8.5

"Discrete Mathematics"

Ex: 8.2 & 8.5

Example 103

Freshman = 1807

$|A| = 453$

$|B| = 567$

$|A \cap B| = 299$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 453 + 567 - 299$

$|A \cup B| = 721$ → Taking courses

Neither course

$|A \cup B|' = 1807 - |A \cup B|$

$= 1807 - 721$

$|A \cup B|' = 1086$

Example 104

$|S| = 1232$

$|F| = 879$

$|R| = 114$

$|S \cap F| = 103$

$|S \cap R| = 23$

$|F \cap R| = 14$

$|S \cup F \cup R| = 2092$

$|S \cap F \cap R| = ?$

$|S \cup F \cup R| = |S| + |F| + |R| - |S \cap F| - |F \cap R| - |S \cap R| + |S \cap F \cap R|$

$2092 = 1232 + 879 + 114 - 103 - 14 - 23 + |S \cap F \cap R|$

$|S \cap F \cap R| = 7$

Example 10.2

$(999 \div 11) = 90$ $(999 \div 7) = 142$

$1, 2, 3, 4, \ldots\ldots, 990, 991, 992, 993, 994, 996, 997, 998, 999, 1000$

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A| = 90$ ($\div$ by 11)

$|B| = 142$ ($\div$ by 7)

$|A \cap B| = 12$ ($\div$ by both '7' and '11')

$|A \cup B| = 90 + 142 - 12$

$\boxed{|A \cup B| = 220}$

Ex: 8.2

d $a_n = a_{n-1} + 2a_{n-2}$; $a_0 = 2$, $a_1 = 7$

Theorem;

$r^2 - c_1 r - c_2 = 0$; $a_n = c_1 a_{n-1} + c_2 a_{n-2}$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

Sol:-

Here; $c_1 = 1$, $c_2 = 2$

Now;

$r^2 - c_1 r - c_2 = 0$

$r^2 - r - 2 = 0$

$r^2 + r - 2r - 2 = 0$

$r(r+1) - 2(r+1) = 0$

$(r+1)(r-2) = 0$

$r_1 = -1$

$r_2 = 2$

Now;

~~$a_n = c_1 a_{n-1} + c_2 a_{n-2}$~~

~~$a_n = a_{n-1} + 2a_{n-2}$~~

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ $\Rightarrow a_n = (-1)(-1)^n + (3)(2)^n$

$2 = \alpha_1 + \alpha_2$

$7 = -\alpha_1 + 2\alpha_2$

$9 = 3\alpha_2$

$\boxed{\alpha_2 = 3}$ $\Rightarrow \boxed{\alpha_1 = -1}$

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Q7 (a) $a_n = a_{n-1} + 6a_{n-2}$; $n \geq 2$, $a_0 = 3$, $a_1 = 6$

Theorem:-

$$r^2 - c_1 r - c_2 = 0$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

Now;

$$c_1 = 1 \quad , c_2 = 6$$

Therefore;

$$r^2 - r - 6 = 0$$

$$r^2 + 2r - 3r - 6 = 0$$

$$r(r+2) - 3(r+2) = 0$$

$$(r+2)(r-3) = 0$$

So $r_1 = -2$

$r_2 = 3$

Now;

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

$$a_0 = \alpha_1(-2)^0 + \alpha_2(3)^0$$

$$3 = \alpha_1 + \alpha_2 \qquad -(i)$$

$$a_1 = \alpha_1(-2)^1 + \alpha_2(3)^1$$

$$6 = -2\alpha_1 + 3\alpha_2 \qquad -(ii)$$

Solving (i) and (ii)

$$(i) \Rightarrow 6 = 2\alpha_1 + 2\alpha_2$$

$$6 = -2\alpha_1 + 3\alpha_2$$

$$12 = +5\alpha_2$$

$$\alpha_2 = 12/5 \quad \Rightarrow \alpha_1 = 3/5$$

$$a_n = (3/5)(-2)^n + (12/5)(3)^n$$

CamScanner

Theorem: 02 $\quad x^2 - c_1 x - c_2 = 0 \quad \because c_2 \neq 0.$

$$\boxed{a_n = \alpha_1 x_0^n + \alpha_2 (n) x_0^n}$$

Q $\quad a_n = 6a_{n-1} - 9a_{n-2} \quad ; a_0 = 1, a_1 = 6$

$x^2 - c_1 x - c_2 = 0 \quad \because c_1 = 6, \; c_2 = -9$

$x^2 - 6x - 9 = 0$

$x^2 - 3x - 3x - 9 = 0$

$(x-3)^2 = 0$

$x - 3 = 0$

$\boxed{x = 3}$

Now;

$a_n = \alpha_1 x_0^n + \alpha_2 n x_0^n$

$a_n = \alpha_1 (3)^0 + \alpha_2 (0)(3)^0$

$a_n = \alpha_1$ ~~$+\alpha_2$~~

$\boxed{1 = \alpha_1}$ ~~$+\alpha_2$~~ $\quad -(i)$

$6 = 3\alpha_1 + 3\alpha_2 \quad -(ii)$

solving (i) and (ii)

$3 = 3\alpha_1 + 3\alpha_2$

$6 = 3\alpha_1 + 3\alpha_2$

(ii) $\Rightarrow \quad 6 = 3 + 3\alpha_2$

$3 = 3\alpha_2$

$\boxed{\alpha_2 = 1}$

~~Now; $a_n = 1(4)$.~~

Therefore; $\quad a_n = (1)(3)^n + (1)(n)(3)^n$

Q2 $a_0 = 0,\ a_1 = -1,\quad a_n - 7a_{n-1} + 12a_{n-2} = 0 \ ;\ n \geq 2$

$$a_n = 7a_{n-1} - 12a_{n-2}$$

$$c_1 = 7,\quad c_2 = -12$$

Recurrence;

$$r^2 - c_1 r - c_2 = 0$$

$$r^2 - 7r + 12 = 0$$

$$r^2 - 3r - 4r + 12 = 0$$

$$r(r-3) - 4(r-3) = 0$$

$$(r-3)(r-4) = 0$$

$$r_1 = 3$$

$$r_2 = 4$$

Now;

$$a_n = \alpha_1(3)^n + \alpha_2(4)^n$$

$$0 = \alpha_1 + \alpha_2 \quad -(i)$$

$$-1 = 3\alpha_1 + 4\alpha_2 \quad -(ii)$$

solving (i) and (ii)

$$(i) \Rightarrow \quad 0 = 3\alpha_1 + 3\alpha_2$$

$$-1 = 3\alpha_1 + 4\alpha_2$$

$$+1 = -\alpha_2$$

$$\alpha_2 = -1 \quad \Rightarrow \quad \alpha_1 = 1$$

Now;

$$a_n = (1)(3)^n + (-1)(4)^n$$

Q3 $a_0 = 1$, $a_1 = 1$ for $n \geq 3$; $a_n + 2a_{n-1} - 15a_{n-2} = 0$

$c_1 = -2$, $c_2 = 15$

Recurrence;

$$r^2 + 2r - 15 = 0$$

$$r^2 - 3r + 5r - 15 = 0$$

$$r(r-3) + 5(r-3) = 0$$

$$(r-3)(r+5) = 0$$

$r_1 = 3$

$r_2 = -5$

Now;

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

$$a_n = \alpha_1(3)^n + \alpha_2(-5)^n$$

$1 \, \cancel{a_0} = \alpha_1 + \alpha_2$ —(i)

$1 = 3\alpha_1 - 5\alpha_2$ —(ii)

solving (i) and (ii)

(i)×3 $\quad 3 = \cancel{3\alpha_1} + 3\alpha_2$

$\quad 1 = 3\alpha_1 - 5\alpha_2$

$\quad - \quad - \quad +$

$\quad 2 = 8\alpha_2$

$\alpha_2 = 1/4 \quad \Rightarrow \quad \alpha_1 = 3/4$

Now;

$$a_n = (3/4)(3)^n + (1/4)(-5)^n$$