# WORKSHEET # VII

1. Evaluate the following integrals

a) $\displaystyle\int \frac{18\tan^2 x \sec^2 x}{(2+\tan^3 x)}\,dx$

b) $\displaystyle\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\,\sin^2\sqrt{\theta}}\,d\theta$

c) $\displaystyle\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$

d) $\displaystyle\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\,d\theta$

e) $\displaystyle\int x^{1/3}\sin(x^{4/3}-8)\,dx$

f) $\displaystyle\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\,d\theta$

g) $\displaystyle\int \frac{1-\cos(6t)}{2}\,dt$

h) $\displaystyle\int \frac{\sin 2x}{\cos^2 x+3}dx$

i) $\displaystyle\int \frac{x+\sin x}{1+\cos x}\,dx$

j) $\displaystyle\int \frac{\sqrt{\tan x}}{\sin x\cos x}\,dx$

k) $\displaystyle\int \frac{dx}{1+\cos x}$

l) $\displaystyle\int \sqrt[3]{\frac{2x^2+3}{x^{11}}}\,dx$

m) $\displaystyle\int \sqrt{1+\cos 3x}\,dx$

n) $\displaystyle\int \frac{(2x-1)\,\cos\sqrt{3(2x-1)^2+6}}{\sqrt{3(2x-1)^2+6}}\,dx$

2. Evaluate the following integrals by computing $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$.

$$\int_1^2 x^2+5x+2dx$$

3. Find the following limits by using the definition of definite integral or sum formulas.

$$\lim_{n\to\infty}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+\sqrt{n+3}+\ldots+\sqrt{2n}}{n^{3/2}}\right\}$$

4. Find the indicated derivatives for the given functions

a) $\displaystyle y(x)=\int_{\sqrt{x}}^{x^2} t\sqrt{t^2+1}\,dt \;\;,\;\; y'=?$

b) $\displaystyle y(x)=\int_0^{\sin x} \sqrt{1-t^2}\,dt \;\;,\;\; y''=?$

c) $\displaystyle y(x)=\int_{\sin x}^{\cos x} \frac{1}{1-t^2}\,dt \;\;,\;\; y'=?$

5. Evaluate the following integrals

a) $\displaystyle\int_2^{-2} |1-x|\,dx$

b) $\displaystyle\int_6^{0} |5-|2x||\,dx$

c) $\displaystyle\int_0^{\pi/4} \sqrt{1-\cos^2(4x-\frac{\pi}{4})}dx$

6. Prove that $\displaystyle\int_0^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt+\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt=\text{constant}.\quad(\sin x\,,\,\cos x\geq 0)$

7. Prove that $\displaystyle\int_0^{\pi} x\,f(\sin x)\,dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$ (Hint: $x=\pi-t$)