Find the values of a and b that make f is continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

Sol// For $x = 2$

1) $f(x)$ is def at $x=2$

$f(x) = ax^2 - bx + 3$, $x = 2$

$f(2) = a(2)^2 - b(2) + 3 \Rightarrow \boxed{f(2) = 4a - 2b + 3}$

2) $\lim_{x \to 2} f(x) = ?$

$L.H.L = \lim_{x \to 2^-} \left(\dfrac{x^2-4}{x-2} \right)$ or $R.H.L = \lim_{x \to 2^+} \left(ax^2 - bx + 3 \right)$

$L.H.L = \lim_{x \to 2^-} \left(\dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} \right)$ or $R.H.L = a(2)^2 - b(2) + 3$

$L.H.L = 2 + 2 = 4$ or $R.H.L = 4a - 2b + 3$

$L.H.L = R.H.L$

$4 = 4a - 2b + 3$

$0 = 4a - 2b + 3 - 4$

$\boxed{0 = 4a - 2b - 1} \rightarrow (I)$

For $x = 3$

1) $f(x)$ is def at $x = 3$

$f(x) = 2x - a + b \Rightarrow f(3) = 2(3) - a + b$

$\boxed{f(3) = 6 - a + b}$

2) $\lim_{x \to 3} f(x) = ?$

$L.H.L = \lim_{x \to 3^-} \left(ax^2 - bx + 3 \right)$ or $R.H.L = \lim_{x \to 3^+} \left(2x - a + b \right)$

$= a(3)^2 - b(3) + 3$ $\quad R.H.L = 2(3) - a + b$

$= a(3)^2 - b(3) + 3$ $\quad$ $R.H.L = 2(3) - a + b$

$L.H.L = 9a - 3b + 3$ $\quad$ $R.H.L = 6 - a + b$

$$9a - 3b + 3 = 6 - a + b$$

$$9a - 3b + 3 - 6 + a - b = 0$$

$$10a - 4b - 3 = 0 \rightarrow (B)$$

$$4a - 2b - 1 = 0 \rightarrow (A)$$

eq^m (A) × by 2 Sub (B)

$(A) \Rightarrow \quad 8a - \cancel{4b} - 2 = 0$

$(B) \Rightarrow \quad 10a - \cancel{4b} - 3 = 0$

$\quad\quad - \quad + \quad +$

$-2a + 1 = 0$, $+2a = +1 \Rightarrow \boxed{a = 1/2}$ put in (A)

$(A) \Rightarrow 4\left(\frac{1}{2}\right) - 2b - 1 = 0 \Rightarrow 2 - 2b - 1 = 0$

$-2b + 1 = 0 \Rightarrow +2b = +1$

$$\boxed{b = \frac{1}{2}}$$

Q#

$$f(x) = \begin{cases} 1.5x, & x \le 20 \\ 1.25x + k, & x > 20 \end{cases}$$

Find $k = ?$ If $f(x)$ is Cont.

Sol

i) $f(x)$ is def at 20.

$f(x) = 1.5x$, $f(20) = 1.5(20)$

$f(20) = 30$

2) $\lim_{x \to 20} f(x) = ?$

$L.H.L = \lim_{x \to 20^-} (1.5x)$ or $R.H.L = \lim_{x \to 20^+} (1.25x + k)$

$= 1.5(20)$ or $= 1.25(20) + k$

$L.H.L = 30$ $\quad$ $R.H.L = 25 + k$

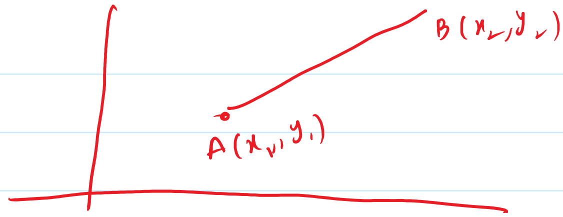
$L\cdot H\cdot L = R\cdot H\cdot L$

$30 = 25 + k \Rightarrow 30 - 25 = k$

$\boxed{k = 5}$

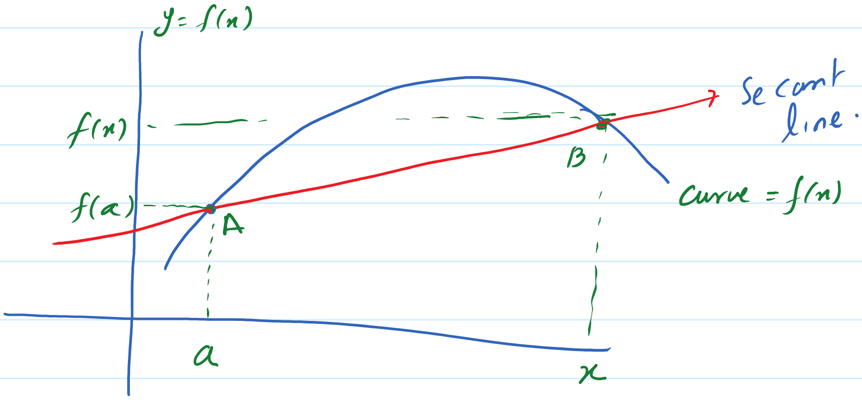
Differentiability of the function :

Wednesday, May 5, 2021 7:52 AM



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Average Rate of Change :

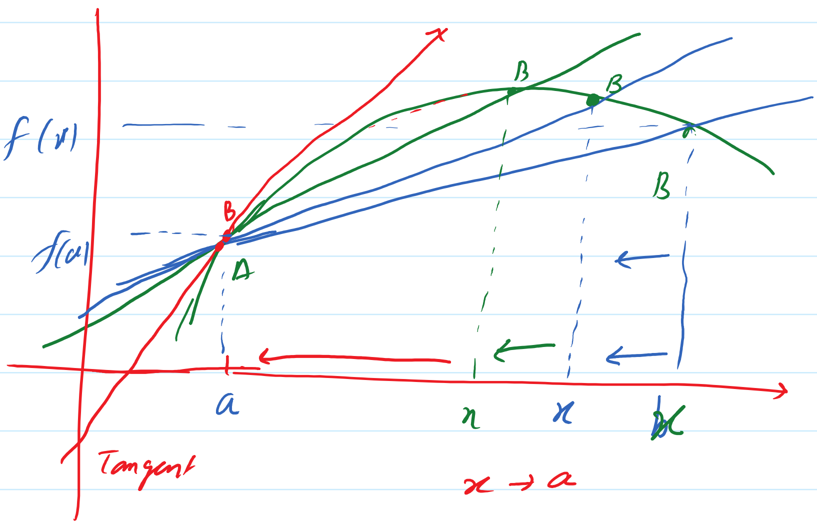


$A(a, f(a))$, $B(x, f(x))$

$\quad x_1, \; y_1 \qquad\qquad x_2, \; y_2$

$$\boxed{A.R.C = \frac{f(x) - f(a)}{x - a}}$$

Instantaneous rate of change:
OR Slope of tangent line
OR Differentiability.



$$m = \frac{f(x) - f(a)}{x - a}$$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

$$\lim_{x \to 3} \frac{\sqrt{2} - \sqrt{x}}{\sqrt{3} - \sqrt{x}} \times \frac{\sqrt{3} + \sqrt{x}}{\sqrt{3} + \sqrt{x}}$$