Function

Shahadat Hossain

October 13, 2016

> Mathematicians just love sigma notation for two reasons. First, it provides a convenient way to express a long or even infinite series. But even more important, it looks really cool and scary, which frightens nonmathematicians into revering mathematicians and paying them more money.
>
> — *Calculus II for Dummies*

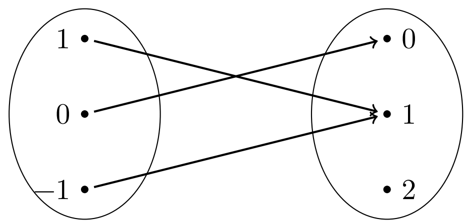
Acknowledgments

Thanks to Zack Chroman, Michael Diao, Steven Hao, Ryan Kim, Kevin Qian, Colin Tang, Michael Tang, Tyler Zhu, for helpful suggestions and comments.

Contents

§1 Definition

Function is a special case of relation, from a non empty set A to a non empty set B, that associates each member of A to a unique member of B. Symbolically, we write $\boldsymbol{f} : A \to B$. We read it as "f is a function from A to B". Set A is called domain of f and set B is called co-domain of f. For example, let $A = \{1, 0, -1\}$ and $B = \{0, 1, 2\}$. Then $A \times B = \{(1,0), (1,1), (1,2), (0,0), (0,1), (0,2), (-1,0), (-1,1), (-1,2)\}$ Now, "$f : A \to B$ defined by $f(x) = x^2$" is the function such that $f = \{(1,1), (0,0), (-1,1)\}$. f can also be show diagramatically by following picture:



Every function say $f : A \to B$ satisfies the following conditions:
(a) $f \subseteq A \times B$, (b) $\forall a \in A \Rightarrow (a, f(a)) \in f$ and (c) $(a,b) \in f \,\&\, (a,c) \in f \Rightarrow b = c$

Example 1.1

Which of the following correspondences can be called a function ?

$$\begin{array}{ll}
(a) f(x) = x^3; & \{-1, 0, 1\} \to \{0, 1, 2, 3\} \\
(b) f(x) = \pm\sqrt{x}; & \{0, 1, 4\} \to \{-2, -1, 0, 1, 2\} \\
(c) f(x) = \sqrt{x}; & \{0, 1, 4\} \to \{-2, -1, 0, 1, 2\} \\
(d) f(x) = -\sqrt{x}; & \{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}
\end{array}$$

Solution. $f(x)$ in (C) & (D) are functions as definition of function is satisfied. while in case of (A) the given relation is not a function, as $f(-1) \notin$ codomain. Hence definition of function is not satisfied. While in case of (B), the given relation is not a function, as $f(1) = \pm 1$ and $f(4) = \pm 2$ i.e. element 1 as well as 4 in domain is related with two elements of codomain. Hence definition of function is not satisfied □