Date: D D M M Y Y

Discrete
Assignment No. 01

23K-3032 (Shah Hunain)

Q1 a) This is a proposition with a truth value 'T'.
b) This is a proposition with a truth value 'F'.
c) This is a proposition with a truth value 'T'.
d) This is a proposition with a truth value 'F'
e) This is not a proposition.
f) This is not a proposition.

Q2 A: Smartphone 'A' has 256 MB RAM.
B: Smartphone 'B' has 288 MB RAM.
C: Smartphone 'C' has 128 MB RAM.
D: Smartphone 'A' has 32 GB ROM.
E: Smartphone 'B' has 64 GB ROM
F: Smartphone 'C' has 32 GB ROM.
G: Smartphone 'A' has 8MP camera resolution
H: Smartphone 'B' has 7MP camera resolution
I: Smartphone 'C' has 5MP camera resolution

a) $B \wedge \neg(A \vee C)$
Truth value:- T

b) $F \vee (I \wedge \neg H)$
Truth value:- T

c) $B \wedge E \wedge H \Rightarrow T \wedge T \wedge F$
Truth value:- F

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) $(B \wedge E) \rightarrow I$

Truth value:- F

$A \leftrightarrow B$

Truth value:- F

3/ A : Annual Revenue of ACME computer was 138 billion Dollars
B : " of Nadir software was 87 billion dollars
C : " of Quixote Media was 111 billion dollars
D : ACME computer net profit was 8 billion dollars
E : Nadir software " 5 billion dollars
F : Quixote Media " 13 billion dollars

a) $C \wedge \neg(A \vee B)$

Truth value:- F

b) $\neg E \wedge A$

Truth value:- T

c) $D \vee F$

Truth value:- T

d) $F \rightarrow A$

Truth value:- T

e) $E \leftrightarrow A$

Truth value:- T

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14 (a) $p \rightarrow q$

If you have the flu, then you miss the final examination.

(b) $\neg q \leftrightarrow r$

You will not miss the final exam if and only if you pass the course.

(c) $q \rightarrow \neg r$

If you miss the final exam, then you will not pass the course.

(d) $p \vee q \vee r$.

You have the flu or you miss the final exam or you pass the course.

(e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

If you have the flu, then you will not pass the course or if you miss the final exam you will not pass the course.

(f) $(p \wedge q) \vee (\neg q \wedge r)$

You have the flu and you miss the final exam or you donot miss the final exam and you pass the course.

15 (a) $r \neg q$

(b) $(p \wedge q) \wedge r$

(c) $p \rightarrow r$

(d) $p \wedge q \wedge r$

(e) $(p \wedge q) \rightarrow r$

(f) $r \leftrightarrow (q \vee p)$

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(a) If you send me an e-mail message, then I will remember to send you the address.

(b) If you were born in the United States, then you would be citizen of this country.

(c) If you keep your textbooks, then it will be useful reference in your future courses.

(d) If Red Wings goalies plays well, then they will win Stanley cup.

(e) If you get the job, then you the best credentials.

(f) If beach erodes, then there is a storm.

(g) If you have a valid password, then you can log onto the server.

(h) If you don ot begin your climb too late you will reach the summit.

Q7 (a)

1- I will go for a walk in the woods tomorrow when its sunny.

2- I will go for a walk in the woods, if it is sunny tomorrow.

3- I will go for a walk in the woods unless it is not sunny tomorrow.

4- I will go for a walk in the woods tomorrow when its sunny.

5- I will go for a walk in the woods tomorrow whenever its sunny.

(b) If I will go for a walk in the woods, then it's sunny tomorrow. (converse)

If it is not sunny tomorrow, then I will not go for a walk in the woods (inverse

If I will not go for a walk in the woods, then it is not sunny tomorrow. (contrapositive

(c) $\neg p \rightarrow \neg q$ (inverse)

$p \rightarrow q$ (inverse of its inverse)

$q \rightarrow p$ (converse)

$\neg q \rightarrow \neg p$ (inverse of its converse)

$\neg q \rightarrow \neg p$ (contrapositive)

$q \rightarrow p$ (inverse of its contrapositive)

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Q8 (a) John is not rich or not happy.
(b) Carlos will not bicycle and will not run tomorrow.
(c) The fan is not slow and it is not very hot.
(d) Akram is not fit or Saleem is not injured.

Q9 (a) Exclusive
(b) Inclusive
(c) Inclusive
(d) Inclusive

Q10 (a) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

$(p \wedge (\neg . \neg p \wedge \neg q)) \vee (p \wedge q)$ ∵ De-Morgan's law
$p \wedge (p \wedge \neg q) \vee (p \wedge q)$ ∵ double negation law
$((p \wedge p) \wedge \neg q) \vee (p \wedge q)$ ∵ associative law
$(p \wedge \neg q) \vee (p \wedge q)$ ∵ Idempotent law
$p \wedge (\neg q \vee q)$ ∵ distributive law
$p \wedge T$ ∵ Identity law
$\equiv p$

(b) $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$
$\neg((p \wedge q) \vee (\neg p \wedge \neg q))$ ∵ biconditional law
$\neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q)$ ∵ de-morgan's law
$(\neg p \vee \neg q) \wedge (p \vee q)$ ∵ demorgan's & double negation law
$(p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$ ∵ biconditional law
$\equiv p \leftrightarrow \neg q$

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c) $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$ ∵ biconditional law

$(p \vee q) \wedge (\neg q \vee \neg p)$ ∵ implication law

$(\neg q \vee \neg p) \wedge (p \vee q)$ ∵ commutative law

$(\neg p \vee \neg q) \wedge (q \vee p)$ ∵ commutative law

$(p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$ ∵ implication law

$\equiv p \leftrightarrow \neg q$ ∵ biconditional law

d) $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

$(p \wedge q) \rightarrow (\neg p \vee q)$ ∵ implication law

$\neg (p \wedge q) \vee (\neg p \vee q)$ ∵ implication law

$(\neg p \vee \neg q) \vee (\neg p \vee q)$ ∵ demorgans law

$(\neg p \vee \neg q) \vee (\neg p \vee q)$

$\neg p \vee (\neg q \vee q)$ ∵ distributive law

$\neg p \vee T$ ∵ domination law

$\equiv T$

e) $\neg (p \vee \neg (p \wedge q)) \equiv F$

$\neg p \wedge \neg . \neg (p \wedge q)$ ∵ demorgans law

$\neg p \wedge (p \wedge q)$ ∵ double negation law

$(\neg p \wedge p) \wedge q$ ∵ associative law

$F \wedge q$ ∵ negation law

$\equiv F$ ∵ domination law

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Q11 a) $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$	$(p \vee q)$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Thus; $((p \to r) \wedge (q \to r)) \equiv ((p \vee q) \to r)$

b) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$q \vee r$	$p \to (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Thus; $((p \to q) \vee (p \to r)) \equiv (p \to (q \vee r))$

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e) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

p	q	r	s	$p \to q$	$r \to s$	$(p \to q) \to (r \to s)$	$p \to r$	$q \to s$	$(p \to r) \to (q \to s)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	F	F
T	T	F	T	T	T	T	F	T	T
T	T	F	F	T	T	T	F	F	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

Thus, not logically equivalent

Q12 a) F

b) T

c) F

d) T

e) F

f) T

Q13 (a) T

(b) F

(c) T

(d) T

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4. (a) $\forall x\, F(x, Bob)$
(b) $\forall y\, F(Alice, y)$
(c) $\forall x \exists y\, F(x, y)$
(d) $\neg \exists x \forall y\, F(x, y)$
(e) $\exists y \forall x\, F(y, x)$

5. (a) $\exists x (P(x) \wedge d(x))$
(b) $\exists x (P(x) \wedge \neg d(x))$
(c) $\forall x (P(x) \vee d(x))$
(d) $\neg \exists x (P(x) \vee d(x))$

Q16 (a) There exsists a student who has sent an e-mail message to atleast one another other student
(b) There exsists a student who has sent an e-mail message to every other student.
(c) Every student has sent an e-mail message to atleast one other student.
(d) There exsists a student who has received an e-mail message from every other student
(e) For every student, there exsists a student who has sent an e-mail message to them.
(f) Every student has sent an e-mail message to every other student.

Q17 (a) there exsists a student in the class who has taken atleast one computer science course.
(b) There exsists a student in the class who has taken all computer science course
(c) Every student has taken atleast one computer science course.
(d) There exsists a computer course such that every student in the class has taken it
(e) For every computer science course, there exsists a student in the class who has taken it.
(f) Every student in the class has taken all computer science courses offered.

Q18 (a) Disjunctive syllogism
(b) Simplification
(c) Modus ponens
(d) Modus tollens
(e) Hypothetical syllogism

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Q19 (a) By applying modus ponens and disjunctive syllogism ; valid
(b) By applying modus tollens and hypothetical syllogism ; valid

Q23 (i) Domain.

Q19 (a)

1- If today is Tuesday, I have a test in Mathematics or Economics.

$T \rightarrow (M \vee E)$

2- If my Economic professor is sick, I will not have a test in Economics.

$S \rightarrow (\neg E)$

3- Today is Tuesday ~~and my Economics sir is sick~~.

T

4- My Economics sir is sick

S

5- Using Modus ponens in statement 1 and 3

$T \wedge (T \rightarrow (M \vee E))$ $\quad \because p \wedge (p \rightarrow q) \rightarrow q$

Thus; $M \vee E$

6- Economics sir is sick ; therefore no test for Economics

~~7- Now; by using disjunctive syllogism~~

~~$M \vee E$~~

7- By (5) and (6), we conclude that I will a test is Mathematic (M)

Thus; (A) is valid.

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(b) 1- If Ali is a lawyer, then he is ambitious

$L \rightarrow A$

2- If Ali is an early riser, then he does not like chocolate

$R \rightarrow (\neg C)$

3- If Ali is ambitious, then he is an early riser

$A \rightarrow R$

4- Ali is a lawyer

L

5- using modus ponens in statement 1 and 4

$L \wedge (L \rightarrow A)$

Thus; A

Ali is ambitious

6- Modus ponens in 5 and 3

$A \wedge (A \rightarrow R)$

Thus; R

Ali is an early riser.

7- From (6) and (2)

We can conclude that, Ali does not like chocolate.

$R \wedge (R \rightarrow \neg C)$ ∵ Modus ponens

Thus; $\neg C$

OR

applying hypothetical syllogism on (1) and (3)

$(L \rightarrow A) \wedge (A \rightarrow R)$

$L \rightarrow R$ $\quad$ $-(B)$

~~again~~ applying hypothetical syllogism on (B) and (2)

$(L \rightarrow R) \wedge (R \rightarrow \neg C)$

$L \rightarrow \neg C$

Thus; proved

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Q23 a)

(i) Domain:- $\{a, b, c, d\}$

co-domain:- $\{a, b, c, d\}$

Range:- $\{a, b, c, d\}$

(ii) bijective function

(iii) $f^{-1}(b) = a$

$f^{-1}(a) = b$

$f^{-1}(c) = c$

$f^{-1}(d) = d$

(b)

(i) Domain:- $\{a, b, c, d\}$

co-domain:- $\{a, b, c, d\}$

Range:- $\{b, c, d\}$

(ii) none

(iii) does not exsists

(c) (i) Domain:- $\{a, b, c, d\}$

co-domain:- $\{a, b, c, d\}$

Range:- $\{b, c, d\}$

(ii) none

(iii) does not exsists

(d)

(i) Domain:- $\{a, b, c, d\}$

co-domain:- $\{a, b, c, d\}$

Range:- $\{a, b, c, d\}$

(ii) bijective function

(iii) $f^{-1}(a) = b$

$f^{-1}(b) = c$

$f^{-1}(c) = a$

$f^{-1}(d) = d$

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Q25 $f(a) = 2a+3$ and $g(a) = 3a+2$

(a) $f \circ g = 2(3a+2)+3$

$= 6a+7+3$

$f \circ g = 6a+7$

$g \circ f = 3(2a+3)+2$

$= 6a+9+2$

$g \circ f = 6a+11$

(b) $f(a)$ and $g(a)$ both are linear functions

(c) $f(a) = 2a+3$

$\cancel{y=}\ f(x) = 2x+3$

$y = 2x+3$

$2x = y-3$

$x = \frac{y-3}{2}$

$f^{-1}(x) = \frac{x-3}{2}$

$f^{-1}(a) = \frac{a-3}{2}$

Now;

$g(a) = 3a+2$

$g(x) = 3x+2$

$y = 3x+2$

$3x = y-2$

$x = \frac{y-2}{3}$

$g^{-1}(x) = \frac{x-2}{3}$

$g^{-1}(a) = \frac{a-2}{3}$; both $f(a)$ and $g(a)$ are invertible

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