Tuesday, May 4, 2021 8:33 AM

$$f(x) = \begin{cases} \dfrac{x^2 - 4x - 5}{x - 5}, & x < 5 \\ x + 1 & x \geq 5 \end{cases}$$

is continuous at $x = 5$.?

Sol//

i) $f(x)$ is def at $x = 5$.

$f(x) = x + 1, \quad x = 5$

$f(5) = 5 + 1 \Rightarrow \boxed{f(5) = 6}$

ii) $\lim_{x \to 5} f(x) = ?$

$L.H.L = \lim_{x \to 5^-} \left(\dfrac{x^2 - 4x - 5}{x - 5} \right) \simeq \dfrac{0}{0}$

$R.H.L = \lim_{x \to 5^+} (x + 1)$

$= \lim_{x \to 5^-} \left[\dfrac{x^2 - 5x + x - 5}{x - 5} \right],$

$R.H.L = 5 + 1$

$\boxed{R.H.L = 6}$

$= \lim_{x \to 5^-} \left[\dfrac{x(x-5) + 1(x-5)}{x - 5} \right],$

$= \lim_{x \to 5^+} \dfrac{\cancel{(x-5)}(x+1)}{\cancel{x-5}}$ A.L

$= 5 + 1$

$L.H.L = 6$

$L.H.L = R.H.L = 6$

$\boxed{\lim_{x \to 5} f(x) = 6}$

③ $\lim_{x \to 5} f(x) = f(5)$

$6 = 6$

$f(x)$ is cont at $x = 5$.

Q #2 $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2 \end{cases}$

Q #2 $\quad f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 5, & x = 2 \end{cases}$

Sol//

1) $f(x)$ is def at $x = 2$.

$f(x) = 5 \quad x = 2$

$$\boxed{f(2) = 5}$$

2) $\lim_{x \to 2} f(x) = ?$

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} \Rightarrow \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} (x+2) \Rightarrow 2 + 2 = \boxed{4}$$ exist

3) $\lim_{x \to 2} f(x) = f(2)$

$4 \neq 5$

$f(x)$ is discontinuous at $x = 2$.

Q #3 $\quad f(x) = \begin{cases} 2 + \dfrac{\sin x}{x} & x \neq 0 \\ 3 & x = 0 \end{cases}$

is cont at $x = 0$?

Sol// is $f(x)$ is def at $x = 0$.

$f(x) = 3$, $\boxed{f(0) = 3}$

2) $\lim_{x \to 0} f(x)$

$$\lim_{x \to 0} \left(2 + \frac{\sin x}{x}\right) = \lim_{x \to 0} 2 + \lim_{x \to 0} \frac{\sin x}{x}$$

$= 2 + 1$

3) $\lim_{x \to 0} f(x) = f(0)$ $= 3$ Exist.

$3 = 3$ $f(x)$ is Cont.

Q #3

Find value of c. If

$$f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$$ is ~~Continou~~

1) Cont every where.

1) $f(x)$ is def:

$f(x) = x^3 - cx$, $x = 2$

$f(2) = 2^3 - c(2)$, $\boxed{f(2) = 8 - 2c}$

2) $\lim_{x \to 2} f(x)$.

$L.H.L = \lim_{x \to 2^-} (cx^2 + 2x)$, $R.H.L = \lim_{x \to 2^+} (x^3 - cx)$

$L.H.L = c(2)^2 + 2(2)$, $R.H.L = (2)^3 - c(2)$

$\boxed{L.H.L = 4c + 4}$, $\boxed{R.H.L = 8 - 2c}$

$L.H.L = R.H.L$ $f(x)$ is cont.

$4c + 4 = 8 - 2c$

$4c + 2c = 8 - 4$

$6c = 4 \Rightarrow c = \frac{4}{6} \Rightarrow \boxed{c = \frac{2}{3}}$ Ans.

Q #2 If $f(x) = \begin{cases} 2x + \dfrac{3e^x + 1}{x} & x \neq 0 \\ m + 5x & x = 0 \end{cases}$

Find m is $f(x)$ is Cont $(-\infty, \infty)$

Sol i) $f(x)$ is def.

$$f(x) = m + 5x \Rightarrow f(0) = m + 5(0) =$$

$$\boxed{f(0) = m}$$

ii) $\lim_{x \to 0} f(x) = ?$

$$= \lim_{x \to 0} \left[2x + \frac{3e^x + 1}{x} \right]$$

$$= \lim_{x \to 0} 2x + 3 \lim_{x \to 0} \frac{e^x + 1}{x} \qquad \lim_{x \to 0} \frac{e^{ax} + 1}{x} = a$$

$$\lim_{x \to 0} f(x) = (0) + 3(1)$$

$$\boxed{\lim_{x \to 0} f(x) = 3}$$

iii) $\lim_{x \to 0} f(x) = f(0)$

$$\boxed{3 = m}$$ Ans.

Q#5

$$f(x) = \begin{cases} k + k\dfrac{\sin 2x}{x}, & x \neq 0 \\ k^2 + 1, & x = 0 \end{cases}$$

Find k if $f(x)$ is Cont.

1) $f(x)$ is def.

$f(x) = k^2 + 1$, $\quad x = 0$

$$\boxed{f(0) = k^2 + 1}$$

2) $\lim_{x \to 0} f(x)$:

$\lim f(x) = \lim \left(k + k\frac{\sin 2x}{x}\right)$

$x \to 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \left(k + k\frac{\sin 2x}{x}\right)$$

$$= \lim_{x\to 0} k + k\lim_{x\to 0} \frac{\sin 2x}{x}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= k + k(2)$$

$$\lim_{x\to 0} \frac{\sin ax}{x} = a$$

$$\boxed{\lim_{x\to 0} f(x) = 3k}$$

(3) $\lim_{x\to 0} f(x) = f(0)$

$$3k = k^2+1$$

$$k^2+1-3k = 0$$

$$k^2-3k+1=0$$ use quadratic formula.

$$k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\boxed{k = \frac{3+\sqrt{5}}{2},\ k = \frac{3-\sqrt{5}}{2}}$$