National University of Computer & Emerging Sciences, Karachi
Multivariable Calculus Assignment - 2 (CLO2, CLO3)
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Submission Date: **May 05, 2024** Total Marks: 50

Q 1: Evaluate. [1+1+1=3 marks]

a. $\int_1^2 \int_4^6 \frac{x}{y^2}\, dxdy$ b. $\int \int x^2 + y^2\, dxdy$ c. $\int_0^1 \int_1^2 \frac{x\, e^x}{y} dydx$

Q 2: A manufacturer has modeled its output by a Cobb-Douglas production function $P(L, K) = 70L^{0.6}K^{0.4}$. Where L is the number of monthly labor hours and K is the capital investment (in units of $1000). If L varies roughly from 5000 to 6000 and monthly capital investment varies evenly between $20, 000 to $30, 000, find the monthly output. [2 marks]

Q 3: Evaluate the following: [1+1+1=3 marks]

a. $\iint_D (x + 2y)dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

b. $\int_0^4 \int_0^{\sqrt{y}} xy^2 dxdy$

c. $\iint_R (4 - y^2)dA$ over the region R which is bounded between $y^2 = 2x$ and $y^2 = 8 - 2x$.

Q 4: Find volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $f(x, y) = 3 - x - y$. [2 marks]

Q 5: Write an equivalent double integral with the order of integration reversed. [1+1+1=3 marks]

a. $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\, dxdy$ b. $\int_0^{3/2} \int_0^{9-4x^2} 16x\, dydx$ c. $\int_0^1 \int_{1-x}^{1-x^2} dy\, dx$

Q 6: Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$. [2 marks]

Q 7: Evaluate the iterated integral by converting to polar coordinates. [2+2+2=6]

a. $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)dx\, dy$ b. $\int_{-1}^0 \int_{-\sqrt{1-x^2}}^0 \frac{2}{1+\sqrt{x^2+y^2}}\, dy\, dx$

c. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dydx$

Q 8: Find the following: [2+2+2=6]

a. The surface area of the portion of $z = x^2 + y^2$ lying under $z = 9$.

b. The surface area of the surface $x^2 + y^2 + z = 4$ above $xy - plane$.

c. The surface area of the portion of $z = x^2 + 2y$ that lies above the triangle in the $xy - plane$ with vertices $(0,0)$, $(1,0)$ and $(1,1)$.

Q 9: Evaluate. [2+2+2+2=8 marks]

a. $\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_{x^2+y^2}^2 x\, dzdydx$

b. $\int_0^3 \int_0^2 \int_0^1 (xyz)^2 \, dxdydz$

c. $\int_0^{\frac{\pi}{4}} \int_0^{lnsect} \int_{-\infty}^{2s} e^r drdsdt$

d. $\iiint yz^2 \sin(xyz) \, dxdydz$

Q 10: Find the following: [2+2+2=6]

a. The volume of the tetrahedron bounded by the planes $y = 0$, $z = 0$, $x = 0$ and $y - x + z = 1$.

b. The volume bounded by the $xy - plane$, the cylinder $x^2 + y^2 = 1$ and the plane

$x + y + z = 3$.

c. The volume of the solid bounded by $y + z = 1$, $y = x^2$ and the $xy - plane$.

Q 11: What will the Jacobian in the general change of coordinates formula be if: [2+2=4 marks]

a. $x = f(u)$, $y = v$ and $z = w$ b. $x = f(u)$, $y = g(v)$ and $z = h(w)$

Q 12: Find the following: [2.5+2.5=5 marks]

a. Let E be the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Use the change of variables $x = au$, $y = bv$ and $z = cw$ to compute the integral $\iiint_E 1 \, dV$.

b. Let Q be the quadrilateral in the $xy - plane$ with vertices $(1,0), (4,0)$, $(0,1)$ and $(0,4)$. Evaluate

$$\iint_Q \frac{1}{x + y} \, dA$$

with the change of variables $x = u - uv$ and $y = uv$.