Physics 170 Week 6, Lecture 1

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Textbook Chapter 8:Section

Learning Goals:

- Learn about the force of static friction.
- Learn how to analyze systems in the state of “impe
motion”.
- Learn about the “coefficient of static friction” and t
friction”.
- Solve an example which illustrates how the coefficie
friction can be used to gain information in a statics
- Learn how to incorporate the force of friction into a
problem which has both forces and moments.
- Solve an example using the force of friction and the
of equilibrium for a static rigid body.

The Force of friction:

The "force of friction" is a force which resists slippage a
interface between two materials.

It depends on the details of

- the shape and nature of the surfaces
- the materials involved

We will **model** friction and use it to get information ab
system.

The threshold force needed to begin slippage at an inter
the normal force between the two surfaces is N is

$$F_f = \mu_s N$$

where μ_s is the *"coefficient of static friction"*

The state of impending motion

Before the box begins to move, force of friction cancels
$\vec{F}_f = -\vec{P}$
At "impending motion", the force $\vec{P}$ is just enough so t
begins to move: $F_s = |\vec{F}_f|_{\rm max} = |\vec{P}| = \mu_s N$
Once it starts moving, friction obeys a different formula
$F_k = |\vec{F}_f| = \mu_k N$ with μ_k is *"coefficient of kinetic fricti*
Usually $\mu_k < \mu_s$.

Coefficients of static friction

μ_s	**Contact materials**
0.03-0.05	metal on ice
0.30-0.70	wood on wood
0.20-0.50	leather on wood
0.30-0.60	leather on metal
1.10-1.70	aluminum on aluminum

Example: The block of weight W is being pulled up th
plane of slope α using a force $\vec{P}$. If $\vec{P}$ acts at an angle ϕ
show that for slipping to occur $P = W \sin(\alpha + \theta)/\cos(\phi$
θ is the angle of friction $\theta = \arctan \mu_s$.

In order to understand what the "angle of friction" is, w
by putting $\vec{P} = 0$ and asking at what angle the block wi
slide?

We will assume the situation of "impending motion" tha
has been increased from zero to its current slope with a
where it is just about to start sliding.

We will begin by analyzing the forces acting on the bloc

The Forces acting on the block are:

- Gravity: $\vec{F}_G = -W\hat{j}$
- $\vec{P} = 0.$
- Normal reaction force: $\vec{N} = N\left(-\sin\alpha\hat{i} + \cos\alpha\hat{j}\right)$
- Friction force: $\vec{F}_f = \mu_s N\left(\cos\alpha\hat{i} + \sin\alpha\hat{j}\right)$

Equilibrium of forces, $\sum_i \vec{F}_i = 0$ implies

$$\vec{F}_G + \vec{N} + \vec{F}_f = 0$$

$$\left(-W\hat{j}\right) + N\left(-\sin\alpha\hat{i} + \cos\alpha\hat{j}\right) + \mu_s N\left(\cos\alpha\hat{i} + \sin$$

Or, in components:

$\sum_i F_{ix} = 0$: $-N\sin\alpha + N\mu_s\cos\alpha = 0$

This is all we need for now! N cancels from this equatio

The remaining equation can be written as

$$-\sin\alpha + \mu_s\cos\alpha = 0\ ,\ \ \mu_s = \frac{\sin\alpha}{\cos\alpha}$$

or $\boxed{\mu_s = \tan\alpha}$ but remember $\mu_s = \tan\theta = \tan$ "angle o

α is equal to the angle of friction $\alpha = \theta$! The angle of fr

angle to which you can increase the slope of the plane b

block starts to slide. This is independent of W or N!

We can measure the coefficient of friction μ_s by

A. Lifting the ramp until the block slides, then measuri
angle α

Then compute $\mu_s = \tan \alpha$.

Back to the example:

The block of weight W is being pulled up the inclined p
slope α using a force $\vec{P}$. If $\vec{P}$ acts at an angle ϕ as show
that for slipping to occur $P = W\frac{\sin(\alpha+\theta)}{\cos(\phi-\theta)}$ where θ is the
friction $\theta = \arctan\mu_s$.

Strategy for finding a solution:

- This is a two-dimensional problem: we shall take th
x-coordinate as horizontal and y-coordinate as verti
- The z-direction is toward the viewer out of the page
not be used here – z-component of all vectors will b
- We recognize that this is a situation of "impending
The force needed to begin sliding should be that wh
overcomes the force of static friction plus gravity.
- The force of static friction is equal to the coefficient
friction, $\mu_s = \tan\theta$, times the normal force.

Free body diagram

Find the forces:

- Gravity: $\vec{F}_G = -W\hat{j}$
- Pull: $\vec{P} = P\left(\cos(\phi+\alpha)\hat{i} + \sin(\phi+\alpha)\hat{j}\right)$
- Normal reaction force: $\vec{N} = N\left(-\sin\alpha\hat{i} + \cos\alpha\hat{j}\right)$
- Friction force: $\vec{F}_f = -\mu_s N\left(\cos\alpha\hat{i} + \sin\alpha\hat{j}\right)$

Equilibrium of forces:

$$\vec{F}_G = -W\hat{j} \quad , \quad \vec{P} = P\left(\cos(\phi+\alpha)\hat{i} + \sin(\phi+\alpha)\hat{j}\right)$$

$$\vec{N} = N\left(-\sin\alpha\hat{i} + \cos\alpha\hat{j}\right) \;,\; \vec{F}_f = -\mu_s N\left(\cos\alpha\hat{i} + \sin\alpha\right.$$

$$\vec{F}_G + \vec{P} + \vec{N} + \vec{F}_f = 0$$

$$-W\hat{j} + P\left(\cos(\phi+\alpha)\hat{i} + \sin(\phi+\alpha)\hat{j}\right) + N\left(-\sin\alpha\hat{i} + c\right.$$

$$-\mu_s N\left(\cos\alpha\hat{i} + \sin\alpha\right.$$

components:

$$P\cos(\phi+\alpha) - N\sin\alpha - \mu_s N\cos\alpha = 0$$

$$-W + P\sin(\phi+\alpha) + N\cos\alpha - \mu_s N\sin\alpha = 0$$

components:

$$P\cos(\phi+\alpha) - N\sin\alpha - \mu_s N\cos\alpha = 0$$

$$-W + P\sin(\phi+\alpha) + N\cos\alpha - \mu_s N\sin\alpha = 0$$

Solve the first equation to get

$$N = P\frac{\cos(\phi+\alpha)}{\sin\alpha + \mu_s\cos\alpha}$$

Then, use the second equation to find

$$P\left[\sin(\phi+\alpha) + (\cos\alpha - \mu_s\sin\alpha)\frac{\cos(\phi+\alpha)}{\sin\alpha + \mu_s\cos\alpha}\right]$$

Bringing forward the last equation from the previous pa

$$P\left[\sin(\phi+\alpha)+(\cos\alpha-\mu_s\sin\alpha)\frac{\cos(\phi+\alpha)}{\sin\alpha+\mu_s\cos\alpha}\right]$$

Now we solve for P:

$$P = W\frac{(\sin\alpha+\mu_s\cos\alpha)}{(\sin\alpha+\mu_s\cos\alpha)\sin(\phi+\alpha)+(\cos\alpha-\mu_s\sin\alpha)}$$

Now we substitute $\mu_s = \tan\theta = \frac{\sin\theta}{\cos\theta}$

$$P = \frac{W(\cos\theta\sin\alpha+\sin\theta\cos\alpha)}{(\cos\theta\sin\alpha+\sin\theta\cos\alpha)\sin(\phi+\alpha)+(\cos\theta\cos\alpha-\sin\theta}$$

Remember the double angle formulas:

$$\cos(a\pm b) = \cos a\cos b \mp \sin a\sin b$$

$$\sin(a\pm b) = \sin a\cos b \pm \cos a\sin b$$

$$P = \frac{W(\cos\theta\sin\alpha + \sin\theta\cos\alpha)}{(\cos\theta\sin\alpha + \sin\theta\cos\alpha)\sin(\phi+\alpha) + (\cos\theta\cos\alpha - \sin\theta}$$

Using

$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$

$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$

we get

$$P = W\frac{\sin(\alpha+\theta)}{\sin(\alpha+\theta)\sin(\phi+\alpha) + \cos(\alpha+\theta)\cos(\phi -}$$

Finally

$$\boxed{P = W\frac{\sin(\alpha+\theta)}{\cos(\phi-\theta)}}$$

Equivalent systems and resultant normal force:

Normal force is usually distributed.

A distributed normal reaction force on flat interface =

force at a single point.

- Consider a flat interface with a variety of forces $\vec{F}_i$ a normal to the interface. (For simplicity, take the for to be parallel.)
- The forces can all be moved to a single point by ad couple moment.
- The point can then be adjusted until the additional moment vanishes. The point at which the force is th is the **"location of the resultant normal force"**

Example:

The crate has a weight of 200lb and a center of gravity
Determine the horizontal force P required to tow it. Als
the location of the resultant normal force measured fron
$h = 4ft$ and $\mu_s = 0.4$.

Strategy for finding a solution:

- This is a two-dimensional problem. We take the x-c
horizontal and y-coordinate vertical with origin at C
z-direction is toward the viewer out of the page.

- We recognize that the force needed to begin towing
should be that which puts it in a state of impending
where the force of friction is $|\vec{F}_f| = \mu_s|\vec{N}|$.

- We will assume that the reaction normal force acts
which is a distance x along the horizontal from the
the bottom right-hand corner of the crate. As we ha
discussed, the normal force is distributed over the c
Placing it all at a single point is the same as finding
equivalent system which has the same total force an
same total moments on the object.

- We must also decide where to place the point of act force of friction. We remember that we can consider equivalent system with this force moved to an point line of action. For a flat interface, we can thus think force as acting at any point on the interface.
- Similarly for the towing force, it is exerted by a rop surrounds the crate and is likely distributed over th Since the rope is located along the line of action of we can consider it a single concentrated force at any along its line of action.
- We will find a mathematical expression for all of the acting on the crate, including the reaction normal fo force of friction.
- We will compute the moments of each of the forces.

- We will then impose the conditions of equilibrium, $\sum$
$\sum_i \vec{M}_{\mathcal{O}i} = 0$
- There are three equations which we can use to solve
unknown quantities.

Free body diagram

Find the forces and the position vectors of point
they act:

- Gravity: $\vec{W} = -(200lb)\hat{j}$ acting at $\vec{r}_G = \vec{0}$
- Towing: $\vec{P} = P\hat{i}$ acting at $\vec{r}_P = (h - (3ft))\hat{j}$
- Reaction: $\vec{N} = N\hat{j}$ acting at $\vec{r}_N = ((2ft) + x)\hat{i} - (3$
- Friction: $\vec{F}_f = -\mu_s N\hat{i}$ acting at $-(3ft)\hat{j}$

Equilibrium of forces:

We can find P and N from the force equations:

$$\vec{W} + \vec{P} + \vec{N} + \vec{F}_f = 0$$

$$-(200lb)\hat{j} + P\hat{i} + N\hat{j} - \mu_s N\hat{i} = 0$$

In components:

$$P = \mu_s N \quad , \quad N = W = 200lbs$$

$\boxed{N = (200lb)}$, $\boxed{P = (80lb)}$

To find the point of action of the resultant normal force
analyze the moments.

Equilibrium of Moments "in two dimensions":

Moment due to Friction $= -3\mu_s N$

Moment due to Weight $=0$

Moment due to Normal $= (2+x)N$

Moment due to Pull $= -(h-3)P$

Total moments $M = -3\mu_s N + (2+x)N - (h-3)P$

Also, we know that $N = W$. $P = \mu_s N$ so $M = (-h\mu_s +$

$M = 0 \;\rightarrow\; \mu_s h = (x+2)/\mu_s$

Note that this equation is independent of W!

Computation of Moments using vectors:

For completeness, let us also compute the moments usin
vector techniques. We compute moments about G:

- Gravity: $\vec{M}_G = \vec{0} \times (-(200lb))\hat{j} = \vec{0}$
- Towing: $\vec{M}_P = ((h - (3ft))\hat{j}) \times P\hat{i} = -(h - (3ft))F$
- Reaction:
  $\vec{M}_N = (((2ft) + x)\hat{i} - (3ft)\hat{j}) \times \left(N\hat{j}\right) = ((2ft) + x)$
- Friction: $\vec{M}_f = (-(3ft)\hat{j}) \times (-\mu_s N\hat{i}) = -\mu_s(3ft)N$

Equilibrium of Moments using vectors:

$$\sum \vec{M} = 0$$

$$\vec{M}_G + \vec{M}_P + \vec{M}_N + \vec{M}_f = 0$$

Explicitly substituting the moment vectors gives

$$\vec{0} - (h - (3ft))P\hat{k} + ((2ft) + x)N\hat{k} - \mu_s(3ft)N\hat{k}$$

or, remembering that $N = W$ and $P = \mu_s N$,

$$[-(h-3)\mu_s + 2 + x - 3\mu_s]N\hat{k} = 0 \ \rightarrow \ h\mu_s = x$$

with x and h in ft. We are given that

$$h = 4 \ ft \ , \ \mu_s = 0.4$$

$\boxed{x = -.400 \ ft}$

Example:

The crate has a weight of 200lb and a center of gravity
Determine the height h of the tow rope so that the crat
tips at the same time. What horizontal force P is requi
this?

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Determine the height h of the tow rope so that the crat
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Strategy for finding a solution:

- The crate will slip and tip at the same time when th
normal reaction force is at A and when the force of
just compensated by P so that the crate is in a stat
impending motion.

- In the previous problem, we found that

$$h\mu_s = x + 2$$

Now, we set $x = 0$ and we get $\boxed{h = 2/\mu_s = \frac{(2)}{(0.4)} = }$

For the next lecture, please

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