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Q1

$x_1 = 40\,km = 40 \times 10^3\,m$

$\vec{v}_1 = 30\,km/h = 8.33\,m/s$

$x_2 = 40\,km = 40 \times 10^3\,m$

$\vec{v}_2 = 60\,km/h = 16.66\,m/s$

(a) average velocity = $\vec{v}_{avg}$ = ? (during full 80km trip)

(b) average speed = v = ?

(c) x-t graph

Sol:

(a) $x_1 = 40 \times 10^3\,m$, $\vec{v}_1 = 8.33\,m/s$

$\vec{v}_1 = \frac{x_1}{t_1} \Rightarrow t_1 = \frac{x_1}{\vec{v}_1} \Rightarrow t_1 = \frac{40 \times 10^3}{8.33}$

$t_1 = 4801.92\,s$ or 1.33 hours

$x_2 = 40 \times 10^3\,m$, $\vec{v}_2 = 16.66\,m/s$

$t_2 = \frac{x_2}{\vec{v}_2} \Rightarrow t_2 = \frac{40 \times 10^3}{16.66} \Rightarrow t = 2400.96\,s$ or 0.666 hours

average velocity = $\vec{v}_{avg} = \frac{x_1 + x_2}{t_1 + t_2}$

$= \frac{40 \times 10^3 + 40 \times 10^3}{4801.9 + 2400.9}$

$\boxed{\vec{v}_{avg} = 11.10\,m/s}$

total time = $1.333 + 0.666 = 1.99$ hours

or 7164 seconds

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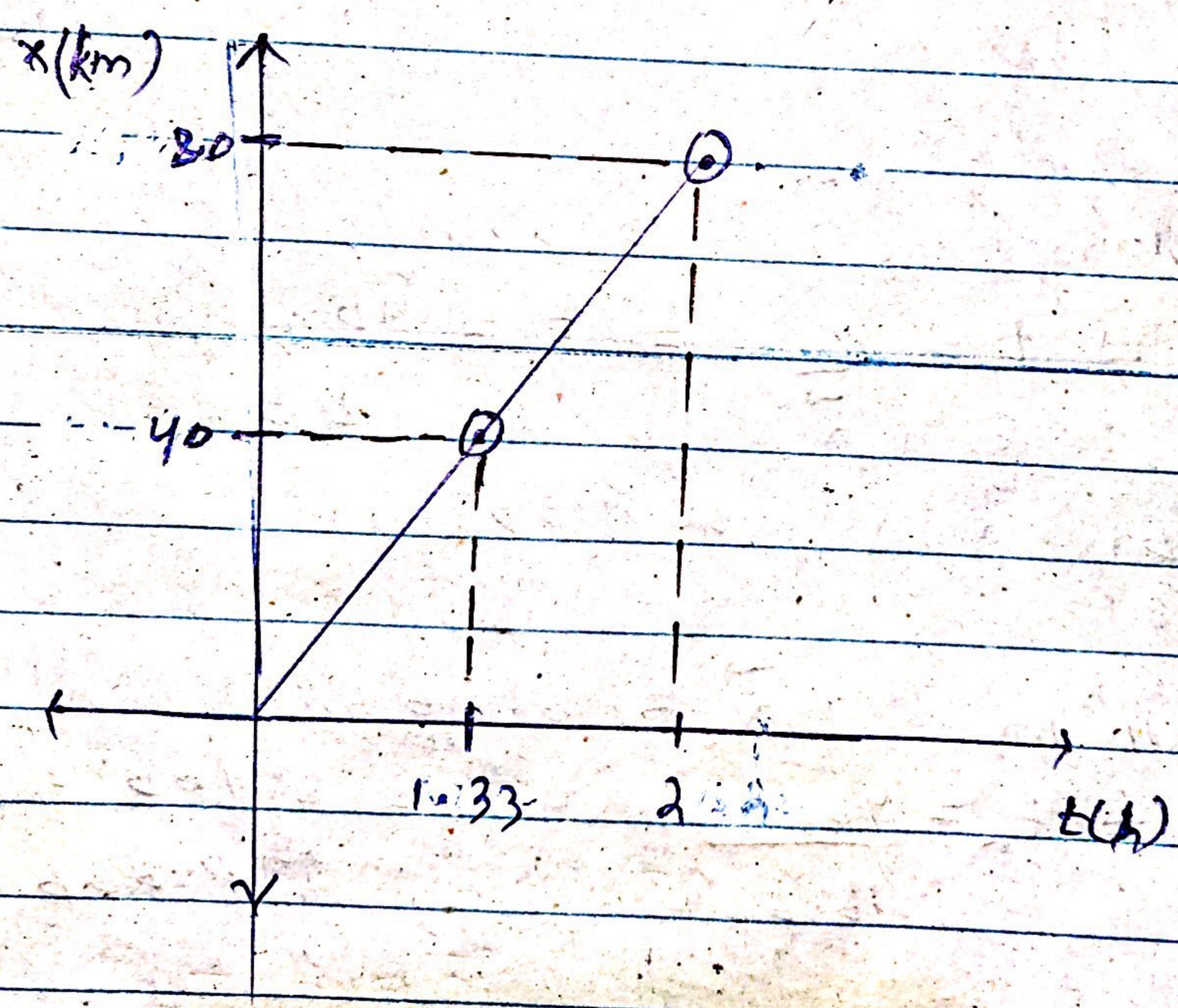
(b) average speed = $\frac{\text{total distance}}{\text{total time}}$

$= \frac{40\times10^3 + 40\times10^3}{4801.9 + 2400.9}$

$\boxed{\text{average speed} = 11.10\ m/s}$

(c) x – t graph



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Q2

initial velocity $= \vec{v}_i = 30 m/s$

time taken = ?

total time before hitting ground = ?

$g = -9.8 m/s^2$

Sol:-

We know that at max. height $\vec{v}_f = 0$

Therefore;

$$v_f = v_i + at$$

$$v_f = v_i + gt$$

$$0 = 30 + (-9.8)t$$

$$+30 = +9.8t$$

$$t = 3.06 \text{ seconds}$$

Now;

total time in air $= 3.06 + 3.06$

$= 6.12$ seconds

because time taken to reach max. height is same as returning back to the ground.

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Q3

$\vec{a} = ?$

$s = 50m$

$t = 5$ seconds

$\vec{v}_f = 16 m/s$

$\vec{v}_i = ?$

Sol:

$s = \vec{v}_i t - \frac{1}{2}\vec{a}t^2$

$50 = (16)(5) - \frac{1}{2}\vec{a}(5)^2$

$+30 = +\frac{25\vec{a}}{2}$

$+60 = 25\vec{a}$

$\vec{a} = \frac{60}{25}$

$\vec{a} = 2.4 m/s^2$

Now;

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$16 = \vec{v}_i + (2.4)(5)$

$16 - 12 = \vec{v}_i$

$\vec{v}_i = 4 m/s$

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Q4

position of an object $\Rightarrow x = 3t - 4t^2 + t^3$

position of object at

(a) $t = 1$, (b) $t = 2$, (c) $t = 3$, (d) $t = 4$,

(e) Object's displacement b/w $t = 0$ and $t = 4$ seconds.

(f) $\vec{v}_{avg}$ for time interval $t = 2$ to $t = 4$ seconds

(g) x-t graph

Sol:-

position of object

(a) at $t = 1$

$x_1 = 3t - 4t^2 + t^3$

$x_1 = 3(1) - 4(1)^2 + (1)^3$

$x_1 = 3 - 4 + 1$

$x_1 = 0$

(b) at $t = 2$

$x_2 = 3t - 4t^2 + t^3$

$x_2 = 3(2) - 4(2)^2 + (2)^3$

$x_2 = 6 - 16 + 8$

$x_2 = -2$

(c) at $t = 3$

$x_3 = 3(3) - 4(3)^2 + (3)^3$

$x_3 = 9 - 36 + 27$

$x_3 = 0$

(d) at $t = 4$

$x_4 = 3(4) - 4(4)^2 + (4)^3$

$x_4 = 12 - 64 + 64$

$x_4 = 12$

(e) Object's displacement b/w $t = 0$ and $t = 4$ seconds is

displacement $= (x_4) - (x_0)$

$= 12 - 0$

$= 12\,m$

(f) average velocity for $t = 2$ to $t = 4$ seconds

$\vec{v}_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_4 - x_2}{4 - 2} = \frac{12 - (-2)}{2} = \frac{12 + 2}{2}$

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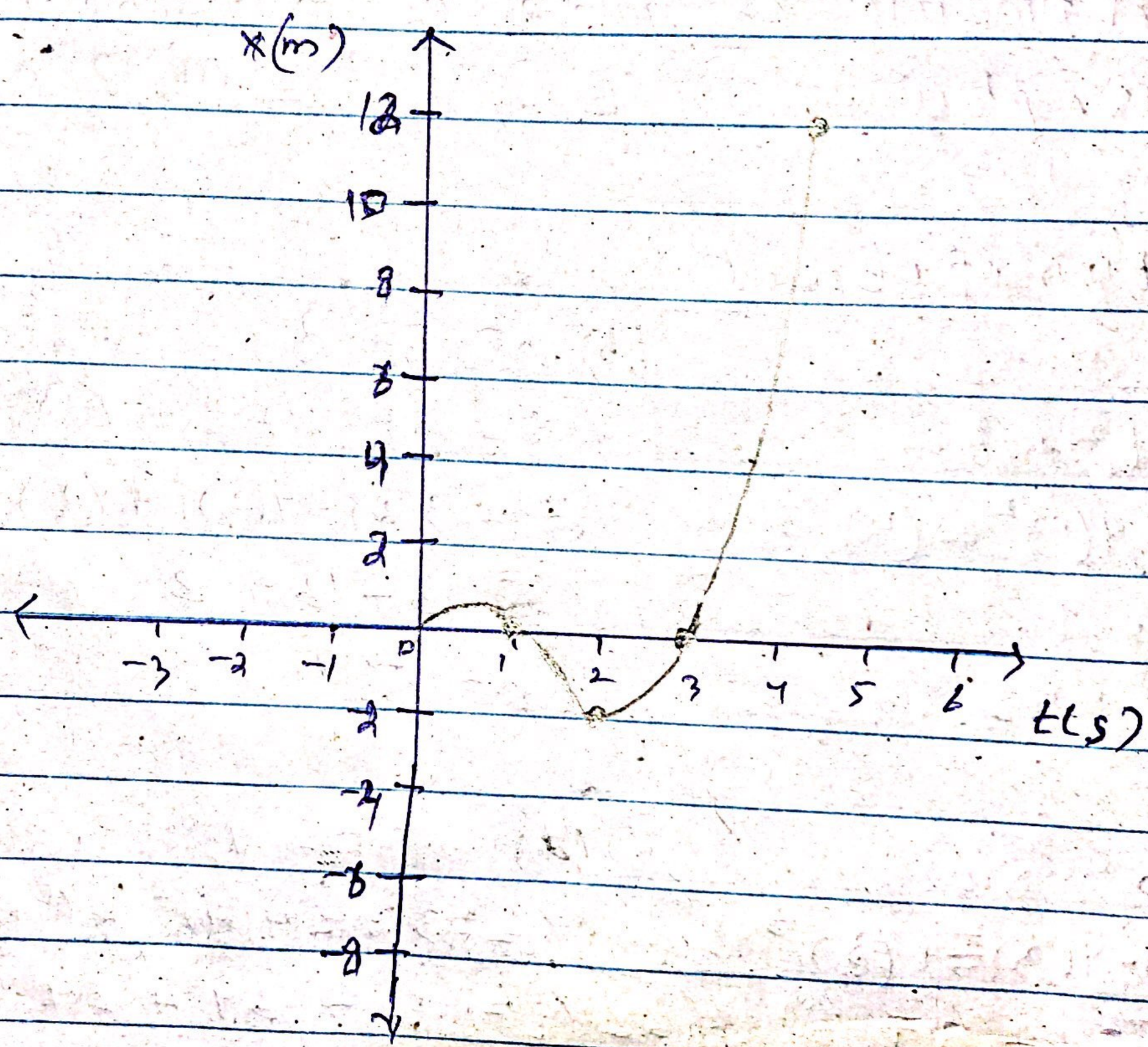
$\vec{v}_{avg} = \frac{14}{2}$

$\vec{v}_{avg} = 7 m/s$

(9)



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Q5 $x = 21t + 5t^2$

(a) $\vec{v}_{avg}$ during first three seconds of motion = ?

(b) instantaneous velocity of particle at t = 3 second = ?

(c) instantaneous acceleration of particle at t = 3 seconds = ?

Solⁿ:

(a) displacement at t = 0

$x = 21(0) + 5(0)^2$

$x = 0\,m$

displacement at t = 3

$x = 21(3) + 5(3)^2$

$x = 63 + 45$

$x = 108\,m$

Now,

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{108 - 0}{3 - 0}$$

$$= \frac{108}{3}$$

$$\vec{v}_x = 36\,m/s$$

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(b) $x = 21t + 5t^2$

$\frac{dx}{dt} = 21(1) + 5(2)t$

$\frac{dx}{dt} = 21 + 10t$

$\left(\frac{dx}{dt}\right)_{at\ t=3} = 21 + 10(3)$

$\vec{v} = 51\ m/s$

(c) $x = 21t + 5t^2$

$\frac{dx}{dt} = 21(1) + (5)(2)t$

$\frac{dx}{dt} = 21 + 10t$

$\frac{d^2x}{dt^2} = 0 + 10(1)$

$\left(\frac{d^2x}{dt^2}\right)_{at\ t=3} = 10\ m/s^2$

$\vec{a} = 10\ m/s^2$

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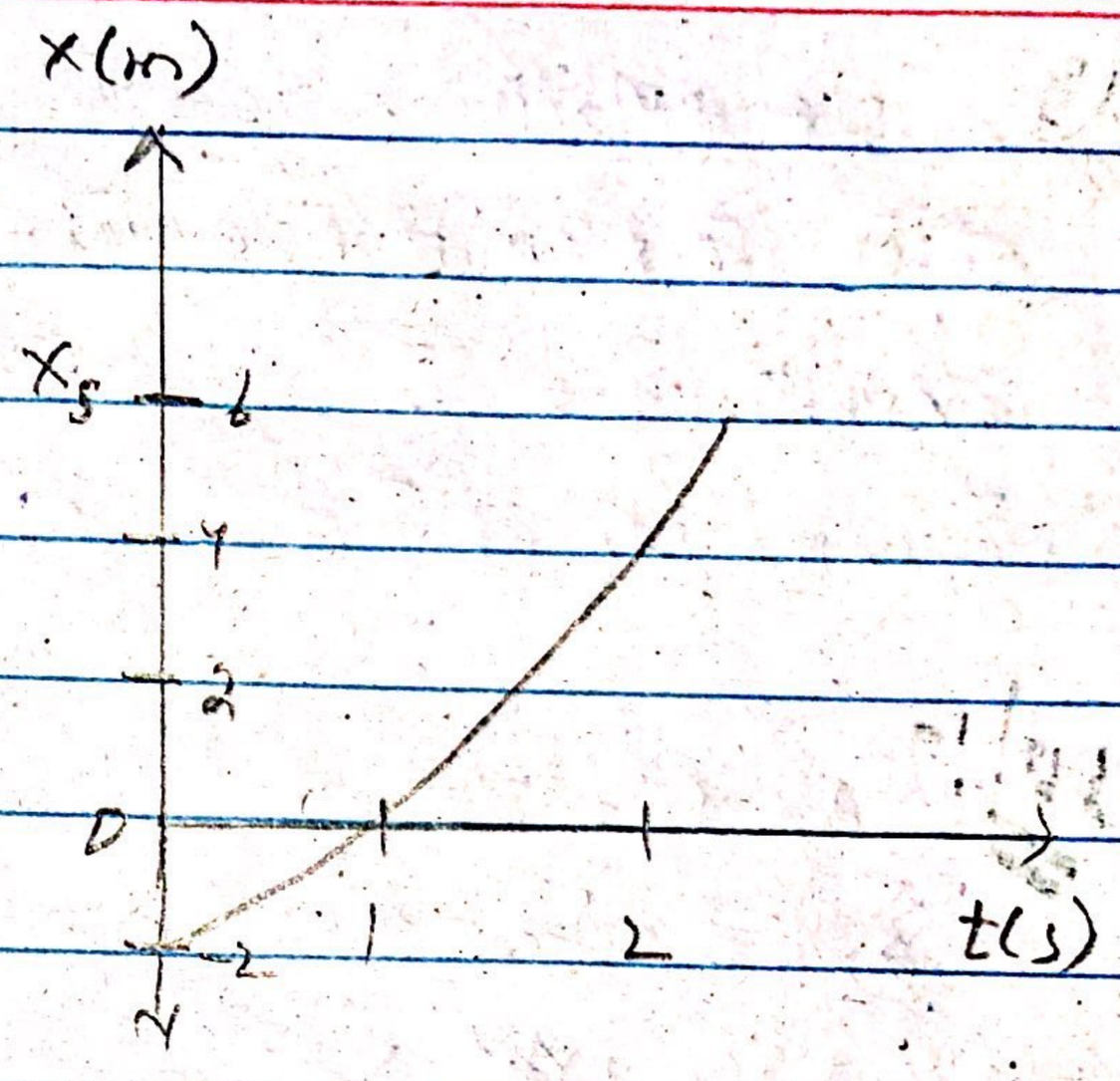
Q7

(a) magnitude of $\vec{a}$ = ?

(b) direction of $\vec{a}$ = ?



Sol:

(a) $x - x_0 = v_0 t + \frac{1}{2} a t^2$

$0 - (-2) = v_0(1) + \frac{1}{2} a (1)^2$

$2 = v_0 + \frac{1}{2} a \quad \text{--(1)}$

Now,

$x - x_0 = v_0 t + \frac{1}{2} a t^2$

$6 - (-2) = v_0(2) + \frac{1}{2} a (2)^2$

$\vec{v} = \frac{\Delta x}{\Delta t}$

$\vec{v} = \frac{6-2}{2-1}$

$6 + 2 = 2\vec{v}_0 + \frac{1}{2} \vec{a} (4)^2$

$8 = 2\vec{v}_0 + 2\vec{a}$

$8 = 2(\vec{v}_0 + \vec{a})$

$4 = \vec{v}_0 + \vec{a}$

$\vec{v}_0 = 4 - \vec{a}$

Now (1) ⇒

$2 = \vec{v}_0 + \frac{1}{2} \vec{a}$

$2 = (4 - \vec{a}) + \frac{1}{2} \vec{a}$

$2 = 4 - \vec{a} + \frac{\vec{a}}{2}$

$2 = \frac{8 - 2\vec{a} + \vec{a}}{2}$

$4 = 8 - \vec{a}$

$\vec{a} = 8 - 4$

$\vec{a} = 4 m/s^2$

(b) direction of $\vec{a}$ is positive.

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Q8 Initial position vector $= 5\hat{i} - 6\hat{j} + 2\hat{k}$

after 10 s position vector $= -2\hat{i} + 8\hat{j} - 2\hat{k}$

$\vec{v}_{avg} = ?$

$\vec{a}_{avg} = ?$

Sol:-

$$\vec{v}_{avg} = \frac{r_2 - r_1}{\Delta t}$$

$$= \frac{(-2\hat{i} + 8\hat{j} - 2\hat{k}) - (5\hat{i} - 6\hat{j} + 2\hat{k})}{10}$$

$$= \frac{-7\hat{i} + 14\hat{j} - 4\hat{k}}{10}$$

$$\vec{a}_{avg} = \frac{\vec{v}_{avg}}{\Delta t} = \frac{\frac{-7\hat{i} + 14\hat{j} - 4\hat{k}}{10}}{10}$$

$$\vec{a}_{avg} = \frac{-7\hat{i} + 14\hat{j} - 4\hat{k}}{100}$$

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Q9 $\vec{r} = (2t^3 - 5t)\hat{i} - (6 - 7t^4)\hat{j}$

(a) r

(b) v

(c) $\vec{a}$ for $t = 2$ seconds

(d) angle b/w +ve x and tangent to particle path at $t = 2$ seconds

Sol:-

(a) $\vec{r} = [2(2)^3 - 5(2)]\hat{i} - [6 - 7(2)^4]\hat{j}$

$= (16 - 10)\hat{i} - (6 - 112)\hat{j}$

$= 6\hat{i} + 106\hat{j}$

(b) $\frac{dr}{dt} = (3t^2 - 5)\hat{i} - (0 - 28t^3)\hat{j}$

$\left(\frac{dr}{dt}\right)_{at\ t=2} = [3(2)^2 - 5]\hat{i} + [28(2)^3]\hat{j}$

$\vec{v} = 19\hat{i} + 224\hat{j}$

(c) $\frac{d^2r}{dt^2} = (12t - 0)\hat{i} + 84t^2\hat{j}$

$\left(\frac{d^2r}{dt^2}\right)_{at\ t=2} = [12(2)]\hat{i} + [84(2)^2]\hat{j}$

$\vec{a} = 24\hat{i} + 336\hat{j}$

(d) $\tan\theta = \frac{v_y}{v_x} \Rightarrow \tan\theta = \left(\frac{224}{19}\right)$

$\theta = 85.151^\circ$

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Q10

height = 9.1 m

$\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ m/s

a) max. height = ? b) total horizontal distance = ?

c) magnitude of $\vec{v}$ = ? d) angle of ball's velocity = ?

Sol:

(a) $2as = v_f^2 - v_i^2$

$2gh = v_y^2 - v_{0y}^2$

$2(9.8)(9.1) + v_{0y}^2 = +v_y^2$

$2(9.8)(9.1) + (6.1)^2 = +v_y^2$

$v_y^2 = 215.57$

$v_y = 17.68$ m/s

$2gh = v_y^2 - v_{0y}^2$

$2(9.8)h = \cancel{(0)^2} - (17.68)^2 - (0)^2$

$h = 10.99$ m

(b) $R = \frac{2v_{0x}v_{0y}}{g} = \frac{2(7.6)(14.68)}{(9.8)}$

$R = 22.76$ m

(c) $|\vec{v}| = \sqrt{(7.6)^2 + (14.7)^2}$

$|\vec{v}| = 16.54$ m/s

(d) $\theta = \tan^{-1}\left(\frac{14.68}{7.6}\right)$

$\theta = 62.66°$

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