Activity

DISCRETE STRUCTURE
C S 1005
SE 2A

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

TABLE 8 Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Showing Nonequivalence

Show that the statement forms $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent.

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$		$\sim p \wedge \sim q$
T	T	F	F	T	F		F
T	F	F	T	F	T	$\neq$	F
F	T	T	F	F	T	$\neq$	F
F	F	T	T	F	T		T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent

Negations of *And* and *Or*: De Morgan's Laws

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Applying De Morgan's Laws

Write negations for each of the following statements:

a. John is 6 feet tall and he weighs at least 200 pounds.

b. The bus was late or Tom's watch was slow.

Solution

a. John is not 6 feet tall or he weighs less than 200 pounds.

b. The bus was not late and Tom's watch was not slow.

an alternative

(b) is "Neither was the bus late nor was Tom's watch slow."

Inequalities and De Morgan's Laws

Use De Morgan's laws to write the negation of $-1 < x \leq 4$.

Solution The given statement is equivalent to

$$-1 < x \quad \text{and} \quad x \leq 4.$$

By De Morgan's laws, the negation is

$$-1 \not< x \quad \text{or} \quad x \not\leq 4,$$

$$-1 \geq x \quad \text{or} \quad x > 4.$$

Activity-1

1.1 Propositional Logic

2. Which of these are propositions? What are the truth values of those that are propositions?

a) Do not pass go.
b) What time is it?
c) There are no black flies in Maine.
d) $4 + x = 5$.
e) The moon is made of green cheese.
f) $2^n \geq 100$.

Solution:

a) This is not a proposition; it's a command.
b) This is not a proposition; it's a question.
c) This is a proposition that is false, as anyone who has been to Maine knows.
d) This is not a proposition; its truth value depends on the value of x.
e) This is a proposition that is false.
f) This is not a proposition; its truth value depends on the value of n.

4. What is the negation of each of these propositions?

a) Janice has more Facebook friends than Juan.
b) Quincy is smarter than Venkat.
c) Zelda drives more miles to school than Paola.
d) Briana sleeps longer than Gloria.

Solution:

a) Janice does not have more Facebook friends than Juan.
b) Quincy is not smarter than Venkat.
c) Zelda does not drive more miles to school than Paola.
d) Briana does not sleep longer than Gloria.

8. Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

a) Smartphone B has the most RAM of these three smartphones.

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

Solution:

a) True, because $288 > 256$ and $288 > 128$.

b) True, because C has 5 MP resolution compared to B's 4 MP resolution. Note that only one of these conditions needs to be met because of the word *or*.

c) False, because its resolution is not higher (all of the statements would have to be true for the conjunction to be true).

d) False, because the hypothesis of this conditional statement is true and the conclusion is false.

e) False, because the first part of this biconditional statement is false and the second part is true.

10. Let p and q be the propositions

p: I bought a lottery ticket this week.
q: I won the million dollar jackpot on Friday.

Express each of these propositions as an English sentence.

a) $\neg p$
b) $p \vee q$
c) $p \rightarrow q$
d) $p \wedge q$
e) $p \leftrightarrow q$
f) $\neg p \rightarrow \neg q$
g) $\neg p \wedge \neg q$
h) $\neg p \vee (p \wedge q)$

Solution:

a) I did not buy a lottery ticket this week.

b) Either I bought a lottery ticket this week, or [in the inclusive sense] I won the million dollar jackpot on Friday.

c) If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

d) I bought a lottery ticket this week, and I won the million dollar jackpot on Friday.

e) I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

f) If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

g) I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

h) Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

12. Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.

a) $\neg p$
b) $p \vee q$
c) $\neg p \wedge q$
d) $q \rightarrow p$
e) $\neg q \rightarrow \neg p$
f) $\neg p \rightarrow \neg q$
g) $p \leftrightarrow q$
h) $\neg q \vee (\neg p \wedge q)$

Solution:

a) The election is not decided.

b) The election is decided, or the votes have been counted.

c) The election is not decided, and the votes have been counted.

d) If the votes have been counted, then the election is decided.

e) If the votes have not been counted, then the election is not decided.

f) If the election is not decided, then the votes have not been counted.

g) The election is decided if and only if the votes have been counted.

h) Either the votes have not been counted, or else the election is not decided and the votes have been counted. Note that we were able to incorporate the parentheses by using the words *either* and *else*.

14. Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

Express each of these propositions as an English sentence.

a) $p \rightarrow q$

b) $\neg q \leftrightarrow r$

c) $q \rightarrow \neg r$

d) $p \vee q \vee r$

e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

f) $(p \wedge q) \vee (\neg q \wedge r)$

Solution:

a) If you have the flu, then you miss the final exam.

b) You do not miss the final exam if and only if you pass the course.

c) If you miss the final exam, then you do not pass the course.

d) You have the flu, or miss the final exam, or pass the course.

e) It is either the case that if you have the flu then you do not pass the course, or the case that if you miss the final exam then you do not pass the course (or both, it is understood).

f) Either you have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

16. Let p, q, and r be the propositions

p: You get an A on the final exam.
q: You do every exercise in this book.
r: You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

c) To get an A in this class, it is necessary for you to get an A on the final.

d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Solution:

a) $r \wedge \neg q$	**b)** $p \wedge q \wedge r$	**c)** $r \rightarrow p$
d) $p \wedge \neg q \wedge r$	**e)** $(p \wedge q) \rightarrow r$	**f)** $r \leftrightarrow (q \vee p)$

18. Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.

a) This is $\mathbf{T} \leftrightarrow \mathbf{T}$, which is true.
b) This is $\mathbf{T} \leftrightarrow \mathbf{F}$, which is false.

The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise.

20. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 3$, then unicorns exist.
b) If $1 + 1 = 3$, then dogs can fly.

a) This is $\mathbf{F} \to \mathbf{F}$, which is true.
b) This is $\mathbf{F} \to \mathbf{F}$, which is true.

32. How many rows appear in a truth table for each of these compound propositions?

a) $(q \to \neg p) \vee (\neg p \to \neg q)$
b) $(p \vee \neg t) \wedge (p \vee \neg s)$
c) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$
d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

A truth table will need 2^n rows if there are n variables.
a) $2^2 = 4$ **b)** $2^3 = 8$ **c)** $2^6 = 64$ **d)** $2^5 = 32$

Activity 2

34. Construct a truth table for each of these compound propositions.

a) $p \to \neg p$
b) $p \leftrightarrow \neg p$
c) $p \oplus (p \vee q)$
d) $(p \wedge q) \to (p \vee q)$
e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

36. Construct a truth table for each of these compound propositions.

c) $p \oplus \neg q$
d) $\neg p \oplus \neg q$
e) $(p \oplus q) \vee (p \oplus \neg q)$
f) $(p \oplus q) \wedge (p \oplus \neg q)$

38. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \vee r$
b) $(p \vee q) \wedge r$
c) $(p \wedge q) \vee r$
d) $(p \wedge q) \wedge r$
e) $(p \vee q) \wedge \neg r$
f) $(p \wedge q) \vee \neg r$

39. Construct a truth table for each of these compound propositions.

a) $p \to (\neg q \vee r)$
b) $\neg p \to (q \to r)$
c) $(p \to q) \vee (\neg p \to r)$
d) $(p \to q) \wedge (\neg p \to r)$

40. Construct a truth table for $((p \to q) \to r) \to s$.

41. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

Solution (36)

c) $p \oplus \neg q$ **d)** $\neg p \oplus \neg q$

e) $(p \oplus q) \vee (p \oplus \neg q)$ **f)** $(p \oplus q) \wedge (p \oplus \neg q)$

(c) and (d)

p	q	$\neg p$	$\neg q$	$p \oplus \neg q$	$\neg p \oplus \neg q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

(e) and (f)

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	T	T	F
T	F	T	F	T	F
F	T	T	F	T	F
F	F	F	T	T	F
				tautology	contradiction

30. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

29. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Solution

a) Converse: If I stay home, then it will snow tonight. Contrapositive: If I do not stay at home, then it will not snow tonight. Inverse: If it does not snow tonight, then I will not stay home.

b) Converse: Whenever I go to the beach, it is a sunny summer day. Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day. Inverse: Whenever it is not a sunny day, I do not go to the beach.

c) Converse: If I sleep until noon, then I stayed up late. Contrapositive: If I do not sleep until noon, then I did not stay up late. Inverse: If I don't stay up late, then I don't sleep until noon.

Equivalences laws

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Logical Equivalences

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \lor q$
$p \to q \equiv \neg q \to \neg p$
$p \lor q \equiv \neg p \to q$
$p \land q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \land \neg q$
$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$
$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Tautologies and Contradictions

p	$\sim p$	$p \lor \sim p$	$p \land \sim p$
T	F	T	F
F	T	T	F

TABLE 5 **A Demonstration That $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ Are Logically Equivalent.**

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Negations of *And* and *Or:* De Morgan's Laws

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Example:

Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

Solution:

$$\begin{aligned} \neg(p \rightarrow q) &\equiv \neg(\neg p \vee q) \\ &\equiv \neg(\neg p) \wedge \neg q \\ &\equiv p \wedge \neg q \end{aligned}$$

4. Use truth tables to verify the associative laws

a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

Solution :

a)

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

5. Use a truth table to verify the distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

6. Use a truth table to verify the first De Morgan law $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

7. Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy.

b) Carlos will bicycle or run tomorrow.

c) Mei walks or takes the bus to class.

d) Ibrahim is smart and hard working.

Practice

8. Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

b) Yoshiko knows Java and calculus.

c) James is young and strong.

d) Rita will move to Oregon or Washington.

Solution :

We need to negate each part and swap "and" with "or."

a) Kwame will not take a job in industry and will not go to graduate school.

b) Yoshiko does not know Java or does not know calculus.

c) James is not young, or he is not strong.

d) Rita will not move to Oregon and will not move to Washington.

Show that

a) $(p \wedge \sim q) \vee (p \wedge q) \equiv p.$

b) $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q.$

Proof:

$$(p \wedge \sim q) \vee (p \wedge q) \equiv p \wedge (\sim q \vee q)$$
$$\equiv p \wedge (q \vee \sim q)$$
$$\equiv p \wedge \mathrm{T}$$
$$\equiv p$$

Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p.$

$$(p \vee \sim q) \wedge (\sim p \vee \sim q)$$
$$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$$
$$\equiv \sim q \vee (p \wedge \sim p)$$
$$\equiv \sim q \vee \mathrm{F}$$
$$\equiv \sim q$$

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q.$

verify the logical equivalence

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p.$$

Solution:

$\sim(\sim p \wedge q) \wedge (p \vee q)$	$\equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q)$	by De Morgan's laws
	$\equiv (p \vee \sim q) \wedge (p \vee q)$	by the double negative law
	$\equiv p \vee (\sim q \wedge q)$	by the distributive law
	$\equiv p \vee (q \wedge \sim q)$	by the commutative law for $\wedge$
	$\equiv p \vee \mathrm{F}$	by the negation law
	$\equiv p$	by the identity law.

10. For each of these compound propositions, use the conditional-disjunction equivalence (Example 3) to find an equivalent compound proposition that does not involve conditionals.

a) $\neg p \to \neg q$
b) $(p \vee q) \to \neg p$
c) $(p \to \neg q) \to (\neg p \to q)$

Solution :

We apply the equivalence $p \to q \equiv \neg p \vee q$ to the conditionals in the original statements.

a) $\neg p \to \neg q \equiv p \vee \neg q$

b)

$(p \vee q) \to \neg p \equiv \neg(p \vee q) \vee \neg p$	by the conditional-disjunction equivalence
$\equiv (\neg p \wedge \neg q) \vee \neg p$	by the second De Morgan's law
$\equiv \neg p$	by the first absorption law

c)

$(p \to \neg q) \to (\neg p \to q) \equiv \neg(p \to \neg q) \vee (\neg p \to q)$	by the conditional-disjunction equivalence
$\equiv \neg(\neg p \vee \neg q) \vee (\neg\neg p \vee q)$	by the conditional-disjunction equivalence
$\equiv (p \wedge q) \vee (p \vee q)$	by the double negation and De Morgan's laws
$\equiv (p \wedge q) \vee p \vee q$	by the associative law
$\equiv p \vee q$	by the absorbtion law

12. Show that each of these conditional statements is a tautology by using truth tables.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$
b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
c) $[p \wedge (p \rightarrow q)] \rightarrow q$
d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

18. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

19. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

Solution :

(a) we have the following table.

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

(b) we have the following table.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

16. Show that each conditional statement in Exercise 12 is a tautology by applying a chain of logical identities as in Example 8. (Do not use truth tables.)

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Solution :

a)

$[\neg p \wedge (p \vee q)] \rightarrow q \equiv \neg[\neg p \wedge (p \vee q)] \vee q$	by the conditional-disjunction equivalence
$\equiv p \vee \neg(p \vee q) \vee q$	by a De Morgan's law
$\equiv (p \vee q) \vee \neg(p \vee q)$	by commutativity and associativity
$\equiv \mathbf{T}$	by a negation law

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$\equiv \neg[(p \rightarrow q) \wedge (q \rightarrow r)] \vee (p \rightarrow r)$	by the conditional-disjunction equivalence
$\equiv \neg(p \rightarrow q) \vee \neg(q \rightarrow r) \vee (p \rightarrow r)$	by a De Morgan's law
$\equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$	by the negation of conditionals and the conditional-disjunction equivalences
$\equiv [\neg p \vee (p \wedge \neg q)] \vee [r \vee (q \wedge \neg r)]$	by associativity
$\equiv [(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee [(r \vee q) \wedge (r \vee \neg r)]$	by a distributive law
$\equiv [\mathbf{T} \wedge (\neg p \vee \neg q)] \vee [(r \vee q) \wedge \mathbf{T}]$	by a negation law
$\equiv (\neg p \vee \neg q) \vee (r \vee q)$	by an identity law
$\equiv (\neg p \vee r) \vee (\neg q \vee q)$	by associativity
$\equiv (\neg p \vee r) \vee \mathbf{T}$	by a negation law
$\equiv \mathbf{T}$	by a domination law

The area of logic that deals with predicates and quantifiers is called the **predicate calculus**.

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.