# Q No 8

(a) $f(x) = ax^2 + bx + c$

$$\frac{d f(x)}{dx} = \frac{d}{dx}(ax^2+bx+c)$$

$$\frac{df(x)}{dx} = 2ax + b$$

$$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}(2ax+b)$$

$$\frac{d^2 f(x)}{dx^2} = 2a$$

(b) $f(x) = -\cos(2\pi x^2) + x^2$

$$\frac{df(x)}{dx} = \frac{d}{dx}\left(-\cos(2\pi x^2) + x^2\right)$$

$$\frac{df(x)}{dx} = \sin(2\pi x^2)(4\pi x) + 2x$$

$$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}\left(\sin(2\pi x^2)(4\pi x) + 2x\right)$$

$$\frac{d^2 f(x)}{dx^2} = 4\pi \sin(2\pi x^2) + 16\pi^2 x^2 \cos(2\pi x^2) + 2$$

(c) $f(x) = \sum_{m=1}^{M} \log(1+e^{-a_m x})$ $\qquad a_1 \ldots a_m$ are constants.

$$\frac{d f(x)}{dx} = \frac{d}{dx}\left(\sum_{m=1}^{M} \log(1+e^{-a_m x})\right)$$

$$\frac{df(x)}{dx} = \sum_{m=1}^{M} \frac{-a_m e^{-a_m x}}{(1+e^{-a_m x})\ln 10}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u \ln a}$$

here $a = 10$

$$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}\left[\sum_{m=1}^{M} \frac{-a_m e^{-a_m x}}{(1+e^{-a_m x})\ln 10}\right]$$

Apply Quotient derivative rule

$$= \sum_{m=1}^{M} \frac{\ln 10(1+e^{-a_m x})\, a_m^2 e^{-a_m x} + a_m e^{-a_m x}(-\ln 10 . a_m e^{-a_m x})}{\left((1+e^{-a_m x})\ln 10\right)^2}$$

$$= \sum_{m=1}^{M} \frac{\ln 10\, a_m^2 e^{-a_m x} + \cancel{\ln 10\, a_m^2 e^{-2a_m x}} - \cancel{\ln 10\, a_m^2 e^{-2a_m x}}}{(1+e^{-a_m x})^2 (\ln 10)^2}$$

$$= \sum_{m=1}^{M} \frac{\cancel{(\ln 10)}\, a_m^2 e^{-a_m x}}{(1+e^{-a_m x})^2 (\ln 10)^{\cancel{2}}}$$

$$\frac{d^2 f(x)}{dx^2} = \sum_{m=1}^{M} \frac{a_m^2 e^{-a_m x}}{\ln 10 \,(1+e^{-a_m x})^2}$$