

Topic 1:

Problem 1:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_1_Problem_1.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 1: Minimum Variance Line (Two Assets)
Problem 1

Part (a)

Covariance Matrix C=
[[0.0196 0.0056]
 [0.0056 0.0256]]

Part (b)

Expected Return= 0.075
Minimum Variance Portfolio= (in %)
[[75.]
 [25.]]

Expected Return= 0.08
Minimum Variance Portfolio= (in %)
[[50.]
 [50.]]

Expected Return= 0.085
Minimum Variance Portfolio= (in %)
[[25.]
 [75.]]

Part (c)

Expected Return (in %)= 7.5 %
Standard Deviation/Risk (in %)= [[12.1346611]]

Expected Return (in %)= 8.0 %
Standard Deviation/Risk (in %)= [[11.87434209]]

Expected Return (in %)= 8.5 %
Standard Deviation/Risk (in %)= [[13.31352696]]

Topic 1:

Problem 2:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_1_Problem_2.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 1: Minimum Variance Line (Two Assets)
Problem 2

Part (a)

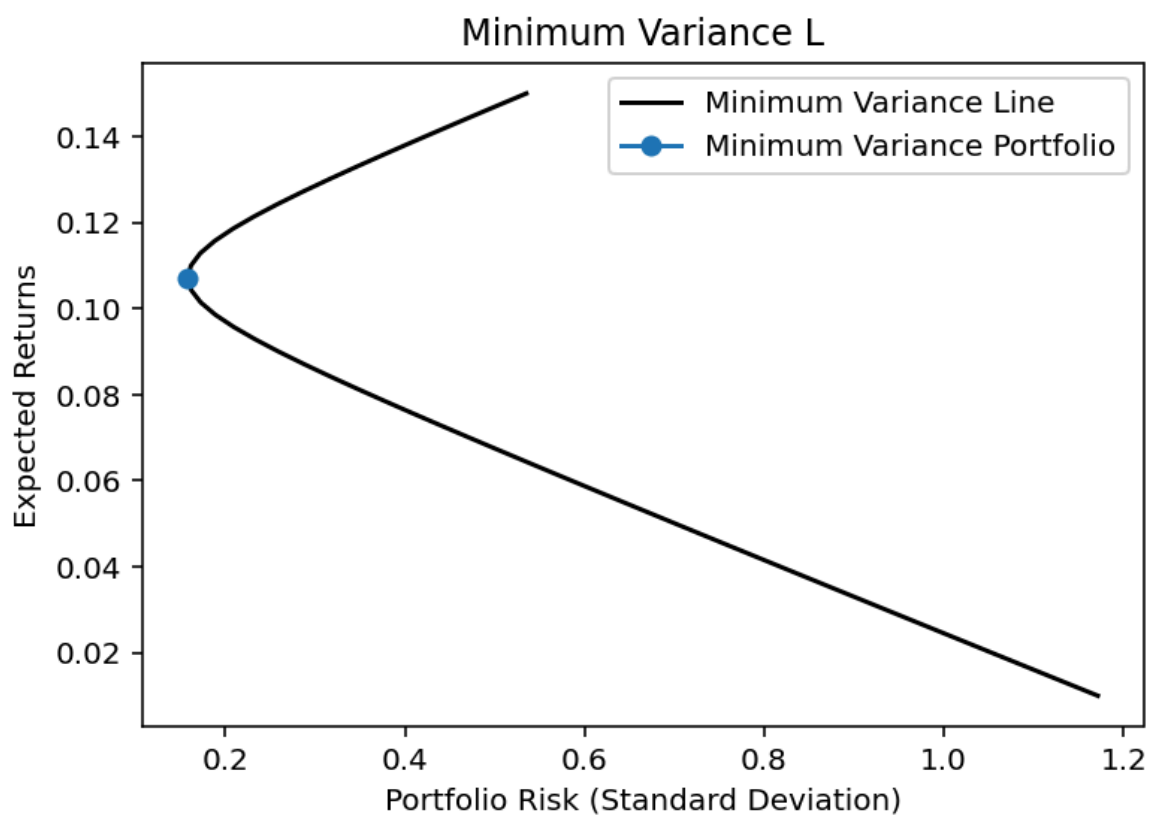
Equation of Minimum Variance Line= $m(a)+b$, where $a=$

$\begin{bmatrix} -50. \\ 50. \end{bmatrix}$

and $b=$

$\begin{bmatrix} 6. \\ -5. \end{bmatrix}$

Part (b)



Topic 2:

Problem 1:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_2_Problem_1.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 2: Three-Asset Minimum Variance Portfolios
Problem 1

Part (a)

Covariance Matrix C=
[[0.0225 0.009 0.0054]
 [0.009 0.04 0.009]
 [0.0054 0.009 0.0324]]

Part (b)

MVP weights (in %)=
[[59.73770492]
 [9.73770492]
 [30.52459016]]

Part (c)

Portfolio Variance=
[[0.01497895]]

Topic 2:

Problem 2:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_2_Problem_2.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 2: Three-Asset Minimum Variance Portfolios
Problem 2

Part (a)

Global Minimum Variance Portfolio, weights (in %)=
[[38.06214393]
[53.63853085]
[8.29932523]]

Part (b)

Errors (in %) of Weights Calculated with $w=m*a+b$ and $w=\alpha*w_1 + (1-\alpha)*w_2$:

Return m= 1.0 %
Error: [[4.79616347e-14]
[-1.27897692e-13]
[1.70530257e-13]]

Return m= 2.0 %
Error: [[4.61852778e-14]
[-9.94759830e-14]
[7.81597009e-14]]

Return m= 3.0 %
Error: [[3.90798505e-14]
[-9.94759830e-14]
[6.75015599e-14]]

Return m= 4.0 %
Error: [[-1.42108547e-14]
[8.52651283e-14]
[6.03961325e-14]]

Return m= 5.0 %
Error: [[3.55271368e-14]
[0.00000000e+00]
[5.55111512e-14]]

Return m= 6.0000000000000001 %
Error: [[2.84217094e-14]
[1.42108547e-14]
[5.50670620e-14]]

Return m= 6.999999999999999 %
Error: [[-2.13162821e-14]
[4.26325641e-14]
[-5.50670620e-14]]

Return m= 8.0 %
Error: [[2.13162821e-14]
[-4.26325641e-14]
[3.19744231e-14]]

Return m= 9.0 %
Error: [[2.84217094e-14]
[6.75015599e-14]
[2.13162821e-14]]

Return m= 10.0 %
Error: [[7.10542736e-15]
[-2.22044605e-14]
[9.94759830e-14]]

Return m= 11.0 %
Error: [[7.10542736e-14]
[-2.13162821e-14]
[1.06581410e-13]]

Return m= 12.0 %
Error: [[4.26325641e-14]
[-8.88178420e-14]
[8.52651283e-14]]

Return m= 13.0 %
Error: [[0.]
[0.]
[0.]]

Return m= 14.000000000000002 %
Error: [[0.00000000e+00]
[0.00000000e+00]
[8.52651283e-14]]

Return m= 15.000000000000002 %
Error: [[0.]
[0.]
[0.]]

Return m= 16.0 %
Error: [[4.26325641e-14]
[-4.26325641e-14]
[9.94759830e-14]]

Return m= 17.0 %
Error: [[1.42108547e-14]
[4.26325641e-14]
[5.68434189e-14]]

Return m= 18.000000000000004 %

Error: [[2.84217094e-14]

[-4.26325641e-14]

[1.42108547e-14]]

Return m= 19.0 %

Error: [[2.84217094e-14]

[-4.26325641e-14]

[2.84217094e-14]]

Return m= 20.0 %

Error: [[1.42108547e-14]

[-8.52651283e-14]

[5.68434189e-14]]

Return m= 21.000000000000004 %

Error: [[0.00000000e+00]

[-2.84217094e-14]

[0.00000000e+00]]

Return m= 22.0 %

Error: [[0.00000000e+00]

[0.00000000e+00]

[5.68434189e-14]]

Return m= 23.0 %

Error: [[4.26325641e-14]

[-8.52651283e-14]

[5.68434189e-14]]

Return m= 24.000000000000004 %

Error: [[1.42108547e-14]

[-2.84217094e-14]

[2.84217094e-14]]

Return m= 25.0 %

Error: [[0.00000000e+00]

[-2.84217094e-14]

[2.84217094e-14]]

Return m= 26.0 %

Error: [[0.00000000e+00]

[5.68434189e-14]

[-2.84217094e-14]]

Return m= 27.0 %

Error: [[0.]

[0.]

[0.]]

Return m= 28.000000000000004 %

Error: [[0.00000000e+00]

[-5.68434189e-14]

[0.00000000e+00]]

Return m= 29.000000000000004 %

Error: [[0.00000000e+00]

[-5.68434189e-14]

[0.00000000e+00]]

Return m= 30.0 %

Error: [[0.]

[0.]

[0.]]

Return m= 31.0 %

Error: [[-2.84217094e-14]

[5.68434189e-14]

[0.00000000e+00]]

Return m= 32.0 %

Error: [[0.]

[0.]

[0.]]

Return m= 33.0 %

Error: [[0.]

[0.]

[0.]]

Return m= 34.0 %

Error: [[-2.84217094e-14]

[-5.68434189e-14]

[0.00000000e+00]]

Return m= 35.0 %

Error: [[-2.84217094e-14]

[0.00000000e+00]

[-5.68434189e-14]]

Return m= 36.000000000000001 %

Error: [[0.00000000e+00]

[-5.68434189e-14]

[-5.68434189e-14]]

Return m= 37.0 %

Error: [[-2.84217094e-14]

[5.68434189e-14]

[0.00000000e+00]]

Return m= 38.0 %

Error: [[-2.84217094e-14]

[0.00000000e+00]

[0.00000000e+00]]

Return m= 39.0 %

Error: [[0.00000000e+00]

[-5.68434189e-14]
[-5.68434189e-14]]

Return m= 40.0 %
Error: [[0.00000000e+00]
[-1.70530257e-13]
[0.00000000e+00]]

Return m= 41.0 %
Error: [[-2.84217094e-14]
[-1.13686838e-13]
[-5.68434189e-14]]

Return m= 42.00000000000001 %
Error: [[-5.68434189e-14]
[-1.13686838e-13]
[-1.13686838e-13]]

Return m= 43.0 %
Error: [[0.00000000e+00]
[0.00000000e+00]
[-5.68434189e-14]]

Return m= 44.0 %
Error: [[5.68434189e-14]
[0.00000000e+00]
[0.00000000e+00]]

Return m= 45.0 %
Error: [[0.]
[0.]
[0.]]

Return m= 46.0 %
Error: [[0.00000000e+00]
[-5.68434189e-14]
[0.00000000e+00]]

Return m= 47.0 %
Error: [[0.00000000e+00]
[1.13686838e-13]
[-5.68434189e-14]]

Return m= 48.00000000000001 %
Error: [[-2.84217094e-14]
[0.00000000e+00]
[-5.68434189e-14]]

Return m= 49.0 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-1.13686838e-13]]

Return m= 50.0 %

Error: [[-2.84217094e-14]
[-1.13686838e-13]
[-5.68434189e-14]]

Return m= 51.0 %
Error: [[-2.84217094e-14]
[0.00000000e+00]
[0.00000000e+00]]

Return m= 52.0 %
Error: [[0.00000000e+00]
[-1.13686838e-13]
[-1.13686838e-13]]

Return m= 53.0 %
Error: [[-5.68434189e-14]
[-1.13686838e-13]
[0.00000000e+00]]

Return m= 54.0 %
Error: [[-2.84217094e-14]
[-1.13686838e-13]
[-5.68434189e-14]]

Return m= 55.000000000000001 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-5.68434189e-14]]

Return m= 56.000000000000001 %
Error: [[0.00000000e+00]
[-1.13686838e-13]
[-1.13686838e-13]]

Return m= 57.000000000000001 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-1.13686838e-13]]

Return m= 58.000000000000001 %
Error: [[-5.68434189e-14]
[-1.13686838e-13]
[-5.68434189e-14]]

Return m= 59.0 %
Error: [[0.00000000e+00]
[0.00000000e+00]
[-2.27373675e-13]]

Return m= 60.0 %
Error: [[0.]
[0.]
[0.]]

Return m= 61.0 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-2.27373675e-13]]

Return m= 62.0 %
Error: [[0.00000000e+00]
[-2.27373675e-13]
[-1.13686838e-13]]

Return m= 63.0 %
Error: [[-1.13686838e-13]
[-1.13686838e-13]
[-1.13686838e-13]]

Return m= 64.0 %
Error: [[-5.68434189e-14]
[2.27373675e-13]
[-1.13686838e-13]]

Return m= 65.0 %
Error: [[0.]
[0.]
[0.]]

Return m= 66.0 %
Error: [[-1.13686838e-13]
[0.00000000e+00]
[-2.27373675e-13]]

Return m= 67.0 %
Error: [[-5.68434189e-14]
[1.13686838e-13]
[-2.27373675e-13]]

Return m= 68.0 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[0.00000000e+00]]

Return m= 69.0 %
Error: [[-5.68434189e-14]
[-2.27373675e-13]
[0.00000000e+00]]

Return m= 70.0 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-2.27373675e-13]]

Return m= 71.00000000000001 %
Error: [[-5.68434189e-14]
[-1.13686838e-13]
[0.00000000e+00]]

Return m= 72.0 %
Error: [[-5.68434189e-14]
[-1.13686838e-13]
[-1.13686838e-13]]

Return m= 73.0 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-2.27373675e-13]]

Return m= 74.0 %
Error: [[0.00000000e+00]
[0.00000000e+00]
[-1.13686838e-13]]

Return m= 75.0 %
Error: [[-5.68434189e-14]
[0.00000000e+00]
[-1.13686838e-13]]

Return m= 76.0 %
Error: [[-5.68434189e-14]
[-2.27373675e-13]
[-1.13686838e-13]]

Return m= 77.0 %
Error: [[-5.68434189e-14]
[-2.27373675e-13]
[-1.13686838e-13]]

Return m= 78.0 %
Error: [[-5.68434189e-14]
[-1.13686838e-13]
[-1.13686838e-13]]

Return m= 79.0 %
Error: [[-5.68434189e-14]
[-2.27373675e-13]
[-1.13686838e-13]]

Return m= 80.0 %
Error: [[-5.68434189e-14]
[-3.41060513e-13]
[-1.13686838e-13]]

Return m= 81.0 %
Error: [[-5.68434189e-14]
[-2.27373675e-13]
[-2.27373675e-13]]

Return m= 82.0 %
Error: [[-1.13686838e-13]
[-3.41060513e-13]

[-2.27373675e-13]]

Return m= 83.0 %

Error: [[-5.68434189e-14]

[0.00000000e+00]

[-1.13686838e-13]]

Return m= 84.000000000000001 %

Error: [[0.00000000e+00]

[-2.27373675e-13]

[-1.13686838e-13]]

Return m= 85.0 %

Error: [[-1.13686838e-13]

[0.00000000e+00]

[-2.27373675e-13]]

Return m= 86.0 %

Error: [[0.00000000e+00]

[-2.27373675e-13]

[-3.41060513e-13]]

Return m= 87.0 %

Error: [[-5.68434189e-14]

[0.00000000e+00]

[-2.27373675e-13]]

Return m= 88.0 %

Error: [[1.13686838e-13]

[0.00000000e+00]

[-2.27373675e-13]]

Return m= 89.0 %

Error: [[-1.13686838e-13]

[-2.27373675e-13]

[-2.27373675e-13]]

Return m= 90.0 %

Error: [[5.68434189e-14]

[-2.27373675e-13]

[0.00000000e+00]]

Return m= 91.0 %

Error: [[0.]

[0.]

[0.]]

Return m= 92.0 %

Error: [[-1.13686838e-13]

[-2.27373675e-13]

[0.00000000e+00]]

Return m= 93.0 %

Error: [[1.70530257e-13]

[-4.54747351e-13]
[1.13686838e-13]]

Return m= 94.0 %
Error: [[0.00000000e+00]
[-2.27373675e-13]
[-2.27373675e-13]]

Return m= 95.0 %
Error: [[0.00000000e+00]
[-2.27373675e-13]
[0.00000000e+00]]

Return m= 96.00000000000001 %
Error: [[0.00000000e+00]
[-2.27373675e-13]
[0.00000000e+00]]

Return m= 97.0 %
Error: [[-2.84217094e-13]
[0.00000000e+00]
[-3.41060513e-13]]

Return m= 98.0 %
Error: [[-1.13686838e-13]
[4.54747351e-13]
[-3.41060513e-13]]

Return m= 99.0 %
Error: [[0.00000000e+00]
[2.27373675e-13]
[-3.41060513e-13]]

Return m= 100.0 %
Error: [[0.]
[0.]
[0.]]

The number of times when error in weights is more than or equal to 1 percent is 0 .

Topic 3:

Problem 1:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_3_Problem_1.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 3: Efficient Frontier Properties
Problem 1

Errors when subtracting values of both sides for different m:

```
Return m= 1.0 % , Error: [[7.80625564e-17]]  
Return m= 2.0 % , Error: [[6.07153217e-17]]  
Return m= 3.0 % , Error: [[4.16333634e-17]]  
Return m= 4.0 % , Error: [[3.98986399e-17]]  
Return m= 5.0 % , Error: [[2.42861287e-17]]  
Return m= 6.000000000000001 % , Error: [[1.04083409e-17]]  
Return m= 6.999999999999999 % , Error: [[5.20417043e-18]]  
Return m= 8.0 % , Error: [[-8.67361738e-18]]  
Return m= 9.0 % , Error: [[-2.25514052e-17]]  
Return m= 10.0 % , Error: [[-3.46944695e-17]]  
Return m= 11.0 % , Error: [[-4.33680869e-17]]  
Return m= 12.0 % , Error: [[-5.55111512e-17]]  
Return m= 13.0 % , Error: [[-8.32667268e-17]]  
Return m= 14.000000000000002 % , Error: [[-9.54097912e-17]]  
Return m= 15.000000000000002 % , Error: [[-1.02348685e-16]]  
Return m= 16.0 % , Error: [[-9.88792381e-17]]  
Return m= 17.0 % , Error: [[-1.1969592e-16]]  
Return m= 18.000000000000004 % , Error: [[-1.1969592e-16]]  
Return m= 19.0 % , Error: [[-1.47451495e-16]]  
Return m= 20.0 % , Error: [[-1.47451495e-16]]  
Return m= 21.000000000000004 % , Error: [[-1.75207071e-16]]  
Return m= 22.0 % , Error: [[-1.68268177e-16]]  
Return m= 23.0 % , Error: [[-1.96023753e-16]]  
Return m= 24.000000000000004 % , Error: [[-2.16840434e-16]]  
Return m= 25.0 % , Error: [[-2.16840434e-16]]  
Return m= 26.0 % , Error: [[-2.4459601e-16]]  
Return m= 27.0 % , Error: [[-2.4459601e-16]]  
Return m= 28.000000000000004 % , Error: [[-2.72351586e-16]]  
Return m= 29.000000000000004 % , Error: [[-3.00107161e-16]]  
Return m= 30.0 % , Error: [[-2.86229374e-16]]  
Return m= 31.0 % , Error: [[-2.72351586e-16]]  
Return m= 32.0 % , Error: [[-2.86229374e-16]]  
Return m= 33.0 % , Error: [[-3.27862737e-16]]  
Return m= 34.0 % , Error: [[-3.41740525e-16]]  
Return m= 35.0 % , Error: [[-3.27862737e-16]]  
Return m= 36.000000000000001 % , Error: [[-3.55618313e-16]]  
Return m= 37.0 % , Error: [[-3.83373888e-16]]  
Return m= 38.0 % , Error: [[-3.97251676e-16]]  
Return m= 39.0 % , Error: [[-4.11129464e-16]]
```

Return m= 40.0 % , Error: [[-3.97251676e-16]]
Return m= 41.0 % , Error: [[-4.80518403e-16]]
Return m= 42.000000000000001 % , Error: [[-4.52762827e-16]]
Return m= 43.0 % , Error: [[-5.36029554e-16]]
Return m= 44.0 % , Error: [[-4.80518403e-16]]
Return m= 45.0 % , Error: [[-5.08273978e-16]]
Return m= 46.0 % , Error: [[-5.08273978e-16]]
Return m= 47.0 % , Error: [[-4.80518403e-16]]
Return m= 48.000000000000001 % , Error: [[-5.91540705e-16]]
Return m= 49.0 % , Error: [[-5.6378513e-16]]
Return m= 50.0 % , Error: [[-5.36029554e-16]]
Return m= 51.0 % , Error: [[-5.91540705e-16]]
Return m= 52.0 % , Error: [[-5.36029554e-16]]
Return m= 53.0 % , Error: [[-5.36029554e-16]]
Return m= 54.0 % , Error: [[-6.19296281e-16]]
Return m= 55.000000000000001 % , Error: [[-6.19296281e-16]]
Return m= 56.000000000000001 % , Error: [[-6.19296281e-16]]
Return m= 57.000000000000001 % , Error: [[-6.19296281e-16]]
Return m= 58.000000000000001 % , Error: [[-7.30318583e-16]]
Return m= 59.0 % , Error: [[-6.74807432e-16]]
Return m= 60.0 % , Error: [[-7.02563008e-16]]
Return m= 61.0 % , Error: [[-6.19296281e-16]]
Return m= 62.0 % , Error: [[-6.74807432e-16]]
Return m= 63.0 % , Error: [[-7.02563008e-16]]
Return m= 64.0 % , Error: [[-7.02563008e-16]]
Return m= 65.0 % , Error: [[-8.41340886e-16]]
Return m= 66.0 % , Error: [[-8.41340886e-16]]
Return m= 67.0 % , Error: [[-8.1358531e-16]]
Return m= 68.0 % , Error: [[-8.1358531e-16]]
Return m= 69.0 % , Error: [[-8.1358531e-16]]
Return m= 70.0 % , Error: [[-8.41340886e-16]]
Return m= 71.000000000000001 % , Error: [[-7.30318583e-16]]
Return m= 72.0 % , Error: [[-8.41340886e-16]]
Return m= 73.0 % , Error: [[-8.41340886e-16]]
Return m= 74.0 % , Error: [[-8.96852037e-16]]
Return m= 75.0 % , Error: [[-8.41340886e-16]]
Return m= 76.0 % , Error: [[-8.41340886e-16]]
Return m= 77.0 % , Error: [[-8.96852037e-16]]
Return m= 78.0 % , Error: [[-8.96852037e-16]]
Return m= 79.0 % , Error: [[-8.41340886e-16]]
Return m= 80.0 % , Error: [[-8.41340886e-16]]
Return m= 81.0 % , Error: [[-8.41340886e-16]]
Return m= 82.0 % , Error: [[-1.00787434e-15]]
Return m= 83.0 % , Error: [[-9.52363188e-16]]
Return m= 84.000000000000001 % , Error: [[-9.52363188e-16]]
Return m= 85.0 % , Error: [[-9.52363188e-16]]
Return m= 86.0 % , Error: [[-1.11889664e-15]]
Return m= 87.0 % , Error: [[-9.52363188e-16]]
Return m= 88.0 % , Error: [[-9.52363188e-16]]
Return m= 89.0 % , Error: [[-8.96852037e-16]]
Return m= 90.0 % , Error: [[-1.06338549e-15]]
Return m= 91.0 % , Error: [[-1.06338549e-15]]
Return m= 92.0 % , Error: [[-1.06338549e-15]]
Return m= 93.0 % , Error: [[-1.06338549e-15]]

Return m= 94.0 % , Error: $[-1.06338549e-15]$
Return m= 95.0 % , Error: $[-1.17440779e-15]$
Return m= 96.00000000000001 % , Error: $[-1.11889664e-15]$
Return m= 97.0 % , Error: $[-1.06338549e-15]$
Return m= 98.0 % , Error: $[-1.17440779e-15]$
Return m= 99.0 % , Error: $[-1.17440779e-15]$
Return m= 100.0 % , Error: $[-1.17440779e-15]$

The number of times when error is more than or equal to 0.01 is 0 .

Topic 3:

Problem 2:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_3_Problem_2.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 3: Efficient Frontier Properties
Problem 2

```
New Portfolio with Target Return  $m = 8.5\%$  is (in %): [[25.]  
[75.]]  
Return of Portfolio 1: [[7.6]] %  
Return of Portfolio 2: [[8.]] %  
Return of New Portfolio: [[8.5]] %
```

Topic 4:

Problem 1:

```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_4_Problem_1.py' --wdir
```

Topic 4: Practical Applications
Problem 1

Optimal Portfolio Weights for 9% Target Returns is: [[60.92307692]
[18.76923077]
[20.30769231]]

Risk associated with portfolio for 9% Target Returns is: [[9.75515643]] %

INTERESTING OBSERVATION

Global Minimum Variance Portfolio Weights is: (in percentages) [[54.07655416]
[14.88553072]
[31.03791512]]

Returns on GMVP is: [[9.39037191]] %

Risk associated with GMVP is: [[9.39436505]] %

THERE EXISTS ANOTHER PORTFOLIO THAT GIVES MORE RETURNS FOR LESS
RISK.

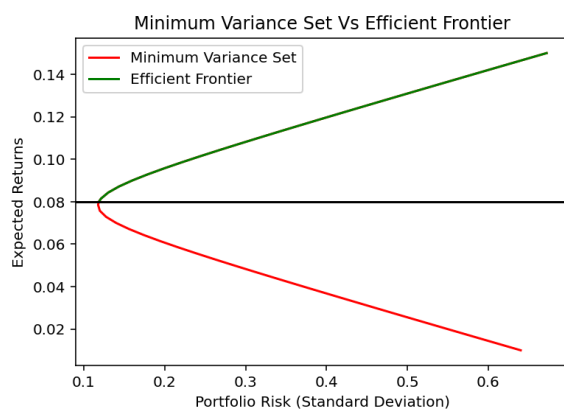
Topic 4:

Problem 2:

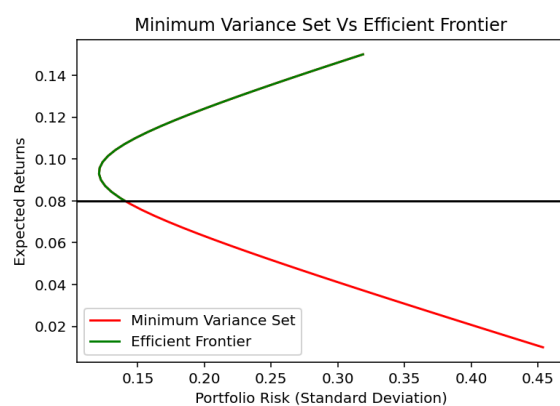
```
%runfile 'D:/MS Semester 2/Financial Mathematics/Python/Problem Sheet  
3/Topic_4_Problem_2.py' --wdir  
Reloaded modules: MV_Functions
```

Topic 4: Practical Applications
Problem 2

Without Almarai:



With Almarai:



Weights without Almarai: (in %) [[50.]
[50.]]

Weights with Almarai: (in %) [[86.42622951]
[-13.57377049]
[27.14754098]]