Find complex Potential between two Parallel plates located at $x=0$ and $x=10$ with potential $\phi_1 = 20$ volts and $\phi_2 = 120$ volts respe

Solution: As according to Laplace equation we ha

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\Rightarrow \quad \frac{\partial^2 \phi}{\partial x^2} = 0 \Rightarrow \frac{d^2 \phi}{d x^2} = 0$$

$$\frac{d\phi}{dx} = A \Rightarrow \phi = Ax + B$$

at $x=0$ $\phi = 20$ $\quad 20 = B$

at $x=10$ $\phi = 120$ $\quad 120 = 10A + B$

As $B=20$ $\quad 10A = 110 \Rightarrow A = 11$

Thus $\phi = 11x + 20$

For Complex Conjugate we h

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \& \quad \frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$$

$$\frac{\partial \psi}{\partial y} = 11 \Rightarrow \psi = 11y + h(x)$$

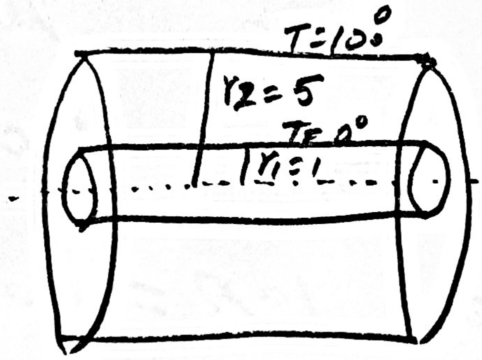
$$\frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y} \Rightarrow h'(x) = -0 \Rightarrow h(x) = c$$

$\psi = 11y + c$

Thus Complex Potential $F = \phi + i\psi$

$F = (11x + 20) + i(11y + C)$

Q. Find complex field for the temprature between two coaxial cylinder of radius $r_1 = 1$ with $T = 0^\circ$ and $r_2 = 5$ with $T = 10^\circ$



Solution: As we know that heat equation in cylindrical coordinate is of the form

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = \frac{1}{c^2}\frac{\partial T}{\partial t}$$

Since T depends on only r so we have

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0$$

we Can solve the differential equation 3 by variable separable method, Cauchy method

In Case of variable separable method

$$\frac{\frac{d^2T}{dr^2}}{\frac{dT}{dr}} = \frac{-1}{r} \Rightarrow \ln\left(\frac{dT}{dr}\right) = -\ln r + \ln A$$

$$\frac{dT}{dr} = A/r \Rightarrow T = A\ln r + B$$

when $r=1$ $T=0$ $\quad 0 = A\ln(1) + B \Rightarrow B=0$

when $r=5$ $T=100$ $\quad 100 = A\ln 5 \Rightarrow A = \frac{100}{\ln 5}$

Thus $T(r,\theta) = \frac{100}{\ln 5}\ln r$

For Complex field we find harmonic Conjugate of T.

$$\frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \& \quad \frac{\partial \psi}{\partial r} = -\frac{1}{r}\frac{\partial T}{\partial \theta}$$

$$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}\frac{100}{\ln 5} \Rightarrow \psi = \frac{100}{\ln 5}\theta + h(r)$$

$$\frac{\partial \psi}{\partial r} = \frac{-1}{r}\frac{\partial T}{\partial \theta}$$

$$h'(r) = 0 \implies h(r) = c$$

Thus $\psi = \frac{100}{\ln 5}\theta + c$

So the required complex field takes the form

$$F = T + i\psi$$

$$F = \frac{100}{\ln 5} r + i\left(\frac{100}{\ln 5}\theta + c\right)$$

Even function:- A function $f(x)$ is even if $f(-x) = f(x)$ $\forall x \in D$

odd function : A function $f(x)$ is odd if

$$f(-x) = -f(x) \quad \forall x \in D$$

Periodic function: $f(x)$ is Periodic of period P if $f(x+P) = f(x)$

where P is a +ve number.

Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

with

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\,dx \qquad \text{with } P = 2\pi$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx$$

Q. Find Fourier Series of $f(x) = \sin x$ with $P = 2\pi$ $\underline{\underline{6}}$

Solution : Here $f(x) = \sin x$

So $$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin x \, dx = \frac{1}{2\pi}\left[\cos x\right]_{-\pi}^{\pi}$$

$$a_0 = \frac{-1}{2\pi}\left[\cos\pi - \cos(-\pi)\right] = \frac{-1}{2\pi}\left[\cancel{-1}+\cancel{1}\right] = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin x \cos nx \, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\sin x \cos nx \, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\left[\sin(x+nx) + \sin(x-nx)\right]\right\}dx$$

$$= \frac{1}{2\pi}\left[-\frac{\cos(x+nx)}{1+n} - \frac{\cos(x-nx)}{1-n}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left[-\left[\frac{\cos(1+n)\pi - \cos(1+n)(-\pi)}{1+n}\right]\right.$$

$$\left. - \left[\frac{\cos(1-n)\pi - \cos(1-n)(-\pi)}{1-n}\right]\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{0}{1+n} - \frac{0}{1-n}\right] = 0$$

$$b_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\sin x \sin nx \, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} [\cos(x - nx) - \cos(x+nx)]\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} [\cos(1-n)x - \cos(1+n)x]\, dx \qquad n \neq 1$$

$$= \frac{1}{2\pi}\left[\frac{\sin(1-n)x}{1-n} - \frac{\sin(1+n)x}{1+n}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left[\left(\frac{\sin(1-n)\pi - \sin(1-n)(-\pi)}{1-n}\right]\right.$$

$$- \frac{1}{2\pi}\left(\frac{\sin(1+n)\pi - \sin(1+n)(-\pi)}{1+n}\right)$$

$b_n = 0$ if $n = 1$

$$\frac{1}{2\pi} x \Big|_{-\pi}^{\pi} = \frac{1}{2\pi}(\pi + \pi) = 1$$

$b_1 = 1$

$\sin x = 0 + 0 + \sin x.$

if $f(x) = x$ odd $-1 \le x \le 1$

as $\int_{-1}^{1} x\,dx = \frac{x^2}{2}\Big|_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$

if $f(x) = x^2$ $-1 \le x \le 1$

Then $\int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Thus (Even)(Even) = Even

(Even)(odd) = odd

(odd)(odd) = Even

AS $\sin x$ is an odd function

So $a_0 = 0$

a_n: $\underbrace{\sin x}_{\text{odd}}\ \underbrace{\cos nx}_{\text{Even}} = \underline{\text{odd}}$

$b_n = \underbrace{\sin x}_{\text{odd}}\ \underbrace{\sin nx}_{\text{odd}} = \underline{\text{Even}}$