

January 3rd 2021 Monday 9:00 am - 12:00 noon

Course Code: MT – 1003	Course Name: Calculus and Analytical Geometry
Instructor Name: Dr. Khusro Mian/ Dr. Fahad Riaz/ Ms. Asma Masood/ Mr. Nadeem Khan/ Ms. Afreen Naz/ Ms. Urooj/ Ms. Alishba Tariq/ Ms. Javeria Iftikhar/ Ms. Sadia Khan	
Student Roll No:	Section No:

Instructions:

- Return the question paper.
- Attempt all questions. There are 10 Questions and 02 pages.
- Solve the paper according to the sequence given in the question paper.
- Graphical Calculator is not allowed.

Time: 180 minutes

Max Marks: 100

Question 01:

03+03+04=10

- (a) For the following function, show that it is continuous or not at $x = 4$

$$f(x) = \begin{cases} 2x + 3 & x \leq 4 \\ 7 + \frac{16}{x} & x > 4 \end{cases}$$

- (b) For the following $f(x) = \begin{cases} |x - 2| & x > 0 \\ x + 2 & x \leq 0 \end{cases}$ evaluate $\int_{-4}^6 f(x) dx$

- (c) Find absolute extrema for $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$

Question02:

05+05=10

Find the derivative of the function. [Any Two]

a) $f(x) = \cos(\sqrt{\sin(\tan \pi x)})$

b) $f(x) = \frac{\tan 3x}{(x+7)^4}$

c) $f(x) = [1 + \sin^3(x^5)]^{12}$

Question 03:

05+05=10

For the functions $x = 2 - y^2$ and $x = y^2$

- (a) Sketch the region and find the area enclosed between two curves
 (b) Find the volume of the solid obtained by revolving above curves along y axis.

Question04:

05+05=10

For the following functions, does the integral diverges or converges?

(a) $\int_{-\infty}^{+\infty} (1 - x)e^{-x} dx$

(b) $\int_0^1 \frac{1}{\sqrt{x(x+1)}} dx$

Question 05:**05+05=10**

For the following functions, integrate using the method of substitution. (Attempt any two)

- a. $\int_0^1 \frac{1}{\sqrt{2x-x^2}} dx$
- b. $\int_5^{10} \frac{\sqrt{x^2-25}}{x} dx$
- c. $\int \frac{1}{1-\sin x + \cos x} dx$

Question 06:**05+05=10**

- a) Use implicit differentiation to find $\frac{dy}{dx}$ for $x^3 + y^3 = 3xy$, and then find the slope of the tangent line at the point $(\frac{3}{2}, \frac{3}{2})$.
- b) By using implicit differentiation find an equation of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the given point $(2, 1)$.

Question 07:**05+05=10**

Evaluate limits:

- (a) $\lim_{x \rightarrow +\infty} \left[\cos\left(\frac{2}{x}\right) \right]^{x^2}$
- (b) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x)$

Question 08:**03+03+04=10**

Integrate the following:

- (a) $\int e^{-3x} \sin 3x dx$
- (b) $\int \frac{x^5 + x^2 + 2}{x^3 - x} dx$
- (c) $\int \sin^5 x dx$

Question09:**05+05=10**

Show that the lines are skew and find the distance between them.

$$L_1: x - 1 = 2t, \quad y = 3t, \quad z + 2 = -5t$$

$$L_2: x = t, \quad y = 4 + t, \quad z = 3t$$

Question10:**05+05=10**

- (a) Find the point of intersection between two lines.

$$L_1: x + 1 = 4t, \quad y - 3 = t, \quad z - 1 = 0$$

$$L_2: x + 13 = 12t, \quad y - 1 = 6t, \quad z - 2 = 3t$$

- (b) Find the angles of intersection and the line of intersection between two planes.

$$P_1: 2x - 4y + 4z = 6$$

$$P_2: 6x + 2y - 3z = 4$$

The End ☺