Rolle’s Theorem and Mean Value Theorem

Rolle’s Theorem

Suppose that a function

i. $f(x)$ is continuous on the closed interval $[a,b]$

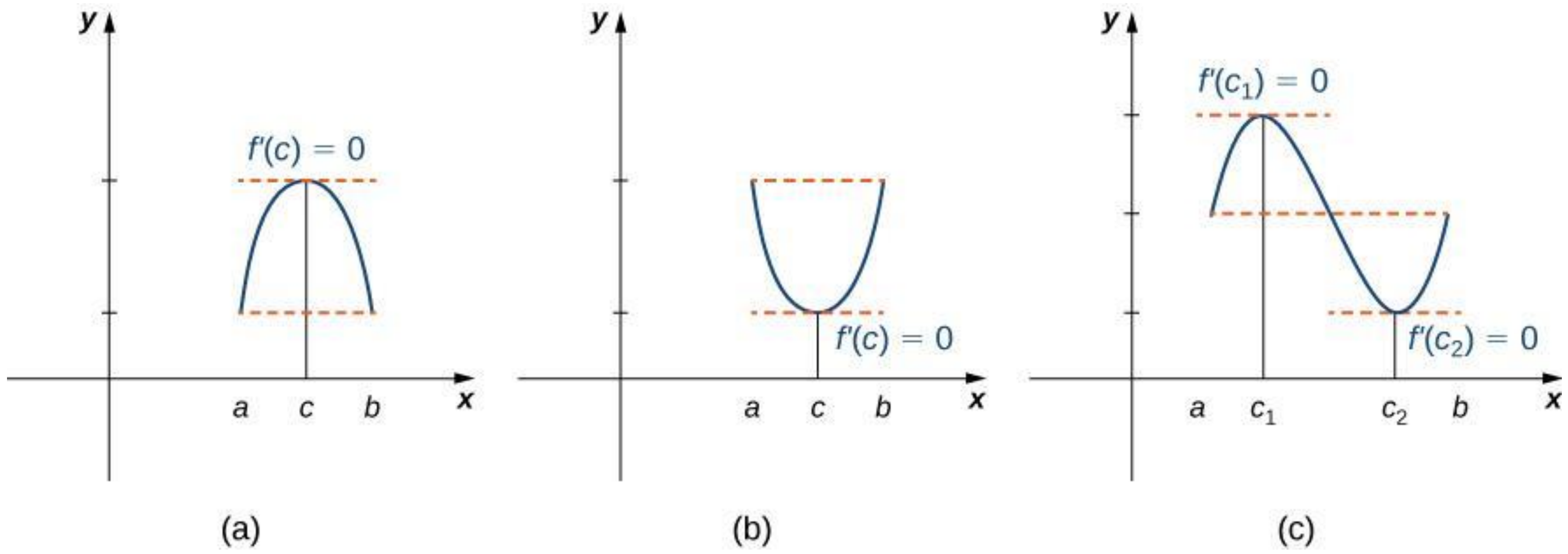
ii. $f(x)$ differentiable on the open interval (a,b).

iii. if $f(a)=f(b)$,

then there exists at least one point c in the open interval (a,b) for which $f'(c)=0$

Geometric interpretation

There is a point c on the interval (a,b) where the tangent to the graph of the function is horizontal.



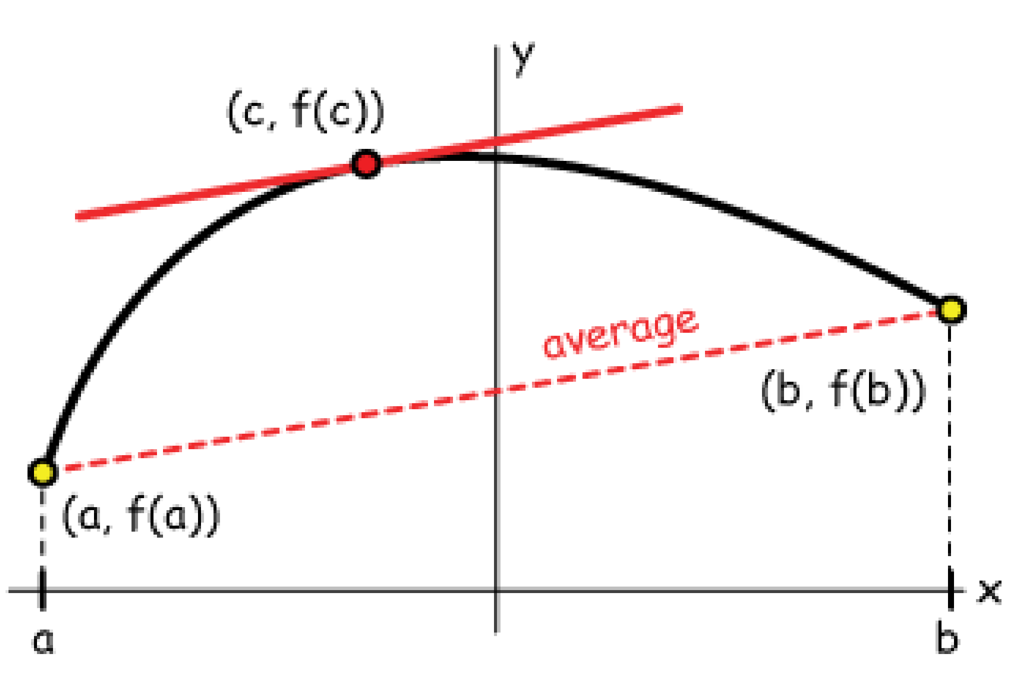
Mean Value Theorem

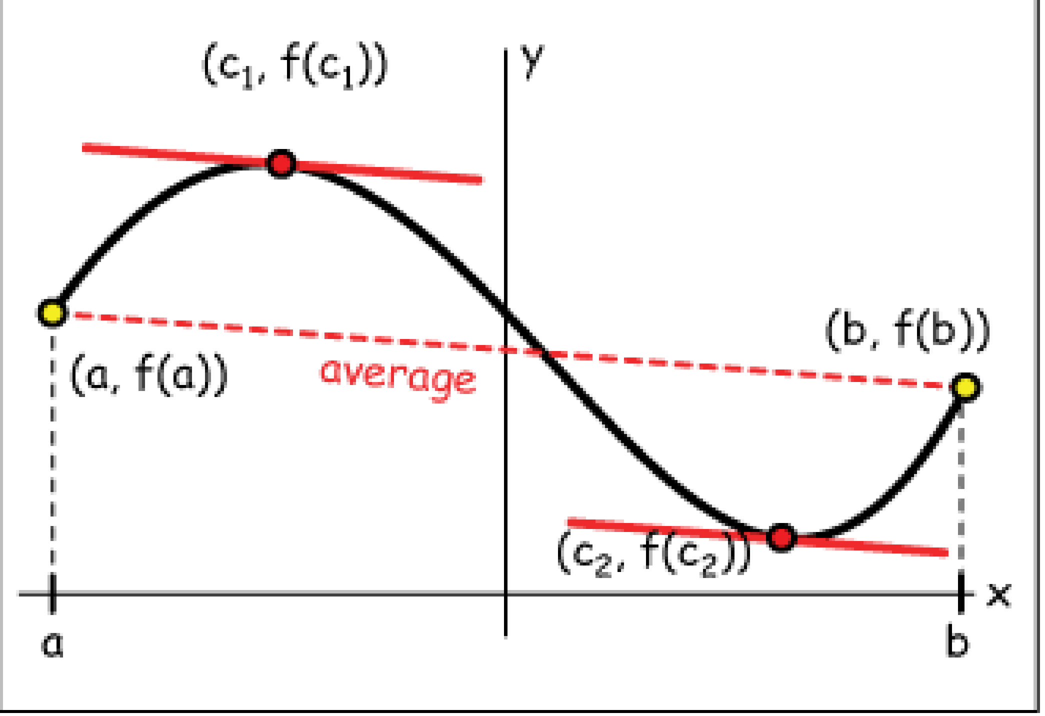
If **f** is a function that is continuous on $[a, b]$ and differentiable on (a, b), then there is a number $c \in [a, b]$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

which can be rearranged to

$$f(b) - f(a) = f'(c)(b - a)$$





1. Find c in [0,1] determined by the Mean Value Theorem for $f(x) = e^x$.
2. Find c in [-2,2] determined by Rolle's Theorem for $f(x) = x^4 - 4x^2$.
3. Find c for Rolle's Theorem for f(x)=$x^3 - 12x$ when $0 \le x \le 2\sqrt{3}$.
4. Find c for Mean Value Theorem for $f(x) = \frac{x-2}{x}$ on [2,4].
5. Find c for Rolle's Theorem for $f(x) = x^3 - 9x$ on [-3,3].
6. Find c for Rolle's Theorem for $f(x) = 5x^2 - 15x$ on [0,3].
7. Find c for the Mean Value Theorem for $f(x) = x^3 + 1$ on [-2,4].
8. Fine c for Rolle's Theorem for $f(x) = x^3 - 4x$ on [-2,2].
9. Find c for the Mean Value Theorem for $f(x) = \frac{x-1}{x}$ on [1,3].

Answers:

1. $c = \ln(e-1)$	2. $c = 0; c = \pm\sqrt{2}$	3. $c = 2$	4. $c = 2\sqrt{2}$
5. $c = \pm\sqrt{3}$	6. $c = \frac{3}{2}$	7. $c = 2$	8. $c = \frac{\pm 2}{\sqrt{3}}$
9. $c = \sqrt{3}$	10. f(x) is not defined on the open interval (-2, 2).	11. f(x) is discontinuous at x = 2.	12. f(x) is not defined on [-2, -1)

EXERCISE SET 4.8 Graphing Utility

1–4 Verify that the hypotheses of Rolle's Theorem are satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the theorem. ■

1. $f(x) = x^2 - 8x + 15;\ [3, 5]$

2. $f(x) = \frac{1}{2}x - \sqrt{x};\ [0, 4]$

3. $f(x) = \cos x;\ [\pi/2, 3\pi/2]$

4. $f(x) = \ln(4 + 2x - x^2);\ [-1, 3]$

5–8 Verify that the hypotheses of the Mean-Value Theorem are satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the theorem. ■

5. $f(x) = x^2 - x;\ [-3, 5]$

6. $f(x) = x^3 + x - 4;\ [-1, 2]$

7. $f(x) = \sqrt{25 - x^2};\ [-5, 3]$

8. $f(x) = x - \dfrac{1}{x};\ [3, 4]$

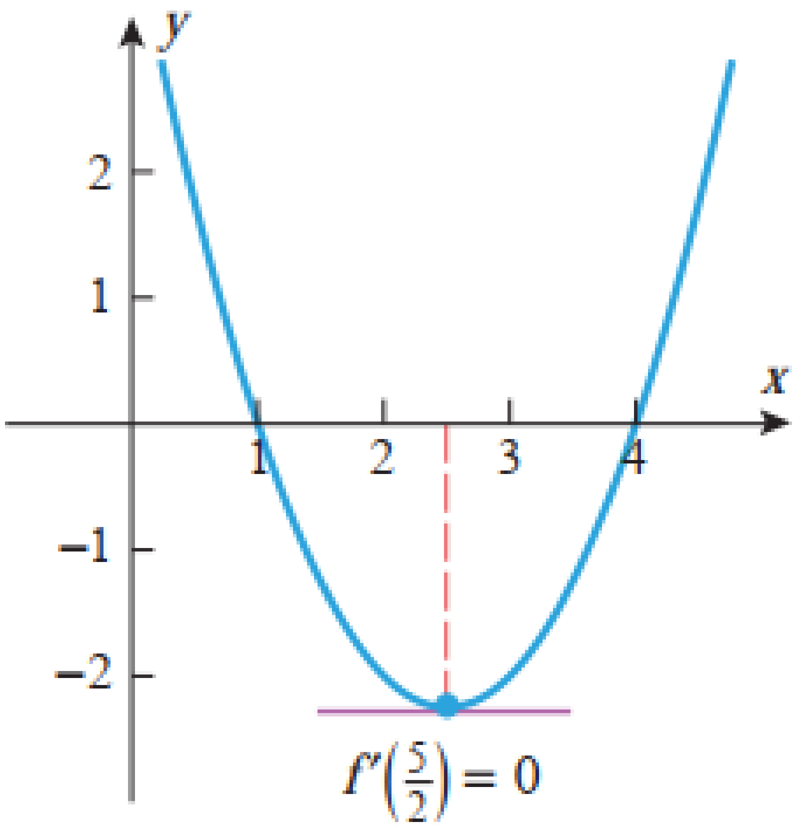
▶ **Example 1** Find the two x-intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.

$$x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$\boxed{\text{so the } x\text{-intercepts are } x = 1 \text{ and } x = 4.}$$

$$\boxed{f'(x) = 2x - 5}$$

Solving the equation $f'(x) = 0$ yields $x = \frac{5}{2}$, so $c = \frac{5}{2}$ is a point in the interval $(1, 4)$



$$\boxed{y = x^2 - 5x + 4}$$