# Question: - I, we must also have Prove that if A and B are



Prove that if $\boldsymbol{A}$ and $\boldsymbol{B}$ are square $N \times N$ matrices, then if $\boldsymbol{AB} = \mathbf{I}$, we must also have $\boldsymbol{BA} = \mathbf{I}$. Some hints to help you get started:

- An equivalent statement to $\boldsymbol{BA} = \mathbf{I}$ is that, for any $\boldsymbol{x} \in \mathbb{R}^N$, $\boldsymbol{BAx} = \boldsymbol{x}$.
- Think about what can you say about $\mathcal{R}(\boldsymbol{AB})$.
- Think about what you can say about the relationship between $\mathcal{R}(\boldsymbol{AB})$ and $\mathcal{R}(\boldsymbol{B})$.
- Recall that for any $\boldsymbol{x} \in \mathcal{R}(\boldsymbol{B})$, we can write $\boldsymbol{x}$ as a linear combination of the columns of $\boldsymbol{B}$, i.e., $\boldsymbol{x} = \boldsymbol{Ba}$ for some vector $\boldsymbol{a} \in \mathbb{R}^N$.

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## Expert Answer

Anonymous
answered this

We are given that A and B are two N×N square matrix which satishfies $AB = I$. **We have to show that** $BA = I$.

Now, the statement $BA = I$ **....(1)** is equivalent to a statement that is for any $x \in \mathbb{R}^N$, $BAx = x$ ....(2). We can prove this equivalent relation very easily.

If (1) holds then $BA = I => BAx = I.x => BAx = x$ for all $x \in \mathbb{R}^N$.

If(2) holds i e. $BAx = x$ for all $x \in \mathbb{R}^N$ . Now, the identity matrix,

$$I = \begin{bmatrix} 1 & 0 & .. & .. & 0 \\ 0 & 1 & .. & .. & 0 \\ : & : & : & : & : \\ 0 & .. & .. & 1 & 0 \\ 0 & .. & .. & 0 & 1 \end{bmatrix} = [e_1 \ \ e_2 \ \ .. \ \ .. \ \ e_N]$$

where, $e_i's$ are the column vectors whose i-th position is 1 and others are zero.

$$BA.I = BA.\left[e_1 \ \ e_2 \ \ ....... \ \ e_N\right]$$

$$=> BA = \left[BA.e_1 \ \ BAe_2 \ \ ....... \ \ BAe_N\right]$$

$$=> BA = \left[e_1 \ \ e_2 \ \ ....... \ \ e_N\right] = I$$

So, (1) and (2) are equivalent .

Now , Range space of a matrix M defined by , $R(M) = \{Mx : x \in \mathbb{R}^N\}$ .

Here,$R(AB) = \{ABx : \ x \in \mathbb{R}^N\} = \{Ix : \ x \in \mathbb{R}^N\} = \mathbb{R}^N$ i.e. dim R(AB)=n .

We know that , $\dim R(AB) \leq \dim R(B) => n \leq \dim R(B)$ .since, R(B) is a subspace of $\mathbb{R}^N$ so, dim R(B)=n and $R(B) = \mathbb{R}^N$ .

So, now, for any $x \in \mathbb{R}^N => x \in R(B) => x = Ba$ for some $a \in \mathbb{R}^N$ .

$BAx = BA.Ba = B(AB)a = B.I.a = Ba = x$ for all $x \in \mathbb{R}^N$ so, **We can conclude that BA = I (Since (1) and (2) are equivalent )**

**Alternating prove :-** It is given that AB = I . Taking determinant both side we get,

$$\det(AB) = \det(I)$$

$=> \det A. \det B = 1$ [ since determinant of an identity matrix is 1].

Since, $\det A. \det B = 1$ so, we can say that $\det \neq 0, \det B \neq 0$ that means both A and B matrices are invertible i.e. $A^{-1} \ and \ B^{-1}$ exist and $A.A^{-1} = I$ , $BB^{-1} = I$ .

Now, $AB = I => B(AB)B^{-1} = B.I.B^{-1} => BA.(BB^{-1}) = BB^{-1} => BA.I = I => BA = I$

(**Proved)**



## Questions viewed by other students

Using the properties of an inner product, prove that the Pythagorean Theorem holds for any “induced norm”. Specifically, show that if ||2|| = V (x,x), then for any x, y satisfying (x, y) = 0, || X + y||2 = || 20 ||2 + || y ||2. = 2 C

See answer

Let D be a diagonal R x R matrix whose diagonal elements are positive. Show that the maximizer @ to maximize ||DB||2 subject to || 8 ||2 = 1 BERR has a 1 in the entry corresponding to the largest diagonal element of D, and is o elsewhere. =

See answer