

Counting [2x5=10]

Q1 Solve with justification

- a) Suppose that a department committee of FAST –NU contains 10 men and 15 women
How many ways are there to form a committee with six members if it must have more women than men . **96460**
- b) How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits.
63273600
- c) There are four blood types, A, B, AB, and O. Blood can also be RH^+ and RH^- Finally, a blood donor can be classified as either male or female.
How many different ways can a donor have his or her blood labeled? **16**
- d) How many different ID cards can be made if there are 6 digits on a card and no digit can be used more than once. **151200**
- e) How many bit strings of length 12 contain
- Exactly three 1s ? **220**
 - At most three 1s ? **299**
 - At least three 1s ? **4017**

Number theory [7+3=10]

Q2 a) Use Euclidean algorithm to express $\gcd(55, 89)$ as a linear combination.

$$1 = 55(34) + 89(-21)$$

b) what are Bezout coefficients and what is inverse of 55 modulo 89

$$34 \text{ and } -21 \text{ are Bezout coeff. and } 34 \text{ is inverse of } 55 \text{ mod } 89$$

c) Solve $55x \equiv 34 \pmod{89}$ using modular inverse and write the general solution.

$$x = 88 \pmod{89} \text{ and } x = 89k + 88, k \in \mathbb{Z}$$

Q3 Determine the check digit for the UPCs that have 11 initial digits 63623991346

$$118 + x_{12} = 0 \pmod{10} \text{ then } x_{12} = 2$$