National University of computer and emerging sciences

Foundation for Advancement of Science and Technology

L'Hospital's Rule Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or ∞/∞.) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

WARNING

Note that in L'Hôpital's rule the numerator and denominator are differentiated individually. This is *not* the same as differentiating $f(x)/g(x)$.

Applying L'Hôpital's Rule

Step 1. Check that the limit of $f(x)/g(x)$ is an indeterminate form of type $0/0$.

Step 2. Differentiate f and g separately.

Step 3. Find the limit of $f'(x)/g'(x)$. If this limit is finite, $+\infty$, or $-\infty$, then it is equal to the limit of $f(x)/g(x)$.

Indeterminate Forms:

$$\frac{\infty}{\infty},\quad \frac{0}{0},\quad \infty-\infty,\quad 0^0,\quad 0\cdot\infty,\quad \infty^0,\quad 1^\infty$$

WARNING

Applying L'Hôpital's rule to limits that are not indeterminate forms can produce incorrect results. For example, the computation

$$\lim_{x\to 0}\frac{x+6}{x+2} = \lim_{x\to 0}\frac{\frac{d}{dx}[x+6]}{\frac{d}{dx}[x+2]} = \lim_{x\to 0}\frac{1}{1} = 1$$

is ***not valid***, since the limit is not an indeterminate form. The correct result is

$$\lim_{x\to 0}\frac{x+6}{x+2} = \frac{0+6}{0+2} = 3$$

3.6.1 THEOREM (L'Hôpital's Rule for Form 0/0) *Suppose that f and g are differentiable functions on an open interval containing $x = a$, except possibly at $x = a$, and that*

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

If $\lim_{x \to a} [f'(x)/g'(x)]$ exists, or if this limit is $+\infty$ or $-\infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Moreover, this statement is also true in the case of a limit as $x \to a^-$, $x \to a^+$, $x \to -\infty$, or as $x \to +\infty$.

3.6.2 THEOREM (L'Hôpital's Rule for Form ∞/∞) *Suppose that f and g are differentiable functions on an open interval containing $x = a$, except possibly at $x = a$, and that*

$$\lim_{x \to a} f(x) = \infty \quad \text{and} \quad \lim_{x \to a} g(x) = \infty$$

If $\lim_{x \to a} [f'(x)/g'(x)]$ exists, or if this limit is $+\infty$ or $-\infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Moreover, this statement is also true in the case of a limit as $x \to a^-$, $x \to a^+$, $x \to -\infty$, or as $x \to +\infty$.

Indeterminate Forms:

$$\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty$$

1–2 Evaluate the given limit without using L'Hôpital's rule, and then check that your answer is correct using L'Hôpital's rule.

1. (a) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 2x - 8}$ (b) $\lim_{x \to +\infty} \frac{2x - 5}{3x + 7}$

2. (a) $\lim_{x \to 0} \frac{\sin x}{\tan x}$ (b) $\lim_{x \to 1} \frac{x^2 - 1}{x^3 - 1}$

7–45 Find the limits.

7. $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$

8. $\lim_{x \to 0} \frac{\sin 2x}{\sin 5x}$

Indeterminate Forms:

$$\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty$$

Type : 0/0

EXAMPLE 1 Find $\lim_{x \to 1} \frac{\ln x}{x - 1}$.

SOLUTION Since

$$\lim_{x \to 1} \ln x = \ln 1 = 0 \qquad \text{and} \qquad \lim_{x \to 1} (x - 1) = 0$$

we can apply l'Hospital's Rule:

$$\lim_{x \to 1} \frac{\ln x}{x - 1} = \lim_{x \to 1} \frac{\dfrac{d}{dx}(\ln x)}{\dfrac{d}{dx}(x - 1)} = \lim_{x \to 1} \frac{1/x}{1}$$

$$= \lim_{x \to 1} \frac{1}{x} = 1$$

Indeterminate Forms:

$$\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty$$

Type : ∞/ ∞

EXAMPLE 2 Calculate $\lim_{x \to \infty} \frac{e^x}{x^2}$.

SOLUTION We have $\lim_{x \to \infty} e^x = \infty$ and $\lim_{x \to \infty} x^2 = \infty$, so l'Hospital's Rule gives

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \lim_{x \to \infty} \frac{\frac{d}{dx}(e^x)}{\frac{d}{dx}(x^2)} = \lim_{x \to \infty} \frac{e^x}{2x}$$

Since $e^x \to \infty$ and $2x \to \infty$ as $x \to \infty$, the limit on the right side is also indeterminate, but a second application of l'Hospital's Rule gives

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \lim_{x \to \infty} \frac{e^x}{2x} = \lim_{x \to \infty} \frac{e^x}{2} = \infty$$

Indeterminate Forms:

$\frac{\infty}{\infty},\ \frac{0}{0},\ \infty - \infty,\ 0^0,\ 0 \cdot \infty,\ \infty^0,\ 1^\infty$

Type : $0.\infty$

We can deal with it by writing the product fg as a quotient:

$$fg = \frac{f}{1/g} \qquad \text{or} \qquad fg = \frac{g}{1/f}$$

This converts the given limit into an indeterminate form of type $\frac{0}{0}$ or ∞/∞ so that we can use l'Hospital's Rule.

Indeterminate Forms:

$$\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty$$

Type : 0. ∞

EXAMPLE 6 Evaluate $\lim_{x \to 0^+} x \ln x$.

SOLUTION The given limit is indeterminate because, as $x \to 0^+$, the first factor (x) approaches 0 while the second factor ($\ln x$) approaches $-\infty$. Writing $x = 1/(1/x)$, we have $1/x \to \infty$ as $x \to 0^+$, so l'Hospital's Rule gives

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{1/x}{-1/x^2} = \lim_{x \to 0^+} (-x) = 0$$

Indeterminate Forms:

$$\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty$$

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Type : $\infty - \infty$

EXAMPLE 7 Compute $\lim_{x\to(\pi/2)^-}(\sec x - \tan x)$.

SOLUTION First notice that $\sec x \to \infty$ and $\tan x \to \infty$ as $x \to (\pi/2)^-$, so the limit is indeterminate. Here we use a common denominator:

$$\lim_{x\to(\pi/2)^-}(\sec x - \tan x) = \lim_{x\to(\pi/2)^-}\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$$

$$= \lim_{x\to(\pi/2)^-}\frac{1-\sin x}{\cos x} = \lim_{x\to(\pi/2)^-}\frac{-\cos x}{-\sin x} = 0$$

Note that the use of l'Hospital's Rule is justified because $1 - \sin x \to 0$ and $\cos x \to 0$ as $x \to (\pi/2)^-$.

Indeterminate Forms:

$\frac{\infty}{\infty}$, $\frac{0}{0}$, $\infty - \infty$, 0^0, $0 \cdot \infty$, ∞^0, 1^∞

Indeterminate Powers

Several indeterminate forms arise from the limit

$$\lim_{x \to a} [\, f(x)]^{g(x)}$$

1. $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ type 0^0
2. $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = 0$ type ∞^0
3. $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \pm\infty$ type 1^∞

Each of these three cases can be treated either by taking the natural logarithm:

let $y = [\, f(x)]^{g(x)}$, then $\ln y = g(x) \ln f(x)$

or by writing the function as an exponential:

$$\boxed{[\, f(x)]^{g(x)} = e^{g(x) \ln f(x)}}$$

Example 6 Find $\lim_{x \to 0} (1 + \sin x)^{1/x}$.

Solution. As discussed above, we begin by introducing a dependent variable

$$y = (1 + \sin x)^{1/x}$$

and taking the natural logarithm of both sides:

$$\ln y = \ln(1 + \sin x)^{1/x} = \frac{1}{x}\ln(1 + \sin x) = \frac{\ln(1 + \sin x)}{x}$$

Thus,

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\ln(1 + \sin x)}{x}$$

which is an indeterminate form of type $0/0$, so by L'Hôpital's rule

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\ln(1 + \sin x)}{x} = \lim_{x \to 0} \frac{(\cos x)/(1 + \sin x)}{1} = 1$$

Since we have shown that $\ln y \to 1$ as $x \to 0$, the continuity of the exponential function implies that $e^{\ln y} \to e^1$ as $x \to 0$, and this implies that $y \to e$ as $x \to 0$. Thus,

$$\lim_{x \to 0} (1 + \sin x)^{1/x} = e \quad \blacktriangleleft$$

EXAMPLE 8 Calculate $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$.

SOLUTION First notice that as $x \to 0^+$, we have $1 + \sin 4x \to 1$ and $\cot x \to \infty$, so the given limit is indeterminate. Let

$$y = (1 + \sin 4x)^{\cot x}$$

Then $$\ln y = \ln[(1 + \sin 4x)^{\cot x}] = \cot x \ln(1 + \sin 4x)$$

so l'Hospital's Rule gives

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln(1 + \sin 4x)}{\tan x} = \lim_{x \to 0^+} \frac{\dfrac{4 \cos 4x}{1 + \sin 4x}}{\sec^2 x} = 4$$

Indeterminate Forms:

$$\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty$$

7–45 Find the limits. ■

7. $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$

8. $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x}$

9. $\lim_{\theta\to 0} \frac{\tan\theta}{\theta}$

10. $\lim_{t\to 0} \frac{te^t}{1 - e^t}$

11. $\lim_{x\to \pi^+} \frac{\sin x}{x - \pi}$

12. $\lim_{x\to 0^+} \frac{\sin x}{x^2}$

13. $\lim_{x\to +\infty} \frac{\ln x}{x}$

14. $\lim_{x\to +\infty} \frac{e^{3x}}{x^2}$

15. $\lim_{x\to 0^+} \frac{\cot x}{\ln x}$

16. $\lim_{x\to 0^+} \frac{1 - \ln x}{e^{1/x}}$

17. $\lim_{x\to +\infty} \frac{x^{100}}{e^x}$

18. $\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$

19. $\lim_{x\to 0} \frac{\sin^{-1} 2x}{x}$

20. $\lim_{x\to 0} \frac{x - \tan^{-1} x}{x^3}$

21. $\lim_{x\to +\infty} xe^{-x}$

22. $\lim_{x\to \pi^-} (x - \pi)\tan \frac{1}{2}x$

23. $\lim_{x\to +\infty} x \sin \frac{\pi}{x}$

24. $\lim_{x\to 0^+} \tan x \ln x$

25. $\lim_{x\to \pi/2^-} \sec 3x \cos 5x$

26. $\lim_{x\to \pi} (x - \pi)\cot x$

27. $\lim_{x\to +\infty} (1 - 3/x)^x$

28. $\lim_{x\to 0} (1 + 2x)^{-3/x}$

29. $\lim_{x\to 0} (e^x + x)^{1/x}$

30. $\lim_{x\to +\infty} (1 + a/x)^{bx}$

31. $\lim_{x\to 1} (2 - x)^{\tan[(\pi/2)x]}$

32. $\lim_{x\to +\infty} [\cos(2/x)]^{x^2}$

33. $\lim_{x\to 0} (\csc x - 1/x)$

34. $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{\cos 3x}{x^2}\right)$

35. $\lim_{x\to +\infty} (\sqrt{x^2 + x} - x)$

36. $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

37. $\lim_{x\to +\infty} [x - \ln(x^2 + 1)]$

38. $\lim_{x\to +\infty} [\ln x - \ln(1 + x)]$

39. $\lim_{x\to 0^+} x^{\sin x}$

40. $\lim_{x\to 0^+} (e^{2x} - 1)^x$

41. $\lim_{x\to 0^+} \left[-\frac{1}{\ln x}\right]^x$

42. $\lim_{x\to +\infty} x^{1/x}$

43. $\lim_{x\to +\infty} (\ln x)^{1/x}$

44. $\lim_{x\to 0^+} (-\ln x)^x$

45. $\lim_{x\to \pi/2^-} (\tan x)^{(\pi/2)-x}$

7. $\lim_{x\to 0} \dfrac{e^x}{\cos x} = 1.$

8. $\lim_{x\to 0} \dfrac{2\cos 2x}{5\cos 5x} = \dfrac{2}{5}.$

9. $\lim_{\theta\to 0} \dfrac{\sec^2\theta}{1} = 1.$

10. $\lim_{t\to 0} \dfrac{te^t + e^t}{-e^t} = -1.$

11. $\lim_{x\to \pi^+} \dfrac{\cos x}{1} = -1.$

12. $\lim_{x\to 0^+} \dfrac{\cos x}{2x} = +\infty.$

13. $\lim_{x\to +\infty} \dfrac{1/x}{1} = 0.$

14. $\lim_{x\to +\infty} \dfrac{3e^{3x}}{2x} = \lim_{x\to +\infty} \dfrac{9e^{3x}}{2} = +\infty.$

15. $\lim_{x\to 0^+} \dfrac{-\csc^2 x}{1/x} = \lim_{x\to 0^+} \dfrac{-x}{\sin^2 x} = \lim_{x\to 0^+} \dfrac{-1}{2\sin x\cos x} = -\infty.$

16. $\lim_{x\to 0^+} \dfrac{-1/x}{(-1/x^2)e^{1/x}} = \lim_{x\to 0^+} \dfrac{x}{e^{1/x}} = 0.$

17. $\lim_{x\to +\infty} \dfrac{100x^{99}}{e^x} = \lim_{x\to +\infty} \dfrac{(100)(99)x^{98}}{e^x} = \cdots = \lim_{x\to +\infty} \dfrac{(100)(99)(98)\cdots(1)}{e^x} = 0.$

18. $\lim_{x\to 0^+} \dfrac{\cos x/\sin x}{\sec^2 x/\tan x} = \lim_{x\to 0^+} \cos^2 x = 1.$

19. $\lim_{x\to 0} \dfrac{2/\sqrt{1-4x^2}}{1} = 2.$

20. $\lim_{x\to 0} \dfrac{1 - \dfrac{1}{1+x^2}}{3x^2} = \lim_{x\to 0} \dfrac{1}{3(1+x^2)} = \dfrac{1}{3}.$

21. $\lim_{x\to +\infty} xe^{-x} = \lim_{x\to +\infty} \dfrac{x}{e^x} = \lim_{x\to +\infty} \dfrac{1}{e^x} = 0.$

22. $\lim_{x\to \pi} (x-\pi)\tan(x/2) = \lim_{x\to \pi} \dfrac{x-\pi}{\cot(x/2)} = \lim_{x\to \pi} \dfrac{1}{-(1/2)\csc^2(x/2)} = -2.$

23. $\lim_{x\to+\infty} x\sin(\pi/x) = \lim_{x\to+\infty} \frac{\sin(\pi/x)}{1/x} = \lim_{x\to+\infty} \frac{(-\pi/x^2)\cos(\pi/x)}{-1/x^2} = \lim_{x\to+\infty} \pi\cos(\pi/x) = \pi.$

24. $\lim_{x\to0^+} \tan x \ln x = \lim_{x\to0^+} \frac{\ln x}{\cot x} = \lim_{x\to0^+} \frac{1/x}{-\csc^2 x} = \lim_{x\to0^+} \frac{-\sin^2 x}{x} = \lim_{x\to0^+} \frac{-2\sin x\cos x}{1} = 0.$

25. $\lim_{x\to(\pi/2)^-} \sec 3x\cos 5x = \lim_{x\to(\pi/2)^-} \frac{\cos 5x}{\cos 3x} = \lim_{x\to(\pi/2)^-} \frac{-5\sin 5x}{-3\sin 3x} = \frac{-5(+1)}{(-3)(-1)} = -\frac{5}{3}.$

26. $\lim_{x\to\pi} (x-\pi)\cot x = \lim_{x\to\pi} \frac{x-\pi}{\tan x} = \lim_{x\to\pi} \frac{1}{\sec^2 x} = 1.$

27. $y = (1-3/x)^x$, $\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln(1-3/x)}{1/x} = \lim_{x\to+\infty} \frac{-3}{1-3/x} = -3$, $\lim_{x\to+\infty} y = e^{-3}.$

28. $y = (1+2x)^{-3/x}$, $\lim_{x\to0} \ln y = \lim_{x\to0} -\frac{3\ln(1+2x)}{x} = \lim_{x\to0} -\frac{6}{1+2x} = -6$, $\lim_{x\to0} y = e^{-6}.$

29. $y = (e^x+x)^{1/x}$, $\lim_{x\to0} \ln y = \lim_{x\to0} \frac{\ln(e^x+x)}{x} = \lim_{x\to0} \frac{e^x+1}{e^x+x} = 2$, $\lim_{x\to0} y = e^2.$

30. $y = (1+a/x)^{bx}$, $\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{b\ln(1+a/x)}{1/x} = \lim_{x\to+\infty} \frac{ab}{1+a/x} = ab$, $\lim_{x\to+\infty} y = e^{ab}.$

31. $y = (2-x)^{\tan(\pi x/2)}$, $\lim_{x\to1} \ln y = \lim_{x\to1} \frac{\ln(2-x)}{\cot(\pi x/2)} = \lim_{x\to1} \frac{2\sin^2(\pi x/2)}{\pi(2-x)} = 2/\pi$, $\lim_{x\to1} y = e^{2/\pi}.$

32. $y = [\cos(2/x)]^{x^2}$, $\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln\cos(2/x)}{1/x^2} = \lim_{x\to+\infty} \frac{(-2/x^2)(-\tan(2/x))}{-2/x^3} = \lim_{x\to+\infty} \frac{-\tan(2/x)}{1/x} =$
$\lim_{x\to+\infty} \frac{(2/x^2)\sec^2(2/x)}{-1/x^2} = -2$, $\lim_{x\to+\infty} y = e^{-2}.$

33. $\lim_{x\to0} \left(\frac{1}{\sin x} - \frac{1}{x}\right) = \lim_{x\to0} \frac{x-\sin x}{x\sin x} = \lim_{x\to0} \frac{1-\cos x}{x\cos x+\sin x} = \lim_{x\to0} \frac{\sin x}{2\cos x - x\sin x} = 0.$

34. $\lim_{x\to 0}\frac{1-\cos 3x}{x^2}=\lim_{x\to 0}\frac{3\sin 3x}{2x}=\lim_{x\to 0}\frac{9}{2}\cos 3x=\frac{9}{2}.$

35. $\lim_{x\to+\infty}\frac{(x^2+x)-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to+\infty}\frac{x}{\sqrt{x^2+x}+x}=\lim_{x\to+\infty}\frac{1}{\sqrt{1+1/x}+1}=1/2.$

36. $\lim_{x\to 0}\frac{e^x-1-x}{xe^x-x}=\lim_{x\to 0}\frac{e^x-1}{xe^x+e^x-1}=\lim_{x\to 0}\frac{e^x}{xe^x+2e^x}=1/2.$

37. $\lim_{x\to+\infty}[x-\ln(x^2+1)]=\lim_{x\to+\infty}[\ln e^x-\ln(x^2+1)]=\lim_{x\to+\infty}\ln\frac{e^x}{x^2+1}$, $\lim_{x\to+\infty}\frac{e^x}{x^2+1}=\lim_{x\to+\infty}\frac{e^x}{2x}=\lim_{x\to+\infty}\frac{e^x}{2}=+\infty$, so $\lim_{x\to+\infty}[x-\ln(x^2+1)]=+\infty$

38. $\lim_{x\to+\infty}\ln\frac{x}{1+x}=\lim_{x\to+\infty}\ln\frac{1}{1/x+1}=\ln(1)=0.$

39. $y=x^{\sin x}, \ln y=\sin x\ln x$, $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln x}{\csc x}=\lim_{x\to 0^+}\frac{1/x}{-\csc x\cot x}=\lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)(-\tan x)=1(-0)=0$, so $\lim_{x\to 0^+}x^{\sin x}=\lim_{x\to 0^+}y=e^0=1.$

40. $y=(e^{2x}-1)^x, \ln y=x\ln(e^{2x}-1)$, $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln(e^{2x}-1)}{1/x}=\lim_{x\to 0^+}\frac{2e^{2x}}{e^{2x}-1}(-x^2)=$
$=\lim_{x\to 0^+}\frac{x}{e^{2x}-1}\lim_{x\to 0^+}(-2xe^{2x})=\lim_{x\to 0^+}\frac{1}{2e^{2x}}\lim_{x\to 0^+}(-2xe^{2x})=\frac{1}{2}\cdot 0=0$, $\lim_{x\to 0^+}y=e^0=1.$

41. $y=\left[-\frac{1}{\ln x}\right]^x, \ln y=x\ln\left[-\frac{1}{\ln x}\right]$, $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln\left[-\frac{1}{\ln x}\right]}{1/x}=\lim_{x\to 0^+}\left(-\frac{1}{x\ln x}\right)(-x^2)=-\lim_{x\to 0^+}\frac{x}{\ln x}=0$, so $\lim_{x\to 0^+}y=e^0=1.$

42. $y=x^{1/x}, \ln y=\frac{\ln x}{x}$, $\lim_{x\to+\infty}\ln y=\lim_{x\to+\infty}\frac{\ln x}{x}=\lim_{x\to+\infty}\frac{1/x}{1}=0$, so $\lim_{x\to+\infty}y=e^0=1.$

43. $y=(\ln x)^{1/x}, \ln y=(1/x)\ln\ln x$, $\lim_{x\to+\infty}\ln y=\lim_{x\to+\infty}\frac{\ln\ln x}{x}=\lim_{x\to+\infty}\frac{1/(x\ln x)}{1}=0$, so $\lim_{x\to+\infty}y=1.$

44. $y=(-\ln x)^x, \ln y=x\ln(-\ln x)$, $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\ln(-\ln x)/(1/x)=\lim_{x\to 0^+}\frac{(1/(x\ln x)}{(-1/x^2)}=\lim_{x\to 0^+}(-\frac{x}{\ln x})=0$, so $\lim_{x\to 0^+}y=1.$

45. $y=(\tan x)^{\pi/2-x}, \ln y=(\pi/2-x)\ln\tan x$, $\lim_{x\to(\pi/2)^-}\ln y=\lim_{x\to(\pi/2)^-}\frac{\ln\tan x}{1/(\pi/2-x)}=\lim_{x\to(\pi/2)^-}\frac{(\sec^2 x/\tan x)}{1/(\pi/2-x)^2}=$
$\lim_{x\to(\pi/2)^-}\frac{(\pi/2-x)}{\cos x}\frac{(\pi/2-x)}{\sin x}=\lim_{x\to(\pi/2)^-}\frac{(\pi/2-x)}{\cos x}\lim_{x\to(\pi/2)^-}\frac{(\pi/2-x)}{\sin x}=1\cdot 0=0$, so $\lim_{x\to(\pi/2)^-}y=1.$