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MULTIPLE LINEAR REGRESSION

UW DIRECT
(Data Intensive Research Enabling Cutting-edge Tech)
https://uwdirect.github.io

Stéphanie Valleau
Chemical Engineering

Multiple linear regression

Concept: independently assess the variation in *Y* with different input features *X*

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \epsilon$$

- Coefficients are determined by setting the partial derivatives to zero and solving the resultant *p+1* linear equations
- There is an exact solution (see e.g. Wikipedia)

What is your intuition on multiple linear regression: $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots \hat{\beta}_p x_p$

Multiple regression is always different from linear regression

All input features $\{x_i\}_{i=1}^{p}$ are equally important

Some input features might matter more than others

There is more than one intercept that is why it is multiple

We must maximize the sum of residual squares to get the optimal coefficients $\{\hat{\beta}_i\}_{i=1}^{p}$

Total Results: 0

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Assumptions

Key assumptions when using a linear regression model

- Errors are **uncorrelated** and normally distributed
- The variance of the error (in Y) is **independent of where we are in X**
- **Linear relationship** between X and Y (the predictor-response relationship)
- Individual contributions of your X's are **piecewise additive** to the response

Let's go to Jupyter …

```
curl –O
https://raw.githubusercontent.com/UWDIRECT/UWDIRECT.github
.io/master/Wi22_content/DSMCER/L8_multiple_linear_regressi
on.ipynb
```

When poll is active, respond at **pollev.com/dsms**

Text **DSMS** to **22333** once to join

About the F-statistic … which statements do you agree with?

It's useful to test whether DSMS is fun-statistically

It helps us figure out whether y depends on the input features

We can only test all coefficients at once

If p is large we can use forward selection

Backward selection can be used if $p > n$

It is statistically fantastic

Total Results: 0

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Questions we looked at

1. Is at least one of the predictors $X_1, X_2, \ldots, X_p$ useful in predicting the response? (**F-statistic**)
2. Do all the predictors help to explain Y, or is only a subset of the predictors useful? (**Step wise feature** selection)
3. How well does the model fit the data? (**R^2 score and RSE**)
4. Given a set of predictor values, what response value should we predict, and how accurate is our prediction?

4 Q directly copied from ISL page 75!

Possible problems

We just saw **non-linearity of the data,** other potential issues include

- Correlation **in error terms**
- **Non-constant variance** in error terms
- **Outliers** – here you can find them by looking at the residuals and remove within a set threshold

Correlation in error

The most common way that error becomes correlated is with time series data

residuals

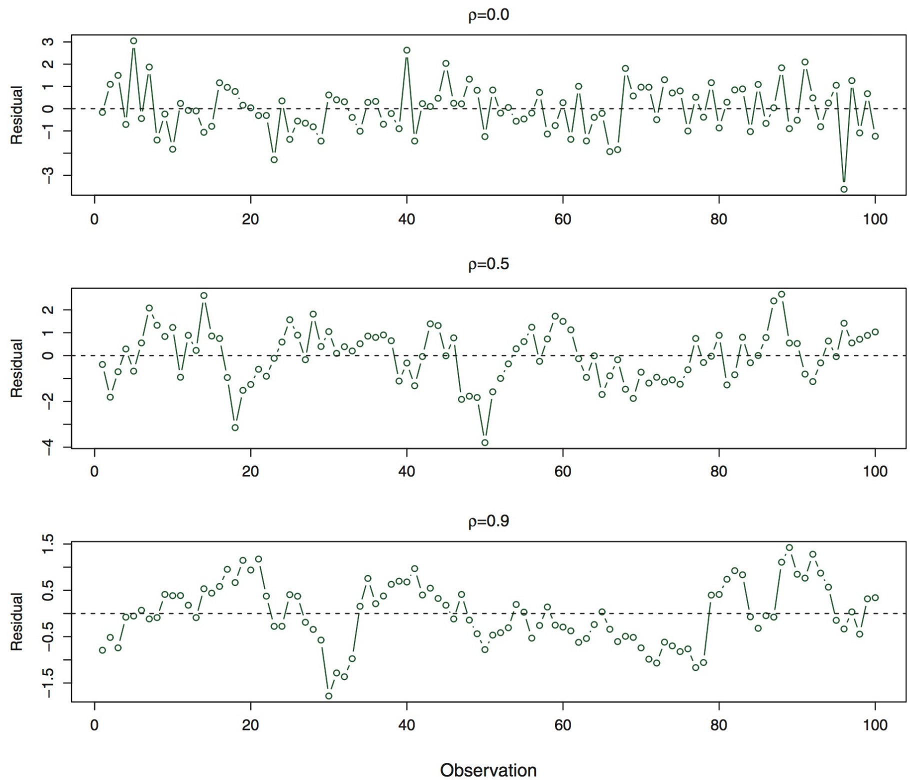


FIGURE 3.10. *Plots of residuals from simulated time series data sets generated with differing levels of correlation ρ between error terms for adjacent time points.*

Identifying correlation in error

Plot the residual vs. observation – see if there is some correlation

$$\rho_{xy} = \mathrm{Cor}(X,Y) = \frac{\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i-\bar{x})^2 \sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

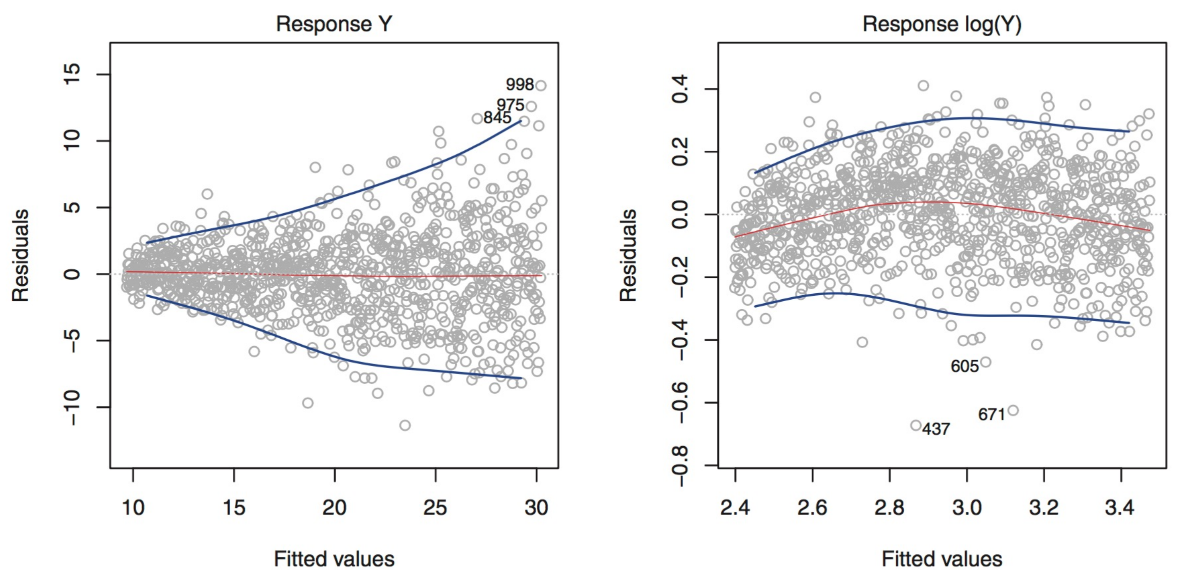
Introduction and practical implementation of methods to deal w/correlated errors is beyond scope of this class. See e.g. https://online.stat.psu.edu/stat462/node/189/

Non-constant variance

$$\text{Var}(\epsilon_i) = \sigma^2 \neq \text{const}$$

This phenomena is known as **heteroscedasticity**

- One solution is to **transform the response** data Y
- Another is to **use weighted least squares** – weights proportional to the variance

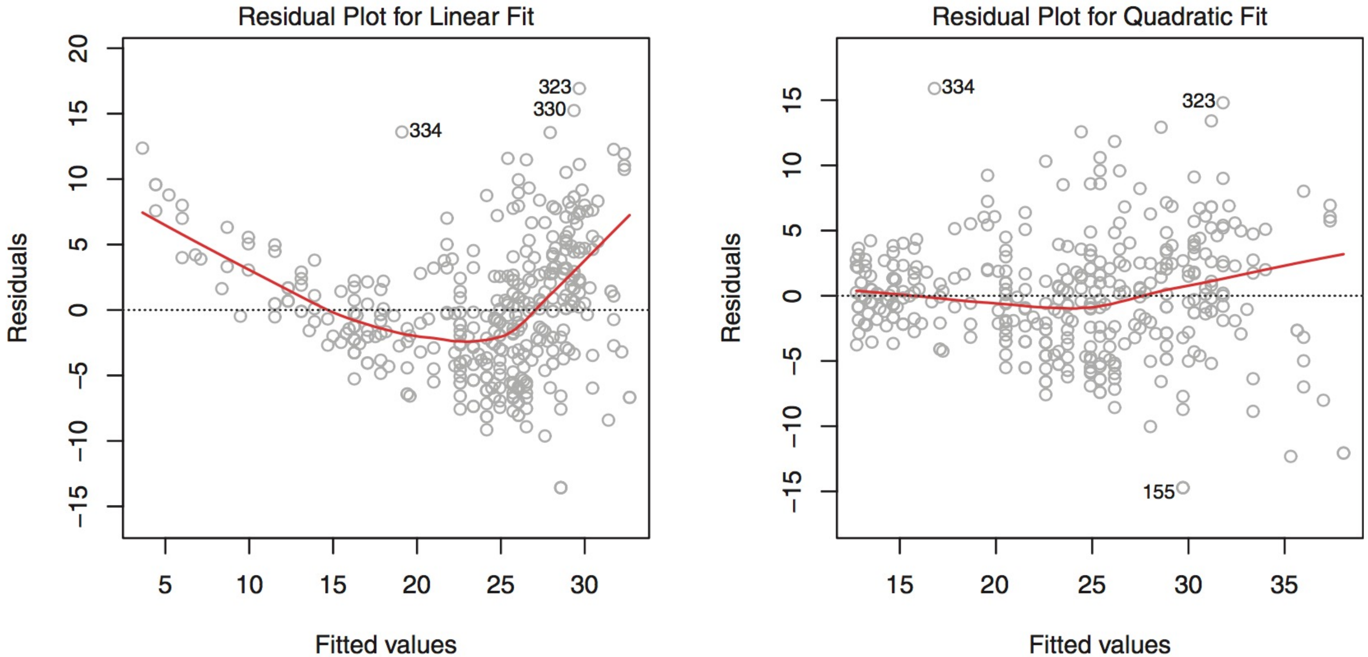


Beyond linear regression

Sometimes your variables have a clear

non-linear dependence on the response

We saw this in part in the notebook – also here Fig. 3.9 from the textbook



Simple nonlinear regression – polynomial

Note that in the case of some simple polynomial regressions, the model is still linear …

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \ldots + \beta_p X_1^p + \epsilon$$

All we need to do is define

$$X_2 = X_1^2$$
$$X_3 = X_1^3$$
$$\ldots$$
$$X_p = X_1^p$$

For more examples, see Section 3.3.2 of the textbook

Other topics / suggestions

Chapter 3 of ISL is strongly suggested to read carefully (maybe multiple times)

Additional topics we didn't cover

- Outliers and high leverage points in your training set
- Collinearity
- More about nonlinear regression
- AND MANY MORE!