11/03/24

Chapter no: 06
Counting Tech

License plate:-

The license plate of a state consists of 3 letter followed by 3-digits.

→ How many different license plate are possible?

→ How many possibilities are there of license plate on which no letter or no digits repeated.

→ How many license plate could begin with 'A' and end with 'D'.

→ How many license plate begin with PdR.

a) | 26 | 26 | 26 | 10 | 10 | 10 |

$= 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 26^3 \times 10^3$

b) | 26 | 25 | 24 | 10 | 9 | 8 |

$= 26 \times 25 \times 24 \times 10 \times 9 \times 8$

c) | A | 26 | 26 | 10 | 10 | 0 |

$26 \times 26 \times 10 \times 10 = 26^2 \times 10^2 =$

d) | P | d | R | 10 | 10 | 10 |

$10 \times 10 \times 10 = 10^3$

Telephone Number :- consists of 10 digits

- consists of 3 digits area code.
- consists of 3 digit office code
- consists of 4 digit station code

let

$X \rightarrow 0$ till 9

$N \rightarrow 2$ till 9

$Y \rightarrow 0$ or 1

8 2 10 8 8 10 10 10 10 10

old phone format NYX-NNX-XXXX

new phone format NXX-NXX-XXXX

→ How many different Telephone number are possible under old and new?

old phone

Area code	Office code	station code
$8 \times 2 \times 10 = 160$	$8 \times 8 \times 10 = 640$	$10 \times 10 \times 10 \times 10 = 10000$

$160 \times 640 \times 10^4 =$

new phone

$8 \times 10 \times 10 = 800$	$8 \times 10 \times 10 = 800$	$10 \times 10 \times 10 \times 10 = 10^4$

$800 \times 800 \times 10^4 =$

$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array}$

Ex:3 There are four blue line b/w A and B and 3 bus line b/w B and C.

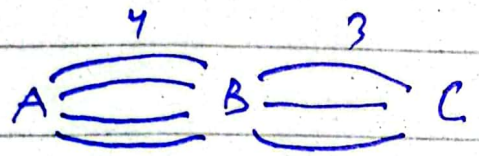
→ Find the number of ways a person can travel.

a) from A to C by way of B.

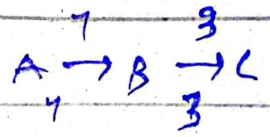
b) round trip by bus from A to C by way of C

c) round trip if person does not repeat more than once.

a)



$4 \times 3 = 12$

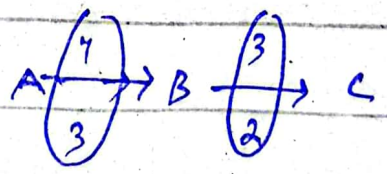


b) $(16)(9)$

$(\cancel{4}6)(\cancel{9}\,\cancel{4}\,3) = 144$

$4 \times 3 = 12$ $\quad$ $3 \times 2 = 6$

c) $12 \times 6 = 72$



$$\begin{array}{r} 24 \\ \times 5 \\ \hline 120 \end{array}$$

Hos — medical

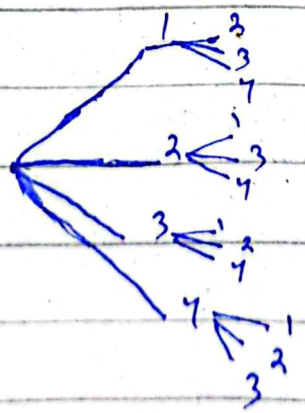
^nP_r permutation $P(n,k) = \dfrac{n!}{(n-k)!}$

^nC_r combination $C(n,k) = \binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

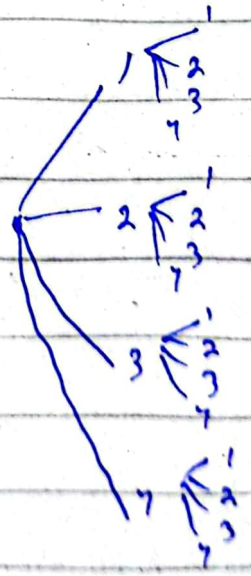
	order matter	does not matter
Repitition	n^k	$\binom{k+n-1}{k}$
non- "	$P(n,k)$ ↗ Permutation	$\binom{n}{k}$ ↗ combination

$$^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$$

$$^6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!(4)!} = \frac{6\times5\times\cancel{4!}}{2!\,\cancel{4!}} = \frac{30}{2} = 15$$



(Not-repeated)



(Repetition)

$S = \{0, 1, 2, 3, 4\}$

write 3 digit numbers

4	5	5

$4 \times 5 \times 5 = 100$

'0' can not be placed here

let group of 12 consists of 5 men and 7 women.

(a) How many five person team can be chosen that consists of 3 men and 2 women?

(b) How many five person team consist of atleast one man.

(c) How many five person team consists of almost one man.

jahan groups kiba adha (combination)

" list " (permutation)

(a) ${}^5C_3 \cdot {}^7C_2 = 210$

(b) ${}^5C_1 \cdot {}^7C_4 + {}^5C_2 \cdot {}^7C_3 + {}^5C_3 \cdot {}^7C_2 + {}^5C_4 \cdot {}^7C_1 + {}^5C_5 \cdot {}^7C_0$

$= 771$

(c) ${}^5C_1 \cdot {}^7C_4 + {}^5C_0 \cdot {}^7C_5 = 196$ ways

ya to nik. ya nik bhi nhi

Q a) How many integers from 1 to 1000 are multiple of 3 or multiple of 5.

5×1 5×2 $3\times5=15 \Rightarrow 15\times66$ 5×200

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ..., 990, ---, 999, 1000

3×1 3×2 3×333

$|A|$ = multiples of 3 = 333

$|B|$ = multiples of 5 = 200

$|A\cap B|$ = multiples of both 3 and 5 = 66

$|A\cup B| = |A| + |B| - |A\cap B|$

$= 333 + 200 - 66$

$|A\cup B| = 467$

(b) neither divisible by 3 nor 5.

$|A\cup B|' = \text{Total} - |A\cup B|$

$= 100 - 467 = 533.$

Q (a) How many different ways can of the letter (no repitition)

BYTES :-

$5! = 120$

(b) the first letter must be B

B				
	4	3	2	1

$= 4! = 24$

Pascals Triangle

0 ←							1								
1 ←						1		1							
2 ←					1		2		1						
3 ←				1		3		3		1					
4 ←			1		4		6		4		1				
5 ←		1		5		10		10		5		1			

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x-y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$ (alternate signs)

Q: The coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

$\binom{25}{13}$ → total 'n' → power of 'y'

$$= \frac{25!}{13!(25-13)!} = 5200300.$$

$$\rightarrow (x+y)^{25} = \sum_{j=0}^{25} \binom{25}{j} x^{25-j} y^j$$

similarly;

$$(2x-3y)^{25} = \sum_{j=0}^{25} \binom{25}{j} (2x)^{25-j} (-3y)^j$$

$$= \binom{25}{13} (2x)^{12} (-3y)^{13}$$

$$= -3.39 \times 10^{16}$$

Pigeonhole Principle

A line from a finite set to a smaller finite set can-not be one-one;

There must be atleast two elements have same image in co-domain.

Generalized:- $k\left(\left\lceil \frac{N}{k} \right\rceil - 1\right) < k\left(\left(\frac{N}{k} + 1\right) - 1\right) = N$

→ among 367 people, there must be atleast $\left\lceil \frac{367}{366} \right\rceil = 2$ people that have same birthday.

→ $(a+b)^2 = a^2 + 2ab + b^2$

$= \binom{2}{0} a^2 + \binom{2}{1} ab + \binom{2}{2} b^2$

$\because \binom{2}{2} = \frac{2!}{2!(2-2)!} = 1$

$\binom{5}{5} b^5$

→ $(a+b)^5 = \binom{5}{0} a^5 + \binom{5}{1} a^4 b + \binom{5}{2} a^3 b^2 + \binom{5}{3} a^2 b^3 + \binom{5}{4} ab^4 +$

$= \sum_{j=0}^{5} \binom{5}{j} a^{5-j} b^j$

⋮

$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} (x)^{n-j} (y)^j$

Chapter no: 06 "Counting"

(6.1, 6.2, 6.3, 6.7)

6.1

Sum Rule:-

$|A \cup B| = |A| + |B|$ ∵ A and B are disjoint sets.

jab kisi kaam ko 'n_1' tareeke se kar sakain aur usse 'n_2' tareeke se bhi kar sakain lekin n_1 and n_2 bilkul alag hon

Example:-

Suppose there are 7 different optional courses in CS and different optional courses in Maths. How many ways student can choose a course.

$n_1 = 7$ different optional courses in CS

$n_2 = 3$ in Math

Therefore;

By sum rule,

There are $7+3 = 10$ choices for a student

Example:-

$n_1 = 37$

$n_2 = 83$

By sum rule;

$37 + 83 = 120$ possible ways to pick a representative.

Example:-

$n_1 = 23$

$n_2 = 15$

$n_3 = 19$

By sum rule;

$23 + 15 + 19 = 57$ ways

The Product Rule :- $n_1 \cdot n_2$ ways

$$A_i \times A_j = (_, _)$$

Example :-

How many ways a student can choose one optional course each from CS and Math courses if there are 7 different optional courses in CS and 3 different optional courses in Maths.

$n_1 = 7$

$n_2 = 3$

$n_1 \cdot n_2 = (7)(3) = 21$ ways

Example :-

The chairs of an auditorium are to be labeled with two characters, a letter followed by a digit. What is the largest number of chairs that can be labeled differently.

Total alphabetic characters $= n_1 = 26$ (A – Z)

Total numeric characters $= n_2 = 10$ (0 – 9)

Therefore;

$n_1 \cdot n_2 = (26)(10) = 260$ ways.

Example :-

Find the number of ways that an organization consisting of 15 members can elect a president, treasurer and secretary. (assuming no person is elected to more than one position).

$= 15 \times (15-1) \times (15-2)$

$= 15 \times 14 \times 13 = 2730$ ways

Example :-

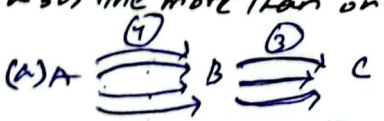
There are four bus lines b/w A and B; and three bus lines b/w B and C.

a) By bus from A to C by way of B.

b) Round trip from A to C by the way of B.

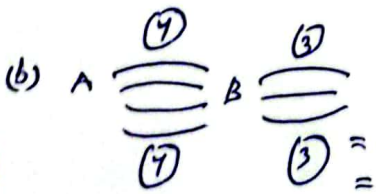
c) Round trip by bus from A to C by way of B, if a person does not want to use a bus line more than once.

(a)



$4 \times 3 = 12$

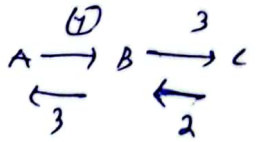
(b)



$= (4 \times 4)(3 \times 3)$

$= (16)(9) = 144$ way

(c)



$= (4 \times 3)(3 \times 2)$

$= (12)(6)$

$= 72$ ways

Example 1-

A bit string is a sequence of 0's and 1's. How many bit string are there of length 4.

2	2	2	2

$2 \times 2 \times 2 \times 2 = 2^7 = 16$

Q How many bit string of length 8 :

(i) begin with a 1 ?

(ii) begin and end with a 1 .

(i)

1	2	2	2	2	2	2	2

$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$

(ii)

1	2	2	2	2	2	2	1

$2^6 = 64$

Q Suppose that an automobile licence plate has three letters followed by three digits

(a) How many different licence plate are possible ?

no. of letter = 26 (A–Z)

no. of digits = 10 (0–9)

26	26	26	10	10	10

$= 26 \times 26 \times 26 \times 10 \times 10 \times 10$

$= 26^3 \times 10^3$

$= 17,576,000$

(b) How many could start with A and end on 0?

A	26	26	10	10	0

$= 26 \times 26 \times 10 \times 10$

$= 26^2 \times 10^2$

$= 67600$

(c) How many begin with PQR ?

P	Q	R	10	10	10

$10 \times 10 \times 10 = 10^3 = 1000$ ways.

(d) How many license plates are possible in which all letters and digits are distinct?

26	25	24	10	9	8

$= 26 \times 25 \times 24 \times 10 \times 9 \times 8$

$= 11,232,000$

(e) How many could begin with AB and have all three letters and digits distinct.

A	B	26	10	9	8

$= 26 \times 10 \times 9 \times 8$

$= 17,280$

4. Determine how many times the inner loop will be iterated when the following algorithm is implemented and run

a) For i: =1 to 4
For j: =1 to 3

[Statement in body of inner loop. None contain branching statements that lead out of the inner loop.]

Sol:-

$n_1 = 4$

$n_2 = 3$

Therefore; $(4)(3) = 12$ ways.

b) For i:=5 to 50
j:=10 to 20

$n_1 = (50-5+1) = 46$

$n_2 = (20-10+1) = 11$

Therefore; $(46)(11) = 506$ ways.

c) For i = 1 to 4
j = 1 to i

$n_1 = 4$

$n_2 = 1$
$n_2 = 2$
$n_2 = 3$
$n_2 = 4$

Therefore; $1+2+3+4 = 10$ times

"Combining sum and product Rule"

Example:-

Suppose statement labels in a programming language can be either a single letter or a letter followed by a digit. Find the number of possible labels.

single letter $= n_1 = 26$ (A-Z)

letter followed by a digit $= n_2 = 26 \times 10 = 260$

Therefore;

$$n_1 + n_2 = 26 + 260 = \boxed{286}$$

Example:-

A computer access code word consists of from one to three letters of English alphabets with repitition allowed. How many different code words are possible?

one English alphabet $= n_1 = 26 = 26$

two " " $= n_2 = 26 \times 26 = 26^2$

three " " $= n_3 = 26 \times 26 \times 26 = 26^3$

$$(26 + 26^2 + 26^3) = 18,278$$

Example:-

Each user on a computer system has a password, which is six to eight characters long, where each character is an upper case letter or a digit. Each password must contain atleast one digit. How many possible passwords are there?

by sum rule;

$$P = P_6 + P_7 + P_8$$

$P_6 = 36^6 - 26^6 = 1,867,866,560$

$P_7 = 36^7 - 26^7 =$

$P_8 = 36^8 - 26^8 =$

$$P = P_6 + P_7 + P_8 = \boxed{2,684,483,063,360}$$

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Subtraction Rule:-

Also, called principle of inclusion-exclusion

$|A \cup B| = |A| + |B| - |A \cap B|$

Q How many bit strings of length eight start either with a 1 bit or end with the two bits 00.

That start with '1' bit

1	2	2	2	2	2	2	2

$2^7 = 128$

That end with bit 00

2	2	2	2	2	2	0	0

$2^6 = 67$

That starts with '1' and end with '00'

1	2	2	2	2	2	0	0

$2^5 = 32$

Hence;

$2^7 + 2^6 - 2^5 = 128 + 67 - 32 = \boxed{160}$

Counting functions:-

How many functions are there from a set with m elements to a set with n elements?

$n \cdot n \cdot n \cdot \ldots \cdot n = n^m$

Counting one-to-one function:-

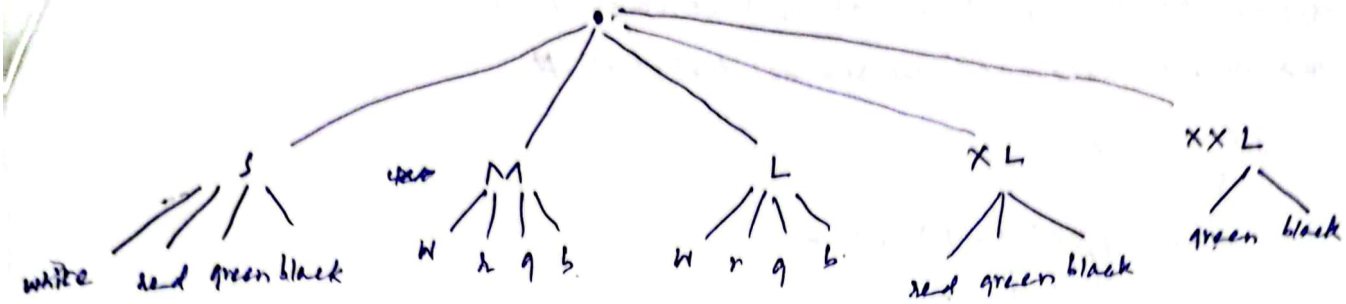
$a_1, a_2, a_3, \ldots, a_m$

There are n way to do a_1, $(n-1)$ ways to do a_2

Thus;

$n(n-1)(n-2) \ldots (n-m+1)$

Tree diagram:- (lesson: 43)



↑ The store must stock 17 T-shirts

Suppose that "I love Discrete Math" T-shirts come in five different sizes S, M, L, XL, XXL. Each size comes in four colors (White, red, green, black) except XL, which comes only in red, green & black and XXL which comes only in green and black. What is the minimum number of shirts that the campus book store need to stock to have one of each size and color available?

"Pigenhole Principle"

Ex: 4.2

(k+1) objects are placed in 'k' boxes; which mean atleast one box contains two or more objects.

Among 100 people there are atleast $\lceil \frac{100}{12} \rceil = 9$ who were born in the same month.

In any set of 27 English, must be atleast two that begin with the same letter.

$\lceil \frac{27}{26} \rceil = 2$ ∵ 26 → bcz there 26 letters in English

Twelve people are running for student government. There are 202 people who vote. What is the minimum number of votes needed for someone to win the elections?

$\lceil \frac{202}{3} \rceil = 68$

38 → different time periods
677 → different classes.

$\lceil 677/38 \rceil = 18$ classes

Q What is the minimum number of students required in a Discrete Mathematics class to be sure that atleast six will receive the same grade, if there are five grades A, B, C, D, F.

$$\left\lceil \frac{N}{5} \right\rceil = 6$$

$$\because N = k\left(\left\lceil \frac{N}{k} \right\rceil - 1\right) + 1 =$$

$$= 5\left(\left\lceil \frac{N}{5} \right\rceil - 1\right) + 1$$

$$= 5(6-1) + 1$$

$$= 30 - 5 + 1$$

$$\boxed{N = 26}$$

Q What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 10-digit telephone no.s? Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive and X represents any digit.

N			N						

$$\left\lceil \frac{25\,000000}{8\,000000} \right\rceil = \left\lceil \frac{25}{8} \right\rceil = 4$$

"Permutation & Combinations."

Ex: 6.?

Q How many way are there to select a first-prize winner, a second prize winner, and a third prize winner from 100 different people who have entered a contest?

$$= (100)(99)(98)$$

$$= 970200 \text{ ways}$$

Q A saleswoman has to visit eight cities. She must begin her trip in a specified city, but she can visit other seven in any order she wishes. How many possible orders can the woman use when visiting cities?

2 cities is decided

then $8-1 = 7!$ ways

1	7	6	5	4	3	2	1

$7! = 7\times6\times5\times4\times3\times2\times1 = 5040$ ways,

Q How many permutations of the letters ABCDEFGH contain string ABC?

string ABC ikkaha iss liye isse combined I consider karo

$$\underset{1}{\underline{ABC}}, \underset{2}{\underline{D}}, \underset{3}{\underline{E}}, \underset{4}{\underline{F}}, \underset{5}{\underline{G}}, \underset{6}{\underline{H}}$$

$6! = \boxed{720}$ or ${}^6P_6 = 720$

Q How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a deck of 52 cards?

(a) $${}^{52}C_5 = \frac{52!}{5!\,(52-5)!} = \frac{52!}{5!.47!} = 2598960$$

(b) $${}^{52}C_{47} = \frac{52!}{47!\,(52-47)!} = \frac{52!}{47!.5!} = 2598960.$$

Q How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school.

$${}^{10}C_5 = \frac{10!}{5!\,(10-5)!} = \frac{10!}{5!.5!} = 252$$

Q A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission?

$${}^{30}C_6 = \frac{30!}{6!\,(30-6)!}$$

$$= \frac{30!}{6!.24!} = 593775$$

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"Binomial co-efficient and Identities"

Ex: 6.7

Q What is expansion of $(x+y)^7$?

$$\binom{7}{0}x^7 + \binom{7}{1}x^6y + \binom{7}{2}x^5y^2 + \binom{7}{3}x^4y^3 + \binom{7}{7}y^7$$

$$x^7 + 7x^6y + 6x^5y^2 + 7x^4y^3 + y^7$$

Q What is coefficient of $x^{12}y^{13}$ in expansion of $(x+y)^{25}$.

$$\binom{25}{13} = \frac{25!}{13!\,(25-13)!} = 5{,}200{,}300$$

Q coefficient of $x^{12}y^{13}$ in expansion of $(2x-3y)^{25}$.

$$= \binom{25}{13}(2)^{12}(-3)^{13}$$

$$= -\frac{25!}{13!\,12!}(2)^{12}(+3)^{13}$$

<u>Pascal's Triangle:-</u>

			1			$\to n=0$
		1		1		$\to n=1$
	1		2		1	$\to n=2$
1		3		3		1 $\to n=3$
1	4		6		4	1 $\to n=4$
						⋮

for $n=4$

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

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20/03/24

Q Solve $3^{100} \bmod 29$.

$3^1 \bmod 29 = 3$

$3^2 \bmod 29 = 9$

$3^4 \bmod 29 = 3^2 \cdot 3^2 \bmod 29 = (3^2 \bmod 29)(3^2 \bmod 29)$

$= 9 \cdot 9 \bmod 29 = 81 \bmod 29 = 23$ or -6

$3^8 \bmod 29 = 3^4 \cdot 3^4 \bmod 29 = -6 \times -6 \bmod 29$

$= 36 \bmod 29 = 7$ or -22

$3^{16} \bmod 29 = 3^8 \cdot 3^8 \bmod 29 = 7 \cdot 7 \bmod 29$

$= 49 \bmod 29 = 20$ or -9

$3^{32} \bmod 29 = 3^{16} \cdot 3^{16} \bmod 29 = -9 \times -9 \bmod 29$

$= 81 \bmod 29 = 23$ or -6

$3^{64} \bmod 29 = 3^{32} \cdot 3^{32} \bmod 29 = -6 \times -6 \bmod 29$

$= 36 \bmod 29 = 7$ or -23

$100 = 64 + 32 + 4$

$3^{100} = 3^{64} \cdot 3^{32} \cdot 3^4$

$3^{100} \bmod 29 = 3^{64} \cdot 3^{32} \cdot 3^4 \bmod 29$

$= (7)(-6)(-6) \bmod 29$

$= 7 \times 36 \bmod 29$

$= 252 \bmod 29$

$= \boxed{20}$

Q Solve $23^3 \bmod 30$

Solve $242^{329} \bmod 273$

Hashing Function

Pseudorandom Numbers:-

$x_{n+1} = (a x_n + c) \bmod m$

a, c, m given non ve

imp

Check digits:- (mod 10)

$3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 3x_7 + x_8 + 3x_9 + x_{10} + 3x_{11} + x_{12} = 0$

Checking an ISBN number:-

$x_{10} = \sum_{i=1}^{9} i x_i \pmod{11}$

$= (x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 + 6x_6 + 7x_7 + 8x_8 + 9x_9) \bmod 11$

21/03/24

Prime Numbers and GCD:-

$P = \{2, 3, 5, 7, \ldots\}$ prime

$C = \{4, 6, 9, \ldots\}$ composite

- $12 = 4 \times 3 = 2^2 \times 3 = 2 \times 2 \times 3$
- $20 = 4 \times 5 = 2^2 \times 5$
- $154 = 77 \times 2 = 11 \times 7 \times 2$ ∴ any number can be written as product of prime.

$gcd(a,b) =$

$a \equiv b \pmod m$

$a = bq + r \qquad a \le r \le b$

∵ Euclid Algorithm

gcd (6, 14) :-

$6 = \boxed{2(1)} \times 3$

$14 = \boxed{2(1)} \times 7$

$14 = 6 \times 2 + 2$

$6 = 3$

exercise

gcd (252, 198) = 18

1) $252 = 198(1) + 54$

2) $198 = 54(3) + 36$

3) $54 = 36(1) + \boxed{18}$ → zero se pehle waale 'gcd'

4) $36 = 18(2) + 0$

↓ Find GCD(a, b) as linear combination

$gcd(a,b) = as + bt$ (bezout identity)

(bezout co-efficients)

from above exercise :- 3) ⇒ $18 = 54 - 36(1)$

$36 = 198 - 54(3)$

$54 = 252 - 198(1)$

$18 = 54 - 36(1)$

$= 54 - [198 - 54(3)]$

$= 54 - 1(198) + 3(54)$

$= 4(54) - 1(198)$

$= 4[252 - 198(1)] - 1(198)$

$= 4(252) - 4(198) - 1(198)$

$= 4(252) - 5(198)$

$(4, -5)$ → bezout co-efficient

if $\gcd(a,b) = 1$

↳ relatively prime

$\gcd(172, 20)$

$172 = 20(8) + 12$

$20 = 12(1) + 8$ $\Rightarrow 12(1) = 20 - 8$

$12 = 8(1) + 4 \rightarrow \gcd$ $\Rightarrow \cancel{8(1) = 12(1) + 4}$

$8 = 4(2) + 0$ $\cancel{4 = 12 - 8(1)}$

$\cancel{12 = 8(1) + 4}$

$\cancel{= (1)(12 - 4) + 4}$

$\cancel{= (1)(20 - 8) + 4}$

$\cancel{= (1)\ 8(172 - 2 - 8) + 4}$

$\cancel{12 = 8(1) + 4}$

$\cancel{= 8(1)}$

$\cancel{4 = 12 - 8(1)}$

$\cancel{= 12 - [20 - 12(1)]}$

$\cancel{= 12 - [172 - 20(8)] - [20 - 172 - 20(8)]}$

$\cancel{= 1[172 - 20(8)] - 20 - 172}$

$4 = 12 - 8(1)$

$8 = 20 - 12(1)$

$12 = 172 - 20(8)$

Therefore; $4 = 12 - (1)[20 - 12(1)]$

$\cancel{= 12 - 20(1) + 1(1)(1)}$

$= 12 - 1(20) + 1(12)$

$= 2(12) - 1(20)$

$= 2[172 - 20(8)] - 1(20)$

$= 2(172) - 16(20) - 1(20)$

$= 2(172) - 17(20)$

$(2, -17) \rightarrow$ bezout coefficient

$\min(a_1, b_1)\ \min(a_2, b_2)\ \min(a_3, b_3)$

$GCD(a, b) = p_1\ p_2 \ldots p_n$

$LCM(a, b) = \dfrac{ab}{gcd(a, b)}$; $LCM = p_1^{\max(a_1, b_1)} \cdot p_2^{\max(a_2, b_2)} \ldots p_n^{\max(a_n, b_n)}$

$gcd(172, 20) =$

$172 = 2 \times 2 \times 43 = 2^2 \times 43$

$20 = 2 \times 2 \times 5 = 2^2 \times 5$

$GCD = 2^{(2,2)} \cdot 5^{(0,1)} \cdot 43^{(1,0)}$

$= 2^2 \cdot 5^0 \cdot 43^0$

$= 2^2 = 4$

Prime number between (1–100)

square root of (100) $= \sqrt{100} = 10$ (below prime)

2, 3, 5, 7

(cancel multiples of above rest will be prime)

16/03/21

Saturday's lecture

"Ex: 8.2"

let c_1 and c_2 be real numbers

① $a_n = c_1 a_{n-1} + c_2 a_{n-2}$

If $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for $n = 0, 1, 2, \ldots$

② let $r^2 - c_1 r - c_2 = 0$ has two distinct roots, then the sequence $\{a_n\}$ is a solution of recurrence relation.

What is the solution of recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} \quad \text{with } a_0 = 2 \; ; \; a_1 = 7$$

$c_1 = 1 \qquad c_2 = 2$

$r^2 - c_1 \; , \; r - c_2 = 0$

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$(r+1)(r-2) = 0$

$r = 2 \, , \, r = -1$ (distinct roots)

$a_0 = \alpha_1 + \alpha_2 \Rightarrow 2 = \alpha_1 + \cancel{\alpha_2}$

$a_1 = 2\alpha_1 - \alpha_2 \Rightarrow \underline{7 = 2\alpha_1 - \cancel{\alpha_2}}$

$9 = 3\alpha_1$

$\alpha_1 = 3$

$\alpha_2 = -1$

The solution of relation is, $a_n = 3 \cdot 2^n - (-1)^n$

Theorem-02

Let c_1 and c_2 are real numbers with $c_2 \neq 0$

let $r^2 - c_1 r - c_2 = 0$ has only one root r_0.

A sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$

iff $a_n = \alpha_1 r_0^n + \alpha_2 r_0^n$

for $n = 0, 1, 2, \ldots$

Ex: (5) Solution of recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial condition $a_0 = 1$; $a_1 = 6$.

$a_n = 6a_{n-1} - 9a_{n-2}$

$c_1 = 6$; $c_2 = -9$

$r^2 - 6r + 9 = 0$

$r^2 - 3r - 3r + 9 = 0$

$(r-3)^2 = 0$; $r = 3$

$\therefore a_n = \alpha_1 r_0^n + \alpha_2 r_0^n$

$\boxed{1 = \alpha_1}$

$6 = 3\alpha_1 + 3\alpha_2$

$\boxed{\alpha_2 = 1}$