WORKSHEET # II

1. If $2 - x^2 \leq g(x) \leq 2\cos x$ for all x, find $\lim\limits_{x \to 0} g(x)$.

2. Prove that $\lim\limits_{x \to 0} x^4 \cos(\dfrac{2}{x}) = 0$

3. Let $f(x) = \dfrac{\sqrt{2x}(x^2 - 1)}{|x - 1|}$. Does $\lim\limits_{x \to 1} f(x)$ exist ?

4. Find the following limits if they exist

a) $\lim\limits_{x \to 64} \dfrac{\sqrt{x} - 8}{\sqrt[3]{x} - 4}$

b) $\lim\limits_{x \to 1} \dfrac{1 - \sqrt{x}}{1 - x}$

c) $\lim\limits_{x \to 5} \dfrac{4 - \sqrt{3x + 1}}{x^2 - 7x + 10}$

d) $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2 + 8} - 3}{x + 1}$

e) $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - x^2}{1 - \sqrt{x}}$

f) $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{\sin x}$

g) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin x - 1}{\cot^2 x}$

h) $\lim\limits_{x \to 0} \sin^{-1}\left(\dfrac{-x^2 + 5x - 1}{x^2 + 2}\right)$

i) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$

j) $\lim\limits_{x \to 0} \dfrac{1 + \sin x - \cos x}{1 - \sin x - \cos x}$

k) $\lim\limits_{x \to \infty} \dfrac{\tan^{-1}(-x) + \sin^{-1}(\frac{-x}{x+1})}{\cos^{-1}(\frac{-x}{x+1})}$

l) $\lim\limits_{x \to \infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 - 2x})$

5. Evaluate $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$ for the function $f(x) = \dfrac{2x}{\sqrt{4x^2 + 1}}$

6. Let $\lim\limits_{x \to 1} f(x) = -1$. Evaluate $\lim\limits_{x \to 1} \dfrac{\sin(1 + f(x))}{1 - f^2(x)}$.

7. Evaluate the following limits (Do not use the L'Hospital's Rule).

a) $\lim\limits_{x \to 0} \dfrac{x}{\tan 3x}$

b) $\lim\limits_{x \to 1^-} \dfrac{1}{\sin^{-1}(\ln x)}$

c) $\lim\limits_{x \to 1^+} \dfrac{\sin(x - 1)}{\sin(x^2 - 1)}$

d) $\lim\limits_{x \to \infty} \dfrac{2x + \sin x}{x + 1}$

e) $\lim\limits_{x \to 1^+} \{\ln[\sin(x^2 - 1)] - \ln(x - 1)\}$

f) $\lim\limits_{x \to 0} \dfrac{2\sin(2x) - \sin(4x)}{x^3}$

g) $\lim\limits_{x \to 0^+} \dfrac{\sqrt{2\sin(4x)} - \sqrt{\sin(2x)}}{\sqrt{x}}$

h) $\lim\limits_{x \to 4} \dfrac{(\sqrt{x^2 - 7} - x + 1)\sin(x - 4)}{(x^2 - 3x - 4)^2}$

8. Find the right-hand and left-hand limits of the following functions at the given point.

(a) $y = \dfrac{|x - 1|}{x - 1} + x^2$, $(x = 1)$

(b) $y = \dfrac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$, $(x = 0)$

9. Find the asymptotes of the following functions.

(a) $f(x) = \dfrac{x^3}{x^2 + 3x - 10}$

(b) $f(x) = \dfrac{x^3 + 2x - 1}{x^3 + 2x^2 - x - 2}$

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