

Course Code: MT-1003	Course Name: Calculus & Analytical Geometry
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Student Roll No:	Section No:

Instructions:

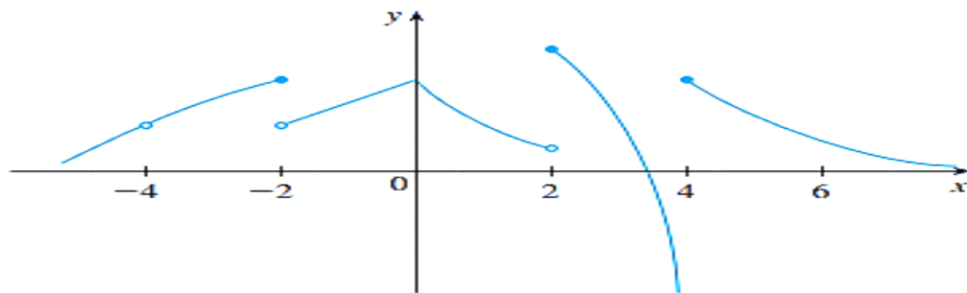
- Attempt all questions in the given sequence and return the question paper.
- Read each question completely before answering it and show all necessary step.
- There are **8 questions and 2 pages**. Graphical calculators are not allowed.

Time: 180 minutes

Max Marks: 100

Question 01: [CLO-03] [6 + 4 = 10]

- a) Let $f(x)$ be the function whose graph is shown below. State which condition of continuity does not hold at $x = \{-4, -2, 4\}$ and also state type of discontinuity.



- b) Find values of the constants k and m , if possible that will make the function $f(x)$ continuous everywhere

$$f(x) = \begin{cases} 2x^3 + x + 7 & x \leq -1 \\ m(x+1) + k & -1 < x \leq 2 \\ x^2 + 5 & x > 2 \end{cases}$$

Question 02: [CLO-03] [5 × 4 = 20]

Evaluate:

a) $\int_{-\infty}^{\infty} \frac{e^{4z}}{1+e^{8z}} dz$

b) $\int \frac{1}{\sin t - \cos t} dt$

c) $\int \frac{5x+6}{x^3-x} dx$

d) Derive the reduction formula for:

$$\int \cos^n x \, dx = ?$$

Question 03: [CLO-03] [03 + 03 + 04 = 10]

Evaluate the following.

a) $f'(z)$ if $f(z) = \sin(\cos(\tan z))$

b) $\frac{dy}{dx} \Big|_{(\frac{\pi}{2}, \frac{\pi}{4})}$ if $y \sin 2x = x \cos 2y$

c) $f'(t)$ if $f(t) = \frac{1-te^t}{t+e^t}$

Question 04: [CLO-04] [2.5 + 2.5 + 5 = 10]

a) Evaluate:

i. $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$

ii. $\lim_{x \rightarrow 0^+} x \ln x$

b) The minute hand of a certain clock is 4 inch long. Starting from the moment when the hand is pointing straight up, how fast is the area of the sector that is swept out by the hand increasing at any instant during the next revolution of the hand?

Question 05: [CLO-04] [2 × 5 = 10]

Let $f(x) = x^{\frac{2}{3}} \left(\frac{5}{2} - x \right)$, find

- a) The intervals in which function is increasing or decreasing
- b) Critical and Stationary points (explicitly mentioned all the points separately if any of them occur)
- c) Inflection points, if any
- d) The intervals in which function is concave up and concave down (if exist)
- e) Relative extrema

Question 06: [CLO-04] [5 × 2 = 10]

- a) Find the area between the curves $y = x$, $y = 4x$ and $y = -x + 2$. (Draw rough sketch)
- b) Find the volume between the curves $y = x^2 + 1$, $y = -x + 3$ revolving around x-axis.

Question 07: [CLO-05] [5 × 2 = 10]

Find whether sequence whose nth term appear converge or diverge.

a) $\left\{ \left(\frac{\pi}{4} \right)^n \right\}_{n=1}^{n=\infty}$

b) $\left\{ \sqrt{n^2 - 3n} - n \right\}_{n=1}^{n=\infty}$

Question 08: [CLO-05] [5 × 4 = 20]

Use any method to determine whether series is convergent or not.

a) $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$

c) $\sum_{k=1}^{\infty} \frac{5}{4^{k+1}}$

b) $\sum_{k=0}^{\infty} \frac{(k+4)!}{4!k!4^k}$

d) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$