

**June 10, 2020, 10:00 am – 11:00 am**

<b>Course Code: CS211</b>	<b>Course Name: Discrete Structures</b>
<b>Instructor Names: Mr. Shoaib Raza</b>	
<b>Student Roll No:</b>	<b>Section No:</b>

**Instructions:**

- Read each question completely before answering it. There are **3 questions and 2 pages**.
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Answer all the questions in given sequence of the question paper. Step by step solution is required.
- Write your NU ID on the top of every page of answer script. Answer script should be of A4 size.
- Scan your answer script properly. If I can't read the text then I can't grade it.

**Total Time:** 60 minutes

**Maximum Marks:** 26

**Question # 1 (Propositional Logic and Rules of Inference)**

**[5x2=10 points]**

(i) Consider the following propositions:

- $p$ : Zafar is a Doctor.                       $q$ : Zafar is an advisor to prime minister.  
 $r$ : Zafar is healthy man.                       $s$ : Zafar do not eat junk food.

Express these statements using the propositions  $p$ ,  $q$ ,  $r$  and  $s$  together with logical connectives (including negations).

- (a) "If Zafar is a doctor, then he is an advisor to prime minister."  
 (b) "If Zafar is healthy man then he does not eat junk food."  
 (c) "If Zafar is an advisor to prime minister then he is healthy man."

(ii) Write in English, the contrapositive of (a), inverse of (b) & converse of (c) using the statements given in part (i).

(iii) Using the premises(statements) from part(i), apply rules of inference to obtain conclusion(s) from these premises.

(iv) Determine whether the following two propositions are logically equivalent:  $a \rightarrow (\neg b \rightarrow c)$  and  $\neg a \vee \neg(c \rightarrow b)$ .

(v) Prove the following logical equivalence using the laws of logic:  $\neg(X \leftrightarrow Y)$  and  $X \leftrightarrow \neg Y$

**Question # 2 (Predicate and Quantifiers)**

**[3x2=6 points]**

(i) Assume that the universe for  $p$  is all the kids and the universe for  $q$  is the set of all cartoon movies. Write the English statement using the following predicates and any needed quantifiers:

- $B(p, q)$ :  $p$  saw  $q$ ,                       $A(q)$ :  $q$  won an award,                       $C(q)$ :  $q$  is a comedy.

- a) Some kids have seen every comedy.  
 b) Bilal has never seen a movie that won an award.

(ii) Let  $F(a, b)$  means " $a + 2b = ab$ ", where  $a$  and  $b$  are integers. Determine the truth value of the statement.

- a)  $\forall a \exists b F(a, b)$ .  
 b)  $\neg \forall a \exists b \neg F(a, b)$ .

(iii) Suppose the variable  $a$  represents students and  $b$  represents courses, and:

$R(b)$ :  $b$  is an aeronautics course,  $Q(a)$ :  $a$  is a freshman,  $P(b)$ :  $a$  is a full-time student,  $S(a, b)$ :  $a$  is taking  $b$ .

Write the statement in good English without using variables in your answers.

a)  $\forall a \exists b S(a, b)$ .

b)  $\forall a \exists b [(P(a) \wedge Q(a)) \rightarrow (R(b) \wedge S(a, b))]$ .

### **Question # 3 (Set theory and Functions)**

**[5x2=10 points]**

(i) Using Set identities, prove or disprove the following set operations:  $P - (Q \cap R) = (P - Q) \cap (P - R)$ .

(ii) Using Venn diagram, determine how many like both cold drinks and hot drinks. If in a group of 1000 people, 370 like cold drinks and 720 like hot drinks and each person likes at least one of the two drinks.

(iii) Find three subsets of  $\{a, b, c, d, e, f, g, h\}$  such that the intersection of two (any two combinations) has size (cardinality) 3 and the intersection of all three has size 2.

(iv) Suppose  $g: A \rightarrow B$  and  $f: B \rightarrow C$  where  $A = \{1, 2, 3, 4\}$ ,  $B = \{a, b, c\}$ ,  $C = \{2, 7, 10\}$ , and  $f$  and  $g$  are defined by  $f = \{(a, 10), (b, 7), (c, 2)\}$  and  $g = \{(1, b), (2, a), (3, a), (4, b)\}$ .

Is Function  $f$  and  $g$  invertible? If yes find  $f^{-1}$  and  $g^{-1}$  or if not why?

(v) How many functions are there from a set with five elements to a set with three elements?

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**ALL THE BEST 😊**