5.3 Recursive Definitions

ACTIVITY

1. Find $f(1), f(2), f(3)$, and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 0, 1, 2, \ldots$

a) $f(n+1) = f(n) + 2$.
b) $f(n+1) = 3f(n)$.
c) $f(n+1) = 2^{f(n)}$.
d) $f(n+1) = f(n)^2 + f(n) + 1$.

2. Find $f(1), f(2), f(3), f(4)$, and $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$ and for $n = 0, 1, 2, \ldots$

a) $f(n+1) = -2f(n)$.
b) $f(n+1) = 3f(n) + 7$.
c) $f(n+1) = f(n)^2 - 2f(n) - 2$.
d) $f(n+1) = 3^{f(n)/3}$.

3. Find $f(2), f(3), f(4)$, and $f(5)$ if f is defined recursively by $f(0) = -1, f(1) = 2$, and for $n = 1, 2, \ldots$

a) $f(n+1) = f(n) + 3f(n-1)$.
b) $f(n+1) = f(n)^2 f(n-1)$.
c) $f(n+1) = 3f(n)^2 - 4f(n-1)^2$.
d) $f(n+1) = f(n-1)/f(n)$.

4. Find $f(2), f(3), f(4)$, and $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$ and for $n = 1, 2, \ldots$

a) $f(n+1) = f(n) - f(n-1)$.
b) $f(n+1) = f(n)f(n-1)$.
c) $f(n+1) = f(n)^2 + f(n-1)^3$.
d) $f(n+1) = f(n)/f(n-1)$.

8. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

a) $a_n = 4n - 2$.
b) $a_n = 1 + (-1)^n$.
c) $a_n = n(n+1)$.
d) $a_n = n^2$.

29. Devise a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1, a_1 = 2$, and $a_n = a_{n-1} \cdot a_{n-2}$, for $n = 2, 3, 4, \ldots$.

32. Devise a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, for $n = 3, 4, 5, \ldots$.

5.3 Recursive Definitions

ACTIVITY

In each of 3–15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use the formulas from Section 5.2 to simplify your answers whenever possible.

3. $a_k = ka_{k-1}$, for all integers $k \geq 1$
$a_0 = 1$

4. $b_k = \dfrac{b_{k-1}}{1 + b_{k-1}}$, for all integers $k \geq 1$
$b_0 = 1$

5. $c_k = 3c_{k-1} + 1$, for all integers $k \geq 2$
$c_1 = 1$

H **6.** $d_k = 2d_{k-1} + 3$, for all integers $k \geq 2$
$d_t = 2$

7. $e_k = 4e_{k-1} + 5$, for all integers $k \geq 1$
$e_0 = 2$

8. $f_k = f_{k-1} + 2^k$, for all integers $k \geq 2$
$f_1 = 1$

H **9.** $g_k = \dfrac{g_{k-1}}{g_{k-1} + 2}$, for all integers $k \geq 2$
$g_1 = 1$

Find the first four terms of each of the recursively defined sequences in 1–8.

1. $a_k = 2a_{k-1} + k$, for all integers $k \geq 2$
$a_1 = 1$

2. $b_k = b_{k-1} + 3k$, for all integers $k \geq 2$
$b_1 = 1$

3. $c_k = k(c_{k-1})^2$, for all integers $k \geq 1$
$c_0 = 1$

4. $d_k = k(d_{k-1})^2$, for all integers $k \geq 1$
$d_0 = 3$

5. $s_k = s_{k-1} + 2s_{k-2}$, for all integers $k \geq 2$
$s_0 = 1, \; s_1 = 1$