| Course Code: MT – 1003 | Course Name: Calculus and Analytical Geometry |
|---|---|
| Instructor Name: Dr. Khusro Mian/ Dr. Fahad Riaz/ Ms. Asma Masood/ Mr. Nadeem Khan/ Ms. Afreen Naz/ Ms. Urooj/ Ms. Alishba Tariq/ Ms. Javeria Iftikhar/ Ms. Sadia Khan | |
| Student Roll No: | Section No: |

Instructions:
- Return the question paper.
- **Attempt all questions.**
- Read each question completely before answering it. **There are 06 Questions and 02 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- **Solve the paper according to the sequence given in the question paper.**
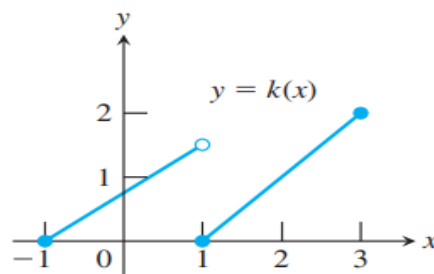- Graphical Calculator is not allowed.

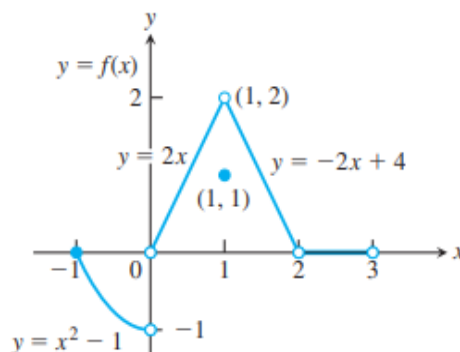**Time: 120 minutes** **Max Marks: 60**

**Question 01:** **[02 : 03 : 04 :10]**

a) Find $\lim\limits_{x \to 0} \dfrac{\sqrt{x+25}-5}{x}$

b) For the given function $k(x)$, is $k(x)$ continuous on $[-1,3]$? If not, where does it fails and what is the type of discontinuity?

c) Consider

$$f(x) = \begin{cases} x^2 - 1 & -1 \le x < 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x + 4 & 1 < x < 2 \\ 0 & 2 < x < 3 \end{cases}$$

i. Does $f(-1)$ exist? If yes, $f(-1)$ ?
ii. Does $\lim\limits_{x \to -1^+} f(x) = f(-1)$ ?
iii. Is $f(x)$ continuous at $x = 1$? If not why?
iv. Is $f(x)$ continuous at $x = 3$? If not why?

**Question 02:** **[02*5=10]**

For the given function $f(x) = x^3 - x^{\frac{1}{3}}$, find

a) all the critical points and identify their type
b) intervals of increasing and decreasing functions
c) inflection points, if any
d) intervals of concavity
e) relative extrema

**Question 03:**                                                                                    [5+5=10]

For given functions, find $f'(x)$

a)  $f(x) = \dfrac{x - \sqrt{x}}{\sqrt[3]{x}}$

b)  $f(x) = \cos^3\left(\dfrac{x}{x+1}\right)$

**Question 04:**                                                                                    [2.5*4=10]

For the given functions, find limit using L'Hospital's rule.

a)  $\displaystyle\lim_{x \to \pi^-} (x - \pi)\tan\frac{1}{2}x$

b)  $\displaystyle\lim_{x \to 0}(1 - \frac{3}{x})^x$

c)  $\displaystyle\lim_{x \to +\infty} \frac{\ln x}{x}$

d)  $\displaystyle\lim_{x \to +\infty} \sqrt{x^2 + x} - x$

**Question 05:**                                                                                    [6+4=10]

a)  For the following functions, find $\dfrac{dy}{dx}$ using implicit differentiation.

   i.   $x\cos y = y$
   ii.  $\tan^3(xy^2 + y) = x$

b)  A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/s. How rapidly is the area enclosed by the ripple increasing at the end of 10 s?

**Question 06:**                                                                                    [04+03+03=10]

a)  For the given functions, $f(x) = (x^2 + x)^{\frac{2}{3}}$ find absolute extrema $on\ [-2,3]$

b)  On the function $f(x) = \ln(4 + 2x - x^2)$ on $[-1,3]$, verify that the hypotheses of Rolle's Theorem are satisfied on the given interval and find all values of $c$ in that interval that satisfy the conclusion of the theorem.

c)  On the function $f(x) = x - \dfrac{1}{x}$ on $[3,4]$, verify that the hypotheses of the Mean-Value Theorem are satisfied on the given interval and find all values of $c$ in that interval that satisfy the conclusion of the theorem.

**The End** ☺