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Question: - I, we must also have Prove that if A and B are

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Prove that if A and B are square $N \times N$ matrices, then if AB = I, we must also have

- An equivalent statement to BA = I is that, for any $x \in \mathbb{R}^N$, BAx = x.
- Think about what can you say about $\mathcal{R}(AB)$.

BA = I. Some hints to help you get started:

- Think about what you can say about the relationship between $\mathcal{R}(AB)$ and $\mathcal{R}(B)$.
- Recall that for any $x \in \mathcal{R}(B)$, we can write x as a linear combination of the columns of B, i.e., x = Ba for some vector $a \in \mathbb{R}^N$.

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Expert Answer



We are given that A and B are two N×N square matrix which satishfies AB=I . We have to show that BA=I .

Now, the statement BA=I(1) is equivalent to a statement that is for any $x\in\mathbb{R}^N$, BAx=x(2). We can prove this equivalent relation very easily.

If (1) holds then BA = I => BAx = I.x => BAx = x for all $x \in \mathbb{R}^N$.

If(2) holds i.e. BAx=x for all $x\in\mathbb{R}^N$. Now, the identity matrix,

$$I = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 1 & 0 \\ 0 & \dots & \dots & 0 & 1 \end{bmatrix} = \begin{bmatrix} e_1 & e_2 & \dots & \dots & e_N \end{bmatrix}$$

where, $e_i^\prime s$ are the column vectors whose i-th position is 1 and

others are zero.

$$BA.I = BA. \begin{bmatrix} e_1 & e_2 & \dots & e_N \end{bmatrix}$$

$$=> BA = \begin{bmatrix} BA.e_1 & BAe_2 & \dots & BAe_N \end{bmatrix}$$

$$=>BA=\begin{bmatrix}e_1&e_2&.....&e_N\end{bmatrix}=I$$

So, (1) and (2) are equivalent.

Now , Range space of a matrix M defined by , $R(M) = \{Mx : x \in \mathbb{R}^N\}$.

Here, $R(AB)=\{ABx:\ x\in\mathbb{R}^N\}=\{Ix:\ x\in\mathbb{R}^N\}=\mathbb{R}^N \ \text{i.e. dim R(AB)=n}$.

So, now, for any $x \in \mathbb{R}^N => x \in R(B) => x = Ba$ for some $a \in \mathbb{R}^N$.

BAx = BA.Ba = B(AB)a = B.I.a = Ba = x for all $x \in \mathbb{R}^N$ so, We can conclude that BA = I (Since (1) and (2) are equivalent)

Alternating prove :- It is given that AB = I. Taking determinant both side we get,

$$\det(AB) = \det(I)$$

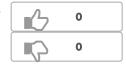
 $=> \det A. \det B = 1$ [since determinant of an identity matrix is 1].

Since, $\det A$. $\det B=1$ so, we can say that $\det \neq 0$, $\det B\neq 0$ that means both A and B matrices are invertible i.e. A^{-1} and B^{-1} exist and $A.A^{-1}=I$, $BB^{-1}=I$.

(Proved)

0 Comments

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Questions viewed by other students

Using the properties of an inner product, prove that the Pythagorean Theorem holds for any "induced norm". Specifically, show that if ||2|| = V(x,x), then for any x, y satisfying (x, y) = 0, $||X + y||^2 = ||20||^2 + ||y||^2 = 2 C$

See answer

Let D be a diagonal R x R matrix whose diagonal elements are positive. Show that the maximizer @ to maximize ||DB||2 subject to ||8||2 = 1 BERR has a 1 in the entry corresponding to the largest diagonal element of D, and is o elsewhere. =

See answer

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