ফাংশন ও লেখচিত্র

1. $f(x) = \dfrac{-1}{|1-x|}$ ফাংশনের রেঞ্জ (The range of the function $f(x) = \dfrac{-1}{|1-x|}$ is) DU:2018-19

   A. $\mathbb{R} - 1$ B. $\mathbb{R} - 0$ C. $\mathbb{R} - 0, 1$ D. $(-\infty, 0)$

2. $f(x) = \dfrac{1}{|\sqrt{x}|}$, $g(x) = x^2$ হলে এর ডোমেন (The domain of $f(x) = \dfrac{1}{\sqrt{|x|}}$ is) DU:2017-18

   A. $[0, +\infty)$ B. $(0, +\infty)$ C. $(-\infty, +\infty)$ D. $(-\infty, 0) \cup (0, +\infty)$

3. $f(x) = \sin x$, $g(x) = x^2$ হলে $f(g(\dfrac{\sqrt{\pi}}{2}))$ এর মান DU:2016-17, 09-10

   A. $\dfrac{\sqrt{2}}{2}$ B. $\dfrac{\sqrt{3}}{2}$ C. $\dfrac{1}{2}$ D. 1

4. $f : \mathbb{R} \to \mathbb{R}$ কে $f(x) = e^{x-3}$ দ্বারা সংজ্ঞায়িত করা হলে $f^{-1}(e)$ এর মান DU:2015-16

   A. 4 B. 3 C. 2 D. 0

5. $y = \dfrac{1}{\sqrt{4-x}}$ ফাংশনের ডোমেন ও রেঞ্জ DU:2015-16

   A. $-\infty < x \leq 4;\ 0 \leq y < \infty$ B. $-\infty < x < 4;\ 0 < y < \infty$ C. $-\infty < x < 4;\ 0 \leq y < \infty$
   D. $-\infty < x \leq 4;\ 0 < y < \infty$

6. $y = \dfrac{1}{\sqrt{4-x^2}}$ বাস্তব ফাংশনটির ডোমেন ও রেঞ্জ DU:2014-15

   A. $x < 2;\ y > \dfrac{1}{2}$ B. $-2 < x < 2;\ y \geq \dfrac{1}{2}$ C. $-2 \leq x \leq 2;\ y < \dfrac{1}{2}$ D. $-x < -2\ \&\ x > 2;\ -2 < y < 2$

7. $f(x) = \sqrt{x^2 - 5x + 6}$ ফাংশনের ডোমেন ও রেঞ্জ DU:2013-14

   A. $x \leq 2,\ 3 \leq x$ and $y \geq 0$ B. $2 \leq x \leq 3$ and $y \geq 0$ C. $x \geq 3$ and $y \geq 0$ D. $x \leq 2,\ x \geq 3$ and $y \geq 0$

8. যদি $f(x) = (x-2)(1-x)$ হয় তবে $f(f(3))$এর মান DU:2013-14

   A. 12 B. 9 C. -12 D. 8

9. $f(x) = 4 - (x-3)^2$ ফাংশনের ডোমেন ও রেঞ্জ DU:2012-13

   A. $\mathbb{R}$, $\mathbb{R}$ B. $\mathbb{R}$, $x \leq 4$ C. $x \geq 4$, $\mathbb{R}$ D. $\mathbb{R}$, $x \geq 4$

10. যদি $f(x) = \dfrac{x-3}{2x+1}$ এবং $x \neq \dfrac{1}{2}$ হয় তবে $f^{-1}(-2)$ এর মান DU:2012-13,08-09

    A. $\dfrac{1}{5}$ B. $\dfrac{1}{2}$ C. 2 D. 5

$fog(x) = f((g(x)))$

11. $x^2 - 2x + 5$ এর নূন্যতম মান DU:2011-12

A. 1 B. 2 C. 3 D. 4

12. $f(x) = 3x^3 + 3$ এবং $g(x) = \sqrt[3]{\frac{x-2}{3}}$ হয় তবে $(f \circ g)(3)$ এর মান DU:2011-12

A. 1 B. 2 C. 3 D. 4

13. $5 - 3x - x^2$ এর সর্বোচ্চ মান DU:2010-11

A. 1 B. 2 C. 3 D. 4

14. $f(x) = \frac{5x+3}{4x-5}$ হলে $f^{-1}(x) = ?$ DU:10-11

A. $\frac{5x+3}{4x-5}$ B. $\frac{4x-5}{5x+5}$ C. $\frac{5x-3}{4x-5}$ D. $\frac{5x+3}{4x+5}$

15. $5 + 3x - x^2$ এর সর্বোচ্চ মান DU:2008-09

A. 3 B. $\frac{11}{4}$ C. $\frac{29}{4}$ D. $\frac{27}{4}$

16. $f(x) = x^2 + 4$ এবং $g(x) = 2x - 1$ হয় তবে $g(f(x))$ এর মান DU:2006-07,05-06

A. $2x^2 + 7$ B. $7x^2 + 2$ C. $x^2 + 2x - 1$ D. $x^2 + 2x + 3$

17. $f(x) = x^2 - 2|x|$ এবং $g(x) = x^2 + 1$ হয় তবে $g(f(-2))$ এর মান DU:2006-07

A. 0 B. 65 C. 5 D. 1

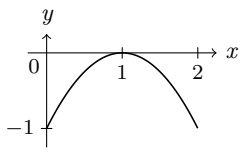
18. $f(x) = \frac{x}{1+x}$ হয় তবে $f\left(\frac{\frac{2}{3}}{\frac{3}{2}}\right)$ এর মান DU:2004-05

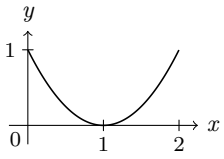
A. 0 B. 65 C. 5 D. 1

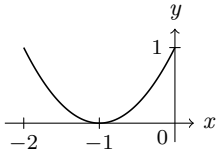
19. $x^2 - 3x + 5$ এর নূন্যতম মান DU:2003-04

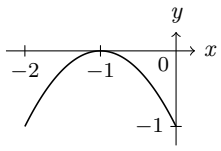
A. 3 B. 5 C. $\frac{15}{4}$ D. $\frac{11}{4}$

20. নিচের কোনটি $y = -(x-1)^2$ এর লেখচিত্র DU:2002-03

A. 

B. 

C. 

D. 

21. $f(x) = x^2 + 3$ এবং $g(x) = \sqrt{x}$ হয় তবে $f(g(x)) = ?$ DU:2001-02

A. $2x + 3,\ x < 0$ B. $x^2 + 1$ C. $3x + 9$ D. $x + 3,\ x \geq 0$

22. $f(x) = \log_{x+1}(2x + 1)$ হলে $f(x)$ এর ডোমেন কোনটি? SUST:2016-17

A. $(-\frac{1}{2}, 0) \cup (0, \infty)$ B. $x > -1$ C. $x \leq -\frac{1}{2}$ D. $(0, \infty)$ E. $(-\frac{1}{2}, -1) \cup (0, \infty)$

$f \circ \phi(x) = f((\phi(x)))$

23. a এর মান কত হলে $f(x) = \begin{cases} \frac{x^2}{x} & \text{when } x \neq 0 \\ a & \text{when } x = 0 \end{cases}$ ফাংশনটি অবিচ্ছিন্ন হবে? SUST:2014-15

A. -1 B. 1 C. 2 D. 4 E. 0

24. $f(x) = x^2 + 1$ এবং $g(x) = \sqrt{2-x}$ দুইটি বাস্তব ফাংশন হলে সংযোজিত ফাংশন gof ডোমেন কত? SUST:2013-14

A. $(-1, i)$ B. $[-1, 1]$ C. $(-\infty, \infty)$ D. $(-\infty, 2)$ E. $[2, \infty]$

25. $f(x-2) = x^2 - 2x + 1$ হলে $f(-4)$ এর মান কত? SUST:2012-13

A. 8 B. 10 C. 12 D. 16 E. 32

26. $f(x) = \sqrt{x} + \frac{1}{x} - 1$ ফাংশনের ডোমেন - SUST:2012-13

A. $(-\infty, 0)$ B. $(0, \infty)$ C. $(1, \infty)$ D. $(-\infty, \infty)$ E. $(-\infty, 0) \cup (0, \infty)$

27. ফাংশনের ডোমেন ও রেঞ্জ $\{a, b, c, d\}$ হলে কোনটি 'এক-এক' ফাংশন? SUST:2012-13

A. $f(a) = b, f(b) = c, f(c) = d, f(d) = b$
B. $f(a) = b, f(b) = c, f(c) = b, f(d) = a$
C. $f(a) = b, f(b) = c, f(c) = d$
D. $f(a) = b, f(b) = c, f(c) = d, f(d) = a$
E. $f(a) = b, f(b) = c, f(c) = d, f(c) = a$

28. $f(x) = 0$ সমীকরণে $f(2 + i\sqrt{3}) = 0$ হলে $f(2 - i\sqrt{3})$ এর মান কত? SUST:2009-10

29. $f(x) = \sqrt{\frac{1-x}{x}}$ ফাংশনের ডোমেন কত? SUST:2008-09

30. $f(x) = \frac{2x-1}{x-2}$ ফাংশনের ডোমেন ও রেঞ্জ এবং বিপরীত ফাংশন কোনটি ? SUST:2007-08

31. $\phi(x) = \frac{1}{1-x}$ হলে $\phi(\phi(\phi(x))) = ?$

32. যদি $f(x) = x + \frac{1}{x}$ হয় তবে $\{f(x)\}^2$ এর মান কত?

A. $3 + f(x^2)$ B. $2 - f(x^2)$ C. $2 + f(x^2)$ D. $4 + f(x^2)$

নিচের ফাংশনগুলোর ডোমেন ও রেঞ্জ বের কর -

33. $f(x) = 1 - 2^{-x}$

34. $f(x) = \log_{10} x$

35. $f(x) = ln\frac{a+x}{a-x}, a > 0$

36. $f(x) = x^2$

$fo\varphi(x) = f(\varphi(x))$

37. $f(x) = \dfrac{x}{|x|}$

38. $f(x) = |x - 10|$

39. $f(x) = e^{-\frac{|x|}{2}}$

40. $f(x) = x + |x|$

41. $f(x) = \sqrt{2x - 1} + \sqrt{3 - 2x}$

42. $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

43. $f(x) = \dfrac{x^2}{x^4 + 1}$

44. $f(x) = \sqrt{3}\sin x + \cos x + 4$

45. $f(x) = \sqrt{\left(\dfrac{1}{\sin x} - 1\right)}$

46. $f(x) = \dfrac{\sin^{-1}(x - 3)}{\sqrt{9 - x^2}}$

47. $f(x) = {}^{7-x}C_{x-2}$

48. $f(x) = \dfrac{x^2 + x + 1}{x^2 - 6x + 8}$

49. $f(x) = \sqrt{x^2 + x - 12}$

50. $f(x) = \dfrac{1}{\sqrt{x^2 - 4x}}$

51. $f(x) = \dfrac{x}{1 + x^2}$

52. $f(x) = \cos x, \quad (-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2})$

53. $f(x) = \sin^{-1}\left(\log_3\left(\dfrac{x}{3}\right)\right)$

$f o \phi(x) = f((\phi(x)))$

Exercise-01

Find the Domain of the following functions:

1. $f(x) = \sqrt{x^2 - 5}$

2. $f(x) = \sin^{-1}(2x - 1)$

3. $f(x) = \sqrt{\sin x} - \sqrt{16 - x^2}$
4. $f(x) = \dfrac{3}{\sqrt{4 - x^2}} \log(x^3 - x)$
5. $f(x) = x^{\cos^{-1} x}$
6. $f(x) = \dfrac{1}{\log(2 - x)} + \sqrt{x + 1}$
7. $f(x) = \sqrt{1 - x} - \sin^1 \dfrac{2x - 1}{3}$
8. $f(x) = \dfrac{x^2 + x + 1}{x^2 + x - 1}$
9. $f(x) = \ln(2x - x^2)$
10. $f(x) = \sec^{-1}(x^2 + 3x + 1)$
11. $f(x) = \dfrac{x^2 - 2x + 5}{x^2 + 2x + 5}$
12. $f(x) = \dfrac{1}{\sqrt{x^2 - x}}$
13. $f(x) = \cot^{-1}(2x - x^2)$
14. $f(x) = \ln \sin^{-1}\left(x^2 + x + \dfrac{3}{4}\right)$
15. $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$
16. $f(x) = \log_7 \log_5 \log_3 \log_2(2x^3 + 5x^2 - 14x)$
17. $f(x) = \ln(\sqrt{x^2 - 5x - 24} - x - 2)$
18. $f(x) = \sqrt{\dfrac{1 - 5^x}{7^{-x} - 7}}$
19. $f(x) = \log_{10} \sin(x - 3) + \sqrt{16 - x^2}$
20. $f(x) = \log_{100x}\left(\dfrac{2 \log_{10} x + 1}{-x}\right)$
21. $f(x) = \dfrac{1}{\sqrt{4x^2 - 1}} + \ln[x(x^2 - 1)]$
22. $f(x) = \sqrt{\log_{\frac{1}{2}} \dfrac{x}{x^2 - 1}}$

$\int \rho\phi(x) = f((\phi(x)))$

23. $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9 - x^2}}$

24. $f(x) = \sqrt{(x^2 - 3x - 10).\ln^2(x - 3)}$

25. $f(x) = \log_x \cos(2\pi x)$

26. $f(x) = \dfrac{\sqrt{\cos x - \dfrac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$

27. $f(x) = \sqrt{\log_{\frac{1}{3}}(\log_4([x]^2 - 5))}$

28. $f(x) = \dfrac{1}{[x]} + \log_{(2\{x\}-5)}(x^2 - 3x + 10) + \dfrac{1}{\sqrt{1 - |x|}}$

29. $f(x) = \log_x \sin x$

30. $f(x) = \log_2 \left[-\log \dfrac{1}{2} \left(1 + \dfrac{1}{\sin\left(\dfrac{x^\circ}{100}\right)} \right] + \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$

31. $f(x) = \dfrac{1}{[x]} + \log_{1-\{x\}}(x^2 - 3x + 10) + \dfrac{1}{\sqrt{2 - |x|}} + \dfrac{1}{\sqrt{\sec(\sin x)}}$

32. $f(x) = \sqrt{(5x - 6 - x^2)[\{\ln\{x\}\}]} + \sqrt{(7x - 5 - 2x^2)} + \left[\ln\left(\dfrac{7}{2} - x\right) \right]^{-1}$

Find the Domain and the Range of the following functions:

Excercise-02

1. $f(x) = \log_{\sqrt{5}} \left(\sqrt{2}(\sin x - \cos x) + 3 \right)$

2. $f(x) = \dfrac{2x}{1 + x^2}$

3. $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$

4. $f(x) = \dfrac{x}{1 + |x|}$

5. $f(x) = \sqrt{2 - x} + \sqrt{1 + x}$

6. $f(x) = \log_{(cosecx-1)}(2 - [\sin x] - [\sin x]^2)$

7. $f(x) = \dfrac{\sqrt{x + 4} - 3}{x - 5}$

8. $f(x) = \sqrt{\sin\left(\log_e\left(\frac{x^2+e}{x^2+1}\right)\right)} + \sqrt{\cos\left(\log_e\left(\frac{x^2+e}{x^2+1}\right)\right)}$

9. $f(x) = \sqrt{\log_{10}\left(\frac{1-2x}{x+3}\right)}$

Exercise-01 Answers:

1. $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$
2. $x \in [0, 1]$
3. $D_f = \{x : x \in [2n\pi, (2n+1)\pi], n \in \mathbb{Z}\} \cap [-4, 4] = [-4, -\pi] \cup [0, \pi]$
4. $D_f = \{(-1, 0) \cup (1, \infty)\} \cap (-2, 2) = (-1, 0) \cup (1, 2)$
5. $(0, 1]$
6. $[-1, 1) \cup (1, 2)$
7. $[-1, 1]$
8. $R_f = (-\infty, -\frac{3}{5}] \cup (1, \infty)$
9. $D_f = (0, 1], \quad R_f = (-\infty, 0]$
10. $D_f = \left[-\frac{5}{4}, -1\right] \cup [1, \infty), \quad R_f = \left[0, \frac{\pi}{2}\right) \cup \left[\sec^{-1}\left(-\frac{5}{4}\right), \pi\right]$
11. $R_f = \left[\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right]$
12. $D_f = \mathbb{R} - [0, 1], \quad R_f = (0, \infty)$
13. $D_f = \mathbb{R}, \quad R_f = \left[\frac{\pi}{4}, \pi\right]$
14. $D_f = \left[\frac{-2-\sqrt{5}}{4}, \frac{-2+\sqrt{5}}{4}\right], \quad R_f = \left[\ln\frac{\pi}{6}, \ln\frac{\pi}{2}\right]$
15. $\left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$
16. $\left(-4, -\frac{1}{2}\right) \cup (2, \infty)$

$f o \phi(x) = f((\phi(x)))$

17. $(-\infty, 3]$

18. $(-\infty, -1) \cup [0, \infty)$

19. $(3 - 2\pi < x < 3 - \pi) \cup (3 < x \leq 4)$

20. $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$

21. $\left(-1 < x < -\frac{1}{2}\right) \cup (x > 1)$

22. $\left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

23. $(-3, -1) \cup \{0\} \cup [1, 3)$

24. $\{4\} \cup [5, \infty)$

25. $\left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right) \cup \{x : x \in \mathbb{N}, x \geq 2\}$

26. $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

27. $[-3, -2) \cup [3, 4)$

28. ϕ

29. $2n\pi < x < (2n+1)\pi$ but $x \neq 1$ where n is non-negative integer.

30. $\{x | 1000 \leq x < 10000\}$

31. $(-2, -1) \cup (-1, 0) \cup (1, 2)$

32. $(1, 2) \cup \left(2, \frac{5}{2}\right)$

$fo\phi(x) = f((\phi(x)))$

Exercise-02 Answers:

1. $D_f = x \in \mathbb{R}, \quad R_f = [0, 2]$

2. $D_f = \mathbb{R}, \quad R_f = [-1, 1]$

3. $D_f = \{x | x \in \mathbb{R}; x \neq -3; x \neq 2\}, \quad R_f = \{f(x) | f(x) \in \mathbb{R},\ f(x) \neq \frac{1}{5};\ f(x) \neq 1\}$

4. $D_f = \mathbb{R}, \quad R_f = (-1, 1)$

5. $D_f = -1 \leq x \leq 2, \quad R_f = [\sqrt{3}, \sqrt{6}]$

6. $D_f = x \in \left(2n\pi, (2n+1)\pi\right) - \left\{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in \mathbb{Z}\right\}$ and
$R_f = \log_a 2;\ a \in (0, \infty) - \{1\} = \{-\infty, \infty\} - \{0\}$

7. $D_f = [-4, \infty) - \{5\}, \quad R_f = \left(0, \frac{1}{6}\right) \cup \left(\frac{1}{6}, \frac{1}{3}\right]$

$f o \phi(x) = f((\phi(x)))$