Q No 4 part (a)

$$p(x) = a_{N-1} x^{N-1} + \dots + a_1 x + a_0$$

Expanding this for $p(x) = y_m$ where $m = 1, 2 \dots M$

$$y_0 = a_{N-1} x_0^{N-1} + \dots + a_1 x_0 + a_0$$

$$y_1 = a_{N-1} x_1^{N-1} + \dots + a_1 x_1 + a_0$$

$$y_2 = a_{N-1} x_2^{N-1} + \dots + a_1 x_2 + a_0$$

$$\vdots$$

$$y_m = a_{N-1} x_m^{N-1} + \dots + a_1 x_m + a_0$$

These equation can be written in matrix as

$$\underset{M \times 1}{\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_m \end{bmatrix}} = \underset{M \times N}{\begin{bmatrix} x_0^{N-1} & x_0^{N-2} & \dots & x_0 & 1 \\ x_1^{N-1} & x_1^{N-2} & \dots & x_1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_m^{N-1} & x_m^{N-2} & \dots & x_m & 1 \end{bmatrix}} \underset{N \times 1}{\begin{bmatrix} a_{N-1} \\ a_{N-2} \\ \vdots \\ a_1 \\ a_0 \end{bmatrix}}$$

this is $y = A x$

in order to solve this

$$\boxed{x = (A^T A)^{-1} A^T y}$$

Q No 4 part (d)

This is same least square optimization problem

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} x_0^{N-1} & x_0^{N-2} & \cdots & x_0 & 1 \\ x_1^{N-1} & x_1^{N-2} & \cdots & x_1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_m^{N-1} & x_m^{N-2} & \cdots & x_m & 1 \end{bmatrix} \begin{bmatrix} a_{N-1} \\ a_{N-2} \\ \vdots \\ a_i \\ a_0 \end{bmatrix}$$

$$\underset{M\times 1}{y} = \underset{M\times N}{A} \; \underset{N\times 1}{x}$$

for $N<M$ A Becomes tall matrix

if we take N close to M or $N=M$ we will get perfect interpolation. This is shown for different values of N and fixed value of M in programming assignment Q4.