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Multivariable Calculus (MVC)

Assignment No : 01

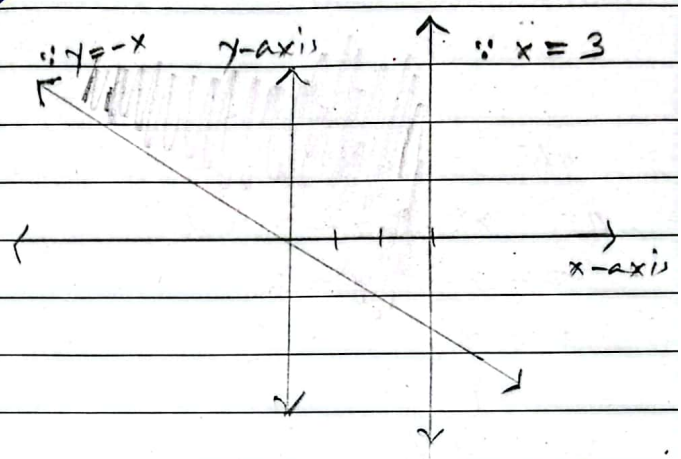
23k-3032 (Shah Hunain)

Q1 A) (i) $f(x,y) = \sqrt{x+y} - \sqrt{x-3}$

$x+y \geq 0$; $x-3 \geq 0$

$x \geq -y$; $x \geq 3$

$\therefore y = -x$; $x = 3$



(ii) $f(x,y) = \sqrt{\frac{1}{x^2} - \frac{1}{y^2}}$

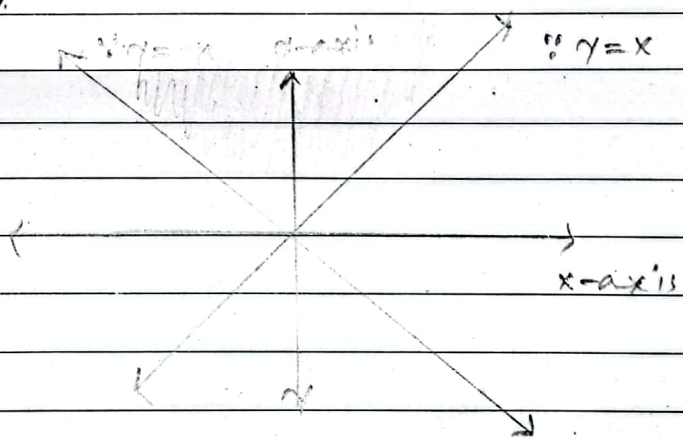
$\frac{1}{x^2} - \frac{1}{y^2} \geq 0$

$\frac{1}{x^2} \geq \frac{1}{y^2}$

$x^2 \leq y^2$

$x \leq \pm y$

$y = \pm x$



(iii) $f(x,y,z) = \frac{1}{x+1} + \frac{1}{y-1} + \frac{1}{x+y-2}$

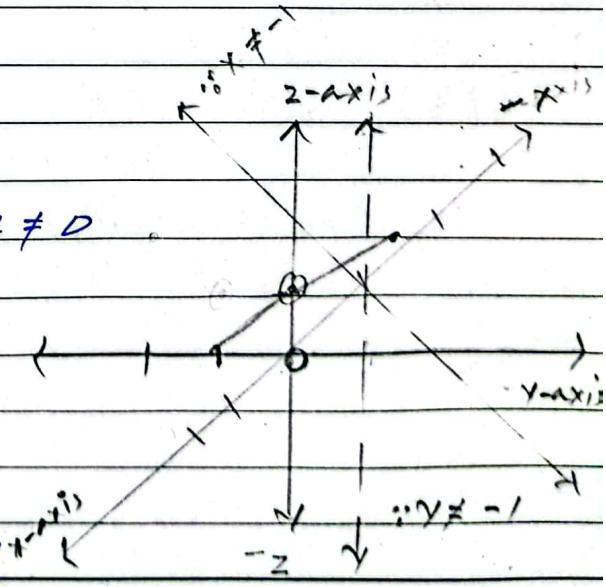
$x+1 = 0$; $y-1 \neq 0$; $x+y-2 \neq 0$

$x \neq -1$; $y \neq 1$

$(0,0,0)$

$(1,1,2)$

$(2,0,2)$



Domain $= \{(x,y,z) \mid x \neq -1 \wedge y \neq 1 \wedge x+y-z \neq 0\}$

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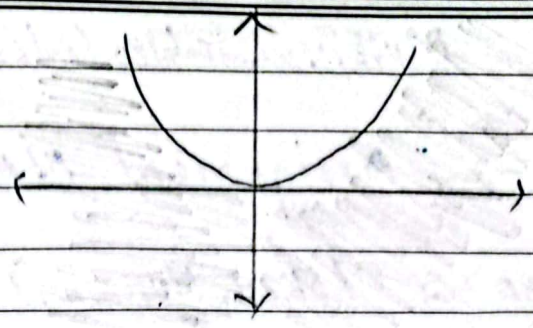
(iv) $f(x,y) = \ln(x^2 - 8y)$

Sol:-

$x^2 - 8y \geq 0$

$x^2 > 8y$

$\because$ parabola along y-axis



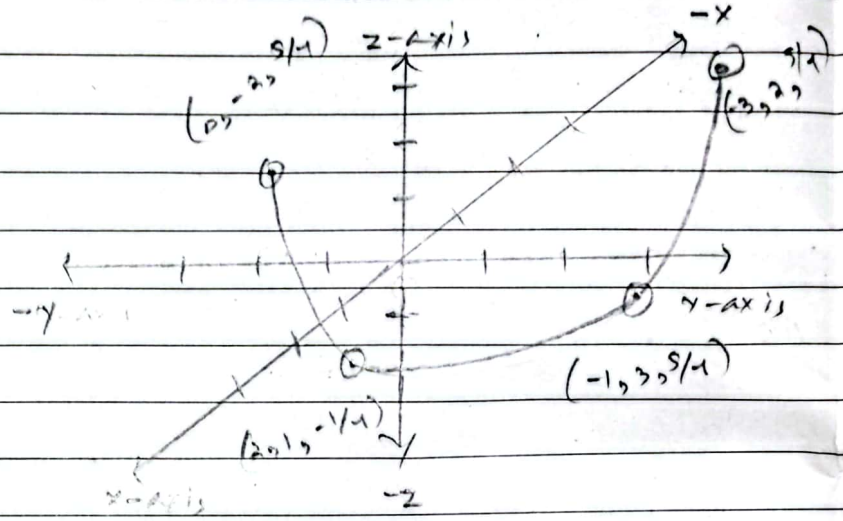
(b)

(i) $x^2 - 4z - y = 2$

$4z = x^2 - y - 2$

$z = \frac{x^2}{4} - \frac{y}{4} - \frac{2}{4}$

$z = \frac{1}{4}(x^2 - y - 2)$



$(0, -2, 5/4)$

$(2, 1, -1/4)$

$(-3, 2, 5/4)$

$(-1, 3, -5/4)$

(ii) $z^2 + 4x^2 = 1 - y^2$

$4x^2 + y^2 + z^2 = 1$

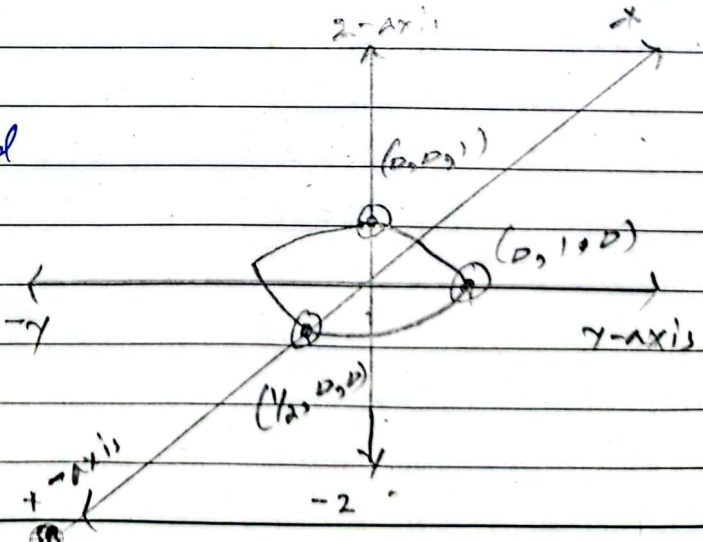
$\frac{x^2}{(1/2)^2} + \frac{y^2}{(1)^2} + \frac{z^2}{(1)^2} = 1$

$\because$ Equation of the ellipsoid

$a = 1/2$

$b = 1$

$c = 1$



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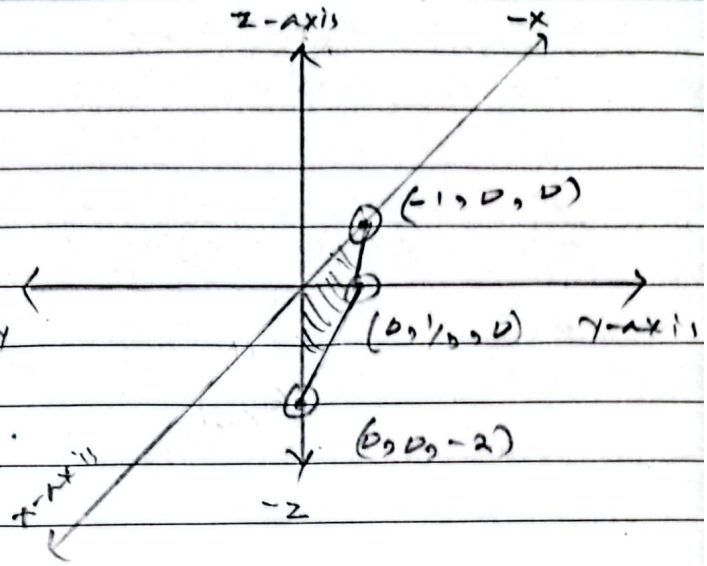
(iii) $2x + z - 6y = -2$

$z + 2x - 6y = -2$

$-x + 3y - \frac{z}{2} = 1$

$\frac{x}{-1} + \frac{y}{1/3} + \frac{z}{-2} = 1$

$\because$ Equation of a 3-D plane



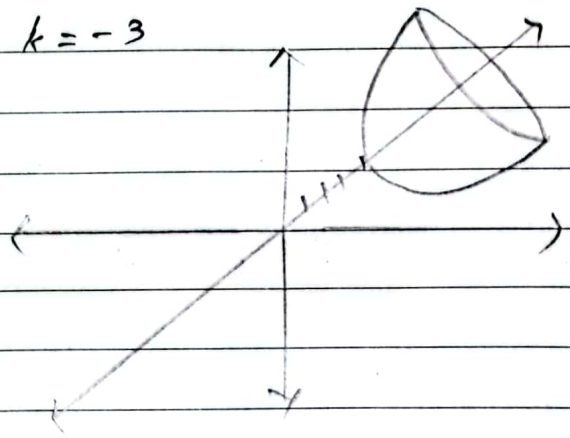
(c)

(i) $f(x, y, z) = x - y^2 - z^2 + 1$, $k = -3$

$-3 = x - y^2 - z^2 + 1$

$y^2 + z^2 = x + 4$

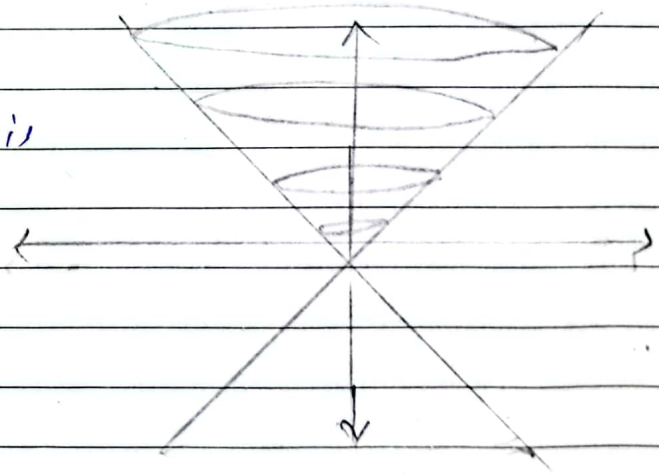
$\because$ Equation of a paraboloid



(ii) $f(x, y, z) = \frac{3x^2 + y^2}{z^2}$, $k = 9$

$9z^2 = 3x^2 + y^2$

$\because$ Equation of a cone along z-axis



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(iii) $f(x,y,z) = 9x^2 + 4y^2 + z^2$; $k = 4$

$4 = 9x^2 + 4y^2 + z^2$

$9x^2 + 4y^2 + z^2 = 4$

$\frac{x^2}{(2/3)^2} + \frac{y^2}{(2/2)^2} + \frac{z^2}{\cancel{(1/2)^2}\,(2)^2} = 1$

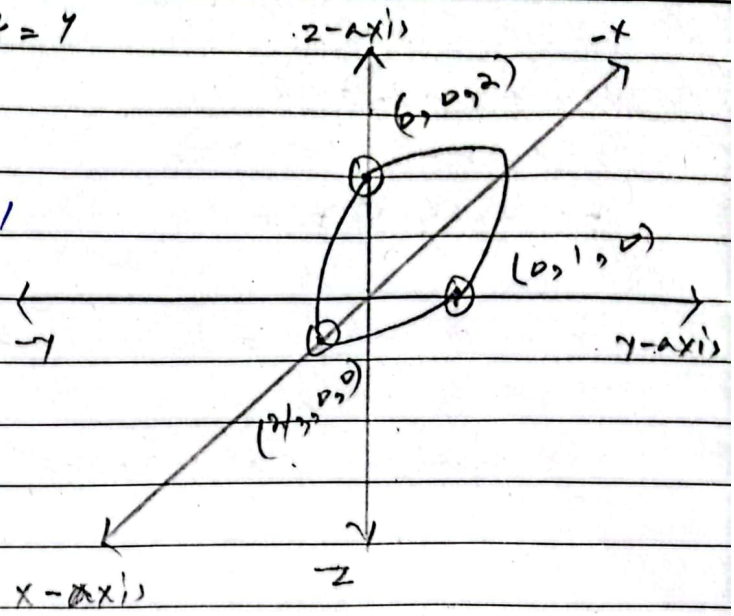
$\frac{x^2}{(2/3)^2} + \frac{y^2}{(1)^2} + \frac{z^2}{(2)^2} = 1$

$\therefore$ Equation of a ellipsoid

$a = 2/3$

$b = 1$

$c = 2$



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Q2 (a)

(i) $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^4+y^2} \approx \frac{0}{0}$

Take $x = 0$

$= \lim_{y\to 0} \frac{(0)^3 y}{x^4+(0)^2}$

$= \lim_{y\to 0} \frac{0}{x^4}$

$= 0 \quad -(A)$

Now put $y = x$

$= \lim_{x\to 0} \frac{x^3 (x)}{x^4+(x)^2} = \frac{x^4}{x^4+x^2}$

$= \lim_{x\to 0} \frac{x^2}{x^2+1} = 0 \quad -(B)$

(A) = (B) ; Thus, the limit exists

(ii) $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2} \approx \frac{0}{0}$

Take $x = 0$

Therefore;

$= \lim_{y\to 0} \frac{(0)^3-y^3}{(0)^2+y^2}$

$= \lim_{y\to 0} \frac{-y^3}{y^2} \approx \frac{0}{0}$

Now; applying L-Hopital rule

$= \lim_{y\to 0} \frac{-3y^2}{2y}$

$= \lim_{y\to 0} \frac{-3y}{2}$

$= \lim_{y\to 0} -3y$

$= 0 \quad -(A)$

Now; put $y = x$

$\lim_{x\to 0} \frac{x^3-(x)^3}{x^2+(x)^2} = \frac{x^3-x^3}{x^2+x^2}$

$= 0 \quad -(B)$

(A) = (B) ; Thus, the limit exists

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(iii) $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)^2}$

Take $x=0$

$= \lim_{y\to 0} \frac{1-\cos[(0)^2+y^2]}{[(0)^2+y^2]^2}$

$= \lim_{y\to 0} \frac{1-\cos(y^2)}{(y^2)^2}$

$= \lim_{y\to 0} \frac{1-\cos y^2}{y^4}$ $\quad \because 1-\cos^2 y = \sin^2 y$

$= \lim_{y\to 0} \frac{\sin^2 y^2}{y^4}$ $\quad \approx \frac{0}{0}$

Applying L-Hopital rule

$= \lim_{y\to 0} \frac{1}{2}\frac{\cancel{2y}\cdot\cos y^2}{4y^3}$ $= \lim_{y\to 0} \frac{\cos y^2}{2y^2}$

$= \lim_{y\to 0} \frac{-\cancel{2y}\sin y^2}{4\cancel{y}}$

$= -\frac{1}{2}\lim_{y\to 0} \sin y^2$

$= -\frac{1}{2}(0)$

$= 0$

Now; applying L-Hopital rule;

$= \lim_{y\to 0} \frac{0-(-\sin y^2\cdot 2y)}{4y^3}$

$= \lim_{y\to 0} \frac{1}{2}\frac{\cancel{2y}\cdot\sin y^2}{4y^3}$

$= \lim_{y\to 0} \frac{\cos y^2\cdot\cancel{2y}}{2\cdot 4\cancel{y}}$

$= \frac{1}{2}\lim_{y\to 0} \cos y^2$

$= \frac{1}{2}(1) = \frac{1}{2}$

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Now; put $y = x$

$$= \lim_{x \to 0} \frac{1 - \cos(x^2 + x^2)}{(x^2 + x^2)^2}$$

$$= \lim_{x \to 0} \frac{1 - \cos 2x^2}{(2x^2)^2}$$

$$= \lim_{x \to 0} \frac{1 - \cos 2x^2}{4x^4}$$

$$= \lim_{x \to 0} \frac{0 - (-\sin 2x^2)(4x)}{16x^3}$$

$$= \lim_{x \to 0} \frac{\sin 2x^2}{4x^2}$$

$$= \lim_{x \to 0} \frac{\cos 2x^2 (4x)}{12x^2 (8x)}$$

$$= \lim_{x \to 0} \frac{-\sin 2x^2 (4x)(4x)}{24x}$$

$$= \lim_{x \to 0} \frac{-\sin 2x^2 (16x^2)}{3\, 24x}$$

$$= \lim_{x \to 0} \frac{-\sin 2x^2 (2x)}{3}$$

$$= 0 \quad -(B)$$

~~(A) ≠ (B); Thus; limit doesnot exsists.~~

$$= \lim_{x \to 0} \frac{\cos 2x^2 (4x)}{12x^2 (8x)}$$

$$= \frac{1}{2} \lim_{x \to 0} \cos 2x^2$$

$$= \frac{1}{2} (1)$$

$$= \frac{1}{2} \quad -(B)$$

(A) = (B); Thus limit exsists

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(b) (i) $\dfrac{xy}{\sqrt{x^2+y^2}}$

$\because$ Put $y = 0$

$= \lim_{x \to 0} \dfrac{x(0)}{\sqrt{x^2+(0)^2}}$

$= \lim_{x \to 0} \dfrac{0}{x^2}$

$= 0$ —(A)

$\because$ Put $y = x$

$= \lim_{x \to 0} \dfrac{(x)(x)}{\sqrt{x^2+(x)^2}}$

$= \lim_{x \to 0} \dfrac{x^2}{\sqrt{2x^2}}$

$= \lim_{x \to 0} \dfrac{x^2}{x\sqrt{2}}$

$= \lim_{x \to 0} \dfrac{x}{\sqrt{2}}$

$= 0$ —(B)

(A) = (B)

$\therefore$ function is continuous

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(ii) $\dfrac{xy}{x^2+y^2}$

$\because$ Put $y=0$

$= \lim_{x\to 0} \dfrac{x(0)}{x^2+(0)^2}$

$= \lim_{x\to 0} \dfrac{0}{x^2}$

$= 0 \quad -(A)$

$\because$ Put $y=x$

$= \lim_{x\to 0} \dfrac{x(x)}{x^2+(x)^2}$

$= \lim_{x\to 0} \dfrac{x^2}{2x^2}$

$= \dfrac{1}{2} \quad -(B)$

$(A) \neq (B)$; $\therefore$ it is not continuous

(iii) $\dfrac{x^4-y^2}{x^4+y^2}$

$\because$ Put $y=0$

$= \lim_{x\to 0} \dfrac{x^4-(0)^2}{x^4+(0)^2}$

$= \lim_{x\to 0} \dfrac{x^4}{x^4}$

$= 1 \quad -(B)$

$\because$ Put $y=x$

$= \lim_{x\to 0} \dfrac{x^4-x^2}{x^4+x^2}$

$= \lim_{x\to 0} \dfrac{x^2(x^2-1)}{x^2(x^2+1)}$

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$$= \lim_{x \to 0} \frac{x^2 - 1}{x^2 + 1}$$

$$= \lim_{x \to 0} \frac{2x - 0}{2x + 0}$$

$$= \lim_{x \to 0} \frac{2x}{2x}$$

$$= 1 \qquad -(B)$$

(A) = (B) ; Thus, The limit exists; but the function is not continuous.

(iv) $\frac{x^2 y}{x^4 + y^2}$

$\because$ Put $y = 0$

$$\lim_{x \to 0} \frac{x^2 (0)}{x^4 + (0)^2}$$

$$= \lim_{x \to 0} \frac{0}{x^4}$$

$$= \lim_{x \to 0} \frac{0}{x^4}$$

$$= 0 \qquad -(A)$$

$\because$ Put $y = x$

$$= \lim_{x \to 0} \frac{(x^2)(x)}{x^4 + x^2}$$

$$= \lim_{x \to 0} \frac{x^3}{x^4 + x^2}$$

$$= \lim_{x \to 0} \frac{x^3}{x^2 (x^2 + 1)}$$

$$= \lim_{x \to 0} \frac{x}{x^2 + 1}$$

$$= \frac{0}{1} = 0 \qquad -(B)$$

(A) = (B) ; Thus, the function is continuous

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Q3 (a) $f(x,y) = xy\left(\frac{x^2-y^2}{x^2+y^2}\right)$

(a) $f_x(0,y) = -y$

$$f_x = \left(\frac{x^2-y^2}{x^2+y^2}\right)(y) + (xy)\left[\frac{(x^2+y^2)(2x-0)-(x^2-y^2)(2x+0)}{(x^2+y^2)^2}\right]$$

$$= y\left(\frac{x^2-y^2}{x^2+y^2}\right) + (xy)\left[\frac{2x(x^2+y^2)-2x(x^2-y^2)}{(x^2+y^2)^2}\right]$$

$$= y\left(\frac{x^2-y^2}{x^2+y^2}\right) + (xy)\left[\frac{2x[(x^2+y^2)-(x^2-y^2)]}{(x^2+y^2)^2}\right]$$

$$f_{x(0,y)} = y\left(\frac{0-y^2}{0+y^2}\right) + (0)\left[\frac{2(0)(0+y^2)-(0-y^2)}{(x^2+y^2)^2}\right]$$

$$= y\left(\frac{-y^2}{y^2}\right) + 0$$

$f_{x(0,y)} = -y$; proved

(b)

$f_y(x,0) = x$

$$f_y = \left(\frac{x^2-y^2}{x^2+y^2}\right)(x) + (xy)\left[\frac{(x^2+y^2)(0-2y)-(x^2-y^2)(0-2y)}{(x^2+y^2)^2}\right]$$

$$= \left(\frac{x^2-y^2}{x^2+y^2}\right)(x) + (xy)\left[\frac{(x^2+y^2)(-2y)-(x^2-y^2)(-2y)}{(x^2+y^2)^2}\right]$$

$$f_{y(x,0)} = \left(\frac{x^2-0}{x^2+0}\right)(x) + (0)\left[\frac{(x^2+0)(0)-(x^2-0)(0)}{(x^2+0)^2}\right]$$

$$= \left(\frac{x^2}{x^2}\right)x$$

$f_{y(x,0)} = x$

; proved

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(b)

$f_x(0,y) = -y$ from (a)

Therefore;

$f_{xy}(0,y)_{\forall y} = -1$

$\boxed{f_{xy}(0,0) = -1}$

Now;

$f_y(x,0) = x$ from (b)

Therefore;

$f_{yx}(x,0)_{\forall x} = 1$

$\boxed{f_{yx}(0,0) = 1}$

(c)

$f_x(x,y) = f_y(x,y) = 0$ for all (x,y)

let

$f_x(x,y) = 30 \cot t$

Integrating

$\int \frac{\partial f}{\partial x} \cdot dx$

$f_x(x,y) = 30 \cot t$

$f_x(x,y) = 0$

also;

$f_y(x,y) = 30 \cot t$

$f_y(x,y) = 0$

$f_x(x,y) = f_y(x,y) = 0$ for all values of (x,y)

Thus; $f(x,y) = a$

where; 'a' is a constant

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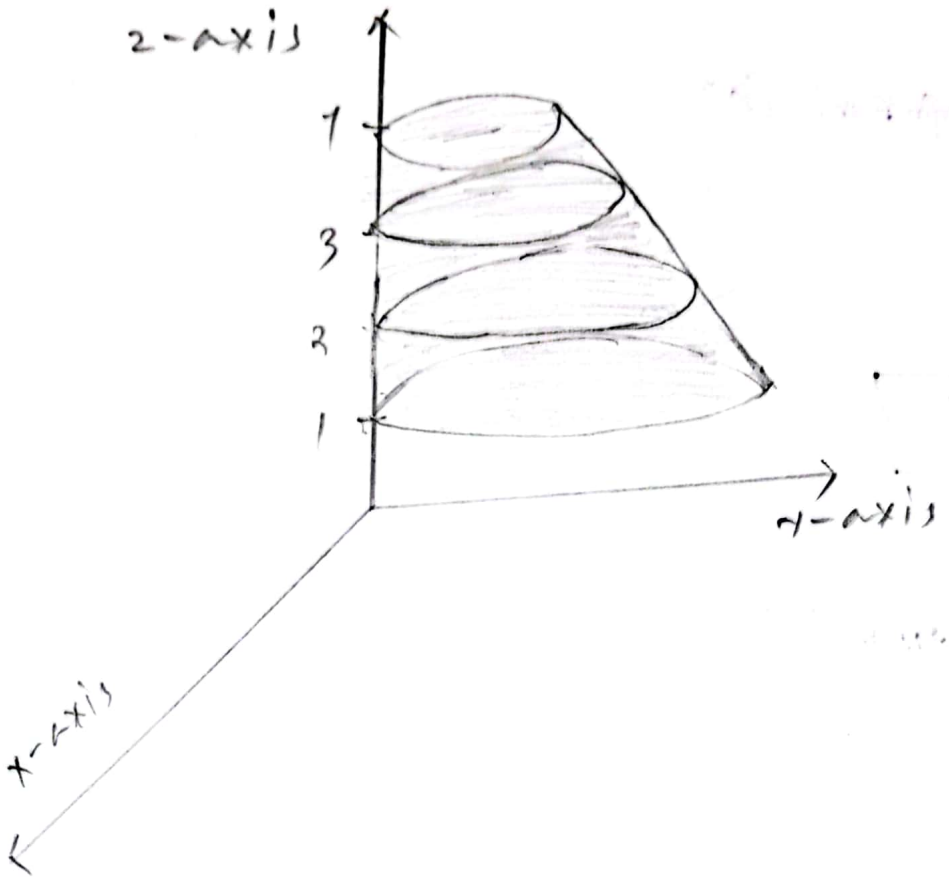
(iii)



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Q4

(b)

$z = f(x, y)$; $x = g(s, t)$; $y = h(s, t)$

$f_x(3, 6) = 7$; $g(1, 2) = 3$; $h(1, 2) = 6$

$f_y(3, 6) = 8$; $g_s(1, 2) = -1$; $h_s(1, 2) = -5$

$g_t(1, 2) = 4$; $h_t(1, 2) = 10$

$\frac{\partial z}{\partial s} = ?$ also $\frac{\partial z}{\partial t} = ?$

Sol:-

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= f_x(3, 6) \cdot g_s(1, 2) + f_y(3, 6) \cdot h_s(1, 2)$$

$$= (7)(-1) + (8)(-5)$$

$$= -7 - 40$$

$$\boxed{\frac{\partial z}{\partial s} = -47}$$

Now; for $\frac{\partial z}{\partial t}$:

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} \Rightarrow f_x(3, 6) \cdot g_t(1, 2) + f_y(3, 6) \cdot h_t(1, 2)$$

$$= (7)(4) + (8)(10)$$

$$= 28 + 80$$

$$\boxed{\frac{\partial z}{\partial t} = 108}$$

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(c) $z = y + f(x^2 - y^2)$

prove that $y \cdot \frac{\partial z}{\partial x} + x \cdot \frac{\partial z}{\partial y} = x$

for $\frac{\partial z}{\partial x}$:-

$$\frac{\partial z}{\partial x} = 0 + (2x \cdot \cancel{z} - y)$$

$$\frac{\partial z}{\partial x} = 2x$$

Now for $\frac{\partial z}{\partial y}$:-

$$\frac{\partial z}{\partial y} = 1 + 0 - 2y$$

$$\frac{\partial z}{\partial y} = 1 - 2y$$

so ; $y \cdot \frac{\partial z}{\partial x} + x \cdot \frac{\partial z}{\partial y}$

$$= y \cdot (2x) + (x)(1 - 2y)$$

$$= \cancel{2xy} + x - \cancel{2xy}$$

$$= x$$

Thus ; proved

$$\boxed{y \cdot \frac{\partial z}{\partial x} + x \cdot \frac{\partial z}{\partial y} = x}$$

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Q5 (a) $f(x, y, z) = x\sqrt[3]{y^2+z^2}$

linear approximation at point $(2, 3, 4)$

Sol:-

$$L(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0)$$

Putting values;

$$= x\sqrt[3]{y^2+z^2} + 3x^2\sqrt{y^2+z^2}\,(x-2) + \frac{x^3}{2}\cdot\frac{1}{\sqrt{y^2+z^2}}\cdot 2y(y-3) + \frac{x^3}{2}\cdot\frac{1}{\sqrt{y^2+z^2}}\cdot(0+2z)(z-4)$$

$$\cancel{= \frac{(2)^3}{2}\cdot\frac{1}{\sqrt{(3)^2+(4)^2}}}$$

$$= (2)^3\sqrt{(3)^2+(4)^2} + 3(2)^2\sqrt{(3)^2+(4)^2}\,(x-2) + \frac{(2)^3}{\cancel{2}}\cdot\frac{1}{\sqrt{(3)^2+(4)^2}}\cdot\cancel{2}(3).(y-3) + \frac{(2)^3}{\cancel{2}}\cdot\frac{1}{\sqrt{(3)^2+(4)^2}}\cdot\cancel{2}(4).(z-4)$$

$$L(x,y,z) = 40 + 60(x-2) + \frac{24}{5}(y-3) + \frac{32}{5}(z-4)$$

Now; $x = 1.98$, $y = 3.01$ and $z = 3.97$

Therefore;

$$L(1.98, 3.01, 3.97) = 40 + 60(1.98-2) + \frac{24}{5}(3.01-3) + \frac{32}{5}(3.97-4)$$

$$= 40 - 1.2 + 0.072 - 0.192$$

$$\boxed{L(1.98, 3.01, 3.97) = 38.656}$$

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Q6

(a)

$a = 5\,m$

$b = 12\,m$

$\partial a = \partial b = 0.2\,cm$ or $0.002\,m$

i) **Area :**

$$\Delta = \frac{1}{2} ab$$

$$\partial \Delta = \frac{1}{2} (a.\partial b + b\,\partial a)$$

$$= \frac{1}{2} \left[(5)(0.002) + (12)(0.002)\right]$$

$$\partial \Delta = 0.017 \text{ square meter}$$

∴ The maximum ~~area~~ error in calculated value of area a is 0.017 sq. meter

ii) **Length of hypotenuse :-**

$$x^2 = a^2 + b^2$$

$$x = \sqrt{a^2 + b^2}$$

$$x = \frac{1}{2\sqrt{a^2+b^2}} . (2a.\partial a + 2b.\partial b)$$

$$x = \frac{1}{\cancel{2}\sqrt{(5)^2+(12)^2}} . \cancel{2}\left[(5)(0.002) + (12)(0.002)\right]$$

$$x = 2.615 \times 10^{-3}$$

or

$$x = 0.002615\,m$$

∴ The maximum error in calculated value of hypotenuse is 0.002615 m.

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