National University of Computer and Emerging Sciences

SOLUTION MID1 (DISCRETE STRUCTURE)

Q1: [6*2=12]

a. Let p, q, and r be the propositions:
P: AI builds systems that do intelligent things. q: ML builds system that learns from experience.
r: NLP builds systems to understand languages.
Write the following propositions using p, q and r and logical connectives (including negation).

i. It is not the case that NLP does not build systems to understand languages but AI builds systems that do intelligent things.

$$\neg(\neg r \wedge p)$$

ii. If AI builds systems that do intelligent things, then neither NLP builds systems to understand languages nor ML builds systems that learn from experience.

$$p \rightarrow (\neg r \wedge \neg q)$$

iii. ML builds systems that learn from experience unless AI does not build systems that do not do intelligent things.

$$p \rightarrow q$$

iv. NLP builds systems to understand languages iff AI builds systems that do intelligent things. $r \leftrightarrow p$

b. Write inverse of the statement (ii) and contrapositive of statement (iii) from Q1(a) in English.
Solution:
(i) **If AI does not builds systems that do intelligent things, then NLP builds systems to understand languages or ML builds systems that learn from experience.**
(ii) If ML does not build systems that learn from experience then AI does not build systems that do intelligent things.

c. Determine using the truth table that the hypothetical syllogism rule forms tautology, contradiction, or contingency.

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$(P \rightarrow Q) \wedge (Q \rightarrow R)$	$P \rightarrow R$	$((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Thus it is tautology.

d. Using rules of inference, show that the following argument is valid. Show proper steps.

$$((\neg q \to (s \to \neg r)) \wedge (\neg t \to s) \wedge (\neg q \vee p) \wedge (\neg p)) \to (r \to t)$$

Solution

$$((\neg q \to (s \to \neg r)) \wedge (\neg t \to s) \wedge (\neg q \vee p) \wedge (\neg p)) \to (r \to t)$$
$$((\neg q \to (s \to \neg r)) \wedge (\neg t \to s) \wedge (\neg q \vee p) \wedge (\neg p)) \ \textit{Disjunctive Syllogism}$$
$$(\neg q \to (s \to \neg r)) \wedge \neg q \ \wedge (\neg t \to s) \ \textit{Modus Ponen}$$
$$(\neg t \to s) \wedge (s \to \neg r) \ \textit{Hypothetical Syllogism}$$
$$(\neg t \to \neg r) \ \textit{ContraPositive}$$
$$r \to t$$

Hence Proved

e. Prove the following logical equivalence using the laws of logics, justify each steps.

$$((a \vee b) \wedge (a \to c)) \to (b \vee c) \equiv T$$

Solution:

The statement is a tautology.

	$((a \vee b) \wedge (a \to c)) \to (b \vee c)$	
$\equiv$	$\neg((a \vee b) \wedge (\neg a \vee c)) \vee (b \vee c)$	Implication equivalence(x2).
$\equiv$	$(\neg(a \vee b) \vee \neg(\neg a \vee c)) \vee (b \vee c)$	De Morgans.
$\equiv$	$((\neg a \wedge \neg b) \vee (\neg\neg a \wedge \neg c)) \vee (b \vee c)$	De Morgans.
$\equiv$	$((\neg a \wedge \neg b) \vee (a \wedge \neg c)) \vee (b \vee c)$	Double negation.
$\equiv$	$(\neg a \wedge \neg b) \vee b \vee (a \wedge \neg c) \vee c$	Assocative and commutative.
$\equiv$	$((\neg a \vee b) \wedge (\neg b \vee b)) \vee ((a \vee c)) \wedge (\neg c \vee c))$	Distributive.
$\equiv$	$((\neg a \vee b) \wedge T) \vee ((a \vee c)) \wedge T)$	Negation.
$\equiv$	$(\neg a \vee b) \vee (a \vee c)$	Identity laws (x2).
$\equiv$	$a \vee \neg a \vee b \vee c)$	Associative and commutative.
$\equiv$	$T \vee b \vee c$	Negation
$\equiv$	T	Domination

f. Let A and B be sets using set identity to show that $(B^c \cup (B^c - A))^c = B$

Solution

$(B^c \cup (B^c - A))^c$	$=$	$(B^c \cup (B^c \cap A^c))^c$	by the set difference law
	$=$	$(B^c)^c \cap (B^c \cap A^c)^c$	by De Morgan's law
	$=$	$B \cap (B^c \cap A^c)^c$	by the double complement law
	$=$	$B \cap ((B^c)^c \cup (A^c)^c)$	by De Morgan's law
	$=$	$B \cap (B \cup A)$	by the double complement law (used twice)
	$=$	B	by the absorption law.

Q2: [6*2=12]

a. Let P(x) means "x is a process", I(x) mean "x generates interrupt", and A(x) mean "x needs API" and domain consists of operating systems process.
Translate the following English statement into logical expression.
   i. Some processes which generate interrupt need API.

$\exists x((P(x) \wedge I(x)) \wedge A(x))$

Translate the following quantifier expression into English.

   ii. $\forall x((P(x) \wedge A(x)) \rightarrow \neg I(x))$

Every process which needs an API does not generate an interrupt.

b. Each student in Liberal Arts at some college has a mathematics requirement A and a science requirement B. A poll of 140 sophomore students shows that 60 completed A, 45 completed B and 20 completed both A and B.
Determine the number of students who have completed neither A nor B.

Solution

by the Inclusion–Exclusion Principle:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 60 + 45 - 20 = 85$$

55 completed neither requirement, i.e.

$$n(A^C \cap B^C) = n[(A \cup B)^C] = 140 - 85 = 55.$$

c. Let $A = \{a, b, c\}$, $B = \{x, y, z\}$, $C = \{r, s, t\}$. Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be defined by: $f = \{(a, y), (b, x), (c, y)\}$ and $g = \{(x, s), (y, t), (z, r)\}$.

Find $f \circ g$ and $g \circ f$, if it does not exist, give reason.

Solution

Use the definition of the composition function to compute:

$$(g \circ f)(a) = g(f(a)) = g(y) = t$$
$$(g \circ f)(b) = g(f(b)) = g(x) = s$$
$$(g \circ f)(c) = g(f(c)) = g(y) = t$$
$$g \circ f = \{(a, t), (b, s), (c, t)\}.$$

fog does not exist because f(r) ,f(s),and f(t) are not available .

d. Let f and g be the function from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$ and from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$,

$f(1) = d$, $f(2) = c$, $f(3) = a$, $f(4) = b$ and $g(a) = 2$, $g(b) = 1$, $g(c) = 3$ and $g(d) = 2$

Determine whether f and g are one-to-one or onto ,if it does not exist, give reason.

Solution

f is one-one and onto

g is not one-one (more than two elements of domain have same images)

g is not onto (neither) , no pre image of the element 4

e. Prove or disprove by contradiction that, let $n \in Z$. If $n^2 - 6n + 5$ is even, then n is odd.

Contrapositive. If n is even, then $n^2 - 6n + 5$ is odd.
n is even
$\Longrightarrow n = 2a$ for some integer a (defn. of even)
$\Longrightarrow n^2 - 6n + 5 = (2a)^2 - 6(2a) + 5$ (substitute $n = 2a$)
$\Longrightarrow n^2 - 6n + 5 = 2(2a^2) - 2(6a) + 2(2) + 1$ (simplify)
$\Longrightarrow n^2 - 6n + 5 = 2(2a^2 - 6a + 2) + 1$ (take 2 common)
$\Longrightarrow n^2 - 6n + 5$ is odd (defn. of odd)
Hence, the proposition is true.

f. Prove using mathematical induction that for all integers $n \geq 1$,

$$\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$$

Solution

Let $P(n)$ denote $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$.

- Basis step. $P(1)$ is true.
- Induction step.

Assume $P(k)$: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{k\cdot(k+1)} = \frac{k}{k+1}$ for some $k \geq 1$

Prove $P(k+1)$: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{(k+1)\cdot(k+2)} = \frac{k+1}{k+2}$

LHS of $P(k+1)$

$= \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{k\cdot(k+1)}\right) + \frac{1}{(k+1)\cdot(k+2)}$

$= \frac{k}{k+1} + \frac{1}{(k+1)\cdot(k+2)}$ ($\because P(k)$ is true)

$= \frac{k^2+2k+1}{(k+1)\cdot(k+2)}$ ($\because$ common denominator)

$= \frac{(k+1)^2}{(k+1)\cdot(k+2)}$ ($\because$ simplify)

$= \frac{k+1}{k+2}$ ($\because$ remove common factor)

$=$ RHS of $P(k+1)$

Good Luck!