Assignment 02

Calculus

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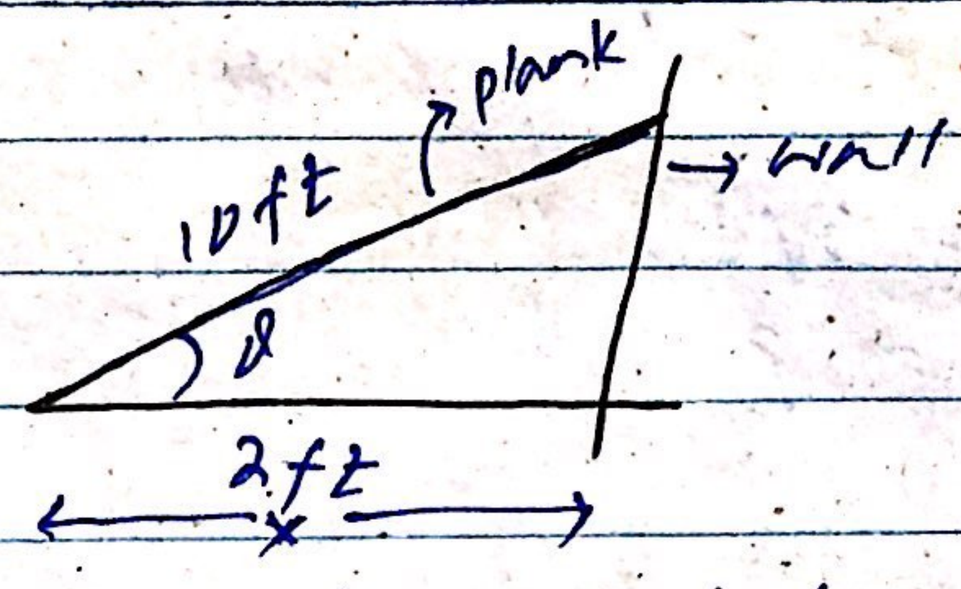
Q1



length of plank is 10 ft

distance from wall = x = 2 ft

rate of being pushed towards wall = $\frac{dx}{dt} = -6$ in/s

$\frac{dx}{dt} = -\frac{1}{2}$ ft/seconds

$\frac{d\theta}{dt} = ?$

sol:-

$\because \cos\theta = \frac{Base}{hyp} \Rightarrow \theta = \cos^{-1}\left(\frac{base}{hyp}\right)$

$\theta = \cos^{-1}\left(\frac{x}{10}\right) \Rightarrow \theta = \cos^{-1}\left(\frac{2}{10}\right)$

$\theta = 78.46^\circ$

Now for $\frac{d\theta}{dt}$:-

$\cos\theta = \frac{base}{hyp}$

$\cos\theta = \frac{x}{10}$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dx}{dt}$ $\because$ differentiating w.r.t 't'.

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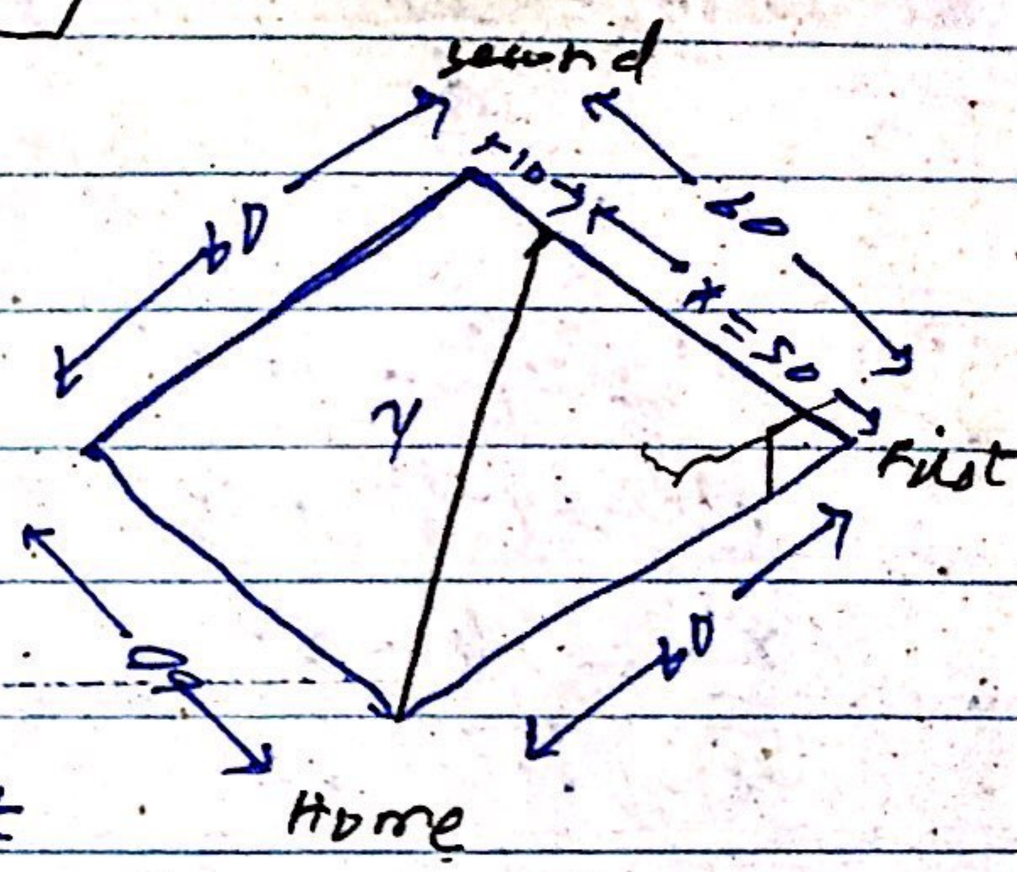
$$-\sin(78.76)\cdot\frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{-1}{2}$$

$$70.979\,\frac{d\theta}{dt} = \frac{-1}{20}$$

$$\boxed{\frac{d\theta}{dt} = 0.0510 \text{ rad/second.}}$$



Q2 A softball diamond is a square whose sides are 60ft long

distance from second phase ~~x~~ = 10 ft

$x = 60 - 10 \Rightarrow \boxed{x = 50\text{ft}}$

$\frac{dx}{dt} = 25\text{ft/s}$

$\frac{dy}{dt} = ?$

Sol:- $y^2 = x^2 + 60^2$

$y^2 = 50^2 + 60^2$

$y = 10\sqrt{61}$ ft/seconds

Now

differentiating w.r.t 't'

$$2y\cdot\frac{dy}{dt} = 2x\cdot\frac{dx}{dt} + 0$$

$$2(10\sqrt{61})\cdot\frac{dy}{dt} = 2(50)(25)$$

$$\boxed{\frac{dy}{dt} = 16.004 \text{ ft/s}}$$

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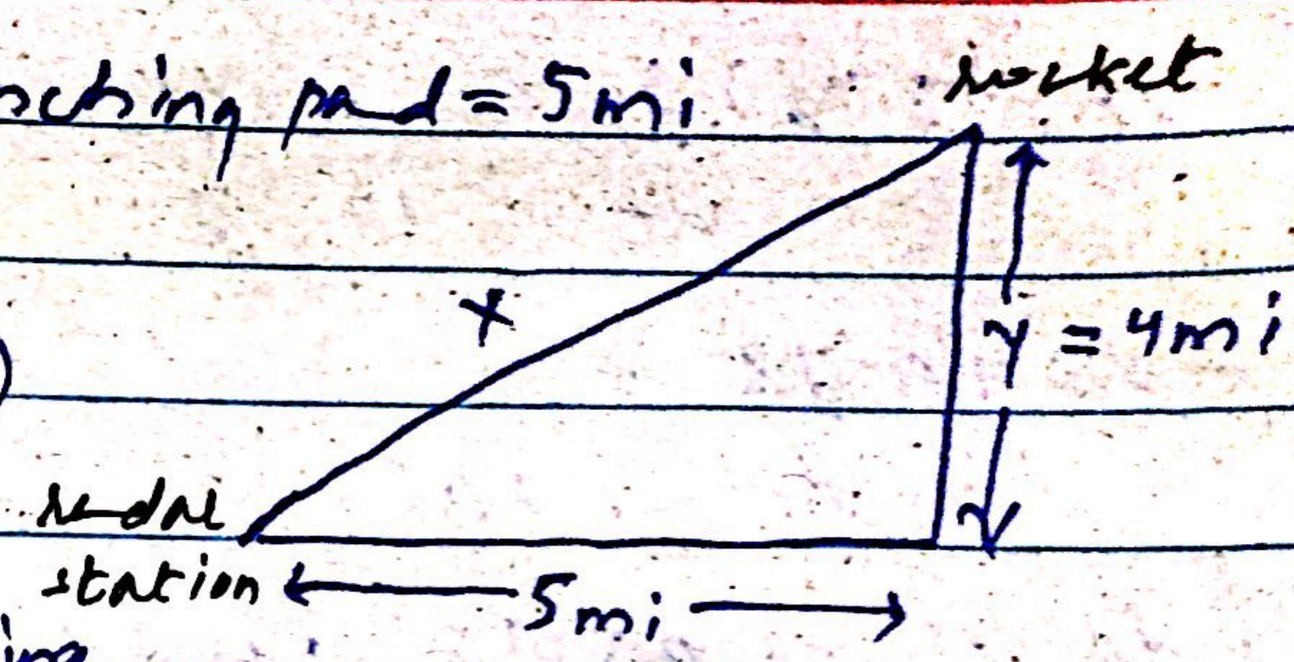
Q3

distance b/w rocket and launching pad = 5mi

height = y = 4mi

$\frac{dy}{dt}$ = ? (rate of rising)



distance from radar is increasing at rate = $\frac{dx}{dt}$ = 2000 mi/h

Sol:

$x^2 = y^2 + (5)^2$

$x^2 = (4)^2 + (5)^2$

$x^2 = 16 + 25$

$x = \sqrt{41}$ mi

Now

differentiating w.r.t 't'

$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt} + 0$

$\cancel{2}(\sqrt{41})(2000) = \cancel{2}(4) \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = 3201.56 \text{ mi/h}}$

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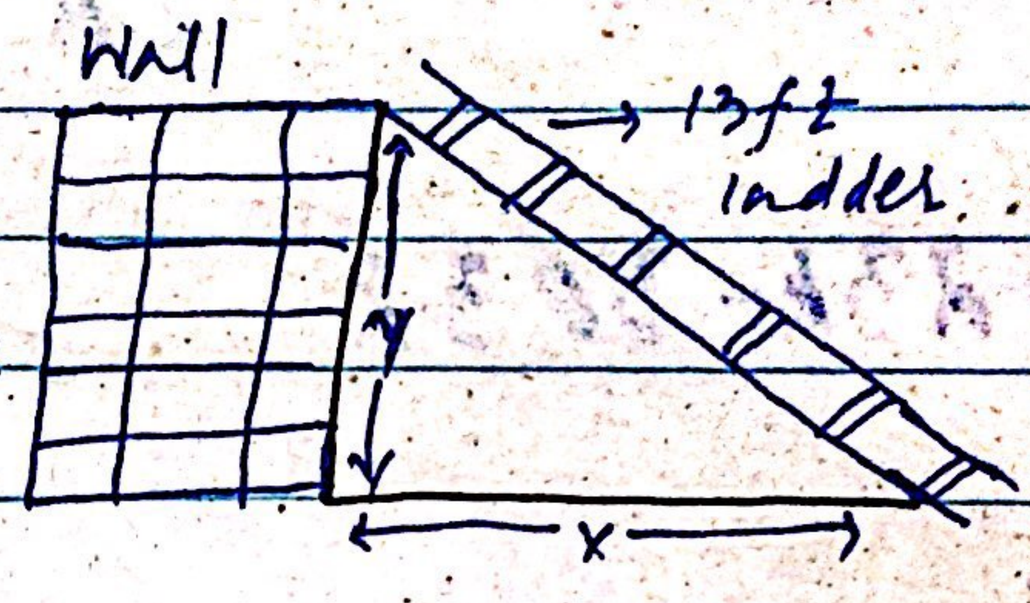
Q4

length of ladder = 13 ft

ladder slipping rate = $\frac{dy}{dt} = -2 ft/s$



$y = 5 ft$

$\frac{dx}{dt} = ?$

Sol

$x^2 + y^2 = (13)^2$

Now differentiating w.r.t 't'

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$2x \cdot \frac{dx}{dt} + 2(5)(-2) = 0 \quad -(i)$

Now; for x

$x^2 + y^2 = (13)^2$

$x^2 + (5)^2 = (13)^2$

$x^2 = 169 - 25$

$x^2 = 144$

$\boxed{x = 12 ft}$

Now

$(i) \Rightarrow 2(12)\frac{dx}{dt} - 20 = 0$

$\frac{dx}{dt} = \frac{20}{24}$

$\boxed{\frac{dx}{dt} = 0.833 \text{ ft/second}}$

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Ex: 7.1 (any 10 qs)

Q15 $\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}} \cdot dx$ —(i)

let $u = \sqrt{x-1}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x-1}}$

$dx = 2du\sqrt{x-1}$

$dx = 2\sqrt{u} \cdot du$

therefore;

(i) $\Rightarrow \int \frac{e^u}{\sqrt{u}} \cdot 2\sqrt{u} \cdot du$

$= 2\int e^u \cdot du$

$= 2e^u + c$

$= 2e^{\sqrt{x-1}} + c$

Q16 $\int (x+1)\cot(x^2+2x)\,dx$ —(i)

let $u = x^2 + 2x$

$\frac{du}{dx} = 2x + 2$

$dx = \frac{du}{2x+2}$

Therefore; (i) $\Rightarrow \int (x+1)\cot u \cdot \frac{du}{2x+2}$

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$= \int \cancel{(x+1)} \cot u \cdot \frac{du}{2\cancel{(x+1)}}$

$= \frac{1}{2} \int \cot u \cdot du$ $\quad \because \int \cot u \cdot du = \ln|\sin u| + c$

$= \frac{1}{2} \ln|\sin u| + c$

$= \frac{1}{2} \ln \sin|x^2 + 2x| + c$ //

Qn 7 $\int \frac{\cosh\sqrt{x}}{\sqrt{x}} dx$ —(i)

let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x} \cdot du$

$dx = 2u \cdot du$

Therefore;

$(i) \Rightarrow \int \frac{\cosh u}{\cancel{u}} \cdot 2\cancel{u} \cdot du$

$= 2\int \cosh u \cdot du$

$= 2 \sinh u + c$

$= 2 \sinh \sqrt{x} + c$ //

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Q18 $\int \frac{dx}{x(\ln x)^2}$ = (i)

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x \cdot du$

Therefore;

(i) $\Rightarrow \int \frac{1}{\cancel{x}(u)^2} \cdot \cancel{x} \cdot du$

$= \int \frac{1}{u^2} \cdot du$

$= \int u^{-2} \cdot du$

$= \frac{u^{-2+1}}{-2+1} + c$

$= \frac{u^{-1}}{-1} + c$

$= \frac{-1}{u} + c$

$= \frac{-1}{\ln x} + c$ //

Q19 $\int \frac{dx}{\sqrt{x}\, 3^{\sqrt{x}}}$ = (i)

let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x} \cdot du$

$dx = 2u \cdot du$

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Therefore;

$(i) \Rightarrow \int \frac{1}{\cancel{u}\, 3^{u}} \cdot 2\cancel{u} \cdot du$

$= 2\int \frac{1}{3^{u}} \cdot du$

$= 2\int 3^{-u} \cdot du \qquad \because \int a^{u} dx = \frac{a^{u}}{\ln a} + c$

$= 2 \frac{3^{-u}}{\ln 3} (-1) + c$

$= \frac{-2 \cdot 3^{-u}}{\ln 3} + c$

$= \frac{-2}{\ln 3} \cdot 3^{-\sqrt{x}} + c$ //

d2) $\int \sec(\sin\theta) \tan(\sin\theta) \cos\theta \cdot d\theta$ — (i)

let $u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$d\theta = \cancel{\cos\theta}\, du / \cos\theta$

Therefore;

$(i) \Rightarrow \int \sec u \cdot \tan u \cancel{\cos\theta} \cdot \frac{du}{\cancel{\cos\theta}}$

$= \int (\sec u \cdot \tan u)\, du$

$\cancel{= \ln(\sec u + \tan u) +}$

$= \sec u + c$

$= \sec(\sin\theta) + c$ //

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d21 $\int \frac{\operatorname{csch}^2(2/x)}{x^2} \cdot dx$ —(i)

let $u = \frac{2}{x}$

$\frac{du}{dx} = \frac{-2}{x^2}$

$dx = \frac{-x^2 \cdot du}{2}$

Therefore;

(i) $\Rightarrow \int \frac{\operatorname{csch}^2 u}{\cancel{x^2}} \cdot \frac{-\cancel{x^2} \cdot du}{2}$

$= \frac{-1}{2} \int \operatorname{csch}^2 u \cdot du$

$= \frac{-1}{2} (-\coth u) + c$

$= \frac{\coth u}{2} + c$

$= \frac{\coth(2/x)}{2} + c$

d22 $\int \frac{dx}{\sqrt{x^2 - 4}}$ —(i)

$\because \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + c$

$= \ln|x + \sqrt{x^2 - 4}| + c$

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Q23 $\int \frac{e^{-x}}{4-e^{-2x}}\,dx$ —(i)

let $u = e^{-x}$

$\frac{du}{dx} = -e^{-x}$

$dx = \frac{du}{-e^{-x}} \Rightarrow dx = \frac{du}{-u}$

Therefore;

(i) $\Rightarrow \int \frac{\cancel{u}}{4-u^2} \cdot \frac{du}{-\cancel{u}}$

$= -\int \frac{du}{4-u^2}$

$\because \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left(\frac{a+u}{a-u}\right) + c$

$= \frac{-1}{4}\ln\left(\frac{2+u}{2-u}\right) + c$

$= \frac{-1}{4}\ln\left(\frac{2+e^{-x}}{2-e^{-x}}\right) + c$ //

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d24 $\int \frac{\cos(\ln x)}{x} \cdot dx$ —(i)

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x \cdot du$

Therefore;

(i) $\Rightarrow \int \frac{\cos u}{\cancel{x}} \cdot \cancel{x} \cdot du$

$= \int \cos u \cdot dx$

$= \sin u + c$

$= \sin(\ln x) + c$ //

d25 $\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx$ —(i)

let $u = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x} \Rightarrow dx = \frac{du}{u}$

Therefore;

(i) $\Rightarrow \int \frac{\cancel{u}}{\sqrt{1-u^2}} \cdot \frac{du}{\cancel{u}}$

$= \int \frac{du}{\sqrt{1-u^2}}$

$\therefore \frac{du}{\sqrt{a^2 - x^2}} = \cancel{\ln|u + \sqrt{u^2 - a^2}| + c}\ \sin^{-1}\frac{u}{a} + c$

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$= \ln|u + \sqrt{u^2 - 1}| + c$

$= \ln|e^x + \sqrt{e^{2x} - 1}| + c$

$= \sin^{-1} u/1 + c$

$= \sin^{-1} \frac{e^x}{1} + c$

$= \sin^{-1} e^x + c$

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