

## QNo 4

Solution:

$N$  different opportunities to invest in " $n$ " asset that is represented by " $x_n$ "

$x_n \rightarrow$  fractions of savings.

$P_n \rightarrow$  estimated data of asset " $n$ " historical data

$P_n = 1.12 \approx$  assets " $n$ " has historical return of 12%

$\rightarrow$  Covariance matrix  $R$  for the assets given.

The variance of overall return is given by

$$\sum_{n=1}^N \sum_{m=1}^N R_{m,n} x_m x_n$$

expected return of at least 8%

we know the portfolio variance:

$$V_p(R_{m,n}, x_m, x_n) = \frac{\sigma^2 R_{m,n}}{\sum_{i,j=1}^N (x_n^{-1})_{ij}}$$

Here in the data we have Covariance matrix inverse correlation matrix is  $x_n^{-1}$

the correlations of returns, the Pearson's formula is

$$x_{ij} := (R_i, R_j) - (R_i)(R_j) / \sigma_i \sigma_j$$

To bring that problem to the optimization problem which is in general as:

$$\min_x \|y - Ax\|_2^2$$

we have  $\sum_{n=1}^N \sum_{m=1}^N R_{m,n} x_m, x_n$

$$v_j = \sum_{n=1}^N \sum_{m=1}^N R_{m,n} x_m, x_n$$

$$z_j = f_j \left( \sum_{n=1}^N \sum_{m=1}^N R_{m,n} x_m, x_n \right)$$

$$y_s = f_s \left( \sum_{j=1}^N R_{m,n} f_j(v_j) x_m, x_n \right)$$

$$F_s(x_1, \dots, x_n)$$

So we have

$$\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} F_1(x_1, \dots, x_n) \\ F_m(x_1, \dots, x_n) \end{pmatrix}$$

$$\text{minimize } \frac{1}{2} \sum_{j=1}^m (y - \hat{y})^2$$