Solution of Final Exam Fall 2023

Question 01: **[CLO-3]** **[10]**

a. **T**/**F:** The improper integral $\int_1^\infty \frac{1}{x^2}\, dx$ represents a finite area

b. The function $f(x) = x^{\frac{5}{11}}$ has a point of inflection with an x-coordinate of

I) $\frac{5}{11}$ II) $-\frac{5}{11}$ **III) 0** IV) Does not exist

c. First derivative of $xy = 90$ is equal to.

I) $\frac{dy}{dx} = \frac{y}{x}$ **II) $\frac{dy}{dx} = -\frac{y}{x}$** III) $\frac{dy}{dx} = xdy + ydx$ IV) $\frac{dy}{dx} = \frac{x}{y}$

d. If a is a simple constant, what is the derivative of $y = x^a$?

I) ax^{a-1} II) $(a-1)^x$ III) x^{a-1} IV) ax

e. Evaluate $\lim_{x\to 2^-} \frac{x^2+2x}{x^2-5x+6}$

I) $-\infty$ **II) -2** III) $\frac{2}{5}$ IV) Does not exist

f. Given that $f(1) = 5$, $f'(1) = 4$ and $g(x) = [f(x)]^{-4}$ find $g'(1)$

I) 2 II) $\frac{93}{31}$ III) $\frac{-37}{4}$ **IV) $\frac{-16}{3125}$**

g. The curves $y = x^4 - 3$ and $y = -x^4 + 5$ enclosed an area. Set up a definite integral which calculates the area of this region.

I) $\int_{-\sqrt{2}}^{\sqrt{2}} 2\, dx$ II) $\int_{-1}^{1} 2\, dx$ **III) $\int_{-\sqrt{2}}^{\sqrt{2}} (8 - 2x^4)dx$** IV) $\int_{-1}^{1} (8 - 2x^4)dx$

h. If $f(x) = \sqrt{1 + \sqrt{1+x}}$ then $f'(8) = ?$

I) $\frac{1}{12}$ II) $\frac{1}{8}$ III) $\frac{1}{9}$ **IV) $\frac{1}{24}$**

i. If $f(x) = sin^{-1}(3x)$ then $\int f(x)dx = ?$

I) $xsin^{-1}(3x) + \frac{\sqrt{1-9x^2}}{9} + c$ II) $xcos^{-1}(3x) + \frac{\sqrt{1-9x^2}}{9} + c$

III) $xsin^{-1}(3x) + \frac{\sqrt{1-9x^2}}{3} + c$ IV) $xsin^{-1}(3x) - \frac{\sqrt{1-9x^2}}{9} + c$

j. If the function f is continuous on $[a, b]$ and if $f(x) \geq 0$ for all x in $[a, b]$, then the area A under the curve $y = f(x)$ over the interval $[a, b]$ is defined as ______________ , with x_k^* as the right endpoint of each subinterval

I) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ II) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_{k-1})\Delta x$

III) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ V) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f[\frac{1}{2}(x_{k-1} + x_k)]\Delta x$

Question 02: **[CLO-3]** **[5+5=10]**

a. Find $\frac{d^2y}{dx^2}$ by using implicit differentiation.

$$x + siny = xy$$

$$1 + \cos y \cdot \frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx}(\cos y - x) = y - 1$$

$$\frac{dy}{dx} = \frac{y-1}{\cos y - x}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}(\cos y - x) - (y-1)\left(-\sin y\frac{dy}{dx} - 1\right)}{(\cos y - x)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{y-1}{\cancel{(\cos y - x)}}\cancel{(\cos y - x)} - (y-1)\left(-\sin y\left(\frac{y-1}{\cos y - x}\right) - 1\right)}{(\cos y - x)^2}$$

$$\frac{(y-1)\left[1 + \sin y\left(\frac{y-1}{\cos y - x}\right) + 1\right]}{(\cos y - x)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{(y-1)\left[\sin y\left(\frac{y-1}{\cos y - x}\right) + 2\right]}{(\cos y - x)^2}}$$

b. Find the derivative of

$$f(x) = cot\left[\frac{csc2x}{x^3 + 5}\right]$$

$$-\csc^2\left(\frac{\csc 2x}{x^3+5}\right)\frac{-2(x^3+5)\csc 2x \cot 2x - 3x^2 \csc 2x}{(x^3+5)^2}$$

Question 03: **[CLO-3]** **[5+5+5+5=20]**

Evaluate the integral of the following

a. $\displaystyle\int \frac{dx}{2+\cos x}$

- We set $t = \tan\left(\frac{x}{2}\right)$, with $\cos(x) = \frac{1-t^2}{1+t^2}$ and $dx = \frac{2}{1+t^2}\,dt$, to obtain

$$\begin{aligned}\int \frac{1}{2+\cos(x)}\,dx &= \int \frac{1}{2+\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt \\ &= \int \frac{2}{2(1+t^2)+(1-t^2)}\,dt \\ &= \int \frac{2}{3+t^2}\,dt = \int \frac{2}{3} \cdot \frac{1}{1+(t/\sqrt{3})^2}\,dt\end{aligned}$$

- In this new integral we set $u = t/\sqrt{3}$ with $du = dt/\sqrt{3}$ to obtain $\int \frac{2}{3}\cdot\frac{\sqrt{3}}{1+u^2}\,du = \frac{2}{\sqrt{3}}\tan^{-1}(u) + C$.
- Substituting back for t and then x yields the answer as $\boxed{\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\left(\frac{x}{2}\right)\right) + C}$

b. $\displaystyle\int_0^5 \frac{w}{w-2}\,dw$

$$\int_0^5 \frac{w}{w-2}\,dw = \int_{-2}^{3} \frac{u+2}{u}\,du = \int_{-2}^{3}\left(1+\frac{2}{u}\right)\,du$$

This is still a Type II integral since function $1+\frac{2}{u}$ is discontinuous at $u=0$. Need to split up the integral:

$$\int_{-2}^{3}\left(1+\frac{2}{u}\right)\,du = \int_{-2}^{0}\left(1+\frac{2}{u}\right)\,du + \int_{0}^{3}\left(1+\frac{2}{u}\right)\,du$$

$$\begin{aligned}&= \lim_{t\to 0^-}\int_{-2}^{t}\left(1+\frac{2}{u}\right)\,du + \lim_{s\to 0^+}\int_{s}^{3}\left(1+\frac{2}{u}\right)\,du = \lim_{t\to 0^-}\left(u+2\ln|u|\right)\Big|_{-2}^{t} + \lim_{s\to 0^+}\left(u+2\ln|u|\right)\Big|_{s}^{3} \\ &= \lim_{t\to 0^-}\left(t+2\ln|t|\right) + 2 - 2\ln 2 + 3 + 2\ln 3 - \lim_{s\to 0^+}\left(s+2\ln|s|\right)\end{aligned}$$

Both of the limits diverge, so the integral diverges. □

c. $\displaystyle\int \frac{5}{x^3+2x^2+5x}\,dx$

Factor: $x^3+2x^2+5x = x(x^2+2x+5)$. The second factor is irreducible.

Set up partial fractions:

$$\frac{5}{x^3+2x^2+5x} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+5},$$

$$\begin{aligned}5 &= A(x^2+2x+5) + Bx^2 + Cx \\ &= (A+B)x^2 + (2A+C)x + 5A, \text{ so}\end{aligned}$$

$$\begin{cases} 0 = A+B \\ 0 = 2A+C \\ 5 = 5A \end{cases}$$

$A=1$, $B=-1$, $C=-2$ and

$$\begin{aligned}\int \frac{5\,dx}{x^3+2x^2+5x} &= \int \frac{dx}{x} - \int \frac{x+2}{x^2+2x+5}\,dx \\ &= \int \frac{dx}{x} - \int \frac{x+2}{(x+1)^2+4}\,dx\end{aligned}$$

Let $y = x+1$, so $dy = dx$ and $x = y-1$, so $\int \frac{x+2}{(x+1)^2+4}\,dx = \int \frac{y+1}{y^2+4}\,dy$ $= \frac{1}{2}\int \frac{2y}{y^2+4}\,dy + \int \frac{dy}{y^2+4} = \frac{1}{2}\ln(y^2+4) + \frac{1}{2}\tan^{-1}\frac{y}{2} + C$. The answer is $\ln|x| - \frac{1}{2}\ln\left((x+1)^2+4\right) - \frac{1}{2}\tan^{-1}\frac{x+1}{2} + C.$

d. $\int \frac{1}{2x^2+4x+7}dx$

$$\int \frac{dx}{2x^2+4x+7}$$

Divide the numerator and denominator by 2

$$\int \frac{\frac{1}{2}dx}{\frac{2x^2}{2}+\frac{4x}{2}+\frac{7}{2}} = \frac{1}{2}\int \frac{dx}{x^2+2x+\frac{7}{2}}$$

Complete the square for the denominator $x^2+2x+\frac{7}{2}$

$$\frac{1}{2}\int \frac{dx}{(x^2+2x+1)+\frac{7}{2}-1}$$

Completed square form

$$\frac{1}{2}\int \frac{dx}{(x+1)^2+\frac{5}{2}}$$

Let $u = x+1 \Rightarrow du = dx$

Apply the substitution

$$\frac{1}{2}\int \frac{\overbrace{dx}^{du}}{\underbrace{(x+1)^2+\frac{5}{2}}_{u^2+\left(\frac{1}{4}\right)^2}} = \frac{1}{2}\int \frac{du}{u^2+\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2}$$

Integrate, apply $\int \frac{du}{u^2+a^2} = \frac{1}{a}tan^{-1}\left(\frac{u}{a}\right)+C$. For $\int \frac{du}{u^2+\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2}$ Let $a = \frac{\sqrt{5}}{\sqrt{2}}$

$$\frac{1}{2}\int \frac{du}{u^2+\left(\sqrt{\frac{5}{2}}\right)^2} = \frac{1}{2}\left(\frac{1}{\frac{\sqrt{5}}{\sqrt{2}}}\tan^{-1}\left(\frac{u}{\frac{\sqrt{5}}{\sqrt{2}}}\right)\right)+C$$

Simplify

$$\frac{\sqrt{2}}{2\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{2}u}{\sqrt{5}}\right)+C$$

Back - substitute $u = x+1$

$$\frac{\sqrt{2}}{2\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{5}}(x+1)\right)+C$$

Rationalizing $\frac{\sqrt{2}}{\sqrt{5}}$. $\frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$. So,

$$\frac{\sqrt{10}}{10}\tan^{-1}\left(\frac{\sqrt{10}}{5}(x+1)\right)+C$$

Question 04: **[CLO-4]** **[10+5+5=20]**

a. A study on optimizing revenue function $\boldsymbol{R}$ from a website is,

$$\boldsymbol{R}(\boldsymbol{x}) = (\boldsymbol{x} - \boldsymbol{1})^{\boldsymbol{2}} \boldsymbol{e}^{\boldsymbol{3x}}$$

where x measures the proportion of the total bandwidth requested by a customer.
Find intervals in which the $\boldsymbol{R}(\boldsymbol{x})$ is decreasing, increasing, concave up and concave down.

$R(x) = (x-1)^2 e^{3x}$

$R'(x) = (x-1)^2 \cdot 3e^{3x} + 2(x-1) \cdot e^{3x}$

$R'(x) = 3(x-1)^2 e^{3x} + 2e^{3x}(x-1)$

$R'(x) = e^{3x}(x-1)(3(x-1)+2)$

$R'(x) = e^{3x}(x-1)(3x-3+2)$

$R'(x) = e^{3x}(x-1)(3x-1)$

For Critical points $R'(x) = 0$

$e^{3x}(x-1)(3x-1) = 0$

$e^{3x} = 0, \quad x-1=0, \quad 3x-1=0$

$\downarrow$ Not exist $\quad \boxed{x=1, \quad x = \frac{1}{3}}$

		$R'(x) \to$ sign	Intervals.
1	$(-\infty, 1/3)$	+ve	Increasing $(-\infty, 1/3]$
	$(1/3, 1)$	−ve	decreasing $[1/3, 1]$
	$(1, \infty)$	+ve	Increasing $[1, \infty)$

For Concave up and down

$R'(x) = e^{3x}(x-1)(3x+1)$

$R''(x) = e^{3x}[(x-1)(3x+1)]' + 3e^{3x}(x-1)(3x-1)$

$R''(x) = e^{3x}[(x-1)(3) + (3x-1)] + 3e^{3x}(x-1)(3x-1)$

$R''(x) = e^{3x}(3x-3+3x-1 + 3(3x^2-x-3x+1)]$

$R''(x) = e^{3x}(6x-4+9x^2-12x+3)$

$R''(x) = e^{3x}(9x^2-6x+1)$

$R''(x) = e^{3x}(9x^2-6x-1) = 0$

$\to$ (B)

$e^{3x} = 0, \quad 9x^2-6x+1 = 0$

$\downarrow$ Not exist, $\quad x = \frac{1+\sqrt{2}}{3} = 0.8$

$x = \frac{1-\sqrt{2}}{3} = -0.$

	$R''(x)$	Results
$(-\infty, -0.13)$	+ve	Concave up
$(-0.13, 0.8)$	−ve	Concave down
$(0.8, \infty)$	+ve	Concave up.

b. Show that the function $\boldsymbol{f(t) = 2t + e^{-2t}}$ satisfies the hypotheses of the Mean-Value Theorem over the interval $[\mathbf{-2, 3}]$ and find all values of **c** in the interval $(\mathbf{-2, 3})$ at which the tangent line to the graph of $\boldsymbol{f(t)}$ is parallel to the secant line joining the points$(\mathbf{-2}, \boldsymbol{f(-2)})$ *and* $(\mathbf{3}, \boldsymbol{f(3)})$.

$f(t) = 2t + e^{-2t}$ is continous function on $[-2, 3]$ and diff at $(-2, 3)$.

$f(t) = 2t + e^{-2t}$

$f(-2) = f(a) = 2(-2) + e^{-2(-2)}$

$f(-2) = f(a) = -4 + e^{4}$

$f(3) = f(b) = 2(3) + e^{-2(3)}$

$f(3) = f(b) = 6 + e^{-6}$

$\because f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$f'(t) = 2 - 2e^{-2t}$

$f'(c) = 2 - 2e^{-2c}$

$2 - 2e^{-2c} = \dfrac{6 + e^{-6} + 4 - e^{4}}{3 + 2}$

$5(2 - 2e^{-2c}) = 10 + e^{-6} - e^{4}$

$\cancel{10} - 10e^{-2c} = \cancel{10} + e^{-6} - e^{4}$

$e^{2c} = \dfrac{e^{4} - e^{-6}}{10}$ take ln

$\ln e^{2c} = \ln\left(\dfrac{e^{4} - e^{-6}}{10}\right)$

$2c = 1.697$

$c = \dfrac{1.697}{2}$, $\boxed{c = 0.8486}$ Ans

$$c = -0.8486$$

c. Use L-Hopital's rule to compute the limit

$$\lim_{x\to 0^+}\left[\frac{1}{x^2}-\frac{1}{\tan x}\right]$$

By combining the fractions, we can write the function as a quotient. Since the least common denominator is $x^2 \tan x$, we have

$$\frac{1}{x^2}-\frac{1}{\tan x}=\frac{(\tan x)-x^2}{x^2\tan x}.$$

As $x\to 0^+$, the numerator $\tan x - x^2 \to 0$ and the denominator $x^2\tan x\to 0$. Therefore, we can apply L'Hôpital's rule. Taking the derivatives of the numerator and the denominator, we have

$$\lim_{x\to 0^+}\frac{(\tan x)-x^2}{x^2\tan x}=\lim_{x\to 0^+}\frac{(\sec^2 x)-2x}{x^2\sec^2 x+2x\tan x}.$$

As $x\to 0^+$, $(\sec^2 x)-2x\to 1$ and $x^2\sec^2 x+2x\tan x\to 0$. Since the denominator is positive as x approaches zero from the right, we conclude that

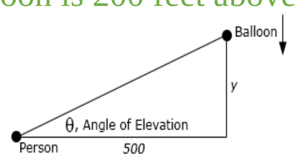
$$\lim_{x\to 0^+}\frac{(\sec^2 x)-2x}{x^2\sec^2 x+2x\tan x}=\infty.$$

Therefore,

$$\lim_{x\to 0^+}\left(\frac{1}{x^2}-\frac{1}{\tan x}\right)=\infty.$$

Question 05: [CLO-4] [5+5+5=15]

a. The angle of elevation is the angle formed by a horizontal line and a line joining the observer's eye to an object above the horizontal line. A person is 500 feet way from the launch point of a hot air balloon. The hot air balloon is starting to come back down at a rate of 15 ft/sec. At what rate is the angle of elevation, θ, changing when the hot air balloon is 200 feet above the ground.



We want to determine θ' when $y = 200$ and we know that $y' = -15$.

There are a variety of equations that we could use here but probably the best one that involves all of the known and needed quantities is,

$$\tan(\theta)=\frac{y}{500}$$

Differentiating with respect to t gives,

$$\sec^2(\theta)\;\theta'=\frac{y'}{500}\qquad\Rightarrow\qquad\theta'=\frac{y'}{500}\cos^2(\theta)$$

To finish off this problem all we need to do is determine the value of θ for the time in question. We can either use the original equation to do this or we could acknowledge that all we really need is $\cos(\theta)$ and we could do a little right triangle trig to determine that.

For this problem we'll just use the original equation to find the value of θ.

$$\tan(\theta)=\tfrac{200}{500}\qquad\Rightarrow\qquad\theta=\tan^{-1}\left(\tfrac{2}{5}\right)=0.38051\text{ radians}$$

The rate of change of the angle of elevation is then,

$$\theta'=\frac{-15}{500}\cos^2(0.38051)=\boxed{-0.02586}$$

b. Find the area of the region bounded by the curves $y = x^4 + \ln(x + 10)$ and $y = x^3 + \ln(x + 10)$

$$y = x^4 + ln(x+10), \text{ and } y = x^3 + \ln(x+10)$$

Let $y = y$

$$x^4 + ln(x+10) = x^3 + \ln(x+10)$$

Solve for x

$$x^4 - x^3 = 0$$

$$x^3(x-1) = 0$$

$$x_1 = 0,\ x_2 = 1$$

Where

$$x^3 + \ln(x+10) \geqslant x^4 + ln(x+10) \text{ on the interval } (0,1)$$

Therefore,

The area is given by

$$\text{Area} = \int_0^1 [(x^3 + \ln(x+10)) - (x^4 + \ln(x+10))]$$

$$\text{Area} = \int_0^1 (x^3 - x^4)$$

Integrate, apply $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, so

$$\text{Area} = \left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$$

Evaluate using The Fundamental Theorem of Calculus

$$\text{Area} = \left[\frac{(1)^4}{4} - \frac{(1)^5}{5}\right] - \left[\frac{()^4}{4} - \frac{()^5}{5}\right]$$

Simplify

$$\text{Area} = \frac{1}{4} - \frac{1}{5}$$

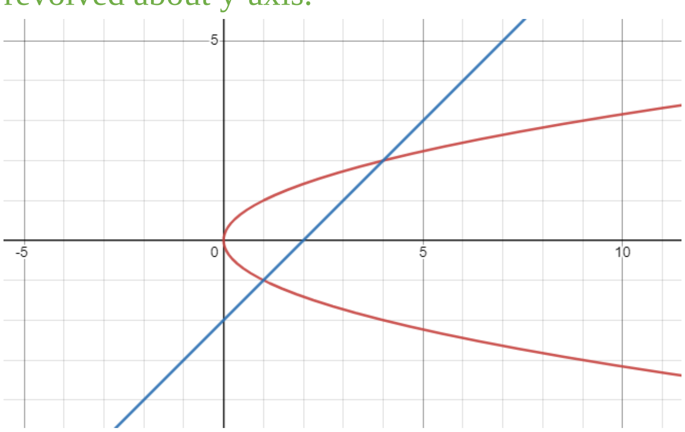
$$\text{Area} = \frac{1}{20}$$

c. If $x = y^2$ and $x = y + 2$

i. Sketch the curves

ii. Determine the point of intersection between two curves

iii. Calculate the volume of the solid that results when region enclosed by the given curves is revolved about y-axis.



The volume of the solid that results when the region enclosed by the given curves is revolved about the $y-$axis.
$g(y) = y^2$ and $f(y) = y + 2$

$$\begin{aligned} y^2 &= y + 2 \\ y^2 - y - 2 &= 0 \\ (y-2)(y+1) &= 0 \\ y &= 2 \text{ and } y = -1 \end{aligned}$$

The volume of the solid

$$\begin{aligned}
\text{Volume} &= \pi \int_c^d ([f(y)]^2 - [g(y)]^2)dy \\
&= \pi \int_{-1}^{2} [(y+2)^2 - (y)^2]dy \\
&= \pi \int_{-1}^{2} (y^2 + 4y + 4 - y^4)dy \\
&= \pi \left[\frac{y^3}{3} + 4\frac{y^2}{2} + 4y - \frac{y^5}{5}\right]_{-1}^{2} \\
&= \pi \left[\frac{y^3}{3} + 2y^2 + 4y - \frac{y^5}{5}\right] \\
&= \pi \left[\left(\frac{2^3}{3} + 2(2)^2 + 4(2) - \frac{2^5}{5}\right) - \left(\frac{(-1)^3}{3} + 2(-1)^2 + 4(-1) - \frac{(-1)^5}{5}\right)\right] \\
&= \pi \left[\left(\frac{8}{3} + 8 + 8 - \frac{32}{5}\right) - \left(-\frac{1}{3} + 2 - 4 + \frac{1}{5}\right)\right] \\
&= \pi \left[\left(\frac{8}{3} + 16 - \frac{32}{5}\right) - \left(-\frac{1}{3} - 2 + \frac{1}{5}\right)\right] \\
&= \pi \left[\left(\frac{40}{15} + \frac{240}{15} - \frac{96}{15}\right) - \left(-\frac{5}{15} - \frac{30}{15} + \frac{3}{15}\right)\right] \\
&= \pi \left[\frac{184}{15} - \left(-\frac{32}{15}\right)\right] \\
&= \frac{216\pi}{15} \\
&= \boxed{\frac{72\pi}{5}}
\end{aligned}$$

Question 06: **[CLO-5]** **[5+5+15=25]**

a. Determine whether or not the following sequence converges. If it does converge, what is its limit? First find a formula for the general term starting with $\boldsymbol{n = 1}$

$$-\frac{4}{13}, \frac{4}{26}, -\frac{4}{39}, \frac{4}{52}, -\frac{4}{65}$$

general term.

$$\left\{ \frac{(-1)^n 4}{13n} \right\}.$$

For Converg.

$$= \frac{4}{13} \lim_{n \to +\infty} \frac{(-1)^n}{n}.$$

$$= \frac{4}{13}(0)$$

$$= 0. \quad \text{Convergent.}$$

b. Determine whether or not the following series converges. If it converges, find its sum

$$\sum_{k=1}^{\infty} \left[\frac{8}{6^{k+1}} + \frac{3}{4^{k+1}} \right]$$

Sum of two geometric Series.

$$= \sum_{k=1}^{\infty} \frac{8}{6^{k+1}} + \sum_{k=1}^{\infty} \frac{3}{4^{k+1}}$$

$$= \left\{ \frac{8}{6^2} + \frac{8}{6^3} + \frac{8}{6^4} + \cdots \right\} + \left\{ \frac{3}{4^2} + \frac{3}{4^3} + \frac{3}{4^4} + \cdots \right\}$$

$$a = \frac{8}{36}, \quad r = \frac{1}{6} < 1 \quad \Bigg| \quad a = \frac{3}{16}, \quad r = \frac{1}{4} < 1$$

Convergent

$$\boldsymbol{s_{n1} = \frac{4}{15}, s_{n2} = \frac{1}{4}}$$

$$\boldsymbol{s_n = \frac{4}{15} + \frac{1}{4} = \frac{31}{60}}$$

c. Use an appropriate convergence test to determine whether or not the following series converges

i. $\sum_{k=1}^{\infty}\left[\frac{\boldsymbol{lnk}}{\boldsymbol{k\sqrt{k}}}\right]$

(a) $\displaystyle\sum_{k=1}^{\infty}\frac{\ln k}{k\sqrt{k}}=\sum_{k=2}^{\infty}\frac{\ln k}{k\sqrt{k}}$ because $\ln 1 = 0$, $\displaystyle\int_{2}^{+\infty}\frac{\ln x}{x^{3/2}}dx=\lim_{\ell\to+\infty}\left[-\frac{2\ln x}{x^{1/2}}-\frac{4}{x^{1/2}}\right]_2^{\ell}=\sqrt{2}(\ln 2+2)$ which implies that $\displaystyle\sum_{k=2}^{\infty}\frac{\ln k}{k^{3/2}}$ converges. (Integral Test, assumptions are true.)

ii. $\sum_{k=1}^{\infty}\left[\frac{\boldsymbol{k^{4/3}}}{\boldsymbol{8k^2+5k+1}}\right]$

(b) Comparison Test: $\displaystyle\frac{k^{4/3}}{8k^2+5k+1}\geq\frac{k^{4/3}}{8k^2+5k^2+k^2}=\frac{1}{14k^{2/3}}$, $\displaystyle\frac{1}{14}\sum_{k=1}^{\infty}\frac{1}{k^{2/3}}$ diverges (p-series with $p = 2/3 < 1$), so the original series also diverges.

iii. $\sum_{k=1}^{\infty}\left[\frac{\boldsymbol{(k+1)!}}{\boldsymbol{5^k k!}}\right]$

$$\rho = \lim_{k\to+\infty}\frac{U_{k+1}}{U_k}$$

$$= \lim_{k\to+\infty}\frac{(k+2)!}{5^{k+1}\cdot(k+1)!}\Big/\frac{(k+1)!}{5^k\cdot k!}$$

$$= \lim_{k\to+\infty}\frac{(k+2)(k+1)!}{5^k\cdot 5\cdot(k+1)\cdot k!}\Big/\frac{(k+1)!}{5^k\cdot k!}$$

$$= \lim_{k\to+\infty}\frac{k+2}{5(k+1)}$$

$$\rho = \frac{1}{5} < 1 \qquad \text{Convergent.}$$