QNo 3 (a)

Weighted least square

$$\sum_{m=1}^{M} w_m \left(a_m^T \underline{x} - y_m\right)^2$$

$$\text{let } \left(a_m^T x - y_m\right)^2 = \gamma_m^2$$

$$\sum_{m=1}^{M} \omega_m \|\gamma_m\|_2^2$$

$$\gamma = \omega_1 \gamma_1^2 + \omega_2 \gamma_2^2 + \omega_M \gamma_m^2$$

$$\|\gamma\|_2^2 = \gamma^T \gamma = \begin{bmatrix} \gamma_1 & \gamma_2 \cdots & \gamma_m \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix}$$

let

$$W = \begin{bmatrix} \omega_1 & & 0 \\ & \omega_2 & \\ & & \ddots \\ 0 & & \omega_m \end{bmatrix}$$

Then

$$\gamma^T W \gamma = \underset{1 \times M}{\begin{bmatrix} \gamma_1 & \gamma_2 & \cdots \gamma_m \end{bmatrix}} \underset{M \times M}{\begin{bmatrix} \omega_1 & & 0 \\ & \omega_2 & \\ & & \ddots \\ 0 & & \omega_m \end{bmatrix}} \underset{M \times 1}{\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix}}$$

$$= \|W \gamma\|_2^2$$

or $= \|W(A\underline{x} - \underline{y})\|_2^2$

Q No 3 (b)

$$\min_{x \in \mathbb{R}^N} \|W(A\underline{x} - y)\|_2^2$$

$$= (Ax - y)^T W (Ax - y)$$

$$= (x^T A^T - y^T) W (Ax - y)$$

$$= x^T A^T W A x - x^T A^T W y - y^T W A x + y^T W y$$

Take derivative w.r.t vector $\underline{x}$ and set it $= 0$

$$\Rightarrow \frac{\partial}{\partial x}(x^T A^T W A x - x^T A^T W y - y^T W A x + y^T W y)$$

$$\Rightarrow 2A^T W A x - A^T W y - y^T W A + 0$$

$$\Rightarrow 2A^T W A x - A^T W y - A^T W y$$

$$\Rightarrow 2A^T W A x - 2A^T W y = 0$$

$$A^T W A x = A^T W y$$

$$\boxed{x = (A^T W A)^{-1} A^T W y}$$