

HEAP

What is a Heap?

A **Heap** is a special tree-based data structure that satisfies the heap property. Heaps can be categorized mainly into two types:

1. **Min-Heap**: The value of each node is greater than or equal to the value of its parent. The smallest value is at the root.
2. **Max-Heap**: The value of each node is less than or equal to the value of its parent. The largest value is at the root.

Structure of a Heap

- **Binary Tree**: Heaps are typically implemented as binary trees, where each node has at most two children.
- **Complete Tree**: Heaps are complete binary trees, meaning all levels are fully filled except possibly for the last level, which is filled from left to right.

Operations on a Heap

1. **Insertion**:

- Add the new element at the end of the tree (bottom-most, right-most position).
- Restore the heap property by "bubbling up" the new element, swapping it with its parent until the correct position is found.

2. **Deletion (typically delete the root)**:

- Replace the root with the last element in the tree.
- Restore the heap property by "bubbling down" the new root, swapping it with its smallest/largest child (depending on min-heap or max-heap) until the correct position is found.

3. **Peek**:

- Return the root element (the smallest element in a min-heap or the largest in a max-heap).

How Heaps are Used

- **Priority Queue**: Heaps are often used to implement priority queues where the highest (or lowest) priority element is accessed first.
- **Heap Sort**: A sorting algorithm that uses a heap to sort elements in $O(n \log n)$ time.

Time Complexity

- **Insertion**: $O(\log n)$
- **Deletion**: $O(\log n)$
- **Peek**: $O(1)$

Example of Operations

1. **Insert into Min-Heap**:

- Insert 3, 1, 6, 5, 2, 4 into an empty min-heap.

...

```
    /\n  2  3\n  /\ /\n5 6 4\n```\n
```

2. ****Delete from Min-Heap****:

- Delete the root (1).

```
```\n\n    2\n    /\n  4  3\n    /\n  5  6\n  ```\n
```

## ### Common Questions and Answers

### 1. **\*\*What is a heap?\*\***

- A heap is a tree-based data structure that maintains a specific order property: min-heap or max-heap.

### 2. **\*\*What are the types of heaps?\*\***

- Min-Heap: Smallest value at the root.
- Max-Heap: Largest value at the root.

### 3. **\*\*How do you insert an element into a heap?\*\***

- Add the element at the end and bubble it up to maintain the heap property.

4. **\*\*How do you delete the root of a heap?\*\***

- Replace the root with the last element and bubble it down to maintain the heap property.

5. **\*\*What is a complete binary tree?\*\***

- A binary tree where all levels are fully filled except possibly the last, which is filled from left to right.

6. **\*\*What is the time complexity of heap operations?\*\***

- Insertion:  $O(\log n)$
- Deletion:  $O(\log n)$
- Peek:  $O(1)$

7. **\*\*What is a priority queue?\*\***

- A data structure where elements are processed based on priority, often implemented using a heap.

8. **\*\*How is heap sort performed?\*\***

- Build a heap from the unsorted array, then repeatedly extract the root (maximum or minimum) and rebuild the heap.

9. **\*\*Why are heaps useful?\*\***

- Efficient for priority-based retrieval and sorting tasks.

### Implementation of a Heap in Golang

Here is a simple implementation of a min-heap in Golang:

```
``go
package main

import (
 "fmt"
)

// Heap represents a min-heap
type Heap struct {
 elements []int
}

// NewHeap creates a new heap
func NewHeap() *Heap {
 return &Heap{elements: []int{}}
}

// Insert adds a new element to the heap
func (h *Heap) Insert(val int) {
 h.elements = append(h.elements, val)
 h.bubbleUp(len(h.elements) - 1)
}

// bubbleUp restores the heap property by moving the element at index up
func (h *Heap) bubbleUp(index int) {
 parentIndex := (index - 1) / 2
```

```
 if index > 0 && h.elements[index] < h.elements[parentIndex] {
 h.elements[index], h.elements[parentIndex] =
h.elements[parentIndex], h.elements[index]
 h.bubbleUp(parentIndex)
 }
}
```

**// ExtractMin removes and returns the smallest element from the heap**

```
func (h *Heap) ExtractMin() int {
 if len(h.elements) == 0 {
 return -1 // or some error value
 }
 min := h.elements[0]
 h.elements[0] = h.elements[len(h.elements) - 1]
 h.elements = h.elements[:len(h.elements) - 1]
 h.bubbleDown(0)
 return min
}
```

**// bubbleDown restores the heap property by moving the element at index down**

```
func (h *Heap) bubbleDown(index int) {
 smallest := index
 leftChild := 2 * index + 1
 rightChild := 2 * index + 2
 if leftChild < len(h.elements) && h.elements[leftChild] <
h.elements[smallest] {
 smallest = leftChild
 }
 if rightChild < len(h.elements) && h.elements[rightChild] <
h.elements[smallest] {
 smallest = rightChild
 }
 if smallest != index {
 h.elements[index], h.elements[smallest] = h.elements[smallest], h.elements[index]
 h.bubbleDown(smallest)
 }
}
```

```

 }
 if rightChild < len(h.elements) && h.elements[rightChild] <
h.elements[smallest] {
 smallest = rightChild
 }
 if smallest != index {
 h.elements[index], h.elements[smallest] = h.elements[smallest],
h.elements[index]
 h.bubbleDown(smallest)
 }
}

```

**// Peek returns the smallest element without removing it**

```

func (h *Heap) Peek() int {
 if len(h.elements) == 0 {
 return -1 // or some error value
 }
 return h.elements[0]
}

```

```

func main() {
 heap := NewHeap()
 heap.Insert(3)
 heap.Insert(1)
 heap.Insert(6)
 heap.Insert(5)
 heap.Insert(2)
 heap.Insert(4)
}

```

```
 fmt.Println(heap.ExtractMin()) // Output: 1
 fmt.Println(heap.Peek()) // Output: 2
 fmt.Println(heap.ExtractMin()) // Output: 2
 fmt.Println(heap.Peek()) // Output: 3
}
'''
```

### ### Summary

A **heap** is a special tree-based data structure that helps in efficient priority-based operations. Min-heaps and max-heaps are its two main types, which maintain specific order properties to facilitate quick insertion, deletion, and retrieval of elements. They are widely used in priority queues and sorting algorithms like heap sort due to their efficient time complexities.

Sure! Let's add more questions and answers related to heapify operations and applications of heaps.

### ### Additional Questions and Answers

#### 1. **What is Heapify?**

- Heapify is the process of converting a binary tree into a heap. There are two main types of heapify operations:

- **Up-Heapify (Bubble Up)**: Ensures the heap property is maintained when a new element is added at the end of the heap.

- **Down-Heapify (Bubble Down)**: Ensures the heap property is maintained when the root element is removed and replaced with the last element.



## 2. **\*\*How does Heapify work?\*\***

- **\*\*Up-Heapify (Bubble Up)\*\***:
  - Insert the new element at the end of the heap.
  - Compare the new element with its parent; if the heap property is violated, swap them.
  - Repeat the process until the heap property is restored or the root is reached.
- **\*\*Down-Heapify (Bubble Down)\*\***:
  - Replace the root with the last element.
  - Compare the new root with its children; if the heap property is violated, swap it with the smaller (for min-heap) or larger (for max-heap) child.
  - Repeat the process until the heap property is restored or a leaf is reached.

## 3. **\*\*What are some applications of Heaps?\*\***

- **\*\*Priority Queues\*\***: Heaps are used to implement priority queues where the highest (or lowest) priority element is accessed first.
- **\*\*Heap Sort\*\***: A sorting algorithm that uses a heap to sort elements efficiently.
- **\*\*Graph Algorithms\*\***: Used in algorithms like Dijkstra's shortest path and Prim's minimum spanning tree.
- **\*\*Scheduling\*\***: Used in CPU scheduling to manage tasks and processes based on priority.
- **\*\*Median Maintenance\*\***: Heaps can be used to keep track of the median of a dynamically changing dataset.
- **\*\*Merging K Sorted Lists\*\***: Heaps can efficiently merge multiple sorted lists.

## 4. **\*\*How does Heap Sort work?\*\***

- **\*\*Build a Heap\*\***: Convert the array into a heap.
- **\*\*Sort\*\***: Repeatedly extract the root (smallest or largest element) and place it at the end of the array, then heapify the remaining elements.

- The time complexity of Heap Sort is  $O(n \log n)$ .

5. **What is the difference between a Heap and a Binary Search Tree (BST)?**

- **Heap:**

- A complete binary tree.
- Maintains the heap property (min-heap or max-heap).
- No order among siblings.
- Efficient for priority-based operations.

- **Binary Search Tree (BST):**

- Not necessarily a complete binary tree.
- Maintains the BST property (left child < parent < right child).
- Ordered structure.
- Efficient for search operations.

6. **What are the advantages and disadvantages of using Heaps?**

- **Advantages:**

- Efficient for priority-based retrieval and dynamic datasets.
- Good for implementing priority queues.
- Useful in graph algorithms and sorting.

- **Disadvantages:**

- More memory overhead compared to arrays.
- Not as fast as hash tables for exact lookups.

7. **How do you convert an array into a heap?**

- Start from the last non-leaf node and apply down-heapify.
- Repeat the process for each node up to the root.
- This process takes  $O(n)$  time.

8. **\*\*What is a Fibonacci Heap?\*\***

- A more advanced heap data structure that supports faster amortized time complexity for operations like decrease-key and delete.
- Used in advanced graph algorithms like Dijkstra's shortest path and Prim's minimum spanning tree.

9. **\*\*How do you handle duplicates in a heap?\*\***

- Heaps can store duplicate values. The heap property is maintained by considering the positions of duplicates, and they are handled like any other value during heapify operations.

### Example: Heapify Operation in Golang

Here is an example of the heapify operation in Golang:

```
``go
package main

import (
 "fmt"
)

// Heap represents a min-heap
type Heap struct {
 elements []int
}

// NewHeap creates a new heap
```

```

func NewHeap() *Heap {
 return &Heap{elements: []int{}}
}

// Insert adds a new element to the heap
func (h *Heap) Insert(val int) {
 h.elements = append(h.elements, val)
 h.upHeapify(len(h.elements) - 1)
}

// upHeapify restores the heap property by moving the element at index up
func (h *Heap) upHeapify(index int) {
 parentIndex := (index - 1) / 2
 if index > 0 && h.elements[index] < h.elements[parentIndex] {
 h.elements[index], h.elements[parentIndex] = h.elements[parentIndex],
h.elements[index]
 h.upHeapify(parentIndex)
 }
}

// ExtractMin removes and returns the smallest element from the heap
func (h *Heap) ExtractMin() int {
 if len(h.elements) == 0 {
 return -1 // or some error value
 }
 min := h.elements[0]
 h.elements[0] = h.elements[len(h.elements) - 1]
 h.elements = h.elements[:len(h.elements) - 1]
}

```

```

 h.downHeapify(0)
 return min
}

// downHeapify restores the heap property by moving the element at index down
func (h *Heap) downHeapify(index int) {
 smallest := index
 leftChild := 2 * index + 1
 rightChild := 2 * index + 2
 if leftChild < len(h.elements) && h.elements[leftChild] <
h.elements[smallest] {
 smallest = leftChild
 }
 if rightChild < len(h.elements) && h.elements[rightChild] <
h.elements[smallest] {
 smallest = rightChild
 }
 if smallest != index {
 h.elements[index], h.elements[smallest] = h.elements[smallest],
h.elements[index]
 h.downHeapify(smallest)
 }
}

// Peek returns the smallest element without removing it
func (h *Heap) Peek() int {
 if len(h.elements) == 0 {
 return -1 // or some error value
 }
}

```

```

 }
 return h.elements[0]
}

func main() {
 heap := NewHeap()
 heap.Insert(3)
 heap.Insert(1)
 heap.Insert(6)
 heap.Insert(5)
 heap.Insert(2)
 heap.Insert(4)

 fmt.Println(heap.ExtractMin()) // Output: 1
 fmt.Println(heap.Peek()) // Output: 2
 fmt.Println(heap.ExtractMin()) // Output: 2
 fmt.Println(heap.Peek()) // Output: 3
}
...

```

### ### Summary

A **heap** is a versatile tree-based data structure used in various applications such as priority queues, sorting, and graph algorithms. Understanding heap operations like insertion, deletion, and heapify, as well as their applications, is crucial for efficient algorithm design and implementation.