National University of Computer and Emerging Sciences, Karachi.

practice (CLO-1), Spring 2024
CS1005-Discrete Structures

FAST School of Computing

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1. Which of these sentences are propositions? What are the truth values of those that are propositions?
   a) 2+x>8
   b) Napoleon won the battle of Waterloo.
   c) 2 + 3
   d) In 1492 Columbus sailed the ocean blue.
   e) Does John love Discrete Mathematics?

2. Express the given statements using logical connectives.
   a) A student gets A in Discrete if and only if his weight total is $\geq 95\%$.
   b) Either Alice is smart, or she is not smart but honest.
   c) $\sqrt{30}$ is greater than 6 or $\sqrt{30}$ is less than 5.
   d) Sam had pizza last night and Chris finished her homework.
   e) Either Chris finished her homework or Pat watched the news this morning, but not both.

3. Let p = "$2 \leq 5$", q = "8 *is an even integer*", and r = "11 *is a prime number*". Express the following as statement in English and determine whether the statement is true or false.
   a) $\neg p \wedge q$
   b) $(p \wedge q) \rightarrow r$
   c) $p \rightarrow (q \vee (\neg r))$
   d) $(\neg p) \rightarrow (\neg q)$

4. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP.
   Express the given statements using Logical Connectives. Also determine the truth value of each of these propositions.
   a) Smartphone B has the most RAM of these three smartphones.
   b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
   c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
   d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
   e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

5. Let p, q, and r be the propositions.
   p: You have the flu. q: You missed the final examination. r: You pass the course.
   Express each of these propositions as an English sentence.
   a) $p \rightarrow q$ b) $\neg q \leftrightarrow r$ c) $q \rightarrow \neg r$
   d) $p \vee q \vee r$ e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ f) $(p \wedge q) \vee (\neg q \wedge r)$

6. Let p, q, and r be the propositions.
   p: You get an A on the final exam. q: You do every exercise in this book. r: You get an A in this class.
   Write these propositions using p, q, and r and logical connectives.
   a) You get an A in this class, but you do not do every exercise in this book.
   b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
   c) To get an A in this class, it is necessary for you to get an A on the final.
   d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
   e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
   f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

7. Write each of these statements in the form "if p, then q" in English. [Hint: Refer to the list of common ways to express conditional statements provided in this section.]
   a) You send me an e-mail message only if I will remember to send you the address.
   b) To be a citizen of this country, it is sufficient that you were born in the United States.
   c) If you keep your textbook, it will be a useful reference in your future courses.
   d) The Red Wings will win the Stanley Cup if their goalie plays well.

8. Use De Morgan's laws to find the negation of each of the following statements.
   a) Jan is rich and happy. b) Carlos will bicycle or run tomorrow.
   c) The fan is slow or it is very hot. d) Akram is unfit and Saleem is injured.
   e) $0 \leq x \leq 4$ f) $-2 < x \leq 5$

9. Prove the following equivalences by using laws of logic:
   a) $(p \mathrm{A} (\neg (\neg p \lor q))) \lor (p \mathrm{A} q) \equiv p$
   b) $\neg (p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$
   c) $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$
   d) $(p \mathrm{A} q) \rightarrow (p \rightarrow q) \equiv T$
   e) $\neg (p \lor \neg (p \mathrm{A} q)) \equiv F$

10. Using Truth table, show that these compound propositions are logically equivalent or not.
   a) $(p \rightarrow r) \mathrm{A} (q \rightarrow r)$ and $(p \lor q) \rightarrow r$
   b) $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$

11. Given propositional function $q(x, y)$: $x + y = 1$, which of the following are propositions; which are not? For those that are, determine their truth values.
   a) $q(x, y)$ b) $q(-6, 7)$ c) $q(x + 1, -x)$
   d) $q(x, 3)$ e) $q(1, 1)$ f) $q(5, -4)$

12. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
   a) $\exists x(x^2 = 2)$ b) $\exists x(x^2 = -1)$ c) $\forall x(x^2 + 2 \geq 1)$ d) $\exists x(x^2 = x)$

13. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a) Everybody can fool Bob.
   b) Alice can fool everybody.
   c) Everybody can fool somebody.
   d) There is no one who can fool everybody.
   e) Everyone can be fooled by somebody.

14. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.
   a) There is a student at your school who can speak Russian and who knows C++.
   b) There is a student at your school who can speak Russian but who doesn't know C++.
   c) Every student at your school either can speak Russian or knows C++.
   d) No student at your school can speak Russian or knows C++.

15. Let $Q(x, y)$ be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.
   a) $\exists x \exists y\, Q(x, y)$ b) $\exists x \forall y\, Q(x, y)$ c) $\forall x \exists y\, Q(x, y)$
   d) $\exists y \forall x\, Q(x, y)$ e) $\forall y \exists x\, Q(x, y)$ f) $\forall x \forall y\, Q(x, y)$

16. What rule of inference is used in each of these arguments?
    a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
    b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
    c) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will get sunburned.

17. Let U= {1, 2, 3, 4, 5, 6, 7,8}, A = {1, 2, 4, 5}, B = {2, 3, 5, 6}, and C = {4, 5, 6, 7}. Find and draw the Venn diagrams for each of these combinations of the sets *A*, *B* and *C*:
    a) $(A \cap B) \cap \overline{C}$ b) $\overline{A} \cup (B \cup C)$ c) $(A - B) \cap C$

18. Solve the following:
    a) Let A= {1,2,3} and B={1,2,3,4}. Determine whether A is a subset of B, and whether A is a proper subset of B.
    b) Given the set C= {x|x is a prime number less than 10), list all the proper subsets of C.
    c) Define the set N= {x|x is a prime number between 10 and 20). What is the cardinality of the power set of N?
    d) Let E = {1, 2, 4} and F = {a, b, c, d}. Find E × F (the Cartesian product of E and F).

19. Prove or disprove the following expression by using the set identities:
    a) $(A - (A \cap B)) \cap (B - (A \cap B)) = \Phi$ b) $(A - B) \cup (A \cap B) = A$

20.
    a) In a university of 1000 students, 350 like Computer Science and 450 like Software Engineering. 100 students like both CS & SE. How many like either of them or how many like neither?
    b) In a survey on the gelato preferences of college students, the following data was obtained:
    78 like mixed berry, 32 like Irish cream, 57 like tiramisu, 13 like both mixed berry and Irish cream, 21 like both Irish cream and tiramisu, 16 like both tiramisu and mixed berry, 5 like all three flavors, and 14 like none of these three flavors. How many students were surveyed?

21. Let A= {a, b, c, d} and B = {a, b, c, d}. Consider the following functions:
    a) *f(a) = b, f(b) = a, f(c) = c, f(d) = d* b) *f(a) = b, f(b) = b, f(c) = d, f(d) = c*

    (i) Determine the Domain, Co-domain and Range of the functions.
    (ii) Determine whether the functions are Injective, Surjective and Bijective or not?
    (iii) Determine the inverse of function if exists.

22. a) *Let* $f(x) = \left\lfloor \frac{x^2}{3} \right\rfloor$, Find f (S) if:
    (i) S = {-2,-1,0,1,2,3} (ii) S = {0,1,2,3,4,5} (iii) S = {1,5,7,11} (iv) S = {2,6,10,14}

    b) Solve the following:
    (i) $\left\lceil \frac{3}{4} \right\rceil$ (ii) $\left\lfloor \frac{7}{8} \right\rfloor$ iii) $\left\lceil -\frac{3}{4} \right\rceil$ (iv) $\left\lfloor -\frac{7}{8} \right\rfloor$
    (v) $\lceil 3 \rceil$ (vi) $\lfloor -1 \rfloor$ (vii) $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor$ (viii) $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$

    c) Prove or disproof that if x is a real number, then $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$.

23. Let f and g be the functions from the set of integers to the set of integers defined by f(a) = 2a + 3 and g(a) = 3a + 2.
    a) What is the composition of f and g? What is the composition of g and f?
    b) Which type of function f and g are?
    c) Are f and g invertible?

24. Solve the following with defined type of proof method.
    a) Prove the statement: There is an integer n > 5 such that $2^n - 1$ is prime.

b) Prove that for any integer a and any prime number p, if p | a, P | (a + 1).

c) Prove the statement: There are real numbers a and b such that $\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$.

d) Prove that if |x| > 1 then x > 1 or x < –1 for all x € R.

e) Find a counter example to the proposition: For every prime number n, n + 2 is prime.

f) Show that the set of prime numbers is infinite.

25. Prove by contradiction method, the statement:

a) If n and m are odd integers, then n + m is an eveninteger.

b) Prove the statement by contraposition: For all integers m and n, if m + n is even then m and nare both even or m and n are both odd.

c) Prove by contradiction that $6 - 7\sqrt{2}$ is irrational.

d) Prove by contradiction that $\sqrt{2} + \sqrt{3}$ is irrational.

26. By mathematical induction,

a) prove that following is true for all positive integral values of n.(a) $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = (n(n+1)(2n+1))/6$

b) $1+2+2^2 + \ldots + 2^n = 2^{n+1} - 1$ for all integers $n \geq 0$

c) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{1}{4} n^2(n+1)^2$