1 Definition

1.0.1 The complex number system

There is no real number x which satisfies the polynomial equation $x^2+1=0$. To permit solutions of this and similar equations, the set of complex numbers is introduced. We can consider a complex number as having the form $a+ib$ where a and b are real number and i, which is called the imaginary unit, has the property that $i^2=1$. It is denoted by z. Therefore, $z=a+ib$ where "a" is called as real part of z which is denoted by $(Re\, z)$ and "b" is called as imaginary part of z which is denoted by $(Im\, z)$.

Any complex number is :

(i) Purely real, if $b=0$ (ii) Purely imaginary, if $a=0$ (iii) Imaginary, if $b \neq 0$.

NOTE :

(a) The set $\mathbb{R}$ of real numbers is a proper subset of the Complex Numbers. Hence the complete number system is $\mathbb{N} \subset \mathbb{W} \subset \mathbb{I} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

(b) Zero is purely real as well as purely imaginary but not imaginary.

(c) $i=\sqrt{-1}$ is called the imaginary unit. Also $i^2=-1,\ i^3=-i,\ i^4=1\ etc.$

(d) $\sqrt{a}\sqrt{b}=\sqrt{ab}$ only if atleast one of a or b is non - negative.

(e) is $z=a+ib$, then $a-ib$ is called complex conjugate of z and written as $\bar{z}=a-ib$.

Self Practice Problems

1. Write the following as complex number $(i)\,\sqrt{-16}$ $(ii)\,\sqrt{x},(x>0)$ $(iii)\,b+\sqrt{-4ac},\ (a,c>0)$

Ans. $(i)\,0+i\sqrt{16}$ $(ii)\,\sqrt{x}+i0$ $(iii)\,-b+i\sqrt{4ac}$

2. Write the following as complex number:

$(i)\,\sqrt{x}\,(x<0)$ (ii) roots of $x^2-(2\cos\theta)x+1=0$

2. Algebraic Operations: Fundamental operations with complex numbers In performing operations with complex numbers we can proceed as in the algebra of real numbers, replacing i^2 by -1 when it occurs.

1. Addition $(a+bi)+(c+di)=a+bi+c+di=(a+c)+(b+d)i$
2. Subtraction $(a+bi)c+di)=a+bicdi=(ac)+(bd)i$
3. Multiplication $(a+bi)(c+di)=ac+adi+bci+bdi=(acbd)+(ad+bc)i$

4. Division

c di a bi = c di a bi . c di c bi = 2 2 2 2 c d i ac adi bci bdi

= 2 2 c d ac bd (bc ad)i

= 2 2 c d ac bd + i c d bc ad 2 2

Inequalities in complex numbers are not defined. There is no validity if we say that complex number is positive or negative. e.g. $z > 0$, $4 + 2i < 2 + 4$ i are meaningless. In real numbers if a2 + b2 = 0 then a = 0 = b however in complex numbers,

z1 2 + z2 2 = 0 does not imply z1 = z2 = 0. Example : Find multiplicative inverse of 3 + 2i. Solution Let z be the multiplicative inverse of 3 + 2i. then

z . (3 + 2i) = 1 z = 3 2i 1 = 3 2i 3 2i 3 2i

z = 13 3 − 13 2 i i 13 2 13 3

Ans.

Self Practice Problem 1. Simplify in+100 + in+50 + in+48 + in+46 , n . Ans. 0 3. Equality In Complex Number: Two complex numbers z1 = a1 + ib1 z2 = a2 + ib2 are equal if and only if their real and imaginary parts

are equal respectively i.e. z1 = z2 Re(z1) = Re(z2) and m (z1) = m (z2The complex number system There is no real number x which satisfies the polynomial equation x2

+ 1 = 0. To permit solutions of this

and similar equations, the set of complex numbers is introduced. We can consider a complex number as having the form a + bi where a and b are real number and i, which is called the imaginary unit, has the property that i2 = − 1.

It is denoted by z i.e. z = a + ib. 'a' is called as real part of z which is denoted by (Re z) and 'b' is called as imaginary part of z which is denoted by (Im z). Any complex number is : (i) Purely real, if b = 0 ; (ii) Purely imaginary, if a = 0 (iii) Imaginary, if b 0. NOTE : (a) The set R of real numbers is a proper subset of the Complex Numbers. Hence the complete number system is N (b) Zero is purely real as well as purely imaginary but not imaginary. (c) i = 1 is called the imaginary unit. Also i2 = 1; i3 = i ; i4 = 1 etc.

(d) a b = a b only if atleast one of a or b is non - negative. (e) is z = a + ib, then a − ib is called complex conjugate of z and written as z = a − ib Self Practice Problems 1. Write the following as complex number (i) 16 (ii) x , $(x > 0)$ (iii) −b + 4ac , (a, c> 0) Ans. (i) 0 + i 16 (ii) x + 0i (iii) −b + i 4ac 2. Write the following as complex number (i) x $(x < 0)$ (ii) roots of x2

− (2 cos)x + 1 = 0

2. Algebraic Operations: Fundamental operations with complex numbers In performing operations with complex numbers we can proceed as in the algebra of real numbers, replacing i2 by − 1 when it occurs. 1. Addition (a + bi) + (c + di) = a + bi + c + di = (a + c) + (b + d) i 2. Subtraction (a + bi) − c + di) = a + bi − c − di = (a − c) + (b − d) i 3. Multiplication (a + bi) (c + di) = ac + adi + bci + bdi2

= (ac − bd) + (ad+ bc)i

4. Division

c di a bi = c di a bi . c di c bi = 2 2 2 2 c d i ac adi bci bdi

= 2 2 c d ac bd (bc ad)i

= 2 2 c d ac bd + i c d bc ad 2 2

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