D\,y_m = 2y_m \cos\frac{\phi}{2}$

$\left(\frac{1.5D}{2}\right) = \cos\frac{\phi}{2}$

$\cos^{-1}\left(\frac{1.5D}{2}\right) = \frac{\phi}{2}$

$\frac{\phi}{2} = 41.4$

$\phi = 82.819^\circ$

(b)
Now in radians;

$\phi = 82.819 \cdot \frac{\pi}{180}$

$\phi = 1.445$ radians

(c) Normal wave length

2π radians $= 1\lambda$

1.445 radians $= x$

$(x)(2\pi) = (\lambda)(1.445)$

$x = \frac{\lambda(1.445)}{2\pi}$

$\boxed{x = 0.230\lambda}$

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Problem 8 :-

$$y_1 = A \sin(kx)$$

$$y_2 = A \sin\left(kx + \frac{\pi}{4}\right)$$

a) λ of $y' = ?$

b) amplitude of $y' = ?$

Sol :-

(b) $y' = y_1 + y_2$

$$y' = A \sin(kx) + A \sin\left(kx + \frac{\pi}{4}\right)$$

$$y' = A\left[\sin(kx) + \sin\left(kx + \frac{\pi}{4}\right)\right]$$

$$y' = A\left[2\sin\left(\frac{2kx + \frac{\pi}{4}}{2}\right)\cos\left(\frac{\frac{\pi}{4}}{2}\right)\right]$$

$$y' = 2A\cos\left(\frac{\pi}{8}\right)\sin\left(kx + \frac{\pi}{8}\right)$$

amplitude of y'

$$y' = 2y\cos\frac{\pi}{8}$$

$$y' = 2(0.9238)$$

or

$$\boxed{y' = 1.847\, y}$$

(a) λ of y :-

2π radians $= 1\lambda$

$\frac{\pi}{4}$ radians $= x$

$(x)(2\pi) = \lambda(\pi/4)$

$$\lambda = \frac{(x)(2\pi)}{(\pi/4)}$$

$$\boxed{\lambda = 8x}$$

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Problem 9

$W = 14.0\,N$

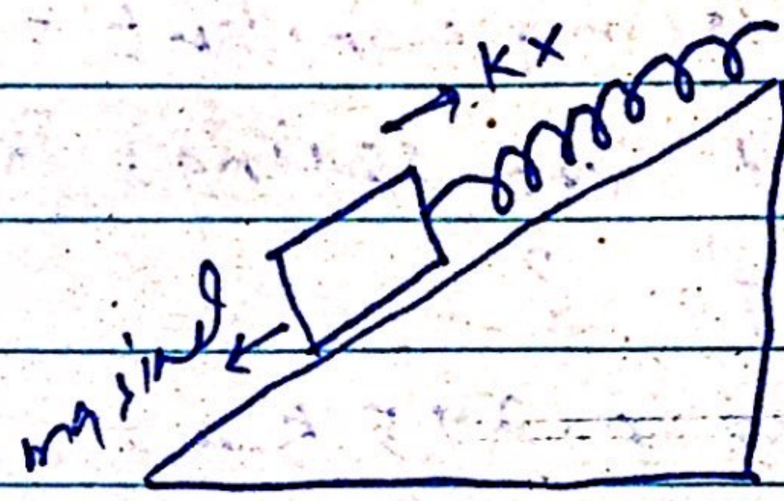
$\theta = 70.0^\circ$

unstretched length $= x = 0.450\,m$

$k = 120\,N/m$

(a) How far from top of incline is block's equilibrium point

(b) period of resulting oscillation $= T = ?$



Sol:

At equilibrium

(a)

$$\Sigma F_{net} = 0$$

$$kx = mg\sin\theta$$

$$x = \frac{mg\sin\theta}{k}$$

$$x = \frac{W\sin\theta}{k} \qquad \because W = mg$$

$$x = \frac{(14.0)\sin(70.0)}{(120)}$$

$$x = 0.0749\,m$$

Now

$$\text{total} = 0.0749 + 0.450$$

$$\boxed{l = 0.524\,m}$$

(b)

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \because W = mg \Rightarrow m = \frac{W}{g}$$

$$T = 2\pi\sqrt{\frac{W}{gk}}$$

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$$T = 2\pi\sqrt{\frac{(14.0)}{(9.8)(120)}}$$

$$\boxed{T = 0.6855 \text{ seconds}}$$

Problem 10 :-

two identical springs

spring constant $= k = 7580\ N/m$

for two springs $k + k = 2k$

$f = ?$

$m = 0.245\ kg$

Sol :-

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{1}{f} = 2\pi\sqrt{\frac{m}{2k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2(7580)}{0.245}}$$

$$\boxed{f = 36.59\ Hz}$$

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