MVC Assignment No. 02

23k-3032
(Shah Hunain)
BSE-2A

Q1

(a) $\int_1^2 \int_4^6 \frac{x}{y^2}\, dx\, dy$

$= \int_1^2 \frac{1}{y^2} \left| \frac{x^2}{2} \right|_4^6 dy$

$= \int_1^2 \frac{1}{2y^2} \left[(6)^2 - (4)^2 \right] dy$

$= \int_1^2 \frac{20}{2y^2}\, dy$

$= \int_1^2 \frac{10}{y^2}\, dy$

$= 10 \left| \frac{y^{-1}}{-1} \right|_1^2$

$= 10 \left| \frac{-1}{y} \right|_1^2$

$= 10 \left(-\frac{1}{2} + 1 \right)$

$= 10 \left(\frac{1}{2} \right)$

$\boxed{= 5}$

(b) $\int\int x^2 + y^2\, dx\, dy$

$= \int \left(\frac{x^3}{3} + xy^2 \right) dy$

$= \frac{x^3 y}{3} + \frac{xy^3}{3} + c$

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c) $\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx$

$= \int_0^1 xe^x \left|\ln y\right|_1^2 dx$

$= \int_0^1 xe^x [\ln(2) - \ln(1)]\, dx$

$= \int_0^1 xe^x [\ln(2) - 0]\, dx$

$= \ln(2) \int_0^1 xe^x\, dx \quad -(A)$

let $u = e^x \Rightarrow \ln u = x$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$= \ln(2) \int_0^1 \ln(u)$

using by parts

$\int_0^1 x e^x\, dx$

here;

$u = x$; $v = e^x$

Now;

$\int u \cdot v\, dx = u \int v\, dx - \int \left((u)' \int v\, dx\right) dx$

Then;

$= x \int_0^1 e^x\, dx - \int_0^1 ((1)\, e^x)\, dx$

$= \left|xe^x - e^x\right|_0^1$

$= [(1)e^1 - e^1] - ((0)e^0 - e^0)$

$= (e^1 - e^1) - (0 - 1)$

$= 0 - 0 + 1$

$= 1$

$(A) \Rightarrow$

$= \ln(2)\ (1)$

$\boxed{= \ln(2)}$

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Q2 $5000 \le L \le 6000$

$20000 \le K \le 30000$

$20 \le K \le 30$ $\because$ (K is capital investment in units of 1000 \$)

The function is

$$P_{avg} = \frac{1}{A(D)} \int_D \int P(L,K)\, dA$$

$$A(D) = (6000-5000)(30-20)$$

$$= (1000)(10)$$

$$\boxed{A(D) = 10{,}000}$$

Now;

$$P_{avg} = \frac{1}{10000} \int_{5000}^{6000} \int_{20}^{30} 70L^{0.6} K^{0.4}\, dK\, dL$$

$$= \frac{1}{10000} \int_{5000}^{6000} 70L^{0.6} \left| \left(\frac{K^{0.4}}{1.4} \right) \right|_{20}^{30} dL$$

$$= \frac{1}{10000} \int_{5000}^{6000} \frac{70L^{0.6}}{1.4} \left[(30)^{1.4} - (20)^{1.4} \right] dL$$

$$= \frac{1}{10000} \int_{5000}^{6000} 50\, L^{0.6} (50.2927)\, dL$$

$$= 0.2532 \int_{5000}^{6000} L^{0.6}\, dL$$

$$= 0.2532 \left| \frac{L^{1.6}}{1.6} \right|_{5000}^{6000}$$

$$= 0.1582 \left[(6000)^{1.6} - (5000)^{1.6} \right]$$

$$\boxed{P_{avg} = 44401.77005 \text{ units}}$$

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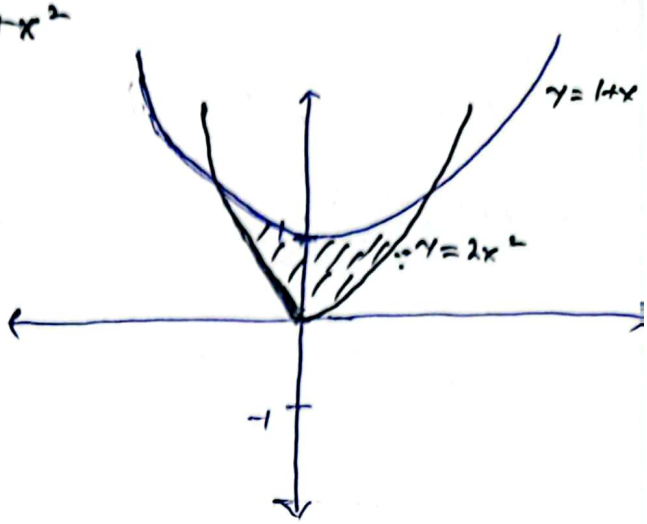
Q3

(a) $\iint_D (x+2y)\,dA$; $y = 2x^2$ and $y = 1+x^2$



$y = 2x^2$ —(i)

$y = 1+x^2$ —(ii)

comparing (i) and (ii)

$2x^2 = 1+x^2$

$x^2 = 1$

$\boxed{x = \pm 1}$

Put in (i)

$y = 2(\pm 1)^2$

$y = 2$

$(-1, 2)$ and $(1, 2)$

$$= \int_{-1}^{1}\int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx$$

$$= \int_{-1}^{1} \left| xy + \frac{2y^2}{2} \right|_{2x^2}^{1+x^2} dx$$

$$= \int_{-1}^{1} (xy + y^2)_{2x^2}^{1+x^2}\,dx$$

$$= \int_{-1}^{1} \left(\left[x(1+x^2) + (1+x^2)^2 \right] - \left[x(2x^2) + (2x^2)^2 \right] \right) dx$$

$$= \int_{-1}^{1} (x + x^3 + 1 + 2x^2 + x^4 - 2x^3 - 4x^4)\,dx$$

$$= \int_{-1}^{1} (-3x^4 - x^3 + 2x^2 + x + 1)\,dx$$

$$= \left| -\frac{3x^5}{5} - \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x \right|_{-1}^{1}$$

$$= \left(\frac{3}{5} - \frac{1}{4} - \frac{2}{3} + \frac{1}{2} - 1\right) - \left(-\frac{3}{5} - \frac{1}{4} + \frac{2}{3} + \frac{1}{2} + 1\right)$$

$$= \frac{3}{5} - \cancel{\frac{1}{4}} - \frac{2}{3} + \cancel{\frac{1}{2}} - 1 + \frac{3}{5} + \cancel{\frac{1}{4}} - \frac{2}{3} - \cancel{\frac{1}{2}} - 1$$

$$\boxed{= \frac{32}{15} \text{ cubic units}}$$

(b) $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy$

$= \int_0^4 y^2 \left| \frac{x^2}{2} \right|_0^{\sqrt{y}} dy$

$= \int_0^4 y^2 \left[\frac{(\sqrt{y})^2}{2} - \frac{(0)^2}{2} \right] dy$

$= \int_0^4 y^2 \left(\frac{y}{2} - 0 \right) dy$

$= \frac{1}{2} \int_0^4 y^3 \, dy$

$= \frac{1}{2} \left| \frac{y^4}{4} \right|_0^4$

$= \frac{1}{8} \left[(4)^4 - (0)^4 \right]$

$\boxed{= 32 \text{ cubic units}}$

(c) $\iint_R (4 - y^2) \, dA$ $\quad \because y^2 = 2x$ and $y^2 = 8 - 2x$

$y^2 = 2x \quad -(i)$

$y^2 = 8 - 2x \quad -(ii)$

comparing (i) and (ii)

$2x = 8 - 2x$

$4x = 8$

$\boxed{x = 2}$

Put in (i)

$y^2 = 2(2)$

$y^2 = 4$

$\boxed{y = \pm 2}$

$(2, 2)$ and $(2, -2)$

Now;

$= \int_{-2}^{2} \int_{\frac{y^2}{2}}^{\frac{8-y^2}{2}} (4 - y^2) \, dx \, dy$

$= \int_{-2}^{2} \left| 4x - xy^2 \right|_{\frac{y^2}{2}}^{\frac{8-y^2}{2}} dy$

$$= \int_{-2}^{2} \left[y\left(\frac{8-y^2}{2}\right) - \left(\frac{8-y^2}{2}\right) y^2 \right] - \left[y\left(\frac{y^2}{2}\right) - \left(\frac{y^2}{2}\right) y^2 \right] dy$$

$$= \int_{-2}^{2} \left(\frac{32-4y^2}{2} - \left(\frac{8y^2 - y^4}{2}\right) - \frac{4y^2}{2} + \frac{y^4}{2} \right) dy$$

$$= \frac{1}{2} \int_{-2}^{2} \left(32 - 4y^2 - 8y^2 + y^4 - 4y^2 + y^4\right) dy$$

$$= \frac{1}{2} \int_{-2}^{2} \left(y^4 + y^4 - 8y^2 - 8y^2 + 32\right) dy$$

$$= \frac{1}{2} \int_{-2}^{2} \left(2y^4 - 16y^2 + 32\right) dy$$

$$= \int_{-2}^{2} \left(y^4 - 8y^2 + 16\right) dy$$

$$= \left| \frac{y^5}{5} - \frac{8y^3}{3} + 16y \right|_{-2}^{2}$$

$$= \frac{256}{15} - \left(-\frac{256}{15}\right)$$

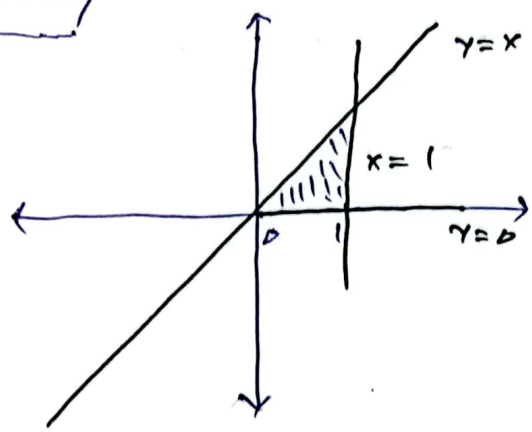
$$\boxed{= 34.133 \text{ cubic units}}$$

$$= \frac{1}{2} \left| \frac{y^5}{5} + \frac{y^4}{4} - \frac{8y^3}{3} - \frac{8y^2}{2} + 32y \right|_{-2}^{2}$$

$$= \frac{1}{2} \left[\frac{552}{15} - \left(\frac{-216}{15}\right) \right]$$

$$= \frac{1}{2} \left(\frac{1472}{15} \right)$$

$$\boxed{= \frac{736}{15} \text{ cubic units}}$$



Ex 2

$$= \int_{0}^{1} \int_{0}^{x} (3 - x - y)\, dy\, dx$$

$$= \int_{0}^{1} \left| 3y - xy - \frac{y^2}{2} \right|_{0}^{x} dx$$

$$= \int_{0}^{1} \left(3x - x^2 - \frac{x^2}{2}\right) dx$$

$$= \frac{1}{2} \int_{0}^{1} \left(6x - 2x^2 - x^2\right) dx$$

$$= \frac{1}{2} \left(\frac{6x^2}{2} - \frac{2x^3}{3} - \frac{x^3}{3} \right)_{0}^{1}$$

$$= \frac{1}{2} \left| 3x^2 - x^3 \right|_{0}^{1}$$

$$= \frac{1}{2} \left([3(1) - (1)] - (0 - 0) \right)$$

$$= \frac{1}{2} (3 - 1)$$

$$= \frac{1}{2} (2)$$

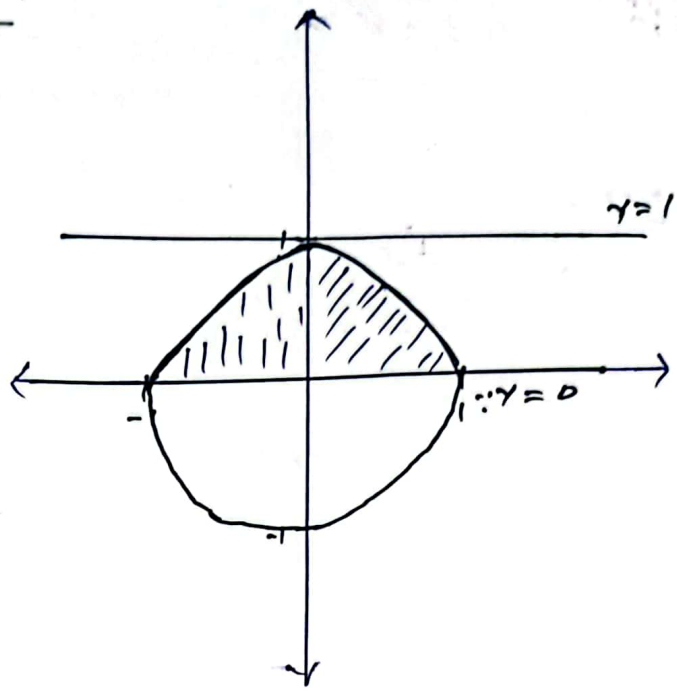
$$\boxed{= 1 \text{ cubic units}}$$

Q5

$x^2 = 1-y^2$ $\quad x^2 - 1 = y^2$

$y = \sqrt{x^2-1}$

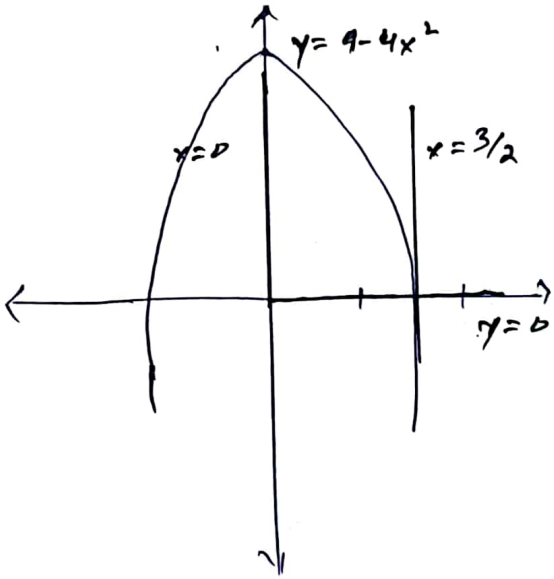
(a) $\displaystyle\int_{y=0}^{y=1}\int_{x=-\sqrt{1-y^2}}^{x=\sqrt{1-y^2}} 3y \, dx \, dy$ (II)



NOW;

$\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 3y \, dy \, dx$ (I)

(b) $\displaystyle\int_{x=0}^{x=3/2}\int_{y=0}^{y=9-4x^2} 16x \, dy \, dx$ (I)



NOW;

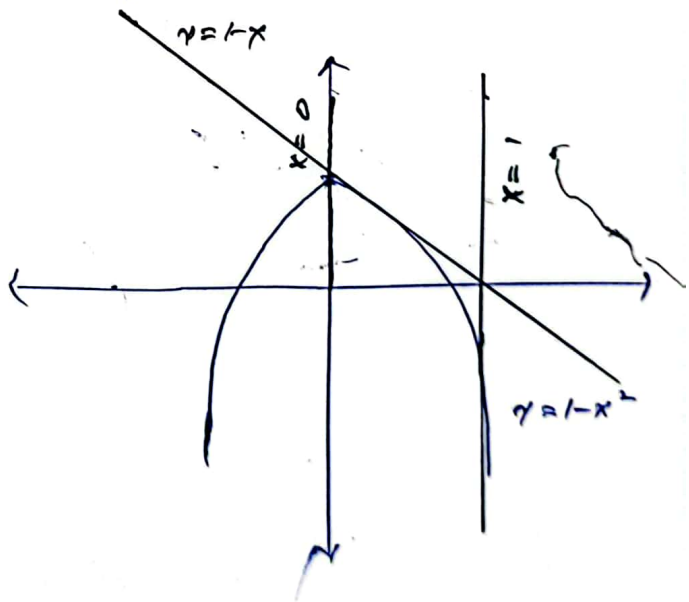
$\displaystyle\int_{0}^{9}\int_{0}^{\frac{9-y}{2}} (16x) \, dx \, dy$ (II)

$x^2 = 1-y \Rightarrow -x^2 = (y-1)$

(c) $\displaystyle\int_{x=0}^{x=1}\int_{y=1-x}^{y=1-x^2} dy \, dx$ (I)

$x = 1-y$

$-x = y-1$



NOW;

$\displaystyle\int_{0}^{1}\int_{1-y}^{\sqrt{1-y}} dx \, dy$

$\underline{\underline{1b}}$

$r^2 = 4\cos 2\theta$

$r = 2\sqrt{\cos 2\theta}$ $\quad$ ∵ sq. root on b.s

$\underline{\underline{\text{Now}}}$;

∵ $x = r\cos\theta$

$x = 2\sqrt{\cos 2\theta}\cos\theta$

∵ $y = x\sin\theta$

$y = 2\sqrt{\cos 2\theta}\sin\theta$

let $r = 0$

$2\sqrt{\cos 2\theta} = 0$

$\sqrt{\cos 2\theta} = 0$

$\cos 2\theta = 0$

$2\theta = \cos^{-1}(0)$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

$\underline{\underline{\text{Now}}}$;

$$\text{Area} = \iint_R r\,dr\,d\theta$$

$$= \int_{3\pi/4}^{5\pi/4}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta$$

$$= \int_{3\pi/4}^{5\pi/4}\left|\frac{r^2}{2}\right|_0^{2\sqrt{\cos 2\theta}} d\theta$$

$$= \left|\frac{1}{2}\int_{3\pi/4}^{5\pi/4}\left[(2\sqrt{\cos 2\theta})^2 - (0)^2\right] d\theta\right.$$

$$= \frac{1}{2}\int_{3\pi/4}^{5\pi/4}(4\cos 2\theta)\,d\theta$$

$$= 2\int_{3\pi/4}^{5\pi/4}\cos 2\theta$$

$$= 2\left|\frac{\sin 2\theta}{2}\right|_{3\pi/4}^{5\pi/4}$$

$$= (\sin 2\theta)_{3\pi/4}^{5\pi/4}$$

$$= 1-(-1) = 1+1$$

$$\text{Area} = 2$$

For lemniscate;

$\text{Area} = 2 \times 2$

$\boxed{\text{Area} = 7 \text{ sq. units}}$

Q7

(a) $\int_{y=0}^{y=1}\int_{x=0}^{x=\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$

$x = r\cos\theta$ —(A) and $y = r\sin\theta$ —(B) also $dx\,dy = r\,dr\,d\theta$

$$\int_0^1\int_0^{\sqrt{1-y^2}} (r^2)\, r\,dr\,d\theta$$

using (B)

~~Put $y=0$~~
~~$0 = r\sin\theta$~~
~~$\theta = 0$~~

~~, Put $y=1$~~
~~, $1 = r\sin\theta$~~

We know that;

$$x^2+y^2 = r^2$$

So; when $x=0$ and $y=0$

$r^2 = 0^2+0^2$

$r = 0$

, when $x=\sqrt{1-y^2}$ and $y=1$

$r^2 = (\sqrt{1-y^2})^2+(1)^2$

$r^2 = 1-y^2+1$

$r^2 = 2-y^2$

$r^2 = 2-(1)^2$

$r^2 = 2-1 = 1$

$\boxed{r=1}$

~~$x = r\cos\theta$~~

~~$\cos\theta = \frac{x}{r}$~~

~~$\theta = \cos^{-1}\left(\frac{0}{0}\right) = \frac{\pi}{2}$~~

~~; $r\cos^{-1}$~~

$\because y = r\sin\theta$

When $y=0$ and $r=0$

$0 = 0\sin\theta$

$\boxed{\theta = 0}$

; when $y=1$ and $r=1$

, $1 = 1\sin\theta$

, $\boxed{\theta = \frac{\pi}{2}}$

Therefore;

$$= \int_0^{\pi/2}\int_0^1 r^3\,dr\,d\theta$$

$$= \int_0^{\pi/2} \left| \frac{\theta^4}{4} \right|_0^1 d\theta$$

$$= \int_0^{\pi/2} \left(\frac{1}{4} - 0 \right) d\theta$$

$$= \left| \frac{\theta}{4} \right|_0^{\pi/2}$$

$$= \frac{\pi}{8} - 0$$

$$\boxed{= \frac{\pi}{8} \text{ cubic units}}$$

(b) $$\int_{x=-1}^{x=0} \int_{y=-\sqrt{1-x^2}}^{y=0} \frac{2}{1+\sqrt{x^2+y^2}} \, dy \, dx$$

when $x=-1$ and $y=-\sqrt{1-x^2}$; when $x=0$ and $y=0$

$\because x^2+y^2=r^2$ $\qquad$ $\because x^2+y^2=r^2$

$(-1)^2+(-\sqrt{1-x^2})^2=r^2$ $\qquad$ $0+0=r^2$

$1+\sqrt{1-(-1)^2}=r^2$ $\qquad$ $\boxed{r=0}$

$1+0=r^2$

$\boxed{r=1}$

Now ;

when $x=-1$ and $r=1$; when $x=0$ and $r=0$

$\because x=r\cos\theta$, $\theta=\cos^{-1}(0)$

$\theta=\cos^{-1}\left(\frac{-1}{1}\right)=\pi$, $\theta=\frac{3\pi}{2}$

Therefore ;

$$= \int_{\pi}^{\frac{3\pi}{2}} \int_0^1 \frac{2r}{1+r} \, dr \, d\theta$$

$$= \int_{\pi}^{3\pi/2} 2 \int_0^1 \frac{r}{1+r} \, dr \, d\theta$$

let $u=1+r \Rightarrow r=u-1$

$\frac{du}{dr}=1 \Rightarrow du=dr$

$$= \int_{\pi}^{\frac{3\pi}{2}} \int_{0}^{1} \frac{(u-1)}{u} \, du \, d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} 2 \int_{0}^{1} \left(1 - \frac{1}{u}\right) du \, d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} 2 \left| u - \ln u \right|_{0}^{1} d\theta$$

$$= 2 \int_{\pi}^{\frac{3\pi}{2}} \left| (1+1) - \ln(1+1) \right|_{0}^{1} d\theta$$

$$= 2 \int_{\pi}^{3\pi/2} \left(\left[(1+1) - \ln(1+1) \right] - \left[(1+0) - \ln(1+0) \right] \right) d\theta$$

$$= 2 \int_{\pi}^{\frac{3\pi}{2}} \left[2 - \ln(2) - 1 + \ln(1) \right] d\theta$$

$$= 2 \int_{\pi}^{3\pi/2} \left(1 - \ln(2) + 0 \right) d\theta$$

$$= 2 \left| \theta - \theta \ln(2) \right|_{\pi}^{3\pi/2}$$

$$= 2 \left[\frac{3\pi}{2} - \frac{3\pi}{2} \ln(2) \right] - \left(\pi - \pi \ln(2) \right)$$

$$= 2 \left[\frac{3\pi}{2} - \frac{3\pi}{2} \ln(2) - \pi + \pi \ln(2) \right]$$

$$= 2 \left[\frac{\pi}{2} - \frac{\pi}{2} \ln(2) \right]$$

$$= \pi - \pi \ln(2)$$

$$\boxed{= \pi \left[1 - \ln(2) \right] \text{ cubic units}}$$

(2)

$$\int_{x=-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$$

when $x=-1$ and $y=-\sqrt{1-x^2}$; when $x=1$ and $y=\sqrt{1-x^2}$

$\because x^2+y^2=r^2$

$(-1)^2+(-\sqrt{1-x^2})^2=r^2$

$1+(\sqrt{1-(-1)^2})^2=r^2$

$1+0=r^2$

$\boxed{r=1}$

, $(1)^2+(\sqrt{1-x^2})^2=r^2$

, $1+(1-1)=r^2$

, $\boxed{r=1}$

~~when $x=-1$ and $r=1$~~

~~$x=r\cos\theta$~~

~~$\theta=\cos^{-1}\left(\frac{-1}{1}\right)=\pi$~~

~~; when $x=1$ and $r=1$~~

~~$x=r\cos\theta$~~

~~$\theta=\cos^{-1}\left(\frac{1}{1}\right)=0$~~

since $r=1$ (not changing)

$0\le\theta\le 2\pi$

<u>Therefore;</u>

$$=\int_0^{2\pi}\int_0^1 \frac{2r}{(1+r^2)^2}\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^1 (1+r^2)^{-2}(2r)\,dr\,d\theta$$

$$=\int_0^{2\pi}\left|\frac{(1+r^2)^{-1}}{-1}\right|_0^1 d\theta$$

$$=\int_0^{2\pi}\left|\frac{-1}{(1+r^2)}\right|_0^1 d\theta$$

$$=\int_0^{2\pi}\left[\frac{-1}{2}-(-1)\right]d\theta$$

$$=\int_0^{2\pi}\left(\frac{-1}{2}+1\right)d\theta$$

$$=\int_0^{2\pi}\left(\frac{1}{2}\right)d\theta$$

$$=\left|\frac{\theta}{2}\right|_0^{2\pi}=\left(\frac{2\pi}{2}-0\right)=\boxed{\pi \text{ cubic units}}$$

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Q8

(A) $z = x^2+y^2$ lying under $z = 9$

$$V = \iint_R f(x,y)\, dA \qquad ; f(x,y) = \text{height}$$

$$V = A \cdot h$$

$$A = \frac{V}{h}$$

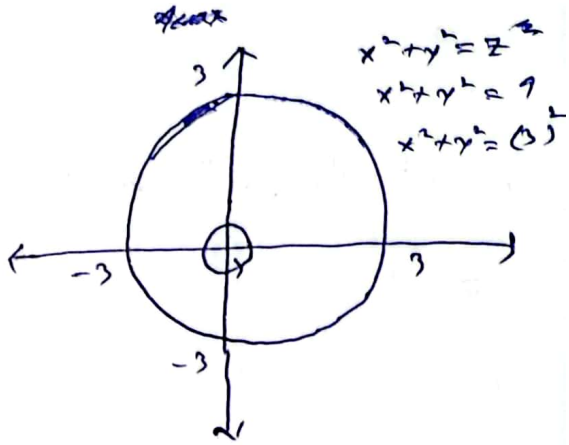
$$A = \iint_R \frac{\cancel{f(x,y)}}{\cancel{f(x,y)}}\, dA$$

$$\boxed{A = \iint_R dA}$$

NOW;

$$z = x^2+y^2$$

$$\frac{\partial z}{\partial x} = 2x \quad \text{and} \quad \frac{\partial z}{\partial y} = 2y$$



NOW;

$$\text{Surface Area} = \iint_R \sqrt{(2x)^2+(2y)^2+1}\, dA$$

$$S = \iint_R \sqrt{4x^2+4y^2+1}\, dA$$

$$S = \iint_R \sqrt{4(x^2+y^2)+1}\, dA$$

$$S = \iint_R \sqrt{4r^2+1}\, dA \qquad \because x^2+y^2 = r^2$$

$\because$ Since every quadrant is touched $0 \le \theta \le 2\pi$

$\because$ Since radius is 3

$0 \le r \le 3$ (-ve; not possible)

Also.

$$S = \int_0^{2\pi}\int_0^3 \sqrt{4r^2+1}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{8} \int_0^3 8r\sqrt{4r^2+1}\, dr\, d\theta$$

$\because$ Multiply and divide by '8' to apply function-function derivative rule of integration

$$= \int_0^{2\pi} \frac{1}{8} \left| \frac{(4r^2+1)^{3/2}}{3/2} \right|_0^3 d\theta$$

$$= \int_0^{2\pi} \frac{1}{\cancel{8}_{12}} \cdot \frac{\cancel{2}}{\cancel{3}_{9}} \left([4(3)^2+1]^{3/2} - [4(0)^2+1]^{3/2} \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{12}\right)\left[(37)^{3/2} - 1\right] d\theta$$

$$= \frac{1}{12} \int_0^{2\pi} \left((37)^{3/2} - 1\right) d\theta$$

$$= \frac{1}{12} \left| \theta (37)^{3/2} - \theta \right|_0^{2\pi}$$

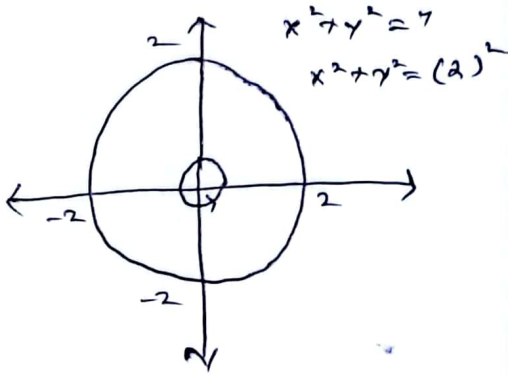
$$= \frac{1}{12} \left(\left[2\pi (37)^{3/2} - 2\pi\right] - \left[0 - 0\right] \right)$$

$$= \frac{\cancel{2}\pi}{\cancel{12}_{6}} \left[(37)^{3/2} - 1\right]$$

$$= \frac{\pi}{6} \left[(37)^{3/2} - 1\right]$$

$$\boxed{= 117.318 \text{ square units}}$$

(b) $x^2 + y^2 + z = 4$ above xy-plane.



$$z = 4 - x^2 - y^2$$

$$\frac{\partial z}{\partial x} = -2x \quad \text{and} \quad \frac{\partial z}{\partial y} = -2y$$

<u>Therefore;</u>

$$S = \iint_R \sqrt{(-2x)^2 + (-2y)^2 + 1}\, dA$$

$$S = \iint_R \sqrt{4x^2 + 4y^2 + 1}\, dA \qquad \because x^2 + y^2 = r^2$$

$$S = \iint_R \sqrt{4r^2 + 1}\, dA \qquad \because dA = r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^{2} (4r^2 + 1)^{1/2} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{8} \int_0^{2} \frac{1}{8} (4r^2 + 1)^{1/2}\, 8r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{12} \left| (4r^2 + 1)^{3/2} \right|_0^{2} d\theta$$

$$= \int_0^{2\pi} \frac{1}{12} \left[(17)^{3/2} - 1 \right] d\theta$$

$$= \frac{1}{12} \int_0^{2\pi} \left[(17)^{3/2} - 1 \right] d\theta$$

$$= \frac{1}{12} \left| \theta (17)^{3/2} - \theta \right|_0^{2\pi}$$

$$= \frac{1}{12} \left[\left(2\pi (17)^{3/2} - 2\pi \right) - (0 - 0) \right]$$

$$= \frac{2\pi}{12} \left[(17)^{3/2} - 1 \right]$$

$$= \frac{\pi}{6} \left[(17)^{3/2} - 1 \right]$$

$$\boxed{S = 36.1769 \text{ sq. units}}$$

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(2) $z = x^2 + 2y$ above triangle in xy-plane with vertices $(0,0), (1,0), (1,1)$

$$\frac{\partial z}{\partial x} = 2x \quad \text{and} \quad \frac{\partial z}{\partial y} = 2$$

Now;

$$S = \iint_R \sqrt{(2x)^2 + (2)^2 + 1}\, dA$$

$$= \iint_R \sqrt{4x^2 + 5}\, dA$$

∵ y is not present thus not solving by polar co-ordinates

$$= \iint_R \sqrt{4x^2+5}\, dy\, dx$$

$$= \int_0^1 \int_0^x \sqrt{4x^2+5}\, dy\, dx$$

$$= \cancel{\int_0^1 \int_0^x \frac{1}{8} \int_0^x}$$

$$= \int_0^1 \left| y\sqrt{4x^2+5} \right|_0^x dx$$

$$= \int_0^1 \left[\left(x\sqrt{4x^2+5} \right) - 0 \right] dx$$

$$= \frac{1}{8} \int_0^1 8x\sqrt{4x^2+5}\, dx$$

$$= \frac{1}{8} \left| \frac{(4x^2+5)^{3/2}}{3/2} \right|_0^1$$

$$= \left(\frac{1}{8}\right)\left(\frac{2}{3}\right) \left[(9)^{3/2} - (5)^{3/2} \right]$$

$$= \left(\frac{1}{12}\right) \left[27 - 5^{3/2} \right]$$

$$= \cancel{\frac{27}{12}} \quad \boxed{= 1.3183 \text{ sq units}}$$

Q9

(A) $\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{x^2+y^2}^{2} x \, dz\, dy\, dx$

$= \int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} x \left| z \right|_{x^2+y^2}^{2} dy\, dx$

$= \int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} x \left(2-x^2-y^2\right) dy\, dx$

$= \int_0^{\sqrt{2}} x \left| 2y - x^2 y - \frac{y^3}{3} \right|_0^{\sqrt{2-x^2}} dx$

$= \int_0^{\sqrt{2}} x \left(2\sqrt{2-x^2} - (\sqrt{2-x^2})x^2 - \frac{(\sqrt{2-x^2})^3}{3} \right) dx$

$= \int_0^{\sqrt{2}} \left(2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - x\frac{(\sqrt{2-x^2})^3}{3} \right) dx$

$= \left| \frac{(2-x^2)^{3/2}}{3/2} \right|_0^{\sqrt{2}} - \int_0^{\sqrt{2}} x^2 \sqrt{2-x^2}\, dx - \int_0^{\sqrt{2}} \frac{x}{3}(\sqrt{2-x^2})^3 dx$

Now; let evaluate $\int_0^{\sqrt{2}} x^2 \sqrt{2-x^2}\, dx$

By applying by parts

$u = x^2$ and $v = x\sqrt{2-x^2}$

Therefore;

$= x^2 \frac{(2-x^2)^{3/2}}{3/2} - \int \left[(2x) \frac{(2-x^2)^{3/2}}{3/2} \right] dx$

$= \frac{2x^2}{3}(2-x^2)^{3/2} - \int 2x (2-x^2)^{3/2} dx$

$-2 \int x \cdot x\sqrt{2-x^2}$

$= \left| \frac{(2-x^2)^{3/2}}{3/2} \right|_0^{\sqrt{2}} - \int_0^{\sqrt{2}} x^2 \cdot x\sqrt{2-x^2}\, dx - \int \frac{x}{3}(2-x^2)^{3/2} dx$ — (A)

Now; lets evaluate $\int_0^{\sqrt{2}} x^2 \cdot x\sqrt{2-x^2}\, dx$

Here;

$u = x^2$ and $v = x\sqrt{2-x^2}$

$(u)' = 2x$, $v = -\frac{1}{3}(2-x^2)^{3/2}$

$$= \frac{-x^2}{3}(2-x^2)^{3/2} + \frac{2}{3}\int x(2-x^2)^{3/2}\,dx$$

$$= \frac{-x^2}{3}(2-x^2)^{3/2} + \left(\frac{2}{3}\right)\left(\frac{-1}{5}\right)(2-x^2)^{5/2}$$

$$(*) \Rightarrow = \left| -\frac{2}{3}(2-x^2)^{3/2} + \frac{x^2}{3}(2-x^2)^{3/2} + \frac{2}{15}(2-x^2)^{5/2} + \frac{1}{15}(2-x^2)^{5/2} \right|_0^{\sqrt{2}}$$

$$= (-0+0+0+0) - \left(\frac{-2}{3}(2)^{3/2} + \frac{3}{15}(2)^{5/2}\right)$$

$$= \frac{2}{3}(2)^{3/2} - \frac{3}{(15)}(2)^{5/2}$$

$$= \frac{2^{7/2}}{15}$$

$$= \frac{(\sqrt{2})^7}{15}$$

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(b) $\int_0^3 \int_0^2 \int_0^1 (xyz)^2 \, dx \, dy \, dz$

$= \int_0^3 \int_0^2 \int_0^1 x^2 y^2 z^2 \, dx \, dy \, dz$

$= \int_0^3 \int_0^2 y^2 z^2 \left| \frac{x^3}{3} \right|_0^1 dy \, dz$

$= \int_0^3 \int_0^2 \frac{y^2 z^2}{3} \, dy \, dz$

$= \int_0^3 \frac{z^2}{3} \left| \frac{y^3}{3} \right|_0^2 dz$

$= \int_0^3 \frac{z^2}{9} (8) \, dz$

$= \frac{8}{9} \int_0^3 z^2 \, dz$

$= \frac{8}{9} \left| \frac{z^3}{3} \right|_0^3$

$= \frac{8}{27} (27)$

$\boxed{= 8}$

(c) $\int_0^{\pi/4} \int_0^{\ln \sec t} \int_{-\infty}^{2s} e^r \, dr \, ds \, dt$

$= \int_0^{\pi/4} \int_0^{\ln \sec t} \left| e^r \right|_{-\infty}^{2s}$

$= \int_0^{\pi/4} \int_0^{\ln \sec t} (e^{2s} - e^{-\infty}) \, ds \, dt$

$= \int_0^{\pi/4} \int_0^{\ln \sec t} (e^{2s} - 0) \, ds \, dt$

$= \int_0^{\pi/4} \left| \frac{e^{2s}}{2} \right|_0^{\ln \sec t} dt$

$= \frac{1}{2} \int_0^{\pi/4} (e^{2 \ln \sec t} - 1) \, dt$

$= \cancel{\frac{1}{2} \left(\frac{e^{2 \ln \sec t}}{2 \cos t \sec t \tan t} - t \right)_0^{\pi/4}}$

$$= \frac{1}{2}\left[\frac{e^{2\ln\sec t}}{2\tan t} - 1\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left(\frac{e^{2\ln\sec t} - 2}{\tan t}\right)_0^{\pi/4}$$

$$= \frac{1}{2}\int_0^{\pi/4} (2\sec t - 1)\,dt$$

$$= \frac{1}{2}\left(2\ln|\sec(t) + \tan(t)| - t\right)_0^{\pi/4}$$

$$= \frac{1}{2}\left[\left(2(0.8813) - \frac{\pi}{4}\right) - (0 - 0)\right]$$

$$= \frac{1}{2}(0.9773)$$

$$\boxed{= 0.4886}$$

(d) $\iiint yz^2 \sin(xyz)\,dx\,dy\,dz$

$$= \iint yz^2\left[\frac{-\cos(xyz)}{yz}\right]dy\,dz$$

$$= \iint -z\cos(xyz)\,dy\,dz$$

$$= \int (-z)\left[\frac{\sin(xyz)}{xz}\right]dz$$

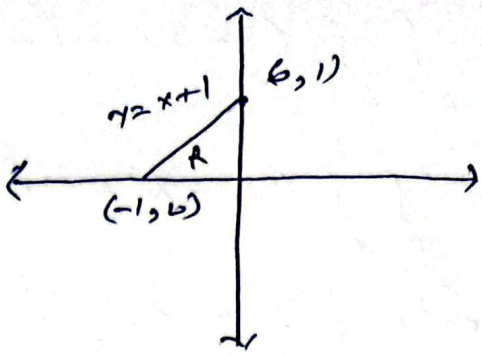
$$= \frac{-1}{x}\int \sin(xyz)\,dz$$

$$= \frac{-1}{x}\left[\frac{-\cos(xyz)}{xy}\right] + c$$

$$\boxed{= \frac{\cos(xyz)}{x^2y} + c}$$

Q10

(a)



$$= \int_{-1}^{0}\int_{0}^{x+1} (1+x-y)\,dy\,dx$$

$$= \int_{-1}^{0} \left| y + xy - \frac{y^2}{2} \right|_{0}^{x+1} dx$$

$$= \int_{-1}^{0} \left(\left[(x+1) + x(x+1) - \frac{(x+1)^2}{2} \right] - \left[0 + 0 - 0 \right] \right) dx$$

$$= \int_{-1}^{0} \left[x+1 + x^2 + x - \frac{(x+1)^2}{2} \right] dx$$

$$= \int_{-1}^{0} \left[x^2 + 2x + 1 - \frac{x^2+2x+1}{2} \right] dx$$

$$= \int_{-1}^{0} \left(x^2 + 2x + 1 - \frac{x^2}{2} - x - \frac{1}{2} \right) dx$$

$$= \left| \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + x - \frac{x^3}{6} - \frac{x^2}{2} - \frac{x}{2} \right|_{-1}^{0}$$

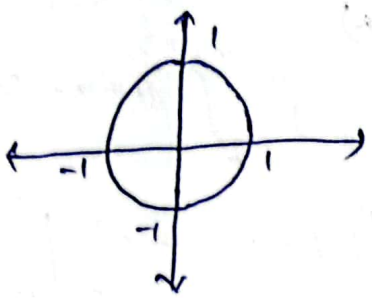
$$= \left| \frac{x^3}{6} + \frac{x^2}{2} + \frac{x}{2} \right|_{-1}^{0}$$

$$= (0 + 0 + 0) - \left(\frac{-1}{6} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} \right)$$

$$\boxed{= \frac{1}{6}}$$ cubic units

(b)

$$= \int_0^{2\pi}\int_0^1 (3 - r\cos\theta - r\sin\theta)\, r\, dr\, d\theta$$



$$= \int_0^{2\pi}\int_0^1 (3r - r^2\cos\theta - r^2\sin\theta)\, dr\, d\theta$$

$$= \int_0^{2\pi} \left| \frac{3r^2}{2} - \frac{r^3\cos\theta}{3} - \frac{r^3\sin\theta}{3} \right|_0^1 d\theta$$

$$= \int_0^{2\pi} \left[\left(\frac{3}{2} - \frac{\cos\theta}{3} - \frac{\sin\theta}{3} \right) - (0 - 0 - 0) \right] d\theta$$

$$= \left| \frac{3\theta}{2} - \frac{\sin\theta}{3} + \frac{\cos\theta}{3} \right|_0^{2\pi}$$

$$= \left(3\pi - 0 + \frac{1}{3}\right) - \left(0 - 0 + \frac{1}{3}\right)$$

$$= 3\pi + \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}}$$

$$= 3\pi$$

$$\boxed{= 9.4215 \text{ cubic units}}$$

(c)

$$= \int_{-1}^{1}\int_{x^2}^{1} (1 - y)\, dy\, dx$$

$$= \int_{-1}^{1} \left| y - \frac{y^2}{2} \right|_{x^2}^{1} dx$$

$$= \int_{-1}^{1} \left[\left(1 - \frac{1}{2}\right) - \left(x^2 - \frac{x^4}{2}\right) \right] dx$$

$$= \left| \frac{x}{2} - \frac{x^3}{3} + \frac{x^5}{10} \right|_{-1}^{1}$$

$$= \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{10}\right) - \left(\frac{-1}{2} + \frac{1}{3} - \frac{1}{10}\right)$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{10} + \frac{1}{2} - \frac{1}{3} + \frac{1}{10}$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$\boxed{= \frac{8}{15} \text{ cubic units}}$$

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