Discrete Structures

Assignment: 03

23k-3032 (Shah Hunain)

BSE-2A

Q1

(i) undirected, multiple edges, no loops ; Multi-graph

(ii) undirected, no multiple edges, no loops ; simple graph

(iii) undirected, multiple edges, 3 loops ; pseudograph

(iv) directed, multiple edges, 2 loops ; directed multi-graph

Q2

(i)

$A_1 \cap A_2 = \{0, 2, 4\}$

$A_1 \cap A_3 = \phi$

$A_1 \cap A_4 = \{6, 8\}$

$A_1 \cap A_5 = \{8\}$

$A_2 \cap A_3 = \{1, 3\}$

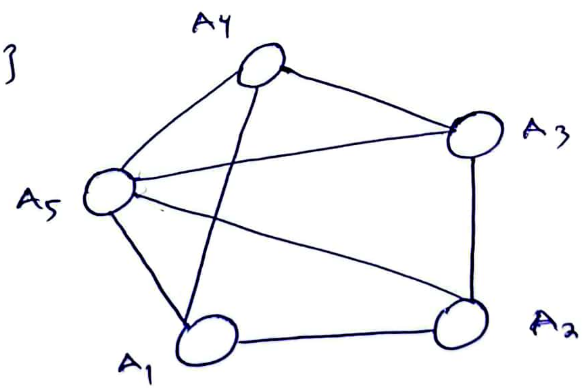
$A_2 \cap A_4 = \phi$

$A_2 \cap A_5 = \{0, 1\}$

$A_3 \cap A_4 = \{5, 7, 9\}$

$A_3 \cap A_5 = \{9\}$

$A_4 \cap A_5 = \{8, 9\}$



(ii) $A_1 \cap A_2 = \{-7,-3,-2,-1,0\}$

$A_1 \cap A_3 = \{-7,-2,0\}$

$A_1 \cap A_4 = \{-5,-3,-7\}$

$A_1 \cap A_5 = \{-6,-3,0\}$

$A_2 \cap A_3 = \{-6,-7,-2,0,2,7,6\}$

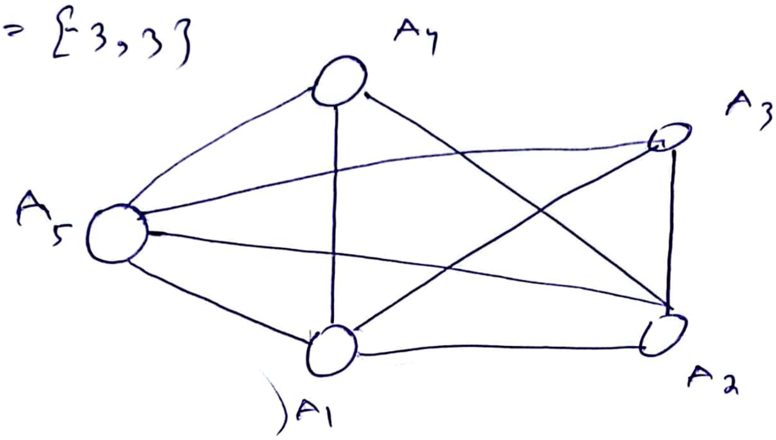
$A_2 \cap A_4 = \{5,-3,-1,1,3,5\}$

$A_2 \cap A_5 = \{-6,-3,0,3,6\}$

$A_3 \cap A_4 = \phi$

$A_3 \cap A_5 = \{-6,0,6\}$

$A_4 \cap A_5 = \{-3,3\}$



Q3 (a)

(i) no. of vertices = 5

no. of edges = 13

$deg(a) = 5$

$deg(b) = 6$

$deg(c) = 5$

$deg(d) = 5$

$deg(e) = 3$

$N(a) = \{b, e\}$

$N(b) = \{a, c, d, e\}$

$N(c) = \{b, d\}$

$N(d) = \{b, c, e\}$

$N(e) = \{a, b, d\}$

(ii) no. of vertices = 9

no. of edges = 12

deg(a) = 3

deg(b) = 2

deg(c) = 4

deg(d) = 0

deg(e) = 6

deg(f) = 0

deg(g) = 4

deg(h) = 2

deg(i) = 3

N(a) = {c, e, i}

N(b) = {e, h}

N(c) = {a, e, g, i}

N(d) = ϕ

N(e) = {a, b, c, g}

N(f) = ϕ

N(g) = {c, e}

N(h) = {b, i}

N(i) = {a, c, h}

(b)

(i) no. of vertices = 5

no. of edges = 13

$deg^-(a) = 6$; $deg^+(a) = 1$

$deg^-(b) = 1$; $deg^+(b) = 5$

$deg^-(c) = 2$; $deg^+(c) = 5$

$deg^-(d) = 4$; $deg^+(d) = 2$

$deg^-(e) = 0$; $deg^+(e) = 0$

(ii)

no. of vertices = 4

no. of edges = 8

$\deg^-(a) = 2$; $\deg^+(a) = 2$

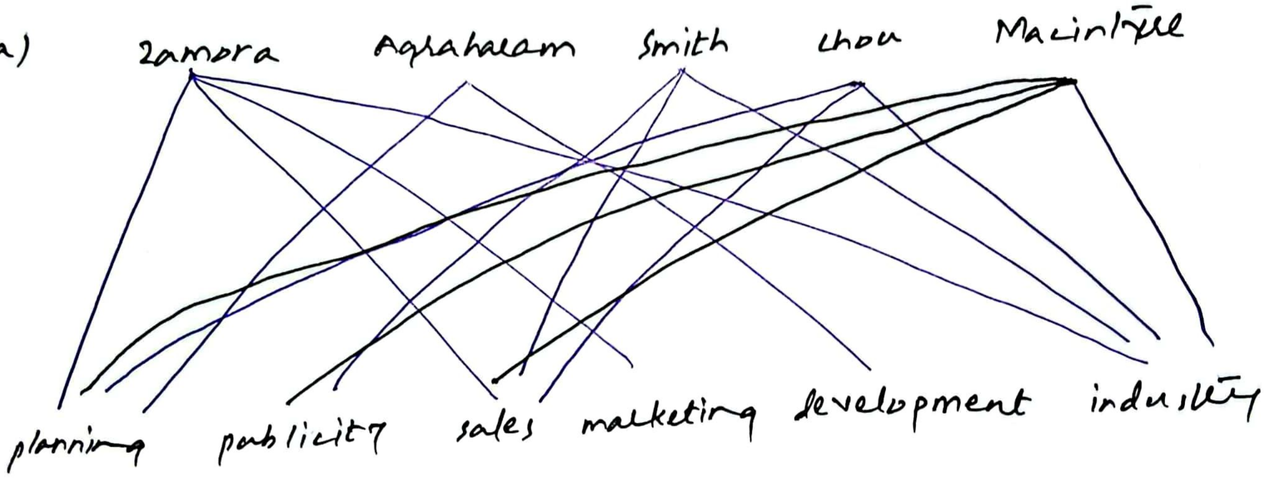
$\deg^-(b) = 3$; $\deg^+(b) = 4$

$\deg^-(c) = 2$; $\deg^+(c) = 1$

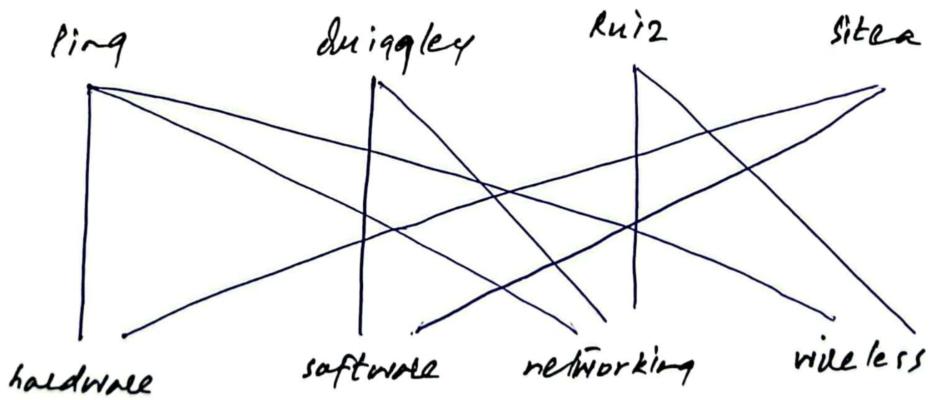
$\deg^-(d) = 1$; $\deg^+(d) = 1$

Q4

(a)



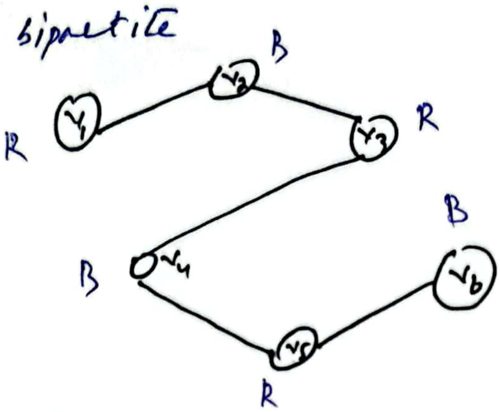
(b)

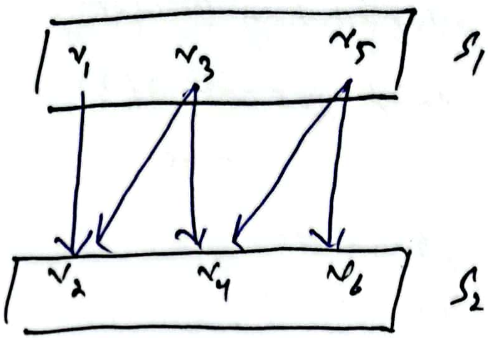


Q5

(i) not bipartite

(ii) bipartite





$S_1 = \{v_1, v_3, v_5\}$

$S_2 = \{v_2, v_4, v_6\}$

(iii) not bipartite

(iv) not bipartite

Q6

(i) $1+1+2+3 = 2m$

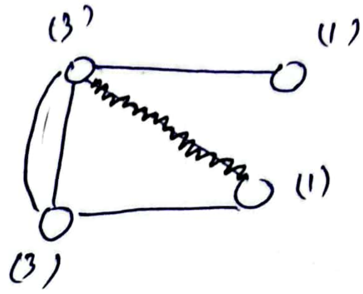
$7 = 2m$

$\boxed{m = \frac{7}{2}}$; no such graph exists

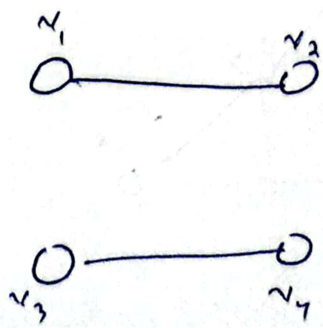
(ii) $1+1+3+3 = 2m$

$8 = 2m$

$\boxed{m = 4}$



(iii) not possible



$1+1+3+3 = 8$

Max. possible edges $= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = \frac{4(3)}{2} = 6$

$8 > 6$;

Thus, not possible

Q7

(a) Every person is a vertex

Applying handshaking Theorem;

Total degree = (15)(3)

= 45

2m = 45

$\boxed{m = \frac{45}{2}}$; not possible

(b) Total degree = (4)(3)

= 12

Apply handshaking;

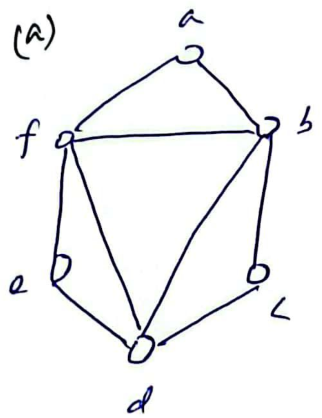
2m = 12

$\boxed{m = 6}$

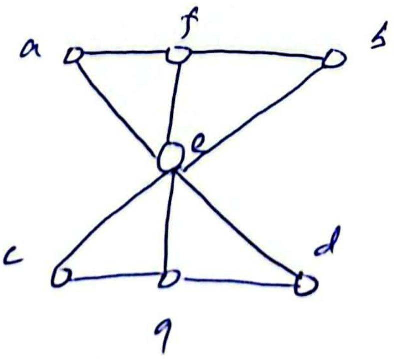
Yes; it is possible

Q8 (a)

(i)



(ii)



(b) edges = 10

degree = 4

vertices = ?

$2e = (4)v$

$(2)(10) = (4)v$

$\boxed{v=5}$

$\therefore$ 5 vertices

2a

(i) no. of vertices : 5 = 5

no. of edges : 7 = 7

degree : (4, 2, 3, 3, 2) = (3, 4, 2, 2, 3)

$f(v_1) = w_2$

$f(v_2) = w_3$

$f(v_3) = w_1$

$f(v_4) = w_5$

$f(v_5) = w_4$

graph is isomorphic

(ii) no. of vertices : 6 = 6

no. of edges : 7 = 7

sequence of degree : (3, 2, 3, 2, 1, 2) = (1, 2, 2, 3, 3, 2)

$f(v_1) = u_5$

$f(v_2) = u_2$

$f(v_3) = u_4$

$f(v_4) = u_3$

$f(v_5) = u_1$

$f(v_6) = u_6$

graph is isomorphic

(iii) $f(v_1) = v_5$

$f(v_2) = u_4$

$f(v_3) = u_3$

$f(v_4) = v_2$

$f(v_5) = v_7$

$f(v_6) = v_1$

$f(v_7) = v_6$

no. of vertices : $7 = 7$

no. of edges : $9 = 9$

degree : $(2, 3, 3, 2, 2, 4, 2) = (4, 2, 3, 3, 2, 2, 2)$

graph is isomorphic

(iv) no. of vertices : $5 = 5$

no. of edges : $7 = 7$

degree ; $(3, 2, 3, 3, 3) = (2, 4, 2, 3, 3)$

Neither vertex of G has degree '4'

not isomorphic graph

CamScanner

Q10 (i)

Step	N'	D(e)	D(f)	D(c)	D(g)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
1	ad	[illegible]	(6,d)	(5,d)	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
2	adb	(3,b)	(6,d)	(5,d)														
3	adbe		(6,d)	(5,d)	(6,e)													
4	adbec		(6,d)	(5,d)	(6,e)													
5	adbecg		(6,d)		(6,e)			(7,g)										
6	adbecgh		(6,d)					(7,g)	(7,h)			(14,h)						
7	adbecghf					(8,f)	(10,f)	(7,g)	(7,h)			(14,h)			(9,k)			
8	adbecghfk					(8,f)	(10,f)		(7,h)		(11,k)	(14,h)			(9,k)			
9	adbecghfkl					(8,f)	(10,f)			(10,l)	(11,k)	(13,l)			(9,k)			
10	" i						(10,f)			(10,l)	(11,k)	(13,l)		(17,l)			(14,l)	
11	" qr						(10,f)			(10,l)	(10,l)	(13,l)		(17,l)			(17,l)	
12	" rj									(10,l)	(10,l)	(13,l)	(12,m)	(17,l)			(17,l)	
13	" m										(10,l)	(13,l)	(12,m)	(12,m)			(17,l)	
14	" n											(13,l)	(12,m)				(17,l)	
15	" q											(13,l)					(17,l)	
16	" p															(14,p)	(13,p)	
17	" o															(14,p)	(13,p)	
18	" t															(14,p)		(21,t)
19	" s																	(16,s)
20	" z																	

The shortest path is adbecghfklijmnqpotsz = $\boxed{168}$

Q10 (i)

Q.10

(ii)

Step	N'	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
1	a	(4, a)	(3, a)	∞	∞	∞	∞	∞
2	ac	(4, a)		(6, c)	(9, c)	∞	∞	∞
3	acb			(6, c)	(9, c)	∞	∞	∞
4	acbd				(7, c)	(11, d)	∞	∞
5	acbde.					(11, d)	(12, e)	∞
6	acbdef						(12, e)	(18, f)
7	acbdefg							(16, g)
8	acbdefgz							

The shortest path is acbdefgz

and the length of the path is $3+4+6+7+11+12+16$

$\boxed{=59}$

Q11

(i) ABCDA = 125

ABDCA = 170

ACBDA = 145

ACDBA = 170

Minimized route:- ABCDA

(ii) ABDCA = 108

ADBCA = 141

ABCDA = 97

ACDBA = 108

Minimized route:- ABCDA

Q12

(a) AHGBCDGFE

(b) AGHIEHFEKJDCB

Q13

(i) Hamilton circuit! $v_0 v_1 v_2 v_6 v_5 v_4 v_7 v_3 v_0$

Hamilton path ! $v_0 v_1 v_2 v_6 v_5 v_4 v_7 v_3$

(ii) Hamilton circuit!- does not exsist

Hamilton path!- badcfehg

(iii) Hamilton circuit!- dabcefgd

Hamilton path!- abcdefg or dabcefg

Q14

(a)

(i) It has Euler circuit because every vertex has an even degree.

(ii) No; because every vertex doesnot have an even degree.

(b)

(i) No (Euler path); because for it exact two vertices should have an odd degree in this case there are more than two

(ii) Yes; two vertices have an odd degree.

$u v_7 v_0 v_1 u v_2 v_3 v_4 v_2 v_6 v_4 w v_6 v_5 w$

Q15

(a)

(i)

$$\begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ v_3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ v_6 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

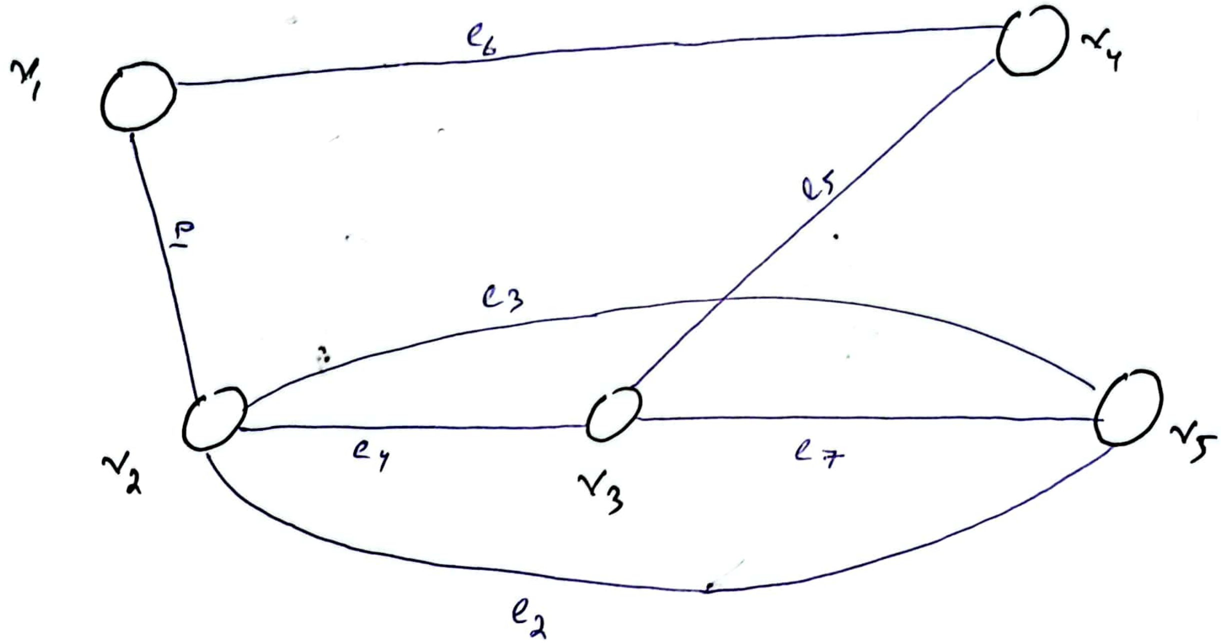
(ii)

$$\begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

(b)

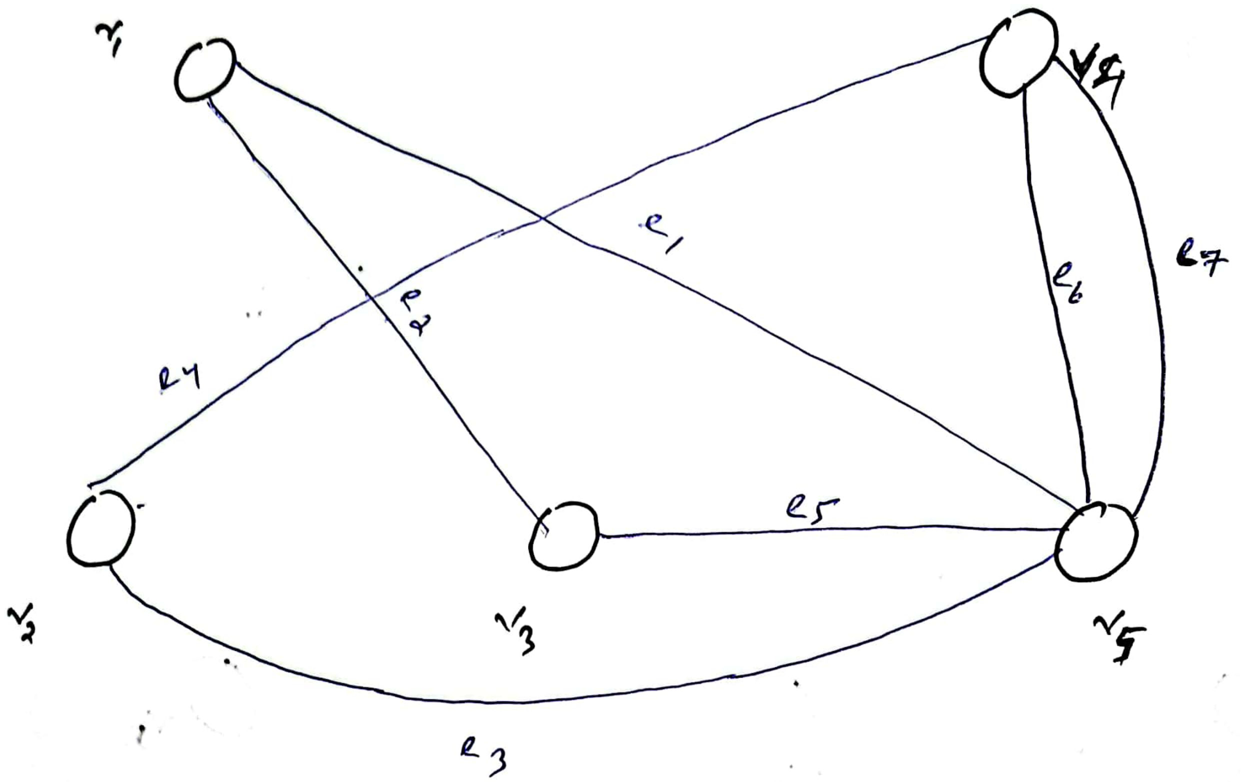
(i)

$$\begin{array}{c|c} & \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



(ii)

$$
\begin{array}{c|c}
 & \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} &
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
\end{array}
$$



Q16

(i)

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & \\ b & \\ c & \\ d & \end{array}\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

initial vertex	terminal vertex
a	{a, b, c, d}
b	{d}
c	{a, b}
d	{c, b, d}

(ii)

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} a \quad b \quad c \quad d \quad e \\ \left[\begin{array}{ccccc} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

initial vertex	terminal vertex
a	{b, d}
b	{a, c, d, e}
c	{b, c}
d	{a, e}
e	{c, e}

(iii)

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} a \quad b \quad c \quad d \\ \left[\begin{array}{cccc} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{array}\right] \end{array}$$

initial vertex	terminal vertex
a	{b, d}
b	{a, c}
c	{b, d}
d	{a, c}

(iv)

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} a \quad b \quad c \quad d \\ \left[\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \end{array}$$

initial vertex	terminal vertex
a	{a, c, d}
b	{b, c, d}
c	{a, b, c}
d	{a, b, d}

Q27

(a) Propositional logic

1- Digital circuit design:-

Propositional logic is utilized in designing digital circuits where logic gates like AND, OR, NOT gates are used to process binary information, enabling the functioning of computers and electronic devices.

2- Automated Reasoning systems:-

In artificial intelligence, propositional logic forms the basis for automated reasoning systems, enabling machines to make logical deductions and infer conclusions from given premises.

(b) Predicates and Quantifiers

1- Database Querying:-

Predicates and quantifiers are fundamentals in database querying, where they are used to formulate complex queries to extract specific information from database efficiently.

2- Mathematical Proofs:-

Predicates and quantifiers play a crucial role in mathematical proofs, especially in fields like set theory and analysis, where quantified statements are used to establish the truth of mathematical assertions.

(c) Number Theory and Cryptography

1- RSA encryption:-

Number theory serves as the foundation for modern cryptographic algorithms like RSA, where the security relies on the difficulty of factoring large composite numbers, a problem rooted in number theory.

2- Cryptocurrency:-

Number-theory principles underpin various aspects of cryptocurrency technologies, such as cryptographic hashing algorithms and digital signatures, ensuring secure transactions and data integrity in decentralized systems.

(d) Functions and Relations:

1- Social Networks:-

Functions and relations are applied in social networks analysis, where they model connections between individuals, helping to understand information flow, influence, and community structures.

2- Database Management:-

Functions and relations are used in relational databases to establish links between tables, enabling efficient data retrieval, manipulation, and management in various applications.

(e) Graph-Theory

1- Transportation Networks:-

Graph Theory is applied in transportation networks to optimize routes, schedules, and resource allocation, enhancing efficiency in systems like urban transit, airline routes, and logistics.

2- Social Network Analysis:-

Graph theory provides tools and algorithms to analyze social networks, identifying influencers, communities, and patterns of interaction among individuals or entities

(f) Trees

1- Hierarchical Data Structures:-

Trees are used in computer science for representing hierarchical data structures like file systems, XML documents, and organizational charts, facilitating efficient storage, retrieval, and manipulation of structured data.

2- Decision Trees:-

In machine learning, decision trees are employed for classification and regression tasks, where they ~~rate~~ recursively partition the feature space based on attribute values, enabling predictive modeling and decision-making.