Limits and Continuity

Exercise Set 1.1

1. **(a)** 3 **(b)** 3 **(c)** 3 **(d)** 3

3. **(a)** -1 **(b)** 3 **(c)** does not exist **(d)** 1

5. **(a)** 0 **(b)** 0 **(c)** 0 **(d)** 3

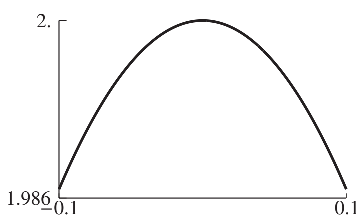
7. **(a)** $-\infty$ **(b)** $-\infty$ **(c)** $-\infty$ **(d)** 1

9. **(a)** $+\infty$ **(b)** $+\infty$ **(c)** 2 **(d)** 2 **(e)** $-\infty$ **(f)** $x = -2$, $x = 0$, $x = 2$

11. **(i)**

-0.1	-0.01	-0.001	0.001	0.01	0.1
1.9866933	1.9998667	1.9999987	1.9999987	1.9998667	1.9866933

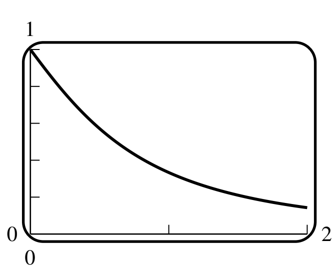
(ii)



The limit appears to be 2.

13. **(a)**

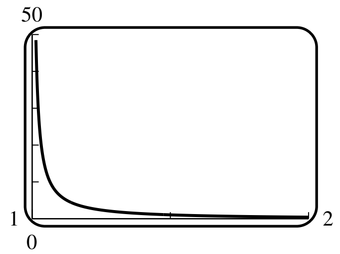
2	1.5	1.1	1.01	1.001	0	0.5	0.9	0.99	0.999
0.1429	0.2105	0.3021	0.3300	0.3330	1.0000	0.5714	0.3690	0.3367	0.3337



The limit is 1/3.

(b)

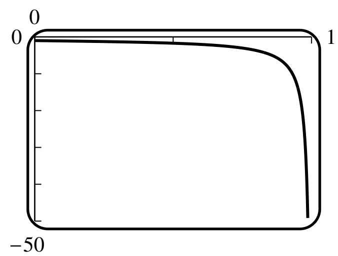
2	1.5	1.1	1.01	1.001	1.0001
0.4286	1.0526	6.344	66.33	666.3	6666.3



The limit is $+\infty$.

(c)

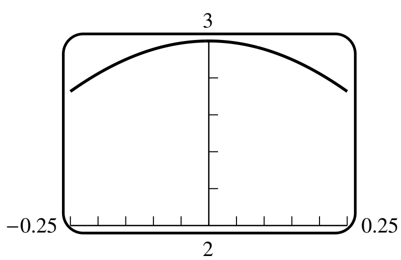
0	0.5	0.9	0.99	0.999	0.9999
-1	-1.7143	-7.0111	-67.001	-667.0	-6667.0



The limit is $-\infty$.

15. (a)

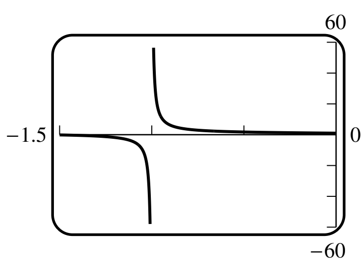
-0.25	-0.1	-0.001	-0.0001	0.0001	0.001	0.1	0.25
2.7266	2.9552	3.0000	3.0000	3.0000	3.0000	2.9552	2.7266



The limit is 3.

(b)

0	-0.5	-0.9	-0.99	-0.999	-1.5	-1.1	-1.01	-1.001
1	1.7552	6.2161	54.87	541.1	-0.1415	-4.536	-53.19	-539.5



The limit does not exist.

17. False; define $f(x) = x$ for $x \neq a$ and $f(a) = a + 1$. Then $\lim_{x\to a} f(x) = a \neq f(a) = a + 1$.

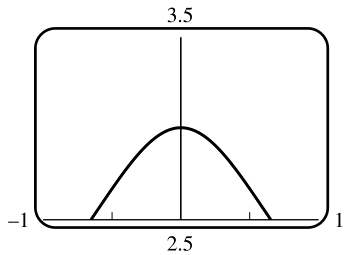
19. False; define $f(x) = 0$ for $x < 0$ and $f(x) = x + 1$ for $x \geq 0$. Then the left and right limits exist but are unequal.

27. $m_{\text{sec}} = \dfrac{x^2 - 1}{x + 1} = x - 1$ which gets close to -2 as x gets close to -1, thus $y - 1 = -2(x + 1)$ or $y = -2x - 1$.

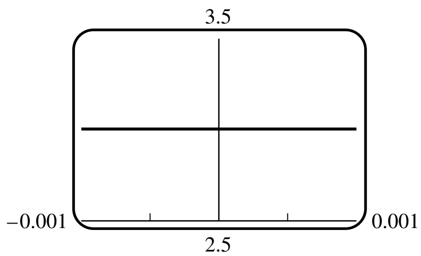
29. $m_{\text{sec}} = \dfrac{x^4 - 1}{x - 1} = x^3 + x^2 + x + 1$ which gets close to 4 as x gets close to 1, thus $y - 1 = 4(x - 1)$ or $y = 4x - 3$.

31. (a) The length of the rod while at rest.

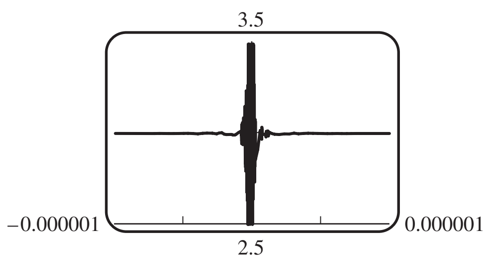
(b) The limit is zero. The length of the rod approaches zero as its speed approaches c.

33. (a) 

The limit appears to be 3.

(b) 

The limit appears to be 3.

(c) 

The limit does not exist.

Exercise Set 1.2

1. (a) By Theorem 1.2.2, this limit is $2 + 2 \cdot (-4) = -6$.

(b) By Theorem 1.2.2, this limit is $0 - 3 \cdot (-4) + 1 = 13$.

(c) By Theorem 1.2.2, this limit is $2 \cdot (-4) = -8$.

(d) By Theorem 1.2.2, this limit is $(-4)^2 = 16$.

(e) By Theorem 1.2.2, this limit is $\sqrt[3]{6+2} = 2$.

(f) By Theorem 1.2.2, this limit is $\dfrac{2}{(-4)} = -\dfrac{1}{2}$.

3. By Theorem 1.2.3, this limit is $2 \cdot 1 \cdot 3 = 6$.

5. By Theorem 1.2.4, this limit is $(3^2 - 2 \cdot 3)/(3+1) = 3/4$.

7. After simplification, $\dfrac{x^4-1}{x-1} = x^3 + x^2 + x + 1$, and the limit is $1^3 + 1^2 + 1 + 1 = 4$.

9. After simplification, $\dfrac{x^2+6x+5}{x^2-3x-4} = \dfrac{x+5}{x-4}$, and the limit is $(-1+5)/(-1-4) = -4/5$.

11. After simplification, $\dfrac{2x^2+x-1}{x+1} = 2x - 1$, and the limit is $2 \cdot (-1) - 1 = -3$.

13. After simplification, $\dfrac{t^3+3t^2-12t+4}{t^3-4t} = \dfrac{t^2+5t-2}{t^2+2t}$, and the limit is $(2^2+5\cdot 2-2)/(2^2+2\cdot 2) = 3/2$.

15. The limit is $+\infty$.

17. The limit does not exist.

19. The limit is $-\infty$.

21. The limit is $+\infty$.

23. The limit does not exist.

25. The limit is $+\infty$.

27. The limit is $+\infty$.

29. After simplification, $\dfrac{x-9}{\sqrt{x}-3} = \sqrt{x}+3$, and the limit is $\sqrt{9}+3=6$.

31. **(a)** 2 **(b)** 2 **(c)** 2

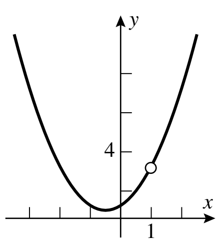
33. True, by Theorem 1.2.2.

35. False; e.g. $f(x)=2x$, $g(x)=x$, so $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} g(x) = 0$, but $\lim\limits_{x\to 0} f(x)/g(x) = 2$.

37. After simplification, $\dfrac{\sqrt{x+4}-2}{x} = \dfrac{1}{\sqrt{x+4}+2}$, and the limit is $1/4$.

39. **(a)** After simplification, $\dfrac{x^3-1}{x-1} = x^2+x+1$, and the limit is 3.

(b)



41. **(a)** Theorem 1.2.2 doesn't apply; moreover one cannot subtract infinities.

(b) $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{x^2}\right) = \lim\limits_{x\to 0^+}\left(\dfrac{x-1}{x^2}\right) = -\infty$.

43. For $x \neq 1$, $\dfrac{1}{x-1} - \dfrac{a}{x^2-1} = \dfrac{x+1-a}{x^2-1}$ and for this to have a limit it is necessary that $\lim\limits_{x\to 1}(x+1-a)=0$, i.e. $a=2$. For this value, $\dfrac{1}{x-1}-\dfrac{2}{x^2-1} = \dfrac{x+1-2}{x^2-1} = \dfrac{x-1}{x^2-1} = \dfrac{1}{x+1}$ and $\lim\limits_{x\to 1}\dfrac{1}{x+1} = \dfrac{1}{2}$.

45. The left and/or right limits could be plus or minus infinity; or the limit could exist, or equal any preassigned real number. For example, let $q(x)=x-x_0$ and let $p(x)=a(x-x_0)^n$ where n takes on the values $0,1,2$.

47. Clearly, $g(x) = [f(x)+g(x)] - f(x)$. By Theorem 1.2.2, $\lim\limits_{x\to a}[f(x)+g(x)] - \lim\limits_{x\to a} f(x) = \lim\limits_{x\to a}[f(x)+g(x)-f(x)] = \lim\limits_{x\to a} g(x)$.

Exercise Set 1.3

1. **(a)** $-\infty$ **(b)** $+\infty$

3. **(a)** 0 **(b)** -1

5. **(a)** $3+3\cdot(-5)=-12$ **(b)** $0-4\cdot(-5)+1=21$ **(c)** $3\cdot(-5)=-15$ **(d)** $(-5)^2=25$

(e) $\sqrt[3]{5+3}=2$ **(f)** $3/(-5)=-3/5$ **(g)** 0

(h) The limit doesn't exist because the denominator tends to zero but the numerator doesn't.

7.

x	10	100	1000	10000	100000	1000000
$f(x)$	0.953463	0.995037	0.999500	0.999950	0.999995	0.9999995

The limit appears to be 1.

9. The limit is $-\infty$, by the highest degree term.

11. The limit is $+\infty$.

13. The limit is $3/2$, by the highest degree terms.

15. The limit is 0, by the highest degree terms.

17. The limit is 0, by the highest degree terms.

19. The limit is $-\infty$, by the highest degree terms.

21. The limit is $-1/7$, by the highest degree terms.

23. The limit is $\sqrt[3]{-5/8}=-\sqrt[3]{5}/2$, by the highest degree terms.

25. $\dfrac{\sqrt{5x^2-2}}{x+3}=\dfrac{\sqrt{5-\frac{2}{x^2}}}{-1-\frac{3}{x}}$ when $x<0$. The limit is $-\sqrt{5}$.

27. $\dfrac{2-y}{\sqrt{7+6y^2}}=\dfrac{-\frac{2}{y}+1}{\sqrt{\frac{7}{y^2}+6}}$ when $y<0$. The limit is $1/\sqrt{6}$.

29. $\dfrac{\sqrt{3x^4+x}}{x^2-8}=\dfrac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$ when $x<0$. The limit is $\sqrt{3}$.

31. $\lim\limits_{x\to+\infty}(\sqrt{x^2+3}-x)\dfrac{\sqrt{x^2+3}+x}{\sqrt{x^2+3}+x}=\lim\limits_{x\to+\infty}\dfrac{3}{\sqrt{x^2+3}+x}=0$, by the highest degree terms.

33. False; if $x/2>1000$ then $1000x<x^2/2, x^2-1000x>x^2/2$, so the limit is $+\infty$.

35. True: for example $f(x)=\sin x/x$ crosses the x-axis infinitely many times at $x=n\pi, n=1,2,\ldots$.

37. It appears that $\lim\limits_{t\to+\infty}n(t)=+\infty$, and $\lim\limits_{t\to+\infty}e(t)=c$.

39. **(a)** $+\infty$ **(b)** -5

41. $\lim\limits_{x\to-\infty}p(x)=+\infty$. When n is even, $\lim\limits_{x\to+\infty}p(x)=+\infty$; when n is odd, $\lim\limits_{x\to+\infty}p(x)=-\infty$.

43. **(a)** No. **(b)** Yes, $\tan x$ and $\sec x$ at $x=n\pi+\pi/2$ and $\cot x$ and $\csc x$ at $x=n\pi, n=0,\pm1,\pm2,\ldots$.

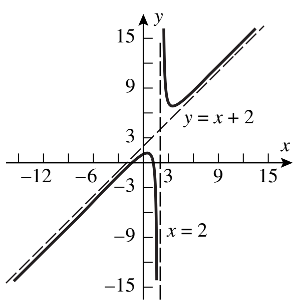
45. **(a)** If $f(t) \to +\infty$ (resp. $f(t) \to -\infty$) then $f(t)$ can be made arbitrarily large (resp. small) by taking t large enough. But by considering the values $g(x)$ where $g(x) > t$, we see that $f(g(x))$ has the limit $+\infty$ too (resp. limit $-\infty$). If $f(t)$ has the limit L as $t \to +\infty$ the values $f(t)$ can be made arbitrarily close to L by taking t large enough. But if x is large enough then $g(x) > t$ and hence $f(g(x))$ is also arbitrarily close to L.

(b) For $\lim\limits_{x\to-\infty}$ the same argument holds with the substitutiion "x decreases without bound" instead of "x increases without bound". For $\lim\limits_{x\to c^-}$ substitute "x close enough to $c, x < c$", etc.

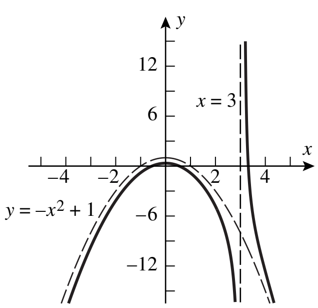
47. $t = 1/x, \lim\limits_{t\to+\infty} f(t) = +\infty$.

49. $t = \csc x, \lim\limits_{t\to+\infty} f(t) = +\infty$.

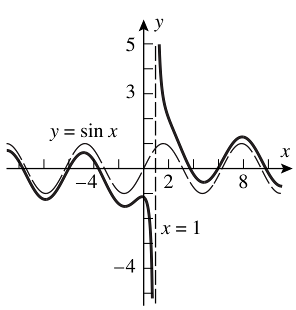
51. After a long division, $f(x) = x + 2 + \dfrac{2}{x-2}$, so $\lim\limits_{x\to\pm\infty}(f(x) - (x+2)) = 0$ and $f(x)$ is asymptotic to $y = x + 2$. The only vertical asymptote is at $x = 2$.



53. After a long division, $f(x) = -x^2+1+\dfrac{2}{x-3}$, so $\lim\limits_{x\to\pm\infty}(f(x)-(-x^2+1)) = 0$ and $f(x)$ is asymptotic to $y = -x^2+1$. The only vertical asymptote is at $x = 3$.



55. $\lim\limits_{x\to\pm\infty}(f(x) - \sin x) = 0$ so $f(x)$ is asymptotic to $y = \sin x$. The only vertical asymptote is at $x = 1$.



Exercise Set 1.4

1. **(a)** $|f(x) - f(0)| = |x + 2 - 2| = |x| < 0.1$ if and only if $|x| < 0.1$.

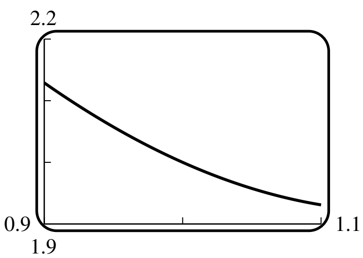
(b) $|f(x) - f(3)| = |(4x - 5) - 7| = 4|x - 3| < 0.1$ if and only if $|x - 3| < (0.1)/4 = 0.025$.

(c) $|f(x) - f(4)| = |x^2 - 16| < \epsilon$ if $|x - 4| < \delta$. We get $f(x) = 16 + \epsilon = 16.001$ at $x = 4.000124998$, which corresponds to $\delta = 0.000124998$; and $f(x) = 16 - \epsilon = 15.999$ at $x = 3.999874998$, for which $\delta = 0.000125002$. Use the smaller δ: thus $|f(x) - 16| < \epsilon$ provided $|x - 4| < 0.000125$ (to six decimals).

3. **(a)** $x_0 = (1.95)^2 = 3.8025, x_1 = (2.05)^2 = 4.2025$.

(b) $\delta = \min\left(|4 - 3.8025|, |4 - 4.2025| \right) = 0.1975$.

5. $|(x^3-4x+5)-2| < 0.05$ is equivalent to $-0.05 < (x^3-4x+5)-2 < 0.05$, which means $1.95 < x^3-4x+5 < 2.05$. Now $x^3-4x+5 = 1.95$ at $x = 1.0616$, and $x^3-4x+5 = 2.05$ at $x = 0.9558$. So $\delta = \min(1.0616 - 1, 1 - 0.9558) = 0.0442$.



7. With the TRACE feature of a calculator we discover that (to five decimal places) $(0.87000, 1.80274)$ and $(1.13000, 2.19301)$ belong to the graph. Set $x_0 = 0.87$ and $x_1 = 1.13$. Since the graph of $f(x)$ rises from left to right, we see that if $x_0 < x < x_1$ then $1.80274 < f(x) < 2.19301$, and therefore $1.8 < f(x) < 2.2$. So we can take $\delta = 0.13$.

9. $|2x - 8| = 2|x - 4| < 0.1$ when $|x - 4| < 0.1/2 = 0.05 = \delta$.

11. If $x \neq 3$, then $\left|\dfrac{x^2 - 9}{x - 3} - 6\right| = \left|\dfrac{x^2 - 9 - 6x + 18}{x - 3}\right| = \left|\dfrac{x^2 - 6x + 9}{x - 3}\right| = |x - 3| < 0.05$ when $|x - 3| < 0.05 = \delta$.

13. Assume $\delta \leq 1$. Then $-1 < x - 2 < 1$ means $1 < x < 3$ and then $|x^3 - 8| = |(x - 2)(x^2 + 2x + 4)| < 19|x - 2|$, so we can choose $\delta = 0.001/19$.

15. Assume $\delta \leq 1$. Then $-1 < x - 5 < 1$ means $4 < x < 6$ and then $\left|\dfrac{1}{x} - \dfrac{1}{5}\right| = \left|\dfrac{x - 5}{5x}\right| < \dfrac{|x - 5|}{20}$, so we can choose $\delta = 0.05 \cdot 20 = 1$.

17. Let $\epsilon > 0$ be given. Then $|f(x) - 3| = |3 - 3| = 0 < \epsilon$ regardless of x, and hence any $\delta > 0$ will work.

19. $|3x - 15| = 3|x - 5| < \epsilon$ if $|x - 5| < \epsilon/3$, $\delta = \epsilon/3$.

21. $\left|\dfrac{2x^2 + x}{x} - 1\right| = |2x| < \epsilon$ if $|x| < \epsilon/2$, $\delta = \epsilon/2$.

23. $|f(x) - 3| = |x + 2 - 3| = |x - 1| < \epsilon$ if $0 < |x - 1| < \epsilon$, $\delta = \epsilon$.

25. If $\epsilon > 0$ is given, then take $\delta = \epsilon$; if $|x - 0| = |x| < \delta$, then $|x - 0| = |x| < \epsilon$.

27. For the first part, let $\epsilon > 0$. Then there exists $\delta > 0$ such that if $a < x < a + \delta$ then $|f(x) - L| < \epsilon$. For the left limit replace $a < x < a + \delta$ with $a - \delta < x < a$.

29. **(a)** $|(3x^2 + 2x - 20 - 300| = |3x^2 + 2x - 320| = |(3x + 32)(x - 10)| = |3x + 32| \cdot |x - 10|$.

(b) If $|x - 10| < 1$ then $|3x + 32| < 65$, since clearly $x < 11$.

(c) $\delta = \min(1, \epsilon/65)$; $\quad |3x+32| \cdot |x-10| < 65 \cdot |x-10| < 65 \cdot \epsilon/65 = \epsilon$.

31. If $\delta < 1$ then $|2x^2 - 2| = 2|x-1||x+1| < 6|x-1| < \epsilon$ if $|x-1| < \epsilon/6$, so $\delta = \min(1, \epsilon/6)$.

33. If $\delta < 1/2$ and $|x-(-2)| < \delta$ then $-5/2 < x < -3/2$, $x+1 < -1/2$, $|x+1| > 1/2$; then $\left|\dfrac{1}{x+1} - (-1)\right| = \dfrac{|x+2|}{|x+1|} < 2|x+2| < \epsilon$ if $|x+2| < \epsilon/2$, so $\delta = \min(1/2, \epsilon/2)$.

35. $|\sqrt{x} - 2| = \left|(\sqrt{x}-2)\dfrac{\sqrt{x}+2}{\sqrt{x}+2}\right| = \left|\dfrac{x-4}{\sqrt{x}+2}\right| < \dfrac{1}{2}|x-4| < \epsilon$ if $|x-4| < 2\epsilon$, so $\delta = \min(2\epsilon, 4)$.

37. Let $\epsilon > 0$ be given and take $\delta = \epsilon$. If $|x| < \delta$, then $|f(x) - 0| = 0 < \epsilon$ if x is rational, and $|f(x) - 0| = |x| < \delta = \epsilon$ if x is irrational.

39. **(a)** We have to solve the equation $1/N^2 = 0.1$ here, so $N = \sqrt{10}$.

(b) This will happen when $N/(N+1) = 0.99$, so $N = 99$.

(c) Because the function $1/x^3$ approaches 0 from below when $x \to -\infty$, we have to solve the equation $1/N^3 = -0.001$, and $N = -10$.

(d) The function $x/(x+1)$ approaches 1 from above when $x \to -\infty$, so we have to solve the equation $N/(N+1) = 1.01$. We obtain $N = -101$.

41. **(a)** $\dfrac{x_1^2}{1+x_1^2} = 1-\epsilon$, $x_1 = -\sqrt{\dfrac{1-\epsilon}{\epsilon}}$; $\quad \dfrac{x_2^2}{1+x_2^2} = 1-\epsilon$, $x_2 = \sqrt{\dfrac{1-\epsilon}{\epsilon}}$

(b) $N = \sqrt{\dfrac{1-\epsilon}{\epsilon}}$ $\qquad$ **(c)** $N = -\sqrt{\dfrac{1-\epsilon}{\epsilon}}$

43. $\dfrac{1}{x^2} < 0.01$ if $|x| > 10$, $N = 10$.

45. $\left|\dfrac{x}{x+1} - 1\right| = \left|\dfrac{1}{x+1}\right| < 0.001$ if $|x+1| > 1000$, $x > 999$, $N = 999$.

47. $\left|\dfrac{1}{x+2} - 0\right| < 0.005$ if $|x+2| > 200$, $-x-2 > 200$, $x < -202$, $N = -202$.

49. $\left|\dfrac{4x-1}{2x+5} - 2\right| = \left|\dfrac{11}{2x+5}\right| < 0.1$ if $|2x+5| > 110$, $-2x-5 > 110$, $2x < -115$, $x < -57.5$, $N = -57.5$.

51. $\left|\dfrac{1}{x^2}\right| < \epsilon$ if $|x| > \dfrac{1}{\sqrt{\epsilon}}$, so $N = \dfrac{1}{\sqrt{\epsilon}}$.

53. $\left|\dfrac{4x-1}{2x+5} - 2\right| = \left|\dfrac{11}{2x+5}\right| < \epsilon$ if $|2x+5| > \dfrac{11}{\epsilon}$, i.e. when $-2x-5 > \dfrac{11}{\epsilon}$, which means $2x < -\dfrac{11}{\epsilon} - 5$, or $x < -\dfrac{11}{2\epsilon} - \dfrac{5}{2}$, so $N = -\dfrac{5}{2} - \dfrac{11}{2\epsilon}$.

55. $\left|\dfrac{2\sqrt{x}}{\sqrt{x}-1} - 2\right| = \left|\dfrac{2}{\sqrt{x}-1}\right| < \epsilon$ if $\sqrt{x} - 1 > \dfrac{2}{\epsilon}$, i.e. when $\sqrt{x} > 1 + \dfrac{2}{\epsilon}$, or $x > \left(1 + \dfrac{2}{\epsilon}\right)^2$, so $N = \left(1 + \dfrac{2}{\epsilon}\right)^2$.

57. **(a)** $\dfrac{1}{x^2} > 100$ if $|x| < \dfrac{1}{10}$ $\qquad$ **(b)** $\dfrac{1}{|x-1|} > 1000$ if $|x-1| < \dfrac{1}{1000}$

(c) $\dfrac{-1}{(x-3)^2} < -1000$ if $|x-3| < \dfrac{1}{10\sqrt{10}}$ **(d)** $-\dfrac{1}{x^4} < -10000$ if $x^4 < \dfrac{1}{10000}$, $|x| < \dfrac{1}{10}$

59. If $M > 0$ then $\dfrac{1}{(x-3)^2} > M$ when $0 < (x-3)^2 < \dfrac{1}{M}$, or $0 < |x-3| < \dfrac{1}{\sqrt{M}}$, so $\delta = \dfrac{1}{\sqrt{M}}$.

61. If $M > 0$ then $\dfrac{1}{|x|} > M$ when $0 < |x| < \dfrac{1}{M}$, so $\delta = \dfrac{1}{M}$.

63. If $M < 0$ then $-\dfrac{1}{x^4} < M$ when $0 < x^4 < -\dfrac{1}{M}$, or $|x| < \dfrac{1}{(-M)^{1/4}}$, so $\delta = \dfrac{1}{(-M)^{1/4}}$.

65. If $x > 2$ then $|x+1-3| = |x-2| = x-2 < \epsilon$ if $2 < x < 2+\epsilon$, so $\delta = \epsilon$.

67. If $x > 4$ then $\sqrt{x-4} < \epsilon$ if $x-4 < \epsilon^2$, or $4 < x < 4+\epsilon^2$, so $\delta = \epsilon^2$.

69. If $x > 2$ then $|f(x)-2| = |x-2| = x-2 < \epsilon$ if $2 < x < 2+\epsilon$, so $\delta = \epsilon$.

71. **(a)** Definition: For every $M < 0$ there corresponds a $\delta > 0$ such that if $1 < x < 1+\delta$ then $f(x) < M$. In our case we want $\dfrac{1}{1-x} < M$, i.e. $1-x > \dfrac{1}{M}$, or $x < 1-\dfrac{1}{M}$, so we can choose $\delta = -\dfrac{1}{M}$.

(b) Definition: For every $M > 0$ there corresponds a $\delta > 0$ such that if $1-\delta < x < 1$ then $f(x) > M$. In our case we want $\dfrac{1}{1-x} > M$, i.e. $1-x < \dfrac{1}{M}$, or $x > 1-\dfrac{1}{M}$, so we can choose $\delta = \dfrac{1}{M}$.

73. **(a)** Given any $M > 0$, there corresponds an $N > 0$ such that if $x > N$ then $f(x) > M$, i.e. $x+1 > M$, or $x > M-1$, so $N = M-1$.

(b) Given any $M < 0$, there corresponds an $N < 0$ such that if $x < N$ then $f(x) < M$, i.e. $x+1 < M$, or $x < M-1$, so $N = M-1$.

75. **(a)** $\dfrac{3.0}{7.5} = 0.4$ (amperes) **(b)** $[0.3947, 0.4054]$ **(c)** $\left[\dfrac{3}{7.5+\delta}, \dfrac{3}{7.5-\delta}\right]$ **(d)** 0.0187

(e) It approaches infinity.

Exercise Set 1.5

1. **(a)** No: $\lim_{x\to 2} f(x)$ does not exist. **(b)** No: $\lim_{x\to 2} f(x)$ does not exist. **(c)** No: $\lim_{x\to 2^-} f(x) \neq f(2)$.

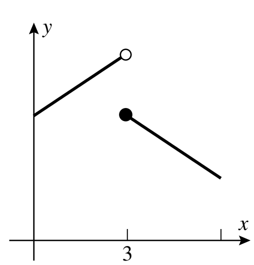
(d) Yes. **(e)** Yes. **(f)** Yes.

3. **(a)** No: $f(1)$ and $f(3)$ are not defined. **(b)** Yes. **(c)** No: $f(1)$ is not defined.

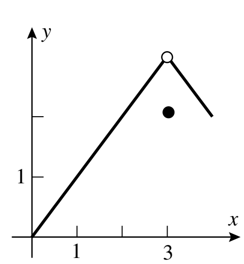
(d) Yes. **(e)** No: $f(3)$ is not defined. **(f)** Yes.

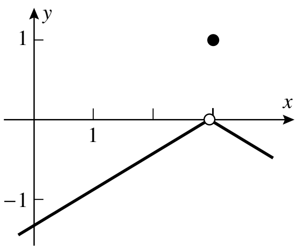
5. **(a)** No. **(b)** No. **(c)** No. **(d)** Yes. **(e)** Yes. **(f)** No. **(g)** Yes.

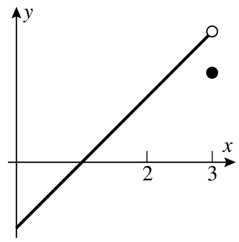
7. **(a)**

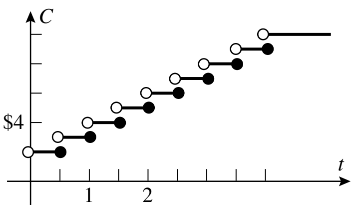


(b)



(c) 

(d) 

9. (a) 

(b) One second could cost you one dollar.

11. None, this is a continuous function on the real numbers.

13. None, this is a continuous function on the real numbers.

15. The function is not continuous at $x = -1/2$ and $x = 0$.

17. The function is not continuous at $x = 0$, $x = 1$ and $x = -1$.

19. None, this is a continuous function on the real numbers.

21. None, this is a continuous function on the real numbers. $f(x) = 2x + 3$ is continuous on $x < 4$ and $f(x) = 7 + \dfrac{16}{x}$ is continuous on $4 < x$; $\lim\limits_{x\to 4^-} f(x) = \lim\limits_{x\to 4^+} f(x) = f(4) = 11$ so f is continuous at $x = 4$.

23. True; by Theorem 1.5.5.

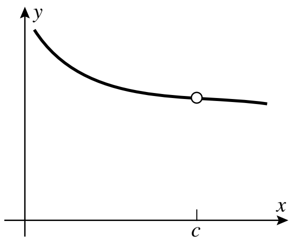
25. False; e.g. $f(x) = g(x) = 2$ if $x \neq 3$, $f(3) = 1$, $g(3) = 3$.

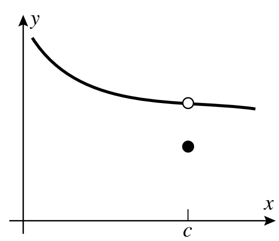
27. True; use Theorem 1.5.3 with $g(x) = \sqrt{f(x)}$.

29. (a) f is continuous for $x < 1$, and for $x > 1$; $\lim\limits_{x\to 1^-} f(x) = 5$, $\lim\limits_{x\to 1^+} f(x) = k$, so if $k = 5$ then f is continuous for all x.

(b) f is continuous for $x < 2$, and for $x > 2$; $\lim\limits_{x\to 2^-} f(x) = 4k$, $\lim\limits_{x\to 2^+} f(x) = 4 + k$, so if $4k = 4 + k$, $k = 4/3$ then f is continuous for all x.

31. f is continuous for $x < -1$, $-1 < x < 2$ and $x > 2$; $\lim\limits_{x\to -1^-} f(x) = 4$, $\lim\limits_{x\to -1^+} f(x) = k$, so $k = 4$ is required. Next, $\lim\limits_{x\to 2^-} f(x) = 3m + k = 3m + 4$, $\lim\limits_{x\to 2^+} f(x) = 9$, so $3m + 4 = 9, m = 5/3$ and f is continuous everywhere if $k = 4$ and $m = 5/3$.

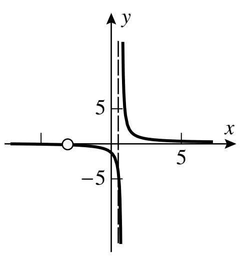
33. **(a)** 

(b) 

35. **(a)** $x = 0$, $\lim_{x \to 0^-} f(x) = -1 \neq +1 = \lim_{x \to 0^+} f(x)$ so the discontinuity is not removable.

(b) $x = -3$; define $f(-3) = -3 = \lim_{x \to -3} f(x)$, then the discontinuity is removable.

(c) f is undefined at $x = \pm 2$; at $x = 2$, $\lim_{x \to 2} f(x) = 1$, so define $f(2) = 1$ and f becomes continuous there; at $x = -2$, $\lim_{x \to -2} f(x)$ does not exist, so the discontinuity is not removable.

37. **(a)** 

Discontinuity at $x = 1/2$, not removable; at $x = -3$, removable.

(b) $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

39. Write $f(x) = x^{3/5} = (x^3)^{1/5}$ as the composition (Theorem 1.5.6) of the two continuous functions $g(x) = x^3$ and $h(x) = x^{1/5}$; it is thus continuous.

41. Since f and g are continuous at $x = c$ we know that $\lim_{x \to c} f(x) = f(c)$ and $\lim_{x \to c} g(x) = g(c)$. In the following we use Theorem 1.2.2.

(a) $f(c) + g(c) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = \lim_{x \to c}(f(x) + g(x))$ so $f + g$ is continuous at $x = c$.

(b) Same as (a) except the $+$ sign becomes a $-$ sign.

(c) $f(c)g(c) = \lim_{x \to c} f(x) \lim_{x \to c} g(x) = \lim_{x \to c} f(x)g(x)$ so fg is continuous at $x = c$.

43. **(a)** Let $h = x - c, x = h + c$. Then by Theorem 1.5.5, $\lim_{h \to 0} f(h + c) = f(\lim_{h \to 0}(h + c)) = f(c)$.

(b) With $g(h) = f(c+h)$, $\lim_{h \to 0} g(h) = \lim_{h \to 0} f(c+h) = f(c) = g(0)$, so $g(h)$ is continuous at $h = 0$. That is, $f(c+h)$ is continuous at $h = 0$, so f is continuous at $x = c$.

45. Of course such a function must be discontinuous. Let $f(x) = 1$ on $0 \le x < 1$, and $f(x) = -1$ on $1 \le x \le 2$.

47. If $f(x) = x^3 + x^2 - 2x - 1$, then $f(-1) = 1$, $f(1) = -1$. The Intermediate Value Theorem gives us the result.

49. For the negative root, use intervals on the x-axis as follows: $[-2, -1]$; since $f(-1.3) < 0$ and $f(-1.2) > 0$, the midpoint $x = -1.25$ of $[-1.3, -1.2]$ is the required approximation of the root. For the positive root use the interval $[0, 1]$; since $f(0.7) < 0$ and $f(0.8) > 0$, the midpoint $x = 0.75$ of $[0.7, 0.8]$ is the required approximation.

51. For the positive root, use intervals on the x-axis as follows: $[2, 3]$; since $f(2.2) < 0$ and $f(2.3) > 0$, use the interval $[2.2, 2.3]$. Since $f(2.23) < 0$ and $f(2.24) > 0$ the midpoint $x = 2.235$ of $[2.23, 2.24]$ is the required approximation of the root.

53. Consider the function $f(\theta) = T(\theta+\pi) - T(\theta)$. Note that T has period 2π, $T(\theta+2\pi) = T(\theta)$, so that $f(\theta+\pi) = T(\theta+2\pi) - T(\theta+\pi) = -(T(\theta+\pi) - T(\theta)) = -f(\theta)$. Now if $f(\theta) \equiv 0$, then the statement follows. Otherwise, there exists θ such that $f(\theta) \neq 0$ and then $f(\theta+\pi)$ has an opposite sign, and thus there is a t_0 between θ and $\theta+\pi$ such that $f(t_0) = 0$ and the statement follows.

55. Since R and L are arbitrary, we can introduce coordinates so that L is the x-axis. Let $f(z)$ be as in Exercise 54. Then for large z, $f(z) =$ area of ellipse, and for small z, $f(z) = 0$. By the Intermediate Value Theorem there is a z_1 such that $f(z_1) =$ half of the area of the ellipse.

57. For $x \geq 0, f$ is increasing and so is one-to-one. It is continuous everywhere and thus by Theorem 1.5.7 it has an inverse defined on its range $[5, +\infty)$ which is continuous there.

Exercise Set 1.6

1. This is a composition of continuous functions, so it is continuous everywhere.

3. Discontinuities at $x = n\pi, n = 0, \pm 1, \pm 2, \ldots$

5. Discontinuities at $x = n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

7. Discontinuities at $x = \dfrac{\pi}{6} + 2n\pi$, and $x = \dfrac{5\pi}{6} + 2n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

9. **(a)** $f(x) = \sin x$, $g(x) = x^3 + 7x + 1$. **(b)** $f(x) = |x|$, $g(x) = \sin x$. **(c)** $f(x) = x^3$, $g(x) = \cos(x+1)$.

11. $\displaystyle\lim_{x\to+\infty} \cos\left(\frac{1}{x}\right) = \cos\left(\lim_{x\to+\infty} \frac{1}{x}\right) = \cos 0 = 1.$

13. $\displaystyle\lim_{\theta\to 0} \frac{\sin 3\theta}{\theta} = 3\lim_{\theta\to 0} \frac{\sin 3\theta}{3\theta} = 3.$

15. $\displaystyle\lim_{x\to 0} \frac{x^2 - 3\sin x}{x} = \lim_{x\to 0} x - 3\lim_{x\to 0} \frac{\sin x}{x} = -3.$

17. $\displaystyle\lim_{\theta\to 0^+} \frac{\sin\theta}{\theta^2} = \left(\lim_{\theta\to 0^+} \frac{1}{\theta}\right) \lim_{\theta\to 0^+} \frac{\sin\theta}{\theta} = +\infty.$

19. $\displaystyle\frac{\tan 7x}{\sin 3x} = \frac{7}{3\cos 7x} \cdot \frac{\sin 7x}{7x} \cdot \frac{3x}{\sin 3x}$, so $\displaystyle\lim_{x\to 0} \frac{\tan 7x}{\sin 3x} = \frac{7}{3\cdot 1} \cdot 1 \cdot 1 = \frac{7}{3}.$

21. $\displaystyle\lim_{x\to 0^+} \frac{\sin x}{5\sqrt{x}} = \frac{1}{5} \lim_{x\to 0^+} \sqrt{x} \lim_{x\to 0^+} \frac{\sin x}{x} = 0.$

23. $\displaystyle\lim_{x\to 0} \frac{\sin x^2}{x} = \left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0} \frac{\sin x^2}{x^2}\right) = 0.$

25. $\displaystyle\frac{t^2}{1 - \cos^2 t} = \left(\frac{t}{\sin t}\right)^2$, so $\displaystyle\lim_{t\to 0} \frac{t^2}{1 - \cos^2 t} = 1.$

27. $\displaystyle\frac{\theta^2}{1 - \cos\theta} \cdot \frac{1 + \cos\theta}{1 + \cos\theta} = \frac{\theta^2(1 + \cos\theta)}{1 - \cos^2\theta} = \left(\frac{\theta}{\sin\theta}\right)^2 (1 + \cos\theta)$, so $\displaystyle\lim_{\theta\to 0} \frac{\theta^2}{1 - \cos\theta} = (1)^2 \cdot 2 = 2.$

29. $\displaystyle\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right) = \lim_{t\to+\infty} \sin t$, so the limit does not exist.

31. $\lim_{x\to 0} \frac{\tan ax}{\sin bx} = \lim_{x\to 0} \frac{a}{b} \frac{\sin ax}{ax} \frac{1}{\cos ax} \frac{bx}{\sin bx} = a/b.$

33. **(a)**

4	4.5	4.9	5.1	5.5	6
0.093497	0.100932	0.100842	0.098845	0.091319	0.076497

The limit appears to be 0.1.

(b) Let $t = x - 5$. Then $t \to 0$ as $x \to 5$ and $\lim_{x\to 5} \frac{\sin(x-5)}{x^2-25} = \lim_{x\to 5} \frac{1}{x+5} \lim_{t\to 0} \frac{\sin t}{t} = \frac{1}{10} \cdot 1 = \frac{1}{10}.$

35. True: let $\epsilon > 0$ and $\delta = \epsilon$. Then if $|x - (-1)| = |x+1| < \delta$ then $|f(x) + 5| < \epsilon$.

37. True; the functions $f(x) = x, g(x) = \sin x$, and $h(x) = 1/x$ are continuous everywhere except possibly at $x = 0$, so by Theorem 1.5.6 the given function is continuous everywhere except possibly at $x = 0$. We prove that $\lim_{x\to 0} x \sin(1/x) = 0$. Let $\epsilon > 0$. Then with $\delta = \epsilon$, if $|x| < \delta$ then $|x \sin(1/x)| \le |x| < \delta = \epsilon$, and hence f is continuous everywhere.

39. **(a)** The student calculated x in degrees rather than radians.

(b) $\sin x^\circ = \sin t$ where x° is measured in degrees, t is measured in radians and $t = \frac{\pi x^\circ}{180}$. Thus $\lim_{x^\circ\to 0} \frac{\sin x^\circ}{x^\circ} = \lim_{t\to 0} \frac{\sin t}{(180t/\pi)} = \frac{\pi}{180}.$

41. $\lim_{x\to 0^-} f(x) = k \lim_{x\to 0} \frac{\sin kx}{kx \cos kx} = k$, $\lim_{x\to 0^+} f(x) = 2k^2$, so $k = 2k^2$, and the nonzero solution is $k = \frac{1}{2}$.

43. **(a)** $\lim_{t\to 0^+} \frac{\sin t}{t} = 1.$

(b) $\lim_{t\to 0^-} \frac{1-\cos t}{t} = 0$ (Theorem 1.6.3).

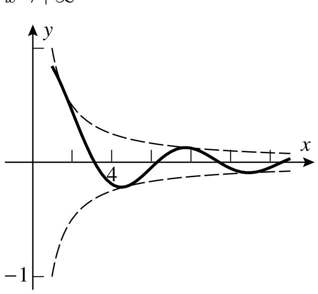
(c) $\sin(\pi - t) = \sin t$, so $\lim_{x\to\pi} \frac{\pi - x}{\sin x} = \lim_{t\to 0} \frac{t}{\sin t} = 1.$

45. $t = x - 1$; $\sin(\pi x) = \sin(\pi t + \pi) = -\sin \pi t$; and $\lim_{x\to 1} \frac{\sin(\pi x)}{x-1} = -\lim_{t\to 0} \frac{\sin \pi t}{t} = -\pi.$

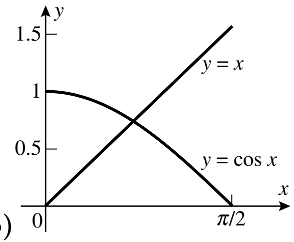
47. $-|x| \le x \cos\left(\frac{50\pi}{x}\right) \le |x|$, which gives the desired result.

49. Since $\lim_{x\to 0} \sin(1/x)$ does not exist, no conclusions can be drawn.

51. $\lim_{x\to +\infty} f(x) = 0$ by the Squeezing Theorem.

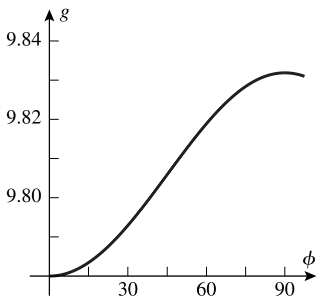


53. **(a)** Let $f(x) = x - \cos x$; $f(0) = -1$, $f(\pi/2) = \pi/2$. By the IVT there must be a solution of $f(x) = 0$.

(b) 

(c) 0.739

55. **(a)** Gravity is strongest at the poles and weakest at the equator.



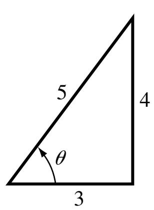
(b) Let $g(\phi)$ be the given function. Then $g(38) < 9.8$ and $g(39) > 9.8$, so by the Intermediate Value Theorem there is a value c between 38 and 39 for which $g(c) = 9.8$ exactly.

Exercise Set 1.7

1. $\sin^{-1} u$ is continuous for $-1 \le u \le 1$, so $-1 \le 2x \le 1$, or $-1/2 \le x \le 1/2$.

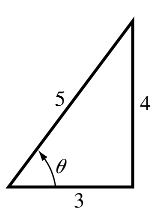
3. $\sqrt{u}$ is continuous for $0 \le u$, so $0 \le \tan^{-1} x$, or $x \ge 0$; $x^2 - 9 \ne 0$, thus the function is continuous for $0 \le x < 3$ and $x > 3$.

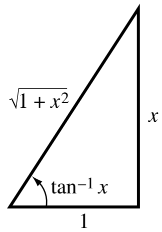
5. $\tan\theta = 4/3$, $0 < \theta < \pi/2$; use the triangle shown to get $\sin\theta = 4/5$, $\cos\theta = 3/5$, $\cot\theta = 3/4$, $\sec\theta = 5/3$, $\csc\theta = 5/4$.

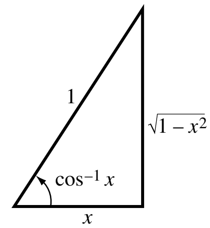


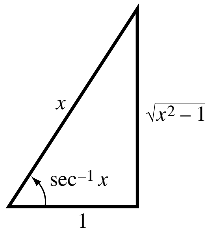
7. **(a)** $0 \le x \le \pi$ **(b)** $-1 \le x \le 1$ **(c)** $-\pi/2 < x < \pi/2$ **(d)** $-\infty < x < +\infty$

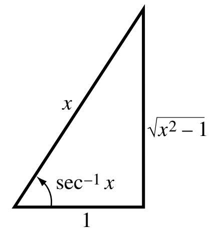
9. Let $\theta = \cos^{-1}(3/5)$; $\sin 2\theta = 2\sin\theta\cos\theta = 2(4/5)(3/5) = 24/25$.

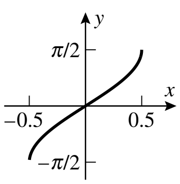


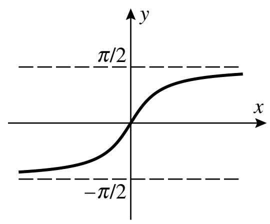
11. (a) $\cos(\tan^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$ 

(b) $\tan(\cos^{-1} x) = \dfrac{\sqrt{1-x^2}}{x}$ 

(c) $\sin(\sec^{-1} x) = \dfrac{\sqrt{x^2-1}}{x}$ 

(d) $\cot(\sec^{-1} x) = \dfrac{1}{\sqrt{x^2-1}}$ 

13. (a) 

(b) 

15. (a) $x = \pi - \sin^{-1}(0.37) \approx 2.7626$ rad **(b)** $\theta = 180^\circ + \sin^{-1}(0.61) \approx 217.6^\circ$.

17. (a) $\sin^{-1}(\sin^{-1} 0.25) \approx \sin^{-1} 0.25268 \approx 0.25545$; $\sin^{-1} 0.9 > 1$, so it is not in the domain of $\sin^{-1} x$.

(b) $-1 \le \sin^{-1} x \le 1$ is necessary, or $-0.841471 \le x \le 0.841471$.

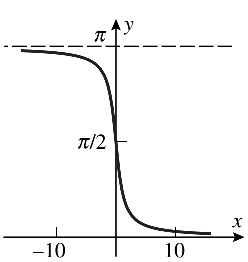
19. $\displaystyle\lim_{x\to+\infty} \sin^{-1}\left(\frac{x}{1-2x}\right) = \sin^{-1}\left(\lim_{x\to+\infty} \frac{x}{1-2x}\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}.$

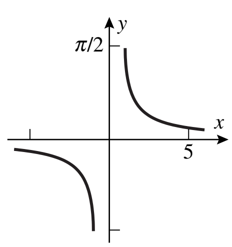
21. False; the range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, so the equation is only true for x in this range.

23. True; the line $y = \pi/2$ is a horizontal asymptote as $x \to \infty$ and as $x \to -\infty$.

25. $\displaystyle\lim_{x\to 0} \frac{x}{\sin^{-1} x} = \lim_{x\to 0} \frac{\sin x}{x} = 1.$

27. $\displaystyle 5 \lim_{x\to 0} \frac{\sin^{-1} 5x}{5x} = 5 \lim_{x\to 0} \frac{5x}{\sin 5x} = 5.$

29. (a) 



(b) The domain of $\cot^{-1} x$ is $(-\infty, +\infty)$, the range is $(0, \pi)$; the domain of $\csc^{-1} x$ is $(-\infty, -1] \cup [1, +\infty)$, the range is $[-\pi/2, 0) \cup (0, \pi/2]$.

31. **(a)** $55.0°$ **(b)** $33.6°$ **(c)** $25.8°$

33. **(a)** If $\gamma = 90°$, then $\sin\gamma = 1$, $\sqrt{1-\sin^2\phi\sin^2\gamma} = \sqrt{1-\sin^2\phi} = \cos\phi$, $D = \tan\phi\tan\lambda = (\tan 23.45°)(\tan 65°) \approx 0.93023374$ so $h \approx 21.1$ hours.

(b) If $\gamma = 270°$, then $\sin\gamma = -1$, $D = -\tan\phi\tan\lambda \approx -0.93023374$ so $h \approx 2.9$ hours.

35. $y = 0$ when $x^2 = 6000v^2/g$, $x = 10v\sqrt{60/g} = 1000\sqrt{30}$ for $v = 400$ and $g = 32$; $\tan\theta = 3000/x = 3/\sqrt{30}$, $\theta = \tan^{-1}(3/\sqrt{30}) \approx 29°$.

37. **(a)** Let $\theta = \cos^{-1}(-x)$ then $\cos\theta = -x$, $0 \le \theta \le \pi$. But $\cos(\pi-\theta) = -\cos\theta$ and $0 \le \pi-\theta \le \pi$ so $\cos(\pi-\theta) = x$, $\pi - \theta = \cos^{-1}x$, $\theta = \pi - \cos^{-1}x$.

(b) Let $\theta = \sec^{-1}(-x)$ for $x \ge 1$; then $\sec\theta = -x$ and $\pi/2 < \theta \le \pi$. So $0 \le \pi-\theta < \pi/2$ and $\pi-\theta = \sec^{-1}\sec(\pi-\theta) = \sec^{-1}(-\sec\theta) = \sec^{-1}x$, or $\sec^{-1}(-x) = \pi - \sec^{-1}x$.

39. $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$,

$$\tan(\tan^{-1}x + \tan^{-1}y) = \frac{\tan(\tan^{-1}x)+\tan(\tan^{-1}y)}{1-\tan(\tan^{-1}x)\tan(\tan^{-1}y)} = \frac{x+y}{1-xy}$$

so $\quad \tan^{-1}x + \tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}$.

41. $\sin(\sec^{-1}x) = \sin(\cos^{-1}(1/x)) = \sqrt{1-\left(\dfrac{1}{x}\right)^2} = \dfrac{\sqrt{x^2-1}}{|x|}$.

Exercise Set 1.8

1. **(a)** -4 **(b)** 4 **(c)** $1/4$

3. **(a)** 2.9691 **(b)** 0.0341

5. **(a)** $\log_2 16 = \log_2(2^4) = 4$ **(b)** $\log_2\left(\dfrac{1}{32}\right) = \log_2(2^{-5}) = -5$ **(c)** $\log_4 4 = 1$ **(d)** $\log_9 3 = \log_9(9^{1/2}) = 1/2$

7. **(a)** 1.3655 **(b)** -0.3011

9. **(a)** $2\ln a + \dfrac{1}{2}\ln b + \dfrac{1}{2}\ln c = 2r + s/2 + t/2$ **(b)** $\ln b - 3\ln a - \ln c = s - 3r - t$

11. **(a)** $1 + \log x + \dfrac{1}{2}\log(x-3)$ **(b)** $2\ln|x| + 3\ln(\sin x) - \dfrac{1}{2}\ln(x^2+1)$

13. $\log\dfrac{2^4(16)}{3} = \log(256/3)$

15. $\ln\dfrac{\sqrt[3]{x}(x+1)^2}{\cos x}$

17. $\sqrt{x} = 10^{-1} = 0.1$, $x = 0.01$

19. $1/x = e^{-2}$, $x = e^2$

21. $2x = 8$, $x = 4$

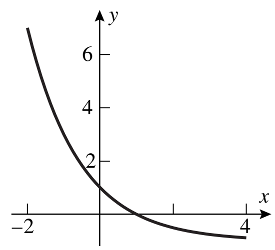
23. $\ln 2x^2 = \ln 3$, $2x^2 = 3$, $x^2 = 3/2$, $x = \sqrt{3/2}$ (we discard $-\sqrt{3/2}$ because it does not satisfy the original equation).

25. $\ln 5^{-2x} = \ln 3,\ -2x\ln 5 = \ln 3,\ x = -\dfrac{\ln 3}{2\ln 5}$

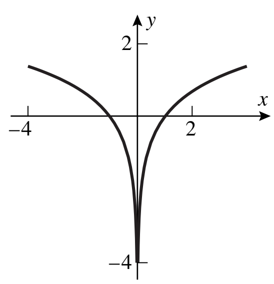
27. $e^{3x} = 7/2,\ 3x = \ln(7/2),\ x = \dfrac{1}{3}\ln(7/2)$

29. $e^{-x}(x+2) = 0$ so $e^{-x} = 0$ (impossible) or $x + 2 = 0,\ x = -2$

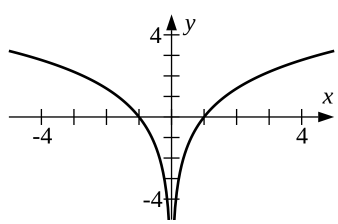
31. (a) Domain: all x; range: $y > -1$.



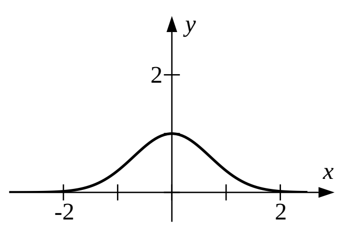
(b) Domain: $x \neq 0$; range: all y.



33. (a) Domain: $x \neq 0$; range: all y.



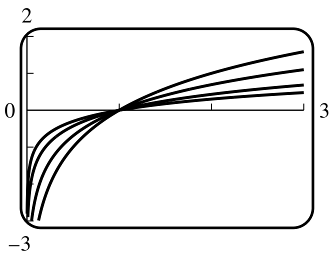
(b) Domain: all x; range: $0 < y \leq 1$.



35. False. The graph of an exponential function passes through $(0, 1)$, but the graph of $y = x^3$ does not.

37. True, by definition.

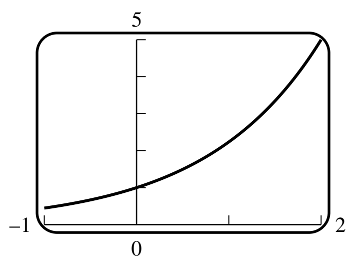
39. $\log_2 7.35 = (\log 7.35)/(\log 2) = (\ln 7.35)/(\ln 2) \approx 2.8777$; $\log_5 0.6 = (\log 0.6)/(\log 5) = (\ln 0.6)/(\ln 5) \approx -0.3174$.



41.

43. $x \approx 1.47099$ and $x \approx 7.85707$.

45. **(a)** No, the curve passes through the origin. **(b)** $y = (\sqrt[4]{2})^x$ **(c)** $y = 2^{-x} = (1/2)^x$ **(d)** $y = (\sqrt{5})^x$



47. $\log(1/2) < 0$ so $3\log(1/2) < 2\log(1/2)$.

49. $\lim\limits_{x\to-\infty} \dfrac{1-e^x}{1+e^x} = \dfrac{1-0}{1+0} = 1$.

51. Divide the numerator and denominator by e^x: $\lim\limits_{x\to+\infty} \dfrac{1+e^{-2x}}{1-e^{-2x}} = \dfrac{1+0}{1-0} = 1$.

53. The limit is $-\infty$.

55. $\dfrac{x+1}{x} = 1 + \dfrac{1}{x}$, so $\lim\limits_{x\to+\infty} \dfrac{(x+1)^x}{x^x} = e$ from Figure 1.3.4.

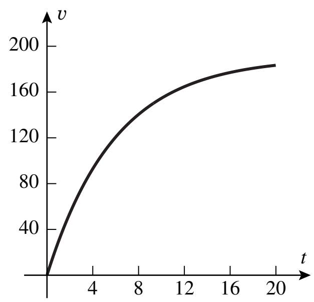
57. $t = 1/x$, $\lim\limits_{t\to+\infty} f(t) = +\infty$.

59. $t = \csc x$, $\lim\limits_{t\to+\infty} f(t) = +\infty$.

61. Let $t = \ln x$. Then t also tends to $+\infty$, and $\dfrac{\ln 2x}{\ln 3x} = \dfrac{t + \ln 2}{t + \ln 3}$, so the limit is 1.

63. Set $t = -x$, then get $\lim\limits_{t\to-\infty} \left(1 + \dfrac{1}{t}\right)^t = e$ by Figure 1.3.4.

65. From the hint, $\lim\limits_{x\to+\infty} b^x = \lim\limits_{x\to+\infty} e^{(\ln b)x} = \begin{cases} 0 & \text{if } b < 1, \\ 1 & \text{if } b = 1, \\ +\infty & \text{if } b > 1. \end{cases}$

67. **(a)** 

(b) $\lim_{t\to\infty} v = 190\left(1 - \lim_{t\to\infty} e^{-0.168t}\right) = 190$, so the asymptote is $v = c = 190$ ft/sec.

(c) Due to air resistance (and other factors) this is the maximum speed that a sky diver can attain.

69. **(a)**

n	2	3	4	5	6	7
$1+10^{-n}$	1.01	1.001	1.0001	1.00001	1.000001	1.0000001
$1+10^{n}$	101	1001	10001	100001	1000001	10000001
$(1+10^{-n})^{1+10^n}$	2.7319	2.7196	2.7184	2.7183	2.71828	2.718282

The limit appears to be e.

(b) This is evident from the lower left term in the chart in part (a).

(c) The exponents are being multiplied by a, so the result is e^a.

71. $75e^{-t/125} = 15, t = -125\ln(1/5) = 125\ln 5 \approx 201$ days.

73. **(a)** 7.4; basic **(b)** 4.2; acidic **(c)** 6.4; acidic **(d)** 5.9; acidic

75. **(a)** 140 dB; damage **(b)** 120 dB; damage **(c)** 80 dB; no damage **(d)** 75 dB; no damage

77. Let I_A and I_B be the intensities of the automobile and blender, respectively. Then $\log_{10} I_A/I_0 = 7$ and $\log_{10} I_B/I_0 = 9.3$, $I_A = 10^7 I_0$ and $I_B = 10^{9.3} I_0$, so $I_B/I_A = 10^{2.3} \approx 200$.

79. **(a)** $\log E = 4.4 + 1.5(8.2) = 16.7, E = 10^{16.7} \approx 5 \times 10^{16}$ J

(b) Let M_1 and M_2 be the magnitudes of earthquakes with energies of E and $10E$, respectively. Then $1.5(M_2 - M_1) = \log(10E) - \log E = \log 10 = 1$, $M_2 - M_1 = 1/1.5 = 2/3 \approx 0.67$.

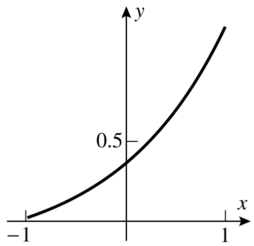
Chapter 1 Review Exercises

1. **(a)** 1 **(b)** Does not exist. **(c)** Does not exist. **(d)** 1 **(e)** 3 **(f)** 0 **(g)** 0

(h) 2 **(i)** 1/2

3. **(a)**

x	-0.01	-0.001	-0.0001	0.0001	0.001	0.01
$f(x)$	0.402	0.405	0.405	0.406	0.406	0.409

(b) 

5. The limit is $\dfrac{(-1)^3-(-1)^2}{-1-1}=1$.

7. If $x\neq -3$ then $\dfrac{3x+9}{x^2+4x+3}=\dfrac{3}{x+1}$ with limit $-\dfrac{3}{2}$.

9. By the highest degree terms, the limit is $\dfrac{2^5}{3}=\dfrac{32}{3}$.

11. **(a)** $y=0$. **(b)** None. **(c)** $y=2$.

13. If $x\neq 0$, then $\dfrac{\sin 3x}{\tan 3x}=\cos 3x$, and the limit is 1.

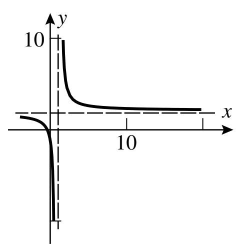
15. If $x\neq 0$, then $\dfrac{3x-\sin(kx)}{x}=3-k\dfrac{\sin(kx)}{kx}$, so the limit is $3-k$.

17. As $t\to\pi/2^+$, $\tan t\to -\infty$, so the limit in question is 0.

19. $\left(1+\dfrac{3}{x}\right)^{-x}=\left[\left(1+\dfrac{3}{x}\right)^{x/3}\right]^{(-3)}$, so the limit is e^{-3}.

21. \$2,001.60, \$2,009.66, \$2,013.62, \$2013.75.

23. **(a)** $f(x)=2x/(x-1)$.

(b) 

25. **(a)** $\lim\limits_{x\to 2} f(x)=5$.

(b) $\delta=(3/4)\cdot(0.048/8)=0.0045$.

27. **(a)** $|4x-7-1|<0.01$ means $4|x-2|<0.01$, or $|x-2|<0.0025$, so $\delta=0.0025$.

(b) $\left|\dfrac{4x^2-9}{2x-3}-6\right|<0.05$ means $|2x+3-6|<0.05$, or $|x-1.5|<0.025$, so $\delta=0.025$.

(c) $|x^2-16|<0.001$; if $\delta<1$ then $|x+4|<9$ if $|x-4|<1$; then $|x^2-16|=|x-4||x+4|\le 9|x-4|<0.001$ provided $|x-4|<0.001/9=1/9000$, take $\delta=1/9000$, then $|x^2-16|<9|x-4|<9(1/9000)=1/1000=0.001$.

29. Let $\epsilon = f(x_0)/2 > 0$; then there corresponds a $\delta > 0$ such that if $|x - x_0| < \delta$ then $|f(x) - f(x_0)| < \epsilon$, $-\epsilon < f(x) - f(x_0) < \epsilon$, $f(x) > f(x_0) - \epsilon = f(x_0)/2 > 0$, for $x_0 - \delta < x < x_0 + \delta$.

31. **(a)** f is not defined at $x = \pm 1$, continuous elsewhere.

(b) None; continuous everywhere.

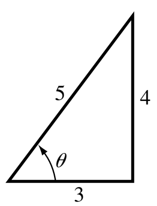
(c) f is not defined at $x = 0$ and $x = -3$, continuous elsewhere.

33. For $x < 2$ f is a polynomial and is continuous; for $x > 2$ f is a polynomial and is continuous. At $x = 2$, $f(2) = -13 \neq 13 = \lim_{x\to 2^+} f(x)$, so f is not continuous there.

35. $f(x) = -1$ for $a \le x < \dfrac{a+b}{2}$ and $f(x) = 1$ for $\dfrac{a+b}{2} \le x \le b$; f does not take the value 0.

37. $f(-6) = 185$, $f(0) = -1$, $f(2) = 65$; apply Theorem 1.5.8 twice, once on $[-6, 0]$ and once on $[0, 2]$.

39. Draw right triangles of sides 5, 12, 13, and 3, 4, 5. Then $\sin[\cos^{-1}(4/5)] = 3/5$, $\sin[\cos^{-1}(5/13)] = 12/13$, $\cos[\sin^{-1}(4/5)] = 3/5$, and $\cos[\sin^{-1}(5/13)] = 12/13$.



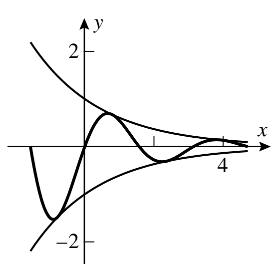
(a) $\cos[\cos^{-1}(4/5) + \sin^{-1}(5/13)] = \cos(\cos^{-1}(4/5))\cos(\sin^{-1}(5/13) - \sin(\cos^{-1}(4/5))\sin(\sin^{-1}(5/13)) = \dfrac{4}{5}\dfrac{12}{13} - \dfrac{3}{5}\dfrac{5}{13} = \dfrac{33}{65}$.

(b) $\sin[\sin^{-1}(4/5) + \cos^{-1}(5/13)] = \sin(\sin^{-1}(4/5))\cos(\cos^{-1}(5/13)) + \cos(\sin^{-1}(4/5))\sin(\cos^{-1}(5/13)) = \dfrac{4}{5}\dfrac{5}{13} + \dfrac{3}{5}\dfrac{12}{13} = \dfrac{56}{65}$.

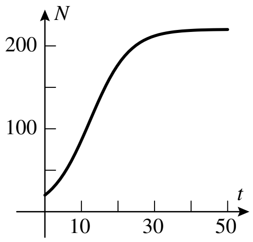
41. $y = 5$ ft $= 60$ in, so $60 = \log x$, $x = 10^{60}$ in $\approx 1.58 \times 10^{55}$ mi.

43. $3\ln\left(e^{2x}(e^x)^3\right) + 2\exp(\ln 1) = 3\ln e^{2x} + 3\ln(e^x)^3 + 2\cdot 1 = 3(2x) + (3\cdot 3)x + 2 = 15x + 2$.

45. **(a)**



(b) The curve $y = e^{-x/2}\sin 2x$ has x−intercepts at $x = -\pi/2, 0, \pi/2, \pi, 3\pi/2$. It intersects the curve $y = e^{-x/2}$ at $x = \pi/4, 5\pi/4$ and it intersects the curve $y = -e^{-x/2}$ at $x = -\pi/4, 3\pi/4$.

47. **(a)** 

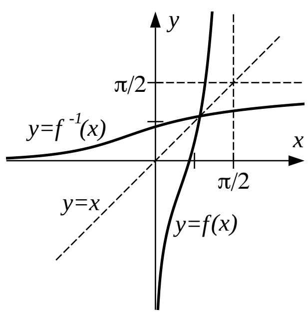
(b) $N = 80$ when $t = 9.35$ yrs.

(c) 220 sheep.

49. **(a)** The function $\ln x - x^{0.2}$ is negative at $x = 1$ and positive at $x = 4$, so by the intermediate value theorem it is zero somewhere in between.

(b) $x = 3.654$ and 332105.108.

51. **(a)** The functions x^2 and $\tan x$ are positive and increasing on the indicated interval, so their product $x^2 \tan x$ is also increasing there. So is $\ln x$; hence the sum $f(x) = x^2 \tan x + \ln x$ is increasing, and it has an inverse.



(b)

The asymptotes for $f(x)$ are $x = 0$, $x = \pi/2$. The asymptotes for $f^{-1}(x)$ are $y = 0, y = \pi/2$.

Chapter 1 Making Connections

1. Let $P(x, x^2)$ be an arbitrary point on the curve, let $Q(-x, x^2)$ be its reflection through the y-axis, let $O(0, 0)$ be the origin. The perpendicular bisector of the line which connects P with O meets the y-axis at a point $C(0, \lambda(x))$, whose ordinate is as yet unknown. A segment of the bisector is also the altitude of the triangle ΔOPC which is isosceles, so that $CP = CO$.

Using the symmetrically opposing point Q in the second quadrant, we see that $\overline{OP} = \overline{OQ}$ too, and thus C is equidistant from the three points O, P, Q and is thus the center of the unique circle that passes through the three points.

3. Replace the parabola with the general curve $y = f(x)$ which passes through $P(x, f(x))$ and $S(0, f(0))$. Let the perpendicular bisector of the line through S and P meet the y-axis at $C(0, \lambda)$, and let $R(x/2, (f(x) - \lambda)/2)$ be the midpoint of P and S. By the Pythagorean Theorem, $\overline{CS}^2 = \overline{RS}^2 + \overline{CR}^2$, or $(\lambda - f(0))^2 = x^2/4 + \left[\dfrac{f(x) + f(0)}{2} - f(0)\right]^2 + x^2/4 + \left[\dfrac{f(x) + f(0)}{2} - \lambda\right]^2$,
which yields $\lambda = \dfrac{1}{2}\left[f(0) + f(x) + \dfrac{x^2}{f(x) - f(0)}\right]$.