Computer Vision (10224)

Lecture 4 – Sampling and Frequency
Dr. Eyal Katz – Spring 2021

This presentation has been updated and reedited by
Dr. Eyal Katz, Afeka College of Engineering

This Lecture is (mainly) based on
Prof. James Hays
Brown University's
Lecture 5: Thinking in Frequency
See additional credits on the last page or on individual slides

Administration – Spring 2021: Midterm Notes

- Since In-class learning is not expected to resume soon, the midterm is replaced by a separate work, per Afeka instructions.
- The midterm opening time and date will be published on AfekaNet (**currently published time is <u>not final</u> - 18-04-2021 07:59**).
- The work should be submitted no later than a week after the midterm opening time.
- Your submitted work should be individual, and original (No workgroups).
- Other details and instructions will be published.

Review: Practice Questions Set 1: 1.4 (reviewed based on last week)

1. Write down a 3x3 filter that returns a positive value if the average value of the 4-adjacent neighbors is less than the center
   and a negative value otherwise

2. Write down a filter that will compute the gradient in the x-direction:

```
gradx(y,x) = im(y,x+1)-im(y,x) for each x, y
```


Slide: Hoiem

Review: Practice Questions Set 1: 1.5

3. Fill in the blanks:

Filtering Operator

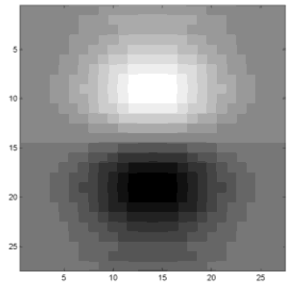
a) _ = D * B

b) A = _ * _

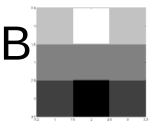
c) F = D * _

d) _ = D * D

A



B



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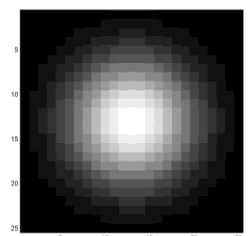

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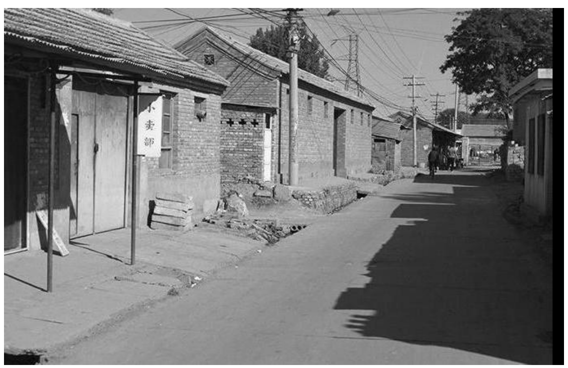
F



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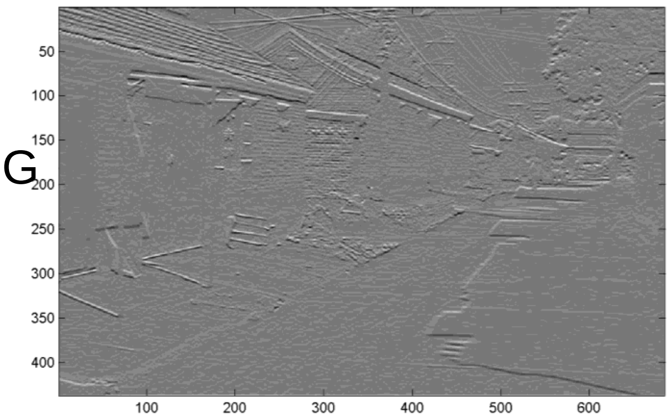

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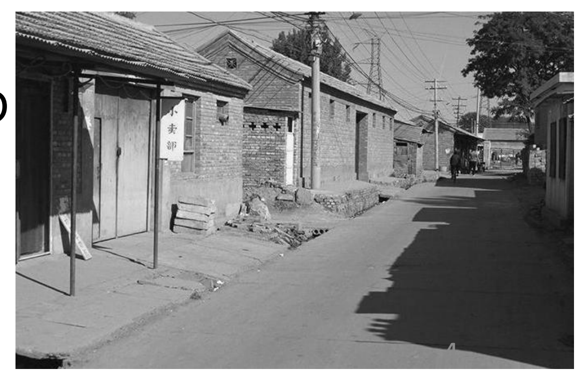
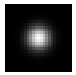
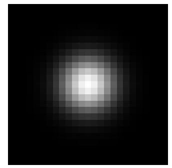


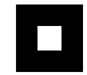
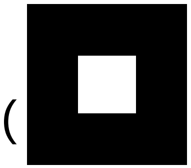
Slide: Hoiem

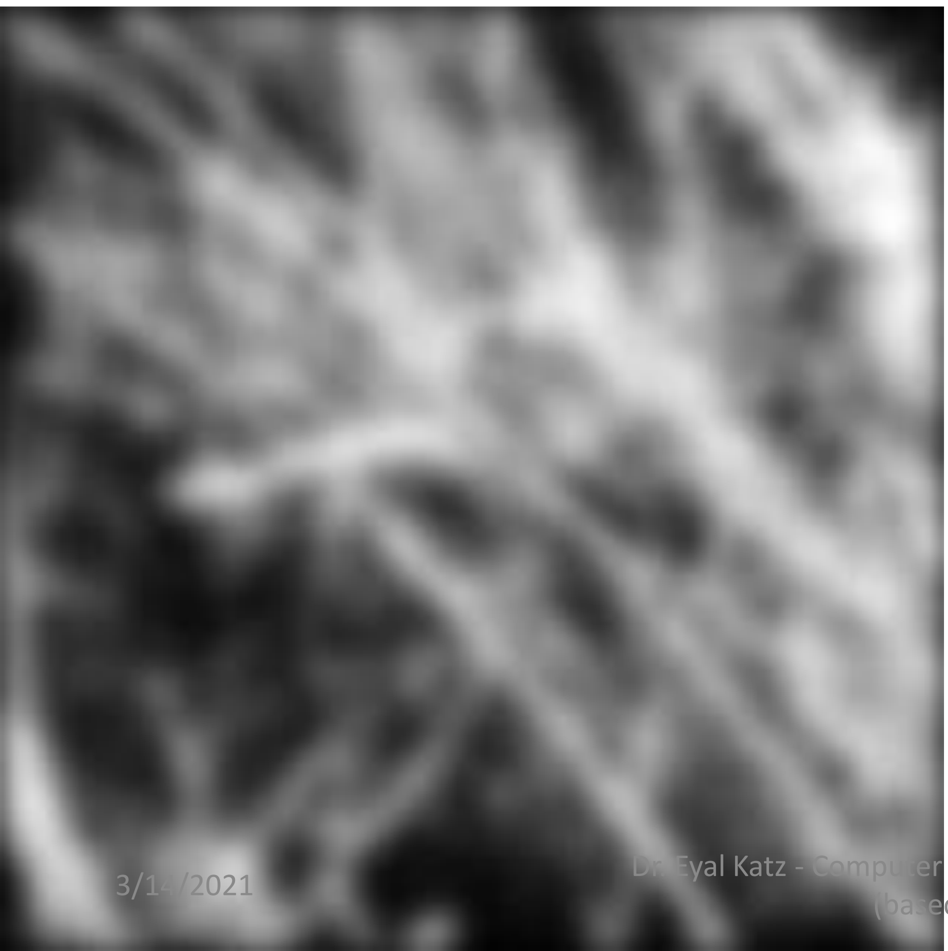
Today's Class

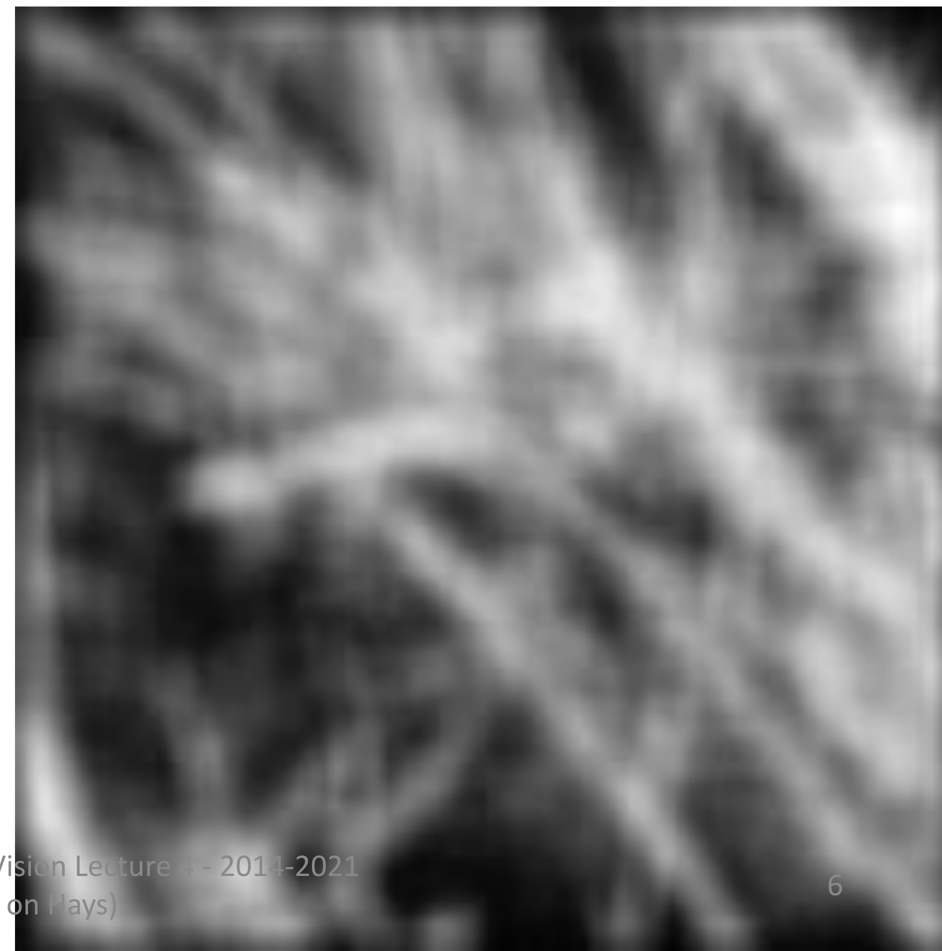
- Fourier transform and the frequency domain
  - Frequency-domain view of filtering (convolution)
  - Sampling
  - Hybrid images

Motivation (I): Why does the Gaussian give a nice smooth image, but the square (box) filter give edgy artifacts?

Gaussian filter  ()

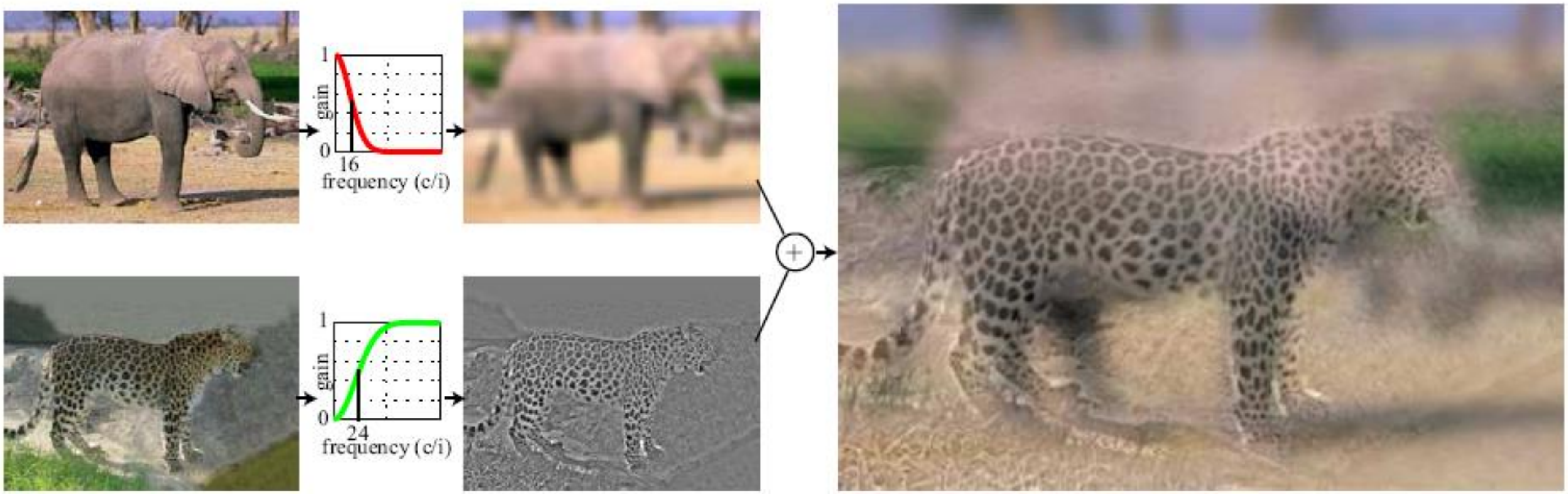
Box filter  ()





Motivation (II)

Motivation (II – cont.)
Hybrid Images



- A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

Motivation (II – cont.)

Why do we get different, distance-dependent interpretations of hybrid images?

?

Slide: Hoiem

Motivation (III):

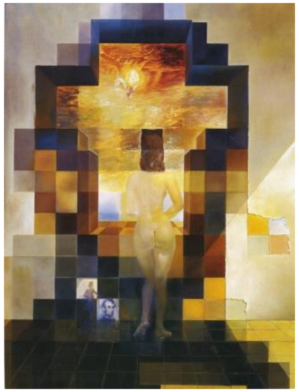
Why does a lower resolution image still makes sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

Slide: Hoiem

Motivation (IV)

Thinking in terms of frequency



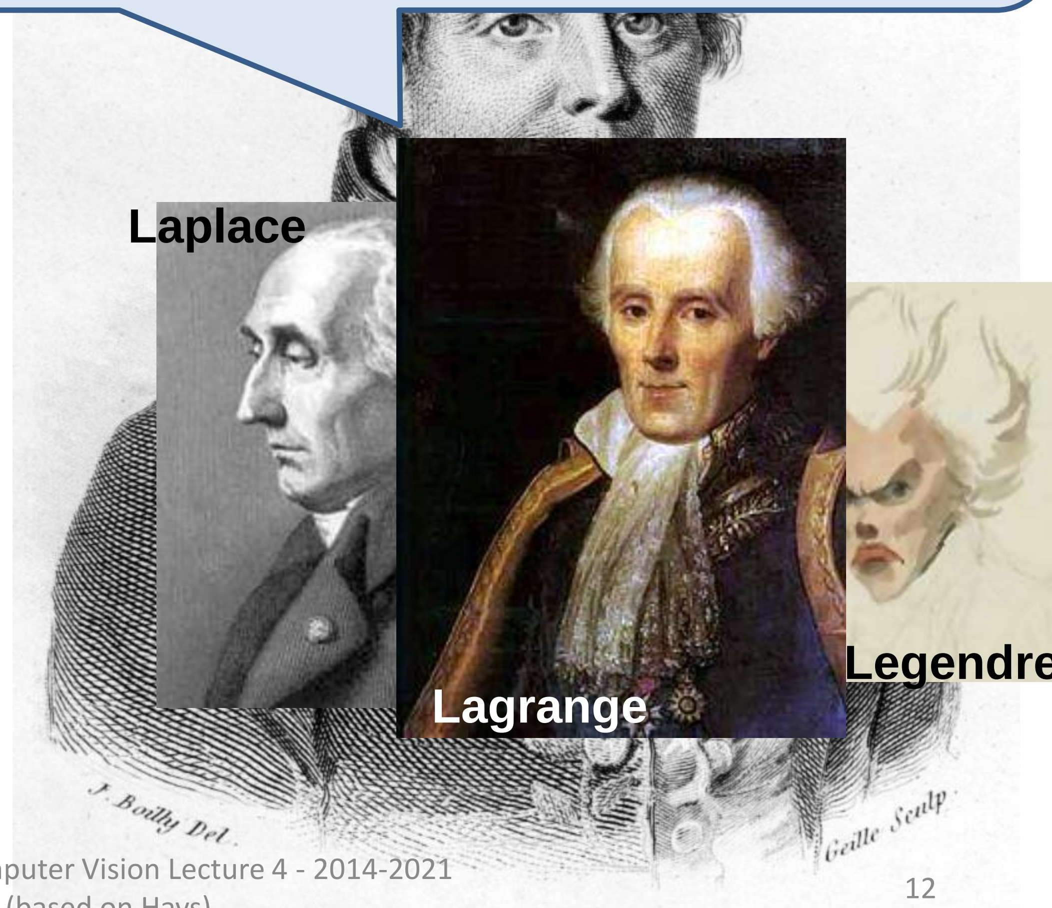
Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

> ***Any** univariate function can
> rewritten as a weighted sum
> sines and cosines of different
> frequencies.*

...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.

- Don’t believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it’s (mostly) true!
  - called Fourier Series
  - there are some subtle restrictions



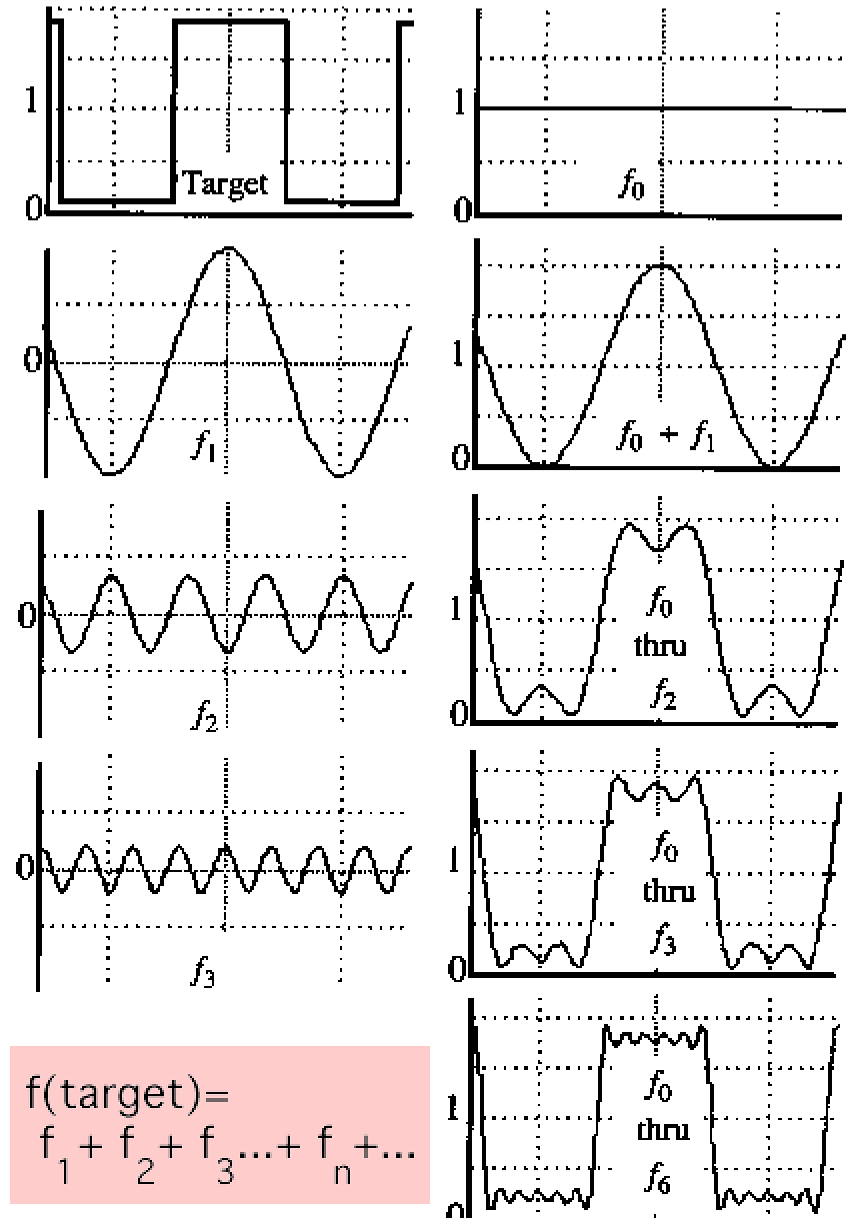
A sum of sines

Our building block:

$$A\sin(\omega x + \phi)$$

Add enough of them to get any signal *f(x)* you want!

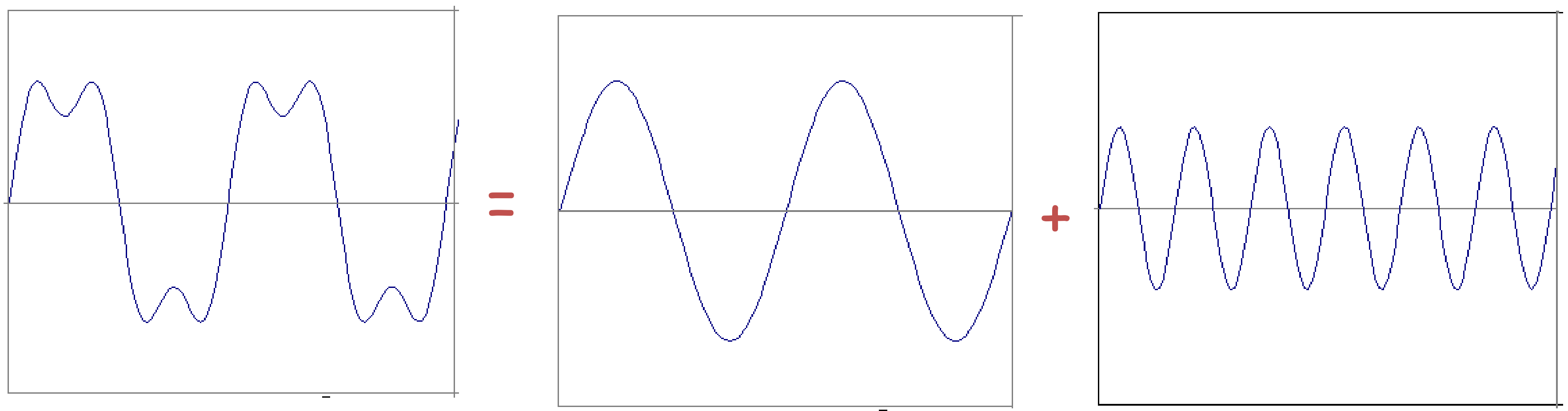
Note: we refer as
a **sinusoidal function**,
or a **sinusoid**, to
sine, cosine, e^{ix}, e^{jx} etc.

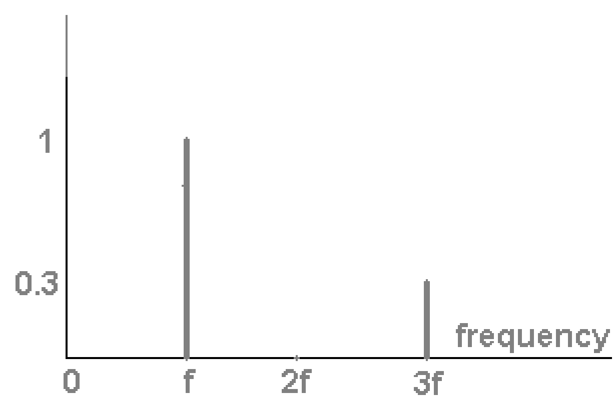


f(target)= $f_1 + f_2 + f_3 ... + f_n + ...$

Frequency Spectra

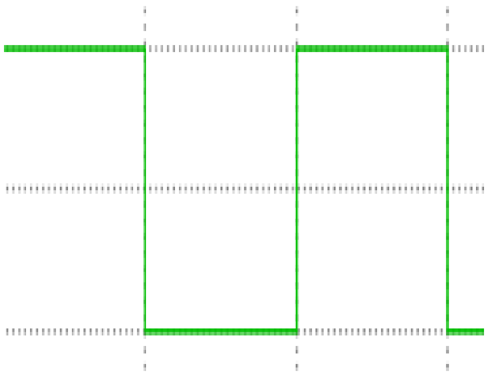
- example : $g(t) = \sin(2\pi f\ t) + (1/3)\sin(2\pi(3f)\ t)$



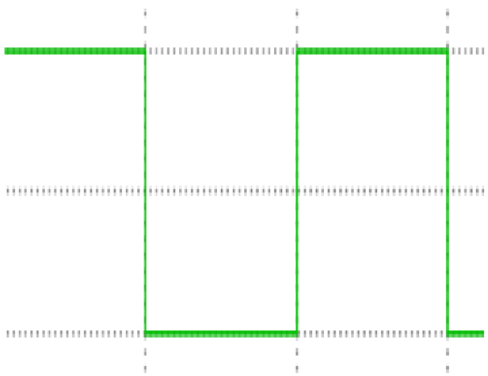


Slides: Efros

Frequency Spectra



Frequency Spectra



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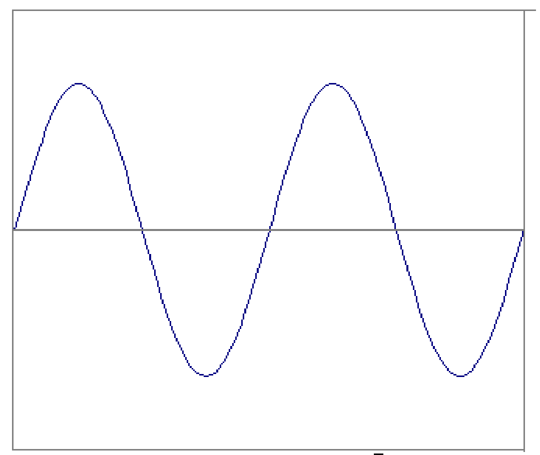

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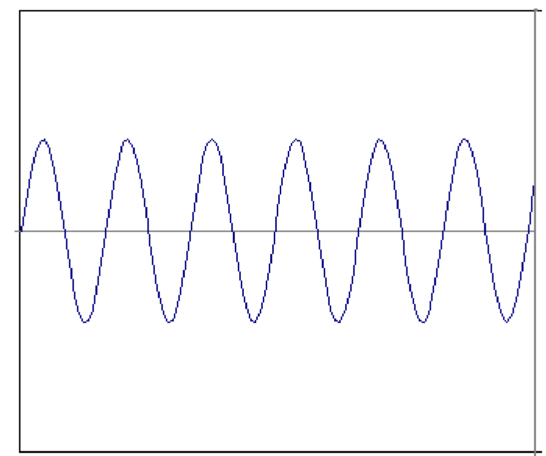

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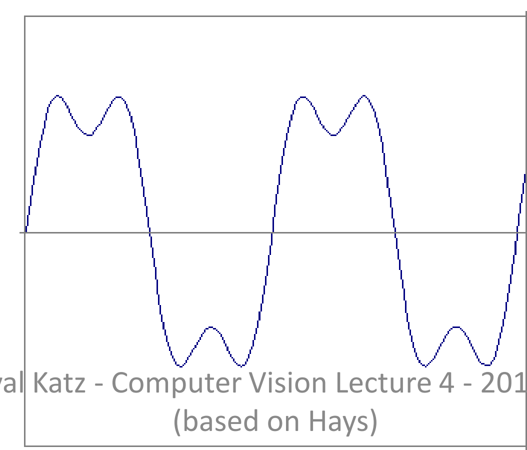
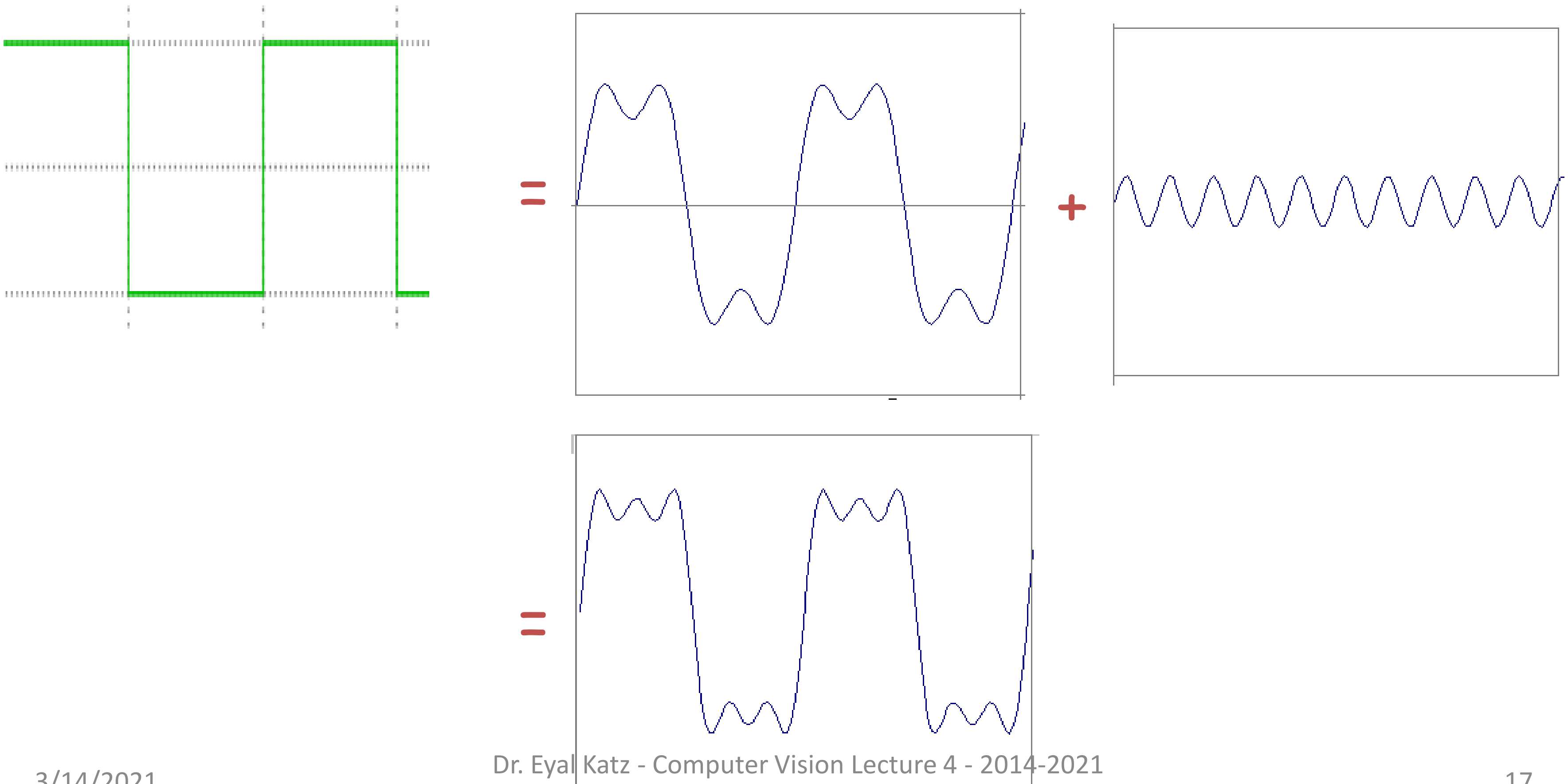
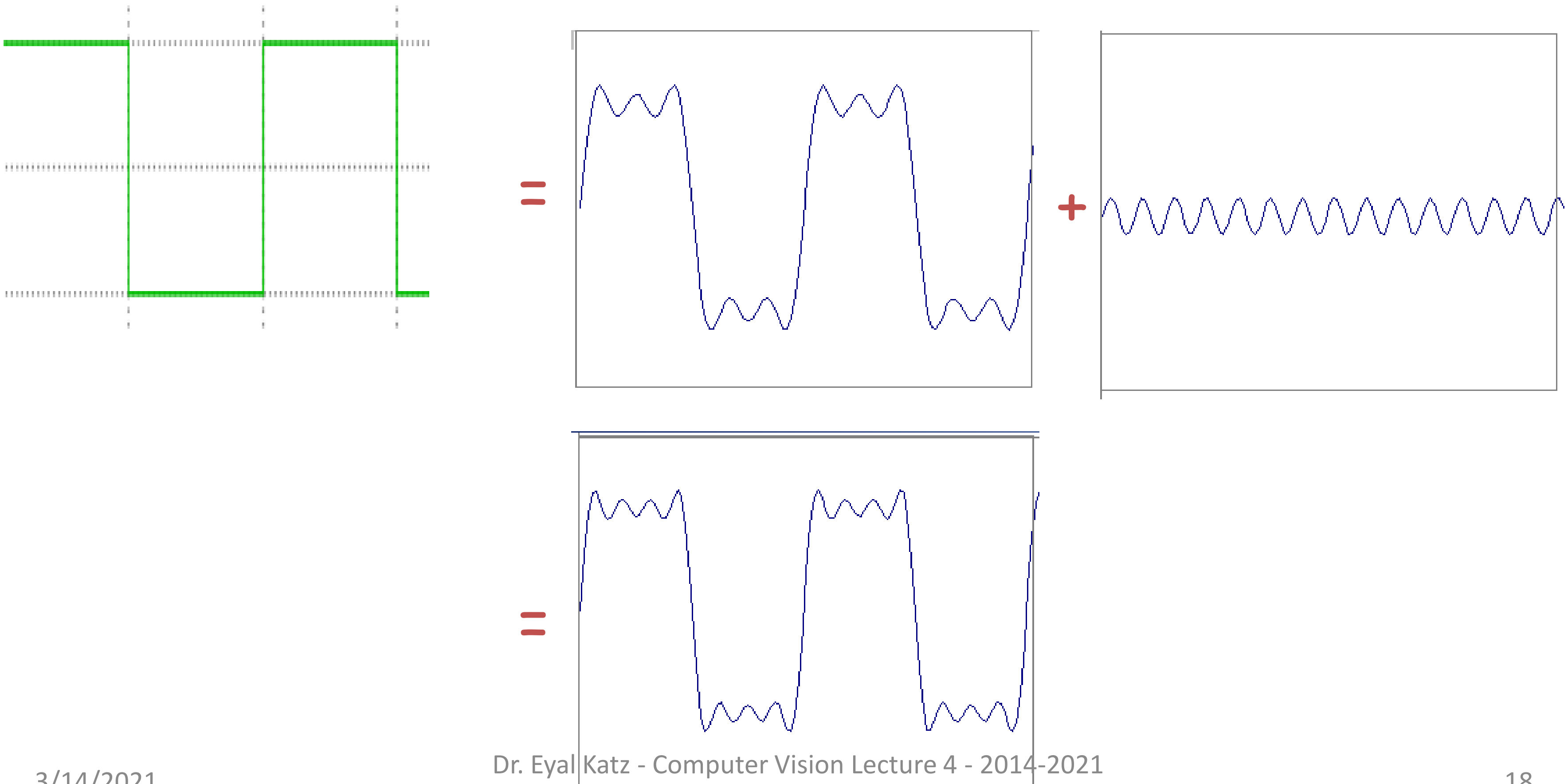


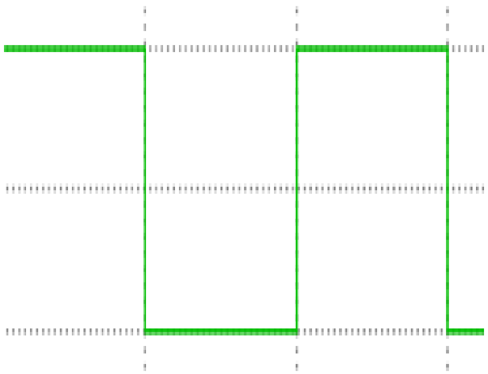
Frequency Spectra



Frequency Spectra



Frequency Spectra



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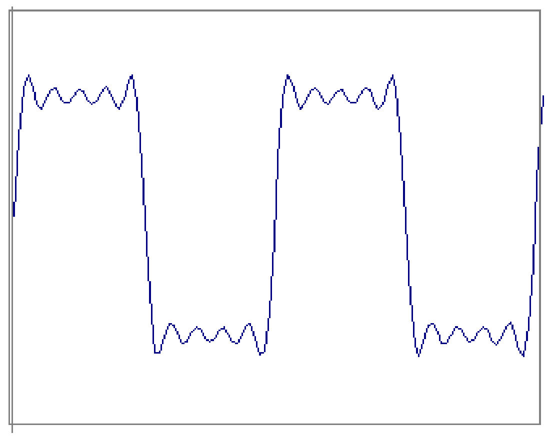

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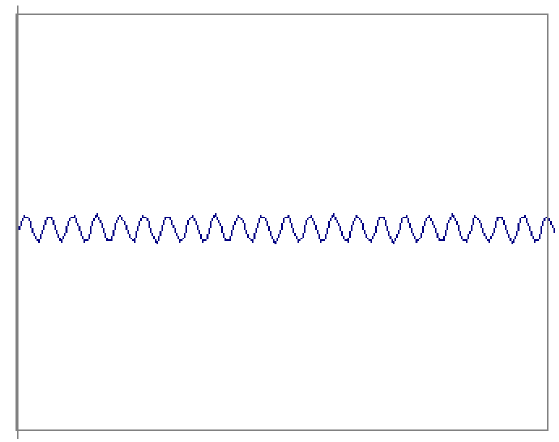

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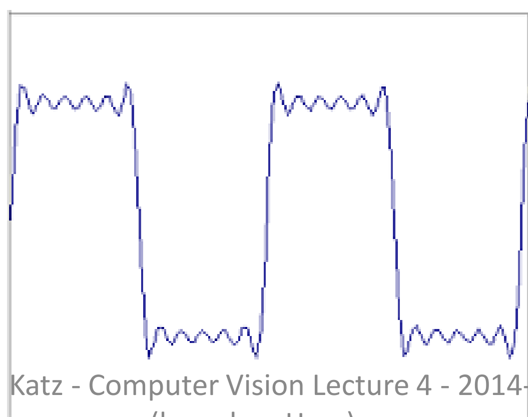
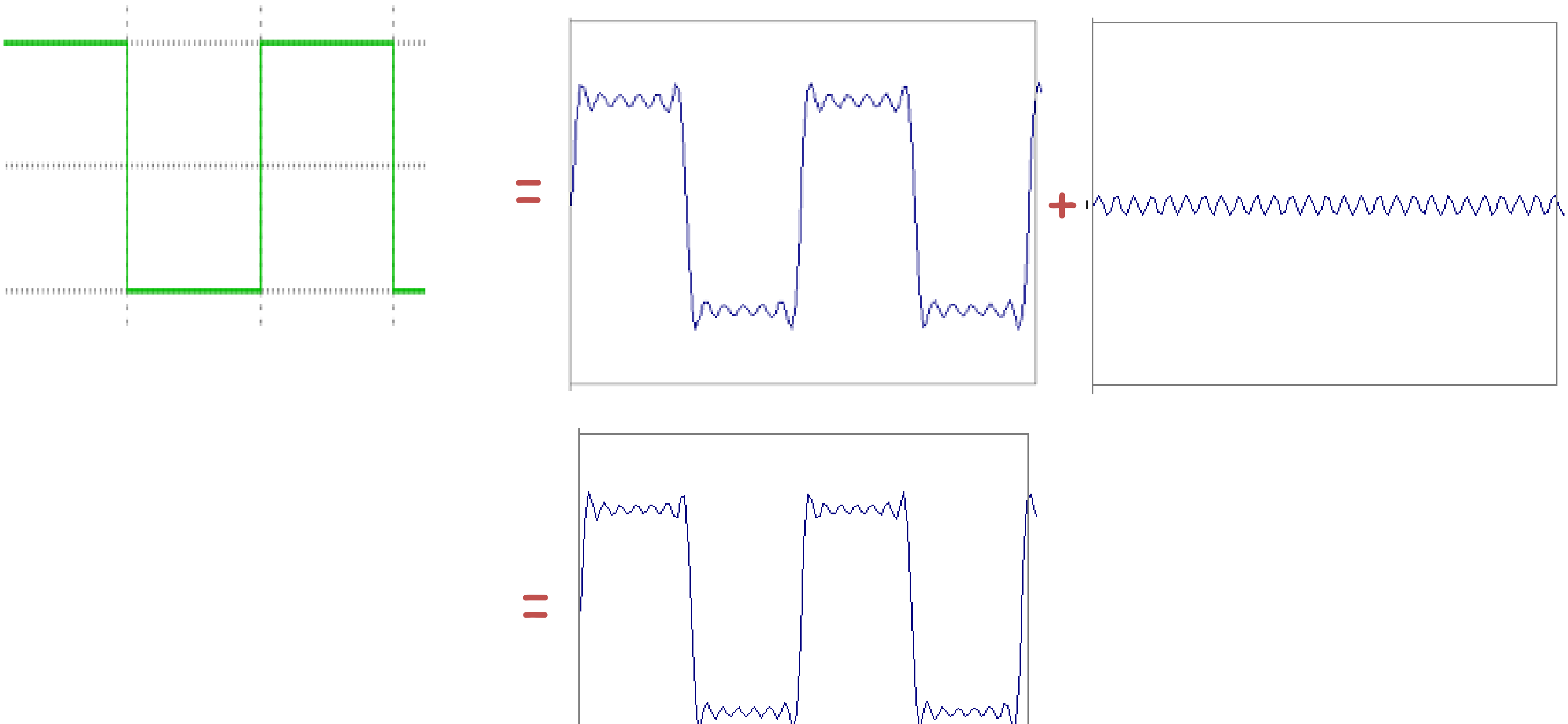
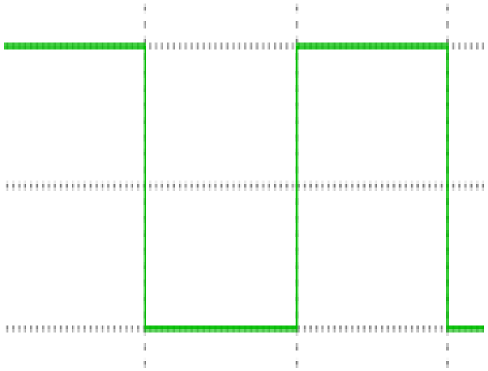


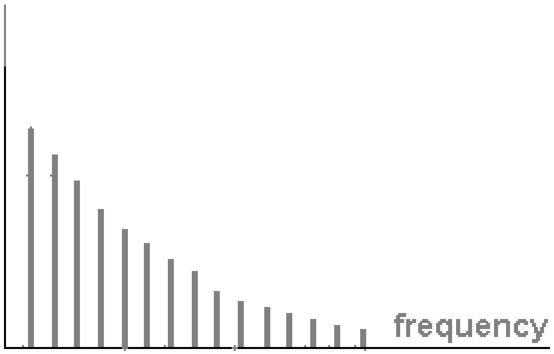
Frequency Spectra



Frequency Spectra

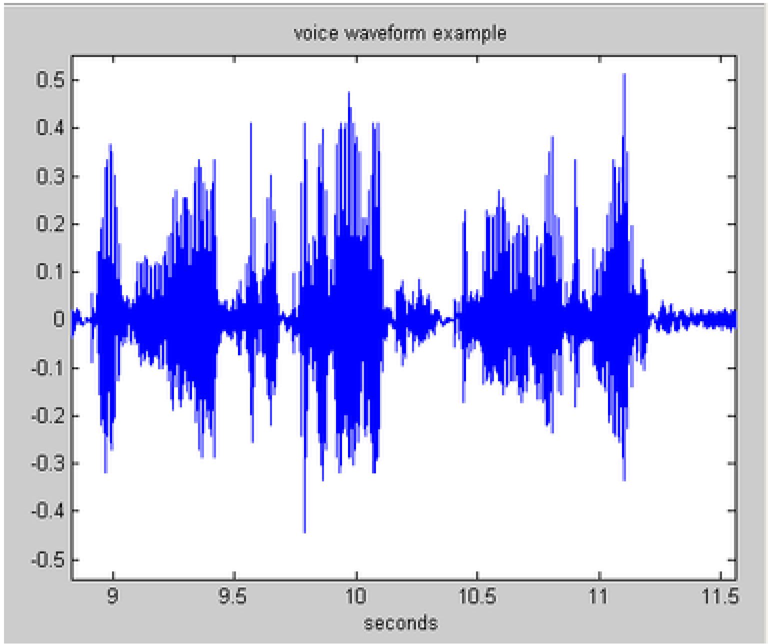


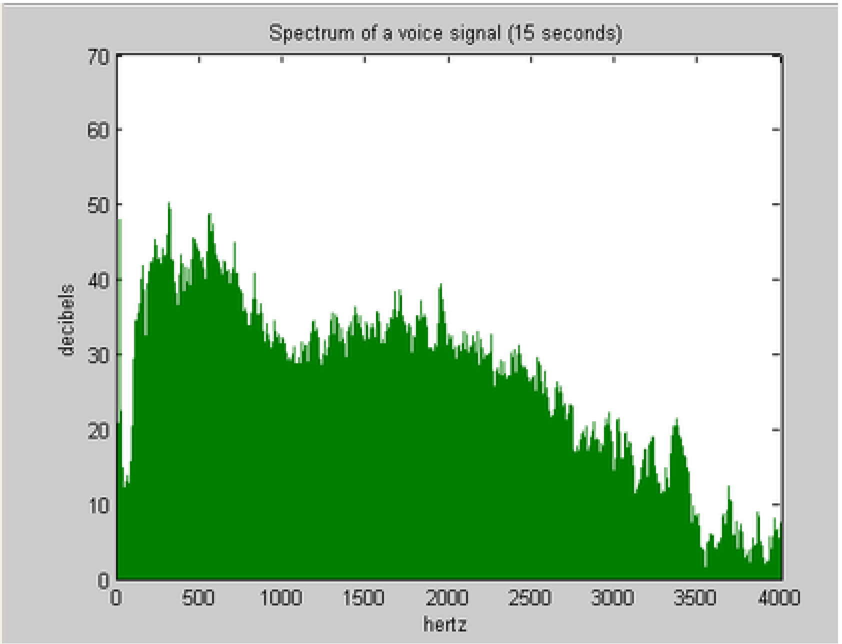
$$= \boxed{A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)}$$



Example: Music

- We think of music in terms of frequencies at different magnitudes

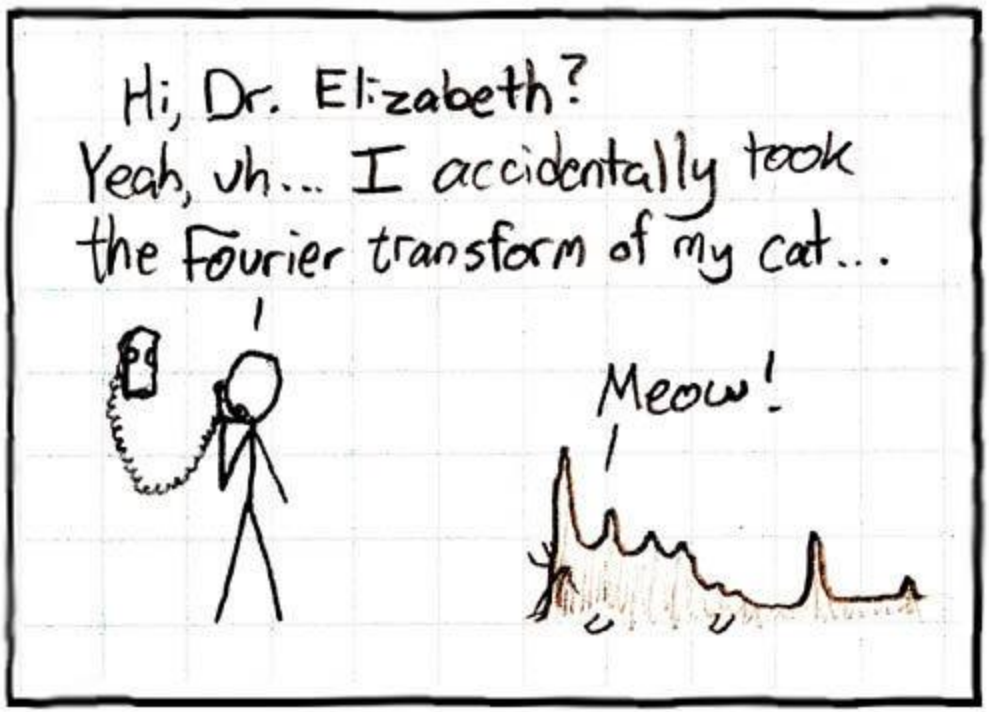




Slide: Hoiem

Other signals

- We can also think of all kinds of other signals the same way

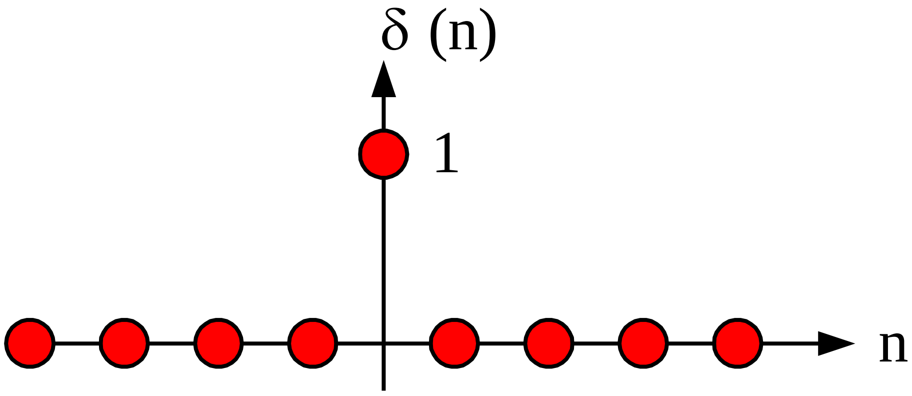


xkcd.com

Recall: Some special functions

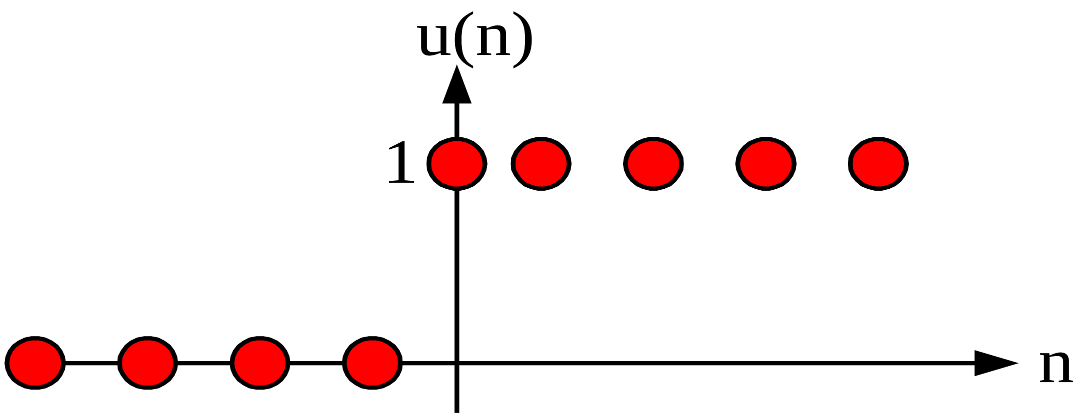
- Discrete **Impulse** Function (or **Kroenecker Delta**):

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$



- Discrete **step** function:

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases}$$



These two are related:

$$\delta(n) = u(n) - u(n-1)$$

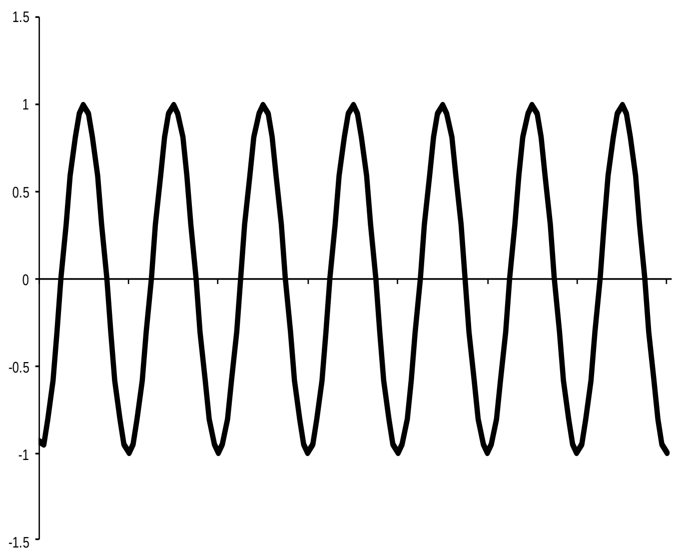
$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

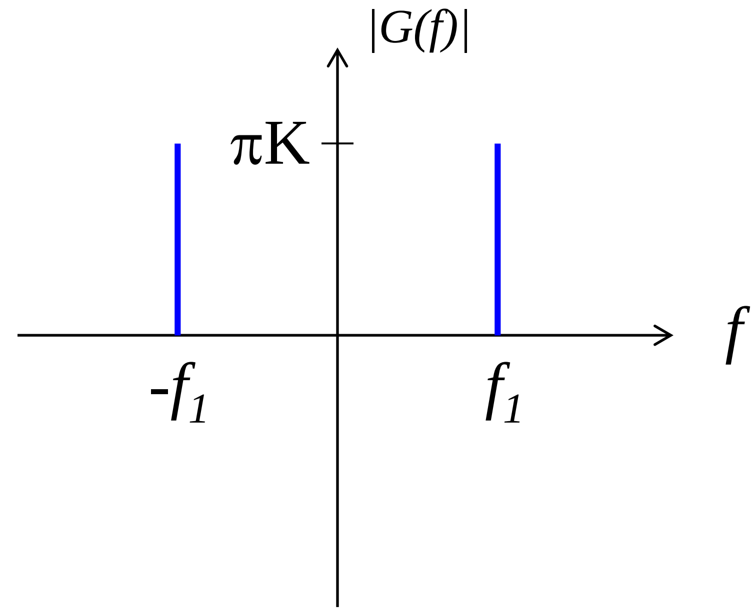
FT Example (a single 1D sinusoid): Cosine and Its Fourier Transform

$g(x)=K\cos(2\pi f_1 x) \Rightarrow \mathrm{F}\{g(x)\}=G(f)$

f - frequency

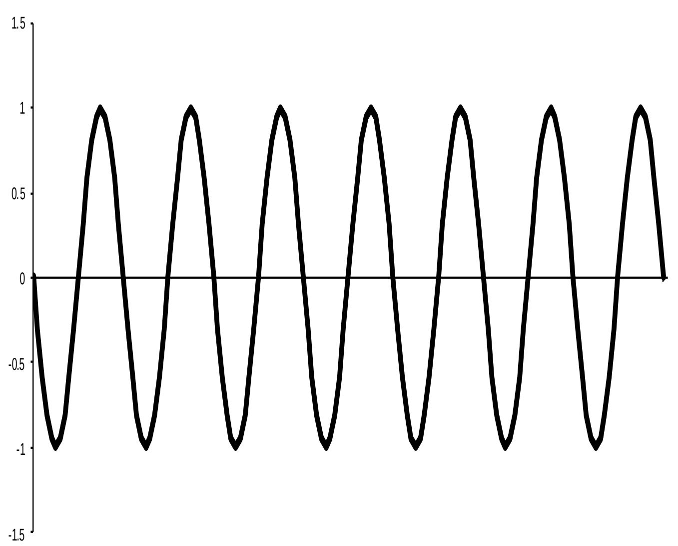
x - ("time") index along the x axis

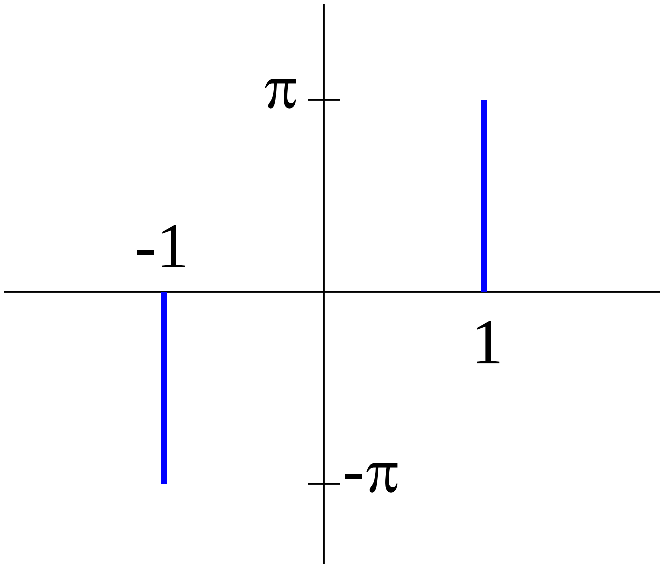




If $g(2\pi fx)$ is even, so is $G(\omega)$

FT Example (a single 1D sinusoid): Sine and Its Transform





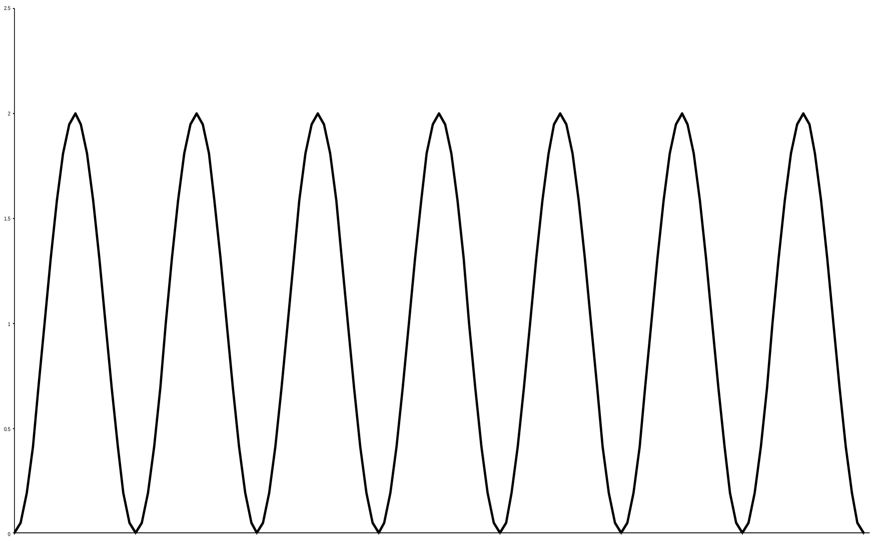
If $f(x)$ is odd, so is $F(\omega)$

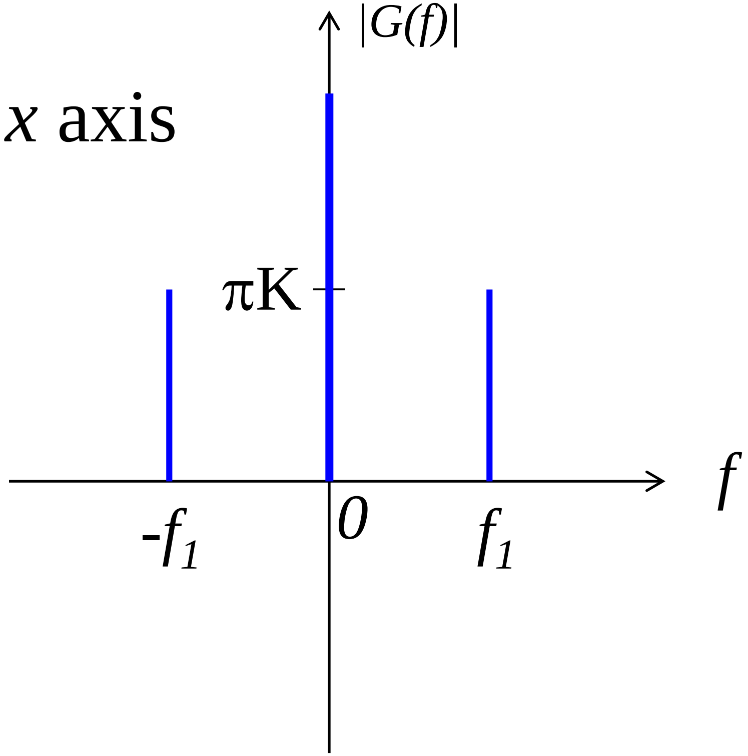
FT Example:
Cosine and Its Fourier Transform

$g(x)=K(1+\cos(2\pi f_1 x))$ ➔ $\mathrm{F}\{g(x)\}=G(f)$

f - frequency

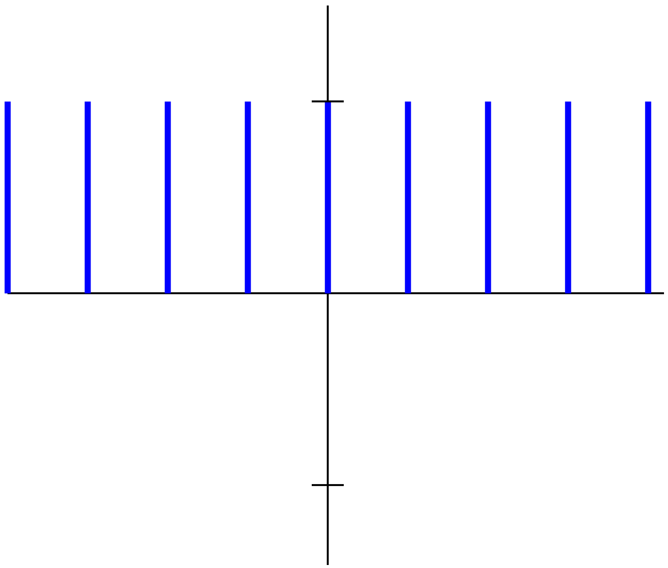
x - (“time”) index along the x axis

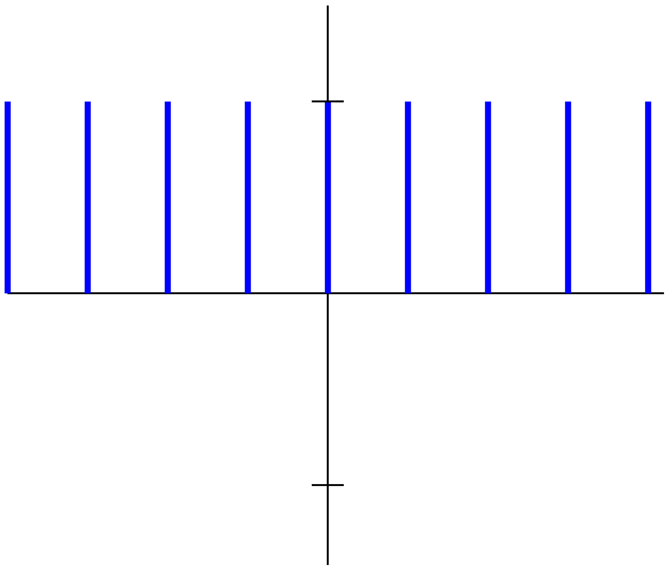




If $g(2\pi f x)$ is even, so is $G(\omega)$

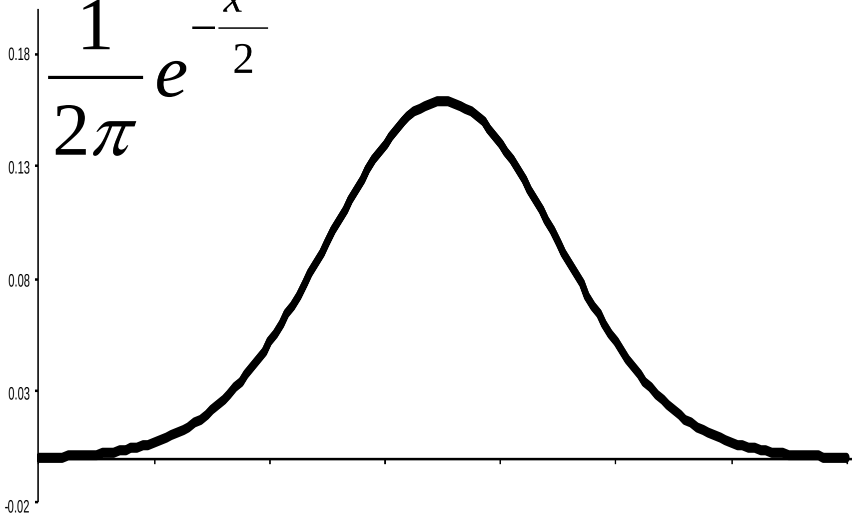
FT Example: Spikes and their Transform

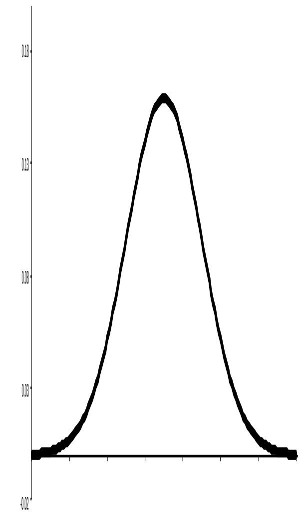




FT Example: Gaussian and Its Transform

$$\frac{1}{2\pi}e^{-\frac{x^2}{2}}$$





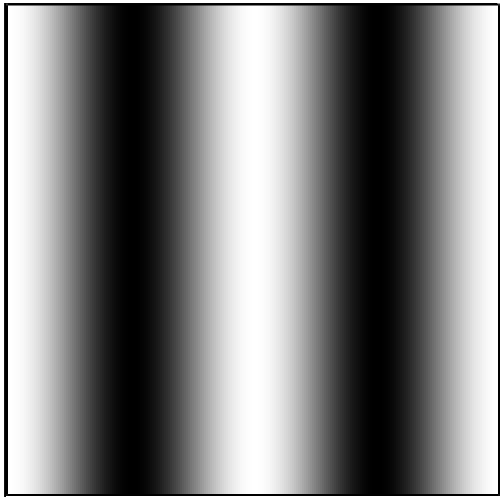
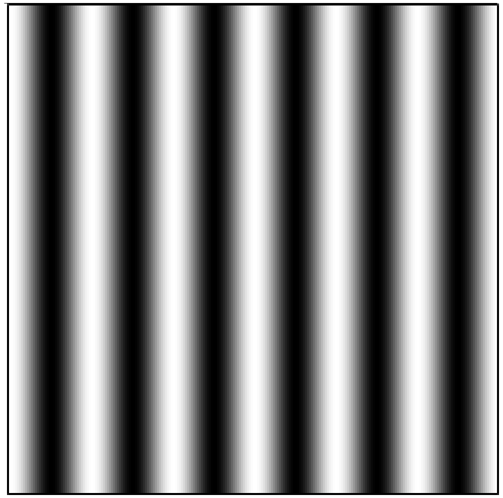
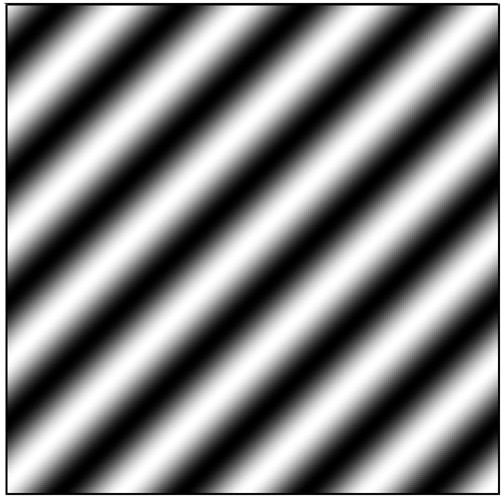
(Gaussian ➔ Gaussian with different width / height)

Fourier analysis in images (a single 2D sinusoid)

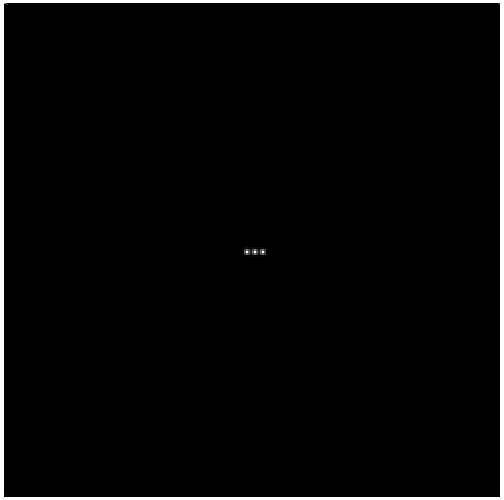
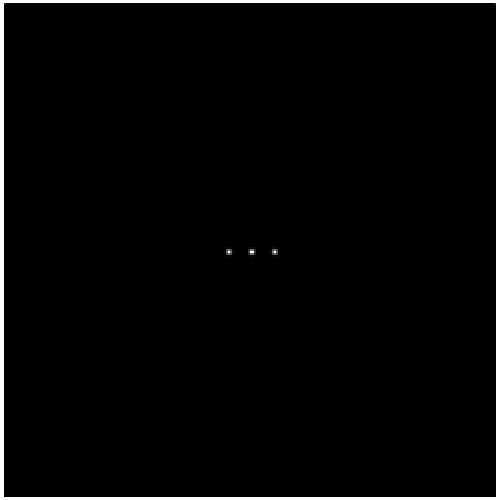
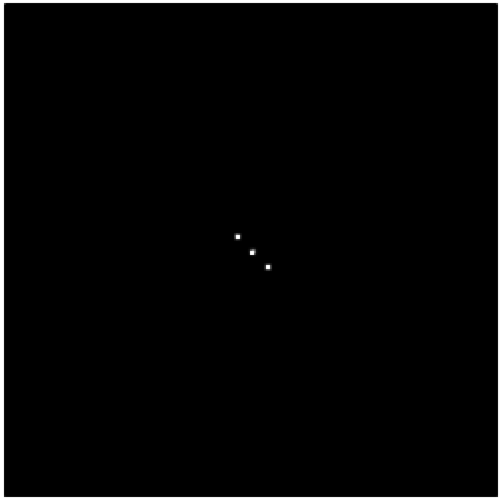
f_x=2[cyc/Img width]
f_y=0[cyc/Img height]

f_x=6[cyc/Img width]
f_y=0[cyc/Img height]

f_x=4[cyc/Img width]
f_y=4[cyc/Img height]

Intensity Image

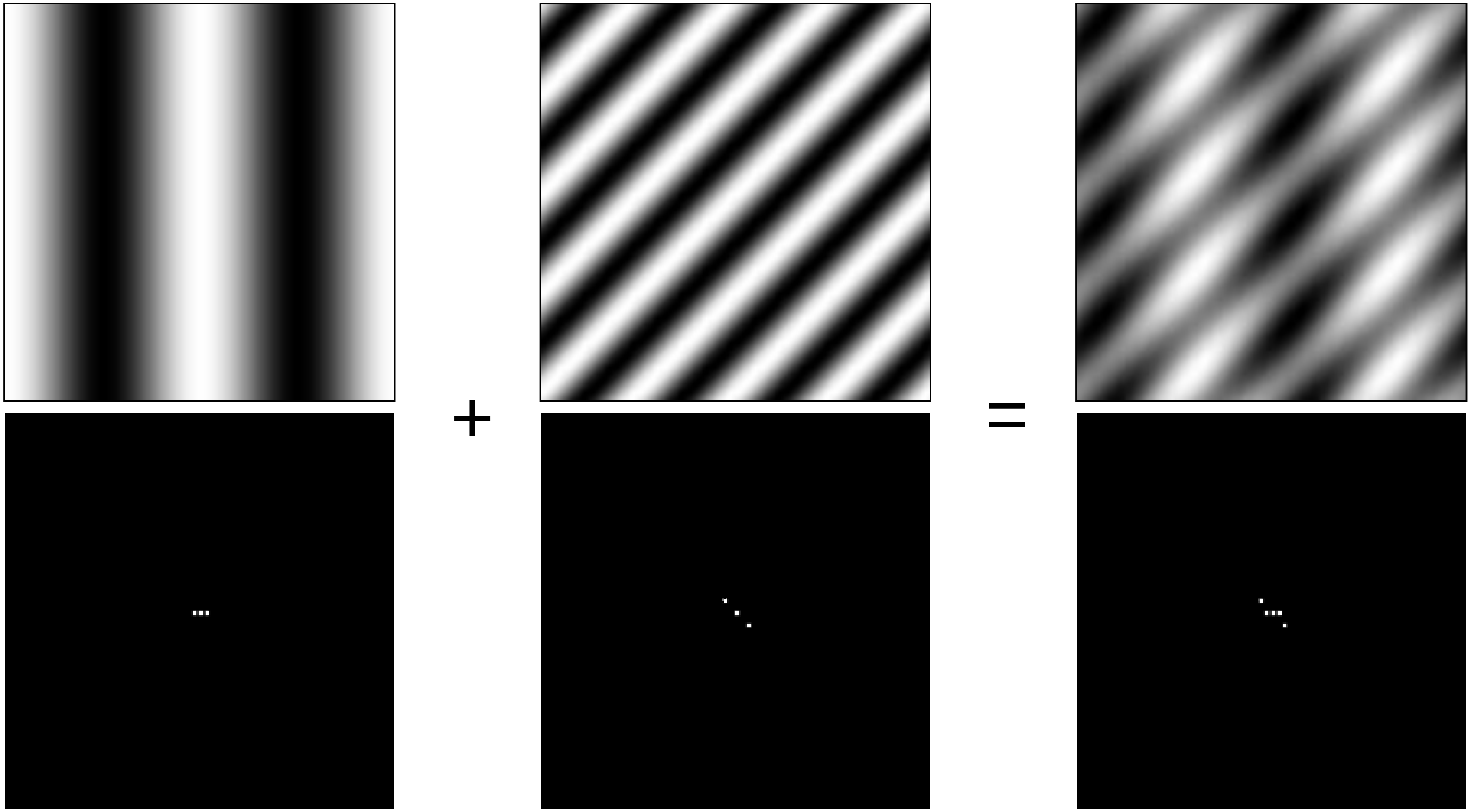




Fourier Image

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering

Signals can be composed

(a single 2D sinusoid) (a single 2D sinusoid) (a sum of 2 sinusoids)



Intensity Image

Fourier Image

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering
More: http://www.cs.unm.edu/~brayer/vision/fourier.html

Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of complex numbers

Magnitude: $$A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$$

Phase: $$\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$$

Computing the Fourier Transform (formal definition)

$$H(\omega) = \mathcal{F}\left\{h(x)\right\} = Ae^{j\phi}$$

Continuous

$$H(\omega) = \int_{-\infty}^{\infty} h(x)e^{-j\omega x}dx$$

Discrete

$$H(k) = \frac{1}{N}\sum_{x=0}^{N-1} h(x)e^{-j\frac{2\pi kx}{N}}$$ *k = -N/2..N/2*

Direct calculation of FT computational complexity: N^2
Fast Fourier Transform (FFT) computational complexity: NlogN
where N is the number of samples (image: number of pixels!)

The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$\mathrm{F}[g * h] = \mathrm{F}[g]\,\mathrm{F}[h]$$

- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$\mathrm{F}^{-1}[gh] = \mathrm{F}^{-1}[g] * \mathrm{F}^{-1}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

Properties of Fourier Transforms

- Linearity $\mathcal{F}[ax(t) + by(t)] = a\mathcal{F}[x(t)] + b\mathcal{F}[y(t)]$

- Fourier transform of a real signal is symmetric about the origin

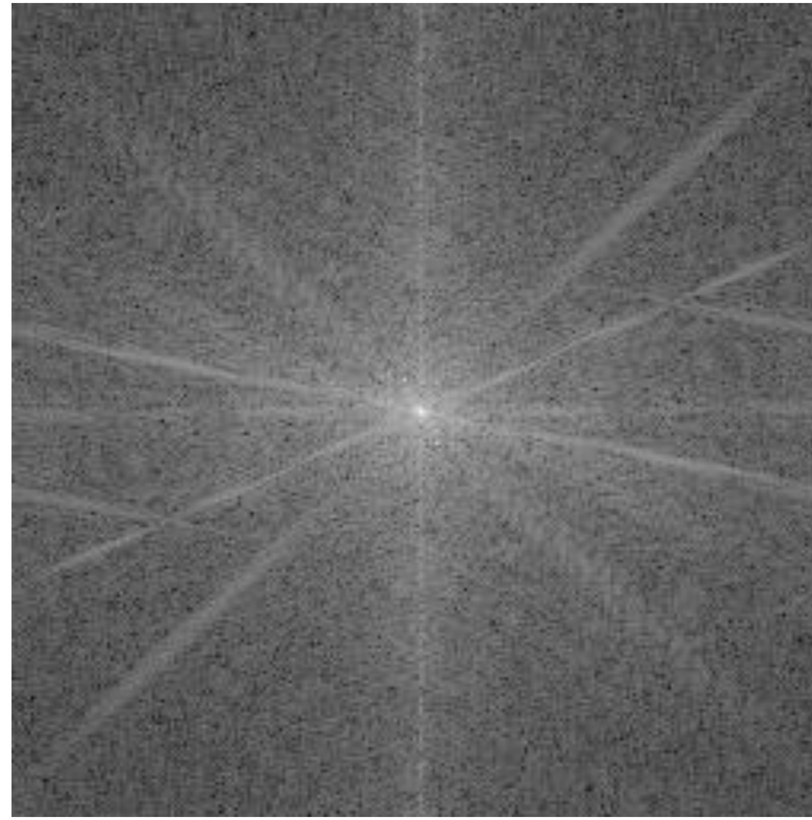
- The energy of the signal is the same as the energy of its Fourier transform

See Szeliski Book (3.4)

Fourier demo (I): sum of signals

cameraman.tif in ///

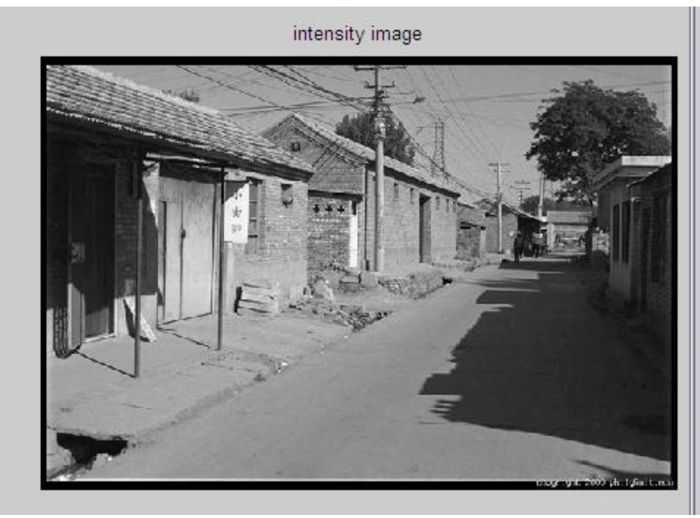
log (abs (fft (cameraman.tif in ///)))



Note that the log is used for display purposes only, of FFT Magnitude (due to the limited dynamic range of an image (0-255)

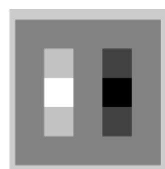
Filtering in spatial domain

1	0	-1
2	0	-2
1	0	-1

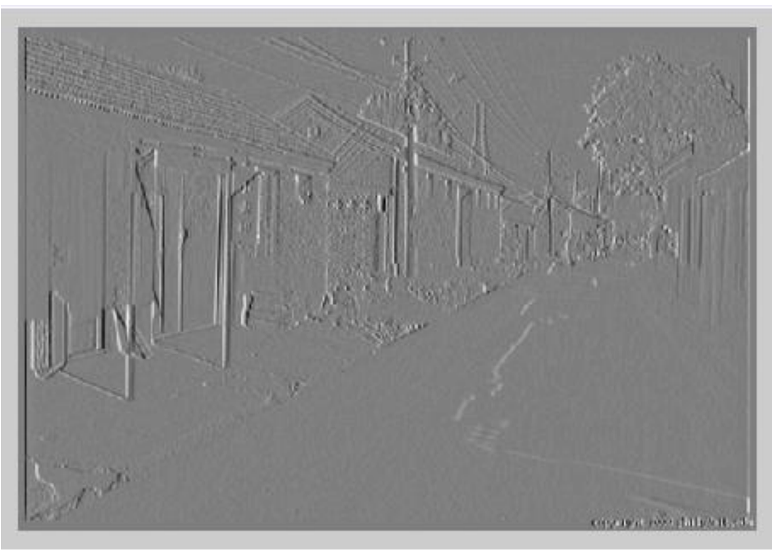


*

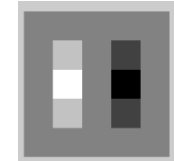
1	0	-1
2	0	-2
1	0	-1

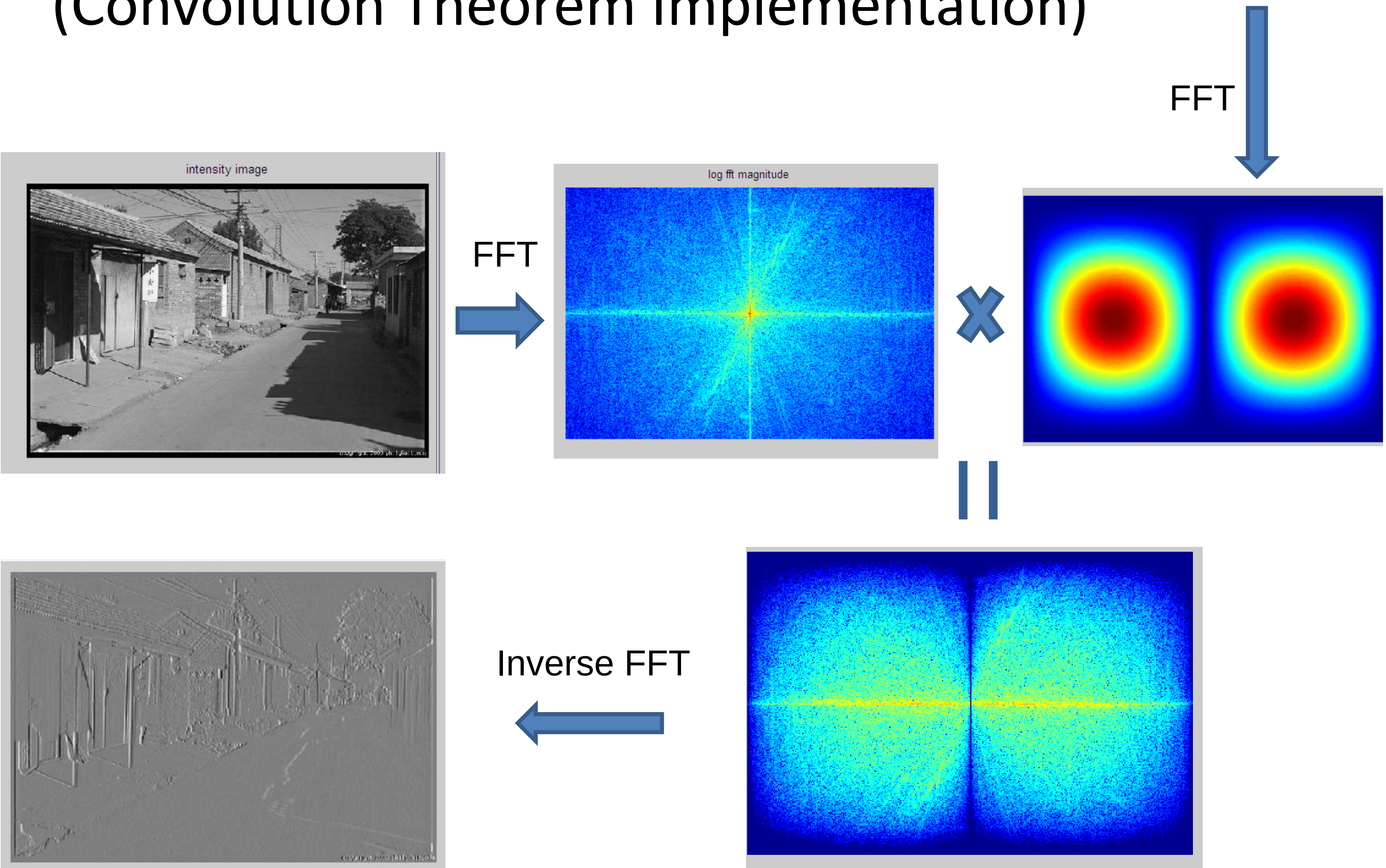


=



Filtering in frequency domain
(Convolution Theorem Implementation)



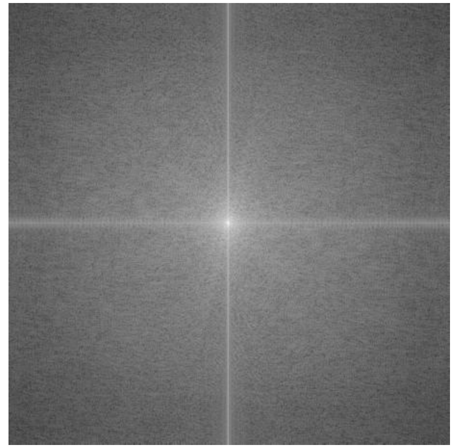


Slide: Hoiem

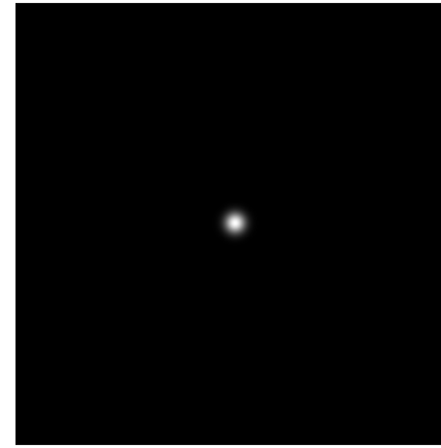
Fourier demo (II):
Convolution Theorem = Filtering in Frequency Domain

orig img

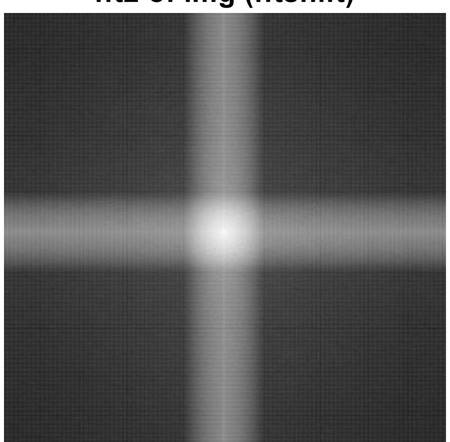
log mag fft2 of img (fftshift)



fft2 of filter (fftshift)



fft2 of img (fftshift)

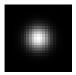


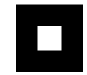
filtered img

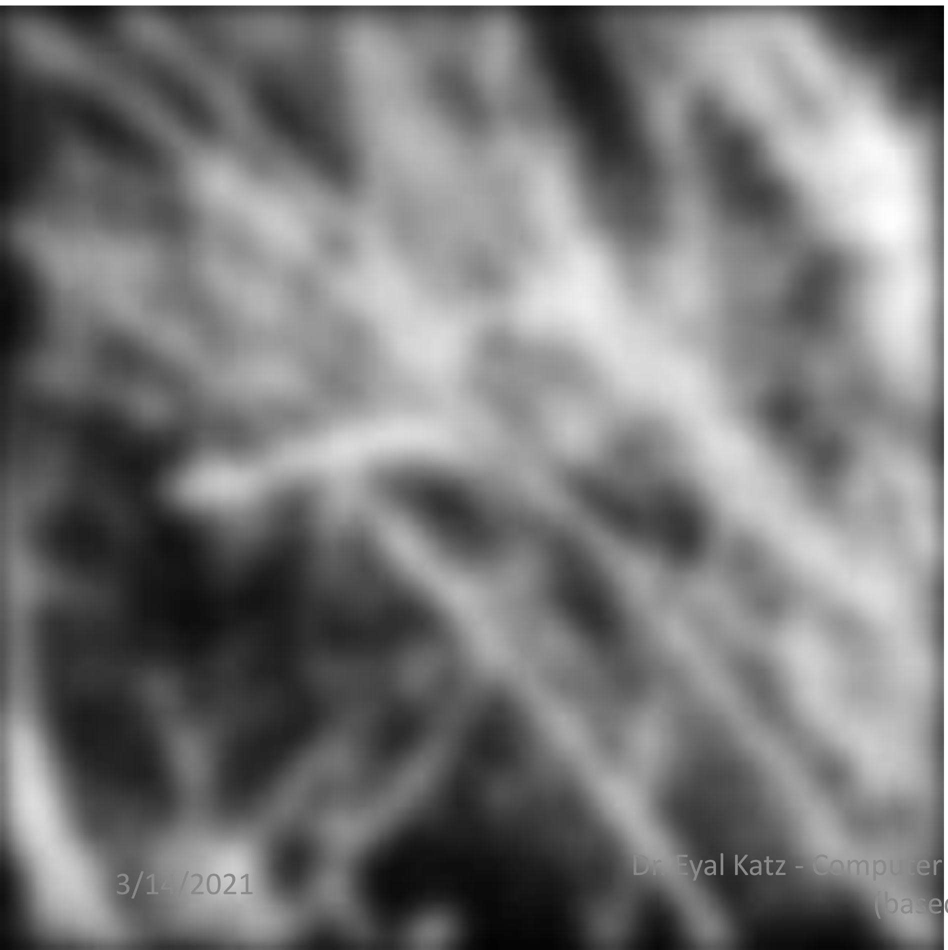
filtered img

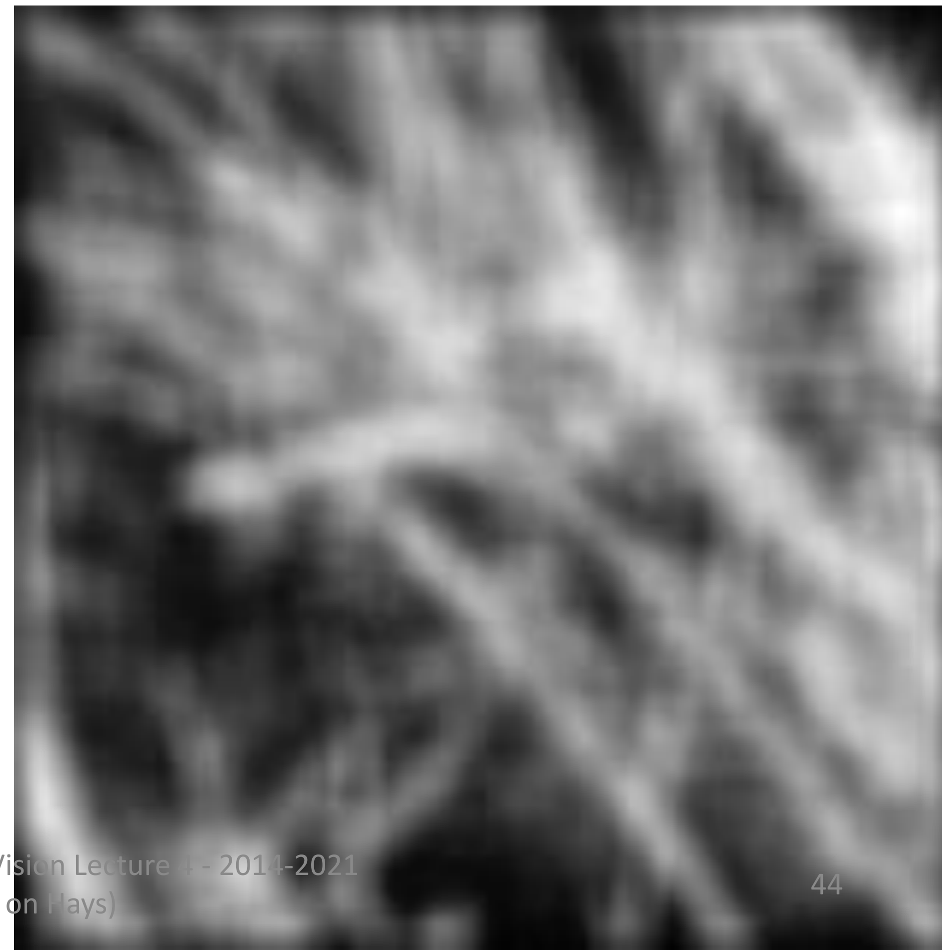
Filtering - Revisited

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian 

Box filter 

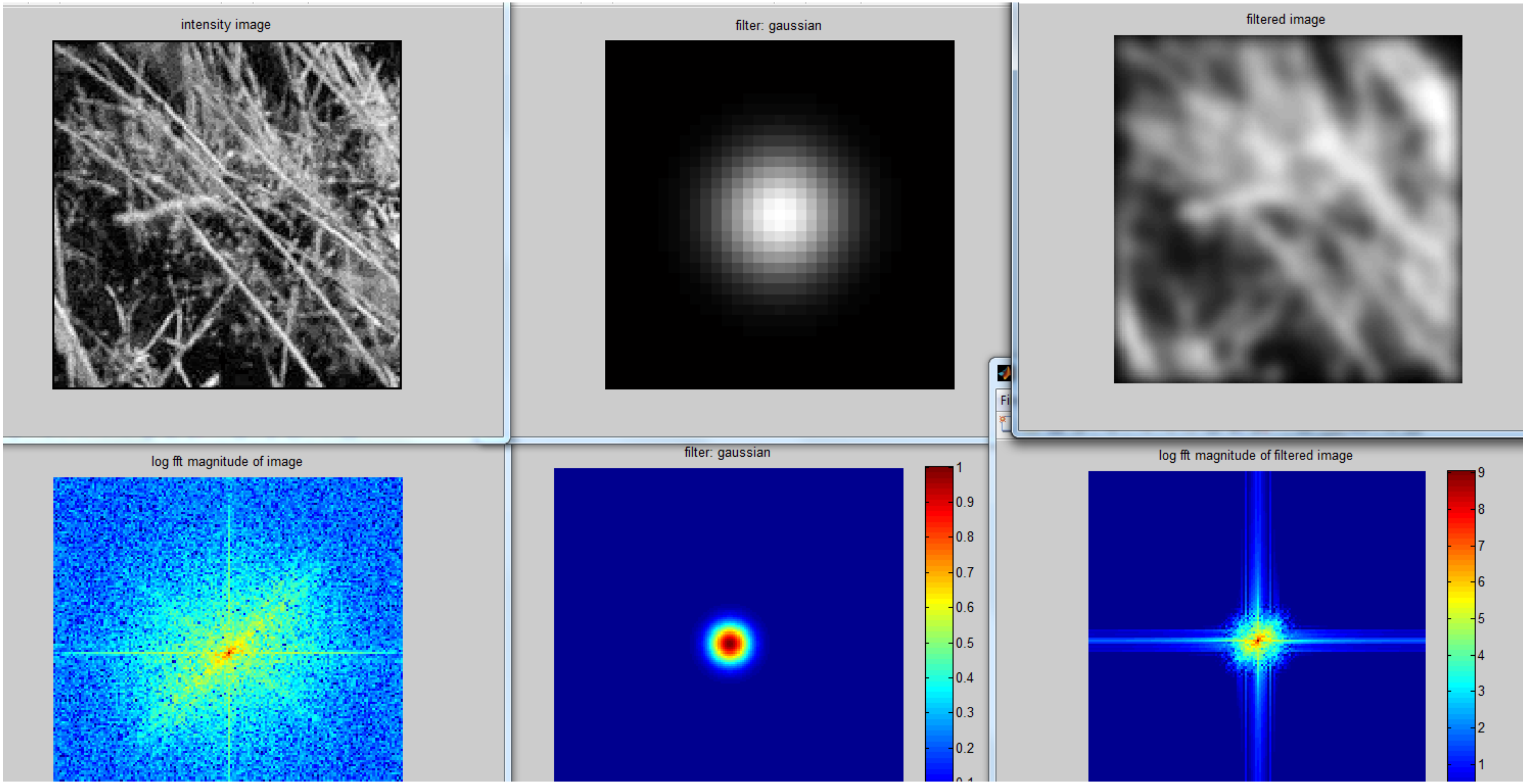




Filtering: Gaussian Filter (Low Pass)

Original Image Gaussian Filter Blurred Image

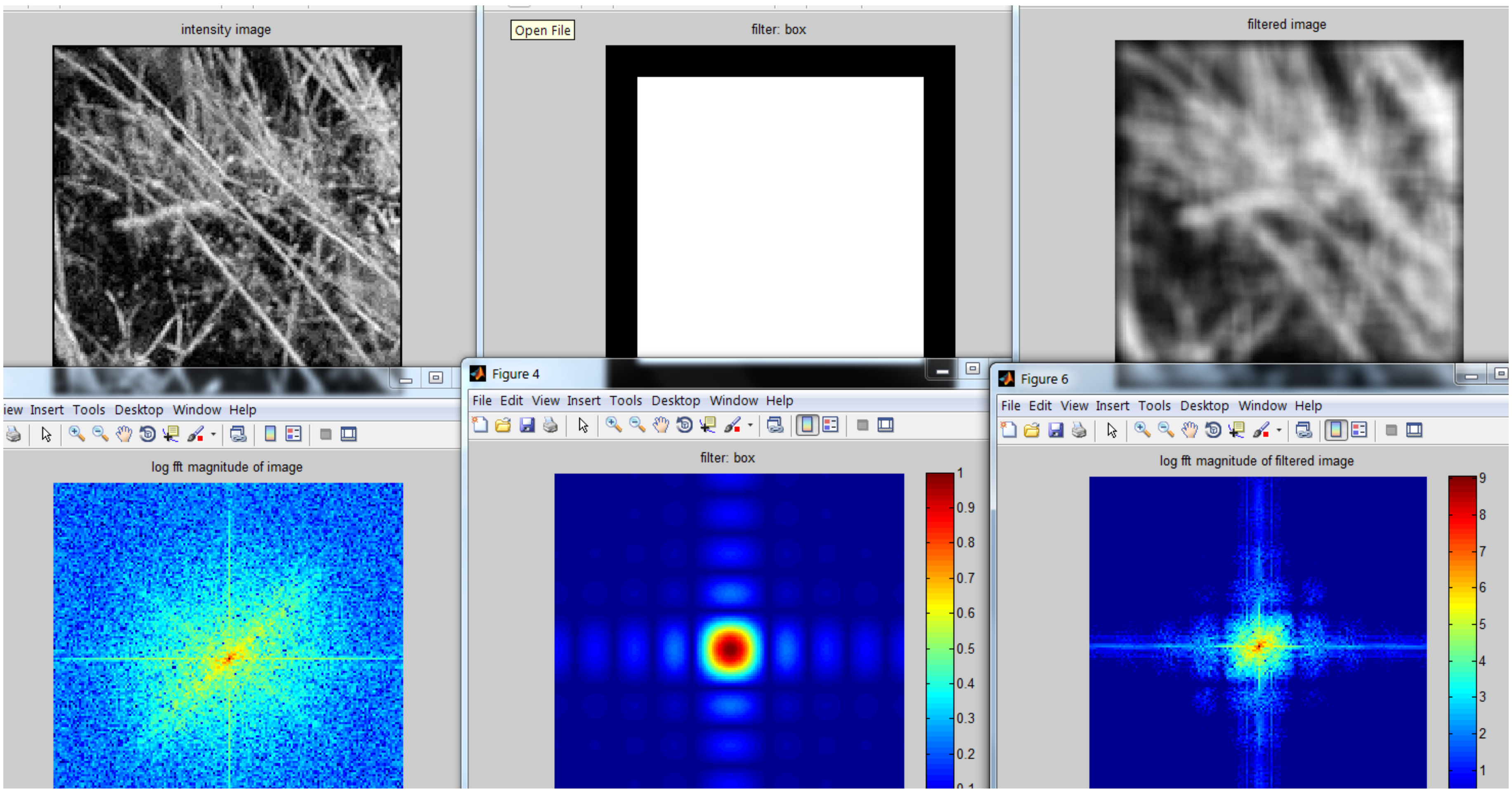
(convolution in space = Multiplication in Frequency)



Filtering: Box Filter (Low Pass Filter)

Original Image | Box Filter | Blurred Image

(convolution in space = Multiplication in Frequency)



Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

Subsampling (Down-Sampling) by a factor of 2

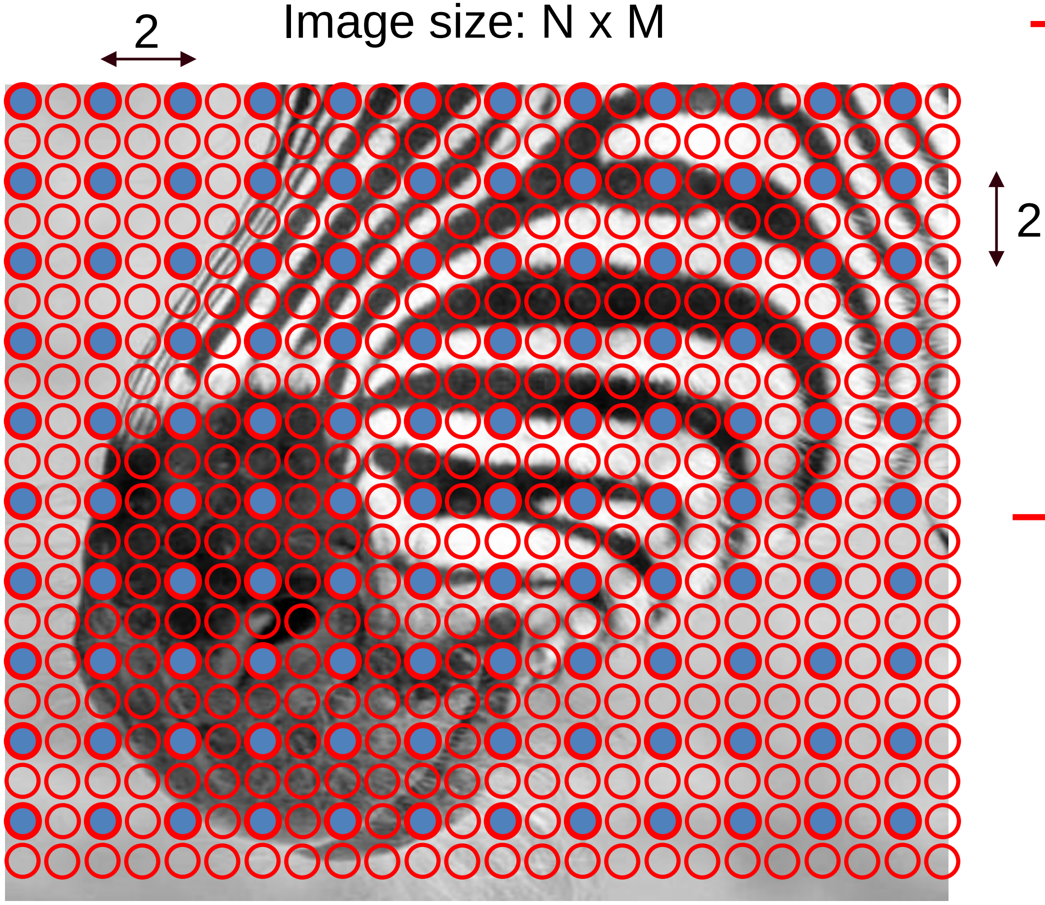
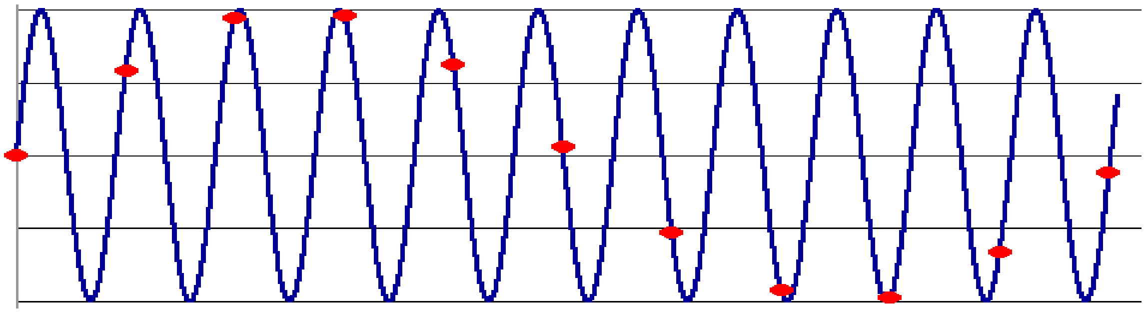


Image size: N/2 x M/2

Throw away every other row and column to create a 1/2 size image in each dimension

Down-sampling



- **Aliasing** can arise when you sample a continuous signal or image
  - occurs when your sampling rate is not high enough to capture the amount of detail in your image
  - Can give you the wrong signal/image—an *alias*
  - formally, the image contains structure at different scales
    - called “frequencies” in the Fourier domain
  - the sampling rate must be high enough to capture the highest frequency in the image

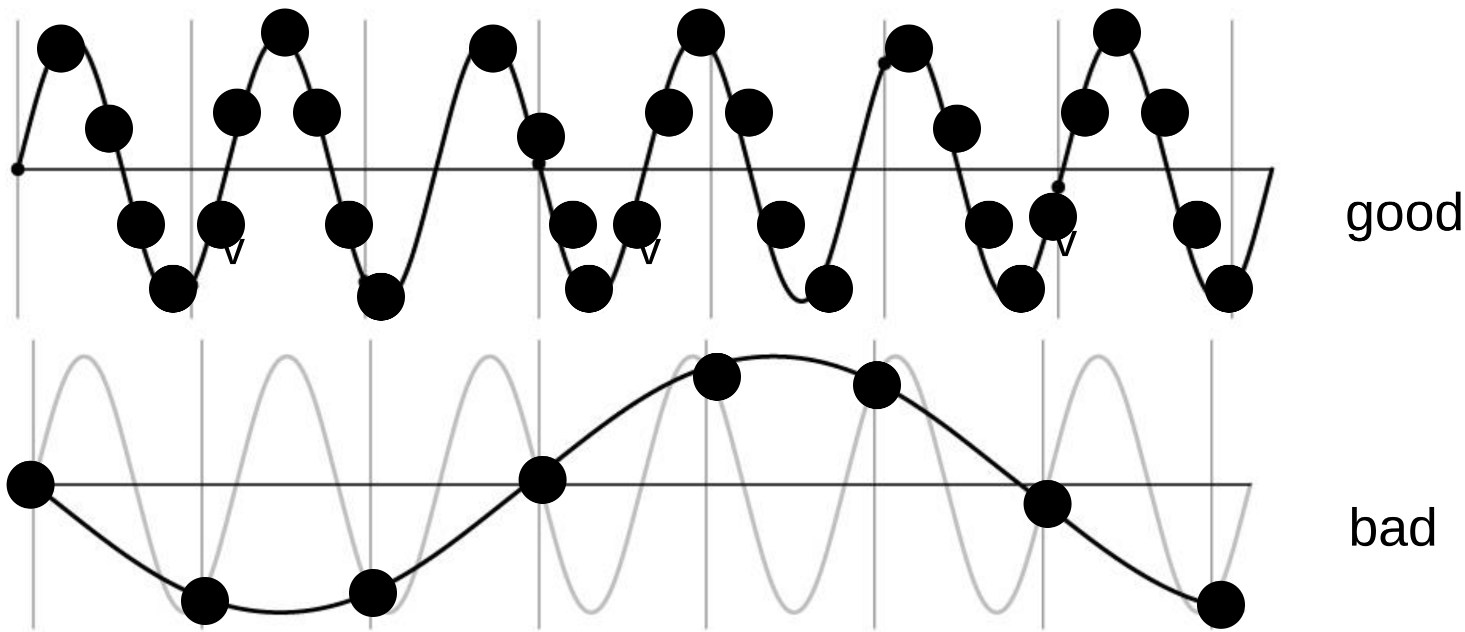
Aliasing problem

- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - “Wagon wheels rolling the wrong way in movies”
  - “Checkerboards disintegrate in ray tracing”
  - “Striped shirts look funny on color television”

Source: D. Forsyth

Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $> 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allow to reconstruct the original perfectly from the sampled version



Anti-aliasing

Possible Solutions:

- Increase sampling rate (sample more often)

- **Anti-aliasing Filtering (pre-filtering)**:
  Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it's better than aliasing
  - Apply a smoothing filter

Algorithm for downsampling by factor of 2

1. Start with image(h, w)
2. Apply low-pass filter

    im_blur = imfilter(image, fspecial('gaussian', 7, 1))

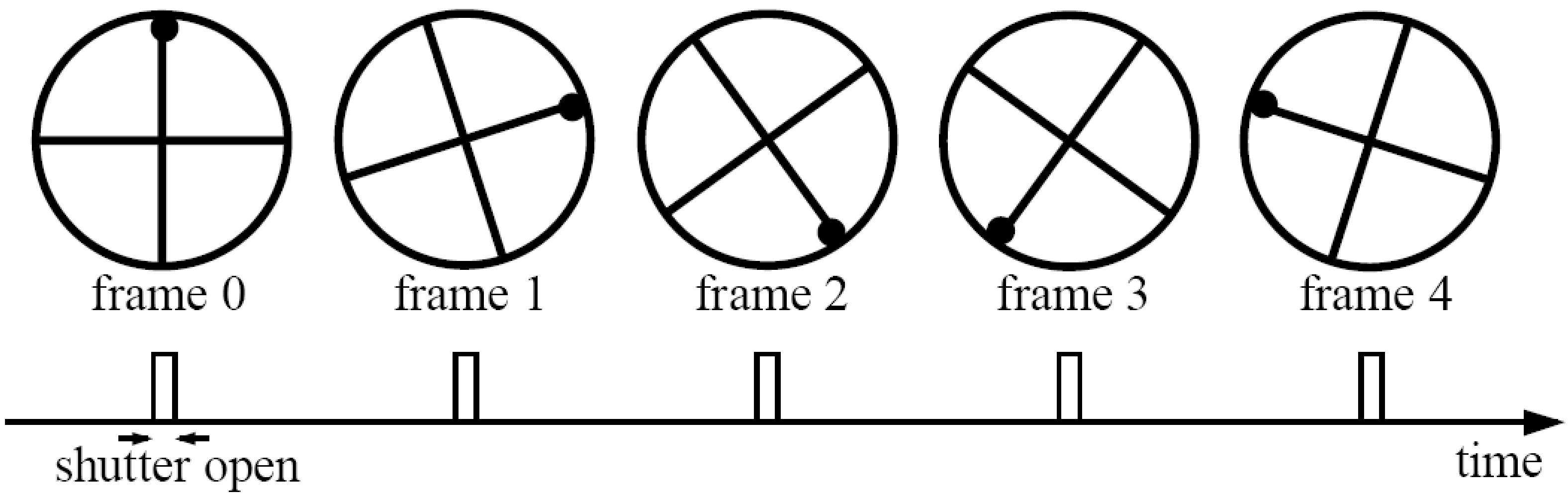
3. Sample every other pixel

    im_small = im_blur(1:2:end, 1:2:end);

Aliasing in video (Temporal Aliasing)

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

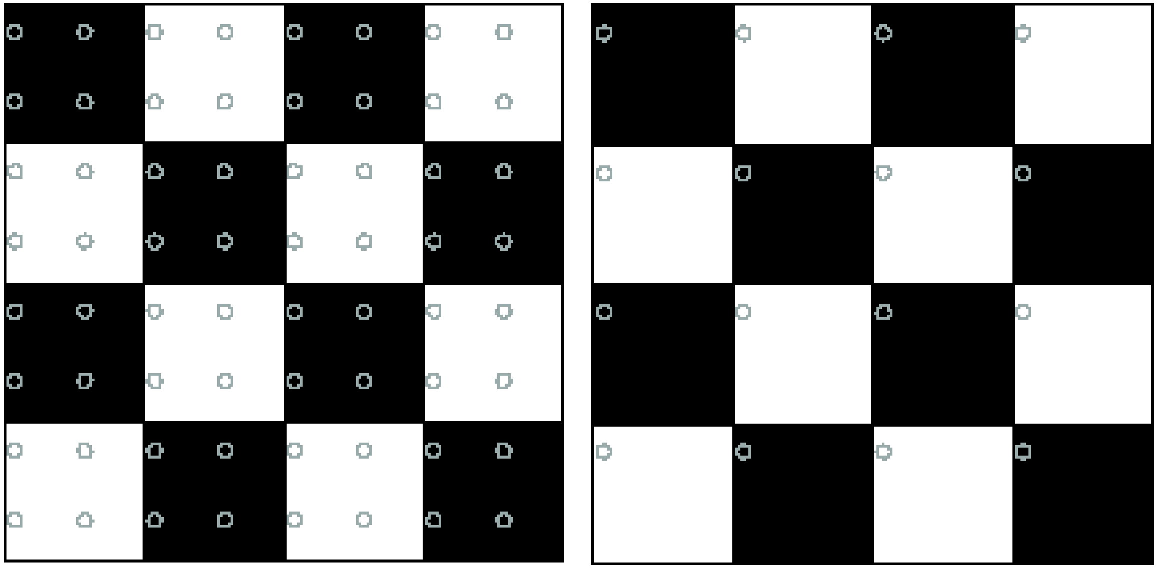
Some links: https://vimeo.com/4041788, www.youtube.com/watch?v=2pbYKDW0myU https://www.youtube.com/watch?v=HqKfCUW17QM

Slide by Steve Seitz

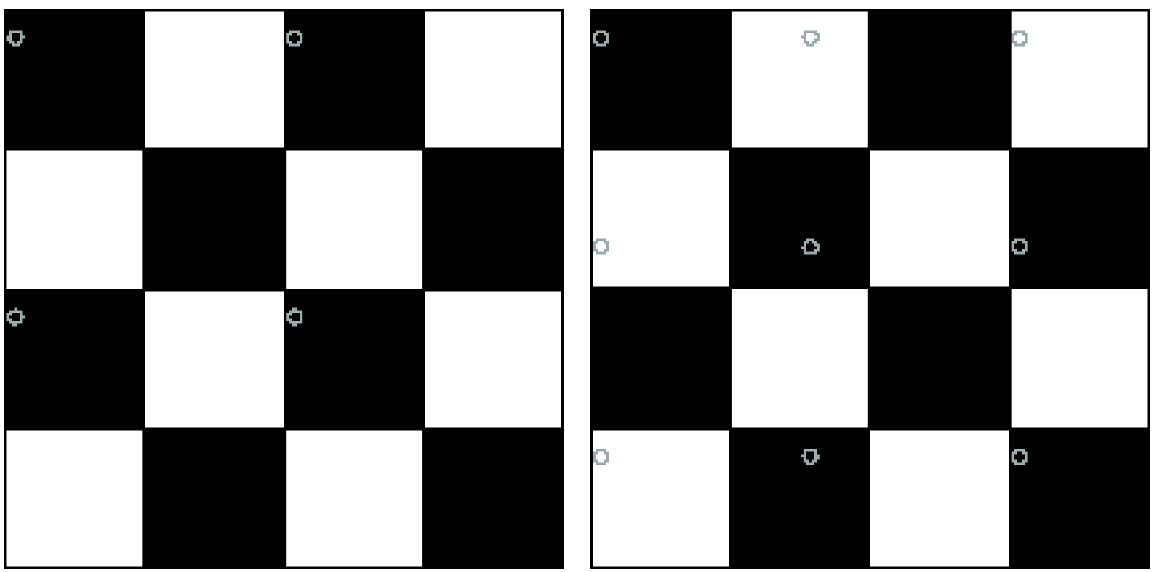
Aliasing in graphics

Source: A. Efros

2D example

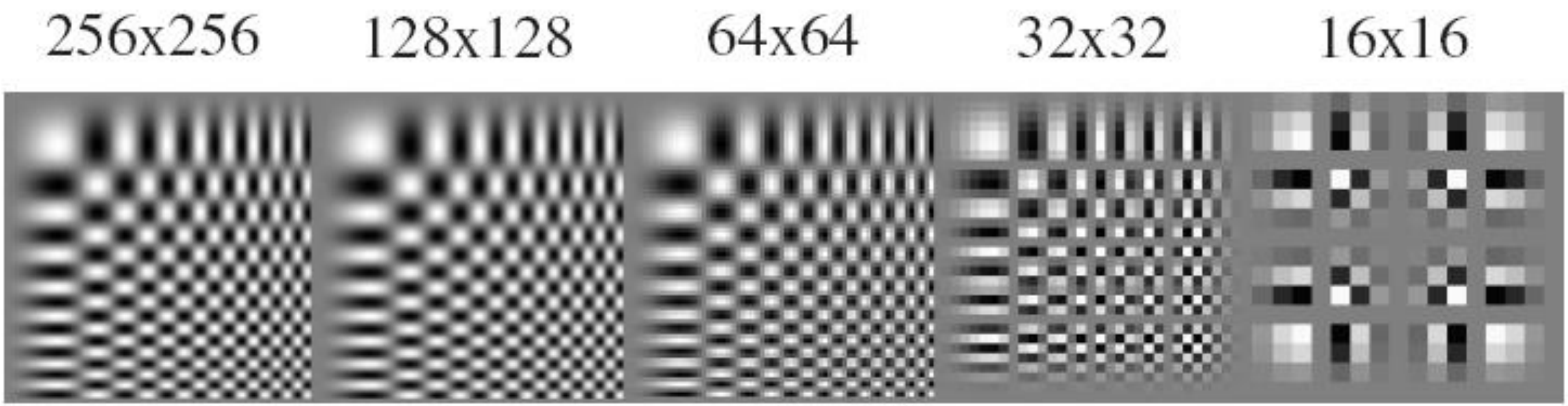


Good sampling

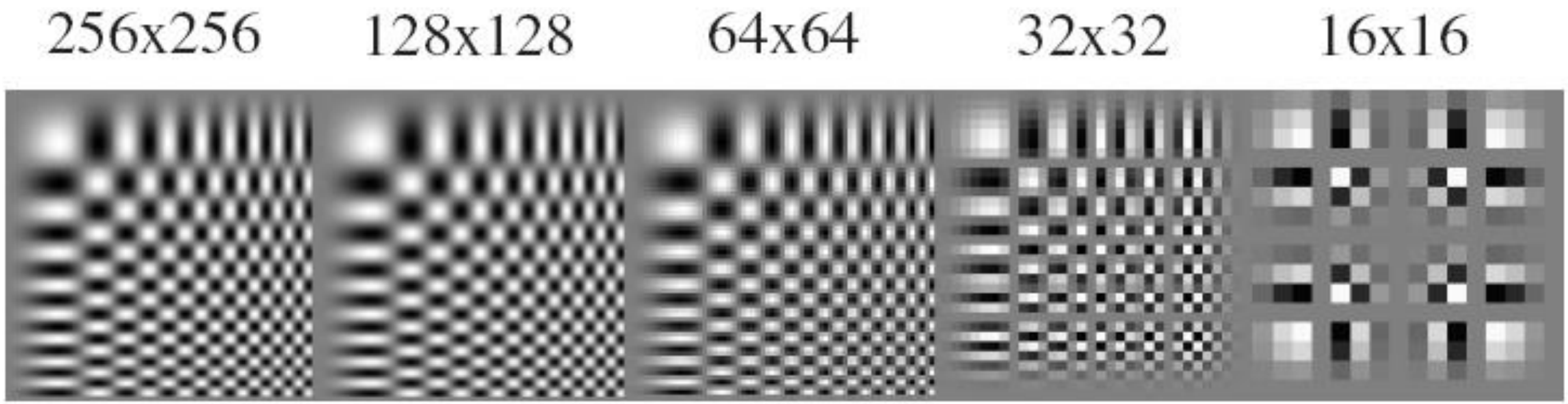


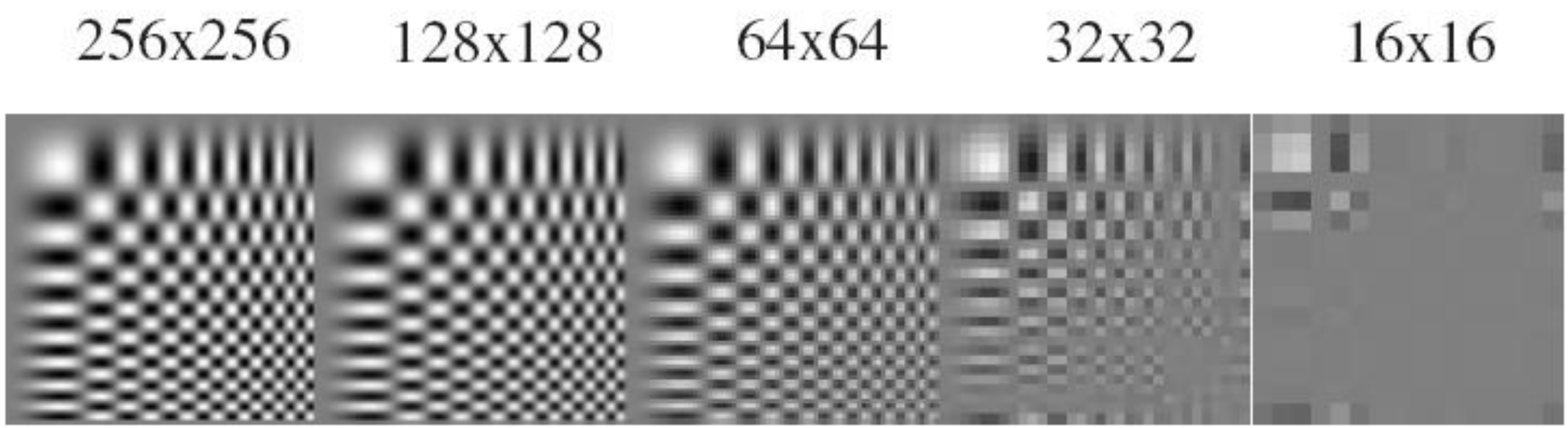
Bad sampling

Sampling and aliasing: no pre-filter



Anti-aliasing





Forsyth and Ponce 2002

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

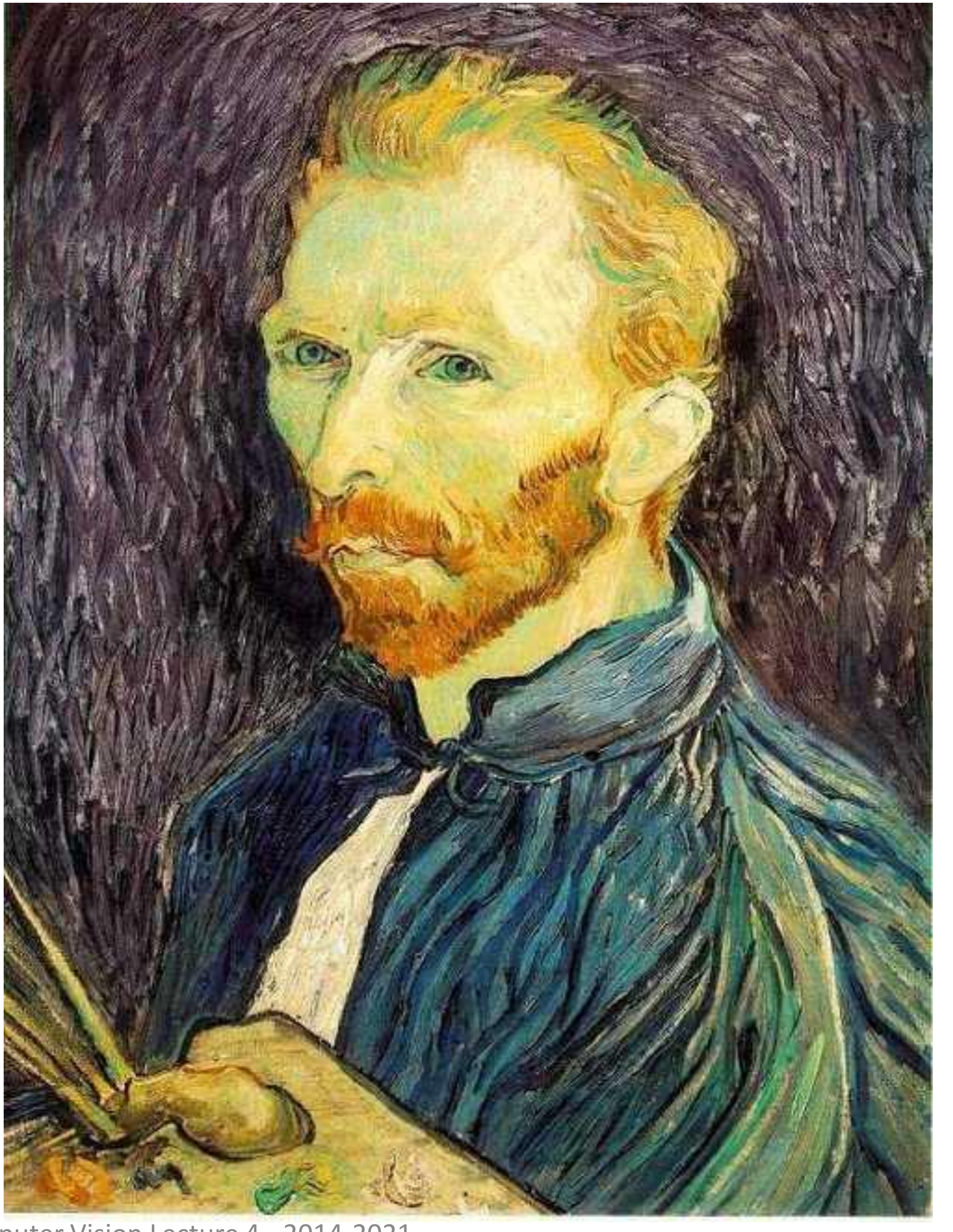
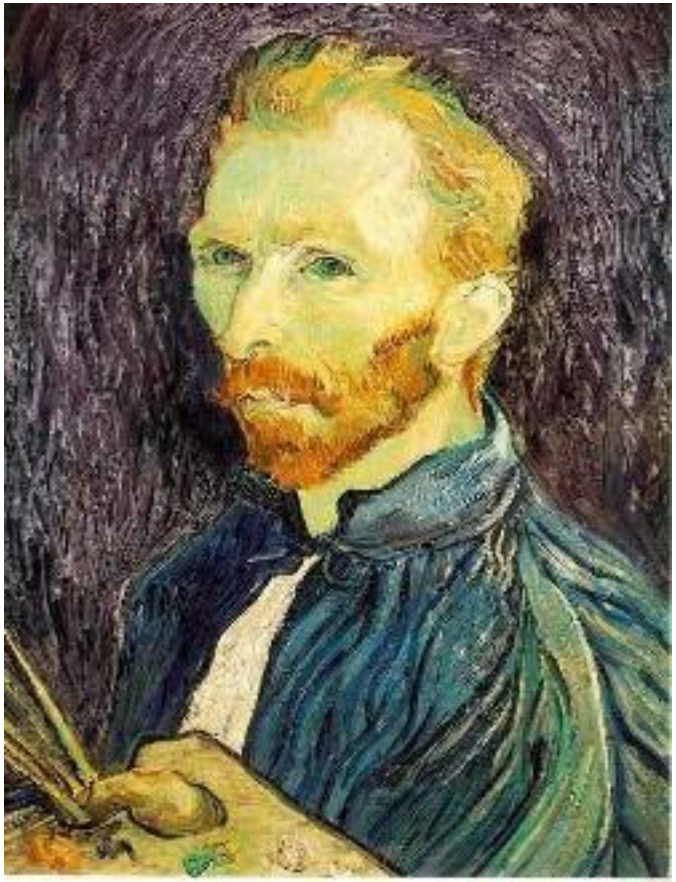


Image sub-sampling:
resizing: 1:2, 1:4, 1:8 of original image

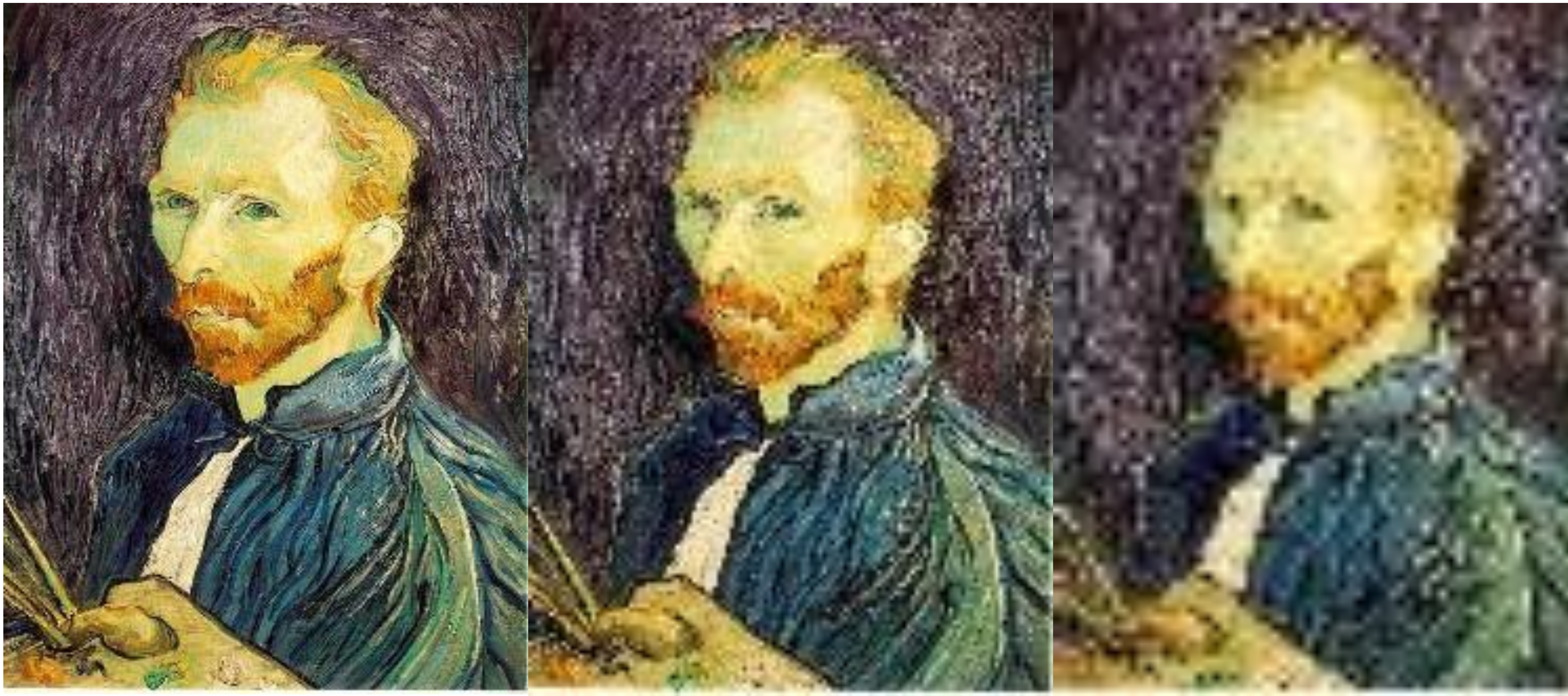


1/4

1/8

Throw away every other row and column to create a **1/2** size image
- called *image sub-sampling*

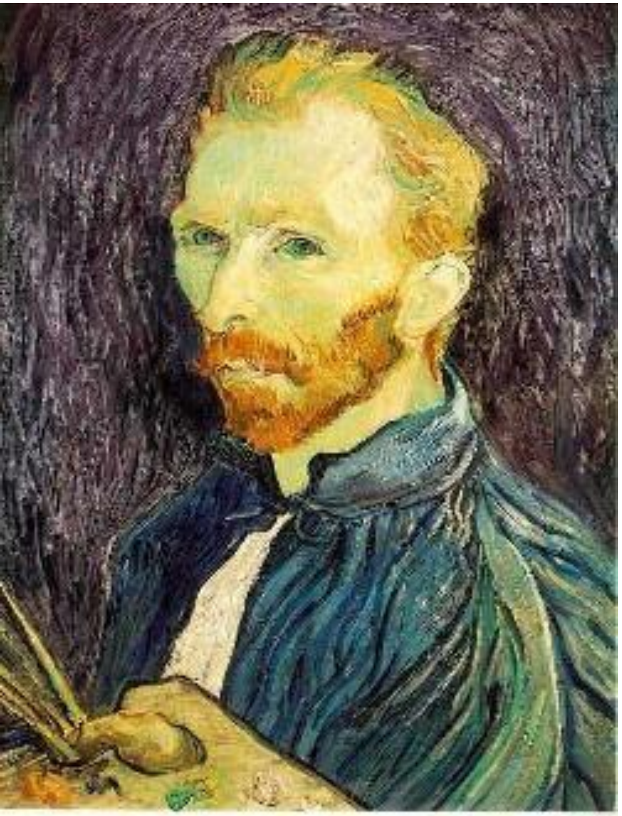
Image sub-sampling (up-sampled for display)



1/2 1/4 (2x zoom) 1/8 (4x zoom)

Why does this look so crufty? … Aliasing!

Subsampling without pre-filtering

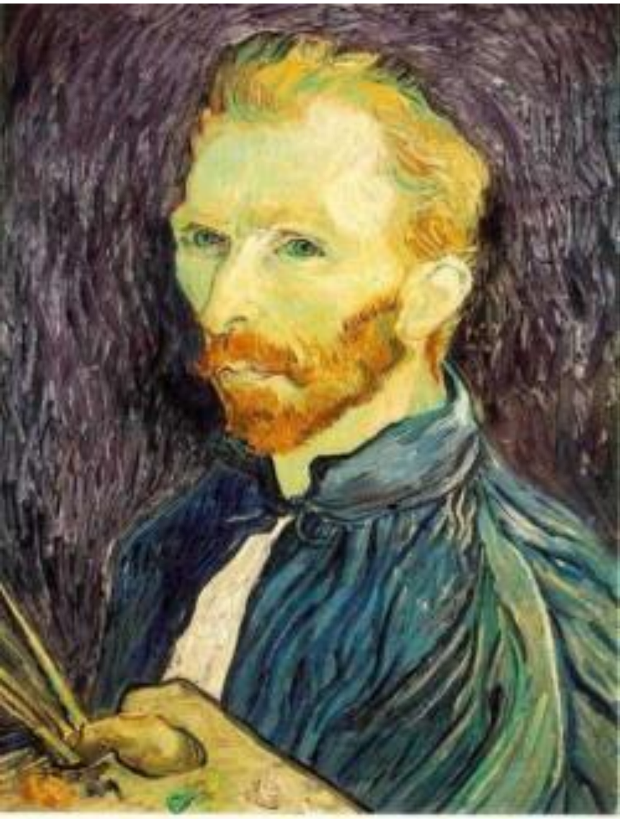


1/2

1/4 (2x zoom)

1/8 (4x zoom)

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

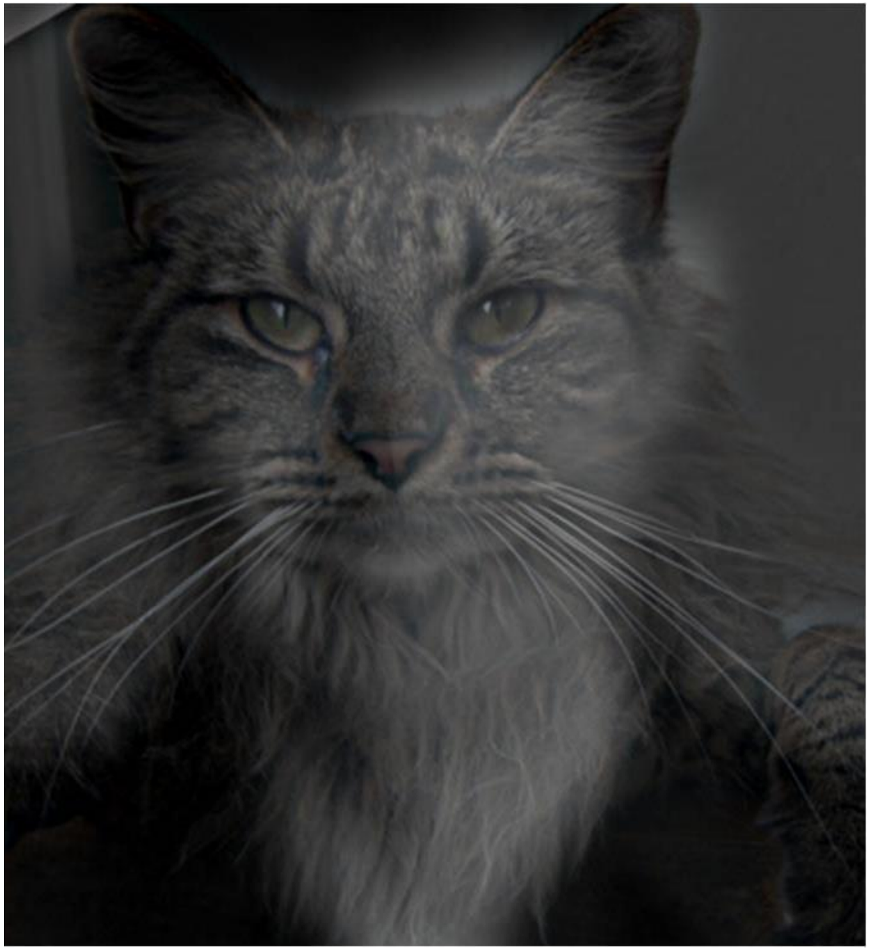
G 1/8

Moire patterns in real-world images. Here are comparison images by Dave Etchells of Imaging Resource using the Canon D60 (with an antialias filter) and the Sigma SD-9 (which has no antialias filter). The bands below the fur in the image at right are the kinds of artifacts that appear in images when no antialias filter is used. Sigma chose to eliminate the filter to get more sharpness, but the resulting apparent detail may or may not reflect features in the image.

More examples – Moire Patterns

Check out Moire patterns on the web.

Why do we get different, distance-dependent interpretations of hybrid images?

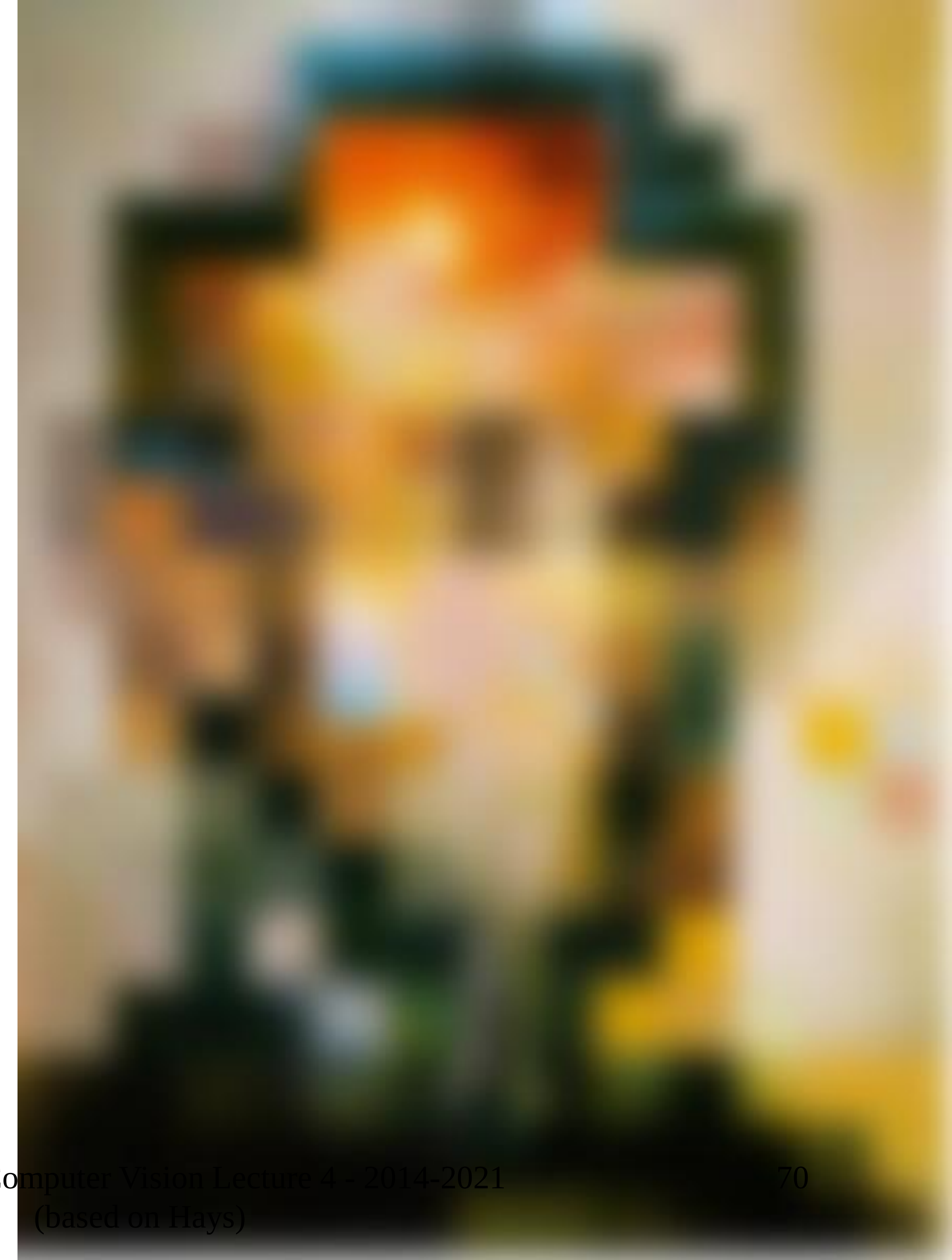


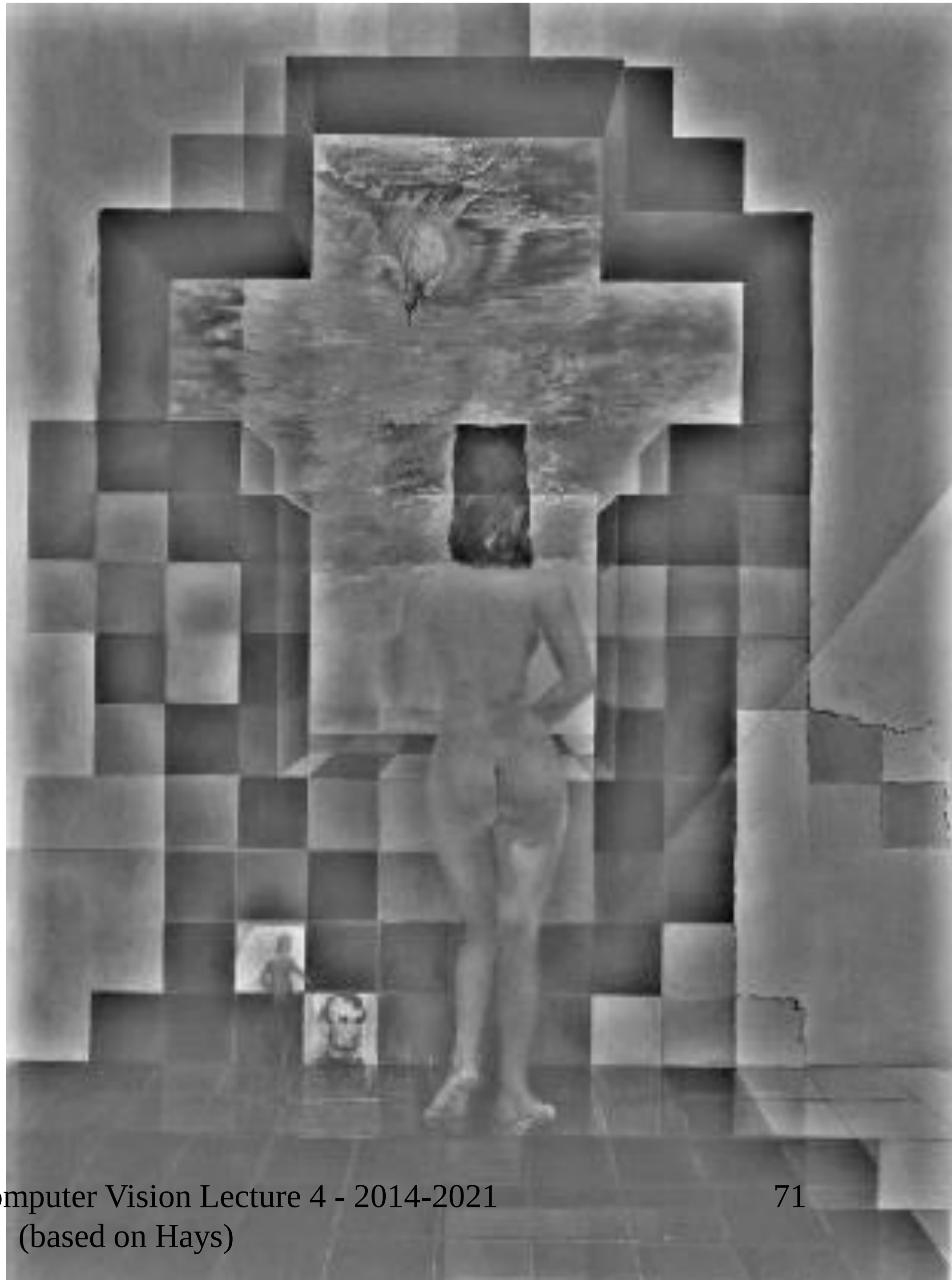
?

Salvador Dali invented Hybrid Images?

Salvador Dali

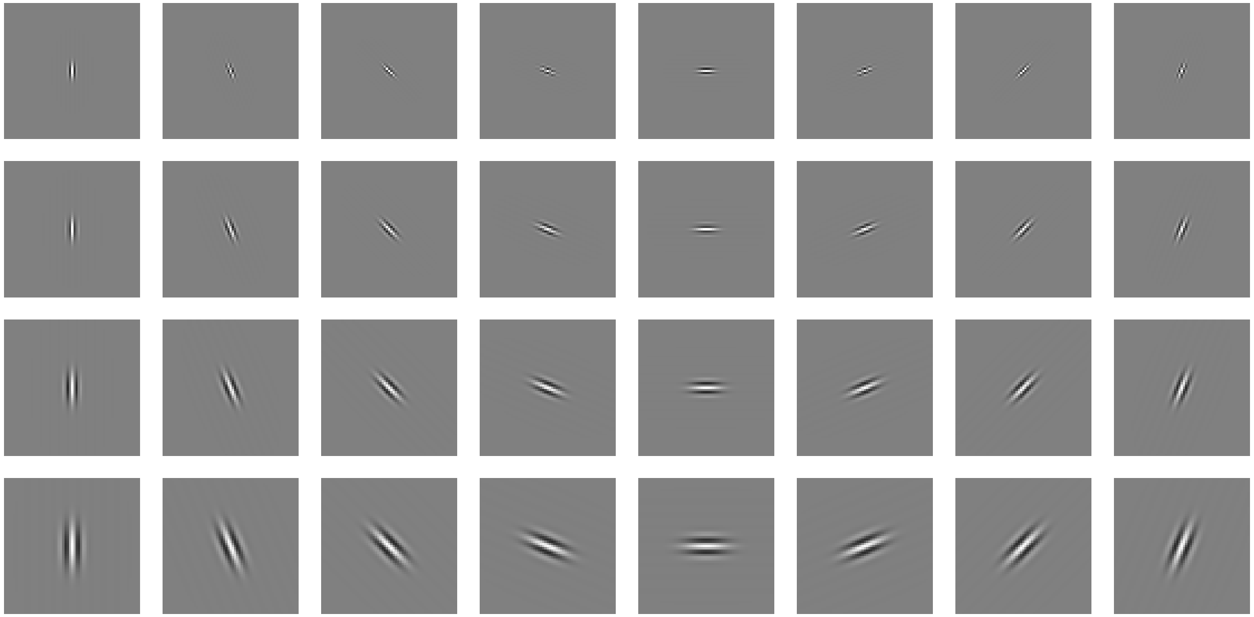
"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976





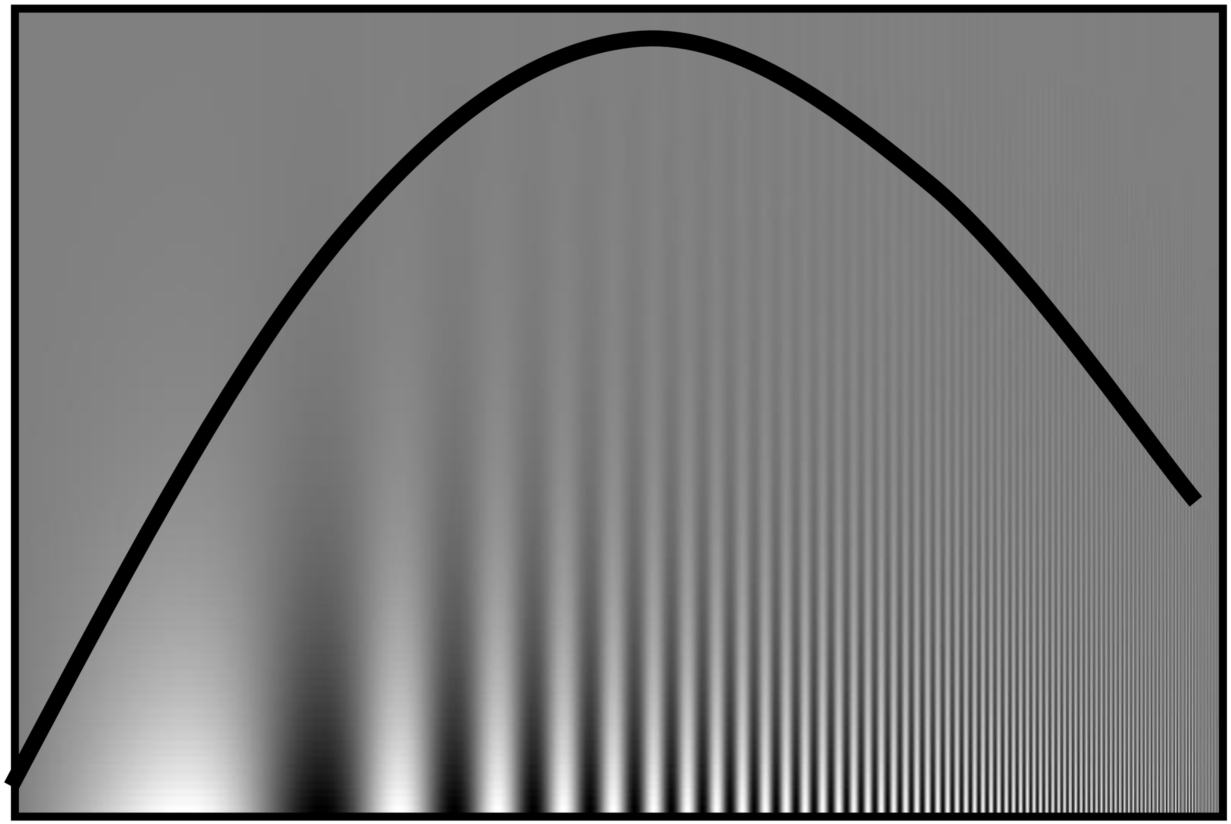
Clues from Human Perception

- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid-high frequencies dominate perception
- When we see an image from far away, we are effectively subsampling it



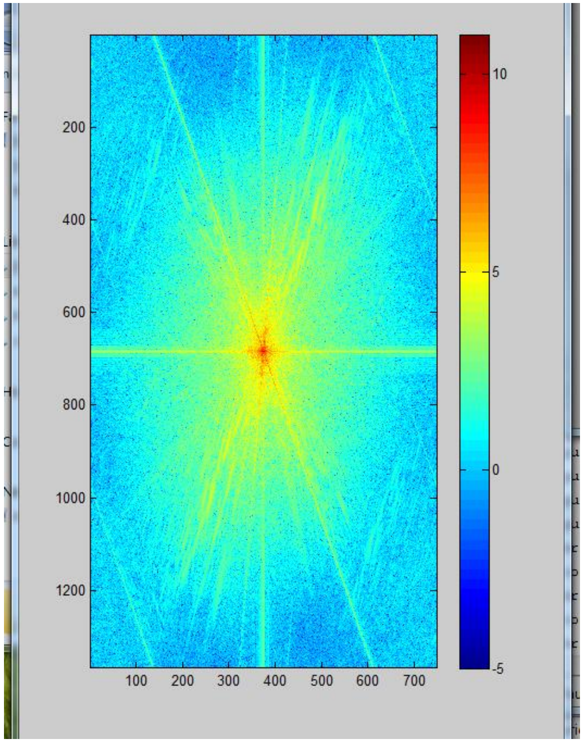
Early Visual Processing: Multi-scale edge and blob filters

Campbell-Robson contrast sensitivity curve

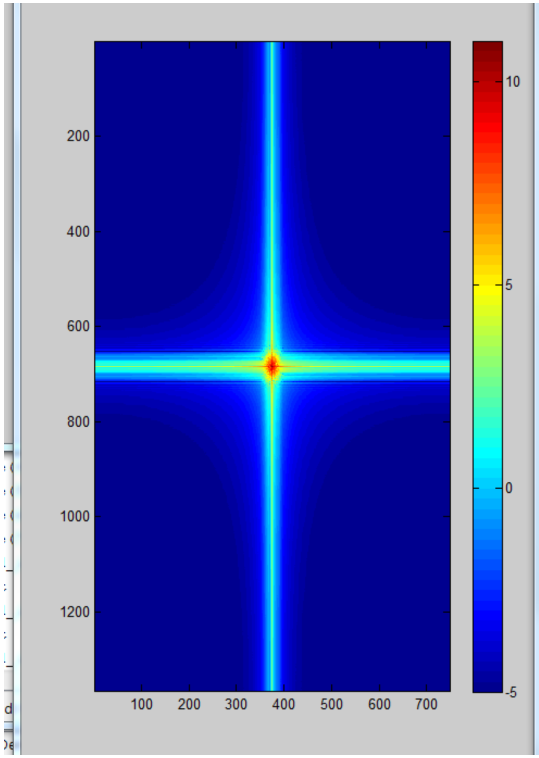


Hybrid Image in FFT

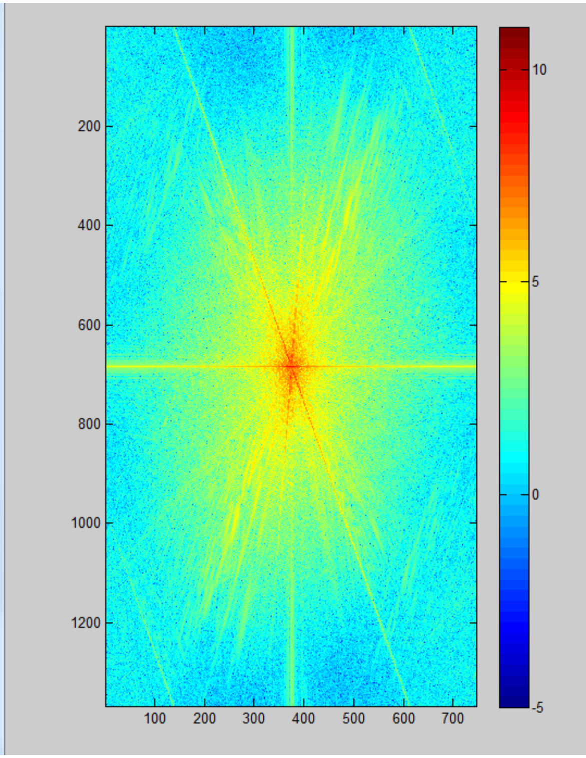
Hybrid Image



Low-passed Image

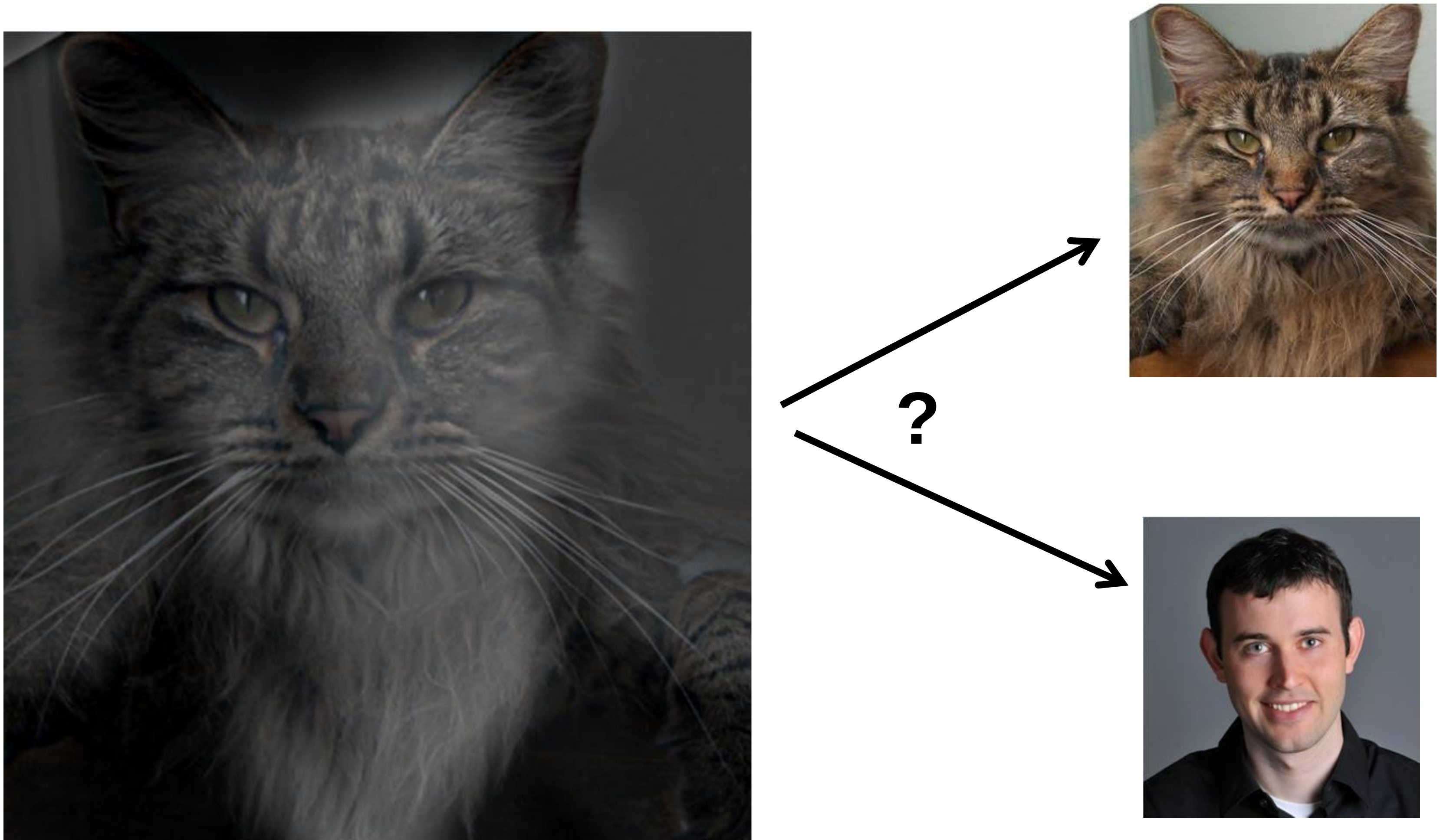


High-passed Image

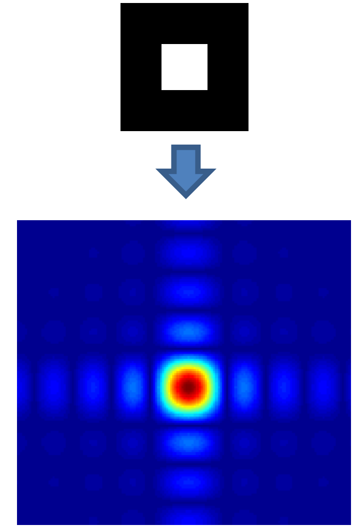


Perception

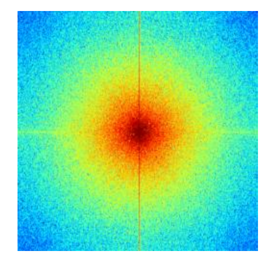
Why do we get different, distance-dependent interpretations of hybrid images?



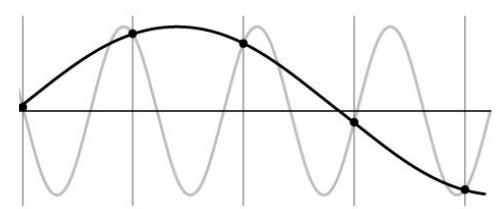
Things to Remember



- Sometimes it makes sense to think of images and filtering in the frequency domain
  - Fourier analysis
- Can be faster to filter using FFT for large images (N logN vs. N^2 for auto-correlation)
- Images are mostly smooth
  - Basis for compression



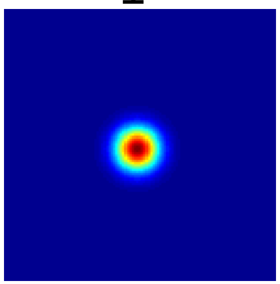
- Remember to low-pass (apply anti-aliasing filter), before down-sampling



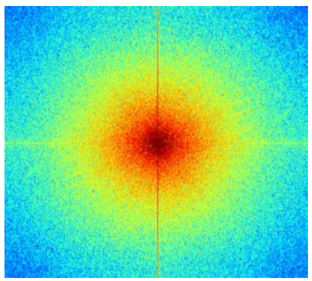
Practice Question
(check at home, discuss next lecture)

1. Match the <u>spatial domain image</u> to the <u>Fourier magnitude image</u>

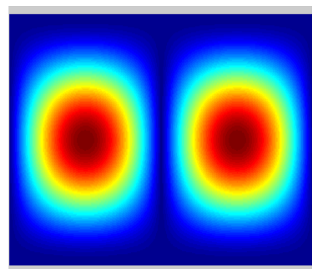
1



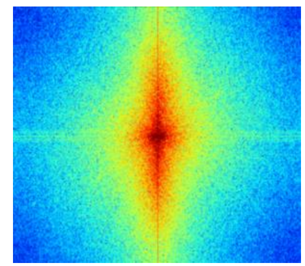
2



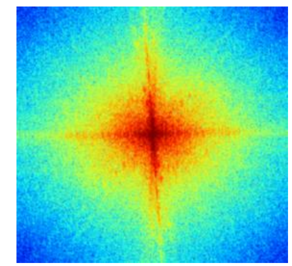
3



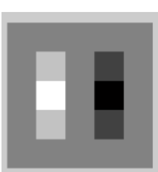
4



5

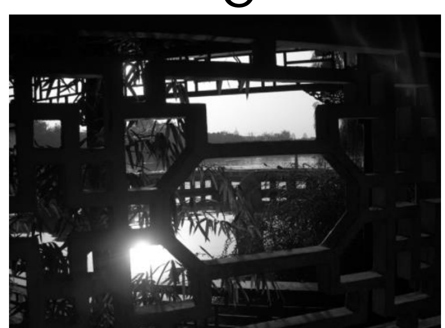


A

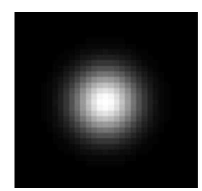


B

C



D



E

Additional Slide Credits

- James Hays
- Derek Hoiem
- Steve Seitz
- Larry Zitnick
- Ali Farhadi