Predicate and Quantifier

1. Given propositional function q(x, y): x + y = 1, which of the following are propositions; which are not? For those that are, determine their truth values.
   a) q(x, y) b) q(-6, 7) c) q(x + 1, -x)
   d) q(x, 3) e) q(1, 1) f) q(5, -4)

2. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
   a) $\exists x(x^2 = 2)$ b) $\exists x(x^2 = -1)$ c) $\forall x(x^2 + 2 \geq 1)$ d) $\exists x(x^2 = x)$

3. Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a) Everybody can fool Bob.
   b) Alice can fool everybody.
   c) Everybody can fool somebody.
   d) There is no one who can fool everybody.
   e) Everyone can be fooled by somebody.

4. Let P(x) be the statement "x can speak Russian" and let Q(x) be the statement "x knows the computer language C++." Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.
   a) There is a student at your school who can speak Russian and who knows C++.
   b) There is a student at your school who can speak Russian but who doesn't know C++.
   c) Every student at your school either can speak Russian or knows C++.
   d) No student at your school can speak Russian or knows C++.

5. Let Q(x, y) be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.
   a) $\exists x \exists y\ Q(x, y)$ b) $\exists x \forall y\ Q(x, y)$ c) $\forall x \exists y\ Q(x, y)$
   d) $\exists y \forall x\ Q(x, y)$ e) $\forall y \exists x\ Q(x, y)$ f) $\forall x \forall y\ Q(x, y)$

Law of inferences

6. What rule of inference is used in each of these arguments?
   a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
   b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
   c) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will get sunburned.

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